

# **ALU HW4: Sign-Extension-Bit Reduction**

# Outlines

- reduction of sign-extension bits for signed multiplier
  - Verilog operator \* for unsigned data \* (operator\_unsigned)
  - Verilog operator for \* for signed data (operator\_signed)
  - partial product (pp) row addition without optimized sign-extension bits for signed data (row)
  - pp row addition with optimized sign extension bits (row\_opt)
  - modified Baugh-Wooley reduction (row\_mbw)
  - addition of radix-4 Booth recoded pp rows without sign-extension bit optimization (row\_booth)
  - addition of radix-4 Booth recoded rows with optimized sign-extension bits

# partial product reduction

- partial product bit (ppb) matrix contains n rows of partial products, including sign-extension bits
  - e.g., n=6 as shown below
- carry save addition (CSA) can efficiently reduce # of partial product rows into 2 using
  - either row-based CSA (in units of partial product rows), or
  - column-based CSA (in unit of ppb, from rightmost column)
- But for easy coding, provided Verilog codes use for-loop to sum up all rows

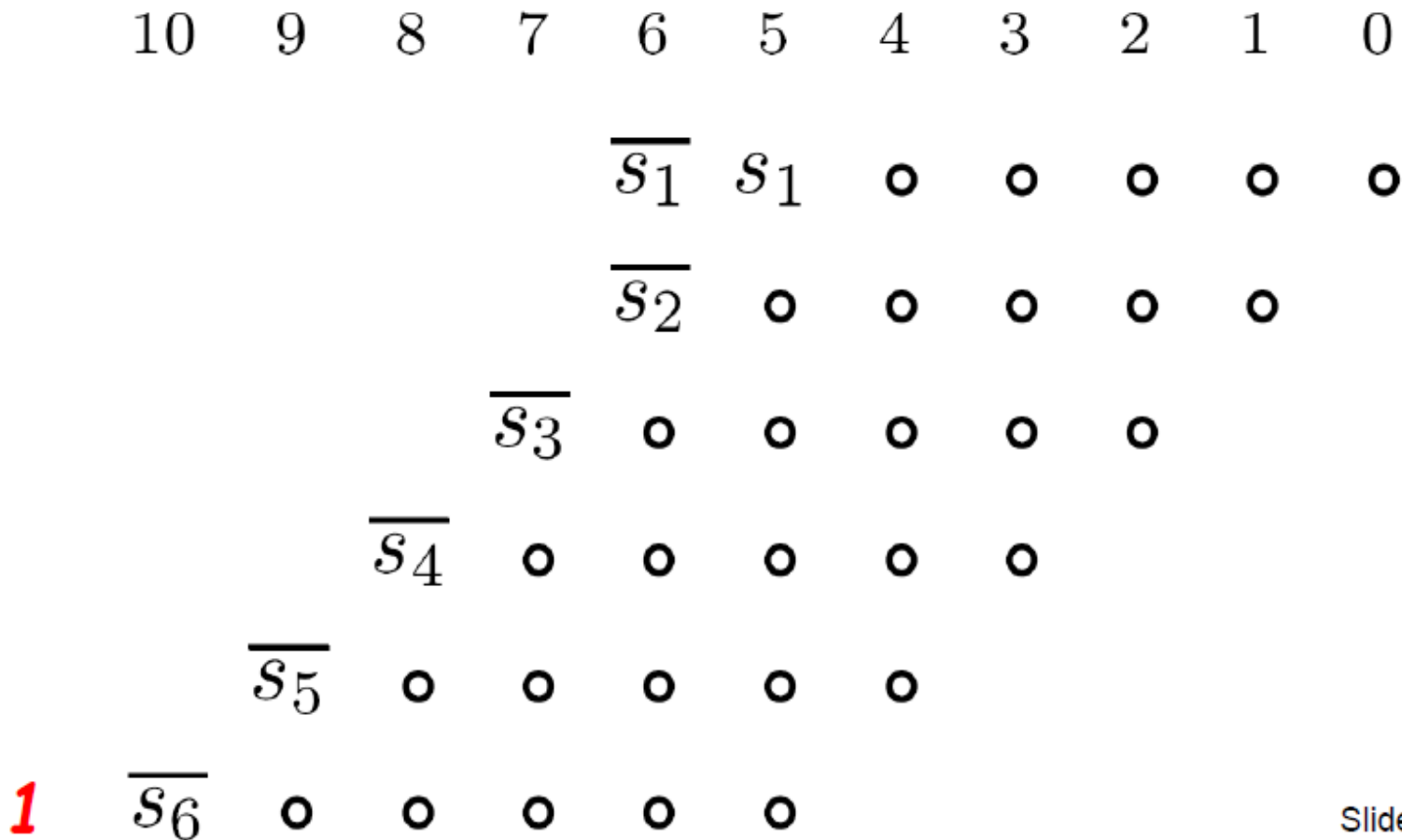
	10	9	8	7	6	5	4	3	2	1	0
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•

# Reduction of Sign-Extension Bits

- modify the partial product generation part in the provided multiplier codes to reduce the sign-extension bits
  - method-1 (ppb): regular partial-product bit (ppb) array
  - method-2 (mbw): modified Baugh-Wooley
  - method-3 (booth): radix-4 Booth
- Do experiments on three different precisions
  - 8x8
  - 8x16
  - 16x16
- compare with direct \* operator for **signed** data

# Method-1: partial product row addition (row\_opt)

- partial product row accumulation with optimized sign extension bits (row\_opt)



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# Method-2: Modified Baugh-Wooley (row\_mbw)

- modified Baugh Wooley for negative partial product bits
- partial product row addition (row\_mbw)

## Using the Negative-Weight Property of the Sign Bit

Sign extension is a way of converting negatively weighted bits (negabits) to positively weighted bits (posibits) to facilitate reduction, but there are other methods of accomplishing the same without introducing a lot of extra bits

Baugh and Wooley have contributed two such methods

**Baugh-Wooley**  
2's-complement multiplication.

a. Unsigned										
					$\times$	$a_4$	$a_3$	$a_2$	$a_1$	$a_0$
						$x_4$	$x_3$	$x_2$	$x_1$	$x_0$
						$a_4 x_0$	$a_3 x_0$	$a_2 x_0$	$a_1 x_0$	$a_0 x_0$
						$a_4 x_1$	$a_3 x_1$	$a_2 x_1$	$a_1 x_1$	$a_0 x_1$
						$a_4 x_2$	$a_3 x_2$	$a_2 x_2$	$a_1 x_2$	$a_0 x_2$
						$a_4 x_3$	$a_3 x_3$	$a_2 x_3$	$a_1 x_3$	$a_0 x_3$
						$a_4 x_4$	$a_3 x_4$	$a_2 x_4$	$a_1 x_4$	$a_0 x_4$
$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	
b. 2's-complement										
					$\times$	$a_4$	$a_3$	$a_2$	$a_1$	$a_0$
						$x_4$	$x_3$	$x_2$	$x_1$	$x_0$
						$a_4 x_0$	$a_3 x_0$	$a_2 x_0$	$a_1 x_0$	$a_0 x_0$
						$a_4 x_1$	$a_3 x_1$	$a_2 x_1$	$a_1 x_1$	$a_0 x_1$
						$a_4 x_2$	$a_3 x_2$	$a_2 x_2$	$a_1 x_2$	$a_0 x_2$
						$a_4 x_3$	$a_3 x_3$	$a_2 x_3$	$a_1 x_3$	$a_0 x_3$
						$a_4 x_4$	$a_3 x_4$	$a_2 x_4$	$a_1 x_4$	$a_0 x_4$
$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	
c. Baugh-Wooley										
					$\times$	$a_4$	$a_3$	$a_2$	$a_1$	$a_0$
						$x_4$	$x_3$	$x_2$	$x_1$	$x_0$
						$a_4 x_0$	$a_3 x_0$	$a_2 x_0$	$a_1 x_0$	$a_0 x_0$
						$a_4 x_1$	$a_3 x_1$	$a_2 x_1$	$a_1 x_1$	$a_0 x_1$
						$a_4 x_2$	$a_3 x_2$	$a_2 x_2$	$a_1 x_2$	$a_0 x_2$
						$a_4 x_3$	$a_3 x_3$	$a_2 x_3$	$a_1 x_3$	$a_0 x_3$
						$a_4 x_4$	$a_3 x_4$	$a_2 x_4$	$a_1 x_4$	$a_0 x_4$
$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	
d. Modified B-W										
					$\times$	$a_4$	$a_3$	$a_2$	$a_1$	$a_0$
						$x_4$	$x_3$	$x_2$	$x_1$	$x_0$
						$a_4 x_0$	$a_3 x_0$	$a_2 x_0$	$a_1 x_0$	$a_0 x_0$
						$a_4 x_1$	$a_3 x_1$	$a_2 x_1$	$a_1 x_1$	$a_0 x_1$
						$a_4 x_2$	$a_3 x_2$	$a_2 x_2$	$a_1 x_2$	$a_0 x_2$
						$a_4 x_3$	$a_3 x_3$	$a_2 x_3$	$a_1 x_3$	$a_0 x_3$
						$a_4 x_4$	$a_3 x_4$	$a_2 x_4$	$a_1 x_4$	$a_0 x_4$
$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	

# Method-2: Modified Baugh-Wooley (row\_mbw)

- $1-s = s'$  for  $s$  in  $\{0, 1\}$

d. Modified B-W

$$-a_4x_0 = (1 - a_4x_0) - 1$$

$$= (a_4x_0)' - 1$$

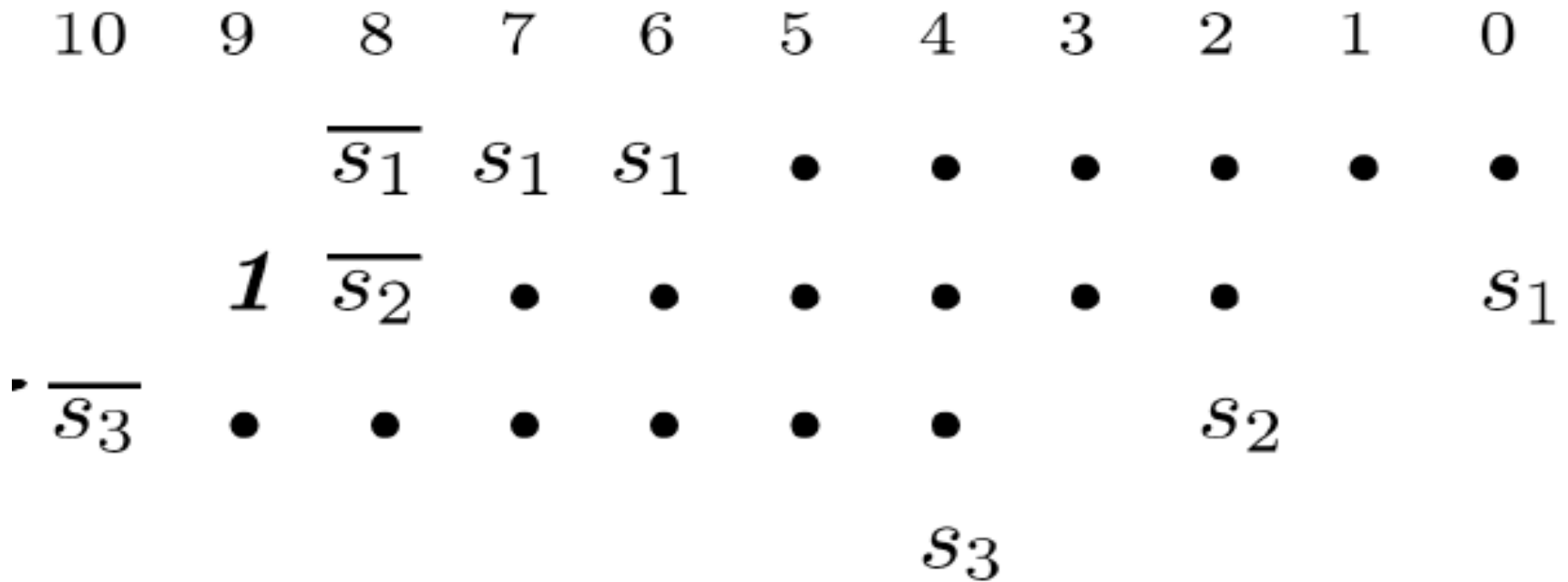
$-1$      $(a_4x_0)'$   
 $\uparrow$      $1$   
 In next column

					$x$	$a_4$ $x_4$	$a_3$ $x_3$	$a_2$ $x_2$	$a_1$ $x_1$	$a_0$ $x_0$
						$\overline{a_4x_0}$	$a_3x_0$	$a_2x_0$	$a_1x_0$	$a_0x_0$
			$\overline{a_4x_2}$	$\overline{a_3x_1}$	$\overline{a_4x_1}$	$\overline{a_3x_1}$	$a_2x_1$	$a_1x_1$	$a_0x_1$	
		$\overline{a_4x_3}$	$\overline{a_3x_2}$	$\overline{a_2x_1}$	$\overline{a_3x_2}$	$\overline{a_2x_2}$	$a_1x_2$	$a_0x_2$		
	$a_4x_4$	$\overline{a_3x_4}$	$\overline{a_2x_3}$	$\overline{a_1x_2}$	$\overline{a_2x_3}$	$\overline{a_1x_3}$	$a_0x_3$			
				$\overline{a_4x_4}$	$\overline{a_3x_4}$	$\overline{a_2x_4}$				
				$1$	$1$					
$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	

# Method-3: Radix-4 Booth

## (row\_booth\_opt)

- row addition of radix-4 Booth recoded partial products with optimized sign extension bits



Scheme (c)

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# Summarized Comparison Table

- operator: using Verilog operator \* directly for signed (unsigned) data
- row: partial product row addition without reduction of sign extension bits
- row\_opt: partial product row addition with optimized sign-extension bits
- row\_booth: radix-4 Booth recoded partial product row addition without reduction of sign extension bits
- row\_booth\_opt: radix-4 Booth recoded pp row addition with optimized sign-extension bits

signed mxn		operator (unsigned)			row			row_opt			row_mbw			row_booth			row_booth_opt		
		area	mid	delay	area	mid	delay	area	mid	delay	area	mid	delay	area	mid	delay	area	mid	delay
area (um2)	CL(total)																		
time (ns)	delay																		
power (uW)	dynamic																		
	leakage																		
	total																		