Financial Engineering
mc-306
Assignment - 1

Kumar Saurav
2K20/mc/73

1) [20] $B(0) = Rs\ 100$ $B(1) = Rs\ 110$

$S(0) = Rs\ 80$

$S(1) = \begin{cases} Rs\ 100 & \text{Probability } 0.80 \\ Rs\ 60 & \text{Probability } 0.20 \end{cases}$

$x = \frac{2}{5} \times \frac{100,000}{80} = 750$

$y = \frac{2}{5} \times \frac{100000}{100} = 400$

$V(0) = Rs\ 100,000$

$V(1) = xS(1) + yB(1)$

$= \begin{cases} 750 \times 100 + 400 \times 110 & p = 0.8 \\ 750 \times 60 + 400 \times 110 & p = 0.2 \end{cases}$

$= \begin{cases} 119,000 & p = 0.8 \\ 89,000 & 1-p = 0.2 \end{cases}$

$K = \frac{V(1) - V(0)}{V(0)}$

$= \begin{cases} 0.19 & p = 0.8 \\ -0.11 & 1-p = 0.2 \end{cases}$

$E(K) = 0.19 \times 0.8 - 0.11 \times 0.2$

$= 0.13$

$Risk = \sqrt{(0.19 - 0.13)^2 \times (0.8) + (-0.11 - 0.13)^2 \times 0.2}$

$= \sqrt{0.00288 + 0.01152}$

$\boxed{Risk = 0.12}$

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2) Sol: $B(0) = Rs\ 90$ $\quad B(1) = Rs\ 100$

$S(0) = Rs\ 25$

$$S(1) = \begin{cases} Rs\ 30 & p \\ Rs\ 20 & 1-p \end{cases}$$

$x = 10, \quad y = 15$

$V(0) = xS(0) + yB(0)$

$= 10 \times 25 + 15 \times 90$

$= 250 + 1350$

$\boxed{V(0) = Rs\ 1600}$

$V(1) = xS(1) + yB(1)$

$$= \begin{cases} 10 \times 30 + 15 \times 100 & p \\ 10 \times 20 + 15 \times 100 & 1-p \end{cases}$$

$$\boxed{V(1) = \begin{cases} 1800 & p \\ 1700 & 1-p \end{cases}}$$

$$k = \begin{cases} 0.125 & p \\ 0.0625 & 1-p \end{cases}$$

(3) Sol: $B(0) = Rs\ 100$ $\quad B(1) = Rs\ 110$

$S(0) = Rs\ 80$

$$S(1) = \begin{cases} Rs\ 100 & p = 0.8 \\ Rs\ 60 & 1-p = 0.2 \end{cases}$$

$x = \frac{5000}{80} = 62.5$

$y = \frac{5000}{100} = 50$

$\boxed{V(0) = Rs\ 10,000}$

3) $V(1) = xS(1) + yB(1)$

$$= \begin{cases} 62.5 \times 100 + 50 \times 110 & p = 0.8 \\ 62.5 \times 60 + 50 \times 110 & 1-p = 0.2 \end{cases}$$

$$= \begin{cases} Rs\ 11,750 & p = 0.8 \\ Rs\ 9250 & 1-p = 0.2 \end{cases}$$

$$K = \frac{V(1) - V(0)}{V(0)}$$

$$= \begin{cases} 0.1750 & p = 0.8 \\ -0.075 & 1-p = 0.2 \end{cases}$$

$$E(K) = 0.175 \times 0.8 - 0.075 \times 0.2$$
$$= 0.125$$

$$\sigma(K) = \sqrt{(0.175 - 0.125)^2 \times 0.8 + (-0.075 - 0.125)^2 \times 0.2}$$

$$\boxed{\sigma(K) = \sqrt{0.002 + 0.008} = 0.1}$$

(4) Sol: $B(0) = Rs\ 90$, $B(1) = Rs\ 100$

$S(0) = Rs\ 25$

$$S(1) = \begin{cases} Rs\ 30 & p \\ Rs\ 20 & 1-p \end{cases}$$

$$V(1) = \begin{cases} Rs\ 1160 & p \\ Rs\ 1040 & 1-p \end{cases}$$

$V(1) = xS(1) + yB(1)$

$$= \begin{cases} x \times 30 + y \times 100 & p \\ x \times 20 + y \times 100 & 1-p \end{cases}$$

$30x + 100y = 1160$
$20x + 100y = 1040$

$10x = 120$

$\boxed{x = 12}$

$30 \times 12 + 100y = 1160$

$100y = 800$

$\boxed{y = 8}$

$V(0) = xS(0) + yB(0)$

$= 12 \times 25 + 8 \times 90$

$= Rs\ 1020$

(5) Sol: $B(0) = Rs\ 100$ $\quad B(1) = Rs\ 110$

$S(0) = Rs\ 80$

$S(1) = \begin{cases} Rs\ 100 & p = 0.8 \\ Rs\ 60 & 1-p = 0.2 \end{cases}$

$\boxed{K = Rs\ 100}$

(6) $C(1) = \begin{cases} 0 & p = 0.8 \\ 0 & 1-p = 0.2 \end{cases}$

$100x + 110y = 0$

$60x + 110y = 0$

$x = 0, y = 0$

$C(0) = 0$

$P(1) = \begin{cases} 0 & p = 0.8 \\ 40 & 1-p = 0.2 \end{cases}$

$xS(1) + yB(1) = P(1)$

$100x + 110y = 0$

$60x + 110y = 40$

$40x = -40$

$\boxed{x = -1}$

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(5)

$-100 + 110y = 0$

$\boxed{y = \frac{10}{11}}$

$P(0) = xS(0) + yB(0)$

$= -1 \times 80 + \frac{10}{11} \times 100$

$\boxed{P(0) = Rs\ 10.91}$

(6) Since wealth is distributed equally in given stock given call and given put

No. of shares $= \frac{300}{80} = 3.75$

for call option money will not be invested as it is not beneficial to use call option

No. of put option when stock option is Rs 60

$= \frac{300}{10.91} = 27.497$

No. of put option when stock price is Rs 100=0

$\therefore V(1) = \begin{cases} 3.75 \times 100 + 300 + 0 \\ 3.75 \times 60 + 300 + 27.49 \times 40 \end{cases}$

$\boxed{V(1) = \begin{cases} Rs\ 675 \\ Rs\ 1624.6 \end{cases}}$

(6) Sol: $B(0) = Rs\ 100$, $B(1) = Rs\ 110$

$S(0) = Rs\ 100$

$S(1) = \begin{cases} Rs\ 120 \\ Rs\ 80 \end{cases}$

$K = Rs\ 100$

$V(0) = 1000$ (split 50-50 b/w stock and option)

$\boxed{x = \frac{500}{100} = 5}$

7) Sol: NO Arbitrage Principle:
There is no admissible portfolio with initial value $V(0) = 0$ such that $V(1) > 0$ with non-zero probability

let suppose $V(0) = 0$
Rs 10000 is borrowed from a bank

(i) we will buy pounds from dealer B
we get $\frac{10000}{80} = 125$ pounds

(ii) we invest it in bank for 1 year
we get $(125 + 125 \times 0.06)$ pound
$= 132.5$ pound

(iii) we sell the pound for Rs 79 to dealer A
we get Rs $(132.5 \times 79) =$ Rs 10467.5

(iv) we return the borrowed amount with interest to the bank i.e, Rs $(10000 + 0.04 \times 10000)$
$=$ Rs 10,400

(v) profit $=$ Rs $(10467.5 - 10400)$
$=$ Rs $67.5 > 0$

Hence, arbitrage opportunity exits.

(8) Sol: $B(0) =$ Rs 100, $B(1) =$ Rs 110
$S(0) =$ Rs 50
let the forward price be f

case 1: short forward contract
If we sell at a fixed price f
- Borrow Rs 50
- Buy asset for $S(0) =$ Rs 50

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→ Portfolio : {1, -1/2, -1}

Now we will sell the asset at f and return the amount Rs 55 to the borrower.

Profit = Rs (f - 55)

Now for no arbitrage condition

$f - 55 <= 0$

$f <= 55$ — (I)

Case 2: long forward contract

If we buy at f at t=1 then

- sell short the asset at Rs 50
- Investing risk free

we get Rs 55 from investment we get the asset at f, we will return the asset to the owner

Profit = Rs (55 - f)

for no arbitrage

$55 - f <= 0$

$55 <= f$ — (II)

from (I) and (II)

f = Rs 55

(4) Strike price = Rs 30

Price of option = Rs 4

Investor is able to make a gain if the price of the price of the commodity (P) becomes less than Rs 34 in future.

Then the investor can sell the commodity at Rs 30 and buying it again at cheaper price making a profit of

(30+4) - P = Rs 34 - P

(b) Sol, $S(0)$ = Rs 5000 per 100gm
Storage cost = Rs 0.5 per gram per year
$r = 0.09$ (compounded quarterly)
forward price for 1kg for delivery in 6 months?
$S(0)$ = Rs 50,000 per kg

Storage cost $= 0.5 \times \frac{1000}{12}$
$=$ Rs 41.67 per month

$r = \frac{0.09}{4} = 0.0225$

$R = 1 + r = 1.0225$

$$F(0,6) = \frac{50000}{(1.0225)^6} + \sum_{i=1}^{6} \frac{41.67}{(1.0225)^i}$$

$= 43751.2136 + 231.4550$
$=$ Rs 43982.6686