



$$\mathbf{s} = [s_1, \dots, s_N]^T \in \mathbb{R}^{N \times T}$$

$$\mathbf{m} = f(\mathbf{s}) \in \mathbb{R}^{1 \times T}$$

$$g: \mathbb{R}^{1 \times T} \mapsto \mathbb{R}^{N \times T}$$

$$g(\mathbf{m}) \cong \mathbf{s}$$

$$\arg \max_{\boldsymbol{\theta}} p_{\boldsymbol{\theta}}(s_1, \dots, s_N | \mathbf{m})$$

$$\log p(\mathbf{m}) = \mathbb{E}_{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})}^N [\log p(\mathbf{m})] \tag{1}$$

$$= \mathbb{E}_{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})}^N \left[\log \frac{p(\mathbf{m}, s_1, \dots, s_N)}{p(s_1, \dots, s_N | \mathbf{m})} \right] \tag{2}$$

$$= \mathbb{E}_{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})}^N \left[\log \frac{p(\mathbf{m} | s_1, \dots, s_N) \cdot \prod_k^N p(s_k)}{\prod_k^N q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})} + \log \frac{\prod_k^N q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})}{p(s_1, \dots, s_N | \mathbf{m})} \right] \tag{3}$$

$$\geq \sum_k^N \mathbb{E}_{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})} \left[\log \frac{p(s_k)}{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})} \right] + \mathbb{E}_{q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})} [p(\mathbf{m} | s_1, \dots, s_N)] \tag{4}$$

$$p(\mathbf{m}, s_1, \dots, s_N) \equiv p(\mathbf{m} | s_1, \dots, s_N) \cdot \prod_k^N p(s_k)$$

$$\mathbf{s}^{(t+1)} = \mathbf{s}^{(t)} + \eta \cdot \nabla_{\mathbf{s}} \left(\log p(\mathbf{s}^{(t)}) + \frac{1}{2\gamma^2} \|\mathbf{m} - g(\mathbf{s}^{(t)})\|^2 \right) + 2\sqrt{\eta}\epsilon_t$$

$$\hat{\mathbf{s}}_k \sim q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m})$$

$$\mathcal{L}(\boldsymbol{\phi}, \mathbf{m}) = \sum_k^N \mathbb{D}_{\text{KL}} [q_{\boldsymbol{\phi}_k}(s_k | \mathbf{m}) \| p(s_k)] + \left(\frac{1}{N} \sum_k^N \hat{\mathbf{s}}_k - \mathbf{m} \right)^2$$