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Unsupervised variational source separation with deep pri-
ors

Contents

<i>Proposal</i>	7
<i>Abstract</i>	7
<i>Research Question</i>	7
<i>Related works</i>	7
<i>Methodology</i>	12
<i>Planning</i>	12

Todo list

add other supervision: knowing k	7
explain free energy	9
Figure: Show WaveNet hidden layer	11
explain PixelCNN++ even if I don't use it know?	11

Proposal

Abstract

Research Question

Source separation is the task of finding a set of latent sources $\mathbf{s} = [s_1, \dots, s_k, \dots, s_N] \in \mathbb{R}^{N \times T}$ to an observed mix of those sources $\mathbf{m} \in \mathbb{R}^{1 \times T}$. The induced model proposes a mixing function $\mathbf{m} = f(\mathbf{s})$. The task is to find an approximate inverse model $g(\cdot)$ which retrieves \mathbf{s} :

$$\mathbf{m} = f(\mathbf{s}) \quad (1)$$

$$g(\mathbf{m}) \cong \mathbf{s} \quad (2)$$

In this learning setting *supervision* can happen in two ways: First the source signals are identified as being from class k ¹. Second the tuples (\mathbf{m}, \mathbf{s}) are supervised giving us examples of mixes and their corresponding sources.

¹ For the setting of music think of the classes being $\{\text{guitar, piano, voice, } \dots\}$

1. Can we learn an source separation model $g(\cdot)$ by learning deep priors for the different source classes.
2. Can we reduce this to an unsupervised setting. Unsupervised relating to the missing pairings of sources and mixes.

add other supervision: knowing k

Related works

In this chapter, we discuss previous research in supervised and semi-supervised source separation.

ICA

Deep Latent-Variable Models

For our process, we have observations from the data space $\mathbf{x} \in \mathcal{D}$ for which there exists an unknown data probability distribution $p^*(\mathcal{D})$.

We collect a data set $\{x_1 \dots x_N\}$ with N samples. We introduce an approximate model with density² $p_\theta(\mathcal{D})$ and model parameters θ . Learning or modeling means finding the values for θ which will give the closest approximation of the true underlying process:

$$p_\theta(\mathcal{D}) \approx p^*(\mathcal{D}) \quad (3)$$

The model p_θ has to be complex enough to be able to fit the data density while little enough parameters to be learned. Every choice for the form of the model will *induce* biases³ about what density we can model, even before we maximize a learning objective using the parameters θ .

In the following described models we assume the sampled data points x to be drawn from \mathcal{D} *independent and identically distributed*⁴. Therefore we can write the data log-likelihood as:

$$p_\theta(\mathcal{D}) = \prod_{x \in \mathcal{D}} p_\theta(x) \quad (4)$$

$$\log p_\theta(\mathcal{D}) = \sum_{x \in \mathcal{D}} \log p_\theta(x) \quad (5)$$

The maximum likelihood estimation of our model parameters maximizes this objective.

To form a latent-variable model we introduce a *latent variable*⁵. The data likelihood now is the marginal density of the joint latent density:

$$p_\theta(x) = \int p_\theta(x, z) dz \quad (6)$$

Typically we introduce a factorization of the joint. Most commonly and simplest:

$$p_\theta(x) = \int p_\theta(x|z)p(z)dz \quad (7)$$

This corresponds to the graphical model in which z is generative parent node of the observed x , see Figure 1. The density $p(z)$ is called the *prior distribution*.

If the latent is small, discrete, it might be possible to directly marginalize over it. If for example, z is a discrete random variable and the conditional $p_\theta(x|z)$ is a Gaussian distribution than the data model density $p_\theta(x)$ becomes a mixture-of-Gaussians, which we can directly estimate by maximum likelihood estimation of the data likelihood.

For more complicated models the data likelihood $p_\theta(x)$ as well as the model posterior $p_\theta(z|x)$ are intractable because of the integration over the latent z in Equation (7).

To formalize the search for an intractable posterior into a tractable optimization problem we follow the *variational principle*⁶ which intro-

² We write density and distribution interchangeably to denote a probability function.

³ called *inductive biases*

⁴ meaning the sample of one datum does not depend on the other data points

⁵ Latent variables are part of the directed graphical model but not observed.

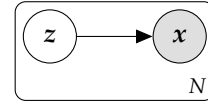


Figure 1: The graphical model with a introduced latent variable z . Observed variables are shaded.

⁶ Michael I. Jordan et al. "An Introduction to Variational Methods for Graphical Models". In: *Machine Learning* 37.2 (1999), pp. 183–233.

duces an approximate posterior distribution $q_\phi(z|x)$, also called the *inference model*. Again the choice of the model here carries inductive biases as such that even in asymptotic expectation we can not obtain the true posterior.

Following the derivation in⁷ we introduce the inference model into the data likelihood⁸:

$$\log p_\theta(x) = \mathbb{E}_{q_\theta(z|x)} [\log p_\theta(x)] \quad (8)$$

$$= \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x, z)}{p_\theta(z|x)} \right] \quad (9)$$

$$= \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x, z)}{q_\phi(z|x)} \frac{q_\phi(z|x)}{p_\theta(z|x)} \right] \quad (10)$$

$$= \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x, z)}{q_\phi(z|x)} \right] + \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{q_\phi(z|x)}{p_\theta(z|x)} \right] \quad (11)$$

$$= \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x, z)}{q_\phi(z|x)} \right] + \mathbb{D}_{\text{KL}}[q_\phi(z|x) \| p_\theta(z|x)] \quad (12)$$

Note that we separated the likelihood into two parts. The second part is the (positive) Kullback-Leibler divergence of the approximate posterior from the true intractable posterior. This unknown divergence states the ‘correctness’ of our approximation⁹.

The first term is the *variational free energy* or *evidence lower bound* (ELBO):

$$\text{ELBO}_{\theta, \phi}(x) = \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x, z)}{q_\phi(z|x)} \right] \quad (13)$$

We can introduce the same factorization as in Equation (7):

$$\text{ELBO}_{\theta, \phi}(x) = \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p_\theta(x|z)p(z)}{q_\phi(z|x)} \right] \quad (14)$$

$$= \mathbb{E}_{q_\theta(z|x)} \left[\log \frac{p(z)}{q_\phi(z|x)} \right] + \mathbb{E}_{q_\theta(z|x)} [\log p_\theta(x|z)] \quad (15)$$

$$= -\mathbb{D}_{\text{KL}}[q_\phi(z|x) \| p(z)] + \mathbb{E}_{q_\theta(z|x)} [\log p_\theta(x|z)] \quad (16)$$

Under this factorization, we separated the lower bound into two parts. First, the divergence of the approximate posterior from the latent prior distribution and second the data posterior likelihood from the latent¹⁰.

The optimization of the $\text{ELBO}_{\theta, \phi}$ allows us to jointly optimize the parameter sets θ and ϕ . The gradient with respect to θ can be estimated with an unbiased Monte Carlo estimate using data samples¹¹. We can *not* though do the same for the variational parameters ϕ , as the expectation of the ELBO is over the approximate posterior which

⁷ Diederik P. Kingma and Max Welling. “An Introduction to Variational Autoencoders”. In: (2019). arXiv: [1906.02691 \[cs, stat\]](#), p. 20.

⁸ The first step is valid as q_θ is a valid density function and thus integrates to one.

⁹ More specifically the divergence marries two errors of our approximate model. First, it gives the error of our posterior estimation from the true posterior, by definition of divergence. Second, it specifies the error of our complete model likelihood from the marginal likelihood. This is called the *tightness* of the bound.

explain free energy

¹⁰ this will later be the reconstruction error. How well can we return to the data density from latent space

¹¹ $\nabla_\theta \text{ELBO}_{\theta, \phi} \cong \nabla_\theta \log p_\theta(x, z)$

depends on ϕ . By a change of variable of the latent variable we can make this gradient tractable, the so called *reparameterization trick*.¹² We express the $z \sim q_\theta$ as a random sample from a unparametrized source of entropy ϵ and a parametrized transformation:

$$z = f_\eta(\epsilon) \quad (17)$$

For example for a Gaussian distribution we can express $z \sim \mathcal{N}(\mu, \sigma)$ as $z = \mu + \sigma \cdot \epsilon$ with $\epsilon \sim \mathcal{N}(0, 1)$ and $\eta = \{\mu, \sigma\}$.

The VAE framework

VAE^{12,13}

The β -VAE¹⁴ extends the VAE objective with an β hyperparameter in front of the KL divergence. The value β gives a constraint on the latent space controlling the capacity of it. Adapting β gives a trade-off between the reconstruction quality of the autoencoder and the simplicity of the latent representations¹⁴. Using such a constraint is similar to the use of in the information bottleneck.¹⁵

Flow based models

Another class of common deep latent models is based on *normalizing flows*.¹⁶ A normalizing flow is a function $f(x)$ that maps the input density to a fixed, prescribed density $p(\epsilon) = p(f(x))$, in that normalizing the density¹⁷. They use a flow for the approximate posterior $q_\phi(z|x)$. Again this is commonly set to be a factorized Gaussian distribution.

For a finite normalizing flow, we consider a chain of invertible, smooth mappings.

NICE¹⁸ - volume preserving transformations - coupling layer - triangular shape

Normalizing Flow¹⁹

RealNVP²⁰ builds on top of NICE creating a more general, non-volume preserving, normalizing flow.

$$y_{1:d} = x_{1:d} \quad (18)$$

$$y_{d+1:D} = x_{d+1:D} \odot \exp(s(x_{1:d})) + t(x_{1:d}) \quad (19)$$

$$\frac{\partial y}{\partial x^T} = \begin{bmatrix} \mathbb{1}_d & 0 \\ \frac{\partial y_{d+1:D}}{\partial x_{1:d}^T} & \text{diag}(\exp(s(x_{1:d}))) \end{bmatrix} \quad (20)$$

Glow²¹ extended the RealNVP by introducing invertible 1×1 -convolutions. Instead of having fixed masks and permutations for the computations of the affine parameters in the coupling layer, Glow

¹² Diederik P. Kingma and Max Welling. "Auto-Encoding Variational Bayes". In: (2014). arXiv: [1312.6114](#) [cs, stat].

¹³ Danilo Jimenez Rezende et al. "Stochastic Backpropagation and Approximate Inference in Deep Generative Models". In: (2014). arXiv: [1401.4082](#) [cs, stat].

¹⁴ Irina Higgins et al. "Beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework". In: (2016).

¹⁵ Christopher P. Burgess et al. "Understanding Disentangling in Beta-VAE". In: (2018). arXiv: [1804.03599](#) [cs, stat].

¹⁶ Esteban Tabak and Cristina V. Turner. "A family of nonparametric density estimation algorithms". In: *Communications on Pure and Applied Mathematics* 66.2 (2013), pp. 145–164.

¹⁷ The extreme of this idea is, of course, an infinitesimal, continuous-time flow with a velocity field.

¹⁸ Laurent Dinh et al. "NICE: Non-Linear Independent Components Estimation". In: (2015). arXiv: [1410.8516](#) [cs].

¹⁹ Danilo Jimenez Rezende and Shakir Mohamed. "Variational Inference with Normalizing Flows". In: (2016). arXiv: [1505.05770](#) [cs, stat].

²⁰ Laurent Dinh et al. "Density Estimation Using Real NVP". In: (2017). arXiv: [1605.08803](#) [cs, stat].

²¹ Diederik P. Kingma and Prafulla Dhariwal. "Glow: Generative Flow with Invertible 1x1 Convolutions". In: (2018). arXiv: [1807.03039](#) [cs, stat].

learns a rotation matrix which mixes the channels. After mixing the input can always be split into the same two parts for the affine transformation. Further, the authors showed that training can be helped by initializing the last layer of each affine parameter network with zeros. This ensures that at the beginning without weight update each coupling layer behaves as an identity.

Modelling raw audio

Deep learning models as used for image applications are unsuitable for raw audio signals (*time-domain*). Digital audio is sampled at high sample rates commonly 16kHz up to 44kHz. The features of interest lie at scales of strongly different magnitudes. Recognizing phase, frequency of a wave might require features at low ms intervals but modelling of speech or music features happens at the scale of seconds to minutes. As such a generative model for this domain has to model at these different scales.

The WaveNet²² introduced an autoregressive generative model for raw audio. It is build upon the similar PixelCNN²³ but adapted for the audio domain. The WaveNet accomplishes this by using dilated causal convolutions a common tool in signal processing.²⁴ A dilated convolution uses a kernel with an inner stride. Using a stack of dilated convolutions increases the receptive field of the deep features without increasing the computational complexity, see Figure 2. Further, the convolutions are gated and the output is constructed from skip connections, refer to ?? . A gated feature, as known from the LSTM,²⁵ computes two outputs: one put through an sigmoid $\sigma(\cdot)$ activation and one through an $\tanh(\cdot)$ activation. The idea being that the sigmoid (with an output range of $[0, 1]$) regulates the amount of information, thereby gating it, while the \tanh (with a range of $[-1, 1]$) gives the magnitude of the feature. The output of the WaveNet is the sum of outflowing skip connections added after each (gated) hidden convolution. This helps fusing information from multiple time-scales (*low-level* to *high-level*). The original authors tested the model on multiple audio generation tasks. For this, they formulated the reconstruction objective as a multi-class recognition problem. Encoding the sound files with μ -law encoding,²⁶ discretizes the range $[-1, 1]$ to allow a set of μ targets. Sound generation with a WaveNet is slow as the autoregressiveness requires the generation value by value²⁷. This can be alleviated by keeping intermediate hidden activations cached.²⁸ The WaveNet can be conditioned by adding the weighted conditionals in the gate and feature activations of the gated convolutions.²⁹

NSynth³⁰

In³¹

²² Aäron van den Oord et al. "WaveNet: A Generative Model for Raw Audio". In: (2016). arXiv: [1609.03499 \[cs\]](#).

²³ Aäron van den Oord et al. "Conditional Image Generation with PixelCNN Decoders". In: (2016). arXiv: [1606.05328 \[cs\]](#).

²⁴ P. Dutilleul. "An Implementation of the Algorithm à Trous to Compute the Wavelet Transform". In: *Wavelets*. Ed. by Jean-Michel Combes et al. Inverse Problems and Theoretical Imaging. Berlin, Heidelberg: Springer, 1990, pp. 298–304.

²⁵ Sepp Hochreiter and Jürgen Schmidhuber. "Long Short-Term Memory". In: *Neural Computation* 9.8 (1997), pp. 1735–1780.

²⁶ Recommendation G. 711. *Pulse Code Modulation (PCM) of Voice Frequencies*. 1988.

²⁷ Why are they doing that then? The WaveNet setting is generating waveforms, by giving the previously generated values as the input and conditioning the process on target classes. Therefore the generation has to happen value-by-value.

²⁸ Tom Le Paine et al. "Fast Wavenet Generation Algorithm". In: (2016). arXiv: [1611.09482 \[cs\]](#).

²⁹ Van den Oord et al., "Conditional Image Generation with PixelCNN Decoders".



Missing

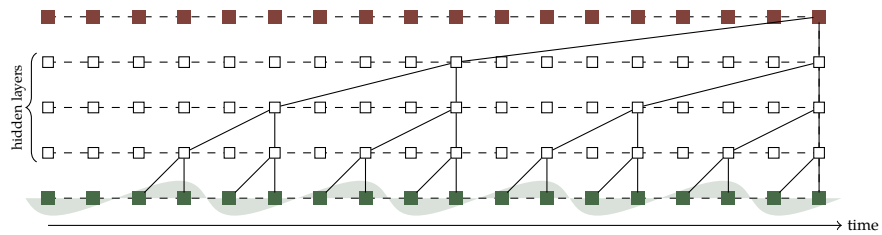


Figure 2: An example of how dilated convolutions are used in the WaveNet. We see three hidden layers with each a kernel size of two. By using the dilations the prediction of the new output element has a receptive field of 18. This convolution is *causal* as the prediction depends only on previous input values.

FloWaveNet³²

Source separation

WaveNet for Speech denoising³³

WaveNet-VAE unsupervised speech rep learning³⁴

³⁵ were the first to use an U-Net architecture (see Figure 3) for source separation. They used a standard U-Net on spectrograms of musical data to extract the singing voice.

The Wave-U-Net³⁶ brings this idea into the time-domain. The downstreaming and upstreaming layers are replaced with WaveNet style layers.

In³⁷ the authors adapted a time-domain WaveNet for musical source separation. The network is non-causal ³⁸ The WaveNet directly outputs the seaparated sources and is trained fully supervised. They show this simple setup performing well against spectrogram based baselines. Also, the experiments show a higher performance with a network that is deeper but less wide ³⁹

Demucs⁴⁰ is another extension building on the U-Net idea. Having a similar structure to the Wave-U-Net they introduce a bidirectional LSTM at the bottleneck. The LSTM is responsible for keeping long-term temporal informational by running over the high-level latent along the time dimension.

Methodology

Datasets

ToyData

MusDB

Planning

³² Sungwon Kim et al. “FloWaveNet : A Generative Flow for Raw Audio”. In: (2019). arXiv: [1811.02155](#) [cs, eess].

³³ Dario Rethage et al. “A Wavenet for Speech Denoising”. In: (2018). arXiv: [1706.07162](#) [cs].

³⁴ Jan Chorowski et al. “Unsupervised Speech Representation Learning Using WaveNet Autoencoders”. In: *IEEE/ACM Transactions on Audio, Speech, and Language Processing* 27.12 (2019), pp. 2041–2053. arXiv: [1901.08810](#).

³⁵ Andreas Jansson et al. “Singing Voice Separation with Deep U-Net Convolutional Networks”. In: *ISMIR*. 2017.

³⁶ Daniel Stoller et al. “Wave-U-Net: A Multi-Scale Neural Network for End-to-End Audio Source Separation” (Paris, France). 2018.

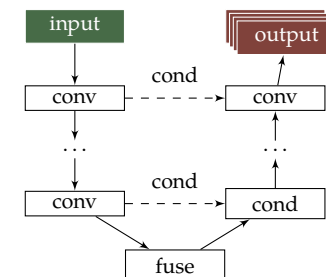


Figure 3: The U-Net

³⁷ Francesc Lluís et al. “End-to-End Music Source Separation: Is It Possible in the Waveform Domain?” In: (2019). arXiv: [1810.12187](#) [cs, eess].

³⁸ In this setting the WaveNet can be non-causal because the prediction happens from the given mix and is not autoregressive.

³⁹ Deepness of a network refers to the number of hidden layers. Wideness refers to the number of kernels/features of each of these hidden layers. Making the WaveNet deeper significantly increases its receptive field.

⁴⁰ Alexandre Défossez et al. “Demucs: Deep Extractor for Music Sources with Extra Unlabeled Data Remixed”. In: (2019). arXiv: [1909.01174](#) [cs, eess, stat].

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