# HW 2

5 a). **3.5.** An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

6

9

15 ball

$$\frac{6}{15}\cdot\frac{5}{14}\cdot\frac{9}{13}\cdot\frac{8}{12}$$

$$=\frac{6}{91}$$

5 b. What is the probability that the first two are black?

$$\frac{9}{15}\cdot\frac{8}{14}=\frac{12}{35}$$

5 c. What is the probability that the last two are black?

$$\frac{9}{15}\cdot\frac{8}{14}=\frac{12}{35}$$

Given no information of the first two, the probability of last two is the same as the first two by symmetric.

10. **3.10.** Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade given that the second and third cards are spades.

$$P(\text{first} \mid \text{second} \ \& \ \text{third}) = \frac{P(\text{first} \ \& \ \text{second} \ \& \ \text{third})}{P(\text{second} \ \& \ \text{third})}$$

$$= \frac{\cancel{\frac{13}{52}}\cdot\cancel{\frac{12}{51}}\cdot\frac{11}{50}}{\cancel{\frac{13}{52}}\cdot\cancel{\frac{12}{51}}}$$

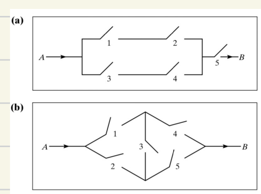
$$= \frac{11}{50}$$

52

3

66. **3.66.** The probability of the closing of the $i$th relay in the circuits shown in Figure 3.4 is given by $p_i$, $i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between $A$ and $B$ for the respective circuits?
*Hint for (b)*: Condition on whether relay 3 closes.

All event independent




a). Event A Event B Event AB

$A\cup B) = A + P(B) - P(AB)$

$$P(p1)\cdot P(p2)\cdot P(p5) + P(p3)\cdot P(p4)\cdot P(p5) - P(p1)P(p2)P(p3)P(p4)P(p5)$$

A B C D AB AC

b). $P(p1)\cdot P(p4) + P(p1)\cdot P(p3)\cdot P(p5) + P(p2)P(p3)P(p4) + P(p2)P(p5) - P(p1)P(p3)P(p4)P(p5) - P(p1)P(p2)P(p3)P(p4$

AD BC BD

$- P(p1)P(p2)P(p4)P(p5) - P(p1)P(p2)P(p3)P(p4)P(p5) - P(p1)P(p2)P(p3)P(p5)$

CD ABC ABD

$- P(p2)P(p3)P(p4)P(p5) + P(p1)P(p2)P(p3)P(p4)P(p5) + P(p1)P(p2)P(p3)P(p4)P(p5)$

ACD BCD ABCD

$+ P(p1)P(p2)P(p3)P(p4)P(p5) + P(p1)P(p2)P(p3)P(p4)P(p5) - P(p1)P(p2)P(p3)P(p4)P(p$

+B+C+D
+B-AC-AD
BC-BD-CD
ABC+ABD
ACD+BCD
+BCD

TR 9.

**3.9.** Consider two independent tosses of a fair coin. Let $A$ be the event that the first toss results in heads, let $B$ be the event that the second toss results in heads, and let $C$ be the event that in both tosses the coin lands on the same side. Show that the events $A$, $B$, and $C$ are pairwise independent—that is, $A$ and $B$ are independent, $A$ and $C$ are independent, and $B$ and $C$ are independent—but not independent.

A: first toss heads
B: second toss head
C: Both toss same side

$P(A): \frac{1}{2}$
$P(B): \frac{1}{2}$
$P(C): \frac{2}{2} \cdot \frac{1}{2} = \frac{1}{2}$

independent: first toss doesn't affect the second toss

$P(A \cap B) = P(A) \cdot P(B)$
P(first & second head) $= P(A) \cdot P(B)$
$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2}$
$\frac{1}{4} = \frac{1}{4}$

$P(A \cap C) = P(A) \cdot P(C)$
P(first head & same side) $= \frac{1}{2} \cdot \frac{1}{2}$
P(first head & second head) $= \frac{1}{4}$
$\frac{1}{4} = \frac{1}{4}$

pairwise independent

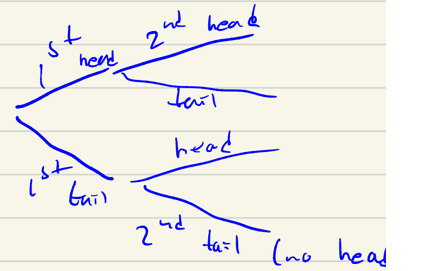
$P(B \cap C) = P(B) \cdot P(C)$
P(same side & second head) $= \frac{1}{2} \cdot \frac{1}{2}$
P(first head & second head) $= \frac{1}{4}$
$\frac{1}{4} = \frac{1}{4}$

$P(ABC) = P(A) \cdot P(B) \cdot P(C)$
P(first & second head) $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$\frac{1}{4} \neq \frac{1}{8}$
not independent

2. 2. A prominent mathematician, Jean d'Alembert, argued that there is a 1/3 probability of two tails when a coin is flipped twice. He reasoned that we could obtain a head on the first flip, a head on the second flip, or no heads. Therefore, the probability of two tails (no heads) is 1/3. Similarly, he reasoned that a coin flipped three times could yield a head on the first flip, a head on the second flip, a head on the third flip, or no heads. Therefore, the probability of three tails is 1/4. Explain the flaw in d'Alembert's reasoning.

Alembert was thinking: 1st head / 2nd head / 3rd no head

two tails no head. This is not correct. He choose 1 from 3 option. but the reality should be




no head should think as $\underset{\frac{1}{2}}{\text{1st tail}} \cdot \underset{\frac{1}{2}}{\text{2nd tail}} = \frac{1}{4}$

similarly as 3 flip $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

3. The Chevalier de Méré wrote to Blaise Pascal in 1654 asking him what was the likelihood of getting at least one pair of sixes if a pair of fair dice is thrown 24 times. What is the answer?

$1 -$ (no pair of sixes in 24 times)
$1 - \left(\frac{5}{6} \cdot \frac{5}{6}\right)^{24}$

4. 4. Give an example of two events that are independent, and two events that are mutually exclusive.

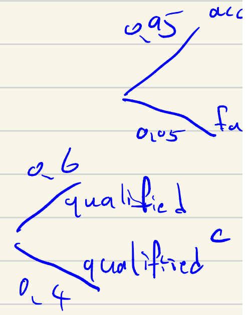
Independent: Event A: roll a dice
Event B: flip a coin

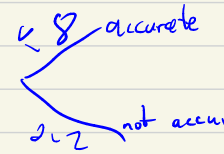
Mutually exclusive: Event A: Person A at Berkeley
Event B: Person A at San Francisco

5. 5. A manager (who happens to be a white Anglo) is responsible for hiring personnel in her department. She is accurate 95% of the time in assessing the qualifications of white Anglo candidates (that is, 95% of the candidates who are qualified she assesses as qualified and 95% of the candidates who are unqualified she assesses as unqualified). She is accurate only 80% of the time for candidates of color. She hires candidates (call them hirees) that she assesses as qualified. In the pool of candidates, 60% of white Anglo candidates and 60% of candidates of color are qualified.

a. What proportion of white Anglo hirees are qualified?
b. What proportion of the persons of color who are hired are qualified?
c. Briefly discuss the implications of your analysis.

For those of you interested in fairness and machine learning, the different accuracies in assessments (and hence in hiring) in this problem violates "equalized odds" as a measure of fairness.




a) • hires candidates that qualified

$$\text{proportion} = \frac{\text{qualified}}{\text{qualified} + \text{unqualified}} = \frac{0.6 \cdot 0.95}{0.6 \cdot 0.95 + 0.4 \cdot 0.05}$$

doesn't hire unqualified unless she assess the as qualified. There is 5% she assess the unqual as qualified

B) $$\text{proportion} = \frac{\text{qualified}}{\text{qualified} + \text{unqualified}} = \frac{0.6 \cdot 0.8}{0.6 \cdot 0.8 + 0.4 \cdot 0.2}$$

c) Manager only hire candidates which qualified. But she can be wrong at qualifying candidates. There is 5% and 20% chance she qualify the wrong candidate for white Anglo and color

6. Giant Pharmaceutical Corp has developed and is testing two new drugs for Swine Flu. Drug B has a higher success rate (percentage of cures) than drug A when given to women, and also when given to men. Is drug B necessarily the drug with the higher success rate when given to people in general? Explain your answer.

6. When give to people in general, it includes elder & children. Their immune system may be weaker than a regular women and men. Also, the drug is only in testing stage. We don't know their testing proportion and testing patient. If they test drug B on people around 20-30 and drug A on 50-60, the result is not comparable.
Therefore, drug B may not have higher success rate when give to people in general.