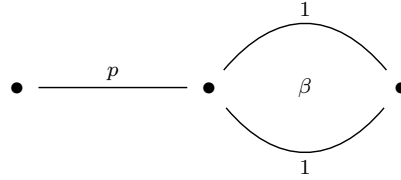


The main idea behind the proof can be stated simply: the connection between the Eckmann-Hilton argument and the Hopf fibration can be found in the naturality condition of certain 2-dimensional operations on 2-paths. We now aim to make this more precise. First we outline how Eckmann-Hilton relates to the above mentioned naturality condition. A 2-loop $\beta : \Omega^2(X, \bullet)$ induces a homotopy of type $\text{id}_{\Omega(X)} \sim \text{id}_{\Omega(X)}$ given by the formula:

$$\text{Whisker}_\beta \equiv \lambda(p).1_p \star \beta$$

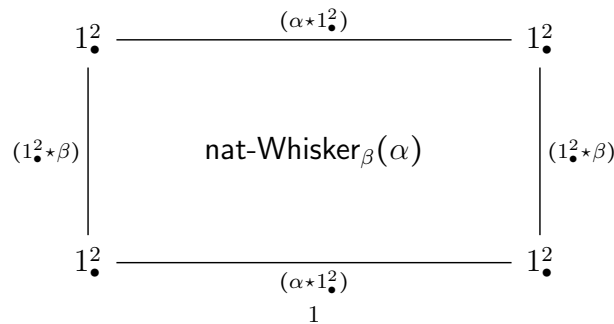
This can be depicted as follows:



We can describe Eckmann-Hilton in the following way:

Eckmann-Hilton is (more or less) the *naturality condition* of this homotopy when *applied to 2-loops*.

Lets see why. The above homotopy has a naturality condition induced by paths in $\Omega(X)$. In particular, for $\alpha : \Omega^2(X)$, the homotopy has naturality condition which can be depicted as:



A standard part of the Eckmann-Hilton proof constructs paths $\beta = 1_{\bullet}^2 \star \beta$ and $\alpha = \alpha \star 1_{\bullet}^2$. Together with the naturality condition, we obtain the Eckmann-Hilton path $\beta \cdot \alpha = \alpha \cdot \beta$. Thus, Eckmann-Hilton ultimately comes from the naturality of $\mathbf{Whisker}_{\beta}$.

What, then, does the Hopf fibration have to do with the naturality condition of $\mathbf{Whisker}$? We will actually highlight a more general connection between fibrations over \mathbb{S}^2 and this naturality condition. Then, in the case of the Hopf fibration, we will see that something extra nice happens. A fibration over \mathbb{S}^2 is equivalent to the descent data:

$$\mathbf{Dscnt}_{\mathbb{S}^2} \equiv \sum_{X:U} \mathrm{id}_X \sim \mathrm{id}_X$$