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2K17/MC/087

<u>FINANCIAL ENGINEERING</u>

<u>ASSIGNMENT-4</u>

Q.1 — — — ?

(a) $E(R_{K_1}) = 0.2 \times (-10) + 0 \times 0.5 + 20 \times 0.3 = 4\times$

$E(R_{K_2}) = 0.2(-30) + 20 \times (0.5) + K \times (0.3) = 8.5\%$.

(b) ∵ 60% of available fund is invested in K_1 then

weight $K_1 = 0.6$, and

weight $K_2 = 0.4$ [∵ $w_{K_1} + w_{K_2} = 1$.].

$E(\text{portfolio}) = w_p E(R_1) + w_{K_2} E(R_{K_2})$.

$= 0.6 \times 4 + 0.4 \times 2.5$

$= 5.8\%$

(c) Suppose $w_{K_1} = n$.

then $w_{K_2} = 1 - n$

$n(4) + (1-n)(8.5) = 20$.

$n = -2.56$.

which isn't possible at [0, 1].

∴ expected return isn't possible.

Q.2 — — — — ?

$E(R_{K_1}) = 0.4(-10) + 0.2(10) + 0.4(20)$

$= 4\%$.

$E(R_{K_2}) = 0.4(20) + 0.2(20) + 0.4(10)$

$= 18\%$.

$s^2(K_1) = \frac{1}{3}(10^{-4})[14^2 + 4^2 + 16^2]$

0.0156

$s^2(K_2) = \frac{1}{3}(10^{-4})[4^2 + 4^2 + 6^2]$

$= 0.02267$.

$\rho_{12} = -0.96309$.

$s_V^2 = (0.4)^2 \times 0.0156 + (0.6)^2 (0.02267)$

$= 0.00023$.

is smaller than S_1^2 is S_2^2. If 80% in stock 1 and 20% in stock 2.

$$S_V^2 = (0.8)^2 \times 0.0156 + (0.2)^2 \times 0.0226$$

$$= 0.008229.$$

Q.3 ———?

To. proove : $S_V^2 \leq \max(S_1^2, S_2^2)$.

If short sales aren't allowed (let us assume that $S_1^2 \leq S_2^2$. If short sales aren't availble/allowed than

$$w_1 = w_2.$$

$$w_1 S_1 + w_2 S_2 \leq (w_1 + w_2) S_2.$$

$$\because (w_1 + w_2 = 1)$$

Also,

$$-1 \leq \rho_{12} \leq 1$$

$$S_V^2 \leq (w_1 S_1 + w_2 S_2)^2 \leq S_2^2.$$

Q.5 ———?

For given correlation, $\rho = -1$

$$\text{So, } H_{min} = \frac{S_1 \mu_1 + S_2 \mu_2}{\mu_1 + \mu_2}.$$

$$= \frac{0.05 \times 0.08 + 0.02 \times 0.1}{0.05 + 0.02}$$

$$0.085 \; 7!$$

$H_{min} = 8.5\%$

$w_1 = 1 - S_{min}$, & $w_2 = S_{min}$.

where $S_{min} = \frac{S_1}{S_1 + S_2}$, $S_1 = \frac{0.05}{0.05 + 0.02} = 0.714$.

$\rho = -1 \rightarrow w = 0.286$ & $w_2 = 0.714$.

$S_{min} = 71.4\%$. $H_{min} = 8.514\%$.

For $\rho = 0.5$

$w_1 = 1 - S_{min}$ & $w_2 = S_{in}$

$S_{min} = 0.7894$

$w_1 = 1 - S_{min} = 0.2106$

$H_{min} = (H_2 - H_1) S_{min} + H_1$

$= 8.42\%$.

$S_{min} = 1.986$.

$\rho = 0.5$, $w_1 = 21.06\%$, $w_2 = 78.04\%$

$H_{min} = 8.421\%$. $m = 1.986\%$.

Hence proved.

Q.4 ——— ——— ——— ?

$m = [\ 0.2 \quad 0.13 \quad 0.04\ ]$

$n = [\ 1 \quad 1 \quad 1\ ]$

$\sigma_1 = 0.25$, $\sigma_2 = 0.28$, $\sigma_3 = 0.2$

$C_{12} = 0.3$, $C_{13} = 0.15$ $C_{23} = 0.4$.

$$C \simeq \begin{bmatrix} 0.0625 & 0.021 & 0.0075 \\ 0.023 & 0.754. & 0.0224 \\ 0.0075 & 0.0224 & 0.04 \end{bmatrix}.$$

$\mu c^{-1} = [12.33 \quad 3.54 \quad 20.72\ ]$

$4\mu^{T}c^{-1} = 36.59$.

$w = \frac{\mu c^{-1}}{4 c^{-1}\mu.} = [0.337 \quad 0.097 \quad 0.568]$.

Expected Result is $HV = m w^{10}$

$= 0.10265$

Std. deviation $= \frac{5v}{\sqrt{w_c\, w^T}}$

$= 0.1653$.

Q.6 ——— ——— ——— ?

for given Condition, $\rho = -1$.

So, $H_{min} = \frac{s_1\mu_1 + s_2\mu_2}{s_1 + s_2}$

$= 0.0857 > 1$

$H_{min} = 8.57\%$.

$w_1 = 1 - S_{min}$ and $w_2 = S_{min}$.

where $S_{min} = \frac{0.05}{0.05 + 0.02} = 0.714$

$w_1 = 0.286$ & $w_2 = 0.714$

$S_m = 71.4\%$ $H_{min} = 8.57\%$.

for $\rho = \frac{0.5}{1 - S_{min}}$, $S_{min} = 0.7894$.

$w_1 = 1 - S_{min} = 0.2106$

$H_{min} = (H_2 - H_1) S_{min} + H_1$

$= 8.421\%$

$S_{min} = 1.986\%$

$\rho = -0.5$.

$w_1 = 1 - S_{min}$, $w_2 = S_{min}$

$S_{min} = 0.5813$.

$w_1 = 1 - S_{min} = 0.4187$

$w_2 = 0.5813$.

$H_{min} = 8.83\%$.

$S_{min} = 1.4\%$,

For $\rho = 0$.

$S_{min} = 86.20\%$.

$w_1 = 1 - S_{min}$.

$= 13.8\%$.

$w_2 = 86.2\%$.

$H_{min} = (H_2 - H_1) S_{min} + H_1$

$= 8.276\%$.

$S_{min} = 1.850\%$.

$\rho = 1$

$S_{min} = \frac{S_1}{S_1 - S_2} = \frac{0.05}{0.05 - 0.02} > 0$.

$\therefore w_1 = 1 - S_{min} < 0$.

$\therefore H_{min} = 66.67\%$.

$S_{min} = 0$

Q.7 — — — ?

Taking $\alpha = 0$, $\beta = 1$

$10 \times V_1 + 4V_2(1) = 1$

$4 V_1(1) + 12 V_2(1) + 6 V_3 = 1$

$6 V_2(1) + 10 V_3(1) = 1$

the solution $V(1) = (X_0, 0, 4_{10})$

$10 V_1(2) + 4 V_2(2) = 5$

$4 V_2(2) + 12(V_2(2) + 6 V_3(2)) = 6$

$6 V_2(2) + 10 V_3(2) = 7$.

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Solving, we get $v(2) = [\frac{3}{10}, \frac{1}{2}, \frac{1}{5}]$

Now, Normalizing $v(1)$.
$w(1) = v(1)$ norm $= (\frac{1}{2}, 0, \frac{1}{2})$.

Normalising $v(3), \bar{w}(2) = Norm(w(2))$

$\mu_1^{(-1)} = m^T w_{13}$.
$= [5.\ 6\ 1]^T [\frac{1}{2}, 0, \frac{1}{2}]$.

$\mu^{(-1)} \to 30/0$,

$\mu^{-2} = m^T w^2 = [5, 6][\frac{1}{2}\ \ 5/6\ \ -\frac{1}{3}]$

$\Rightarrow \lambda\mu^{-1} + (1-\lambda)\mu^{-2} = 28$

$\lambda = \frac{28 - \mu(-2)}{\mu^{2(1)} - \mu^{-1(-2)}} = 1.048$

$w = \lambda(w)(1) + (1-\lambda)w(2)$

$= 1 - \frac{1}{25} \times \frac{27}{60}$

this isn't the most efficient portfolio.

0.9 — — — ?

The security market lin is:

we have

$0.06 - r_1 = 0.5(r_m - r_f)$

$0.12 - r_f = (0.5(r_m - r_f))$

solving, $r_m = 0.09$ & $r_f = 0.03$

Security market line is

$r_1 = 0.03 + k(0.06)$

Hence, when $\beta_p = 2$

$r_i = 0.03 + 2 \times 0.06 = 0.15$

Expected return on asset is 15%.

Q.10 — —

given

$H_1 = 9.5\%$ B_1 0.8%.

$H_2 = 13.3\%$ $B_2 = 1.3\%$.

now

$0.095 - r_f = 0.8(r_m - r_f)$

$0.135 - r_i = 1.3(r_m - r_f)$

by solving $r_f = 0.031$, $r_m = 0.111$

∴ Risk free Return $= 3.1\%$.

Return on market portfolio $= 11.1\%$.

Q.12 — — —?

$w(s) + w(t) = [w(s) - w(0)] + w(s) + w(t) - w(s)$

$= 2 \times [w(s) - w(0)] + (w(t) - w(s))$

and $[w(s) - w(0)]$ is independent from $[w(t) - w(s)]$, so

$w(s) + w(t) = 2[w(s) - w(0)] + [w(t) - w(s)]$

$\sim N(0, 2^2 s + (t - s))$

$= N(0, 3s + t)$