ASSIGNMENT – 3

1 Consider $\Omega = \{a, b, c, d\}$. Construct 4 distinct $\sigma - field$ $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ such that $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}_3 \subset \mathcal{F}_4$. CO4

2 What is the distribution of $W(s) + W(t)$, for $0 \leq s \leq t$? CO4

3 Let X *and* Y be i.i. d. random variables each having uniform distribution on the interval$(-\pi, \pi)$. Let $Z(t) = cos(tX + Y)$, $t \geq 0$. *Is* $\{Z(t), t \geq 0\}$ wide sense stationary process? CO4

4 Let Z be a normally distributed random variable, with mean 0 and variance 1, $Z \sim N(0,1)$. Then consider the continuous time stochastic process $X = \sqrt{t}Z$, Show that the distribution of X is normal with mean 0 and variance t. Is $X(t)$ a Brownian motion? CO4

5 Let $\{W(t), t \geq 0\}$be a Wiener process. Is $\exp\{\sigma W(t) - \frac{\sigma^2}{2}t\}$ a martingale where σ is a positive constant? CO4

6 Find the stochastic differential of $W^2(t)$. CO4

7 Consider a stock whose value $S(t)$ follows sde $dS = r.Sdt + \sigma.SdW$ and has a current price S(0). What is the probability that a call option is in the money based on a strike price $K = 1.25\,S(0)$ at time of expiration T? Given that $T = 0.5, r = 0.04$ *and* $\sigma = 0.10$. CO4

Use the first version of Ito-Doeblin formula to evaluate $\int_0^T W^2(t)dW(t)$ CO4

Find the stochastic differentials of $sin(W(t))$. CO4

10 A stock price is currently Rs.50. Assume that the expected return from the stock is 18% per annum and its volatility is 30% per annum. What is the probability distribution for the stock price in two years? Calculate the mean and standard deviation of the distribution. Determine the 95% confidence interval. CO4