Ans. 1) a) Expected return of a stock $= \mu_j = \sum_{i=1}^{3} p_i k_i$

Exp. return of $K_1 = 0.2\times(-10) + 0.5\times 0 + 0.3\times 20$

$= 4\% \quad = 0.04$

of $K_2 = 0.2\times(-30) + 0.5\times(20) + 0.3\times(15)$

$= -6+10+4.5 = 8.5\% = 0.085$

(b) $W_1 = 0.6 \quad W_2 = 0.4$

Exp return of the portfolio : $\sum_{i=1}^{n} w_i \mu_i$

$= 0.6\times 0.04 + 0.4\times 0.085$

$= 0.058 = 5.8\%$

(c) let wt. of K_2 be s. $\therefore$ $wt(K_1) + wt(K_2) = 1$

$wt(K_1) = 1-s$

Expected return $= 0.2 = (1-s)\times 0.04 + s\times 0.085$

$\Rightarrow 0.04 + 0.045s = 0.2$

$s = 3.56$

$1-s = -2.56$

Ans. 2) Expected return of $K_1 = \mu_1 = 0.04$

$K_2 = \mu_2 = 0.16$

$W_1 = 0.4 \quad W_2 = 0.6$

Risk on portfolio $\sigma^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2\rho W_1 W_2 \sigma_1 \sigma_2$

$\sigma_1^2 = E(K_1^2) - E^2(K_1) = 0.0184$

$\sigma_2^2 = 0.0024$

Now, assuming K_1 & K_2 are not correlated, $\rho = 0$

$\therefore \sigma^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 = 0.003808 \approx 0.38\%$

Risk on portfolio is less than that on K_1 but more than that on K_2

Now, if $W_1 = 80\% = 0.8$ & $W_2 = 0.2$

$\sigma^2 = 0.011872 = 1.1872\%$

$\therefore$ risk is greater than the prev. situation

Ans. 3) for 2-asset portfolio model

$$\sigma^2 = (1-s)\sigma_1^2 + s\sigma_2^2 + 2\rho\sigma_1\sigma_2 s(1-s)$$

we need to prove that

$$\sigma^2 \le \max(\sigma_1^2, \sigma_2^2)$$

Since short selling is not allowed, $0 < s < 1$

or $w_1, w_2 > 0$

Thus, we have

$$w_1\sigma_1 + w_2\sigma_2$$

assume that $\sigma_1^2 \le \sigma_2^2$

$$w_1\sigma_1 + w_2\sigma_2 \le \underbrace{(w_1 + w_2)}_{1}\sigma_2 \quad \text{— (1)}$$

$$\le \sigma_2$$

$$-1 \le \rho \le 1$$

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_{12}$$

which is less than $(w_1\sigma_1 + w_2\sigma_2)^2$

$\le \sigma_2^2$ — from eqn (1)

Similerily, if $\sigma_1^2 \ge \sigma_2^2$, total risk $\le \sigma_1^2$

Hence proved

Ans. 4) $m^T = [\mu_1 \ \mu_2 \ \mu_3] = [0.2 \ 0.13 \ 0.04]$

$e^T = [1 \ 1 \ 1]$

C = Covariance Variance Matrix :

$$= \begin{bmatrix} 0.0625 & 0.021 & 0.0075 \\ 0.021 & 0.0784 & 0.0224 \\ 0.0075 & 0.0224 & 0.04 \end{bmatrix}$$

we need to minimise the risk σ^2 subject to the condition $e^T w = 1$

for this, the soln is given by

$$w = \frac{C^{-1} e}{e^T C^{-1} e}$$

$\therefore \quad W^T = [0.33 \quad 0.1 \quad 0.57]$

the portfolio with these weights gives minimum risk.

Return $= m^T w \simeq 0.0999\%$

Standard deviation $= \sigma = \sqrt{W^T C W} = 0.152$

Ans.5) Return of $K_1 = \mu_1 = 0.175$

$K_2 = \mu_2 = 0.167$

$K_3 = \mu_3 = 0.212$

$\therefore m^T = [\mu_1 \ \mu_2 \ \mu_3] = [0.175 \ \ 0.167 \ \ 0.212]$

$\sigma_1^2 = 0.003825 \quad \sigma_2^2 = 0.013681 \quad \sigma_3^2 = 0.20776$

$\therefore$ Assuming the net stocks are uncorrelated

$$C = \begin{bmatrix} 0.003825 & 0 & 0 \\ 0 & .0013681 & 0 \\ 0 & 0 & 0.20776 \end{bmatrix}$$

Now, for min^m σ^2,

$$W^T = \frac{C^{-1}e}{e^T C^{-1} e}$$

where $W^T = \frac{C^{-1}e}{e^T C^{-1} e}$ is the absolute weights of the assets in the portfolio

$W^T = [0.68 \quad 0.19 \quad 0.13]$

expected return $= m^T W = 0.17829$

$\sigma = \sqrt{W^T C W} = 0.0511 \approx 5\%$

Ans. 6) $\sigma_1 = 5\% = 0.05$

$\sigma_2 = 2\% = 0.02$

$\frac{\sigma_2}{\sigma_1} = \frac{2}{5} = 0.4$

Risk is minimum when the value of s is:

$$\delta^2 = \frac{\sigma_1^2 \sigma_2^2 (1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}$$

for $$\delta = \frac{\sigma_1(\sigma_1 - \sigma_2\rho)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}$$

(i) $\rho = -1$, $s = 0.2857$, $\mu = 0.0857$

$\sigma = 0$

(ii) $\rho = -0.5$

$$s_{min} = \frac{\sigma_1(\sigma_2 - \rho\sigma_1)}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)} = 0.7692$$

i.e $w_2 = .7692$ $\quad w_1 = 1 - s_{min} = 0.2308$

$\mu_{min} = w_1\mu_1 + w_2\mu_2 = 0.0846$

$$\sigma_{min}^2 = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)} = 0.00019231$$

(iii) $\rho = 0.5$

$s_{min} = 1.0526$ & $w_1 = -0.0526$

$\mu_{min} = 0.0789$ & $\sigma_{min} = 0.019868$

(iv) $\rho = 0$

$s_{min} = 0.8621 = w_2$

$\mu_{min} = 0.0828$ & $\sigma_{min} = 0.1857$

w) for $p = 1$

$\gamma_{min} = w_2 = 1.6667$ & $w_2 = -0.6667$

$\mu_{min} = 0.066\ 67$ & $\sigma_{min} = 0$

let $\alpha = 0$ & $\beta = 1$

$$10v_1^{(1)} + 4v_2^{(1)} = 1$$

$$4v_1^{(1)} + 12v_2^{(1)} + 6v_3^{(1)} = 1$$

$$6v_2^{(1)} + 10v_3^{(1)} = 1$$

Solving above set of eqn we get,

$$v^{(1)} = (0.1, 0, 0.1)^T$$

Now, $\alpha = 1$, $\beta = 0$

$$10v_1^{(2)} + 4v_2^{(2)} = 5$$

$$4v_1^{(2)} + 12v_2^{(2)} + 6v_3^{(2)} = 6$$

$$6v_2^{(2)} + 10v_3^{(2)} = 1$$

$$v^{(2)} = (0.3, 0.5, -0.2)^T$$

and

$w^{(1)} = (0.5, 0, 0.5)$ after normalization

$$w^{(2)} = \left(\frac{1}{2}, \frac{5}{6}, \frac{-1}{3}\right)$$

Exp return $\bar{\mu}^{(1)} = m^T w^{(1)} = 3\%$

$$\bar{\mu}^{(2)} = m^T w^{(2)} = 7.16667\%$$

As for λ, 1.048, we get $\lambda\bar{\mu}^{(1)} + (1-\lambda)\bar{\mu}^{(2)} = 2.8$

Thus, ~~by~~ we have

$$w = \lambda w^{(1)} + (1-\lambda) w^{(2)}$$

$$= (0.5, -0.04, 0.54)^T$$

which is reqd. portfolio weights

which is not efficient

$m^T = (10, 12, 18)$, $\mu_{rf} = 6$

$$C = \begin{bmatrix} 4 & 20 & 40 \\ 20 & 10 & 70 \\ 40 & 70 & 14 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} -0.0667 & 0.0353 & 0.0140 \\ 0.0353 & -0.0216 & 0.0073 \\ 0.0140 & 0.0073 & -0.0050 \end{bmatrix}$$

thus, wt. vector of market portfolio is:

$$W_M = \frac{C^{-1}(m - \mu_{rf} e)}{e^T C^{-1}(m - \mu_{rf} e)} = (0.4505, 0.3934, 0.1561)^T$$

Exp. return, $\mu_M = m^T W_M = 12.0357\%$

Std. deviation, $\sigma_M = (W_M^T C W_M)^{1/2} = 4.9004$

$$\frac{\mu_M - \mu_{rf}}{\sigma_M} = \frac{12.0357 - 6}{4.9004} = 1.2317$$

CML :

$$\mu = \mu_{rf} + \left(\frac{\mu_M - \mu_{rf}}{\sigma_M}\right)\sigma \Rightarrow \mu = 6 + 1.2317\sigma$$

all the points on CML are efficient points for investors which lie on Markowitz Bullet as well as the CML

Ans. 9) $\mu_1 = 6\%$, $\beta_1 = 0.5$, r_f = risk free return
$\mu_2 = 12\%$, $\beta_2 = 1.5$, r_m = return of mkt pf.

$$0.06 - r_f = 0.5(r_m - r_f)$$
$$0.5 r_f + 0.5 r_m = 0.06$$
$$r_f + r_m = 0.12 \quad \text{— (1)}$$

$$0.12 - r_f = 1.5(r_m - r_f)$$
$$-0.5 r_f + 1.5 r_m = 0.12 \quad \text{— (2)}$$

using (1) & (2)

$r_m = 0.09$ & $r_f = 0.03$

Expected return on asset with $\beta = 2$

$$= 0.03 + 2(0.06)$$
$$= 0.15 = 15\%$$

Ans. 10) $\mu_1 = 9.5\%$, $\beta_1 = 0.8$, r_f

$\mu_2 = 13.5\%$, $\beta_2 = 1.3$, r_m

$0.095 - r_f = 0.8(r_m - r_f)$

$0.2r_f + 0.8r_m = 0.095$ — (1)

$0.135 - r_f = 1.3(r_m - r_f)$

$-0.3r_f + 1.3r_m = 0.135$ — (2)

solving (1) & (2)

$r_m = 0.111$ & $r_f = 0.031$

$r_m = 11.1\%$ and $r_f = 3.1\%$