The main idea behind the proof can be stated simply: the connection between the Eckmann-Hilton argument and the Hopf fibration can be found in the naturality condition of certain 2-dimensional operations on 2-paths. We now aim to make this more precise. First we outline how Eckmann-Hilton relates to the above mentioned naturality condition. A 2-loop $\beta : \Omega^2(X, \bullet)$ induces a homotopy of type $\mathsf{id}_{\Omega(X)} \sim \mathsf{id}_{\Omega(X)}$ given by the formula:

$$\mathsf{Whisker}_\beta :\equiv \lambda(p).1_p \star \beta$$

This can be depicted as follows:

$$\begin{tikzcd}
\bullet \arrow[r, no head, "p"] & \bullet \arrow[r, no head, bend left, "1"] \arrow[r, no head, bend right, "1"'] \arrow[r, phantom, "\beta"] & \bullet
\end{tikzcd}$$

We can describe Eckmann-Hilton in the following way:

> Eckmann-Hilton is (more or less) the *naturality condition* of this homotopy when *applied to 2-loops*.

Lets see why. The above homotopy has a naturality condition induced by paths in $\Omega(X)$. In particular, for $\alpha : \Omega^2(X)$, the homotopy has naturality condition which can be depicted as:

$$\begin{tikzcd}
1^2_\bullet \arrow[rr, no head, "(\alpha \star 1^2_\bullet)"] \arrow[d, no head, "(1^2_\bullet \star \beta)"'] & & 1^2_\bullet \arrow[d, no head, "(1^2_\bullet \star \beta)"] \\
1^2_\bullet \arrow[rr, no head, "(\alpha \star 1^2_\bullet)"'] \arrow[urr, phantom, "\mathsf{nat\text{-}Whisker}_\beta(\alpha)"] & & 1^2_\bullet
\end{tikzcd}$$

1

A standard part of the Eckmann-Hilton proof constructs paths $\beta = 1^2_\bullet \star \beta$ and $\alpha = \alpha \star 1^2_\bullet$. Together with the naturality condition, we obtain a the Eckmann-Hilton path $\beta \cdot \alpha = \alpha \cdot \beta$. Thus, Eckmann-Hilton ultimately comes from the naturality of $\mathsf{Whisker}_\beta$.

What, then, does the Hopf fibration have to do with the naturality condition of $\mathsf{Whisker}$? We will actually highlight a more general connection between fibrations over $\mathbb{S}^2$ and this naturality condition. Then, in the case of the Hopf fibration, we will see that something extra nice happens. A fibration over $\mathbb{S}^2$ is equvalent to the descent data:

$$\mathsf{Dscnt}_{\mathbb{S}^2} \equiv \sum_{X:U} \mathsf{id}_X \sim \mathsf{id}_X$$