MS4212 Predictive Analytics and Forecasting

Project Report

City University of Hong Kong
2017-2018

Group 23: Private Domestic Premises

Chow Sai Yat Morris (54796760)

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Content

Management Summary

Hong Kong is one of the most densely populated cities in the world. The population is growing larger and larger but land that are suitable and available for construction are gradually declining due to its limited amount and constant development. Hence, the cost required for owning a private house has been continuously rising to a level that many citizens may not be able to afford. From the government's perspective, a decant forecast of the average price of private domestic premises is crucial. A sound prediction facilitates the coming up of comprehensive, effective and efficient plan to hinder the surging housing price.

We have fitted numerous forecasting models including naive, moving average, exponential smoothing, decomposition models and regression to the average price of private domestic premises from January 2007 to December 2017, there is a total of 132 observations. The results generated by the several forecasting models are subsequently compared using classical accuracy measures, for instance sum squared error (SSE), mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) based on the within sample residuals for 106 observations (from January 2007 to October 2015) and the beyond sample residuals for the remaining 26 observations (from November 2015 to December 2017).

After the comparison on the performance of the forecasting models, it is discovered that Holt's method enables both the minimization of MSE, MAE and MAPE and the minimization of Theil's U statistics beyond the sample. Holt's method appears to be a adequate and suitable model to perform the forecasting action for the future average price of private domestic premises (variable Y). Based on the fitted values generated by the forecasting model, it can be seen that the average price of the premises is forecasted to have an increasing trend in the future.

For the average rents of fresh lettings of private domestic premises (variable X1), we have chosen Brown's Method since it has small MSE, MAE, MAPE and Theil's U statistics beyond the sample. According to the forecasted value for the coming year (2018), the average rent is expected to decline at a slow rate in the foreseeable future.

The performance of the forecasting models are compared and Holt's method is finally chosen as it gives the best MSE, MAE, MAPE and Theil's U statistics beyond the sample. Referring to the fitted values by Holt's method, the future amount of agreements for sale and purchase of building units (Residential) is expected to increase gradually in the future.

Introduction

Background and Problem

As we all know, the property prices in Hong Kong is high and keep surging. According to the Demographia International Housing Affordability Survey conducted in 2017, the property market of Hong Kong has been ranked 1 of the most unaffordable in the world for a consecutive 8-year, with 18.1 times of the median of pre-tax household income. In this case, one in four residents in Hong Kong are not able to and have no hope of being able to afford a local home (SCMP, 2018). Although the “spicy” measures (the SSD and BSD) were carried out, the effectiveness was limited as there was only little adjustment in property prices. The imminent housing problem is not addressed or alleviated efficiently.

Available Data

In this project, our focus is the private housing market. To better understand its trend, we gathered a total of 132 periods monthly data of year 2007 to 2017 from Hong Kong Monthly Digest of Statistics. We will:

i. Conduct data analysis base on the first 80% (106 periods), and further estimate the remaining 20% (26 periods)

ii. Compare the estimated values with the original figures, compute the error statistics to examine the accuracy of different forecasting methods

Objectives

Through this project, we aim to find out the strength and direction of relationship between our Y variable and the two X variables, as well as to identify their patterns and trends in order to come up with practical and effective recommendations in helping to slow down the rising housing prices.

Y variable: Average Prices of Private Domestic Premises ($/ square metre)

Reason of choosing: We identified that two groups of people, the owners and the tenants, are most affected by the prices of housing. From the owners' perspective, Y variable directly reflect the housing condition by showing the affordability to ownership.

X1 variable: Average Rents of Fresh Lettings of Private Domestic Premises ($/ square metre per month)

Reason of choosing: From the owners' perspective, X1 variable directly reflect the housing condition by showing the affordability to rental.

X2 variable: Agreements for Sale and Purchase of building units

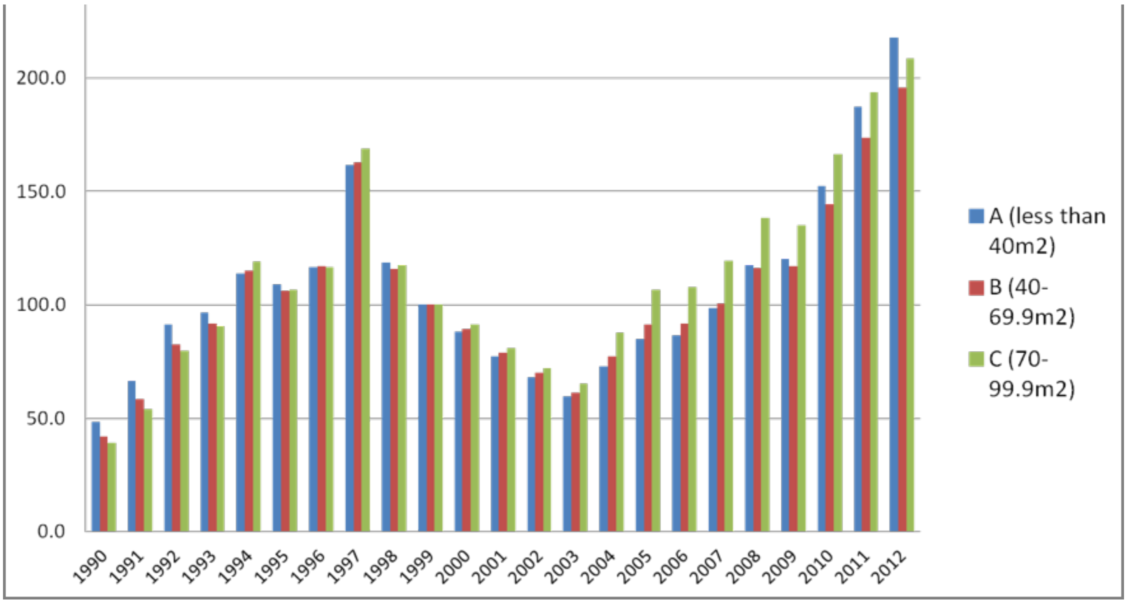
Reason of choosing: Even though the government attempted to lower the housing prices through increment in stamp duty, this action only turned out to effective on suppressing the amount of transaction volumes instead of the rising housing prices. As a result, we decided to choose the Agreements for Sale and Purchase of building units as our X2 variable.

Literature Review

This section will introduce the literature of the linkage between housing prices, rental prices and the number of transactions of the primary private residential market focusing on the period after 1997.

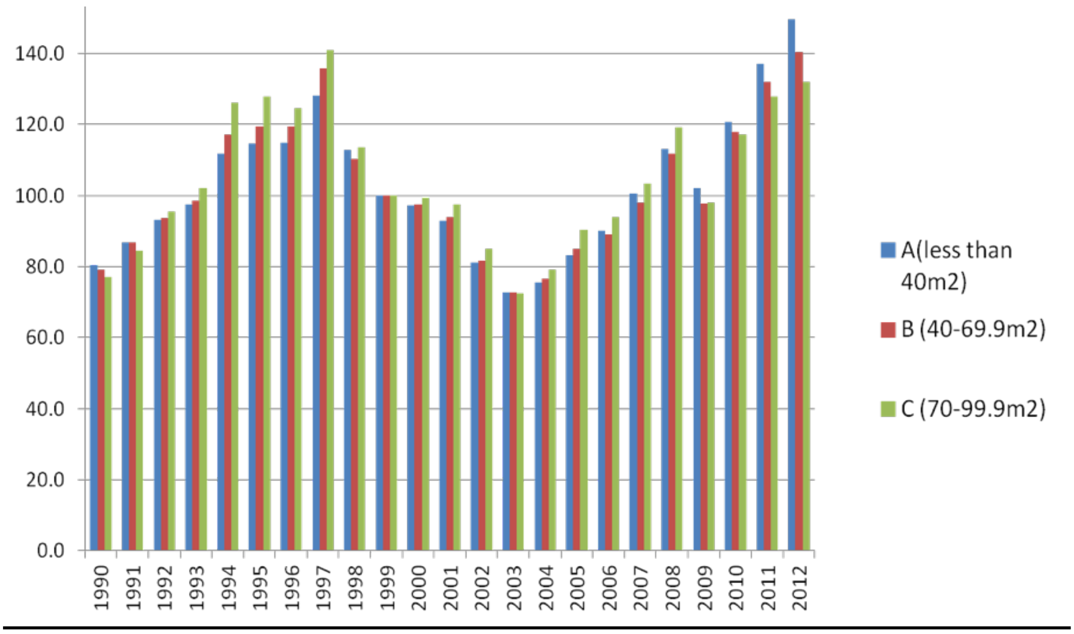
Private Housing Price

With reference to the data of Private Domestic Property Price Index (1990-2012) from the Rating and Valuation Department (shown as below), we can see that after 1997, the price had a significant drop and kept a decreasing trend from 1997-2003 (lowest). This may be caused by the bursting of housing market bubble during the Asian Financial Crisis. Starting from 2003, due to the government policies such as land auctions and Application List System, the prices gradually increased and reached the peak in 2012.



Private Housing Rental Price

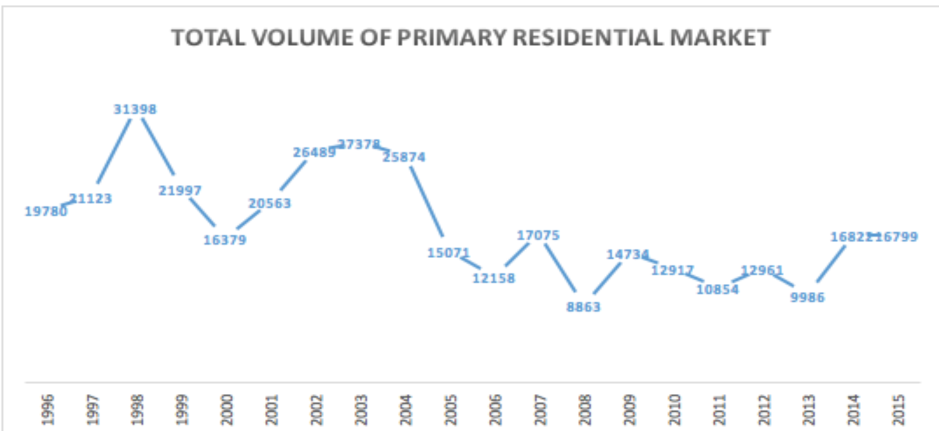
With reference to the data of Private Domestic Property Rental Index (1990-2012) from the Rating and Valuation Department (shown as below), we can see that the trend was similar to the purchase prices: significant drop in 1998, decreasing trend from 1997-2003, increasing trend from 2003-2012, peak in 2012. We believed the reasons behind were also related to the financial crisis. Based on the above, we can assume that the growth of the private housing price is approximately equals to the growth of the rental price. These two items possess a positive correlation.



Volume of Primary Residential Transactions

The figure shown below displays the registration volume of primary private residential market from 1996-2015. We generated a graph based on it for better data visualization.

			1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Volume of Overall Properties			**147,423**	**205,461**	**111,489**	**98,466**	**85,744**	**88,190**	**85,921**	**87,309**	**123,480**	**123,697**	**99,087**	**145,691**	**113,298**	**133,962**	**162,739**	**108,814**	**115,533**	**70,503**	**81,489**	**76,159**
Value of Overall Properties($100M)			**4,746.6**	**8,680.2**	**3,409.1**	**2,566.4**	**2,225.2**	**1,928.1**	**1,853.9**	**1,894.2**	**3,517.9**	**3,955.1**	**3,152.0**	**5,256.3**	**4,131.1**	**5,157.0**	**6,894.8**	**5,878.9**	**6,539.6**	**4,562.8**	**5,474.6**	**5,486.5**
Volume of Primary Residential Market	**Private**	HK	2,314	2,239	1,688	2,653	3,035	3,757	2,231	4,023	4,086	2,922	1,316	2,746	472	1,164	2,019	1,502	1,462	1,102	2,150	1,377
		KLN	2,259	3,400	6,433	4,443	3,107	4,717	9,387	6,709	7,709	4,421	2,691	4,815	2,839	5,688	3,506	2,019	3,185	2,363	2,832	4,454
		NT	15,207	15,484	23,277	14,901	10,237	12,089	14,871	16,646	14,079	7,728	8,151	9,514	5,552	7,882	7,392	7,333	8,314	6,521	11,840	10,968
		Total	**19,780**	**21,123**	**31,398**	**21,997**	**16,379**	**20,563**	**26,489**	**27,378**	**25,874**	**15,071**	**12,158**	**17,075**	**8,863**	**14,734**	**12,917**	**10,854**	**12,961**	**9,986**	**16,822**	**16,799**



Source: Midland Realty Statistics

With reference to the above statistics, we can see that the trend of the transaction volume fluctuates a lot, and we identified several peaks and troughs.

The first peak as well as the highest point was in 1997, the reason for that may be the significant increment of residential land disposal (around 120 hectares). As more private units can be built, the volume would also be higher. The year of 2000 was one of the troughs. This may because of the burst of the technology stocks and affected the purchasing intention of the public. After 2006, the fluctuations of the trend were less dramatic compared with the period before. We reckon that was owing to the high average housing prices throughout these years.

Methodology

The following forecasting models are applied to the data:

a) Naive Method

There are five types of Naive method, which are Naive Model, Naive Trend Model, Naive Rate of Change Model, Naive Seasonal Model and Naive Trend and Seasonal Model. Naive Method relies on the actual values of previous periods to forecast the values of future periods. It considers the value of the upcoming period to be identical to the value of previous period without making further adjustments. As a result, this method is simple but only suitable for time series data, and in our model all of the variables are time series data so Naive Method is still appropriate for us to do forecasting. The other four methods can be use do find out the seasonal or trend or both seasonal and trend patterns. Although Naive Method is rather simple to handle, it is still competitive when comparing with other models in some situations. As a result, we rely on Theil's U statistic to evaluate whether naive method or other forecasting techniques is more suitable in particular model.

b) Moving Average Method

Moving average is a technique to get an overall idea of the trends in a data set. It is an average of any subset of numbers and is commonly used with time series data to smooth out short-term fluctuations and highlight longer-term trends or cycles. Simple moving average method is used in this time series analysis, so that variability can be reduced and systematic patterns can be illuminated, while double moving average method can further amplify these effects. Short term moving average is sensitive to changes while long term moving average is less sensitive, and long term moving average will have a larger smoothing effect, however both of them suffer from the problem of lagging behind. The forecast result will lag behind for a longer period if the period of moving average increases, vice versa.When interpreting the result, a rising moving average indicates the price or value are rising on average and in long term it reflects a long term uptrend, while a falling moving average shows that the price or value are falling on average and in long term it implies a long term downtrend.

c) Exponential Smoothing

Exponential Smoothing Method assigns heavier weights to recent observations unlike Simple Moving Average. The weights used are determined by 3 smoothing constants, namely alpha (α), beta (β) and gamma(γ) which lies between 0 and 1, the sum of all the weights by a smoothing constant is approximately 1. Moreover, the closer the smoothing constant is to 1, the higher weight is given the recent observations and vice versa. A smoothing constant which minimizes the mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) should be chosen.

For data having no trend, Simple (Single) Exponential Smoothing is used, α is used as the smoothing constant for the level of the series. Examples show that a higher α are more responsive and can capture the trend better.

Double Exponential Smoothing is for data having linear trend. Brown's method only uses α while Holt's method uses both α and β (which acts as the smoothing constant for the trend) as the smoothing constant. Holt's method assigns different weights to actual data and trend, yet it is harder to specify 2 parameters. Whereas Winter's multiplicative and additive methods are Triple Exponential Smoothing for data having trend and seasonality. All the 3 smoothing constants are used with γ as the smoothing constant for seasonality. Winter's multiplicative method is suitable for series having multiplicative seasonality while Winter's additive method is suitable for series having additive seasonality as suggested by their names.

d) Decomposition Method

In this project, we used two types of decomposition methods, which were the additive decomposition and multiplicative decomposition method. The time series were decomposed into four components: trend, seasonal, cyclical and irregular terms.

Compared to the complex ARIMA model, decomposition model is easier to be understood by the public. Moreover, for seasonal data, decomposition methods can perform as good as ARIMA methods with high accuracy. They can also display the trend and cycle information that are not provided in the ARIMA models. Besides, as the cyclical terms are not estimated by the algorithm, people can input the assumed constant values or even use their current business figures for forecasting, and get a more fitted model for their business.

e) Regression Model

In Regression, it consists of several models and approaches such as simple regression model and multiple regression model in order to fit in different data that are with no trend, with linear trend, with quadratic trend or those with seasonality. Secondly, in regression it allows us to consider the lagged independent variables in our model, so that we can come up with forecasting result that does not consist of lagging problem. Thirdly, we can use Durbin-Watson test statistic to test for autocorrelation that may be caused by omission of important variable or incorrect functional form and the variance inflation factor to test for multicollinearity that is caused by correlation of independent variables. Due to the above reasons, using regression in forecasting helps us to suit in different types of data, eliminates lagging problem, identifies autocorrelation, recognizes multicollinearity and produce a more accurate forecasting result. The result of regression model is considered to be desirable if residuals are randomly scattered without showing any systematic patterns. We can evaluate the residuals using a scatterplot of residuals against our two X variables. If the pattern shows in the scatterplot is not random, it indicates that non-linear relationship may be needed, or non-constant variance in residuals is present.

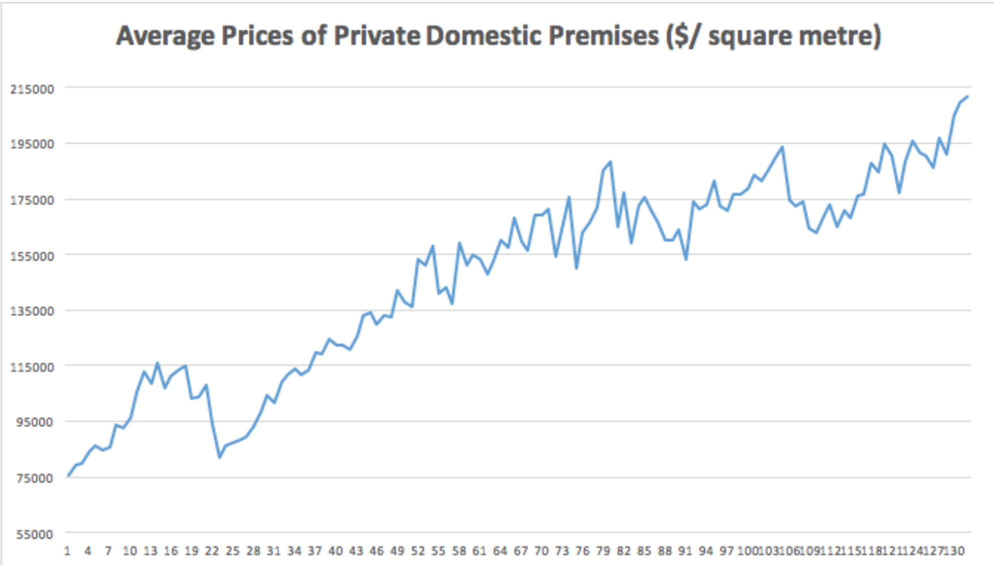
As we considered that the price of private domestic premises is greatly related and affected by both independent variables - Average Rents of Fresh Lettings of Private Domestic Premises ($/ square metre per month) and Agreements for Sale and Purchase of building units ($ million) due to the demand and supply effect. Therefore, in our regression model, we will regress Y variable using three equations: (1) Time series, (2) X1 and X2 variables, (3) Time series, Dummy variables for representing months, X1 and X2 variables so as to investigate the relationship of Y between these variables. Moreover, we will regress X1 and X2 variables with the time series respectively.

Results

Time series 1: Average Prices of Private Domestic Premise

The time series plot of the data from 2007 to 2017 is given in Figure 1, the time series exhibits a generally increasing trend across the 132 monthly period, with the mean and sample standard deviation of 146888.6705 and 35433.50632 respectively. Yet, the seasonality is not significant and there is not much extreme values.

Figure 1: Time series plot of the Average Prices of Private Domestic Premises



a) <u>Naive Method</u>

Simple Naive Method

The n - period simple naive method forecast for the time period t+m at time period t is:

$$F_{t+m} = Y_t \qquad m = 1, 2, 3,\ldots, 26$$

Naive Trend Method

The n - period naive trend method forecast the time period t+m at time period t is:

$$F_{t+m} = Y_t + (Y_t - Y_{t-1})*m \qquad m = 1, 2, 3,\ldots, 26$$

Naive Seasonal Method

The n - period naive seasonal method forecast the time period t+m at time period t is:

$$F_{t+m} = \hat{Y}_{t+m-L} \qquad m = 1, 2, 3,\ldots, 26$$

Use actual values for $\hat{Y}_{t+m-L}$ when possible; otherwise use forecasts.

Naive Trend and Seasonal Method

The n - period naive trend and seasonal method forecast the time period t+m at time period t is:

$$F_{t+m} \qquad m = 1, 2, 3,\ldots, 26$$

Use actual values for $\hat{Y}_{t+m-L,}$ $\hat{Y}_{t+m-L}$, $\hat{Y}_{t+m-1-L}$ when possible; otherwise use forecasts.

Naive Rate of Change Method

The n - period naive rate of change method forecast the time period t+m at time period t is:

$$F_{t+m} = Y_t(Y_t / Y_{t-1})^m \qquad m = 1, 2, 3,\ldots, 26$$

b) <u>Moving Average</u>

Simple Moving Average

The n - period Moving Average forecast for time period t+m made at time period t+m-1 is:

$$F_{t+m} = M_{t+m-1} = (y_{t+m-1} + y_{t+m-2} + y_{t+m-3})/n \qquad m = 1, 2, 3,\ldots, 26$$

Double Moving Average

The current level of data at time t is:

$$a_t = M_t + (M_t - M_t'') = 2M_t - M_t''$$

The 3-period double moving average a_t values of Average Prices of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	182,615	Aug 2016	183,386	May 2017	183,395
Dec 2015	179,325	Sep 2016	183,375	Jun 2017	183,396
Jan 2016	184,695	Oct 2016	183,412	Jul 2017	183,396
Feb 2016	183,886	Nov 2016	183,391	Aug 2017	183,396
Mar 2016	182,635	Dec 2016	183,393	Sep 2017	183,396
Apr 2016	183,739	Jan 2017	183,399	Oct 2017	183,396
May 2016	183,420	Feb 2017	183,394	Nov 2017	183,396
Jun 2016	183,265	Mar 2017	183,395	Dec 2017	183,396
Jul 2016	183,475	Apr 2017	183,396	Jan 2018	183,395

The 12-period double moving average a_t values of Average Prices of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	191,355	Aug 2016	180,627	May 2017	180,667
Dec 2015	183,311	Sep 2016	180,570	Jun 2017	180,662
Jan 2016	185,397	Oct 2016	180,580	Jul 2017	180,664
Feb 2016	184,650	Nov 2016	180,751	Aug 2017	180,664
Mar 2016	182,505	Dec 2016	180,634	Sep 2017	180,663
Apr 2016	182,427	Jan 2017	180,655	Oct 2017	180,664
May 2016	181,707	Feb 2017	180,680	Nov 2017	180,664
Jun 2016	180,997	Mar 2017	180,656	Dec 2017	180,664
Jul 2016	180,854	Apr 2017	180,664	Jan 2018	180,667

The slope β_1 of the series at time t is:

$$b_t = [2/ (n-1)](M_t - M_t'')$$

The 3-period double moving average b_t values of Average Prices of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	-2,059.7111	Aug 2016	-1.2208	May 2017	-0.1624
Dec 2015	-2,384.4444	Sep 2016	-13.6627	Jun 2017	-0.0107
Jan 2016	601.2074	Oct 2016	9.5154	Jul 2017	0.0613
Feb 2016	394.0099	Nov 2016	-1.7894	Aug 2017	-0.0373
Mar 2016	-463.0757	Dec 2016	-1.9789	Sep 2017	0.0044
Apr 2016	177.3805	Jan 2017	1.9157	Oct 2017	0.0095
May 2016	36.1049	Feb 2017	-0.6175	Nov 2017	-0.0078
Jun 2016	-83.1968	Mar 2017	-0.2269	Dec 2017	0.0020
Jul 2016	43.4295	Apr 2017	0.3571	Jan 2018	-0.1624

The 12-period double moving average b_t values of Average Prices of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	1,546	Aug 2016	-7	May 2017	1
Dec 2015	733	Sep 2016	-15	Jun 2017	0
Jan 2016	826	Oct 2016	-16	Jul 2017	0
Feb 2016	664	Nov 2016	16	Aug 2017	0
Mar 2016	387	Dec 2016	-5	Sep 2017	0
Apr 2016	306	Jan 2017	-2	Oct 2017	0
May 2016	182	Feb 2017	3	Nov 2017	0
Jun 2016	71	Mar 2017	-1	Dec 2017	0
Jul 2016	30	Apr 2017	0	Jan 2018	1

The n - period Double Moving Average forecast for the time period t+m made at time period t+m-1 is:

$F_{t+m} = M_{t+m-1}$" $= a_t + b_t\, m$ m = 1, 2, 3,…, 26

c) <u>Exponential Smoothing</u>

Simple Exponential Smoothing

The alpha (α) value that minimizes SSE is solved to be 0.6273 by Solver.

The forecast value for time period *t+m* made at time *t* is

$F_{t+m} = 180{,}802.0343$ $m = 1, 2, 3, \ldots, 26$

Brown's Method

The value of α to be optimal at 0.2971 using Solver.

The forecast value for time period *t+m* made at time *t* is

$F_{t+m} = 184{,}055.0215 + 466.0072*m$ $m = 1, 2, 3, \ldots, 26$

Holt's Method

Solver has found the optimal α and β value to be 0.6016 and 0.0546 respectively.

The forecast value for time period *t+m* made at time *t* is

$F_{t+m} = 181{,}893.5533 + 623.5869*m$ $m = 1, 2, 3, \ldots, 26$

Winter's Multiplicative Method

α, β and γ are found to be 0.4205, 0 and 0.6638 respectively by Solver.

Estimates of the seasonal factors (SN*t*) of the average prices of private domestic premises.

January	1.0102	*May*	0.9764	*September*	0.9589

February	0.9990	*June*	1.0018	*October*	0.9842
March	0.9513	*July*	0.9661	*November*	0.9444
April	0.9700	*August*	1.0120	*December*	0.9769

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = (188{,}241.9443 + 1{,}204.5819*m)*SN_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

Winter's Additive Method

According to Solver, the optimal α, β and γ are 0.4597, 0 and 0.5642 respectively.

Estimates of the seasonal factors (S_t) of the average prices of private domestic premises.

January	4,747.8862	*May*	-1,737.6285	*September*	-4,614.8523
February	2,180.0150	*June*	1,869.7352	*October*	319.4344
March	-5,590.7234	*July*	-3,507.7264	*November*	-4,936.2105
April	-2,080.5098	*August*	2,939.5931	*December*	331.7040

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 184{,}896.3406 + 1{,}204.5819*m + S_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

d) <u>Decomposition Method</u>

Additive Decomposition Method

Estimates of the seasonal factors (Sn_t) of the average prices of private domestic premises are shown below.

January	468.718192	*May*	-135.1300223	*September*	135.193192
February	342.5140253	*June*	3704.044978	*October*	2135.655692
March	-4214.904725	*July*	-2042.533891	*November*	-1160.113058
April	-203.1120164	*August*	2647.637984	*December*	-1677.97035

The estimated trend line is

$$\text{Tr} = 83119.21961 + 1023.467138*\text{t} \qquad t = 1, 2, 3, \ldots, 26$$

Multiplicative Decomposition Method

Estimates of the seasonal factors (Sn_t) of the average prices of private domestic premises are shown below.

January	1.001646634	*May*	1.002889979	*September*	1.004590426
February	1.001846058	*June*	1.029874746	*October*	1.011251406
March	0.971861225	*July*	0.982808897	*November*	0.986615986
April	1.000158636	*August*	1.018446177	*December*	0.98800983

The estimated trend line is

Tr = 83124.51476 + 1023.137639*t *t* = 1, 2, 3, …, 26

<u>e) Regression</u>

Estimates of the seasonal factors (Qn_t) of the average prices of private domestic premises are shown below.

January	2104	*May*	2685	*September*	3536
February	2410	*June*	4903	*October*	3027
March	-1563	*July*	1008	*November*	648.0259
April	2322	*August*	5470	*T*	1011
Intercept	80574				

The regression equation for **Y on time and dummy variables** is

Y = 80574 + 1011*t + 2104*Q1 +2410*Q2 -1563*Q3 +2322*Q4 + 2685*Q5 + 4903*Q6 +1008*Q7 + 5470*Q8 + 3536*Q9 + 3027*Q10 + 648.0259*Q11 *t* = 1, 2, 3, …, 26

Estimates of the independent factors (X1 and X2) of the average prices of private domestic premises are shown below.

Intercept	102606	*X1*	79.1372	*X2*	0.0411

The regression equation for **Y on X1 and X2 variables** is

$$Y = 102606 + 79.1372*X1 + 0.0411*X2 \qquad t = 1, 2, 3, \ldots, 26$$

Estimates of the seasonal factors (Qn_t) of the average prices of private domestic premises are shown below.

January	1613	*May*	1945	*September*	1936
February	1941	*June*	3641	*October*	1430
March	-1712	*July*	-1010	*November*	-504.9944
April	1967	*August*	2908	*T*	887.8747
Intercept	60002	*X1*	76.0484	*X2*	0.0818

The regression equation for **Y on time, dummy, X1 and X2 variables** is

$$Y = 60002 + 887.8747*t + 1613*Q1 + 1941*Q2 - 1712*Q3 + 1967*Q4 + 1945*Q5 + 3641*Q6 - 1010*Q7 + 2908*Q8 + 1936*Q9 + 1430*Q10 - 887.8747*Q11 + 76.0484*X1 + 0.0818*X2 \qquad t = 1, 2, 3, \ldots, 26$$

Evaluation of Forecasting Models

106 within sample and 26 beyond sample residuals are calculated by the forecasting models using the equations and formulae included in the previous section. The corresponding MSE, MAE and MAPE for the within sample residuals are as follows.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	68109521.81	6133.02	4.41%	7151499790	1.000000
Naive Trend Method	68109521.81	6133.017	4.41%	7151499790	1.013473
Naive Seasonal Method	393013394.6	17138.12	12.87 %	3694325909 3	2.671953
Naive Trend and Seasonal Method	336611611.2	15889.08	11.83 %	3130487984 5	2.438013
Naive Rate of Change Method	179727785.6	9939.80	6.98%	1869168970 2	1.581092
Simple Moving Average (n=3)	65574190.94	6513.26	4.81%	6754141667. 67	1.052582747
Simple Moving Average (n=12)	144049754.37	10422.56	7.82%	1354067691 1.48	1.641370422
Double Moving Average (n=3)	87433411.18	6823.75	4.94%	8830774530. 16	1.127146158
Double Moving Average (n=12)	111903031.82	8575.69	6.31%	9287951641. 82	1.42403184
Simple Exponential Smoothing	59357670.41	5913.80	4.32%	6232555393	0.977380
Brown's Method	62920989.21	5971.87	4.44%	6606703867	1.011503
Holt's Method	60535692.73	5755.34	4.25%	6356247736	0.982684
Winter's Multiplicative Method	68399174.87	9067.45	6.81%	1285904486 9	1.549508
Winter's Additive Method	61720493.12	8506.21	6.44%	1285904486 9	1.465022

Additive Decomposition	108734383.20	8375.01	6.59%	11525844622	1.540352
Multiplicative Decomposition	109047395.80	8390.40	6.60%	11559023958	1.541099
Regression (Y on T)	48,939,871.68	5,282.83	3.86%	5,187,626,398.32	0.875373041
Regression (Y on X1 & X2)	88,502,384.85	6,480.73	4.91%	9,381,252,794.29	0.980094429
Regression (Y on All)	51196598.24	5454.18	3.94%	5426839413.52	1.094756869

From the table above, **Regression (Y on All)** performs the best with the smallest MSE, MAE and MAPE, following by **Holt's Method** which has similar performance while **Naive Seasonal Method** performs the worst with the largest MSE, MAE and MAPE within the sample.

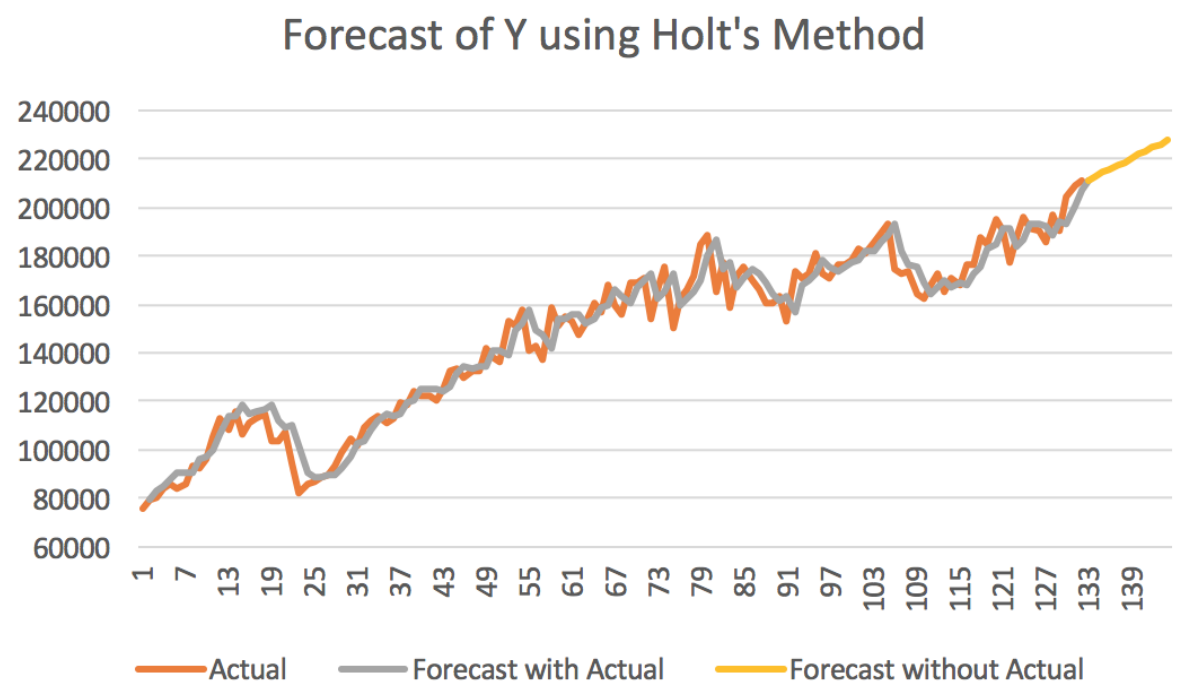
The MSE, MAE and MAPE for the beyond sample residuals computed by the forecasting models are shown in the table below.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	267139706.30	12983.53	6.78%	6945632365.00	2.151670
Naive Trend Method	27580451073.00	156015.11	83.73%	717092000000.00	22.772691
Naive Seasonal Method	266290667.80	13344.43	7.11%	6923557362.00	2.186295
Naive Trend and Seasonal Method	251288584.10	12820.11	6.85%	6533503185.00	2.120887
Naive Rate of Change Method	18783711237.00	126086.69	67.15%	488376000000.00	18.540642
Simple Moving Average (n=3)	188842645.53	11915.84	6.50%	4909908783.67	1.868417515
Simple Moving Average (n=12)	194868160.57	12056.17	6.48%	5066572174.95	1.879175307

Double Moving Average (n=3)	651767394.30	21035.81	11.03%	16945952251.91	3.348988697
Double Moving Average (n=12)	529097339.69	22058.08	12.28%	13756530831.99	3.256961831
Simple Exponential Smoothing	194061093.50	12010.31	6.46%	5045588431	1.875352
Brown's Method	159896217.20	10575.42	5.98%	4157301646	1.815666
Holt's Method	140204471.80	9975.21	5.83%	3645316266	1.706870
Winter's Multiplicative Method	349565443.30	16677.51	9.37%	9088701525	2.693137
Winter's Additive Method	354812792.20	16664.75	9.39%	9225132597	2.697728
Addictive Decomposition	537440545.10	21785.65	12.20%	13973454172	3.301743
Multiplicative Decomposition	541397623.10	21672.71	12.14%	14076338199	3.320528
Regression (Y on T)	368,599,599.6908730	17,575.6426923	9.8290558833355%	9,583,589,591.9627000	2.743619604
Regression (Y on X1 & X2)	529740089.76	18032.90	9.33%	13773242333.87	3.025809059
Regression (Y on All)	367830167.07	17538.55	9.80%	9563584343.91	2.747181566

From the table above, **Holt's Method** performs the best with the smallest MSE, MAE and MAPE while **Naive Trend Method** performs the worst with the largest MSE, MAE and MAPE beyond the sample. **Regression (Y on All)** appear not to have a good performance beyond the samples.

The plot of the data and fitted values generated by Holt's Method for the periods are given below.



The fitted value for time period *t+m* made at time *t* is

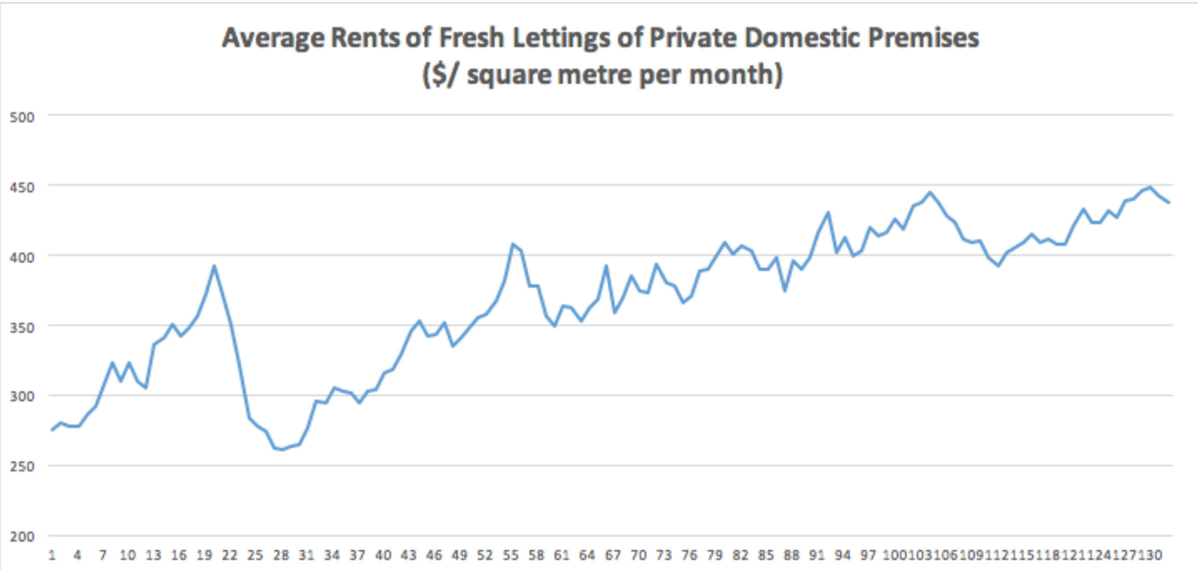
$$F_{t+m} = 209{,}959.4288 + 1{,}500.7737 * m \qquad m = 1, 2, 3, \ldots, 12$$

2018	*Fitted Values*		*Fitted Values*
January	211,460.20	*July*	220,464.84
February	212,960.98	*August*	221,965.62
March	214,461.75	*September*	223,466.39
April	215,962.52	*October*	224,967.17
May	217,463.30	*November*	226,467.94
June	218,964.07	*December*	227,968.71

Time series 2: Average Rents of Fresh Lettings of Private Domestic Premises

The time series plot of the data from 2007 to 2017 is given in Figure 2, the time series exhibits a generally increasing trend across the 132 monthly period, with the mean and sample standard deviation of 368.5181818 and 50.78062956 respectively. This X1 variable shows an increasing trend throughout the years and it does not seem to exhibit any seasonal trends. There is no obvious extreme or missing values but the private domestic premises experienced a drastic decrease in average rents from around period 20 to period 24.

Figure 2: Time series plot of the Average Rents of Fresh Lettings of Private Domestic Premises



a) <u>Naive Method</u>

Simple Naive Method

The n - period simple naive method forecast for the time period t+m at time period t is:

$$F_{t+m} = Y_t \qquad m = 1, 2, 3, \ldots, 26$$

Naive Trend Method

The n - period naive trend method forecast the time period t+m at time period t is:

$$F_{t+m} = Y_t + (Y_t - Y_{t-1})*m \qquad m = 1, 2, 3, \ldots, 26$$

Naive Seasonal Method

The n - period naive seasonal method forecast the time period t+m at time period t is:

$F_{t+m} = \hat{Y}_{t+m-L}$ $\quad\quad m = 1, 2, 3,\ldots, 26$

Use actual values for $\hat{Y}_{t+m-L}$ when possible; otherwise use forecasts.

Naive Trend and Seasonal Method

The n - period naive trend and seasonal method forecast the time period t+m at time period t is:

F_{t+m} $\quad\quad m = 1, 2, 3,\ldots, 26$

Use actual values for $\hat{Y}_{t+m-L,}$ $\hat{Y}_{t+m-L}$, $\hat{Y}_{t+m-1-L}$ when possible; otherwise use forecasts.

Naive Rate of Change Method

The n - period naive rate of change method forecast the time period t+m at time period t is:

$F_{t+m} = Y_t(Y_t / Y_{t-1})^m$ $\quad\quad m = 1, 2, 3,\ldots, 26$

b. <u>Moving Average</u>

Simple Moving Average

The n - period Moving Average forecast for time period t+m made at time period t+m-1 is:

$F_{t+m} = M_{t+m-1} = (y_{t+m-1} + y_{t+m-2} + y_{t+m-3})/n$ where m = 1, 2, 3, …. is the forecast horizon

Double Moving Average

The current level of data at time t is:

$a_t = M_t + (M_t - M_t'') = 2M_t - M_t''$

The 3-period double moving average a_t values of Average Rents of Fresh Lettings of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	19	Aug 2016	19	May 2017	19
Dec 2015	26	Sep 2016	19	Jun 2017	19
Jan 2016	17	Oct 2016	19	Jul 2017	19
Feb 2016	18	Nov 2016	19	Aug 2017	19
Mar 2016	20	Dec 2016	19	Sep 2017	19
Apr 2016	18	Jan 2017	19	Oct 2017	19
May 2016	19	Feb 2017	19	Nov 2017	19
Jun 2016	19	Mar 2017	19	Dec 2017	19
Jul 2016	19	Apr 2017	19	Jan 2018	19

The 12-period double moving average a_t values of Average Rents of Fresh Lettings of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	17	Aug 2016	19	May 2017	18
Dec 2015	27	Sep 2016	19	Jun 2017	18
Jan 2016	19	Oct 2016	18	Jul 2017	18
Feb 2016	21	Nov 2016	18	Aug 2017	18
Mar 2016	22	Dec 2016	19	Sep 2017	18
Apr 2016	20	Jan 2017	18	Oct 2017	18
May 2016	20	Feb 2017	18	Nov 2017	18
Jun 2016	20	Mar 2017	18	Dec 2017	18
Jul 2016	19	Apr 2017	18	Jan 2018	18

The slope β_1 of the series at time t is:

$$b_t = [2/ (n-1)](M_t - M_t'')$$

The 3-period double moving average b_t values of Average Rents of Fresh Lettings of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	2.740741	Aug 2016	-0.002006	May 2017	0.000259
Dec 2015	4.024691	Sep 2016	0.024828	Jun 2017	0.000035
Jan 2016	-0.855967	Oct 2016	-0.015883	Jul 2017	-0.000110
Feb 2016	-0.770919	Nov 2016	0.002313	Aug 2017	0.000061
Mar 2016	0.799268	Dec 2016	0.003753	Sep 2017	-0.000004
Apr 2016	-0.275873	Jan 2017	-0.003273	Oct 2017	-0.000018
May 2016	-0.082508	Feb 2017	0.000931	Nov 2017	0.000013
Jun 2016	0.146963	Mar 2017	0.000470	Dec 2017	-0.000003
Jul 2016	-0.070473	Apr 2017	-0.000624	Jan 2018	0.000259

The 12-period double moving average b_t values of Average Rents of Fresh Lettings of Private Domestic Premises from November 2015 to January 2018:

Nov 2015	0.177189	Aug 2016	0.181317	May 2017	-0.000565
Dec 2015	1.054433	Sep 2016	0.073624	Jun 2017	0.000684
Jan 2016	0.251450	Oct 2016	0.004957	Jul 2017	-0.000268
Feb 2016	0.414812	Nov 2016	-0.027846	Aug 2017	-0.000050
Mar 2016	0.495282	Dec 2016	0.016912	Sep 2017	0.000122
Apr 2016	0.288696	Jan 2017	-0.001993	Oct 2017	-0.000065
May 2016	0.283435	Feb 2017	-0.004309	Nov 2017	0.000003
Jun 2016	0.261949	Mar 2017	0.003537	Dec 2017	0.000020
Jul 2016	0.195535	Apr 2017	-0.000922	Jan 2018	-0.000565

The n - period Double Moving Average forecast for the time period t+m made at time period t+m-1 is:

$$F_{t+m} = M_{t+m-1}" = a_t + b_t m \qquad m = 1, 2, 3, \ldots, 26$$

c. Exponential Smoothing

Simple Exponential Smoothing

Solver has generated the optimal α value to be 1.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 428.8000 \qquad m = 1, 2, 3, \ldots, 26$$

Brown's Method

The optimal value for α is found to be around 0.7761 using Solver.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 429.1919 + (-0.7751)*m \qquad m = 1, 2, 3, \ldots, 26$$

Holt's Method

Solver has found the optimal α and β value to be 1 and 0.0050 respectively.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 428.8000 + 2.4795*m \qquad m = 1, 2, 3, \ldots, 26$$

Winter's Multiplicative Method

α, β and γ has the optimal value of 0.9543, 0.0250 and 0.8330 as found by Solver.

Estimates of the seasonal factors (SN_t) of the Average Rents of Fresh Lettings of Private Domestic Premises.

January	1.3121	*May*	1.3101	*September*	1.3504
February	1.3383	*June*	1.3383	*October*	1.3888
March	1.2597	*July*	1.4009	*November*	1.3538
April	1.3330	*August*	1.4472	*December*	1.3114

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = (309.4626 + 0.3309*m)*SN_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

Winter's Additive Method

According to Solver, the optimal α, β and γ are 0.8561, 0.0187 and 0.8037 respectively.

Estimates of the seasonal factors (S_t) of Average Rents of Fresh Lettings of Private Domestic Premises

January	93.6182	*May*	93.0182	*September*	92.4254
February	101.4182	*June*	101.4182	*October*	111.4207
March	78.0182	*July*	88.0121	*November*	106.0182
April	99.8182	*August*	112.3259	*December*	93.4182

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 343.9298 + 1.7748*m + S_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

d) <u>Decomposition Method</u>

Additive Decomposition Method

Estimates of the seasonal factors (Sn_t) of the Average Rents of Fresh Lettings of Private Domestic Premises are shown below.

January	-7.541542659	*May*	-4.822197421	*September*	8.505332341
February	-6.773834325	*June*	3.773040675	*October*	8.288665675
March	-12.59570933	*July*	11.29283234	*November*	-2.565500992
April	-7.712375992	*August*	21.37304067	*December*	-11.22175099

The estimated trend line is

$$Tr = 285.9931326 + 1.302229005*t \qquad t = 1, 2, 3, \ldots, 26$$

Multiplicative Decomposition Method

Estimates of the seasonal factors (Sn_t) of the Average Rents of Fresh Lettings of Private Domestic Premises are shown below.

January	0.977341486	*May*	0.985397737	*September*	1.02569527
February	0.980031767	*June*	1.009064433	*October*	1.025338443
March	0.964431149	*July*	1.031827592	*November*	0.994127604
April	0.977160759	*August*	1.061901936	*December*	0.967681822

The estimated trend line is

$$Tr = 285.8906661 + 1.302930565*t \qquad t = 1, 2, 3, \ldots, 26$$

e) Regression

Estimates of the seasonal factors (Qn_t) of the Average Rents of Fresh Lettings of Private Domestic Premises are shown below.

January	3.7705	*May*	6.4849	*September*	20.4285
February	4.9235	*June*	15.2175	*October*	19.2507
March	-0.7732	*July*	22.9895	*November*	8.5434
April	3.4584	*August*	32.7117	*T*	1.3063
Intercept	273.8434				

The regression equation for X1 on time and dummy variables is

$$X1 = 273.8434 + 1.3063*t + 3.7705*Q1 + 4.9235*Q2 - 0.7732*Q3 + 3.4584*Q4 + 6.4849*Q5 + 15.2175*Q6 + 22.9895*Q7 + 32.7117*Q8 + 20.4285*Q9 + 19.2507*Q10 + 8.5434*Q11 \qquad t = 1, 2, 3, \ldots, 26$$

Evaluation of Forecasting Models (X1 variable)

106 within sample and 26 beyond sample residuals are calculated by the forecasting models using the equations and formulae included in the previous section. The corresponding MSE, MAE and MAPE for the within sample residuals are as follows.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	174.87	10.61	2.98%	18360.92	1.000000
Naive Trend Method	174.87	10.61	2.98%	18360.92	0.999670
Naive Seasonal Method	1900.41	35.72	10.55%	178638.64	3.773851
Naive Trend and Seasonal Method	1603.72	32.80	9.68%	149145.72	3.453836
Naive Rate of Change Method	344.51	14.69	4.10%	35829.16	1.359946
Simple Moving Average (n=3)	291.14	12.89	3.71%	29987.54	1.337527
Simple Moving Average (n=12)	800.91	22.27	6.59%	75285.14	2.449634
Double Moving Average (n=3)	274.94	13.30	3.80%	27768.98	1.244023
Double Moving Average (n=12)	857.94	20.75	6.42%	71208.67	2.796392
Simple Exponential Smoothing	174.87	10.61	2.98%	18360.92	1.000000
Brown's Method	173.45	10.47	2.94%	18212.19	0.991812
Holt's Method	175.44	10.32	2.92%	18421.61	0.994129
Winter's Multiplicative Method	178.15	14.44	4.05%	33487.56	1.377077

Winter's Additive Method	358.35	20.33	5.71%	67364.47	2.032860
Addictive Decomposition	563.94	16.62	5.05%	59778.08	2.068655
Multiplicative Decomposition	570.30	16.91	5.13%	60451.07	2.073737
Regression	112.09	8.01	2.28%	11881.89	0.811796

From the table above, **Regression** performs the best with the smallest MSE, MAE and MAPE while **Naive Seasonal Method** performs the worst with the largest MSE, MAE and MAPE within the sample. However, Theil's U statistics for both of the model exceeded 1, meaning that the model is worse than the naive one.

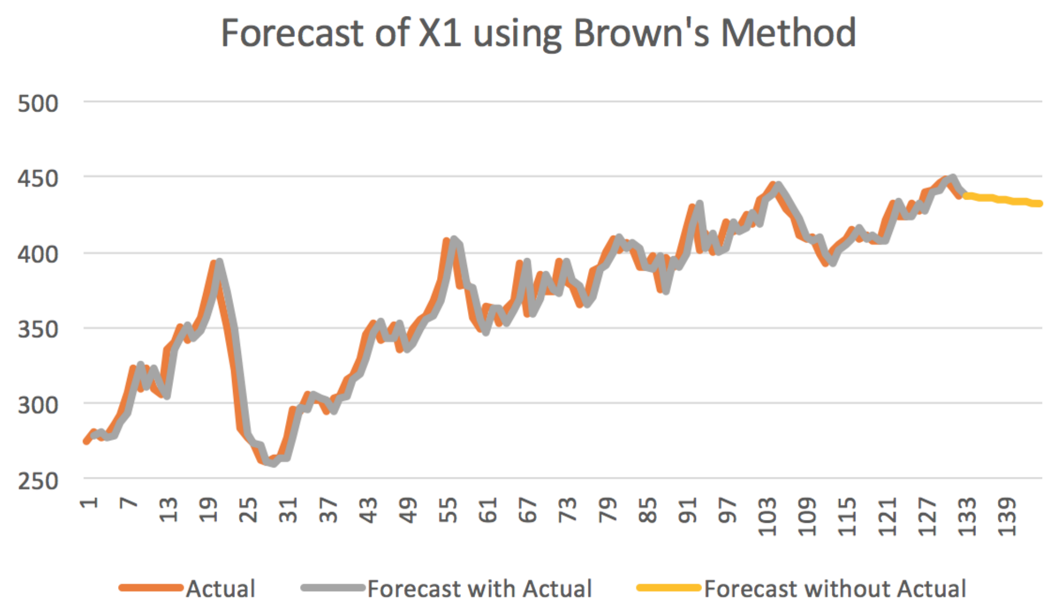
The MSE, MAE and MAPE for the beyond sample residuals computed by the forecasting models are shown in the table below.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	309.97	15.35	3.71%	8059.12	2.567067
Naive Trend Method	18471.14	110.40	25.69%	717092000000	18.838670
Naive Seasonal Method	395.28	16.16	3.86%	10277.16	2.765944
Naive Trend and Seasonal Method	387.77	16.00	3.82%	10082.00	2.748395
Naive Rate of Change Method	12211.39	90.68	21.12%	317496.26	15.340240
Simple Moving Average (n=3)	435.96	17.85	4.34%	11,335.00	3.045747
Simple Moving Average (n=12)	293.35	14.81	3.57%	7,627.15	2.494218
Double Moving Average (n=3)	746.44	23.21	5.45%	19407.53	3.795093
Double Moving Average (n=12)	1970.078	43.41	10.36%	51222.04	6.365981

Simple Exponential Smoothing	309.97	15.35	3.71%	8059.12	2.567067
Brown's Method	424.65	18.14	4.29%	11040.80	0.042893
Holt's Method	1863.06	41.87	9.98%	48439.48	6.177607
Winter's Multiplicative Method	284.69	12.78	3.05%	7401.84	2.424645
Winter's Additive Method	2318.60	46.47	11.10%	60283.69	6.856175
Addictive Decomposition	566.65	20.73	4.98%	14732.88	3.458894
Multiplicative Decomposition	612.14	20.66	4.95%	15915.64	3.585680
Regression	525.42	19.85	4.76%	13660.85	3.458044

From the table above, **Winter's Multiplicative Method** performs the best with the smallest MSE, MAE and MAPE while **Naive Trend Method** performs the worst with the largest MSE, MAE and MAPE beyond the sample. Yet, **Brown's Method** seems to be a better model as its Theil's U statistics is the smallest among the models.

The plot of the data and fitted values generated by Brown's Method for the periods are given below.



The fitted value for time period $t+m$ made at time t is

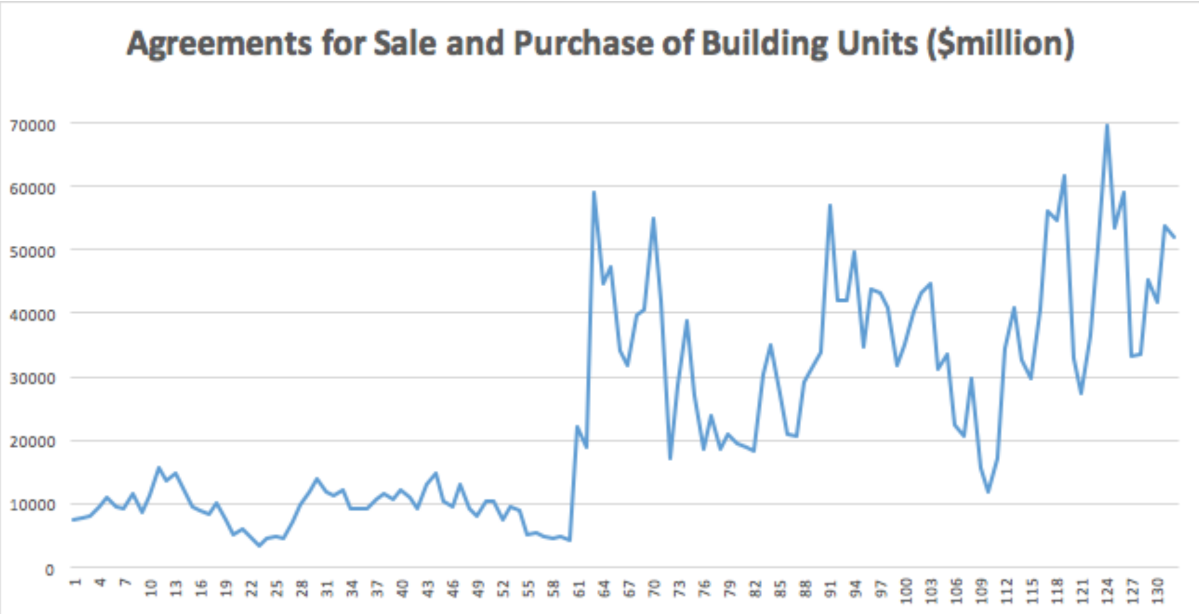
$$F_{t+m} = 438.1169 + (-0.4842)*m \qquad m = 1, 2, 3, \ldots, 12$$

2018	*Fitted Values*		*Fitted Values*
January	437.63	*July*	434.73
February	437.15	*August*	434.24
March	436.66	*September*	433.76
April	436.18	*October*	433.27
May	435.70	*November*	432.79
June	435.21	*December*	432.31

Time series 3: Agreements for Sale and Purchase of Building Units

The time series plot of the data from 2007 to 2017 is given in Figure 3, the time series exhibits a relatively steady trend at the beginning, following with a surge in number at around period 60. After that, the number of agreements fluctuates with a gradual increasing trend until period 132. The mean and sample standard deviation is 23790.77273 and 16517.02066 respectively.

Figure 3: Time series plot of the Agreements for Sale and Purchase of Building Units



a) Naive Method

Simple Naive Method

The n - period simple naive method forecast for the time period t+m at time period t is:

$$F_{t+m} = Y_t \qquad m = 1, 2, 3, \ldots, 26$$

Naive Trend Method

The n - period naive trend method forecast the time period t+m at time period t is:

$$F_{t+m} = Y_t + (Y_t - Y_{t-1})*m$$

Naive Seasonal Method

The n - period naive seasonal method forecast the time period t+m at time period t is:

$F_{t+m} = \hat{Y}_{t+m-L}$ $\qquad$ $m = 1, 2, 3,\ldots, 26$

Use actual values for $\hat{Y}_{t+m-L}$ when possible; otherwise use forecasts.

Naive Trend and Seasonal Method

The n - period naive trend and seasonal method forecast the time period t+m at time period t is:

F_{t+m} $\qquad$ $m = 1, 2, 3,\ldots, 26$

Use actual values for $\hat{Y}_{t+m-L},$ $\hat{Y}_{t+m-L}$, $\hat{Y}_{t+m-1-L}$ when possible; otherwise use forecasts.

Naive Rate of Change Method

The n - period naive rate of change method forecast the time period t+m at time period t is:

$F_{t+m} = Y_t(Y_t / Y_{t-1})^m$ $\qquad$ $m = 1, 2, 3,\ldots, 26$

b) <u>Moving Average</u>

Simple Moving Average

The n - period Moving Average forecast for time period t+m made at time period t+m-1 is:

$F_{t+m} = M_{t+m-1} = (y_{t+m-1} + y_{t+m-2} + y_{t+m-3})/n$ $\qquad$ $m = 1, 2, 3,\ldots, 26$

Double Moving Average

The current level of data at time t is:

$a_t = M_t + (M_t - M_t") = 2M_t - M_t"$

The 3-period double moving average a_t values of Agreements for Sale and Purchase of building units from November 2015 to January 2018:

Nov 2015	19	Aug 2016	19	May 2017	19
Dec 2015	26	Sep 2016	19	Jun 2017	19
Jan 2016	17	Oct 2016	19	Jul 2017	19
Feb 2016	18	Nov 2016	19	Aug 2017	19
Mar 2016	20	Dec 2016	19	Sep 2017	19
Apr 2016	18	Jan 2017	19	Oct 2017	19
May 2016	19	Feb 2017	19	Nov 2017	19
Jun 2016	19	Mar 2017	19	Dec 2017	19
Jul 2016	19	Apr 2017	19	Jan 2018	19

The 12-period double moving average a_t values of Agreements for Sale and Purchase of building units from November 2015 to January 2018:

Nov 2015	17	Aug 2016	19	May 2017	18
Dec 2015	27	Sep 2016	19	Jun 2017	18
Jan 2016	19	Oct 2016	18	Jul 2017	18
Feb 2016	21	Nov 2016	18	Aug 2017	18
Mar 2016	22	Dec 2016	19	Sep 2017	18
Apr 2016	20	Jan 2017	18	Oct 2017	18
May 2016	20	Feb 2017	18	Nov 2017	18
Jun 2016	20	Mar 2017	18	Dec 2017	18
Jul 2016	19	Apr 2017	18	Jan 2018	18

The slope β_1 of the series at time t is:

$b_t = [2/ (n-1)](M_t - M_t'')$

The 3-period double moving average b_t values of Agreements for Sale and Purchase of building units from November 2015 to January 2018:

Nov 2015	2.740741	Aug 2016	-0.002006	May 2017	0.000259
Dec 2015	4.024691	Sep 2016	0.024828	Jun 2017	0.000035
Jan 2016	-0.855967	Oct 2016	-0.015883	Jul 2017	-0.000110
Feb 2016	-0.770919	Nov 2016	0.002313	Aug 2017	0.000061
Mar 2016	0.799268	Dec 2016	0.003753	Sep 2017	-0.000004
Apr 2016	-0.275873	Jan 2017	-0.003273	Oct 2017	-0.000018
May 2016	-0.082508	Feb 2017	0.000931	Nov 2017	0.000013
Jun 2016	0.146963	Mar 2017	0.000470	Dec 2017	-0.000003
Jul 2016	-0.070473	Apr 2017	-0.000624	Jan 2018	0.000259

The 12-period double moving average b_t values of Agreements for Sale and Purchase of building units from November 2015 to January 2018:

Nov 2015	0.177189	Aug 2016	0.181317	May 2017	-0.000565
Dec 2015	1.054433	Sep 2016	0.073624	Jun 2017	0.000684
Jan 2016	0.251450	Oct 2016	0.004957	Jul 2017	-0.000268
Feb 2016	0.414812	Nov 2016	-0.027846	Aug 2017	-0.000050
Mar 2016	0.495282	Dec 2016	0.016912	Sep 2017	0.000122
Apr 2016	0.288696	Jan 2017	-0.001993	Oct 2017	-0.000065
May 2016	0.283435	Feb 2017	-0.004309	Nov 2017	0.000003
Jun 2016	0.261949	Mar 2017	0.003537	Dec 2017	0.000020
Jul 2016	0.195535	Apr 2017	-0.000922	Jan 2018	-0.000565

The n - period Double Moving Average forecast for the time period t+m made at time period t+m-1 is:

$$F_{t+m} = M_{t+m-1}'' = a_t + b_t\, m \qquad m = 1, 2, 3, \ldots, 26$$

c) **<u>Exponential Smoothing</u>**

Simple Exponential Smoothing

The optimal α value is solved to be 0.5815 using Solver.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 34{,}602.5850 \qquad m = 1, 2, 3, \ldots, 26$$

Brown's Method

The α value that minimizes SSE is around 0.4930 as suggested by Solver.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 24{,}842.0626 + (-64.5831)*m \qquad m = 1, 2, 3, \ldots, 26$$

Holt's Method

Solver has found the optimal α and β value to be 0.7047 and 0.0059 respectively.

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = 26{,}137.0343 + 534.8569*m \qquad m = 1, 2, 3, \ldots, 26$$

Winter's Multiplicative Method

The optimal values for α, β and γ are found to be 0.4788, 0 and 0 using Solver.

Estimates of the seasonal factors (SN_t) of the Agreements for Sale and Purchase of Building Units.

January	0.7268	*May*	1.0789	*September*	0.8500
February	0.7578	*June*	0.9374	*October*	1.0945
March	0.7806	*July*	0.8922	*November*	1.5303
April	0.9254	*August*	1.1148	*December*	1.3112

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = [29,647.8679 + (-191.9722)*m]*SN_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

Winter's Additive Method

According to Solver, the optimal α, β and γ are 0, 0.0006 and 0.6667 respectively.

Estimates of the seasonal factors (St) of the Agreements for Sale and Purchase of Building Units..

January	26,579.8649	*May*	34,046.4862	*September*	46,223.7536
February	24,787.4188	*June*	49,489.3828	*October*	38,281.0951
March	30,807.0037	*July*	40,538.7076	*November*	33,929.3452
April	31,433.4188	*August*	39,711.4367	*December*	34,134.0589

The forecast value for time period $t+m$ made at time t is

$$F_{t+m} = (-7,747.4722) + (-191.9722)*m + S_{t-L+m} \qquad m = 1, 2, 3, \ldots, 26$$

d) <u>Decomposition Method</u>

Additive Decomposition Method

Estimates of the seasonal factors (Sn_t) of the Agreements for Sale and Purchase of Building Units are shown below.

January	-238.7534722	*May*	1816/684722	*September*	-961.6753472
February	-608.5659722	*June*	-645.1604663	*October*	1091.090278
March	1360.684028	*July*	1292.980903	*November*	-318.6753472
April	-110.1545139	*August*	29.17361111	*December*	-2707.628472

The estimated trend line is

$$Tr = 1583.298457 + 341.1246799*t \qquad t = 1, 2, 3, \ldots, 26$$

Multiplicative Decomposition Method

Estimates of the seasonal factors (Sn_t) of the Agreements for Sale and Purchase of Building Units are shown below.

January	0.987517084	*May*	1.122578003	*September*	0.916426669
February	0.982989036	*June*	1.07634785	*October*	0.934439269
March	1.084330524	*July*	1.039760879	*November*	0.970114483
April	1.021638566	*August*	0.993437751	*December*	0.870419886

The estimated trend line is

Tr = 1504.765294 + 343.2082052*t $\qquad$ t = 1, 2, 3, …, 26

e) <u>Regression</u>

Estimates of the seasonal factors (Qn_t) of the Agreements for Sale and Purchase of Building Units are shown below.

January	2925	*May*	5248	*September*	2214
February	2732	*June*	3480	*October*	2795
March	4518	*July*	5388	*November*	1916
April	3316	*August*	2819	*T*	315.6598
Intercept	-389.7720				

The regression equation for X2 on time and dummy variables is

X2 = -389.7720 + 315.6598*t + 2925*Q1 + 2732*Q2 + 4518*Q3 + 3316*Q4 + 5248*Q5 + 3480*Q6 + 5388*Q7 + 2819*Q8 + 2214*Q9 + 2795*Q10 + 1916*Q11

t = 1, 2, 3, …, 26

Evaluation of Forecasting Models (X2 variable)

106 within sample and 26 beyond sample residuals are calculated by the forecasting models using the equations and formulae included in the previous section. The corresponding MSE, MAE and MAPE for the within sample residuals are as follows.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	57503334.90	4517.64	21.54%	6037850164.00	1.000000
Naive Trend Method	57503334.90	4517.64	21.54%	6037850164.00	1.002919
Naive Seasonal Method	281327363.20	11983.81	108.46%	25705856655.00	1.679726
Naive Trend and Seaonal Method	249918949.70	11213.05	81.05%	22542189598.00	1.590054
Naive Rate of Change Method	392231497.50	8894.85	39.27%	40605807470.00	1.475268
Simple Moving Average (n=3)	64195531.76	4734.63	23.62%	6612139770.89	1.026034267
Simple Moving Average (n=12)	119901878.27	7286.85	39.12%	11270776557.26	1.180282239
Double Moving Average (n=3)	89641055.46	5846.72	26.48%	9053746601.44	1.036316316
Double Moving Average (n=12)	160136937.39	8,413.58	37.44%	13291365803.44	1.247971894
Simple Exponential Smoothing	79200156.94	5470.08	27.66%	8316016479	1.079212
Brown's Method	55087516.89	4403.50	21.45%	5784189273	0.994060
Holt's Method	54121098.23	4412.84	24.35%	5682715314	0.971379
Winter's Multiplicative Method	99016387.52	6151.32	31.87%	9307540427	1.166917
Winter's Additive Method	240176029.00	10128.19	46.88%	22576546728	1.350682
Addictive Decomposition	94636215.91	7570.36	60.27%	10031438887	1.767463
Multiplicative Decomposition	95240890.43	7552.92	59.45%	10095534386	1.729588

Regression	46854879.89	4599.19	28.89%	4966617268.75	0.886296106

From the table above, **Brown's Method**, **Simple Naive Method** and **Naive Trend Method** produces similar MSE, MAE and MAPE while **Naive Seasonal Method** gives the largest MSE, MAE and MAPE within the 106 samples.

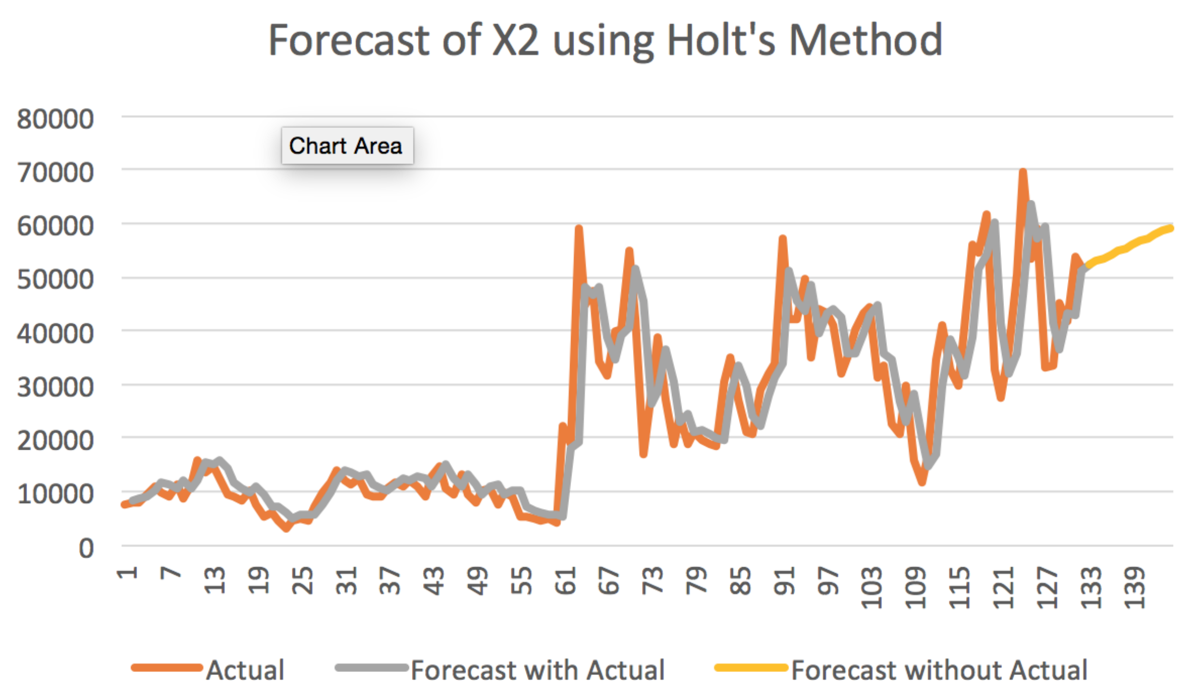
The MSE, MAE and MAPE for the <u>beyond sample </u>residuals computed by the forecasting models are shown in the table below.

Forecasting Methods	***MSE***	***MAE***	***MAPE***	***SSE***	***Theil's U***
Simple Naive Method	518440895.20	19180.23	44.87%	13479463276	1.551052
Naive Trend Method	1789761626	39344.73	97.80%	46533802279	3.203373
Naive Seasonal Method	319994252.30	15478.15	44.56%	8992766673	1.698333
Naive and Trend Method	325833067.50	15556.71	45.28%	9102830323	1.705743
Naive Rate of Change Method	1737911771.00	38034.49	778851.93%	42672727910	3.051283
Simple Moving Average (n=3)	368759545.78	15679.25	39.50%	9587748190.36	1.406386943
Simple Moving Average (n=12)	291995529.96	13652.523	38.05%	7591883779.06	1.411764042
Double Moving Average (n=3)	12515380675.44	97421.44	230.66%	325399897561.36	7.753300025
Double Moving Average (n=12)	373797659.82	15837.28	43.59%	9718739155.29	1.581946771
Simple Exponential Smoothing	246696871.30	12585.57	37.75%	6414118654	1.473429
Brown's Method	479275796.50	18380.63	44.10%	12461170710	1.511385

Holt's Method	206040149.30	11307.31	30.88%	5357043883	1.180812
Winter's Multiplicative Method	381461814.80	16225.24	40.53%	9918007186	1.327528
Winter's Additive Method	416459802.60	16276.82	36.67%	10827954869	1.374131
Addictive Decomposition	94636215.91	7570.356034	60.27%	10031438887	1.682922
Multiplicative Decomposition	95240890.43	7552.924293	59.45%	10095534386	1.761170
Regression	161844213.08	10476.84	34.06%	4207949540.03	1.368076

From the table above, **Holt's Method** has the best predictive power as its MSE, MAE and MAPE are the smallest whereas **Naive Rate of Change Method** seems to be the worse among the models beyond samples.

The plot of the data and fitted values generated by Holt's Method for the periods are given below.



The forecast value for time period t+m made at time t is

$$F_{t+m} = 51{,}736.212 + 618.5929*m \qquad m = 1, 2, 3, \ldots, 12$$

2018	*Fitted Values*		*Fitted Values*
January	52354.80	*July*	56066.36
February	52973.40	*August*	56684.96
March	53591.99	*September*	57303.55
April	54210.58	*October*	57922.14
May	54829.18	*November*	58540.73
June	55447.77	*December*	59159.33

Conclusion

For Y variable:

Holt's method is recommended for forecasting the Y. The main reasons is that this method considers linear trend series and the forecasts produced from this method are closer to the actual values, which can also be seen from MSE, MAE and MAPE. Although these three indicators are relatively low in in sample by using regression method and the Theil's U statistic of them is lower than one, the performance of them in beyond sample are not satisfactory and the Theil's U statistic in that sample is higher than one. On the contrary, for the Holt's method, the MSE, MAE and MAPE are the smallest in beyond sample and they are relatively small in the in sample. Therefore, we choose the Holt's method to predict the price of private domestic premises.

For X1 variable:

Brown's method is recommended for forecasting the X1. The main reasons is that this method considers linear trend series and the forecasts produced from this method are closer to the actual values, which can also be seen from MSE, MAE and MAPE. Besides, the Theil's U statistics of brown's method in both sample are smaller than one which means that the forecasting method is better than naive methods. Although these three indicators in brown's method are not the smallest value among the in sample and the beyond sample, they are relatively small and consistent in both in sample and beyond sample.

For X2 variable:

Holt's method is recommended for forecasting the X2. The main reason is that this method considers linear trend series by applying different weight to the actual data and trend. And the forecasts produced from this method are closer to the actual values, which can be seen from MSE, MAE AND MAPE. Besides, the Theil's U statistics of Holt's method in both sample are the smallest and it is smaller than one in the in sample. After analysing the results, Holt's method has the smallest MAPE in beyond sample and the second highest in in sample. Also, the MSE of this method among the two samples is relatively low. Therefore, it can show that the it is a better method to forecasting the future value of the rent of fresh lettings of private domestic premises.

Recommendations

Based on our forecasts, we expect that the price of the private housing will keep on climbing. On the upside, we can see the private residential property market has a big potential for investment as it generates handsome returns, yet the downside is that it will be much more unaffordable for the general public to purchase a house to fulfil their needs for a shelter. For the rental price, we predict that it will slow down and drop a bit, yet the price is still remaining at a high level and way beyond the financial ability of the tenants. In order to alleviate these problems, we suggest the government to have a better plan for housing. For instances, the government can introduce policies and measures focusing on countering the speculating actions and increasing land supply to cater the demand.

For the number of private housing units' transactions, we suppose that it will have a continuous growth. With a view to maintaining a stable private property market, regulations should be imposed to monitor and enhance the transparency of each transaction.

In fact, we reckon that this forecasting and analysis project is not a comprehensive one since we only used a few variables for prediction. Therefore, for more accurate and reliable prediction, we suggest including more variables involving both external (e.g. worldwide) and internal (local) factors like interest rates and demand in future projects.

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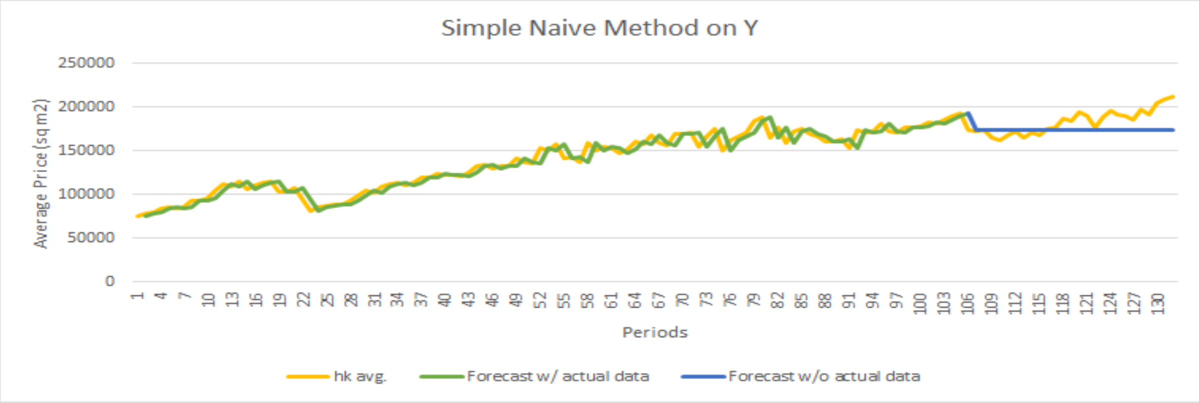
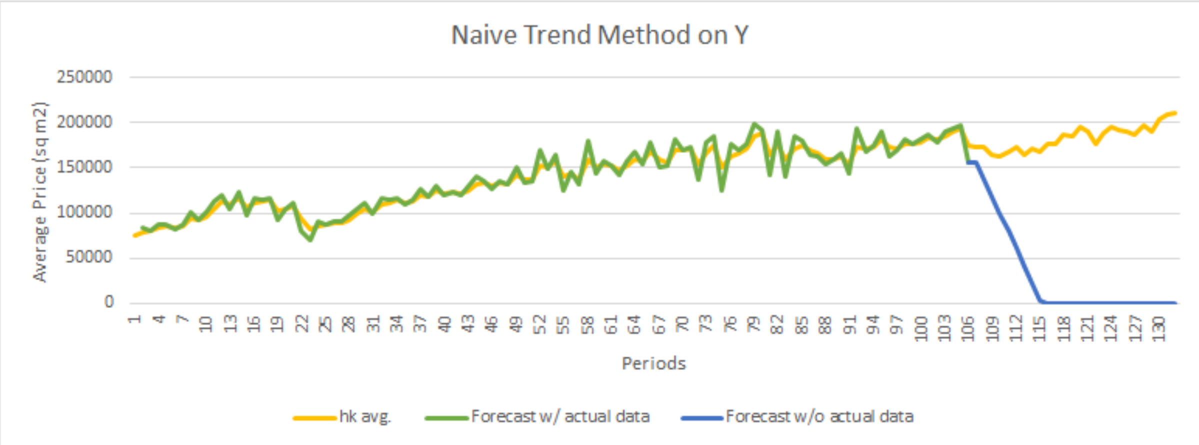
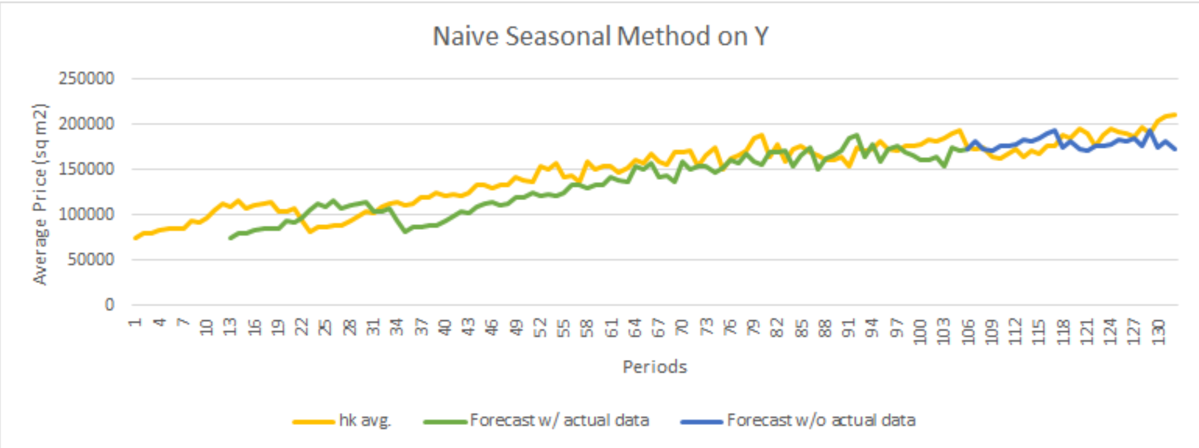
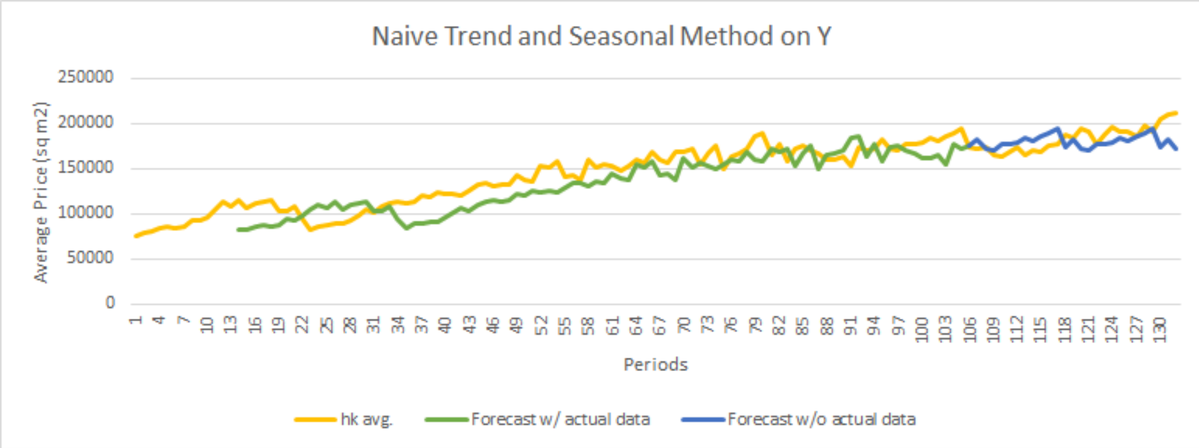
Appendices

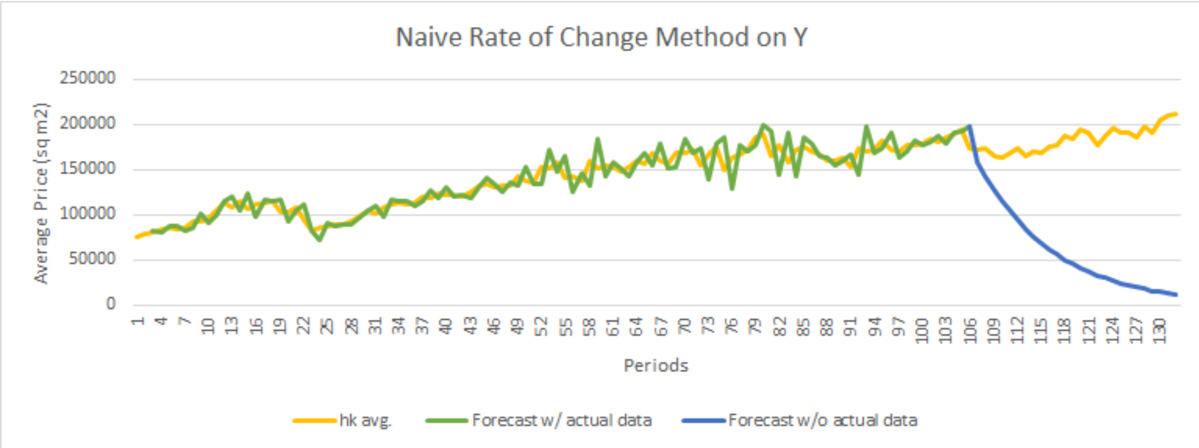
Graphs for forecasting Y variable

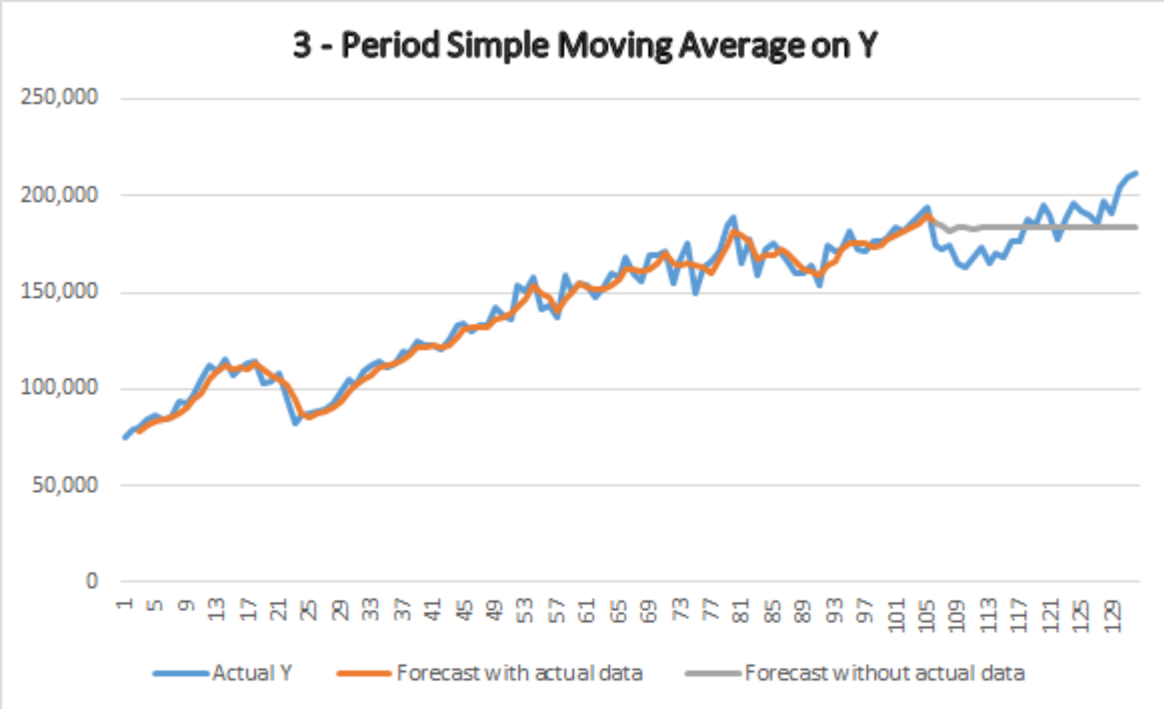


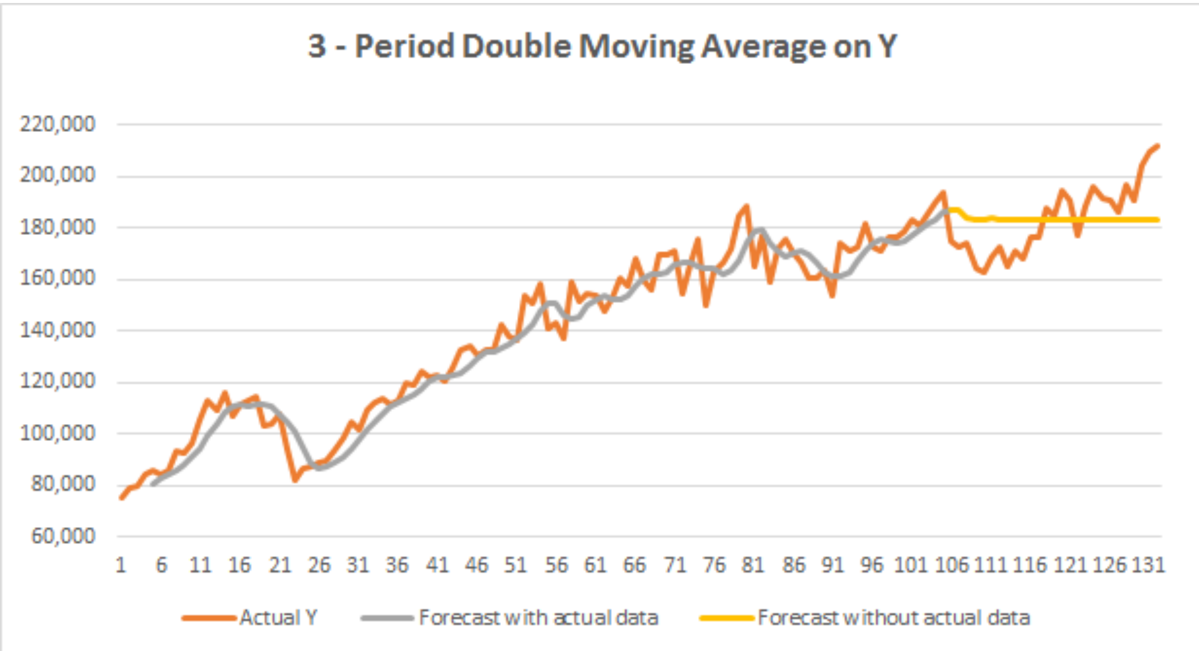


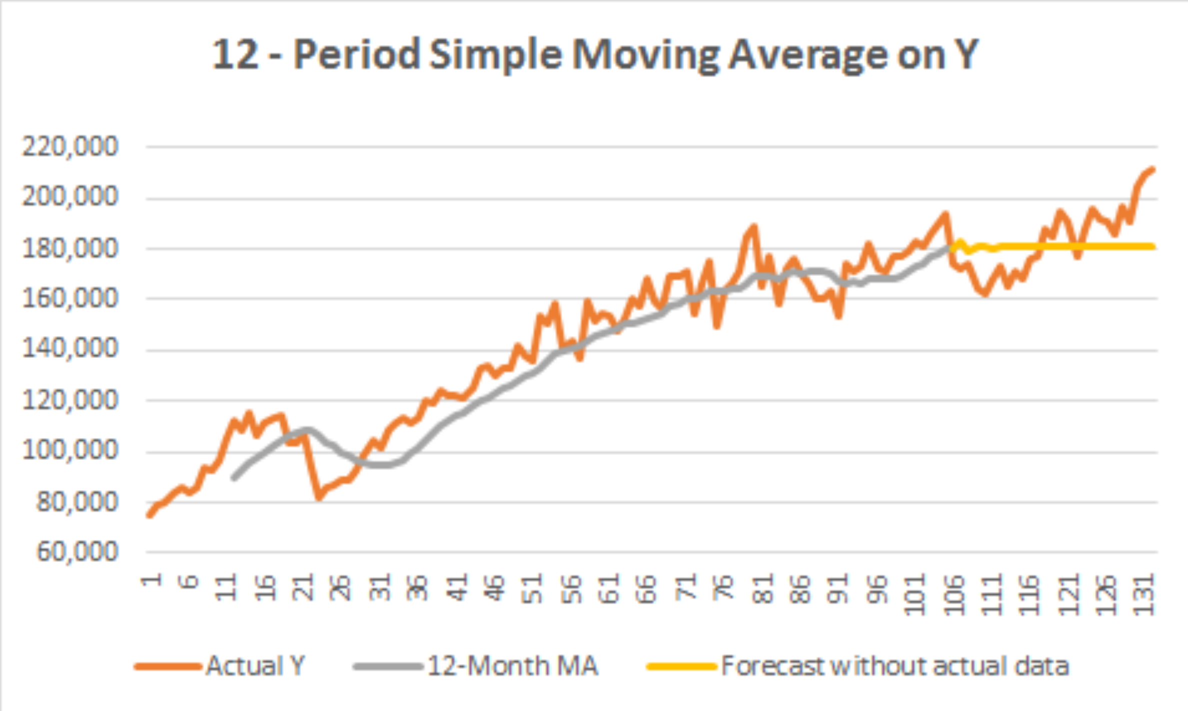


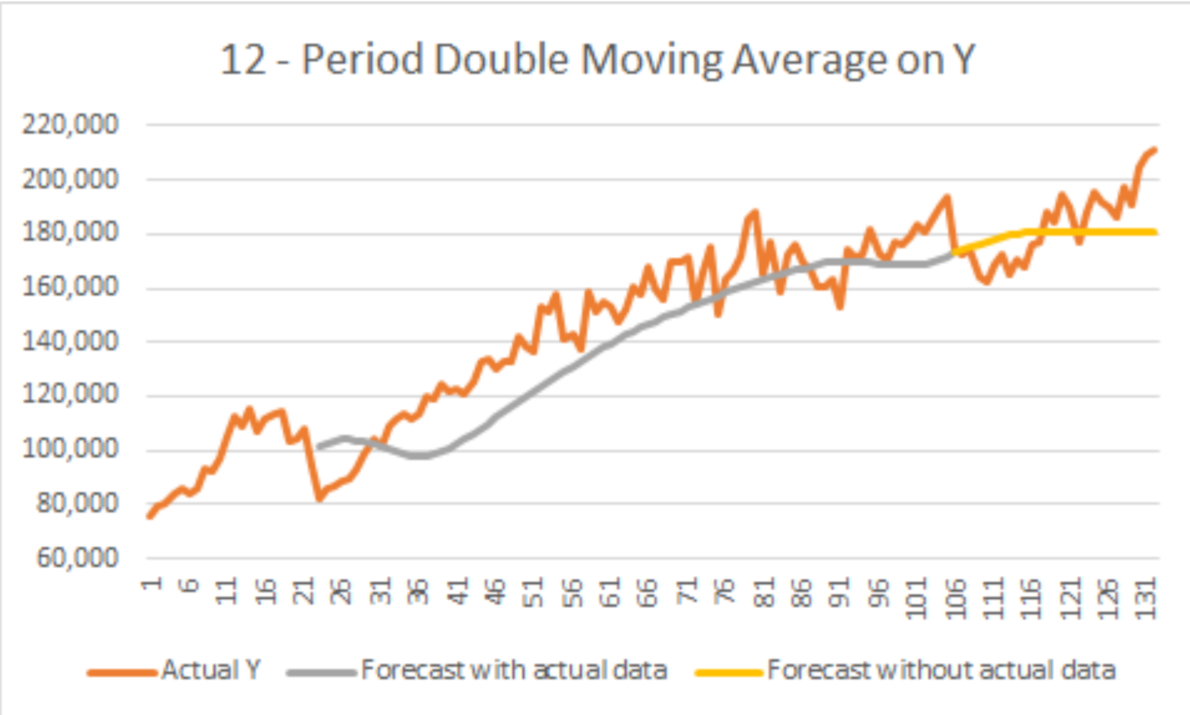

Naive Trend and Seasonal Method on Y
Average Price (sq m2)
Periods
hk avg.
Forecast w/ actual data
Forecast w/o actual data

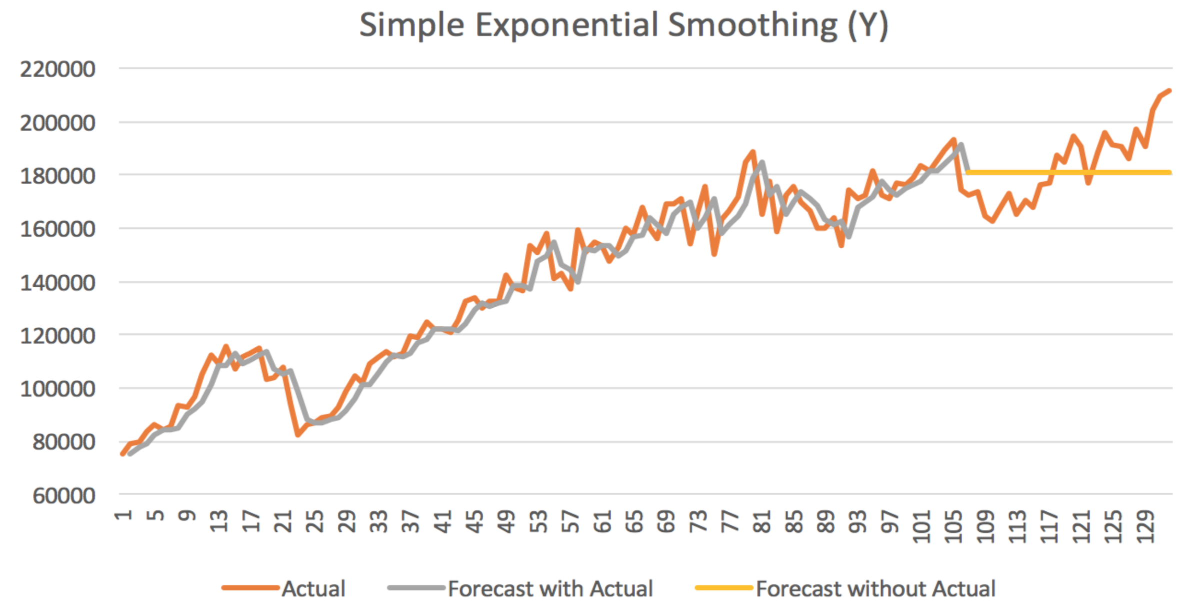

Naive Rate of Change Method on Y
Average Price (sq m2)
Periods
hk avg.
Forecast w/ actual data
Forecast w/o actual data


3 - Period Simple Moving Average on Y
250,000
200,000
150,000
100,000
50,000
0
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
101
105
109
113
117
121
125
129
Actual Y
Forecast with actual data
Forecast without actual data


3 - Period Double Moving Average on Y
220,000
200,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual Y
Forecast with actual data
Forecast without actual data


12 - Period Simple Moving Average on Y
220,000
200,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual Y
12-Month MA
Forecast without actual data


12 - Period Double Moving Average on Y
220,000
200,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual Y
Forecast with actual data
Forecast without actual data


Simple Exponential Smoothing (Y)
220000
200000
180000
160000
140000
120000
100000
80000
60000
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
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93
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117
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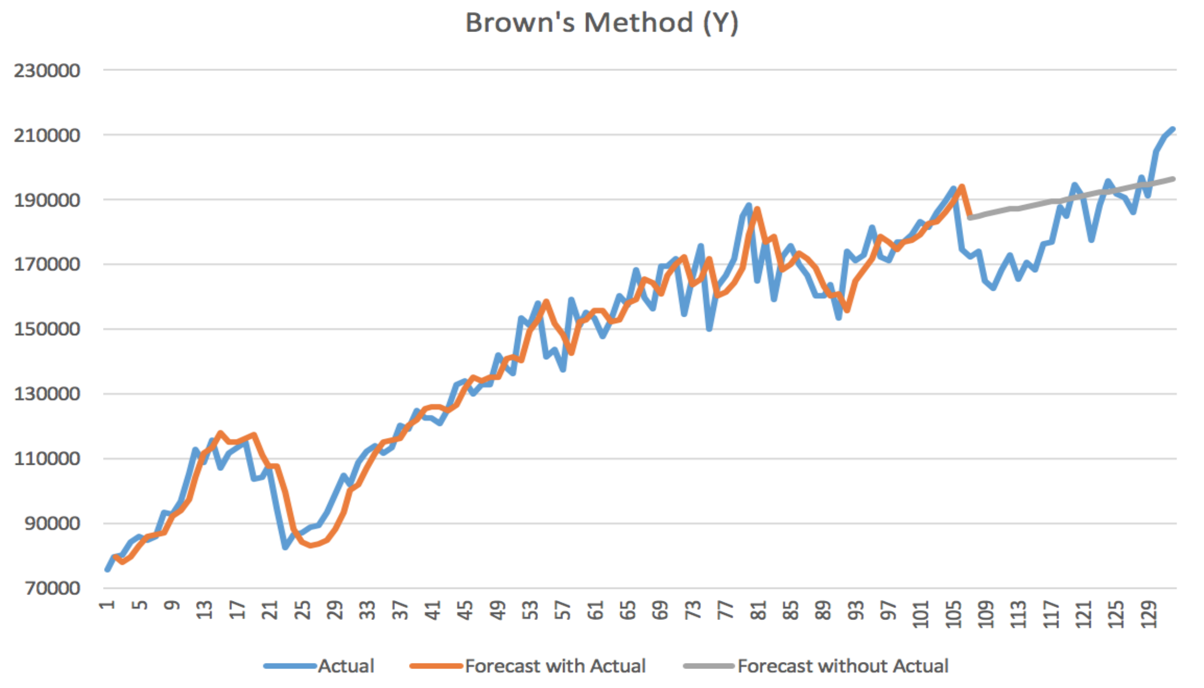
125
129
Actual
Forecast with Actual
Forecast without Actual


Brown's Method (Y)
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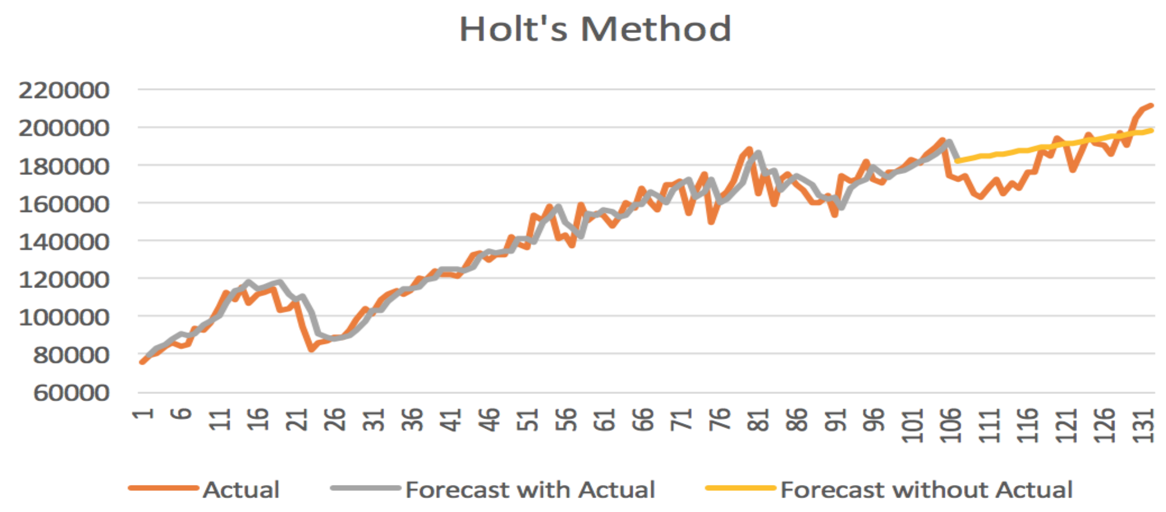
210000
190000
170000
150000
130000
110000
90000
70000
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
101
105
109
113
117
121
125
129
Actual
Forecast with Actual
Forecast without Actual


Holt's Method
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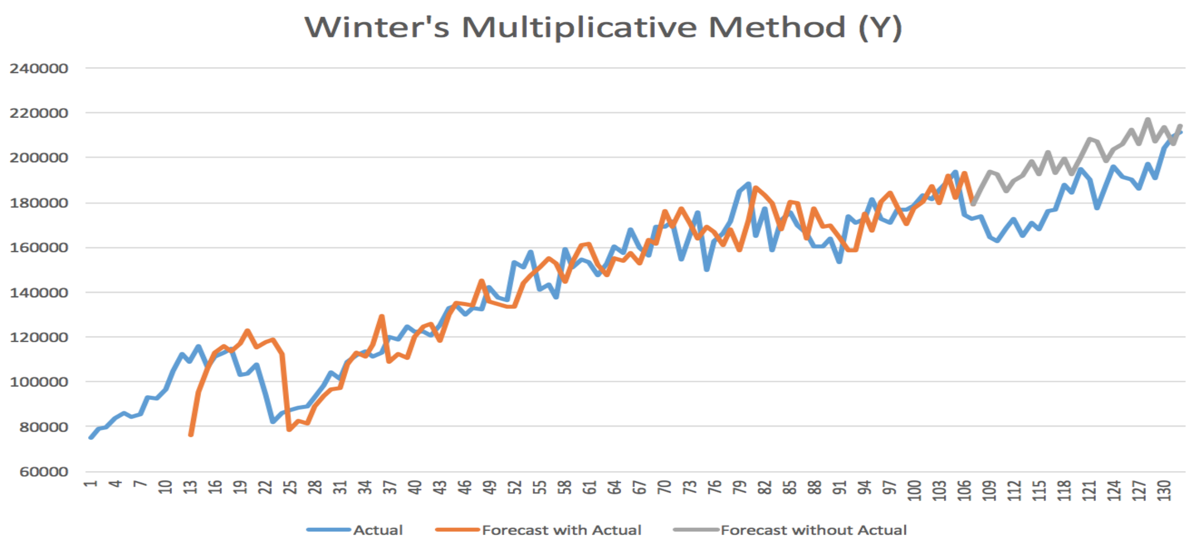
200000
180000
160000
140000
120000
100000
80000
60000
Actual
Forecast with Actual
Forecast without Actual


Winter's Multiplicative Method (Y)
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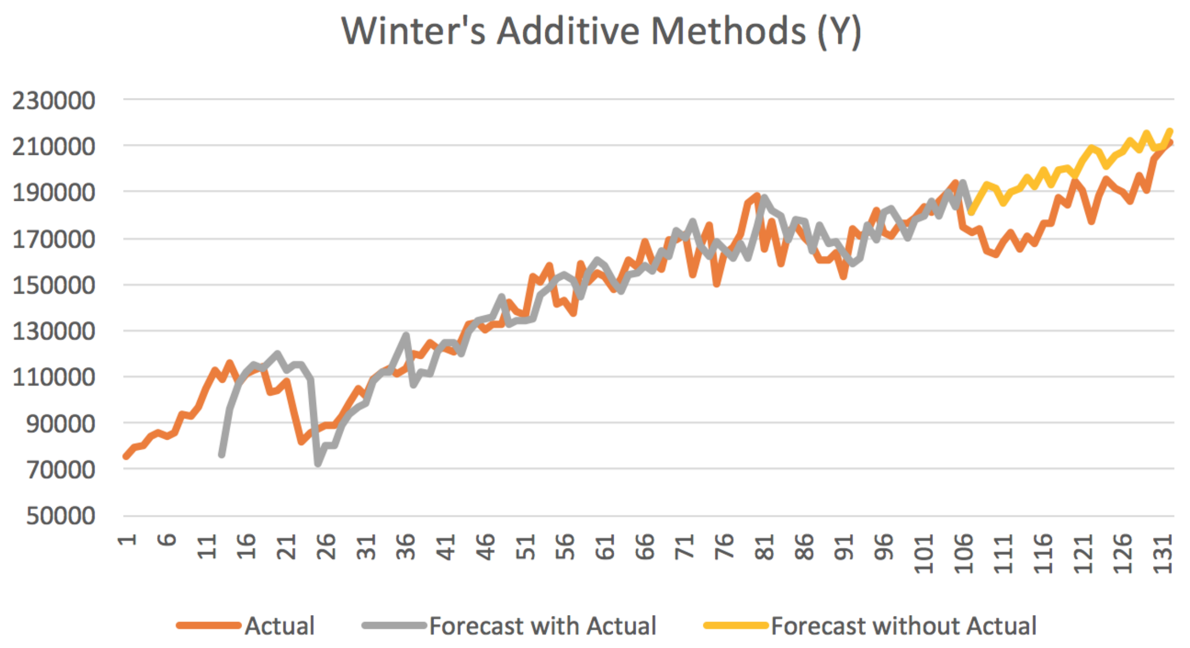
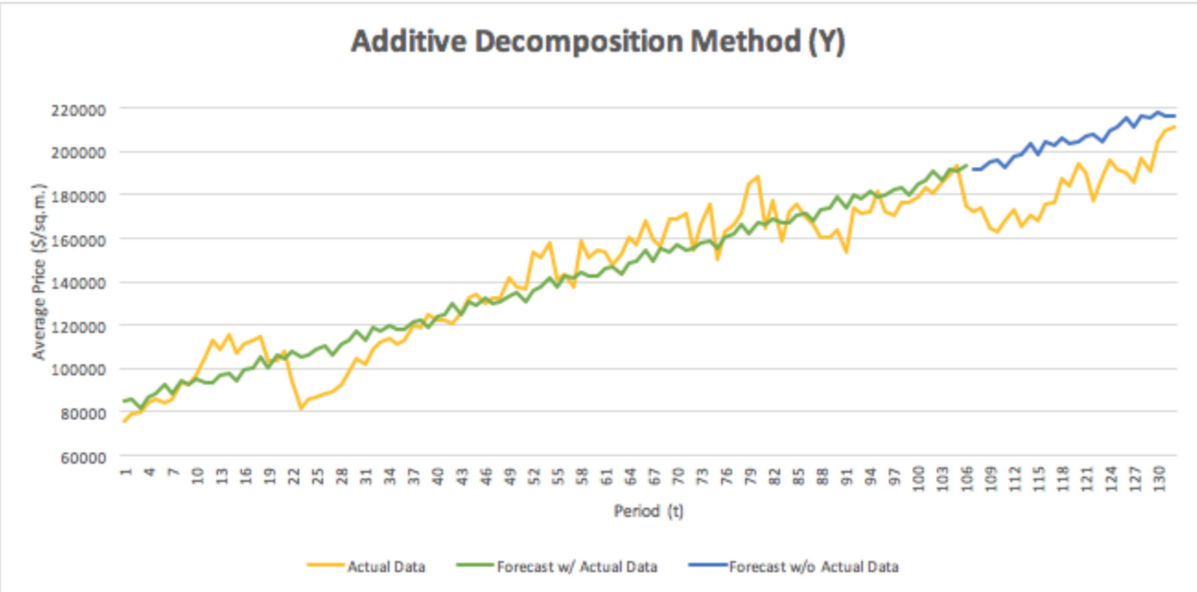
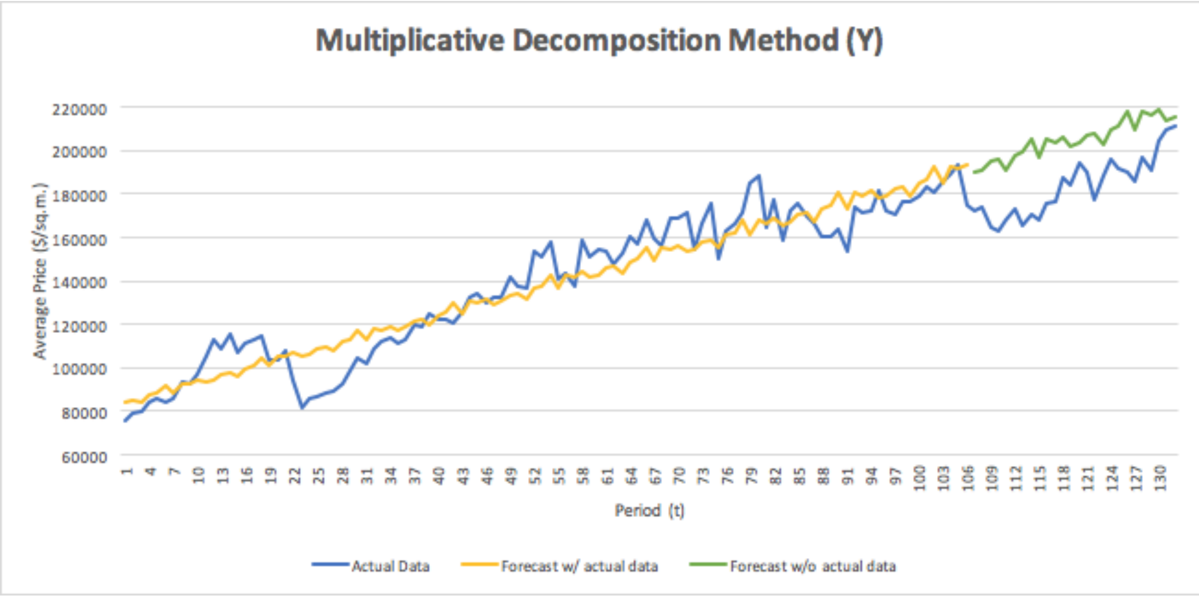
200000
180000
160000
140000
120000
100000
80000
60000
Actual
Forecast with Actual
Forecast without Actual

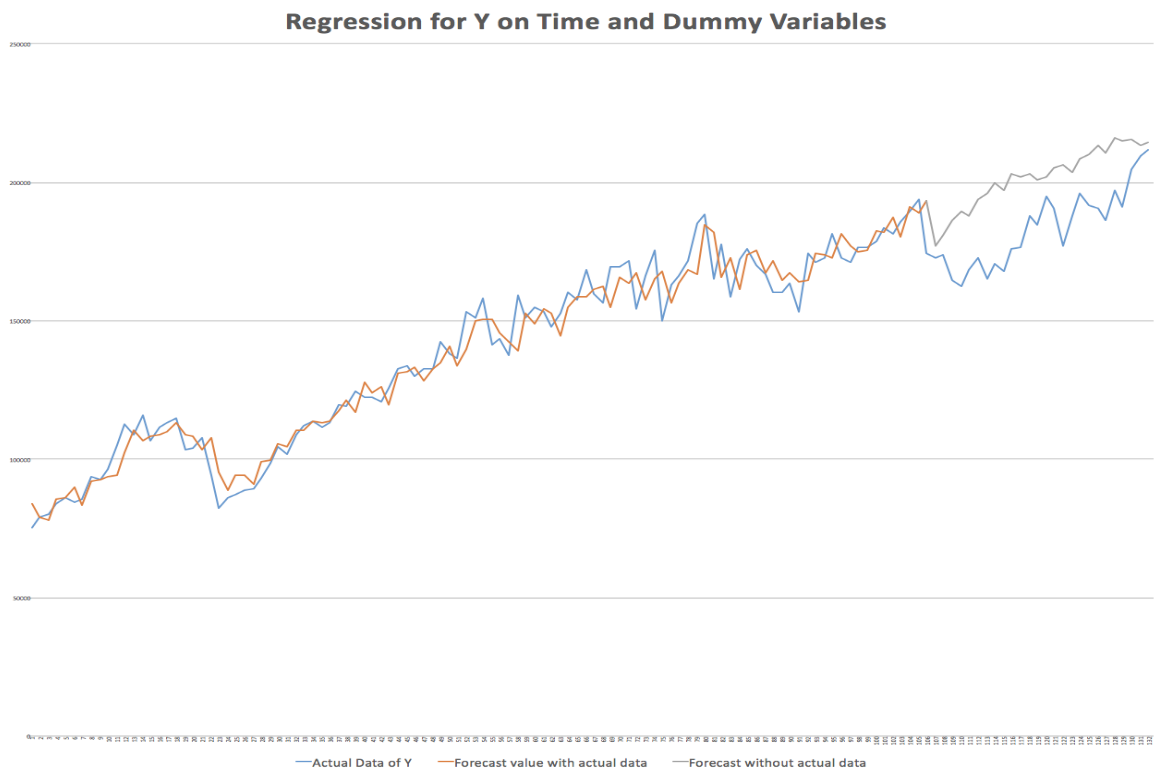


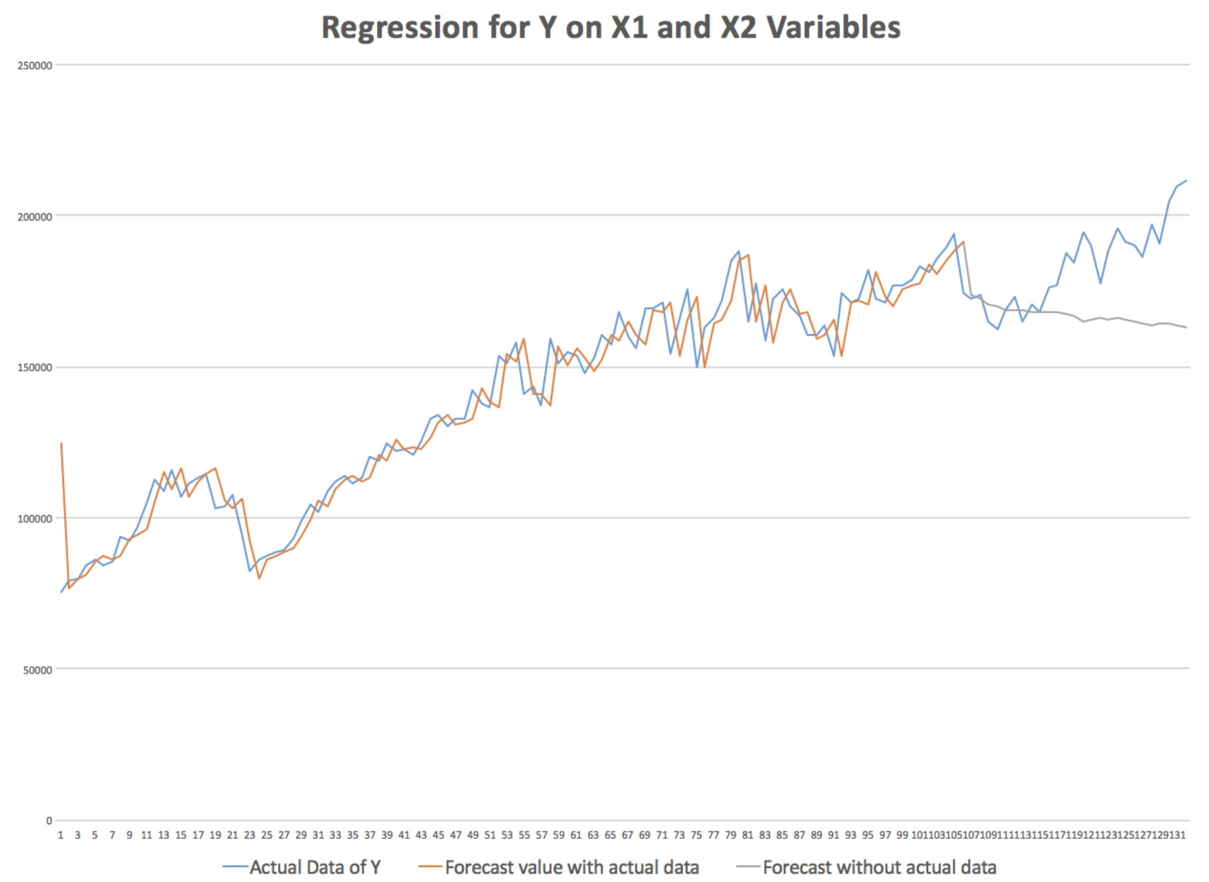
Winter's Additive Methods (Y)
230000
210000
190000
170000
150000
130000
110000
90000
70000
50000
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual
Forecast with Actual
Forecast without Actual

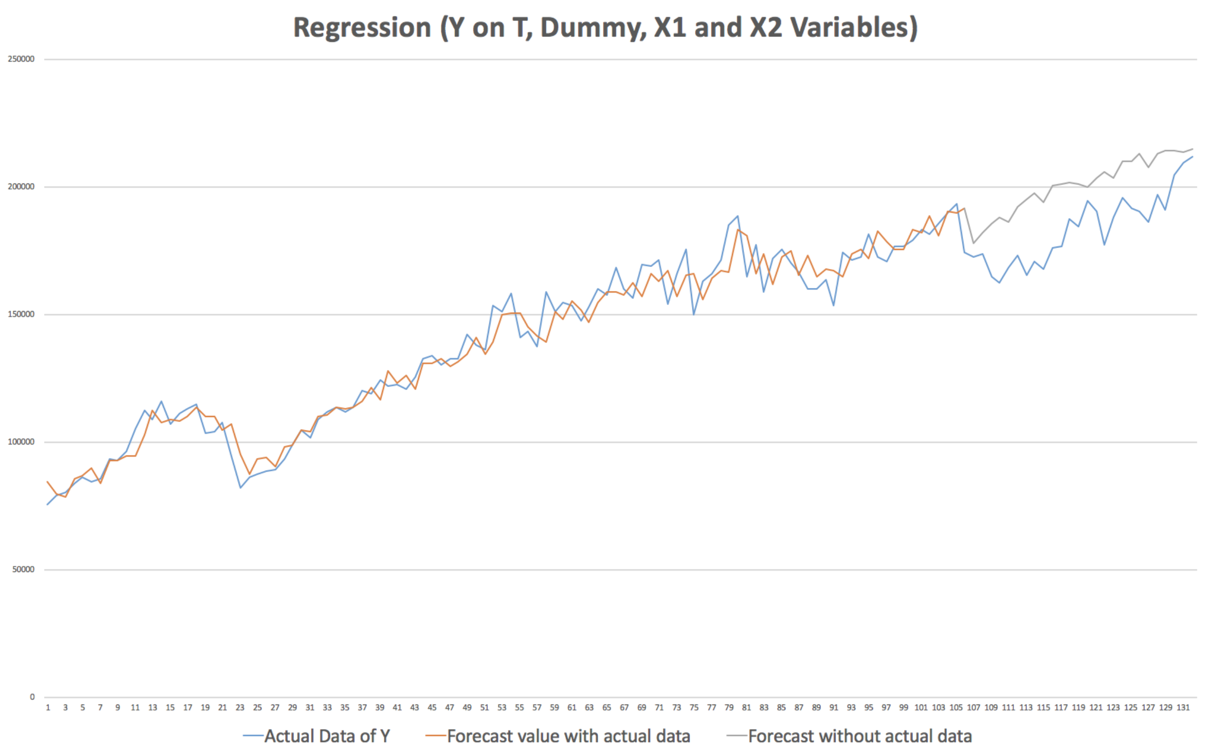


Additive Decomposition Method (Y)
Average Price ($/sq.m.)
220000
200000
180000
160000
140000
120000
100000
80000
60000
Period (t)
Actual Data
Forecast w/ Actual Data
Forecast w/o Actual Data

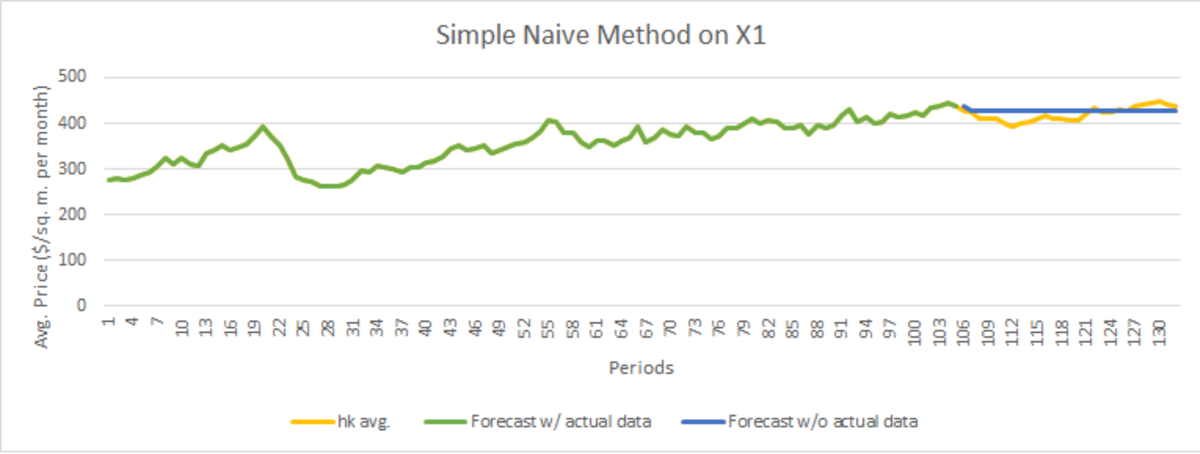

Multiplicative Decomposition Method (Y)
220000
200000
180000
160000
140000
120000
100000
80000
60000
Average Price ($/sq.m.)
Period (t)
Actual Data
Forecast w/ actual data
Forecast w/o actual data

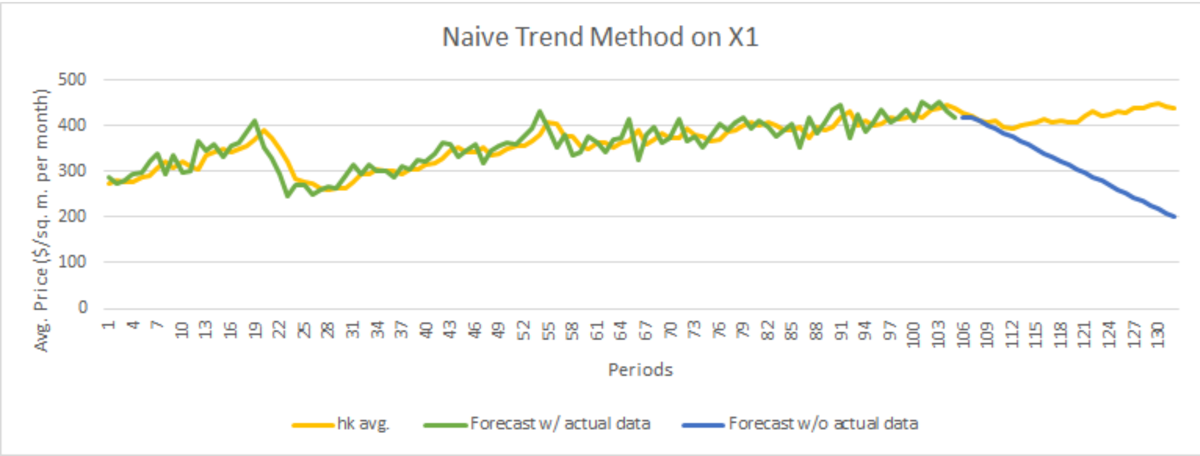

Regression for Y on Time and Dummy Variables
Actual Data of Y
Forecast value with actual data
Forecast without actual data

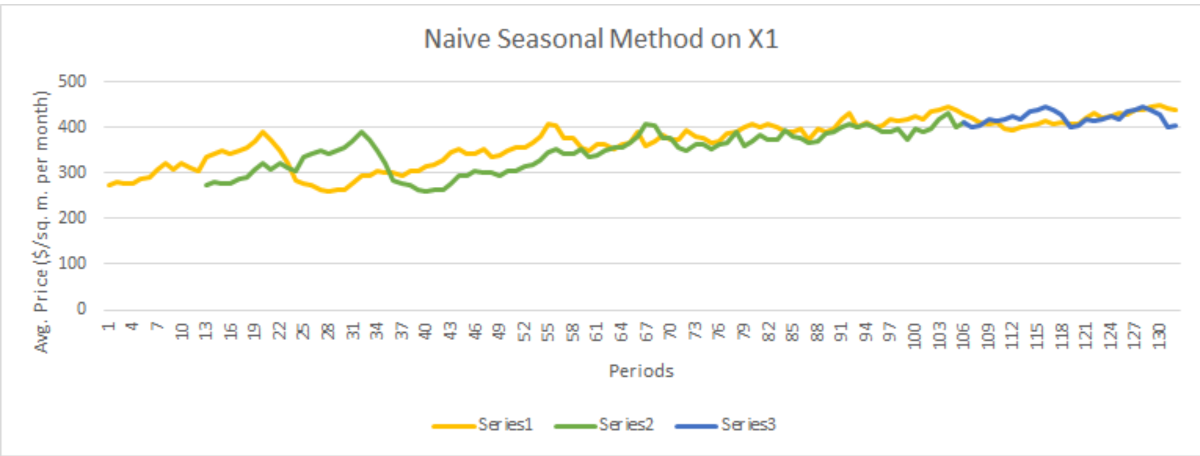


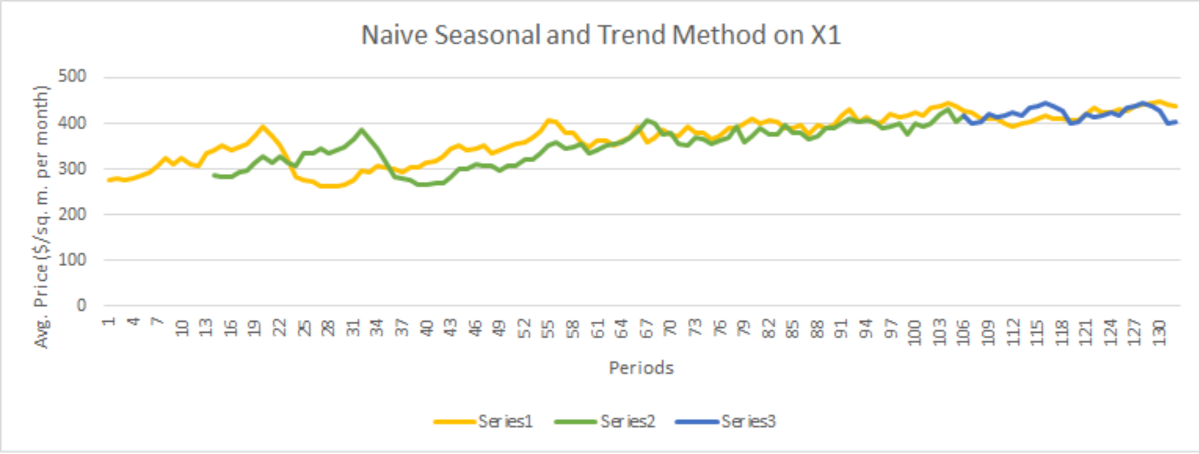


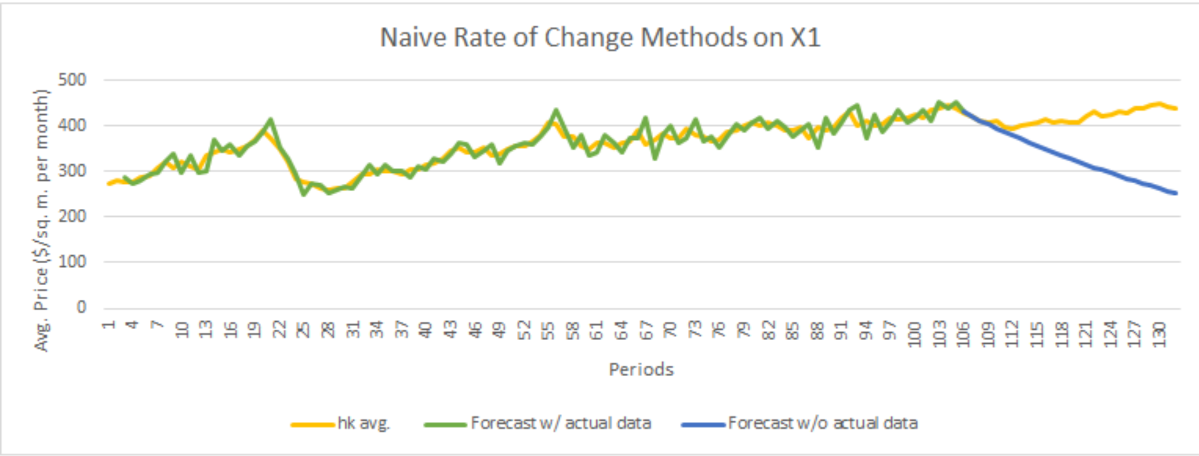
<u>**Graphs for forecasting X1 variable**</u>

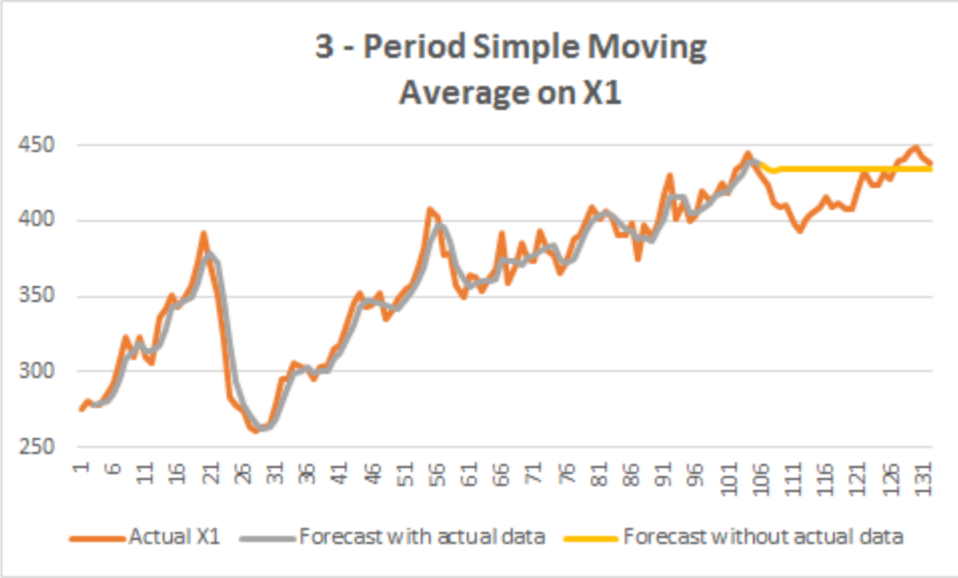

Simple Naive Method on X1
Avg. Price ($/sq. m. per month)
Periods
hk avg.
Forecast w/ actual data
Forecast w/o actual data

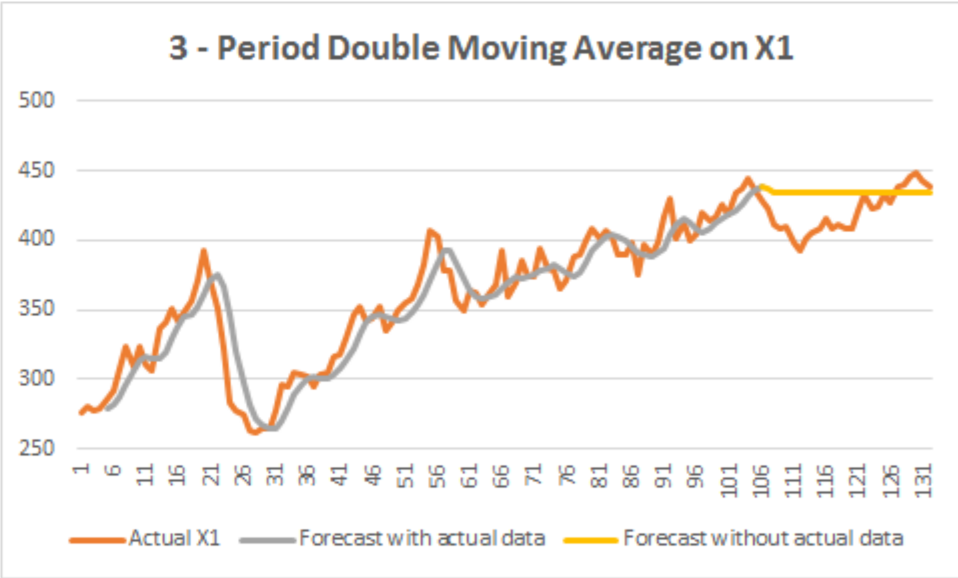

Naive Trend Method on X1
Avg. Price ($/sq. m. per month)
Periods
hk avg.
Forecast w/ actual data
Forecast w/o actual data

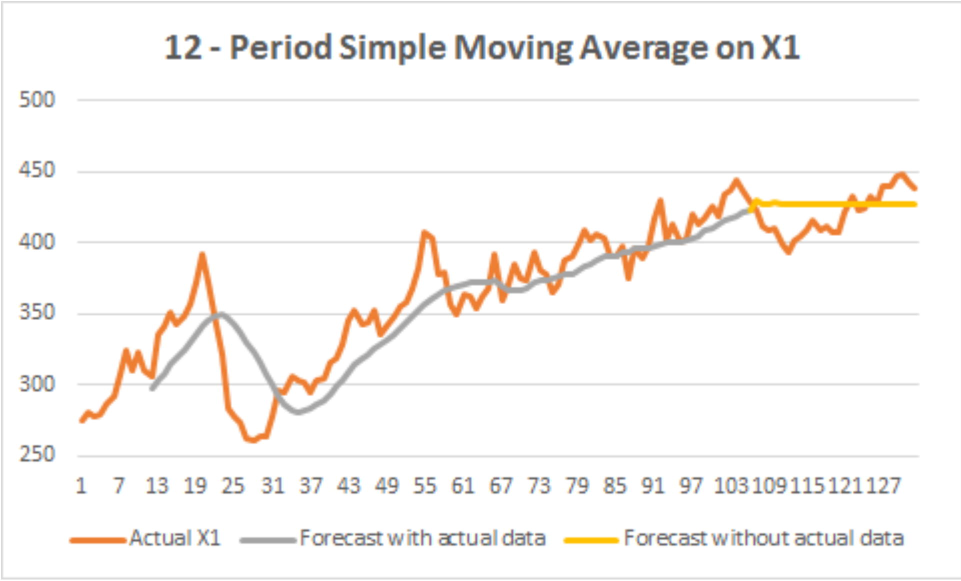

Naive Seasonal Method on X1
Avg. Price ($/sq. m. per month)
Periods
Series1
Series2
Series3

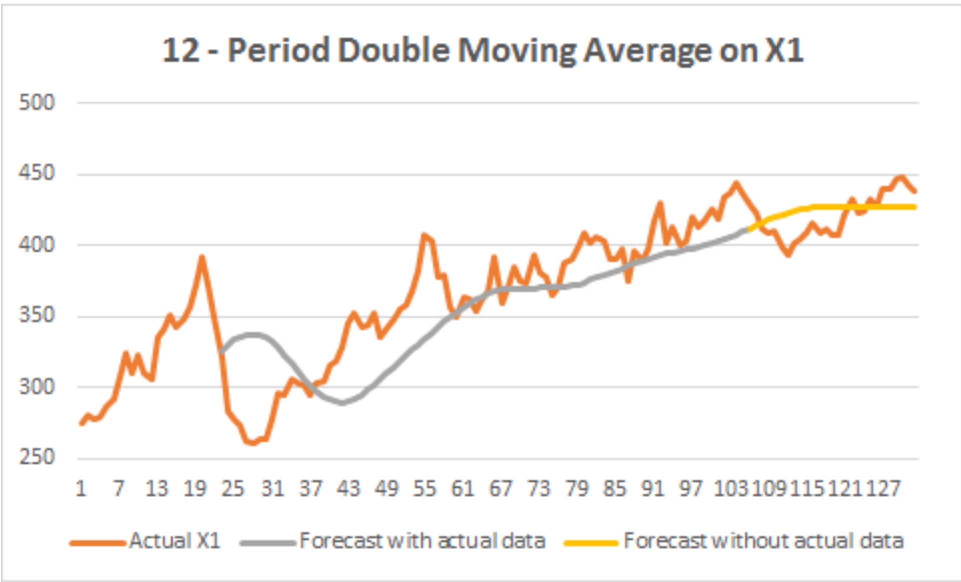

Naive Seasonal and Trend Method on X1
Avg. Price ($/sq. m. per month)
Periods
Series1
Series2
Series3

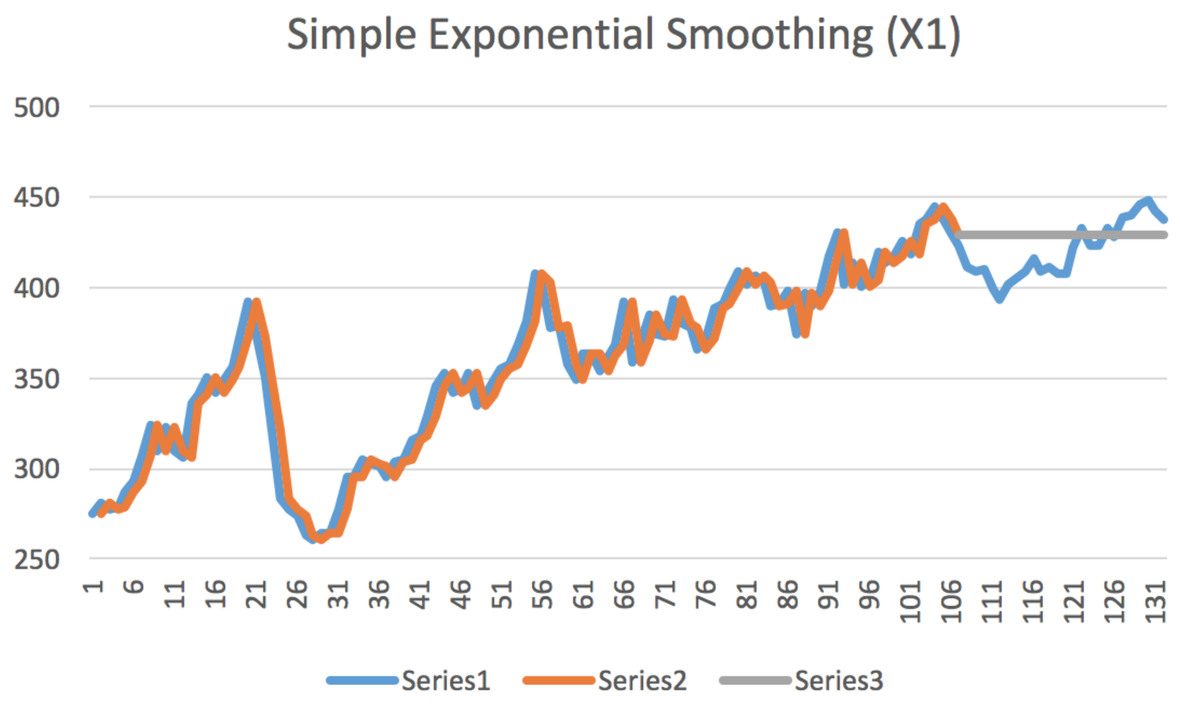

Naive Rate of Change Methods on X1
Avg. Price ($/sq. m. per month)
Periods
hk avg.
Forecast w/ actual data
Forecast w/o actual data

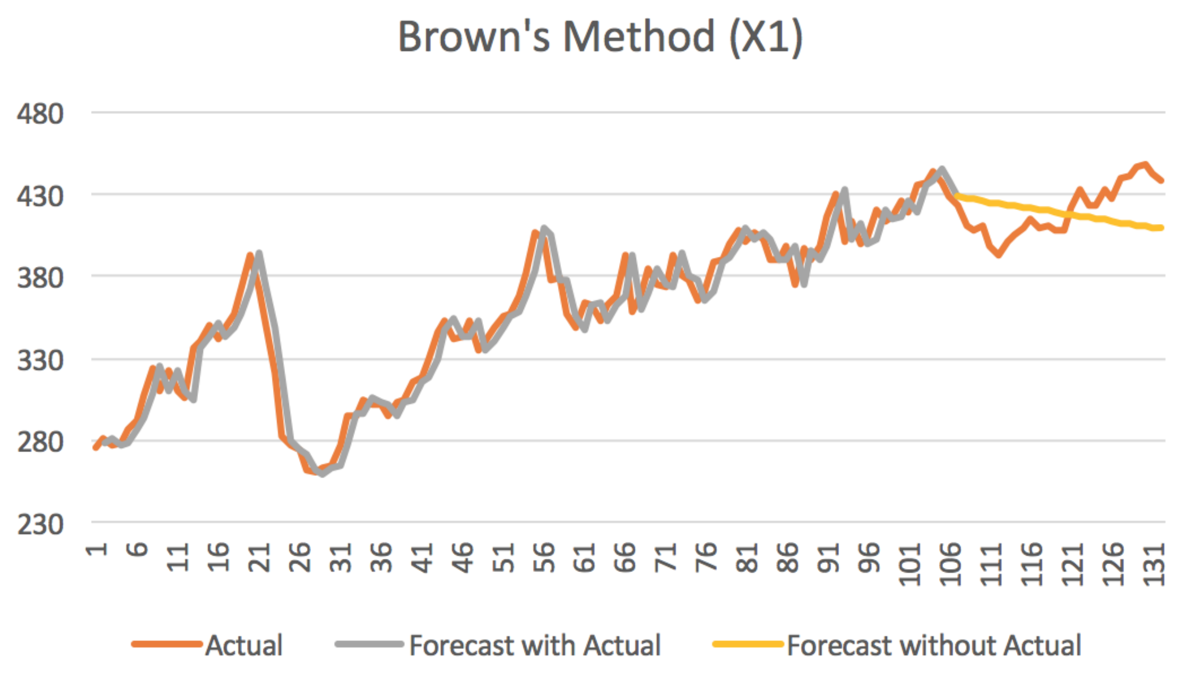

3 - Period Simple Moving Average on X1
450
400
350
300
250
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual X1
Forecast with actual data
Forecast without actual data

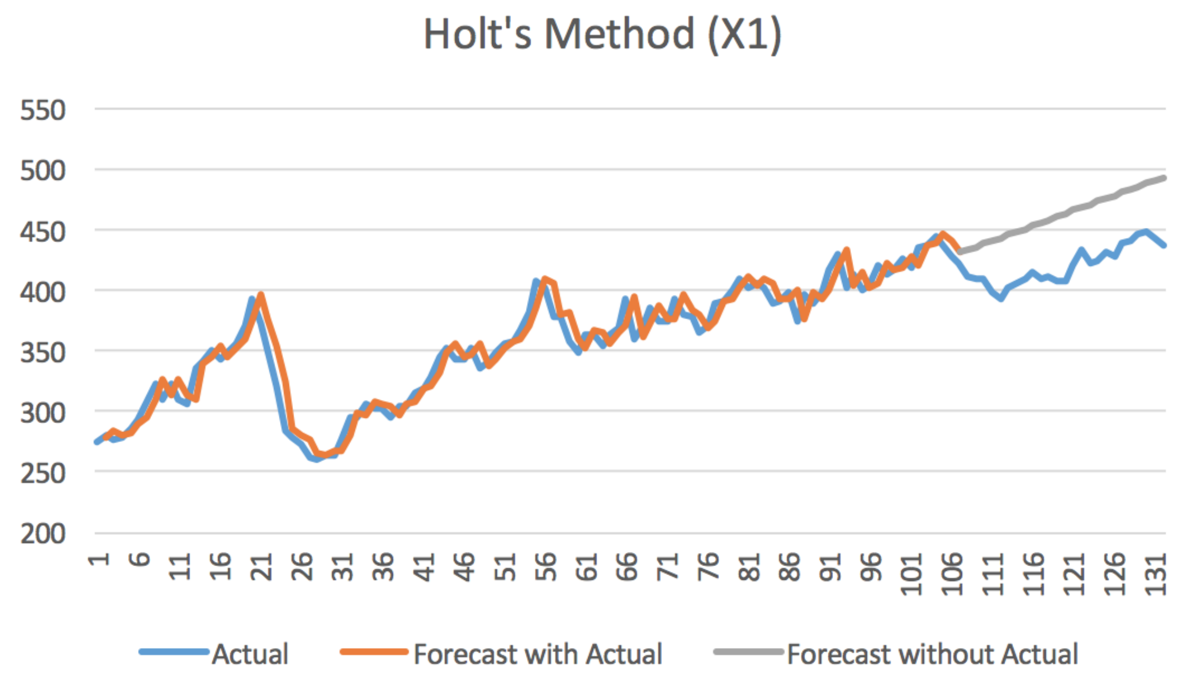

3 - Period Double Moving Average on X1
500
450
400
350
300
250
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual X1
Forecast with actual data
Forecast without actual data


12 - Period Simple Moving Average on X1
500
450
400
350
300
250
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97 103 109 115 121 127
Actual X1
Forecast with actual data
Forecast without actual data

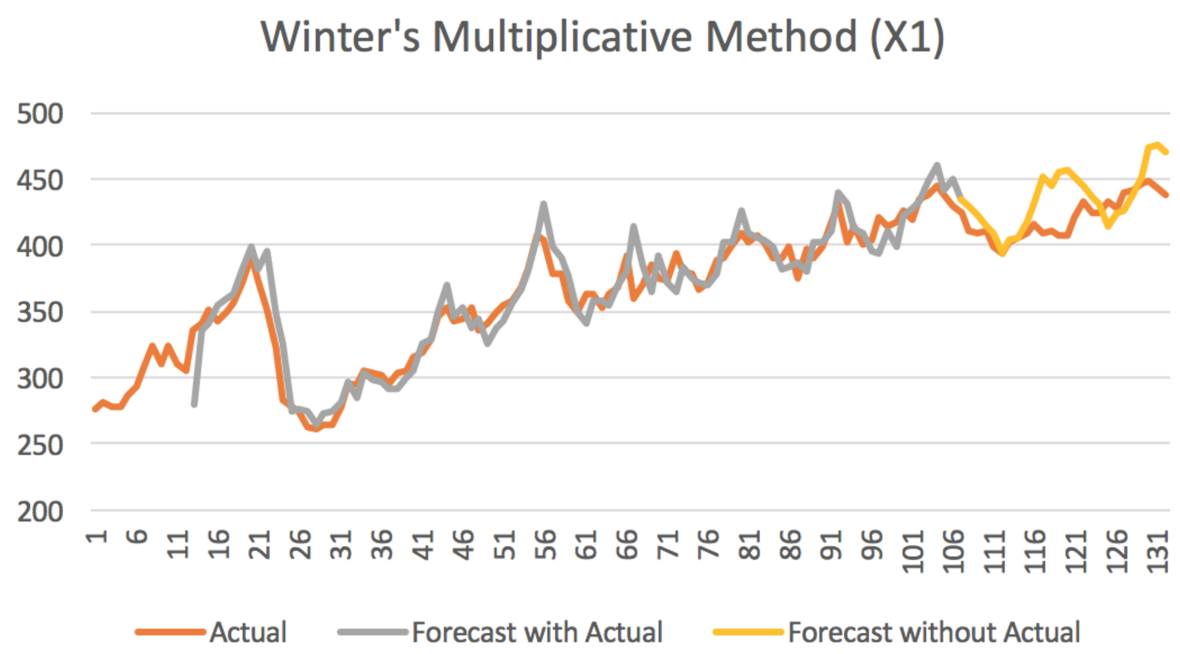

12 - Period Double Moving Average on X1
500
450
400
350
300
250
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97 103 109 115 121 127
Actual X1
Forecast with actual data
Forecast without actual data


Simple Exponential Smoothing (X1)
500
450
400
350
300
250
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Series1
Series2
Series3

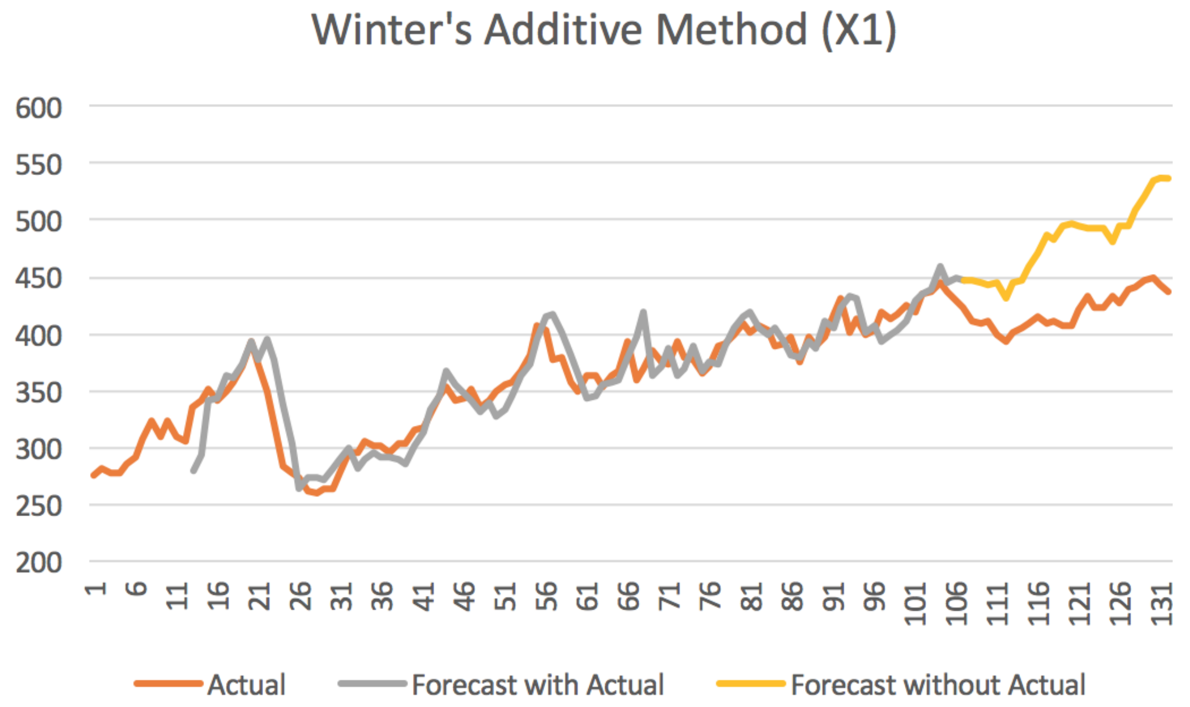

Brown's Method (X1)
480
430
380
330
280
230
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Actual
Forecast with Actual
Forecast without Actual

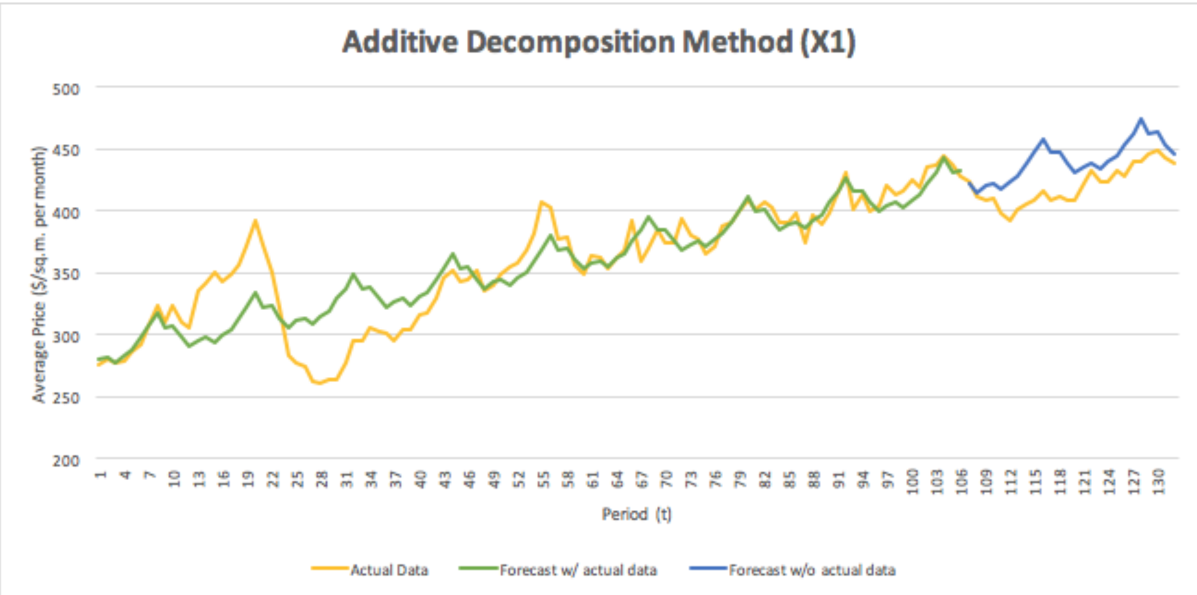


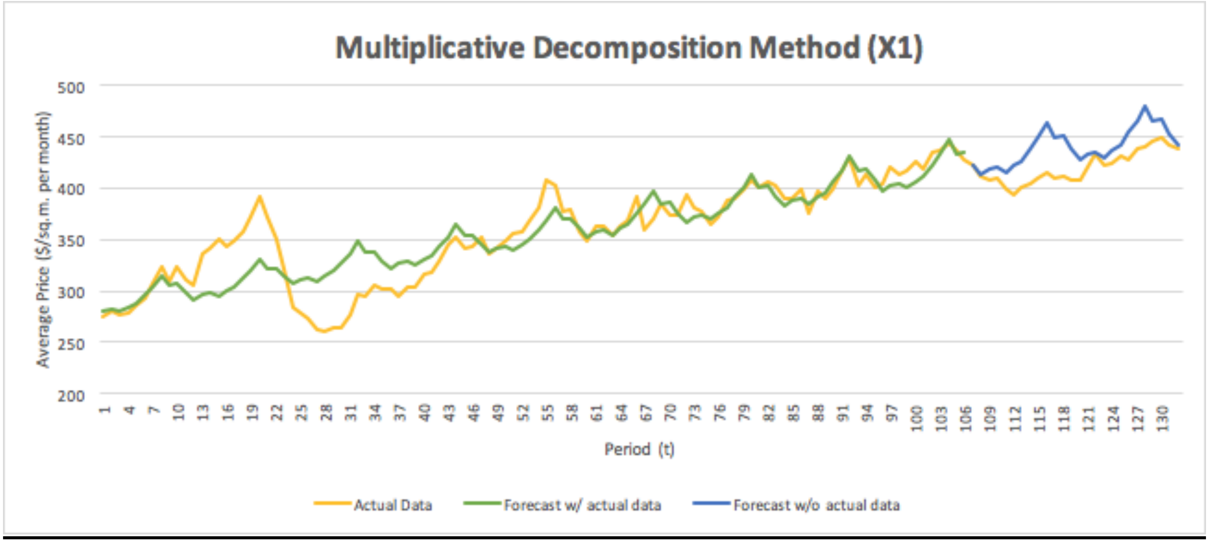
Holt's Method (X1)
550
500
450
400
350
300
250
200
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual
Forecast with Actual
Forecast without Actual

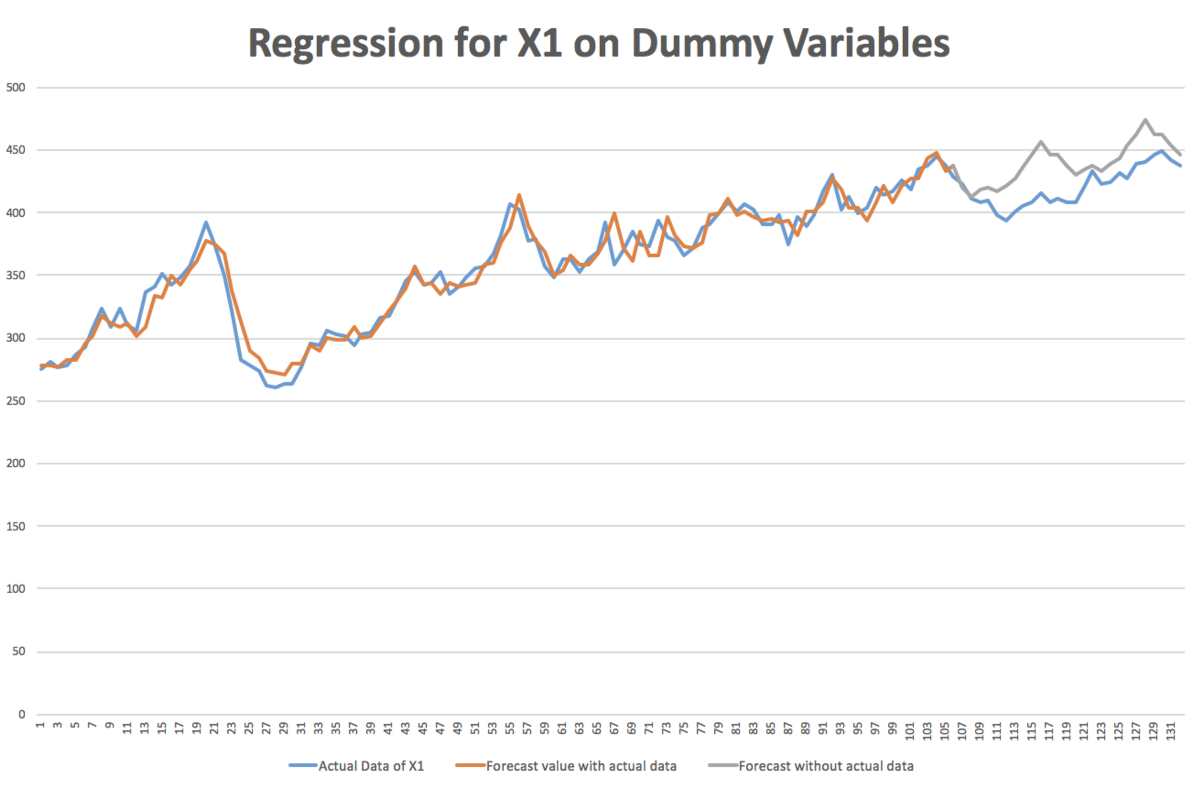


Winter's Multiplicative Method (X1)
500
450
400
350
300
250
200
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
76
81
86
91
96
101
106
111
116
121
126
131
Actual
Forecast with Actual
Forecast without Actual

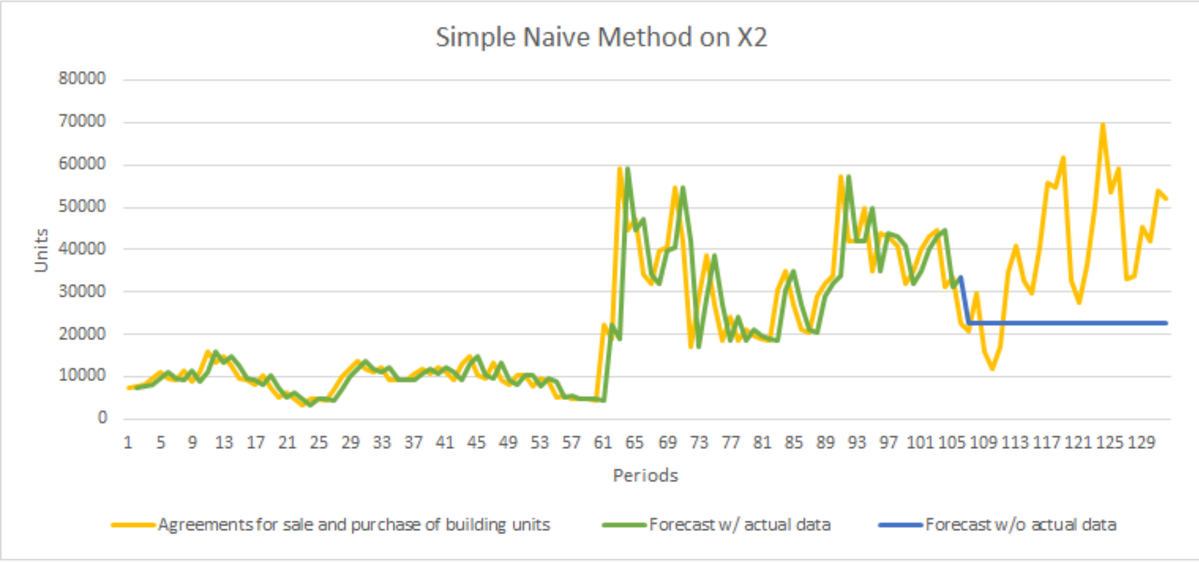

Winter's Additive Method (X1)
600
550
500
450
400
350
300
250
200
Actual
Forecast with Actual
Forecast without Actual

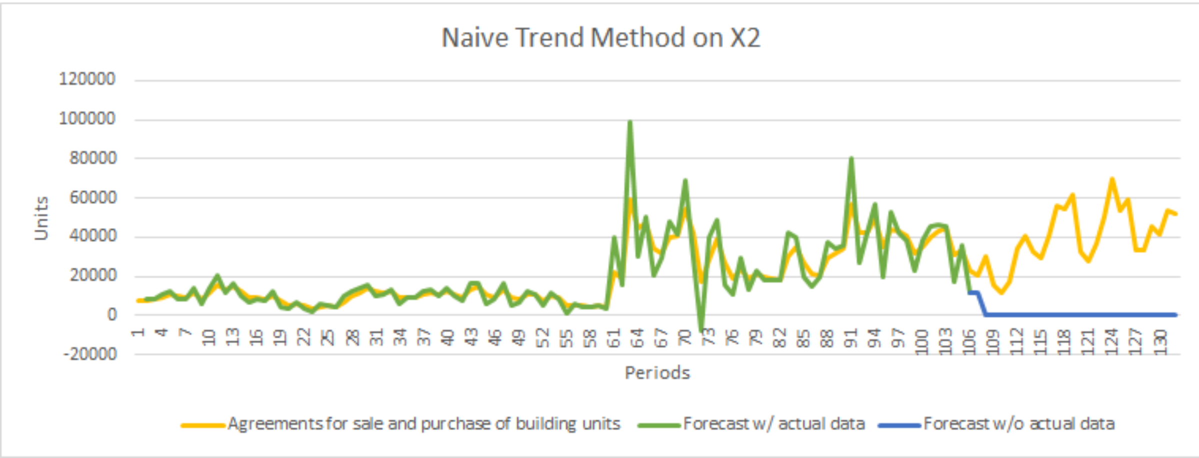

Additive Decomposition Method (X1)
Average Price ($/sq.m. per month)
500
450
400
350
300
250
200
Period (t)
Actual Data
Forecast w/ actual data
Forecast w/o actual data

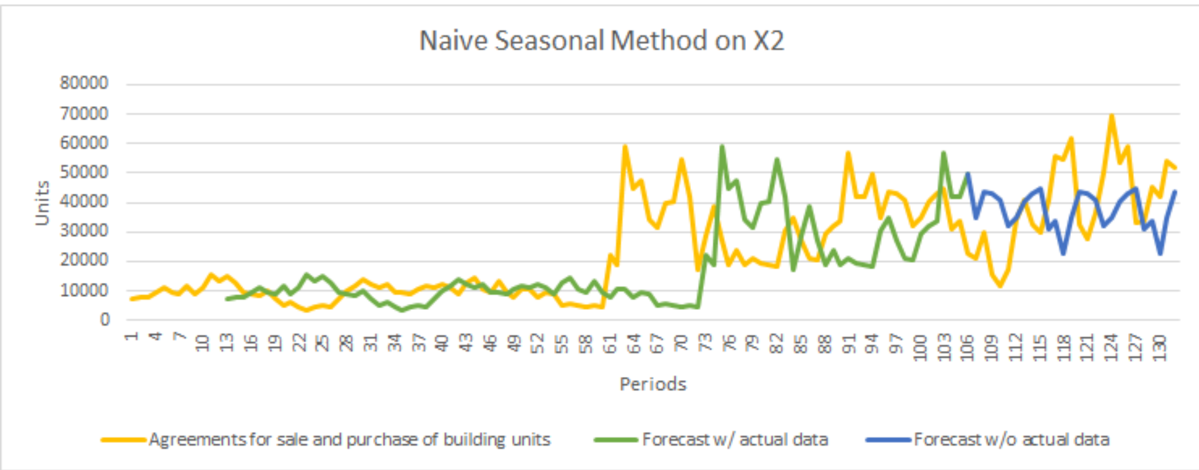

Multiplicative Decomposition Method (X1)
Average Price ($/sq.m. per month)
Period (t)
Actual Data
Forecast w/ actual data
Forecast w/o actual data

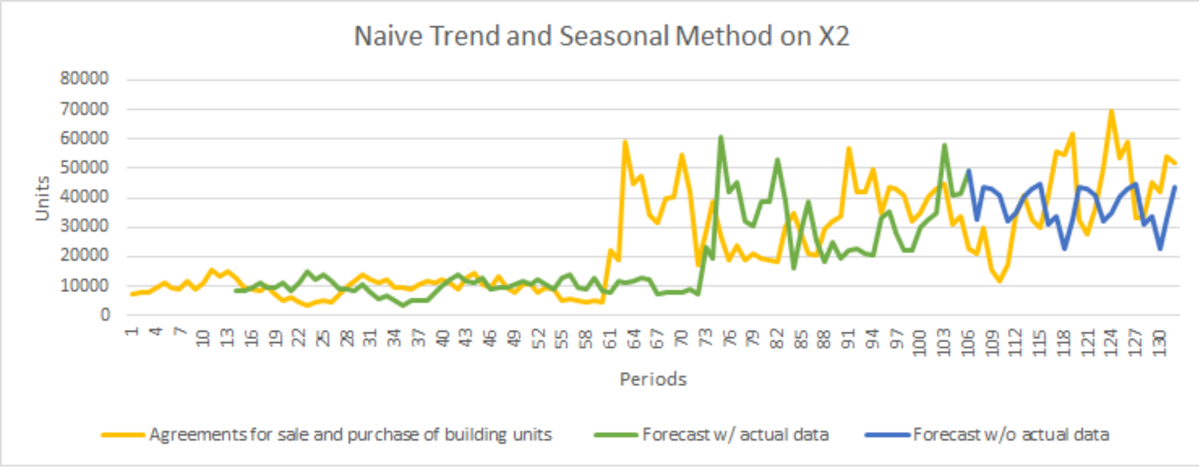

Regression for X1 on Dummy Variables
Actual Data of X1
Forecast value with actual data
Forecast without actual data

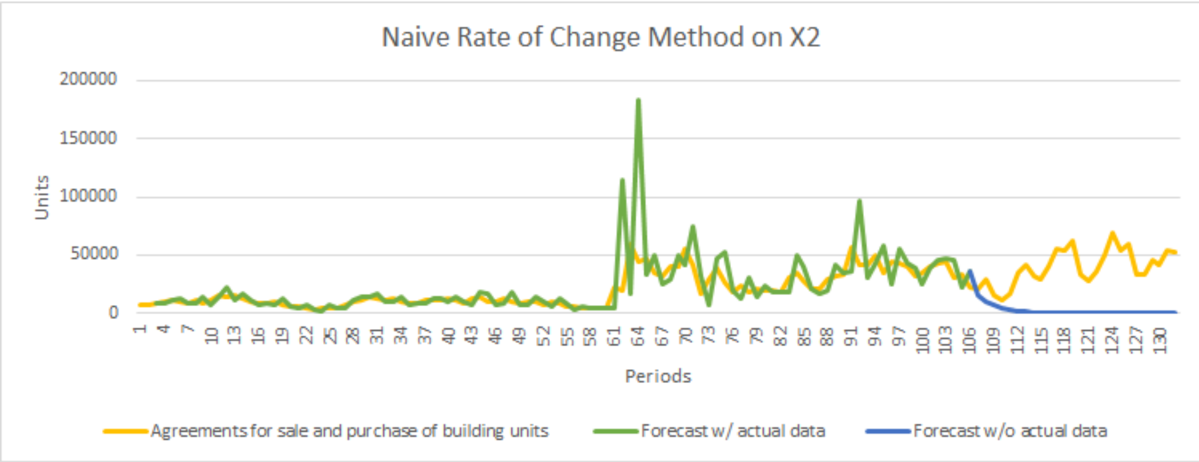
<u>**Graphs for forecasting X2 variable**</u>

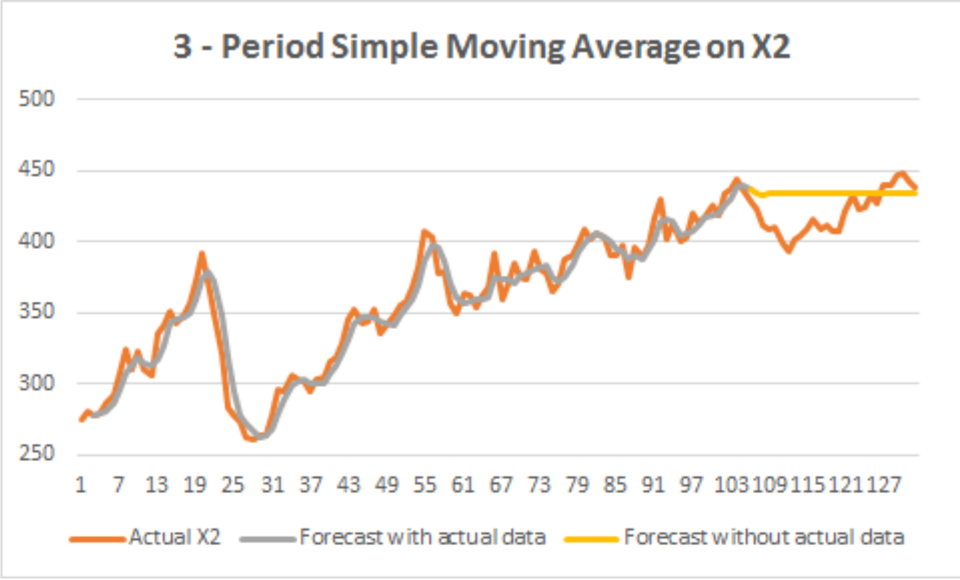


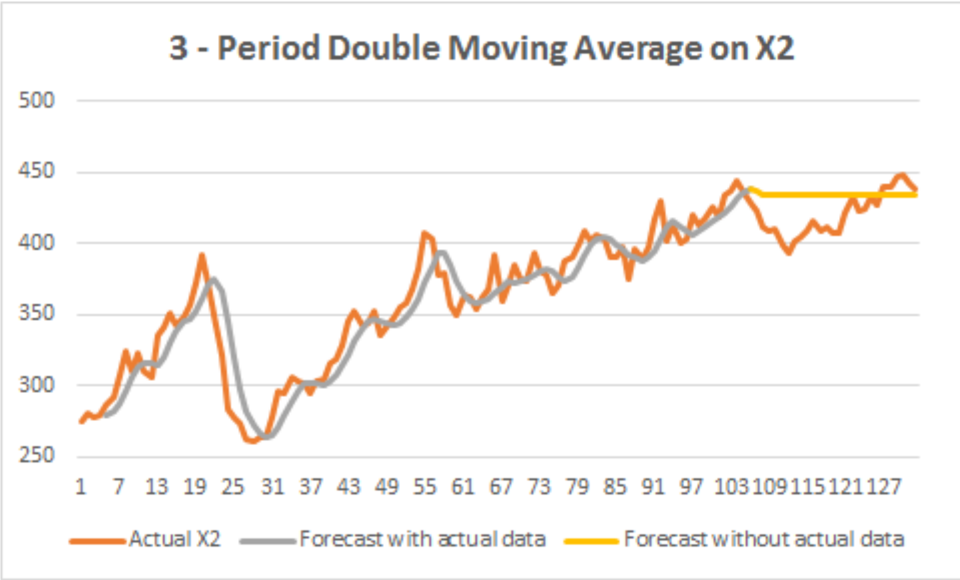


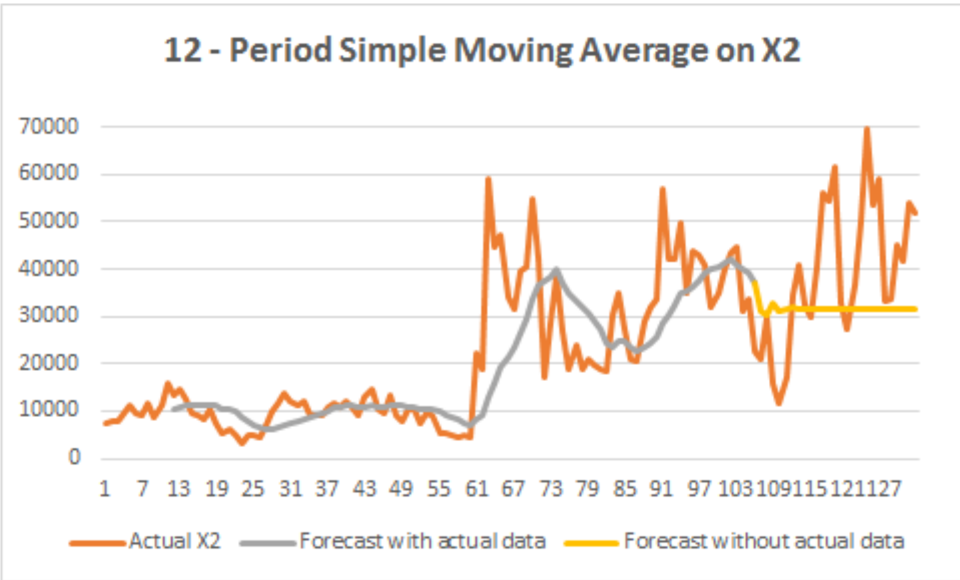


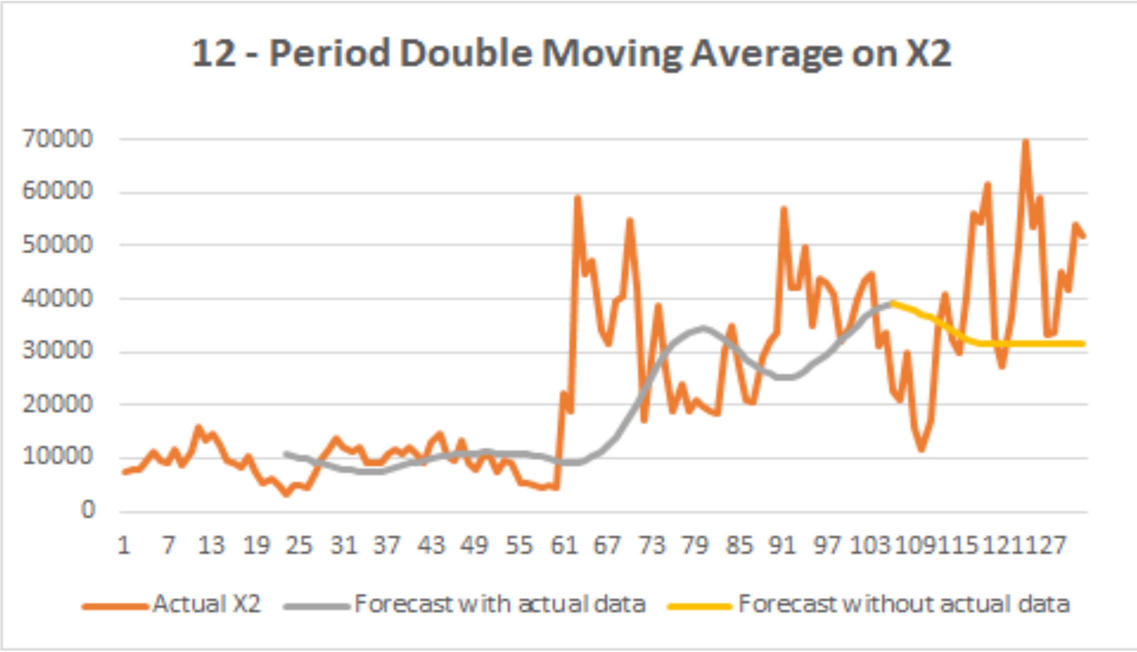

Naive Trend and Seasonal Method on X2
Units
Periods
Agreements for sale and purchase of building units
Forecast w/ actual data
Forecast w/o actual data

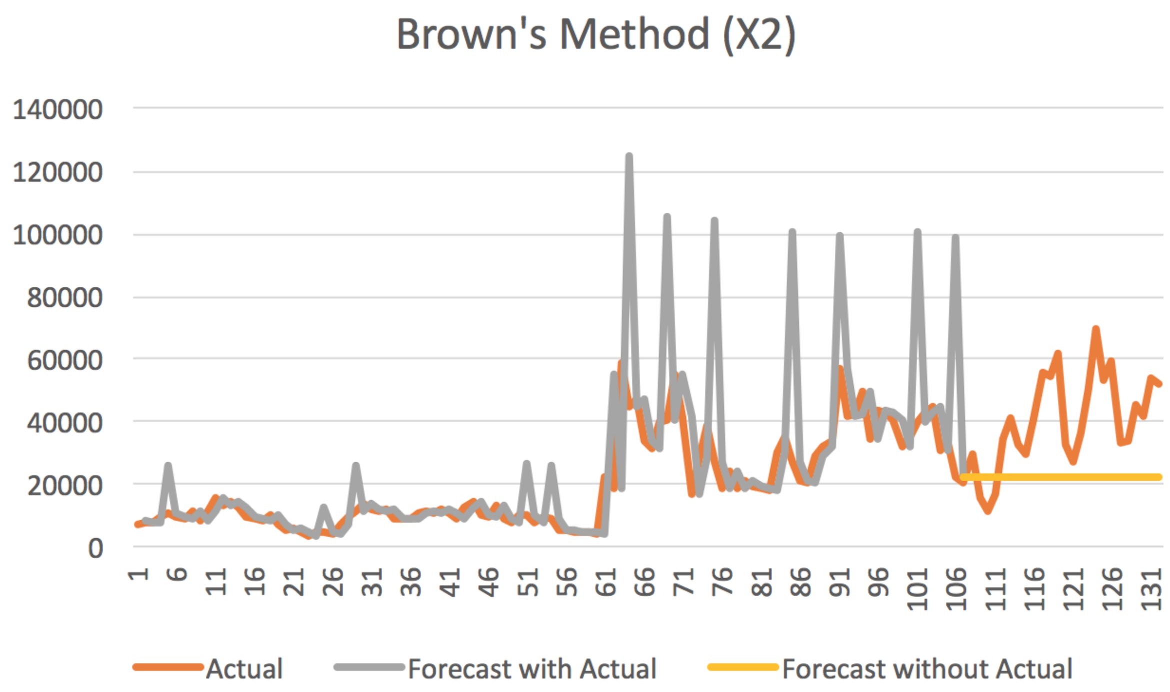

Naive Rate of Change Method on X2
Units
Periods
Agreements for sale and purchase of building units
Forecast w/ actual data
Forecast w/o actual data

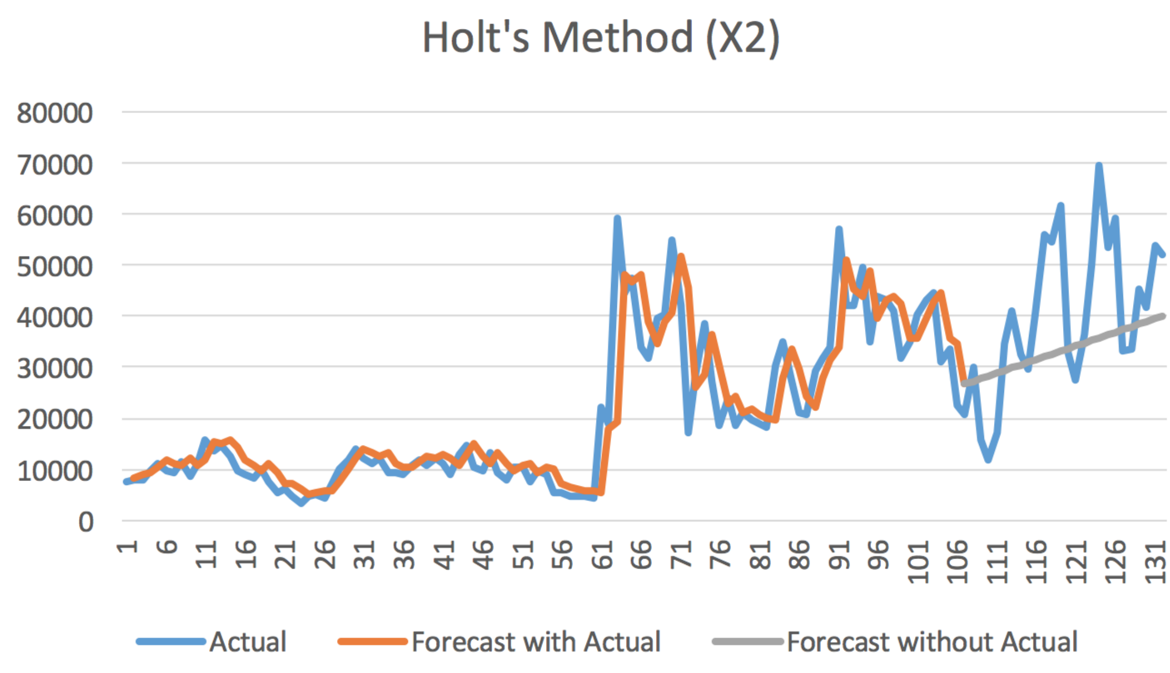

3 - Period Simple Moving Average on X2
500
450
400
350
300
250
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97 103 109 115 121 127
Actual X2
Forecast with actual data
Forecast without actual data

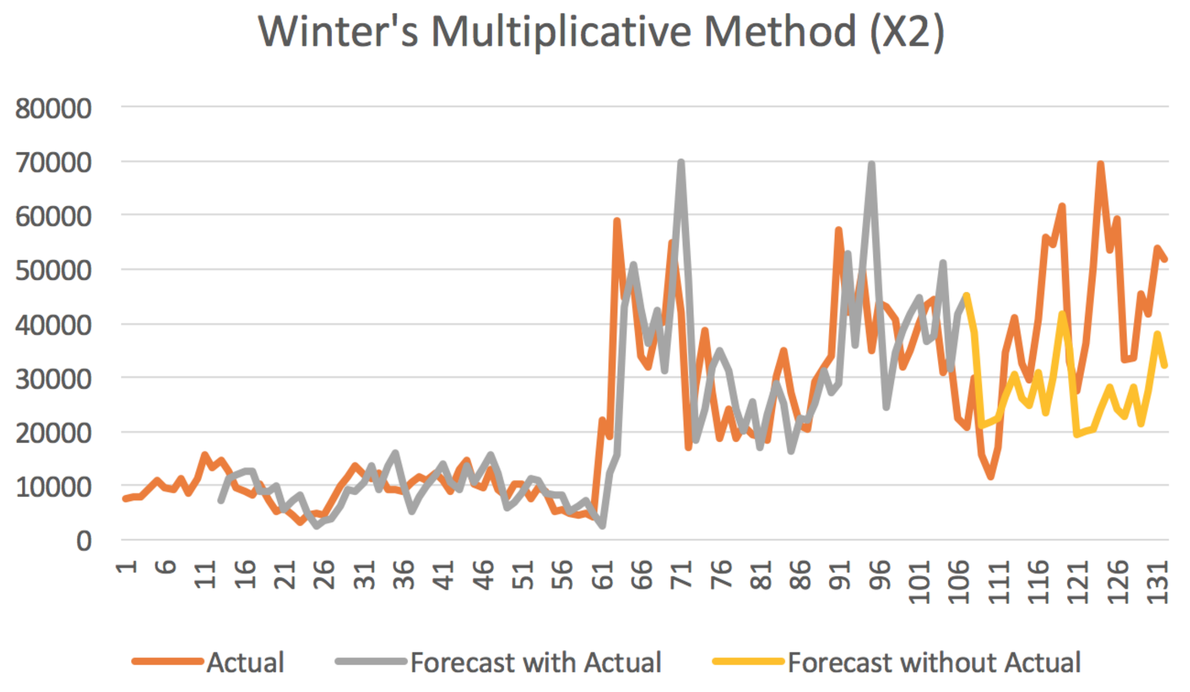

3 - Period Double Moving Average on X2
500
450
400
350
300
250
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97 103 109 115 121 127
Actual X2
Forecast with actual data
Forecast without actual data

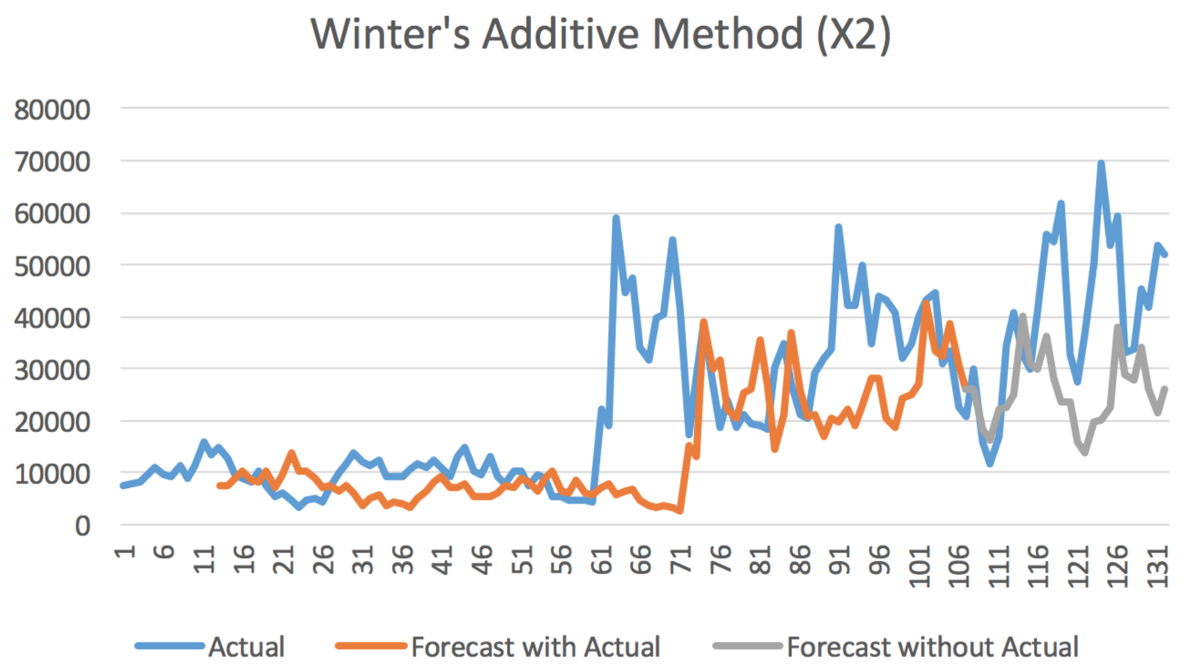

12 - Period Simple Moving Average on X2
70000
60000
50000
40000
30000
20000
10000
0
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97 103 109 115 121 127
Actual X2
Forecast with actual data
Forecast without actual data

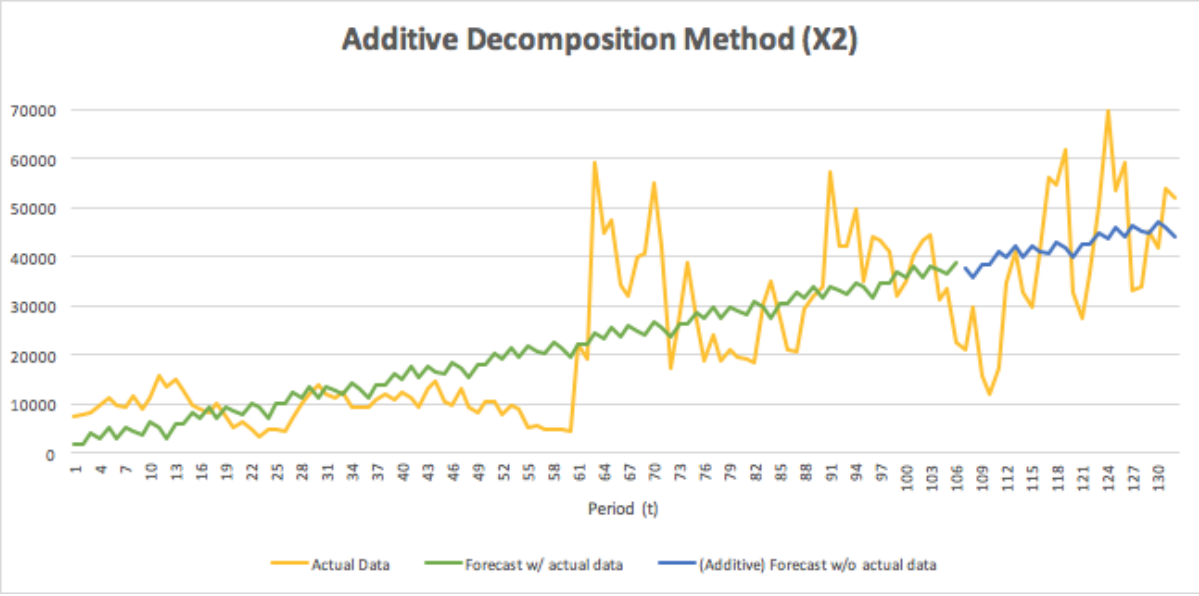

12 - Period Double Moving Average on X2
70000
60000
50000
40000
30000
20000
10000
0
1
7
13
19
25
31
37
43
49
55
61
67
73
79
85
91
97
103
109
115
121
127
Actual X2
Forecast with actual data
Forecast without actual data

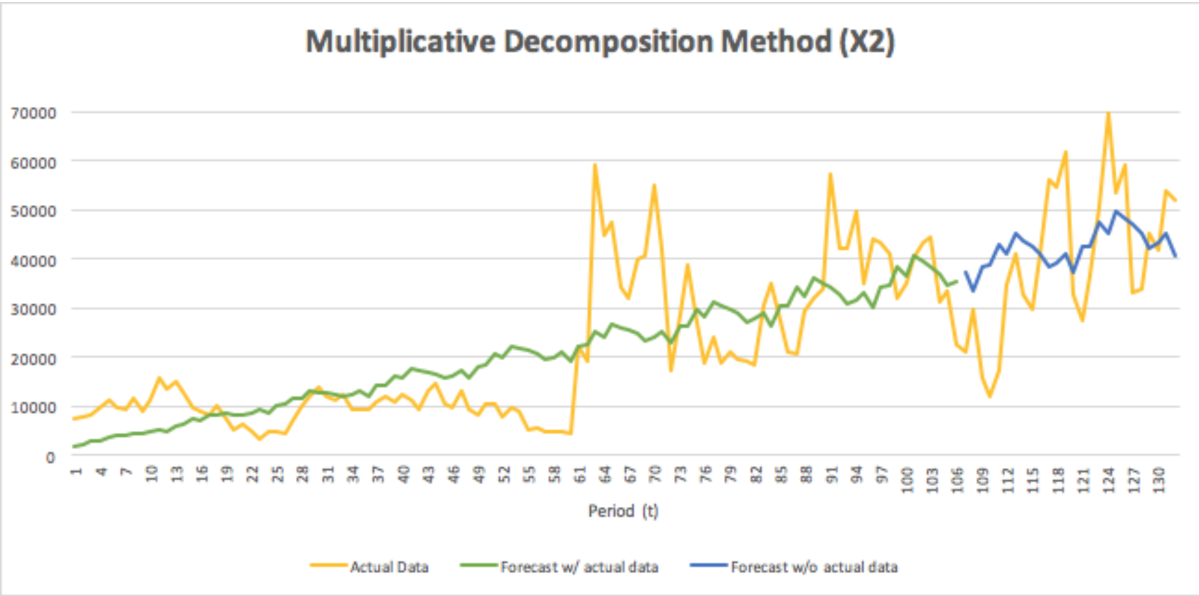

Brown's Method (X2)
140000
120000
100000
80000
60000
40000
20000
0
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Actual
Forecast with Actual
Forecast without Actual

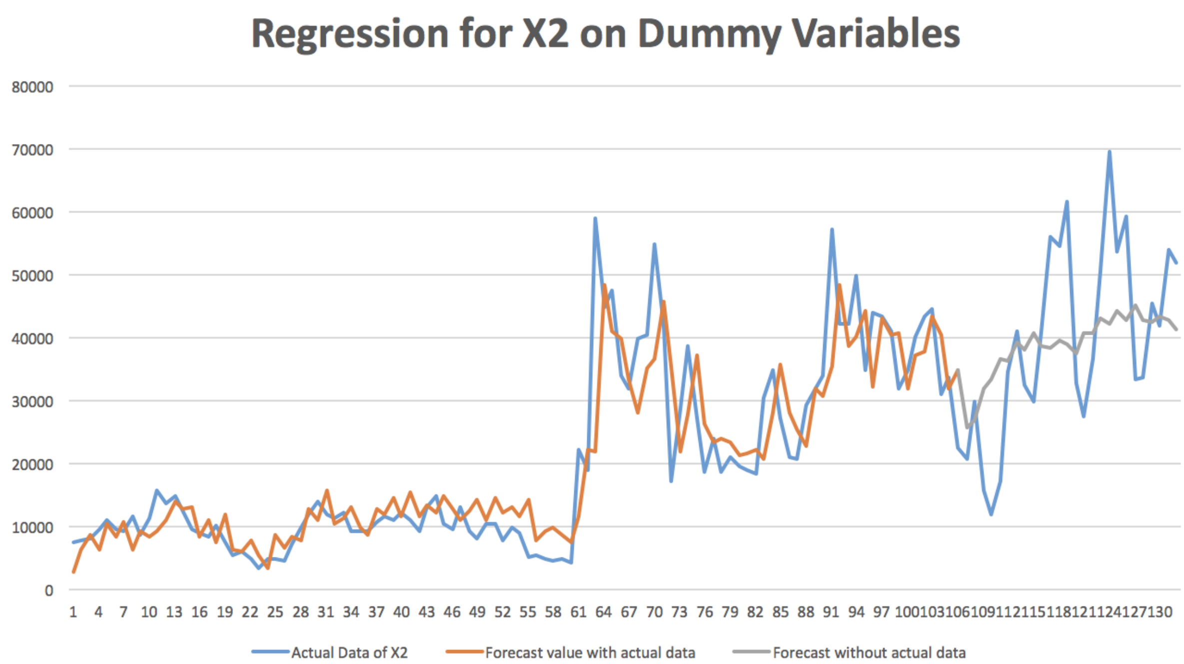

Holt's Method (X2)
80000
70000
60000
50000
40000
30000
20000
10000
0
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Actual
Forecast with Actual
Forecast without Actual


Winter's Multiplicative Method (X2)
80000
70000
60000
50000
40000
30000
20000
10000
0
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Actual
Forecast with Actual
Forecast without Actual


Winter's Additive Method (X2)
80000
70000
60000
50000
40000
30000
20000
10000
0
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131
Actual
Forecast with Actual
Forecast without Actual


Additive Decomposition Method (X2)
70000
60000
50000
40000
30000
20000
10000
0
Period (t)
Actual Data
Forecast w/ actual data
(Additive) Forecast w/o actual data

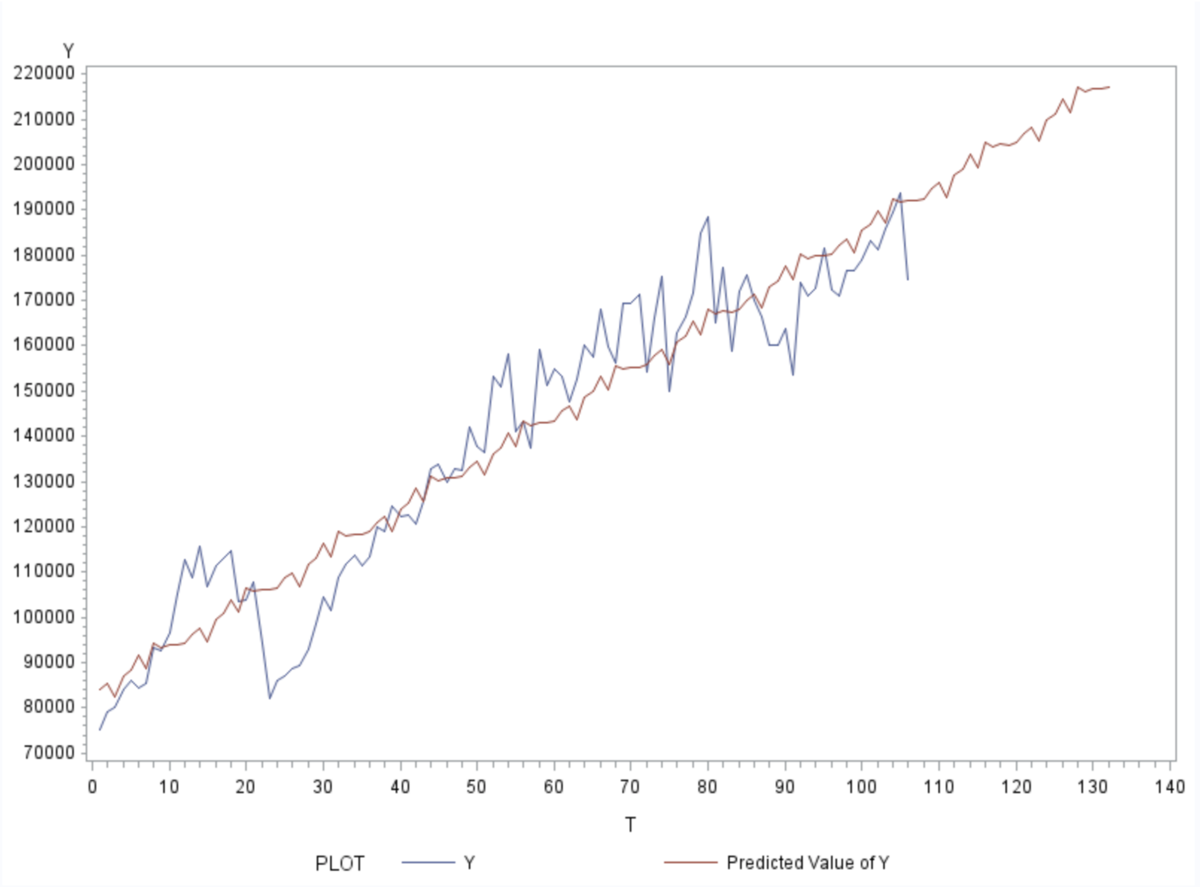

Multiplicative Decomposition Method (X2)
70000
60000
50000
40000
30000
20000
10000
0
Period (t)
Actual Data
Forecast w/ actual data
Forecast w/o actual data


Regression for X2 on Dummy Variables
80000
70000
60000
50000
40000
30000
20000
10000
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100103106109112115118121124127130
Actual Data of X2
Forecast value with actual data
Forecast without actual data

Graphs for SAS Regression Model

Regression for Y on T and Dummy Variables

Graph for forecast Y after Autoregression



The SAS System

The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	5479319650	DFE	92
MSE	59557822	Root MSE	7717
SBC	2249.53128	AIC	2212.24313
MAE	5481.16852	AICC	2216.85852
MAPE	3.9827051	HQC	2227.35622
Durbin-Watson	2.2779	Transformed Regression R-Square	0.6178
		Total R-Square	0.9528

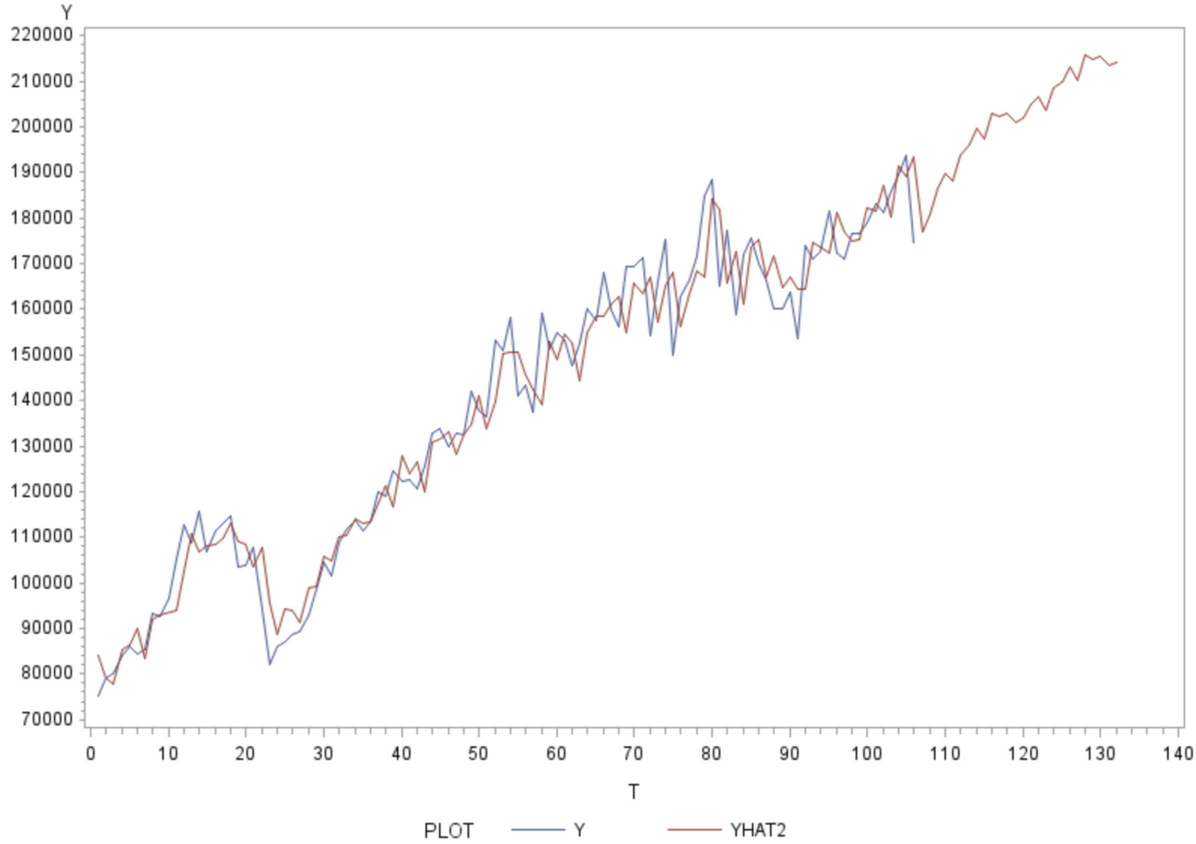
Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	2.2779	0.9111	0.0889

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	80761	6037	13.38	<.0001
T	1	1011	87.0875	11.60	<.0001
Q1	1	2175	2889	0.75	0.4535
Q2	1	2581	3697	0.70	0.4869
Q3	1	-1349	4165	-0.32	0.7468
Q4	1	2566	4445	0.58	0.5651
Q5	1	2944	4597	0.64	0.5236
Q6	1	5167	4649	1.11	0.2693
Q7	1	1269	4609	0.28	0.7836
Q8	1	5721	4469	1.28	0.2037
Q9	1	3768	4203	0.90	0.3723
Q10	1	3223	3752	0.86	0.3926
Q11	1	319.5177	2894	0.11	0.9123
AR1	1	-0.7367	0.0730	-10.09	<.0001

Autoregressive parameters assumed given					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	80761	6031	13.39	<.0001
T	1	1011	86.5776	11.67	<.0001
Q1	1	2175	2889	0.75	0.4534
Q2	1	2581	3697	0.70	0.4869
Q3	1	-1349	4165	-0.32	0.7468
Q4	1	2566	4445	0.58	0.5651
Q5	1	2944	4597	0.64	0.5235
Q6	1	5167	4648	1.11	0.2692
Q7	1	1269	4607	0.28	0.7835
Q8	1	5721	4465	1.28	0.2033
Q9	1	3768	4198	0.90	0.3717
Q10	1	3223	3746	0.86	0.3918
Q11	1	319.5177	2893	0.11	0.9123

Graph for forecast Y after Autoregression



Regression for Y on X1 and X2 Variables

The SAS System

The CORR Procedure

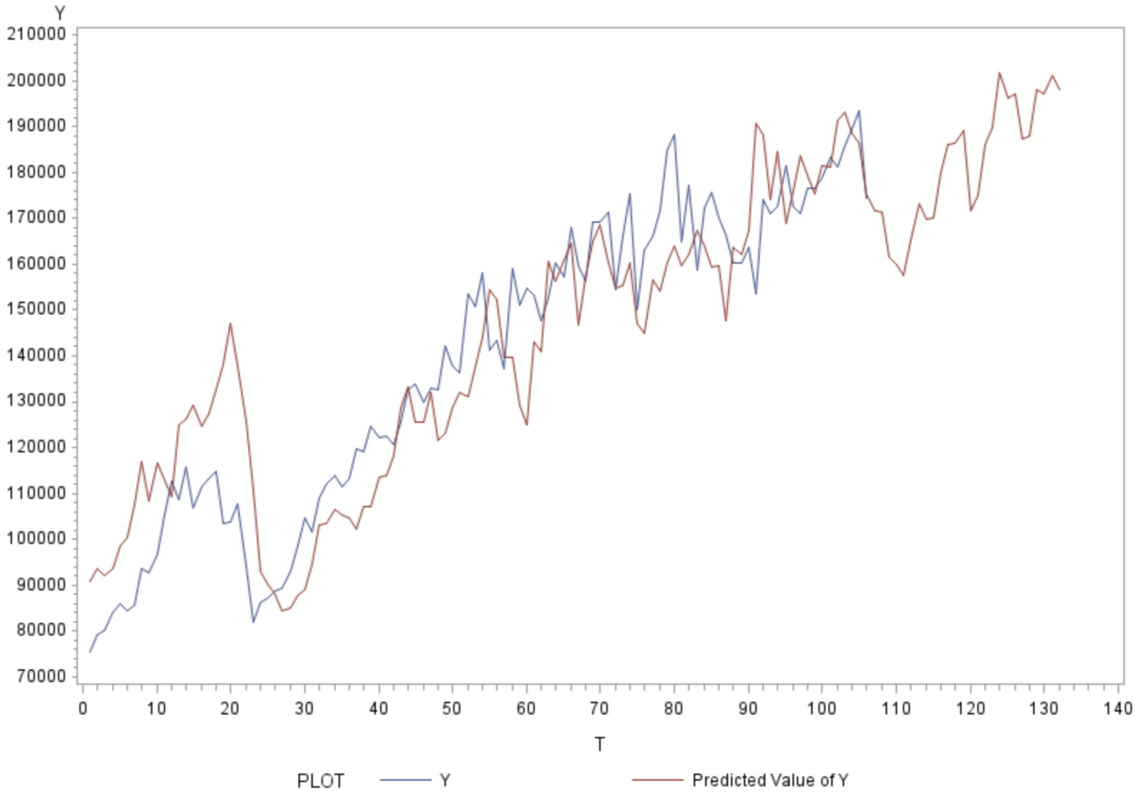
3 Variables:	Y X1 X2

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
Y	106	137901	33260	14617558	75317	193570
X1	132	368.51818	50.78063	48644	261.00000	448.80000
X2	132	23791	16517	3140382	3264	69582

Pearson Correlation Coefficients
Prob > |r| under H0: Rho=0
Number of Observations

	Y	X1	X2
Y	1.00000 106	0.87665 <.0001 106	0.70323 <.0001 106
X1	0.87665 <.0001 106	1.00000 132	0.66856 <.0001 132
X2	0.70323 <.0001 106	0.66856 <.0001 132	1.00000 132



The SAS System

The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	7027345213	**DFE**	102
MSE	68895541	**Root MSE**	8300
SBC	2231.84127	**AIC**	2221.18752
MAE	6102.01412	**AICC**	2221.58356
MAPE	4.40934776	**HQC**	2225.50554
Durbin-Watson	2.2621	**Transformed Regression R-Square**	0.0171
		Total R-Square	0.9395

Durbin-Watson Statistics

Order	DW	Pr < DW	Pr > DW
1	2.2621	0.9097	0.0903

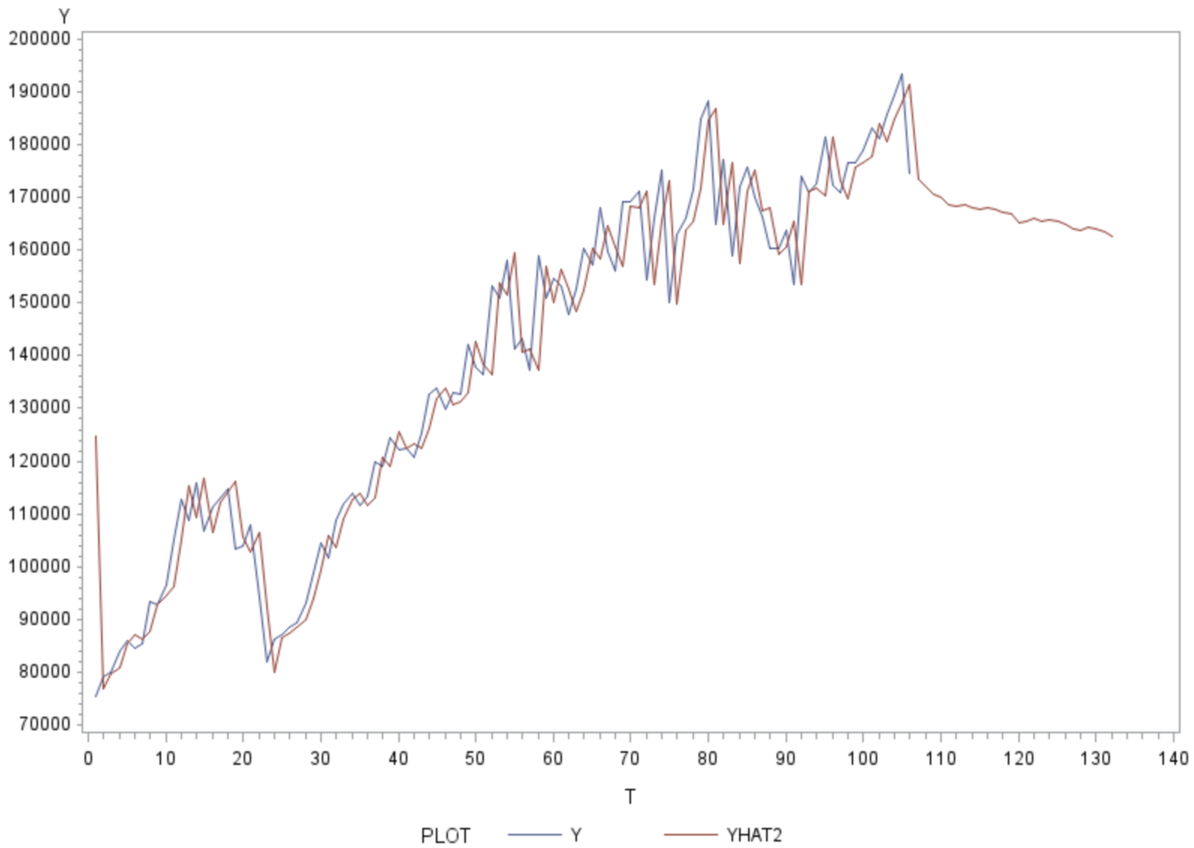
NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	102606	41901	2.45	0.0160
X1	1	79.1372	62.3148	1.27	0.2070
X2	1	0.0411	0.1078	0.38	0.7040
AR1	1	-0.9823	0.0224	-43.80	<.0001

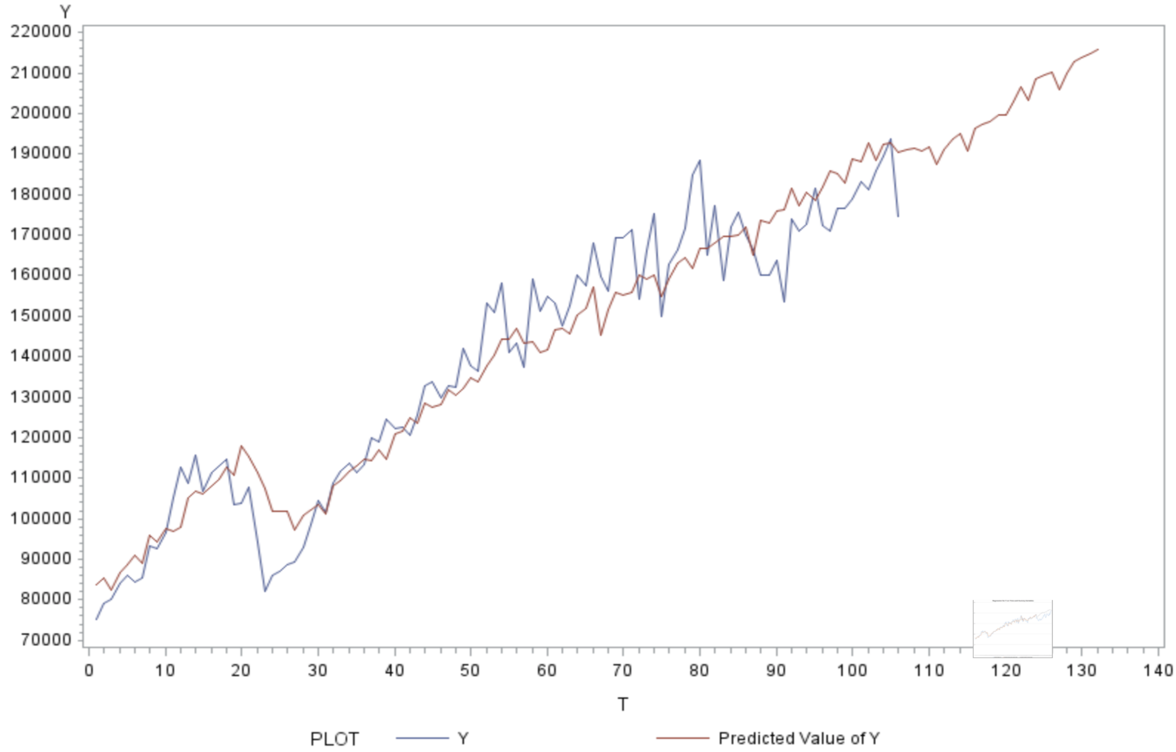
Autoregressive parameters assumed given

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	102606	38642	2.66	0.0092
X1	1	79.1372	61.3642	1.29	0.2001
X2	1	0.0411	0.1078	0.38	0.7039



Y
200000
190000
180000
170000
160000
150000
140000
130000
120000
110000
100000
90000
80000
70000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
T
PLOT
Y
YHAT2

Regression for Y on Time, Dummy, X1 and X2 Variables



The SAS System

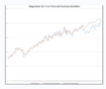
The AUTOREG Procedure

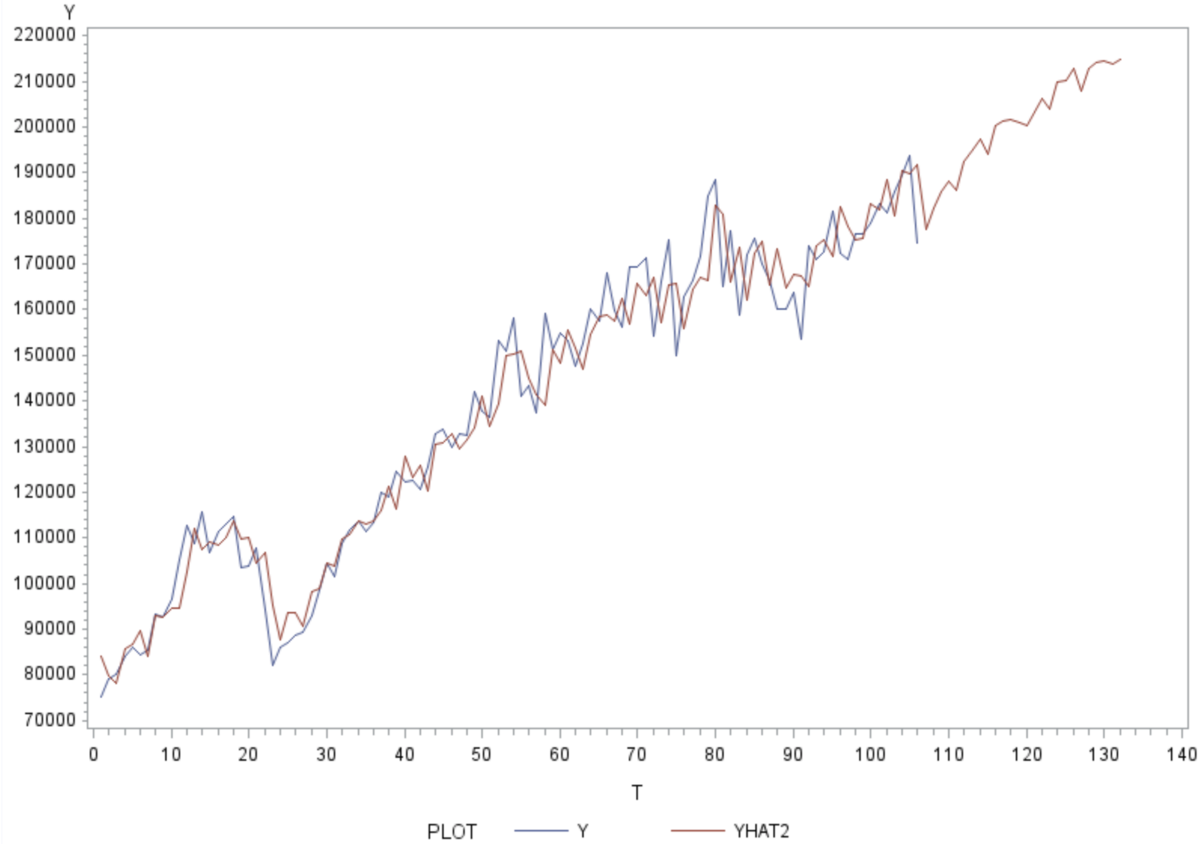
Unconditional Least Squares Estimates			
SSE	5390324969	DFE	90
MSE	59892500	Root MSE	7739
SBC	2256.9897	AIC	2214.37468
MAE	5431.30421	AICC	2220.48704
MAPE	3.91422256	HQC	2231.64678
Durbin-Watson	2.2745	Transformed Regression R-Square	0.6854
		Total R-Square	0.9536

Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	2.2745	0.9016	0.0984

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	60002	20268	2.96	0.0039
T	1	887.8747	122.4726	7.25	<.0001
Q1	1	1613	2968	0.54	0.5881
Q2	1	1941	3754	0.52	0.6064
Q3	1	-1712	4194	-0.41	0.6841
Q4	1	1967	4442	0.44	0.6590
Q5	1	1945	4618	0.42	0.6746
Q6	1	3641	4754	0.77	0.4458
Q7	1	-1010	4907	-0.21	0.8373
Q8	1	2908	5056	0.58	0.5666
Q9	1	1936	4475	0.43	0.6663
Q10	1	1430	4056	0.35	0.7252
Q11	1	-504.9944	3020	-0.17	0.8676
X1	1	76.0484	70.9297	1.07	0.2865
X2	1	0.0818	0.1102	0.74	0.4603
AR1	1	-0.6913	0.0864	-8.00	<.0001




Y
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200000
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180000
170000
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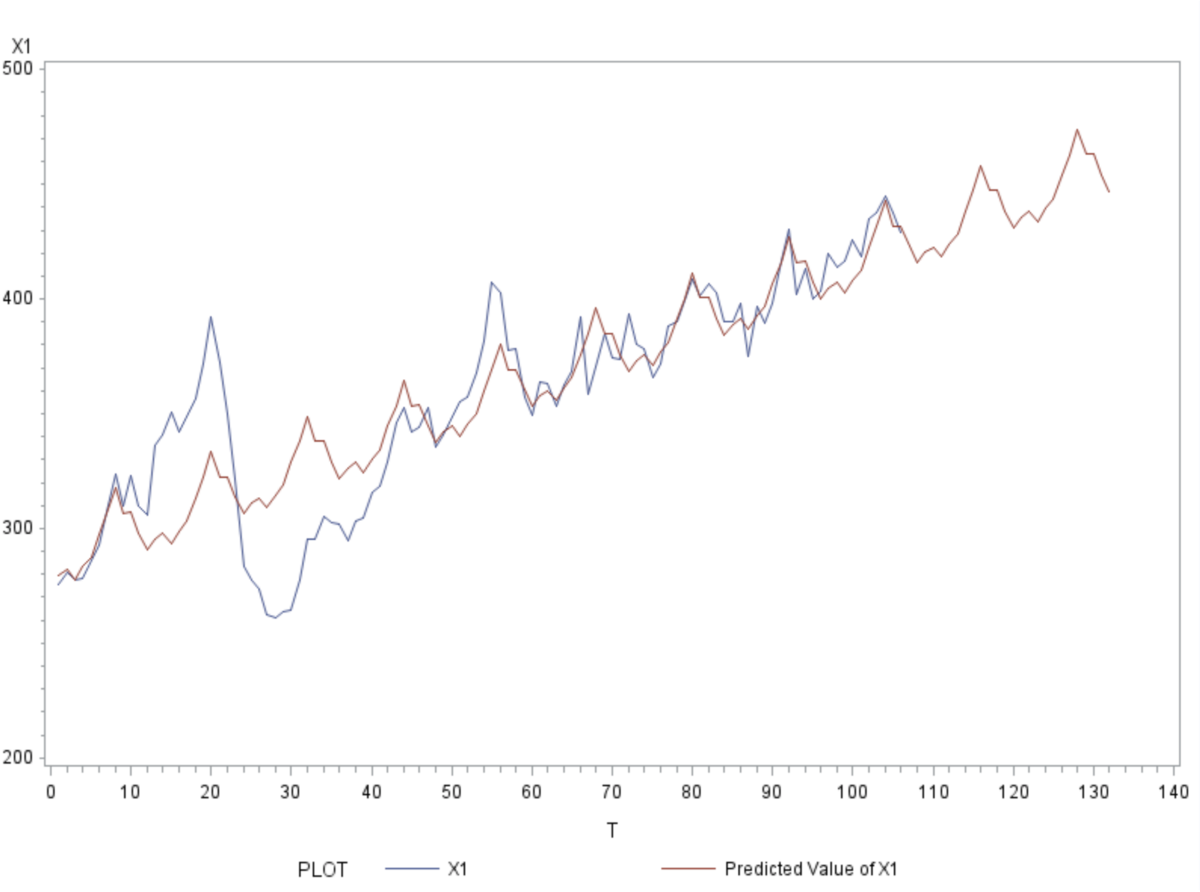
T
PLOT
Y
YHAT2

4. Regression for X1 on Time and Dummy Variables



The SAS System

The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	11872.0161	DFE	92
MSE	129.04365	Root MSE	11.35974
SBC	867.882351	AIC	830.594204
MAE	7.98753087	AICC	835.209589
MAPE	2.27357529	HQC	845.707293
Durbin-Watson	2.0083	Transformed Regression R-Square	0.4128
		Total R-Square	0.9514

Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	2.0083	0.5117	0.4883

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	273.8434	18.1178	15.11	<.0001
T	1	1.3063	0.2770	4.72	<.0001
Q1	1	3.7705	4.0161	0.94	0.3503
Q2	1	4.9235	5.2493	0.94	0.3507
Q3	1	-0.7732	6.0247	-0.13	0.8982
Q4	1	3.4584	6.5184	0.53	0.5970
Q5	1	6.4849	6.7993	0.95	0.3427
Q6	1	15.2175	6.8974	2.21	0.0299
Q7	1	22.9895	6.8217	3.37	0.0011
Q8	1	32.7117	6.5646	4.98	<.0001
Q9	1	20.4285	6.0978	3.35	0.0012
Q10	1	19.2507	5.3573	3.59	0.0005
Q11	1	8.5434	4.0168	2.13	0.0361
AR1	1	-0.8954	0.0464	-19.28	<.0001

The SAS System

The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	11872.0161	DFE	92
MSE	129.04365	Root MSE	11.35974
SBC	867.882351	AIC	830.594204
MAE	7.98753087	AICC	835.209589
MAPE	2.27357529	HQC	845.707293
Durbin-Watson	2.0083	Transformed Regression R-Square	0.4128
		Total R-Square	0.9514

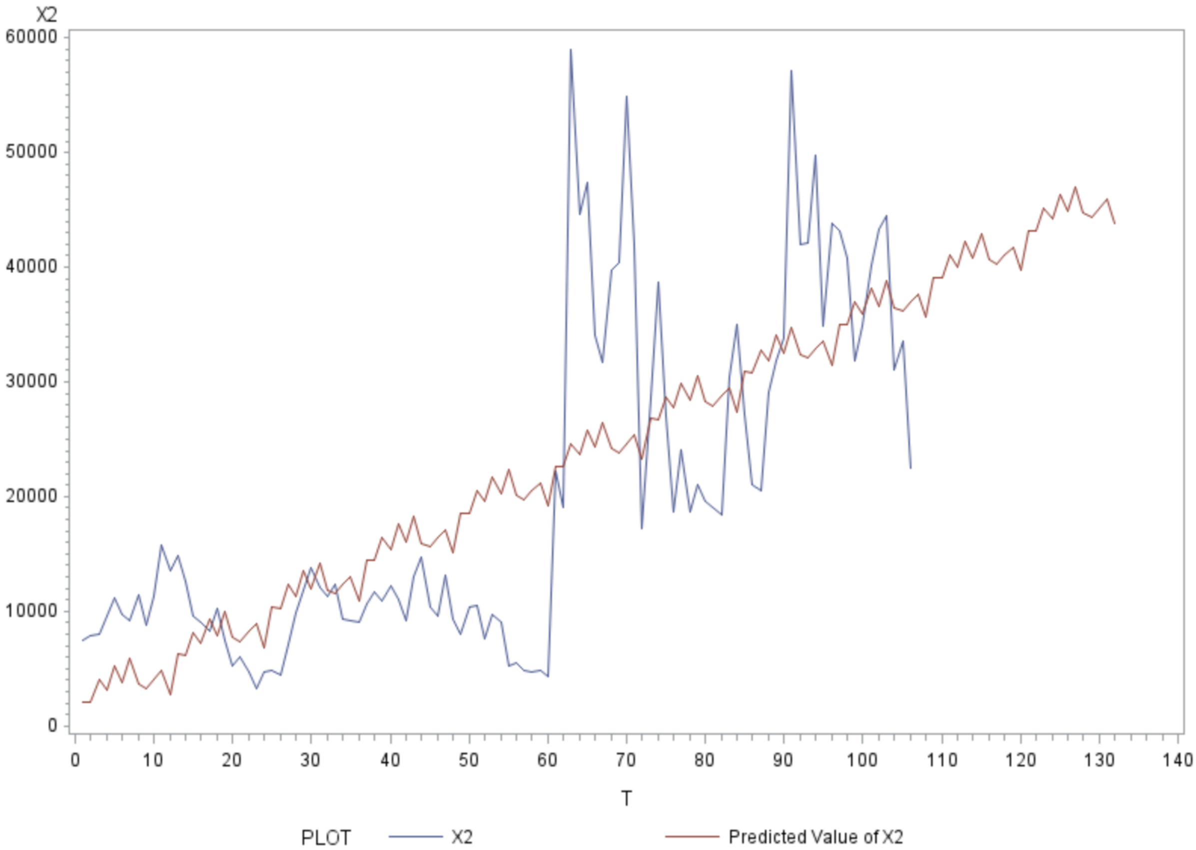
Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	2.0083	0.5117	0.4883

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	273.8434	18.1178	15.11	<.0001
T	1	1.3063	0.2770	4.72	<.0001
Q1	1	3.7705	4.0161	0.94	0.3503
Q2	1	4.9235	5.2493	0.94	0.3507
Q3	1	-0.7732	6.0247	-0.13	0.8982
Q4	1	3.4584	6.5184	0.53	0.5970
Q5	1	6.4849	6.7993	0.95	0.3427
Q6	1	15.2175	6.8974	2.21	0.0299
Q7	1	22.9895	6.8217	3.37	0.0011
Q8	1	32.7117	6.5646	4.98	<.0001
Q9	1	20.4285	6.0978	3.35	0.0012
Q10	1	19.2507	5.3573	3.59	0.0005
Q11	1	8.5434	4.0168	2.13	0.0361
AR1	1	-0.8954	0.0464	-19.28	<.0001

Autoregressive parameters assumed given					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	273.8434	18.1168	15.12	<.0001
T	1	1.3063	0.2769	4.72	<.0001
Q1	1	3.7705	4.0160	0.94	0.3503
Q2	1	4.9235	5.2493	0.94	0.3507
Q3	1	-0.7732	6.0247	-0.13	0.8982
Q4	1	3.4584	6.5184	0.53	0.5970
Q5	1	6.4849	6.7993	0.95	0.3427
Q6	1	15.2175	6.8974	2.21	0.0299
Q7	1	22.9895	6.8217	3.37	0.0011
Q8	1	32.7117	6.5645	4.98	<.0001
Q9	1	20.4285	6.0978	3.35	0.0012
Q10	1	19.2507	5.3572	3.59	0.0005
Q11	1	8.5434	4.0168	2.13	0.0361

5. Regression for X2 on Time and Dummy Variables



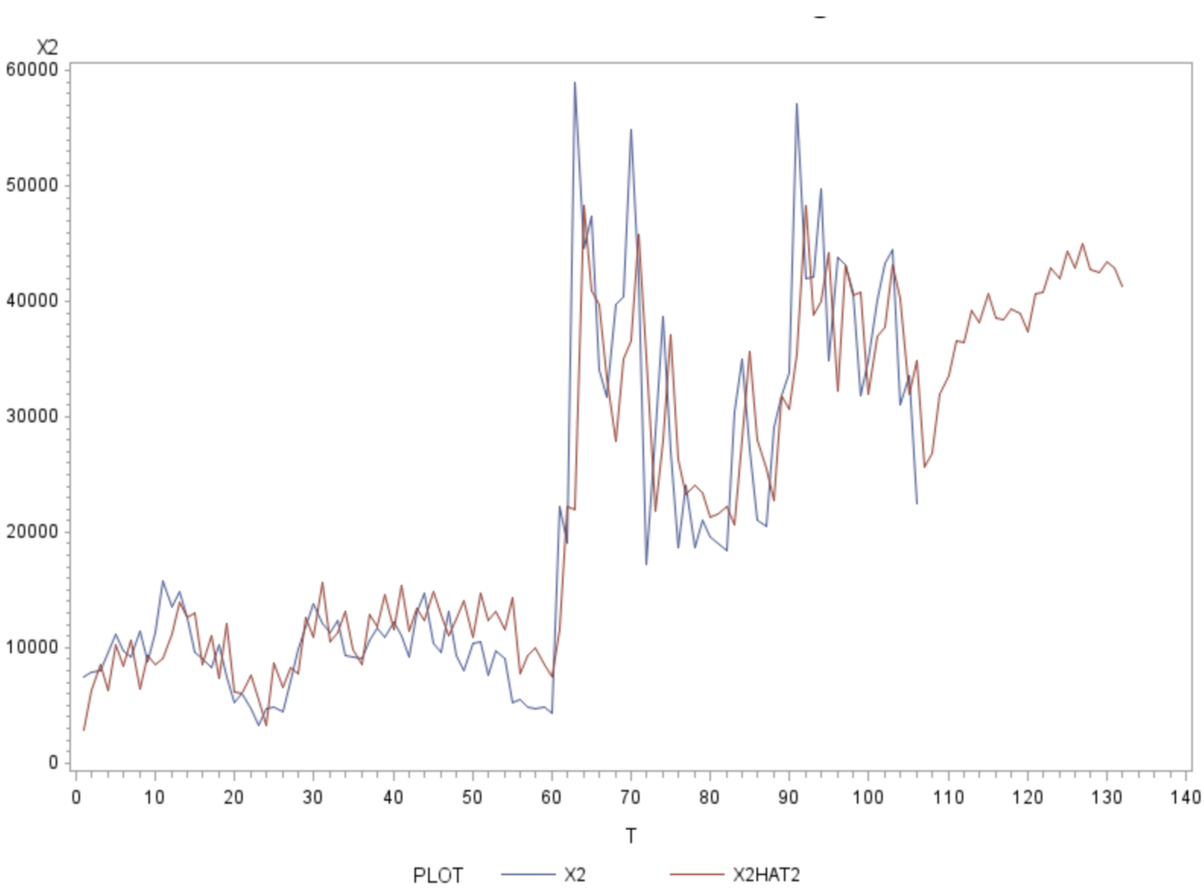
The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	4955499309	DFE	92
MSE	53864123	Root MSE	7339
SBC	2238.82675	AIC	2201.53861
MAE	4585.83653	AICC	2206.15399
MAPE	28.7113863	HQC	2216.6517
Durbin-Watson	2.1182	Transformed Regression R-Square	0.1856
		Total R-Square	0.7719

Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	2.1182	0.7137	0.2863

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-389.7720	5484	-0.07	0.9435
T	1	315.6598	78.0764	4.04	0.0001
Q1	1	2925	2762	1.06	0.2924
Q2	1	2732	3523	0.78	0.4400
Q3	1	4518	3958	1.14	0.2566
Q4	1	3316	4215	0.79	0.4334
Q5	1	5248	4354	1.21	0.2311
Q6	1	3480	4401	0.79	0.4312
Q7	1	5388	4364	1.23	0.2201
Q8	1	2819	4235	0.67	0.5073
Q9	1	2214	3990	0.55	0.5803
Q10	1	2795	3571	0.78	0.4359
Q11	1	1916	2768	0.69	0.4905
AR1	1	-0.7195	0.0742	-9.70	<.0001


X2
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140
T
PLOT
X2
X2HAT2