MC-306

financial Engineering

Name: Ishan Bhateja
Roll No: 2K20/MC/61

Assignment - 2

Ans. 1 (i) One call / put option

a. if $S(T) > K$ call option will be exercised
put option will not be exercised

Payoff for call opⁿ = $S(T) - K$
put opⁿ = 0

b. if $S(T) = K$
neither option will be exercised
payoff = 0

c. if $S(T) < K$
call opⁿ will not be exercised : Payoff = 0
put opⁿ will be exercised : Payoff = $K - S(T)$

(ii) 2 calls & short sold one share of stock

once again, i. $S(T) > K$
we will return the short sold stock & exercise the call opⁿ
Payoff : $2(S(T) - K) - S(T)$
$= S(T) - 2K$

ii. $S(T) \leq K$
call opⁿ not exercised
Return the short sold stock
Payoff = $-S(T)$

(iii) 1 share of stock & has sold one call

$S(T) > K$: call opⁿ exercised
Payoff = $K - S(T) + S(T)$
$= K$

$S(T) \leq K$: Call opⁿ not exercised
Payoff = $S(T)$

(iv) One call with $K = K_1$ & sold one put with $K = K_2$

Payoff = Call option profit - put option loss
$= \max(0, S(T) - K_1) - \max(0, K_2 - S(T))$

Ans.2) Given:

$S(0) = ₹50$

$u = 0.1, \; d = -0.1$

$r = 0.01 \Rightarrow R = 1 + r = 1.01$

$N = 2$

$K = ₹48$

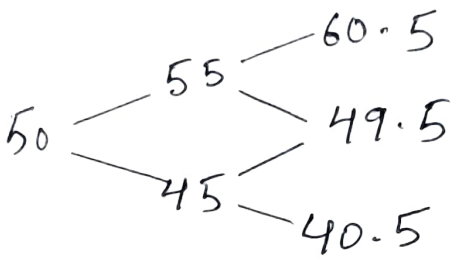
$S^u = S(0)(1+u) = ₹55$

$S^d = S(0)(1+d) = ₹45$

$S^{uu} = S(0)(1+u)^2 = ₹60.5$

$S^{ud} = S^u(1+d) = ₹49.5$

$S^{dd} = S(0)(1+d)^2 = ₹40.5$



$$p^* = \frac{R-(1+d)}{u-d} = 11/20 \Rightarrow 1-p^* = 9/20$$

$c^{uu} = [S^{uu} - K]^+ = ₹12.5$

$c^{ud} = [S^{ud} - K]^+ = ₹1.5$

$c^{dd} = [S^{dd} - K]^+ = ₹0$

$$c(0) = \frac{1}{R}\left[\sum_{i=0}^{N} {}^{N}C_0 (p^*)^i (1-p^*)^{N-i}\right]$$

$$= \frac{1}{R}\left[(p^*)^2 c^{uu} + 2p^*(1-p^*) c^{ud} + (1-p)^* c^{dd}\right]$$

$$= \frac{1}{1.01}\left[\left(\frac{11}{20}\right)^2 \times 12.5 + 2 \times \frac{11}{20} \times \frac{9}{20} \times 1.5 + 0\right]$$

$$= ₹4.48$$

Ans. 3) $S(0) = 60$ $u = 1.1$ $r = 0.03$

$K = 62$ $d = 0.95$ $T = 3$

$S^u = S(0) \times u = ₹66$

$S^d = S(0) \times d = ₹57$

$S^{uu} = S^u \times u = ₹72.6$

$S^{ud} = S^u \times d = ₹62.7$

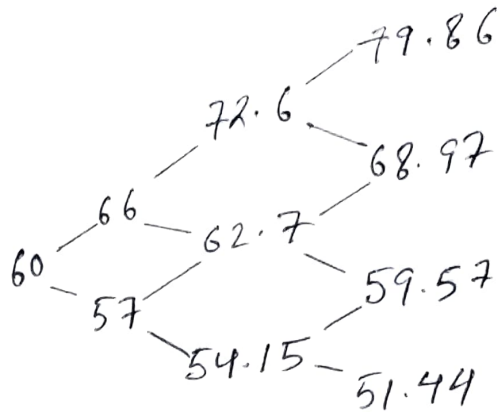
$S^{dd} = S^d \times d = ₹54.15$

$S^{uuu} = S^{uu} \times u = ₹79.86$

$S^{uud} = S^{uu} \times d = ₹68.97$

$S^{udd} = S^{ud} \times d = ₹59.565$

$S^{ddd} = S^{dd} \times d = ₹51.44$



$$p^* = \frac{R-d}{u-d} = \frac{1.03-0.95}{1.1-0.95} = \frac{8}{15} \Rightarrow 1-p^* = \frac{7}{15}$$

$C^{uuu} = [S^{uuu} - K]^+ = ₹17.86$

$C^{uud} = [S^{uud} - K]^+ = ₹6.97$

$C^{udd} = [S^{udd} - K]^+ = C^{ddd} = [S^{ddd} - K]^+ = 0$

$$C(0) = \frac{1}{R^N}\left[\sum_{i=0}^{N} {}^{N}C_i (p^*)^i (1-p^*)^{N-i} C^{u^i d^{N-i}}\right]$$

$$= \frac{1}{1.03^3}\left[\left(\frac{8}{15}\right)^3 \times 17.86 + 3 \times \left(\frac{8}{15}\right)^2 \frac{7}{15} \times 6.97 + 0 + 0\right]$$

$= ₹5.02$

using put-call parity

$C(0) - P(0) = S(0) - Ke^{-rT}$

$P(0) = 5.02 - 60 + 62 \times e^{-0.03 \times 3}$

$= ₹1.68$

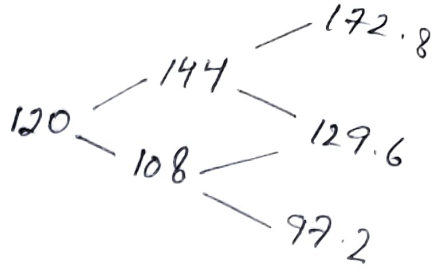
Ans. 4) $S^u = S(0) \times u = ₹144$

$S^d = S(0) \times d = ₹108$

$S^{uu} = S(0) \times u^2 = ₹172.8$

$S^{ud} = S(0)\, ud = ₹129.6$

$S^{dd} = S(0)\, d^2 = ₹97.2$



$C^{uu} = [S^{uu} - K]^+ = ₹52.8$

$C^{ud} = [S^{ud} - K]^+ = ₹9.6$

$C^{dd} = [S^{dd} - K]^+ = 0$

$$p^* = \frac{R-d}{u-d} = \frac{1.01-0.9}{1.2-0.9} = \frac{11}{30}, \quad 1-p^* = \frac{19}{30}$$

$$C(0) = \frac{1}{R^2}\left[(p^*)^2 C^{uu} + 2p^*(1-p^*)C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{1.01}\left[\left(\frac{11}{30}\right)^2 \times 52.8 + 2 \times \frac{11}{30} \times \frac{19}{30} \times 9.6 + 0\right]$$

$$= ₹11.32$$

Replicating portfolio (x, y):

$$xS^u + y(1+r) = f(S^u)$$

$$xS^d + y(1+r) = f(S^d)$$

$$f(S^u) = \frac{1}{1.01}\left[\frac{11}{30} \times 52.8 + \frac{19}{30} \times 9.6\right] = ₹25.18$$

$$f(S^d) = \frac{1}{1.01}\left[\frac{11}{30} \times 9.6 + \frac{19}{30} \times 0\right] = ₹3.48$$

$$144x + 1.01y = 25.18$$

$$108x + 1.01y = 3.48$$

$$\Rightarrow x = 0.6, \quad y = -60.61$$

↓ ↓

costs ₹72 need to borrow ₹7273.20

Ans. 5) $S(0) = ₹100$, $\sigma = 0.18$, $r = 0.04$, $K = ₹80$

$T = 3$, $T/n = T' = 1.5$

$u = e^{\sigma\sqrt{T}} = 1.24$, $d = 1/u = 0.81$

$S^u = S(0) \times u = ₹124$

$S^d = S(0) \times d = ₹81$

$S^{uu} = S(0) \times u^2 = ₹153.76$

$S^{ud} = S(0) \times ud = ₹100$

$S^{dd} = S(0) \times d^2 = ₹65.61$

$C^{uu} = [S^{uu} - K]^+ = ₹53.76$

$C^{ud} = [S^{ud} - K]^+ = 0$

$C^{dd} = [S^{dd} - K]^+ = 0$

$R = e^{rT'} = 1.06$

$$p^* = \frac{R-d}{u-d} = \frac{1.06-0.81}{1.24-0.81} = 0.6, \quad 1-p^* = 0.4$$

$$C(0) = \frac{1}{R^2}\left[p^{*2}C^{uu} + 2p^*(1-p^*)C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{1.06^2}\left[0.6^2 \times 53.76 + 0 + 0\right]$$

$$= ₹17.22$$

Ans. 6) $S(0) = ₹51$, $K = ₹50$, $\sigma = 0.3$, $r = 0.08$, $T = 3/12 = 0.25$

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(51/50) + \left(0.08 + \frac{0.3^2}{2}\right) \times 0.25}{0.3 \times \sqrt{0.25}}$$

$$= 0.3404$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.3404 - 0.3\sqrt{0.25} = 0.1904$$

$\phi(d_1) = 0.6332$

$\phi(d_2) = 0.5755$

$\phi(-d_1) = 0.3668$

$\phi(-d_2) = 0.4245$

$C(0) = S(0)\phi(d_1) - Ke^{-rt}\phi(d_2)$

$= 50 \times e^{-0.08 \times 0.25} \times 0.4245 - 51 \times 0.3668$

$= ₹ 2.09$

Using put-call parity

$P(0) = C(0) - S(0) + Ke^{-rT}$

$= 4.08 - 51 + 50 \times e^{-0.08 \times 0.25}$

$= ₹ 2.09$

Put option & call option are the same.

Ans.7) $S(0) = ₹ 260$, $K = ₹ 256$, $r = 0.04$, $\sigma = 0.25$

$T = 6/12 = 0.5$

$$d_1 = \frac{\ln(S/K) + \left(\frac{\sigma^2}{2} + r\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(260/256) + \left(\frac{0.25^2}{2} + 0.04\right) \times 0.5}{0.25 \times \sqrt{0.5}}$$

$= 0.2892$

$d_2 = d_1 - \sigma\sqrt{T} = 0.2892 - 0.25\sqrt{0.5}$

$= 0.1124$

$\phi(d_1) = 0.6138$

$\phi(d_2) = 0.5447$

$C(0) = S(0)\phi(d_1) - Ke^{-rT}\phi(d_2)$

$= ₹ 22.9$

Ans. 8) Given

$S(0) = ₹42$, $K = ₹40$, $r = 0.05$, $\sigma = 0.22$,

$T = 6/12 = 0.5$

$X = 100$

$$d_1 = \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = \frac{\ln(\frac{42}{40}) + (0.05 + \frac{0.22^2}{2})0.5}{0.22\sqrt{0.5}}$$

$$= 0.5521$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.5521 - 0.22\sqrt{0.5}$$

$$= 0.3965$$

$$\phi(-d_1) = 0.2904$$

$$\phi(-d_2) = 0.3458$$

$$p(0) = Ke^{-rT}\phi(-d_2) - S(0)\phi(-d_1)$$

$$= 40 \times e^{-0.05} \times 0.3458 - 42 \times 0.2904$$

$$= ₹1.29 \text{ for one option}$$

for 100 ₹129.36

Ans. 9) $S(0) = ₹20$, $K = ₹25$, $r = 0.05$, $\sigma = 0.24$, $T = 3/12 = 0.25$

$r_{div} = 0.03$ continous

$$S_d = S(0)e^{-r_d t}$$

$$= 20 \times e^{-0.03 \times 0.25}$$

$$= ₹19.85$$

$$d_1 = \frac{\ln(\frac{S_d}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = \frac{\ln(\frac{19.85}{25}) + (0.05 + \frac{0.24^2}{2})0.25}{0.24\sqrt{0.5}}$$

$$= -1.7579$$

$$d_2 = d_1 - \sigma\sqrt{T} = -1.7579 - 0.24\sqrt{0.25}$$

$$= -1.8779$$

$\phi(d_1) = 0.0394 \qquad \phi(d_2) = 0.0302$

$$C(0) = S_0 \phi(d_1) - K e^{-rT} \phi(d_2)$$

$$= 19.85 \times 0.0394 - 25 \times e^{-0.05 \times 0.25} \times 0.0302$$

$$= £\,0.0365 \times 100 = £\,3.65$$

Ans. 10) $S(0) = £100, \quad \sigma = 0.25, \quad r = 0.05 \quad T = 6/12 = 0.05$

$K = £80$

Δ = shares shorted

$$\frac{\partial C}{\partial S} = \Delta = \phi(d_1)$$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{100}{80}\right) + \left(0.05 + \frac{0.25^2}{2}\right)0.5}{0.25\sqrt{0.5}}$$

$$= 1.4921$$

$$\Delta = \phi(d_1) = 0.9322$$