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Unsupervised variational source separation with deep priors

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Research Question

Source separation is the task of finding a set of latent sources $\boldsymbol{s} = [\boldsymbol{s}_1, \ldots \boldsymbol{s}_k, \ldots, \boldsymbol{s}_n]$ to an observed mix of those sources $\boldsymbol{m}$. The induced model proposes a mixing function $\boldsymbol{m} = f(\boldsymbol{s})$. The task is to find an approximate inverse model $g(\cdot)$ which retrieves $\boldsymbol{s}$:

$$\mathbf{m} = f(\boldsymbol{s}) \tag{1}$$

$$g(\boldsymbol{m}) \approxeq \boldsymbol{s} \tag{2}$$

In this learning setting *supervision* can happen in two ways: First the source signals are identified as being from class k [1]. Second the tuples $(\boldsymbol{m}, \boldsymbol{s})$ are supervised giving us examples of mixes and their corresponding sources.

[1] For the setting of music think of the classes being $\{$*guitar*,*piano*,*voice*,...$\}$

add other supverision: knowing k

1. Can we learn an source separation model $g(\cdot)$ by learning deep priors for the different source classes.
2. Can we reduce this to an unsupervised setting. Unsupervised relating to the missing pairings of sources and mixes.

Related works

In this chapter we discuss previous research in supervised and semi-supervised source separation.

ICA

Deep Latent-Variable Models

For our process we have observations from the data space $\boldsymbol{x} \in \mathcal{D}$ for which there exists an unknown data probability distribution $p^*(\mathcal{D})$.

We collect a data set $\{\boldsymbol{x}_1 \dots \boldsymbol{x}_N\}$ with N samples. We introduce an approximate model with density[2] $p_{\boldsymbol{\theta}}(\mathcal{D})$ and model parameters $\boldsymbol{\theta}$. Learning or modelling means finding the values for $\boldsymbol{\theta}$ which will give the closest approximation of the true underlying process:

[2] We write density and distribution interchangeably to denote a probability function.

$$p_{\boldsymbol{\theta}}(\mathcal{D}) \approx p^*(\mathcal{D}) \tag{3}$$

The model $p_{\boldsymbol{\theta}}$ has to be complex enough to be able to fit the data density while little enough parameters to be learnable. Every choice for the form of the model will *induce* biases[3] about what density we can model, even before we maximize a learning objective using the parameters $\boldsymbol{\theta}$.

[3] called *inductive biases*

In the following described models we assume the sampled data points $\boldsymbol{x}$ to be drawn from $\mathcal{D}$ *independent and identically distributed*[4]. Therefore we can write the data log-likelihood as:

[4] meaning the sample of one datum does not depend on the other data points

$$p_{\boldsymbol{\theta}}(\mathcal{D}) = \prod_{\boldsymbol{x} \in \mathcal{D}} p_{\boldsymbol{\theta}}(\boldsymbol{x}) \tag{4}$$

$$\log p_{\boldsymbol{\theta}}(\mathcal{D}) = \sum_{\boldsymbol{x} \in \mathcal{D}} \log p_{\boldsymbol{\theta}}(\boldsymbol{x}) \tag{5}$$

The maximum likelihood estimation of our model parameters maximizes this objective.

To form a latent-variable model we introduce a *latent variable*[5]. The data likelihood now is the marginal density of the joint latent density:

[5] Latent variables are part of the directed graphical model but not observed.

$$p_{\boldsymbol{\theta}}(\boldsymbol{x}) = \int p_{\boldsymbol{\theta}}(\boldsymbol{x}, \boldsymbol{z}) d\boldsymbol{z} \tag{6}$$

Typically we introduce a factorization of the joint. Most commonly and simplest:

$$p_{\boldsymbol{\theta}}(\boldsymbol{x}) = \int p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{z}) p(\boldsymbol{z}) d\boldsymbol{z} \tag{7}$$

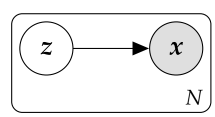


Figure 1: The graphical model with a introduced latent variable $\boldsymbol{z}$. Observed variables are shaded.

This corresponds to the graphical model in which $\boldsymbol{z}$ is generative parent node of the observed $\boldsymbol{x}$, see Figure 1. The density $p(\boldsymbol{z})$ is called the *prior distribution*.

If the latent is small, discrete, it might be possible to directly marginalize over it. If for example $\boldsymbol{z}$ is a discrete random variable and the conditional $p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{z})$ is a Gaussian distribution than the data model density $p_{\boldsymbol{\theta}}(\boldsymbol{x})$ becomes a mixture-of-Gaussians, which we can directly estimate by maximum likelihood estimation of the data likelihood.

For more complicated models the data likelihood $p_{\boldsymbol{\theta}}(\boldsymbol{x})$ as well as the model posterior $p_{\boldsymbol{\theta}}(\boldsymbol{z}|\boldsymbol{x})$ are intractable because of the integration over the latent $\boldsymbol{z}$ in Equation (7).

To formalize the search for an intractable posterior into a tractable optimization problem we follow the *variational principle*[6] which intro-

[6] Michael I. Jordan et al. "An Introduction to Variational Methods for Graphical Models". In: *Machine Learning* 37.2 (1999), pp. 183–233.

duces an approximate posterior distribution $q_{\boldsymbol{\phi}}(z|x)$, also called the *inference model*. Again the choice of model here carries inductive biases as such that even in asymptotic expectation we can not obtain the true posterior.

Following the derivation in[7] we introduce the inference model into the data likelihood [8]:

[7] Diederik P. Kingma and Max Welling. "An Introduction to Variational Autoencoders". In: (2019). arXiv: 1906.02691 [cs, stat], p. 20.

[8] The first step is valid as $q_{\boldsymbol{\theta}}$ is a valid density function and thus integrates to one.

$$
\begin{aligned}
\log p_{\boldsymbol{\theta}}(x) &= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log p_{\boldsymbol{\theta}}(x)\right] && (8)\\
&= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x,z)}{p_{\boldsymbol{\theta}}(z|x)}\right] && (9)\\
&= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x,z)}{q_{\boldsymbol{\phi}}(z|x)}\frac{q_{\boldsymbol{\phi}}(z|x)}{p_{\boldsymbol{\theta}}(z|x)}\right] && (10)\\
&= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x,z)}{q_{\boldsymbol{\phi}}(z|x)}\right] + \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{q_{\boldsymbol{\phi}}(z|x)}{p_{\boldsymbol{\theta}}(z|x)}\right] && (11)\\
&= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x,z)}{q_{\boldsymbol{\phi}}(z|x)}\right] + \mathbb{D}_{\mathrm{KL}}[q_{\boldsymbol{\phi}}(z|x)\|p_{\boldsymbol{\theta}}(z|x)] && (12)
\end{aligned}
$$

Note that we separated the likelihood into two parts. The second part is the (positive) Kullback-Leibler divergence of the approximate posterior from the true intractable posterior. This unknown divergence states the 'correctness' of our approximation [9].

[9] More specifically the divergence marries two errors of our approximate model. First it gives the error of our posterior estimation from the true posterior, by defintion of divergence. Second it specifies the error of our complete model likelihood from the marginal likelihood. This is called the *tightness* of the bound.

The first term is the *variational free energy* or *evidence lower bound* (ELBO):

explain free energy

$$
\mathrm{ELBO}_{\boldsymbol{\theta},\boldsymbol{\phi}}(x) = \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x,z)}{q_{\boldsymbol{\phi}}(z|x)}\right] \qquad (13)
$$

We can introduce the same factorization as in Equation (7):

$$
\begin{aligned}
\mathrm{ELBO}_{\boldsymbol{\theta},\boldsymbol{\phi}}(x) &= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p_{\boldsymbol{\theta}}(x|z)p(z)}{q_{\boldsymbol{\phi}}(z|x)}\right] && (14)\\
&= \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log \frac{p(z)}{q_{\boldsymbol{\phi}}(z|x)}\right] + \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log p_{\boldsymbol{\theta}}(x|z)\right] && (15)\\
&= -\mathbb{D}_{\mathrm{KL}}[q_{\boldsymbol{\phi}}(z|x)\|p(z)] + \mathbb{E}_{q_{\boldsymbol{\theta}}(z|x)}\left[\log p_{\boldsymbol{\theta}}(x|z)\right] && (16)
\end{aligned}
$$

Under this factorization we separated the lower bound into two parts. First the divergence of the approximate posterior from the latent prior distribution and second the data posterior likelihood from the latent [10].

[10] this will later be the reconstruction error. How well can we return to the data density from latent space

The optimization of the $\mathrm{ELBO}_{\boldsymbol{\theta},\boldsymbol{\phi}}$ allows us to jointly optimize the parameter sets $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$. The gradient with respect to $\boldsymbol{\theta}$ can be estimated with an unbiased Monte Carlo estimate using data samples [11]. We can *not* though do the same for the variational parameters $\boldsymbol{\phi}$, as the expectation of the ELBO is over the approximate posterior which

[11] $\nabla_{\boldsymbol{\theta}} \mathrm{ELBO}_{\boldsymbol{\theta},\boldsymbol{\phi}} \approxeq \nabla_{\boldsymbol{\theta}} \log p_{\boldsymbol{\theta}}(x,z)$

depends on $\boldsymbol{\phi}$. By a change of variable of the latent variable we can make this gradient tractable, the so called *reparameterization trick*.[12] We express the $z \sim q_{\boldsymbol{\theta}}$ as an random sample from a unparametrized source of entropy $\boldsymbol{\epsilon}$ and a parametrized transformation:

[12] Diederik P. Kingma and Max Welling. "Auto-Encoding Variational Bayes". In: (2014). arXiv: 1312.6114 [cs, stat].

$$\boldsymbol{z} = f_{\boldsymbol{\eta}}(\boldsymbol{\epsilon}) \tag{17}$$

For example for a Gaussian distribution we can express $z \sim \mathcal{N}(\mu, \sigma)$ as $z = \mu + \sigma \cdot \epsilon$ with $\epsilon \sim \mathcal{N}(0,1)$ and $\eta = \{\mu, \sigma\}$.

The VAE framework

VAE [12] [13]

The β-VAE[14] extends the VAE objective with an β hyperparameter in front of the KL divergence. The value β gives a constraint on the laten space controlling hte capacity of it. Adapting β gives an trade-off between reconstruction quality of the autoencoder and the simplicity of the latent representations[14]. Using such a constraint is similar to the use of in the information bottleneck.[15]

[13] Danilo Jimenez Rezende et al. "Stochastic Backpropagation and Approximate Inference in Deep Generative Models". In: (2014). arXiv: 1401.4082 [cs, stat].

[14] Irina Higgins et al. "Beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework". In: (2016).

[15] Christopher P. Burgess et al. "Understanding Disentangling in Beta-VAE". In: (2018). arXiv: 1804.03599 [cs, stat].

Flow based models

Another class of common deep latent models are based on *normalizing* flows.[16] A normalizing flow is a function $f(\boldsymbol{x})$ that maps the input density to a fixed, prescribed density $p(\epsilon) = p(f(\boldsymbol{x}))$, in that normalizing the density [17]. They use a flow for the approximate posterior $q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})$. Again this is commonly set to be a factorized Gaussian distribution.

[16] Esteban Tabak and Cristina V. Turner. "A family of nonparametric density estimation algorithms". In: *Communications on Pure and Applied Mathematics* 66.2 (2013), pp. 145–164.

[17] The extreme of this idea is, of course, an infinitesimal, continuos-time flow with a velocity field.

For a finite normalizing flow we consider a chain of invertible, smooth mappings.

NICE[18] - volume preserving transformations - coupling layer - triangular shape

Normalizing Flow[19]

RealNVP[20] build on top of NICE creating a more general, non-volume preserving, normalizing flow.

[18] Laurent Dinh et al. "NICE: Non-Linear Independent Components Estimation". In: (2015). arXiv: 1410.8516 [cs].

[19] Danilo Jimenez Rezende and Shakir Mohamed. "Variational Inference with Normalizing Flows". In: (2016). arXiv: 1505.05770 [cs, stat].

[20] Laurent Dinh et al. "Density Estimation Using Real NVP". In: (2017). arXiv: 1605.08803 [cs, stat].

$$y_{1:d} = x_{1:d} \tag{18}$$

$$y_{d+1:D} = x_{d+1:D} \odot \exp\left(s(x_{1:d})\right) + t(x_{1:d}) \tag{19}$$

$$\frac{\partial y}{\partial x^T} = \begin{bmatrix} \mathbb{1}_d & 0 \\ \frac{\partial y_{d+1}:D}{\partial x^T_{1:d}} & \operatorname{diag}\left(\exp\left(s(x_{1:d})\right)\right) \end{bmatrix} \tag{20}$$

Glow[21] extended the RealNVP by introducing invertible 1×1-convolutions. Instead of having fixed masks and permutations for the computations of the affine parameters in the coupling layer, Glow

[21] Diederik P. Kingma and Prafulla Dhariwal. "Glow: Generative Flow with Invertible 1x1 Convolutions". In: (2018). arXiv: 1807.03039 [cs, stat].

learns a rotation matrix which mixes the channels. After mixing the input can always be split in the same two parts for the affine transformation. Further the authors showed that training can be helped by initializing the last layer of each affine parameter network with zeros. This ensures that at in without weight update each coupling layer behaves as an identity.

Modelling raw audio

Deep learning models as used for image applications are unsuitable for raw audio signals (*time-domain*). Digital audio is sampled at high sample rates commonly 16kHz up to 44kHz. The features of interest lie at scales of strongly different magnitudes. Recognizing phase, frequency of a wave might require features at low ms intervals but modelling of speech or music features happens at the scale of seconds to minutes. As such a generative model for this domain has to model at these different scales.

The WaveNet[22] introduced an autoregressive generative model for raw audio. It is build upon the similar PixelCNN[23] but adapted for the audio domain. The WaveNet accomplishes this by using dilated causal convolutions a common tool in signal processing.[24] A dilated convolution uses a kernel with an inner stride. Using a stack of dilated convolutions increases the receptive field of the deep features without increasing the computational complexity, see Figure 2. Further the convolutions are gated and the output is constructed from skip connections, refer to **??** . A gated feature, as known from the LSTM,[25] computes two outputs: one put through an sigmoid $\sigma(\cdot)$ activation and one through an $\tanh(\cdot)$ activation. The idea being that the sigmoid (with an output range of $[0, 1]$) regulates the amount of information, thereby gating it, while the tanh (with a range of $[-1, 1]$) gives the magnitude of the feature. The output of the WaveNet is the sum of outflowing skip connections added after each (gated) hidden convolution. This helps fusing information from multiple time-scales (*low-level* to *high-level*). The original authors tested the model on multiple audio generation tasks. For this they formulated the reconstruction objective as an multi-class recognition problem. Encoding the sound files with μ-law encoding,[26] discretizes the range $[-1, 1]$ to allow a set of μtargets. Sound generation with a WaveNet is slow as the autoregressiveness requires the generation value by value. This can be alleviated by keeping intermediate hidden activations cached.[27]

W

NSynth[28]

In[29]

FloWaveNet[30]

[22] Aäron van den Oord et al. "WaveNet: A Generative Model for Raw Audio". In: (2016). arXiv: 1609.03499 [cs].

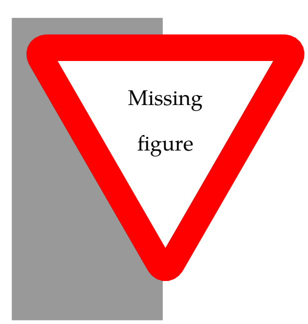
[23] Aäron van den Oord et al. "Conditional Image Generation with PixelCNN Decoders". In: (2016). arXiv: 1606.05328 [cs].

[24] P. Dutilleux. "An Implementation of the Algorithme à Trous to Compute the Wavelet Transform". In: *Wavelets*. Ed. by Jean-Michel Combes et al. Inverse Problems and Theoretical Imaging. Berlin, Heidelberg: Springer, 1990, pp. 298–304.

[25] Sepp Hochreiter and Jürgen Schmidhuber. "Long Short-Term Memory". In: *Neural Computation* 9.8 (1997), pp. 1735–1780.

[26] *Recommendation G. 711. Pulse Code Modulation (PCM) of Voice Frequencies*. 1988.

[27] Tom Le Paine et al. "Fast Wavenet Generation Algorithm". In: (2016). arXiv: 1611.09482 [cs].



Show WaveNet hidden layer

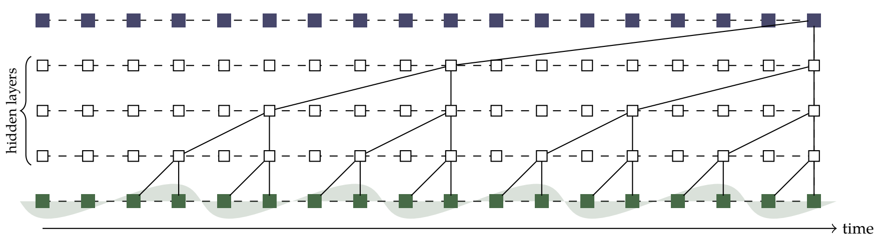


Figure 2: An example of how dilated convolutions are used in the WaveNet. We see three hidden layers with each a kernel size of two. By using the dilations the prediction of the new output element has an receptive field of 18. This convolution is *causal* as the prediction depends only on previous input values.

Source separation

WaveNet for Speech denoising[31]

WaveNet-VAE unsupervised speech rep learning[32]

Andreas Jansson et al. “Singing Voice Separation with Deep U-Net Convolutional Networks”. In: *ISMIR*. 2017[33] were the first to use an U-Net architecture for source separation.

Wave-U-Net[34]

DeMucs[35]

Source Sep in Time Domain[36]

Methodology

Datasets

ToyData

MusDB

Planning

[31] Dario Rethage et al. “A Wavenet for Speech Denoising”. In: (2018). arXiv: 1706.07162 [cs].

[32] Jan Chorowski et al. “Unsupervised Speech Representation Learning Using WaveNet Autoencoders”. In: *IEEE/ACM Transactions on Audio, Speech, and Language Processing* 27.12 (2019), pp. 2041–2053. arXiv: 1901.08810.

[33] Andreas Jansson et al. “Singing Voice Separation with Deep U-Net Convolutional Networks”. In: *ISMIR*. 2017.

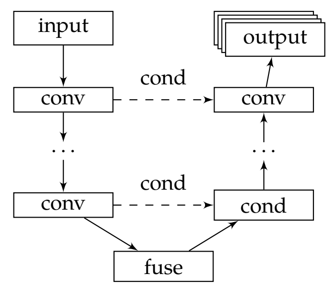


Figure 3: The U-Net

[34] Daniel Stoller et al. “Wave-U-Net: A Multi-Scale Neural Network for End-to-End Audio Source Separation” (Paris, France). 2018.

[35] Alexandre Défossez et al. “Demucs: Deep Extractor for Music Sources with Extra Unlabeled Data Remixed”. In: (2019). arXiv: 1909.01174 [cs, eess, stat].

[36] Francesc Lluís et al. “End-to-End Music Source Separation: Is It Possible in the Waveform Domain?” In: (2019). arXiv: 1810.12187 [cs, eess].

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