## 20250423 Progress Report

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## Outline

- Introduction
- System Model
- Problem Formulation
- Algorithm
- Next Steps
- Appendix



# Introduction

### Introduction

- Cooperative caching scheme for multi-view 3D videos in Low-earth-orbit (LEO) satellite networks that optimizes content placement and delivery by leveraging inter-satellite links (ISL) and DIBR synthesis
- Multi-view 3D Video
- Cooperative Caching on LEO satellite network
- DIBR Synthesis



[1] M. Yeh, C. -H. Wang, D. -N. Yang, J. -T. Lee and W. Liao, "Mobile Proxy Caching for Multi-View 3D Videos With Adaptive View Selection," in IEEE Transactions on Mobile Computing, vol. 21, no. 8, pp. 2909-2921, 1 Aug. 2022

[2] R. Zhao, Y. Ran, J. Luo and S. Chen, "Towards Coverage-Aware Cooperative Video Caching in LEO Satellite Networks," GLOBECOM 2022 - 2022 IEEE Global Communications Conference, Rio de Janeiro, Brazil, 2022

[3] S. Liu, X. Hu, Y. Wang, G. Cui and W. Wang, "Distributed Caching Based on Matching Game in LEO Satellite Constellation Networks," in IEEE Communications Letters, vol 22. no. 2. pp. 300-303. Feb. 2018

#### Related Works

	[1]	[2]	This Work
Content Type	Multi-view 3D video	2D video content	Multi-view 3D video
Network Type	Mobile proxy networks	LEO satellite networks	LEO satellite networks
DIBR Synthesis	V	X	V (on user)
Coverage Scope	Localized to urban or high- demand areas	Global Coverage	Global Coverage

- Previous works [1] utilize DIBR to boost transmission efficiency
  - Caching on servers and managing caches by centralized algorithm
- ▶ [2], [3] use ISL to achieve wide coverage areas and low access latency
- ▶ [4] manages cache with the information provided by ground station



### Research Challenges

- Moving caching from the terrestrial to LEO satellites introduce some challenges
- Limited on-board computation and power
  - Terrestrial servers often have powerful hardwares and enough power
  - LEO satellites have limited computing resources and rely on solar power
- Unstable and limit bandwidth
  - Links between satellites and between satellite and user are wireless and long distance
  - The LEO satellite's number of simultaneous connections has a maximum limit.
- Each satellite performs independently and manages its cache
- View fetching and DIBR synthesis trade-off



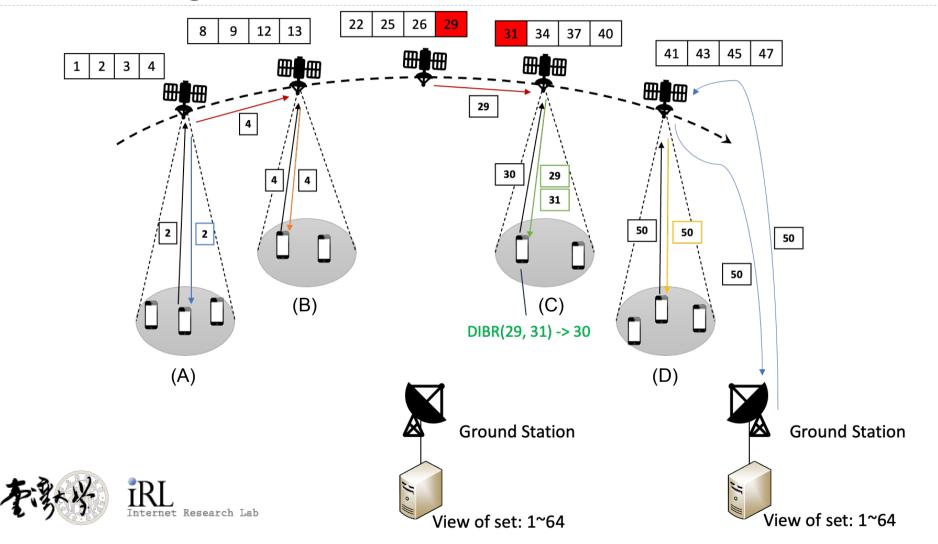
### Motivation

- Accessibility high bandwidth services in remote and underserved regions
- Repeated transmission of the same views to multiple users introduces redundant data transfer
  - Caching multi-view videos on LEO satellites can effectively reduce video transmission
- Traditional caching replacement policies (LRU, LFU) would not yield good results
- Existing LEO caching problems do not consider the energy and bandwidth



# System Model

## LEO Caching for Multi-View 3D Videos with Inter-satellite Links



### Views Caching Policy

When LEO satellite receives the user's request

#### Cache Hit

- A. If requested view cached in access LEO
- B. If the access LEO has not cache, but nearby LEOs have could sharing requested view through intersatellite link
- C. If requested view is not cached but its access LEO and nearby LEOs can return left and right views to user and be synthesized requested view by user

#### Cache Miss

D. if the above cases not satisfied and should fetch the request view from remote server (ground station)



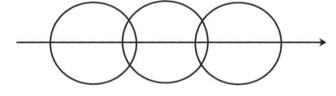
### System Model

- K users:  $\mathbf{K} = \{1, 2, ..., k\}$
- ▶ *N* LEO satellites:  $N = \{1, 2, ..., n\}$
- *G* Ground stations:  $G = \{1,2,...,g\}$
- $\triangleright$  *V*: the universal set of views in a multi-view 3D video
- $V_n(t)$ : the state of the cache storing a subset of views in LEO satellite n at time t
- $\triangleright$   $z_i$ : the size of view i
- $\sum_{i \in V_n(t)} z_i \le Z$ : The total size of cached view in a LEO satellite no more than Z
  - Every LEO satellite have the same cache storage
  - Z: the size of the LEO cache size



## System Model: Satellite Model

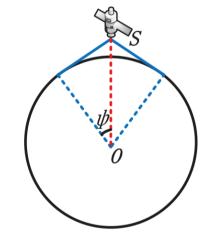
- ▶ Service Area Model:  $A_n(t) = \{x(R, \theta, \Phi) \mid \sin \theta \sin \theta_n \cos(\Phi \Phi_n) + \cos \theta \cos \theta_n \le \cos \psi\}$
- $\triangleright$  R: the radius of earth
- $\Phi_n$ : the longitude of satellite n,  $\theta_n = \left| \frac{\pi}{2} n_{latitude} \right|$
- $\psi$ : half cone angle between covered area and Earth's core



- The point set of all points in the area covered by satellites:  $A_{SOC}(t) = \bigcup_{n=1}^{n} A_n(t)$
- Coverage Model:

$$\psi = \arccos(\frac{R}{R+h}\cos\theta_{min}) - \theta_{min}$$

- The coverage area  $S = 2\pi R^2 (1 \cos \psi)$
- A larger value of  $\psi$  implies the decrease in the coverage







## System Model: Communication Model

•  $LEO_n$  to user k transmission:

$$R_{n,k} = B_{n,k} log_2 \left( 1 + \frac{P_n \cdot G_{n,k}}{\sigma^2} \right)$$

Inter-satellite link transmission:

$$R_{n,n+1} = B^{ISL} log_2 \left( 1 + \frac{P_{ISL} \cdot G_{n,n+1}}{\sigma^2} \right)$$

•  $GS_g$  to  $LEO_n$  transmission:

$$R_{g,n} = B^{GS} log_2 \left( 1 + \frac{P_{GS} \cdot G_{g,s}}{\sigma^2} \right)$$

- ▶ *B*: the channel bandwidth
- ightharpoonup P: the transmit power
- lacksquare G: the channel gains

$$d_{n,k} = -R\sin\theta_{n,k} + \sqrt{(R\sin\theta_{n,k})^2 + h^2 + 2hR}$$

- the distance between  $LEO_n$  and user k
- $\theta_{n,k}$ : the elevation angle of user k with respect to  $LEO_n$





[2] R. Zhao, Y. Ran, J. Luo and S. Chen, "Towards Coverage-Aware Cooperative Video Caching in LEO Satellite Networks," *GLOBECOM* 2022 - 2022 IEEE Global Communications Conference, Rio de Janeiro, Brazil, 2022

[7] H. Zhou, L. Liu and H. Ma, "Coverage and Capacity Analysis of LEO Satellite Network Supporting Internet of Things," *ICC 2019 - 2019 IEEE International Conference on Communications (ICC)*, Shanghai, China, 2019, pp. 1-6

## System Model: Satellite Energy Model

- The limited resources of LEO satellites pose challenges to maintaining service continuity
- Battery Variables:
  - $E_n^H$ : the energy that reaches satellite n
  - $\blacktriangleright$   $E_n(t)$ : battery level of satellite n at the start of time slot t
  - $\triangleright$   $E_n^{max}$ : maximum battery capacity
- Energy Consumption:
  - Transmission:  $E_n^{trans}(t) = P(t) \cdot \frac{z_i}{R}$ 
    - $\Box$  transmit power P(t) of satellite n by <u>downlink view to user</u> or <u>ISL</u>
  - **Cache Policy Processing (const.)**:  $E_n^{cache}$
- ▶ Energy Constraint:  $0 \le E_n^{trans}(t) + E_n^{cache}(t) \le E_n(t)$ ,  $\forall n, t$
- The battery level of  $LEO_n$  for the next time slot:  $E_n(t+1) = E_n(t) E_n^{trans}(t) E_n^{cache} + E_n^H(t)$





## System Model: View Request Model

- Arriving request:  $R_n(t)$  (UTs are uniformly distributed in SOC)
  - A request here is a range of views for users to adaptively change the view angle at different time
  - From a newly joined user
- From a leaving user (the last request before leaving)
- From an existing user
- LEO satellite n receives a request in time t if user is covered by satellite:  $k(R, \theta, \Phi) \in A_{SOC}(t)$
- The request range of the users:  $\left[R_n(t) \lfloor \frac{B}{2} \rfloor, R_n(t) + \lfloor \frac{B}{2} \rfloor\right]$ 
  - ▶ *B* is the size of a request range
  - Fetch a range of views for each user to **support the rapid view change**



• An instantaneous cost function C(t) at time t:

$$C(t) = c_m \sum_{i} 1_{\{i \notin V_n(t) \cup V_n^s(t)\}} + \sum_{i \in V_n^f(t)} c_{n,f}(z_i) + \sum_{i \in V_n^s(t)} c_{n,s}(z_i, d_i)$$

$$+ c_p(v_l, v_r) \sum_{j} 1_{j \in \{V_n^{DIBR}(t)\}} + \sum_{i \in V_n^{ISL}(t)} h \cdot c_{ISL}(z_i, d_i^{ISL})$$

- 1.  $c_m$ : the penalty cost for a cache miss (the additional processing latency for cache miss)
- 2.  $c_{n,f}$ : the cost to fetch view i with size  $z_i$  from remote server(GS)
- 3.  $c_{n,s}$ : the cost to return a view with size  $z_i$  to the user by data rate  $d_i$
- 4.  $c_p$ : DIBR synthesis cost:  $c_p(v_l, v_r) = \alpha \cdot |v_l, v_r| + T_{DIBR}$
- 5.  $c_{ISL}$ : the cost to transmit view i utilizing ISL with size  $z_i$  by data rate  $d_i^{ISL}$
- 6. h: the number of hops



- $V_n^s(t)$ : the views that can be synthesized by  $V_{n-h}(t), V_n(t), V_{n+h}(t)$  according to the DIBR constraint D

  - $\neg$  *h* : the number of hops
- $V_n^f(t) \subseteq V \setminus V_n(t)$ : the view set required by  $LEO_n$  to be fetched from the GS
- $V_n^{DIBR}(t)$ : views are synthesized on the user equipment side
  - LEO satellite only needs to transmit the two reference views (left and right) to the user



- $V_n^e(t) \subseteq V_n(t) \cup V_n^f(t)$ : the view to set be replaced
  - when the cache space of  $LEO_n$  is full and ensuring  $\sum_{i \in V_n(t)} z_i \leq Z$
- $V_n^s(t) \subseteq V_n(t) \cup V_n^f(t) \cup V_n^{ISL}(t)$ : the set of views returned to user
  - where the range of returned views needs to be in  $\left[R_n(t) \lfloor \frac{B}{2} \rfloor, R_n(t) + \lfloor \frac{B}{2} \rfloor\right]$
- $V_n^{ISL}(t)$ : the view set utilizing inter-satellite links (ISL)
  - $V_n^{ISL}(t) = \left\{ i \mid i \in V_{n-1}(t) \cup V_{n+1}(t) \setminus V_n(t) \right\}$



• Objective function: **minimize** the total cost C(t) in T time slots for each user request

$$min \sum_{t=1}^{T} C(t)$$

**Constraint:** 

1. LEO caching storage: 
$$\sum_{i \in V_n(t)} z_i \le Z$$

- 2. DIBR synthesis constraint:  $|v_l v_r| \le D$ ,  $\forall v_l, v_r$  used for synthesis
- 3. Coverage Constraint:  $k(R, \theta, \Phi) \in A_{SOC}(t)$



• Objective function: **minimize** the total cost C(t) in T time slots

$$min \sum_{t=1}^{T} C(t)$$

- **Constraint:** 
  - 4. Power Constraint:  $0 \le E_n^{trans}(t) + E_n^{cache}(t) \le E_n(t)$ ,  $\forall n, t$
  - 5. Bandwidth Constraint:  $\sum_{k} B_{n,k}(t) + \sum_{m} B_{n,m}(t) + \sum_{g} B_{g,n}(t) \le B_{n}^{max}$



# Algorithm

### **Problem Hardness**

Theorem. The LEO Satellite Caching for Multi-View 3D Videos With Inter-Satellite Links Problem (LEO-MV3D-ISL) is NP-hard.

*Proof.* We will prove this theorem by reducing the Partition Problem to an offline version of the LEO-MV3D-ISL.

Given a multi-set  $S = \{a_1, a_2, \dots, a_{|S|}\}$  of positive integers (with sum 2S), the partition problem is to decide whether Scan be partitioned into two subsets  $S_1$  and  $S_2$  such that the sum of the elements in  $S_1$  equals the sum of the elements in  $S_2$ . Given the instance of the partition problem, we map this to an instance of LEO-MV3D-ISL as follows:

- $z_i = a_i$  for  $i = \{1, 2, ..., |V|\}.$
- cache size Z to S.
- 3) Let B=0 and D=0, meaning each request corremiss occurs. sponds to a single view and cannot be obtained through synthesis.
- 4) Set h = 1 to ensure that if a requested view is not stored retrieved via one inter-satellite link transmission.

To complete the proof, it suffices to show that S is a **YES**instance if and only if the constructed instance has no cache miss.

If S is a YES-instance, it can be partitioned into two subsets such that the sum of the elements in each subset is S. Thus, the corresponding views can be divided into two groups and stored separately in the caches of the two LEO satellites. As a result, regardless of the incoming request, it can always be retrieved from one of the caches, ensuring that no cache miss occurs.

In an opposite way, we prove this by contradiction. Suppose that S is not a **YES-instance**, meaning it cannot be divided 1) For each view i, the size  $z_i$  corresponds to the partition into two subsets whose sums are both S. This implies that problem instance's  $a_i$ . Specifically, let  $|V|=|\mathcal{S}|$  and the constructed instance cannot store all the views within the given cache constraints. As a result, there will be at least one 2) Set |N|=2 (representing two LEO satellites) and the request that cannot be served from the caches, leading to a cache miss, which contradicts the assumption that no cache

Since we have demonstrated that solving the offline version of LEO-MV3D-ISL is at least as difficult as solving the Partition Problem, its NP-hardness is established. Given that the on the directly connected LEO satellite, it can still be original LEO-MV3D-ISL is an online problem, it inherently retains the complexities of the offline version.

## Online Algorithm

#### Algorithm 1 LEO Satellite Cooperative Caching

```
Require: Local cache state V_n(t), neighbor caches \{V_{n-1}(t), V_{n+1}(t)\}, Request range R_n(t) = [h, l],
      available energy E_n(t) and bandwidth B_n(t).
 Ensure: V_n^f(t), V_n^{DIBR}(t), V_n^{ISL}(t), V_n^s(t), V_n^e(t), total cost C(t).
   1: Calculate the possibility of view v as P(v).
   2: for all j in range [h, l] do
          if j \in V_n(t) then
   3:
               \tau_i \leftarrow c_{n,s}(z_i,d_i);
           else if j \in (V_{n-1}(t) \cup V_{n+1}(t)) \setminus V_n(t) then
               \tau_i \leftarrow c_{ISL}(z_i, d_i^{ISL}) + c_{n,s}(z_i, d_i);
   6:
           else
   7:
               \tau_j \leftarrow c_m + c_{n,f}(z_i) + c_{n,s}(z_i, d_i);
   8:
           end if
   9:
           Calculate \mu_{h,j};
  10:
  11: end for
  12: (V_n^f(t)^*, V_n^{DIBR}(t)^*, V_n^{ISL}(t)^*, V_n^s(t)^*) = argmin\mu_{k,l};
  13: V_n^e(t)^* \leftarrow argminP(v) if the cache space is not sufficient;
' 14: return (V_n^f(t)^*, V_n^{DIBR}(t)^*, V_n^{ISL}(t)^*, V_n^s(t)^*, V_n^e(t)^*) and cost C;
```

# Next Steps

## **Next Steps**

- Deal with the energy and bandwidth constraints in Online Algorithm
- Complete the competitive ratio analysis
- Performance evaluation of the proposed algorithm through simulation



# Thanks for your listening

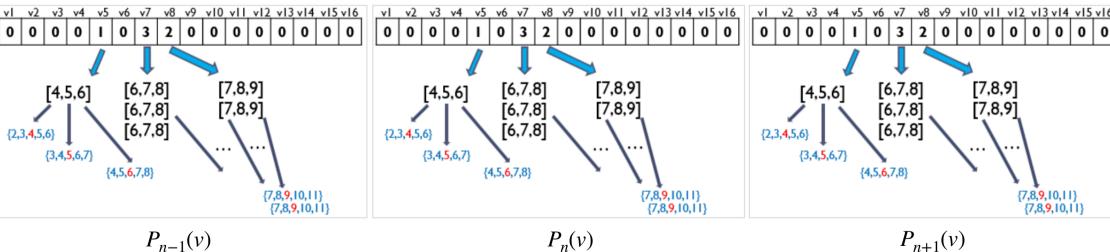
# Appendix

### Online Algorithm

- $m{\tau}_i$  is the total cost to fetch view j
- $\mu_{i,j}$  be the minimum cost of view set [i,j]

$$\mu_{h,j} = \min_{\max\{j-D,h\} \le i < j} \left( \tau_j + \mu_{h,i} + (\alpha(j-i) + T_{DIBR})(j-i-1) \right)$$

 $P_n(v)$  be the possibility that view v at  $LEO_n$  is subscribed in the next time slot

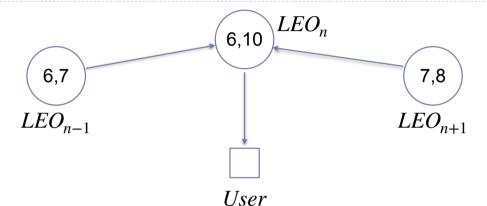


P<sub>n-1</sub>(v)

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$$\mu_{h,j} = \min_{\max\{j-D,h\} \le i < j} \left( \tau_j + \mu_{h,i} + (\alpha(j-i) + T_{DIBR})(j-i-1) \right)$$

$$R_n(t) = [h, l] = [5,10]$$



1. 
$$\tau_5 = c_m + c_{n,f}(z_5) + c_{n,s}(z_5, d_5)$$

• 
$$\mu_{5.5} = \tau_5$$
 (cache miss and transfer 5)

2. 
$$\tau_6 = c_{n,s}(z_6, d_6)$$

• 
$$\mu_{5,6} = \tau_6 + \mu_{5,5}$$
 (transfer 5,6)

3. 
$$\tau_7 = c_{ISL}(z_7, d_7^{ISL}) + c_{n,s}(z_7, d_7)$$

• 
$$\mu_{5.7} = \tau_7 + \mu_{5.5} + [2\alpha + T_{DIBR}]$$
 (transfer 5,7 and synthesis 6 at user)

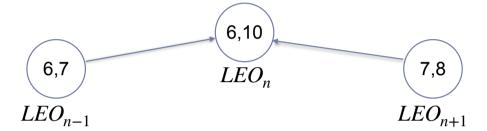
• 
$$\mu_{5,7} = \tau_7 + \mu_{5,6}$$
 (transfer 5,6,7)



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$$\mu_{h,j} = \min_{\max\{j-D,h\} \le i < j} \left( \tau_j + \mu_{h,i} + (\alpha(j-i) + T_{DIBR})(j-i-1) \right)$$

 $R_n(t) = [h, l] = [5,10]$ 



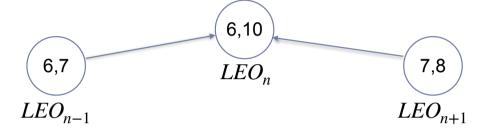
4. 
$$\tau_8 = c_{ISL}(z_8, d_8^{ISL}) + c_{n,s}(z_8, d_8)$$

- $\mu_{5.8} = \tau_8 + \mu_{5.5} + [3\alpha + T_{DIBR}] \cdot 2$  (transfer 5,8 and synthesis 6,7 at user)
- $\mu_{5.8} = \tau_8 + \mu_{5.6} + [2\alpha + T_{DIBR}]$  (transfer 5,6,8 and synthesis 7 at user)
- $\mu_{5,8} = \tau_8 + \mu_{5,7}$  (transfer 5~7,8) -> (5,7,8 or 5,6,7,8)



$$\mu_{h,j} = \min_{\max\{j-D,h\} \le i < j} \left( \tau_j + \mu_{h,i} + (\alpha(j-i) + T_{DIBR})(j-i-1) \right)$$

$$R_n(t) = [h, l] = [5,10]$$



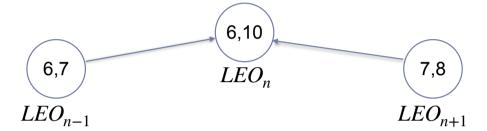
5. 
$$\tau_9 = c_m + c_{n,f}(z_9) + c_{n,s}(z_9, d_9)$$

- $\mu_{5.9} = \tau_9 + \mu_{5.6} + [3\alpha + T_{DIBR}] \cdot 2$  (transfer 5,6,9)
- $\mu_{5,9} = \tau_9 + \mu_{5,7} + [2\alpha + T_{DIBR}]$  (transfer 5~7,9)
- $\mu_{5.9} = \tau_9 + \mu_{5.8}$  (transfer 5~8,9)



$$\mu_{h,j} = \min_{\max\{j-D,h\} \le i < j} \left( \tau_j + \mu_{h,i} + (\alpha(j-i) + T_{DIBR})(j-i-1) \right)$$

 $R_n(t) = [h, l] = [5,10]$ 



6. 
$$\tau_{10} = c_{n,s}(z_{10}, d_{10})$$

- $\mu_{5,10} = \tau_{10} + \mu_{5,7} + [3\alpha + T_{DIBR}] \cdot 2$  (transfer 5~7,10)
- $\mu_{5,10} = \tau_{10} + \mu_{5,8} + [2\alpha + T_{DIBR}]$  (transfer 5~8,10)
- $\mu_{5,10} = \tau_{10} + \mu_{5,9} + \text{ (transfer 5~9,10)}$
- If the final  $\mu_{5,10}$  decide to transfer 5,7,10
  - $V_n^f(t) * = \{5\}, V_n^{DIBR}(t) = \{6,8,9\}, V_n^{ISL}(t) * = \{7\}, V_n^s(t) * = \{5,7,10\}$
  - $V_n^e(t)$  \* would choose the lowest popularity view to evict (5,6,7,10) if cache storage is full



### Competitive Ratio Analysis

- For view  $i, j \in V$ , denoting  $M_n(i, j)$  the **minimum total size of views** in  $V_n(t) \subseteq V$  such that  $i, j \in V_n(t)$  and  $V_n(t) \cup V_n^s(t) = \{i \pmod{|V|}, i+1 \pmod{|V|}, \dots, j \pmod{|V|}\}$ 
  - $V_n^s(t)$  denotes the set of views that can be synthesized by  $V_{n-1}(t)$ ,  $V_n(t)$ ,  $V_{n+1}(t)$  based on D
  - $_{\square} \quad M_n(i,j) = \min_{j-D \leq k \leq j-1} \left\{ M_n(i,k) + z_j \right\}, \text{ where } M_n(i,i) = z_i \text{ for } i \in V$
- **Example:** 
  - $V_n(t) = \{1,3,6,15\}, V_{n-1}(t) = \{5,6,8,9\}, V_{n+1}(t) = \{1,3,4,5\} = V_n^s(t) = \{2,7\}$
  - $V_n(t) \cup V_n^s(t) = \{1,2,3,4,5,6,7,8,9,15\}$
  - Find  $M_n(1,9)$ , which represents the minimum total size of a contiguous set of views covering the range from view 1 to view 9 (Maybe view 1,4,7,9)



### Competitive Ratio Analysis

Define  $\Delta_n = \max_{(i,j)} \{j-i+1 \pmod{|V|} : M_n(i,j) \le Z\}$  as the **maximum number of views** (including synthesized views) that can be hit in the cache

- The view popularity (the distribution of the user request) follows Zipf distribution, denoted by  $f_n(i,z,|V|) = \frac{1/i^z}{\sum_{n=1}^{|V|} (1/n^z)}$ 
  - *i* is the preference rank of a view
  - z is the Zipf factor



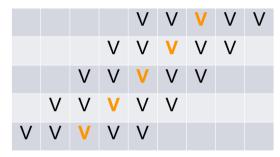
## Competitive Ratio Analysis (Case 1: $\Delta_n = |V|$ )

- Consider the instance with cache state  $V_n(0)$  corresponding to  $\Delta_n$ , which view [i,j] are in  $V_n(0)$  or can be synthesized by cached view in  $V_{n-1}(0)$ ,  $V_n(0)$ ,  $V_{n+1}(0)$
- ▶ *OPT* would be able to synthesizes all views and directly return to the user without cache miss
- ▶ *ALG* always find a view i (for  $j D \le i < j$ ) fitting the minimum cost by exploring the chance to synthesize the view in the range [j D, j 1]
- Since  $|V| = \Delta_n$ , ALG generates an optimal cache storing the views to cover the views  $\{1,2,...,|V|\}$ , such that there is no cache miss at any time



# Competitive Ratio Analysis (Case 2: $\Delta_n < |V|$ )

- ▶ There are at most  $\Delta_n (B 1)$  kinds of request that would not cause any cache miss
- Example:  $(B = 5, \Delta_n = 9)$  5 kinds of requests would not make cache miss



Since the view popularity follows Zipf distribution, the percentage of views that cache hits occur is at most  $\sum_{n=0}^{\Delta_n-(B-1)} f_n(i,z,|V|)$ 

The percentage of views that **cache misses** occur is at least  $p_n = 1 - \sum_{i=1}^{n} f_n(i, z, |V|)$ 



# Competitive Ratio Analysis (Case 2: $\Delta_n < |V|$ )

- We first derive an upper bound of the percentage of views for which any cache miss does not happen, to obtain the lower bound of optimal solution *OPT*
- The cost with optimal policy is **at least**  $\left[ (c_m + \underline{c_{n,f}} + \underline{c_{n,s}}) \cdot p + \underline{c_{n,s}} \cdot (1-p) \right] \cdot T$
- ▶ *OPT* also has a fundamental lower bound  $\underline{c_{n,s}} \cdot \left( \lceil \frac{B-1}{D} \rceil + 1 \right) \cdot T$  (those views are not in *LEO*<sub>n</sub>) with no cache miss
- ▶ **The lower bound of** *OPT* is

$$\max\left\{\left(\left[\left(c_{m}+\underline{c_{n,f}}+\underline{c_{n,s}}\right)\cdot p+\underline{c_{n,s}}\cdot (1-p)\right]\cdot T\right),\left(\underline{c_{n,s}}\cdot \left(\lceil \frac{B-1}{D}\rceil+1\right)\cdot T\right)\right\}$$



# Competitive Ratio Analysis (Case 2: $\Delta_n < |V|$ )

- For each request range [h, l] consisting of B views, considering each view would cache miss  $(c_m \cdot B)$
- Also the inter-satellite links would not help to cache hit
- The number of fetched views is at most  $\lceil \frac{B-1}{D} \rceil + 1$  in each time slot
- ▶ The cost of the solution with *ALG* is at most

$$\left(c_m \cdot B + \overline{c_{n,f}} \cdot (\lceil \frac{B-1}{D} \rceil + 1) + \overline{c_{n,s}} \cdot (\lceil \frac{B-1}{D} \rceil + 1)\right) \cdot T$$

▶ The upper bound of ALG is  $\left(c_m \cdot B + \overline{c_{n,f}} \cdot (\lceil \frac{B-1}{D} \rceil + 1) + \overline{c_{n,s}} \cdot (\lceil \frac{B-1}{D} \rceil + 1)\right) \cdot T$ 



### Competitive Ratio Analysis

$$\frac{\text{ALG}}{\text{OPT}} \leq \frac{(c_m \cdot B + \overline{c_{n,f}} \cdot (\lceil \frac{B-1}{D} \rceil + 1) + \overline{c_{n,s}} \cdot (\lceil \frac{B-1}{D} \rceil + 1)) \cdot T}{\max\{\left(\left[(c_m + \underline{c_{n,f}} + \underline{c_{n,s}}) \cdot p + \underline{c_{n,s}} \cdot (1-p)\right] \cdot T\right), \left(\underline{c_{n,s}} \cdot (\lceil \frac{B-1}{D} \rceil + 1) \cdot T\right)\}}$$

$$\leq \frac{(c_m \cdot B + c_m \cdot (\lceil \frac{B-1}{D} \rceil + 1) + c_m \cdot (\lceil \frac{B-1}{D} \rceil + 1)) \cdot T}{\max\{(\underline{c_{n,s}} \cdot p + \underline{c_{n,s}} \cdot (1-p)) \cdot T, (\underline{c_{n,s}} \cdot (\lceil \frac{B-1}{D} \rceil + 1) \cdot T)\}}$$

$$\leq \frac{3c_m \cdot B \cdot T}{\max\{(\underline{c_{n,s}} \cdot T), (\underline{c_{n,s}} \cdot \frac{B}{D} \cdot T)\}} = \min\left\{\frac{3c_m B}{\underline{c_{n,s}}}, \frac{3c_m D}{\underline{c_{n,s}}}\right\}$$

- ▶ Second inequality holds because  $c_m > \overline{c_{n,f}}$  and  $c_m > \overline{c_{n,s}}$
- The last inequality holds because  $\frac{B}{D} \le \frac{B-1}{D} + 1 \le \lceil \frac{B-1}{D} \rceil + 1 \le B$

