Roll rate calculation

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Due to a fun cocktail of factors, the rocket will rotate along its roll axis in flight. According to Structures, the rotation needs to be less than 5 rotations per second in order to use the IMU. Structures tried to find out how much the rocket would rotate and found that it tops out at 10 rotations per second. We in Trajectory/Aerodynamics then had to approach the problem in our own way to see if this was realistic.

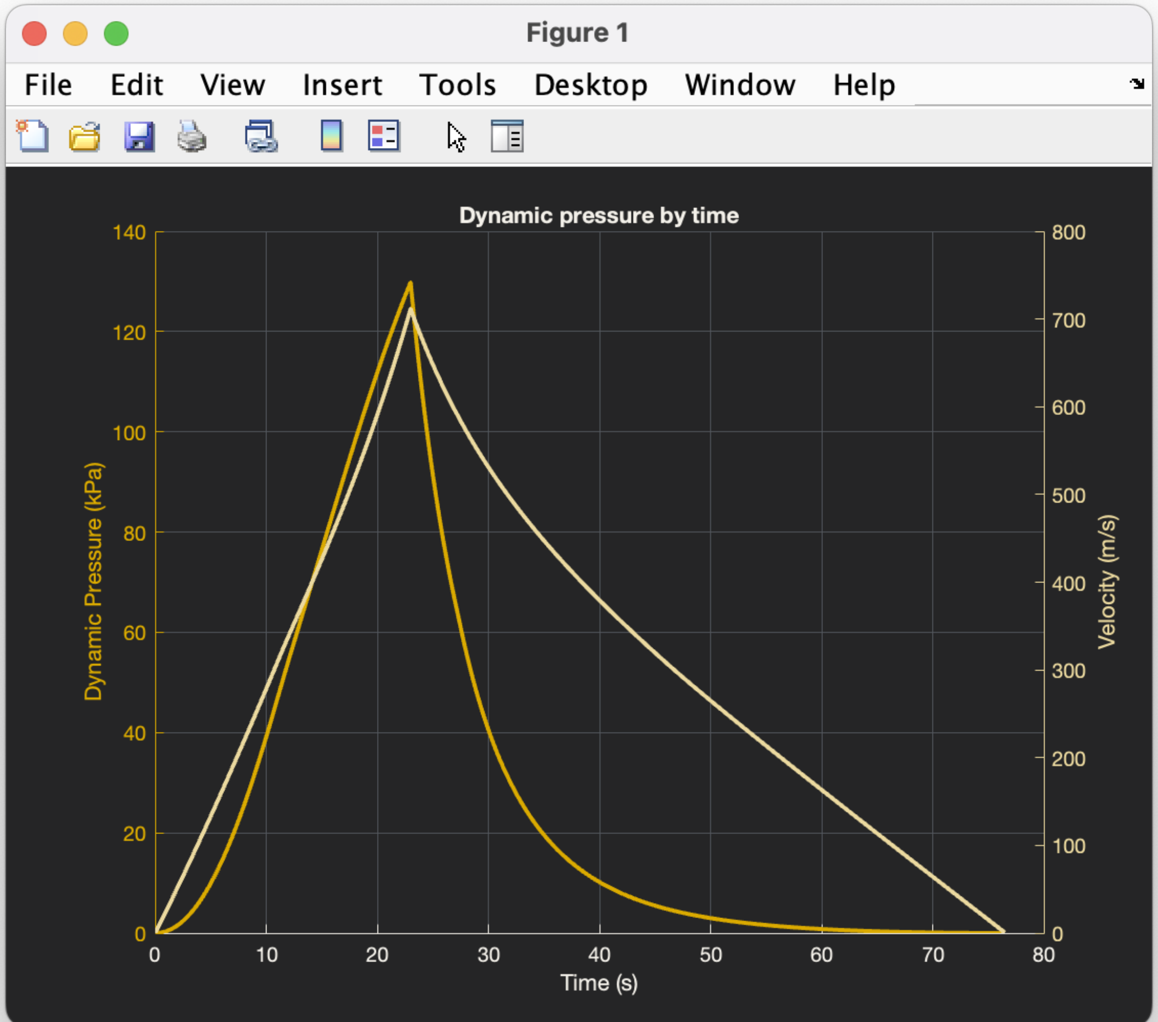
Many factors contribute to roll. But implementing them all means we need a fully functioning 6DOF, which we do not have. But propulsion is confident thrust asymmetry won't be a problem, and the Aerodynamics RE says wind will about even out, so the main factor will be fin misalignment. We found a NASA paper with an equation that gives roll rate acceleration by fin geometry and misalignment (Madden, 1972).

$$I\dot{p} = \frac{C_{l_p} dSqp}{2U} + \sum_{j=1}^{m} C_{l_\delta} \cdot \delta_j \cdot d \cdot S \cdot q$$

There are a lot of parameters here, but the interesting ones are, the moment of inertia of the rocket *I*, the roll acceleration *p_dot*, the angle of misalignment of each fin , the roll damping derivative Clp, and the roll forcing derivative Cl. We can assume that all the fins are the same and have the same misalignment, replacing the sum with a multiplication by the number of fins m. To get a maximum, we can also ignore the roll damping term. This gives a simpler equation for the roll acceleration.

$$\dot{p} = m\frac{C_{l_\delta} \cdot \delta \cdot d \cdot S \cdot q}{I}$$

This is only the derivative of the function we're looking for, but using ode45 with an initial condition of p(0) = 0 to integrate we can get the desired p. We assume the maximum misalignment Structures is confident they can achieve of = 0.0115°. S and d are measurements of the fin, being the chord length d = 0.2286 m, and the area S = 0.12609 m^2. q is the dynamic pressure, with $q = \frac{1}{2}\rho \cdot U$. Taking the velocity and altitude by time from the 1DOF, along with a standard atmosphere model for the air density by altitude, we get a function for dynamic pressure by time.

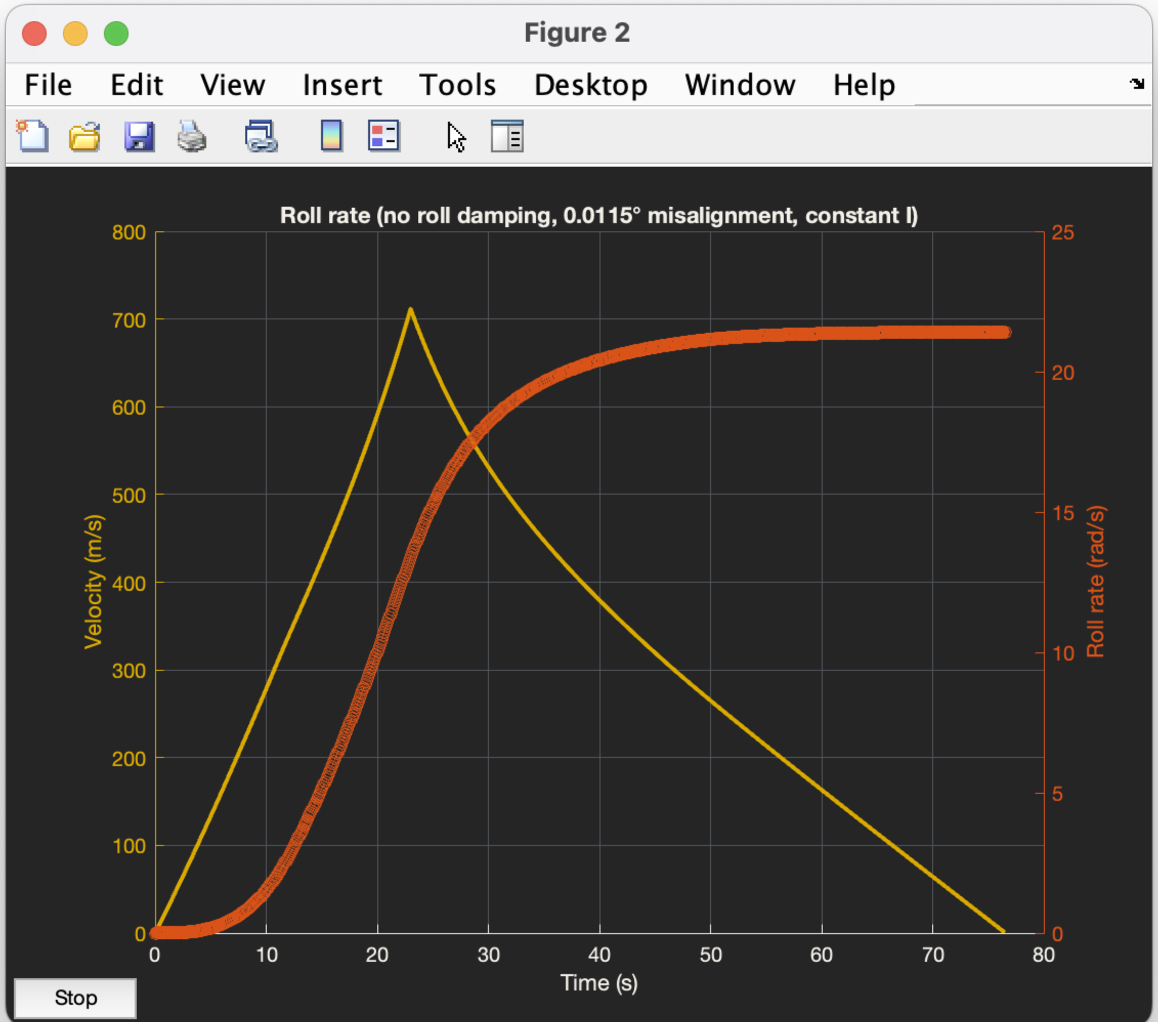


The real challenge comes with finding the roll forcing derivative. When researching into roll forcing and roll damping coefficients, the most useful research paper to use was Barrowman's report from 1967 (Barrowman, 1967). Though this was very useful in terms of understanding the equations we were using, there was a lot of terminology that was confusing in the report itself. The roll forcing derivative is the quantity that tells us how much roll is being forced by a singular fin. On the other hand, the roll damping derivative is the quantity that tells us how much roll is being damped by a singular fin. The equations are listed below:

$$C_{\ell_\delta} = N \left(C_{N_\delta}\right)_1 \frac{\bar{Y}_T}{L_r}$$

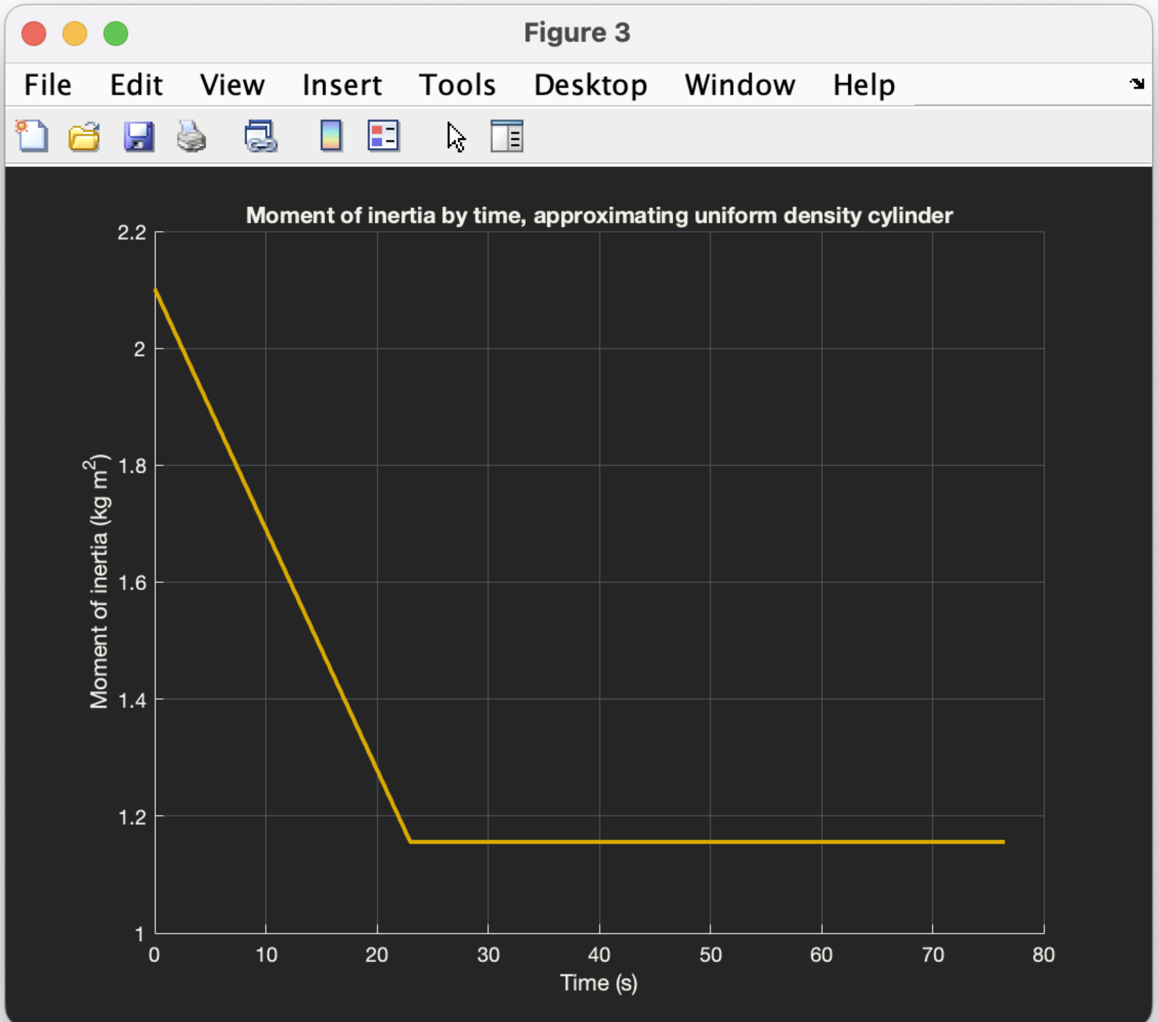
$$C_{\ell_p} = -\frac{N c_r S}{6L_r^2}\left[(1 + 3\lambda)\, s^2 + 4(1 + 2\lambda)\, s r_t + 6(1 + \lambda)\, r_t^2\right]\left(C_N\right)_1$$

The next problem is finding the rocket's moment of inertia. Seizing a value called "inertia" from the sDOF of I = 870.4137679386765, we can run the program and plot the output.

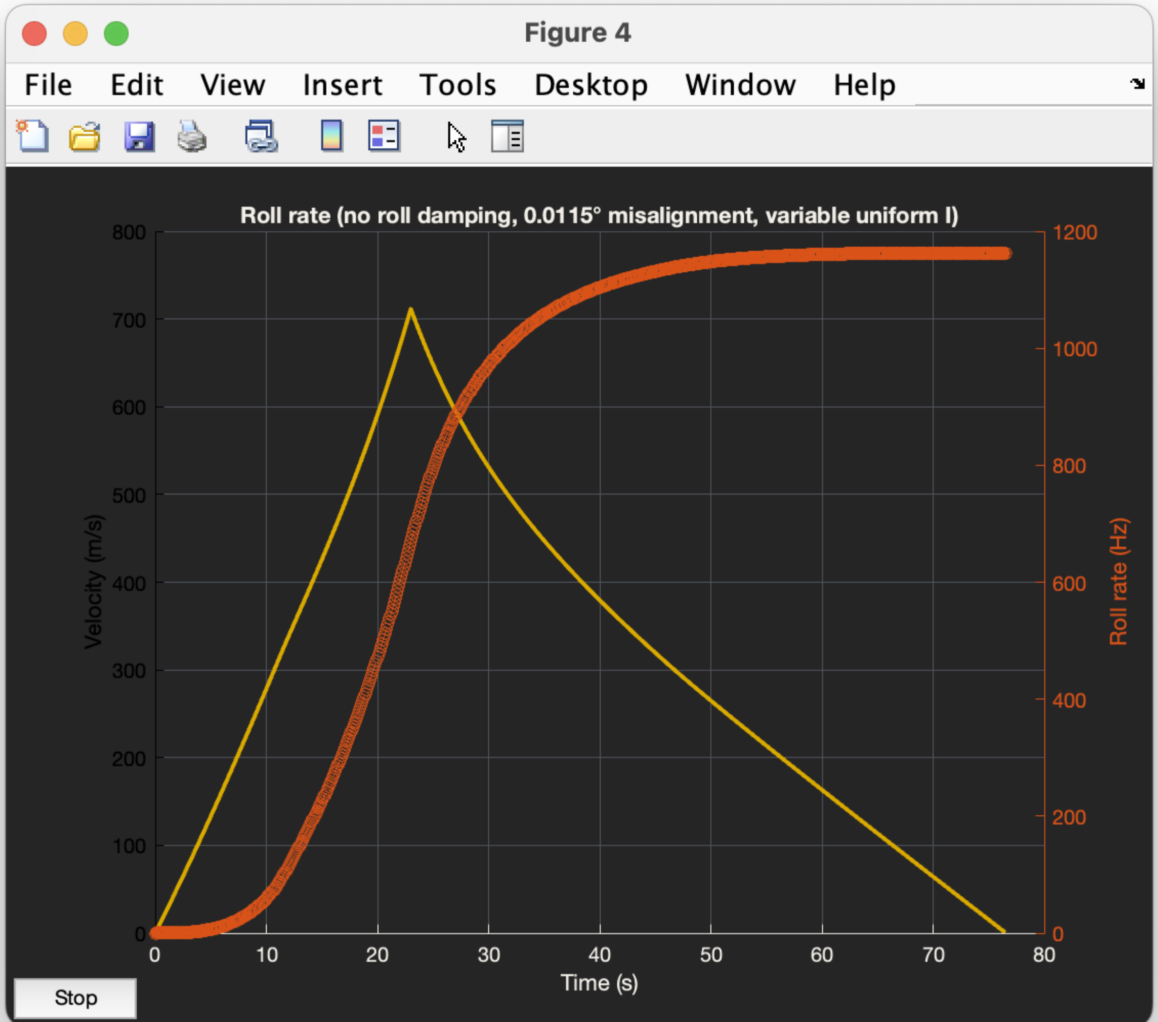


This maxes out at about 21.4 rad/s, about 3.4 Hz. This is well within the limit of 5 Hz.

The problems come when we have a variable moment of inertia. Due to the moment of inertia decreasing as time increases and us not having a concrete model of the moment of inertia due to not fully knowing the mass distribution of our rocket, it seemed to be a very difficult task, proving that overall roll rate equation from the NASA research paper that we had originally found would be not as useful as we expected. We solved for the moment of inertia using a conservative approach, coming up with a much lower moment of inertia than initially anticipated.

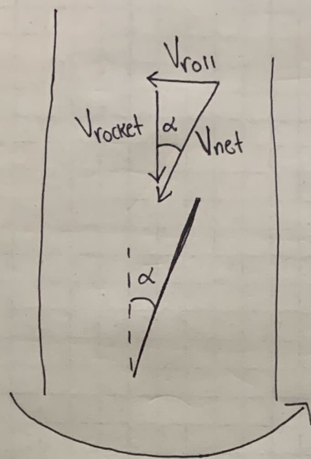


Using this far lower moment of inertia, the roll rate gets a lot worse.



Something seemed wrong, so this is where we chose to trust more than a 50-year-old article. In an 85-reply thread, Tyler Trostle laid out an approach to find an upper bound by assuming the fins would have zero angle of attack when the rocket is rolling at its maximum roll rate.

Tyler Trostle



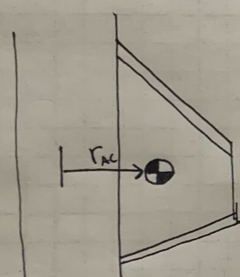
α = fin misalignment angle

$V_{roll} = r_{AC} \cdot \omega$ = radius $\cdot$ roll rate $\left(\frac{rad}{s}\right)$

$$\tan(\alpha) = \frac{V_{roll}}{V_{rocket}} = \frac{r \cdot \omega}{V_{rocket}}$$

For a given rocket velocity and misalignment angle, calculate roll rate at each time step as:

$$\boxed{\omega = \frac{V_{rocket}}{r_{AC}} \tan(\alpha)}$$



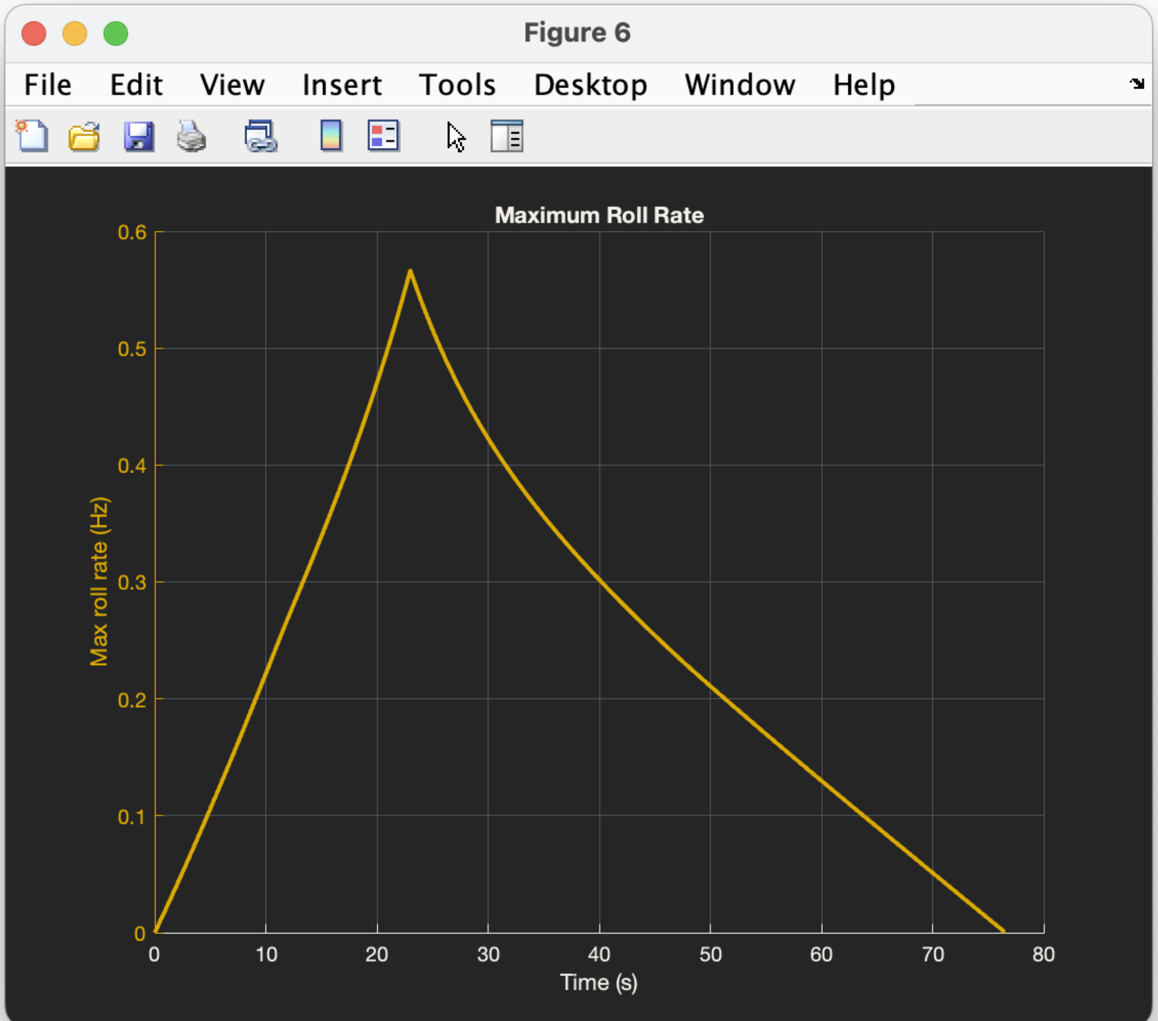
r_{AC} = radius of "aerodynamic center"

* Note: effective angle of attack changes along r for the fin since your V_{roll} increases with r. This model assumes the aerodynamic center of the fin is where $\alpha_{eff} = 0°$

He derived this equation for this upper bound.

$$\omega = \frac{V_{rocket}}{r_{AC}} \cdot tan(\alpha)$$

Implementing this with the velocity function from earlier, we get a relatively low maximum roll rate.



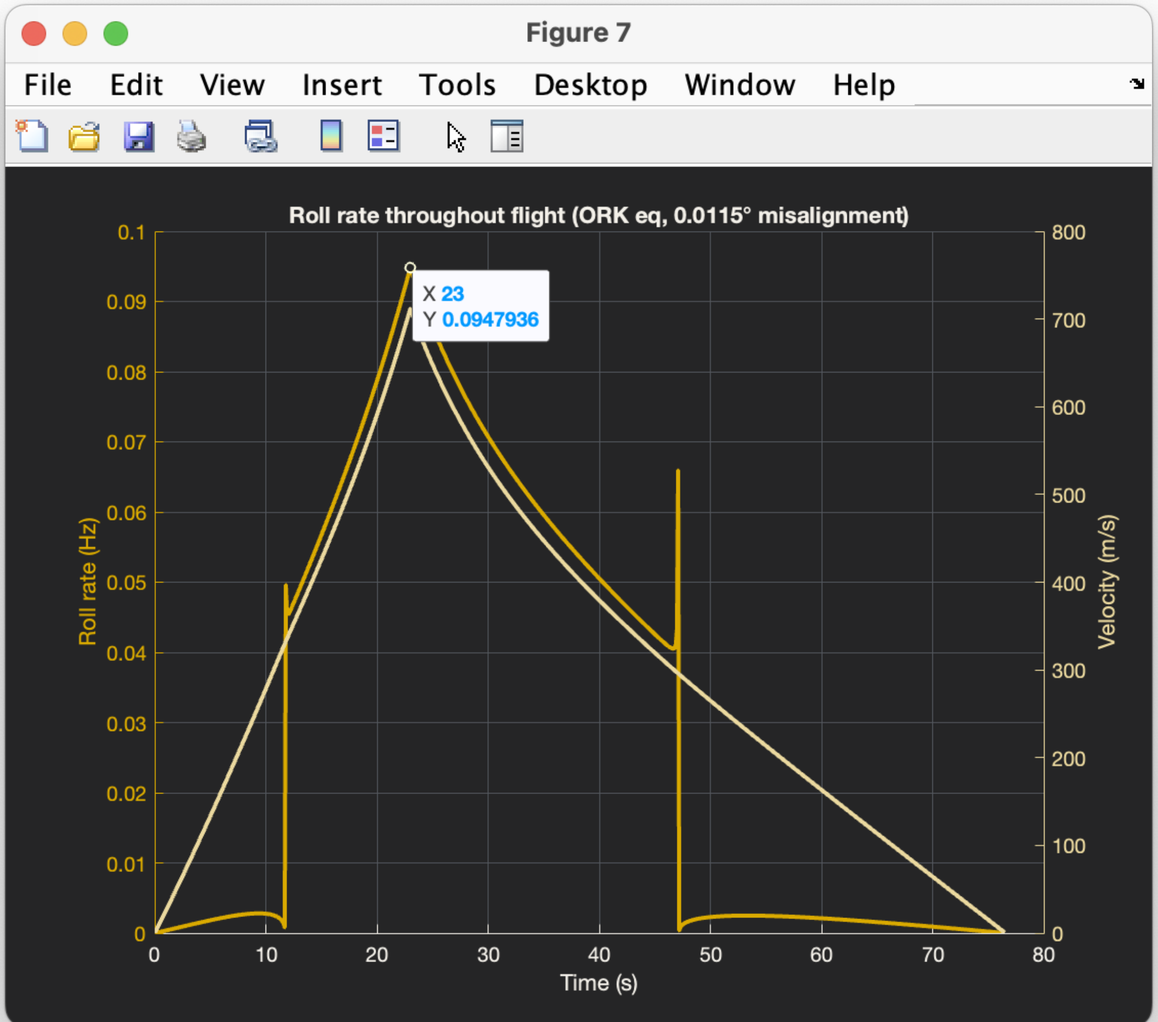
With two very conflicting results, we went for a third approach to break the tie. No-one knows fins like the Finns , so we implemented the approach described in the OpenRocket technical documentation (Niskanen, 2009).

This centers around two slightly different equations, one for subsonic and one for supersonic.

$$f_{subsonic} = \frac{A_{ref}\beta v y_{MAC} C_{N_\alpha}\delta}{4\pi^2 \sum c\xi^2 \Delta\xi}$$

$$f_{supersonic} = \frac{A_{ref}\beta v y_{MAC} C_{N_\alpha}\delta}{4\pi \sum c\xi^2 \Delta\xi}$$

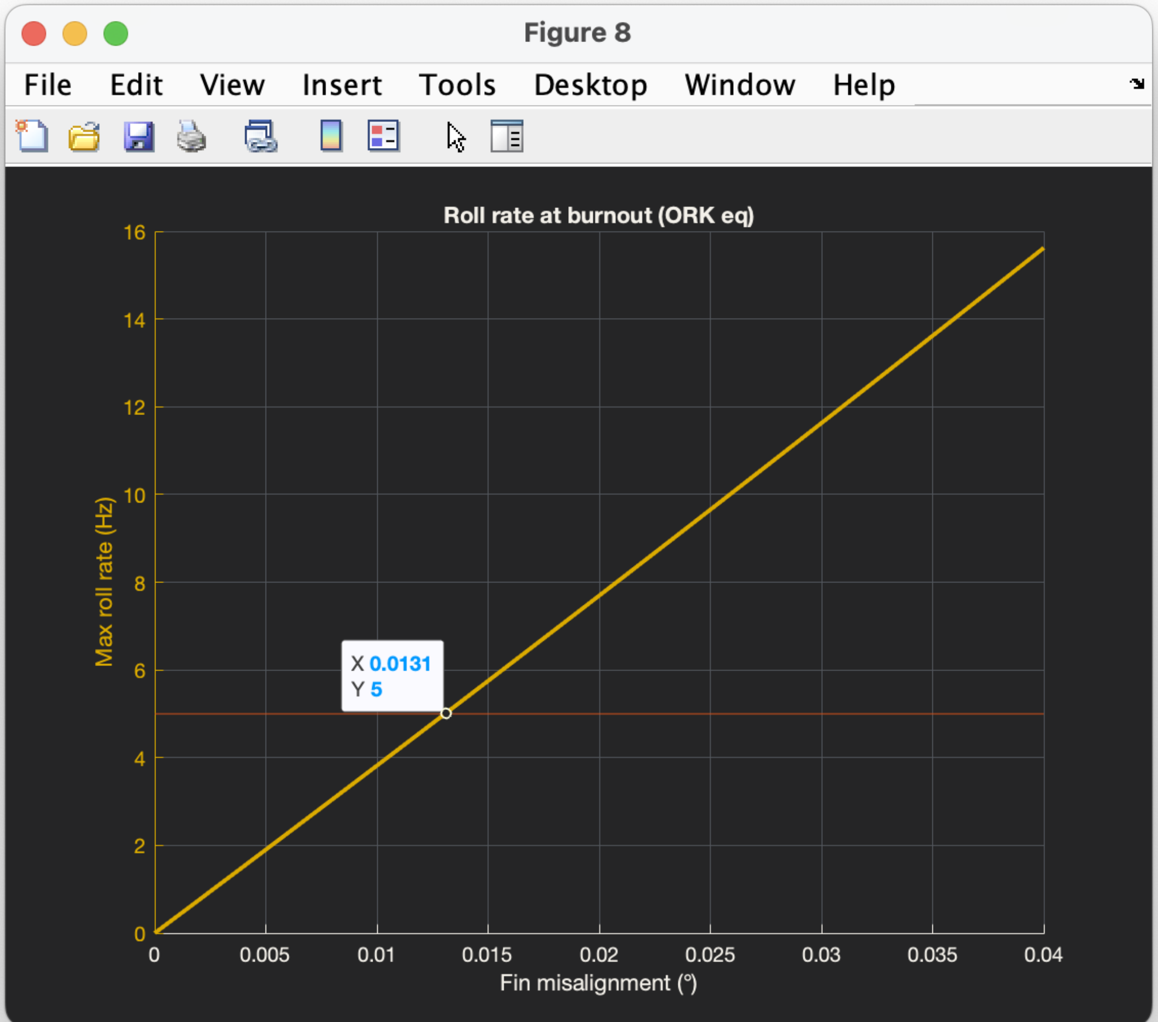
Implementing these, we get this plot:



The odd peaks are where the rocket crosses mach 1, and the equations for f, beta, and CNa change. When supersonic, the roll rate is directly proportional to velocity.

Most importantly, the max roll rate of about 0.6 Hz lines up with the maximum roll rate in Tyler's model. With two models in agreement, we concluded that the conservative strategy we used to approach our calculations would suffice.

To get more specific constraints, we can plot the fin misalignment against the max roll rate is causes. This gives that any misalignment less than 0.0131° should be fine.



We have tried three different models now. The first suffered from being poorly documented and hard to implement, leading to simplifications on our part that may have impeded the accuracy. The other two are simpler and agree for the most part.

All this work is in the 6DOF/Roll Rate.

Sources

Madden, R. (1972) A Statistical Analysis of the Roll Rate of a Launch Vehicle under the Influence of Random Fin Misalignments. *AIAA, 10*(3) 324-325. DOI: 10.2514/3.50094

Barrowman, J. S. (1967). *The practical calculation of the aerodynamic characteristics of slender finned vehicles* (No. NAS 1.15: 209983).

Niskanen, S. (2009). *Development of an Open Source model rocket simulation software* (Master's thesis).