

Supplementary Pseudocode

Figure S1 demonstrates the algorithm for the naïve all-against-all LCA parser, with pseudocode given in Algorithm S1. Algorithm S2, Algorithm S3, and Algorithm S4 spell out the pseudocode for the Hashing-Intersection algorithm demonstrated in main text Figure 4 of our paper.

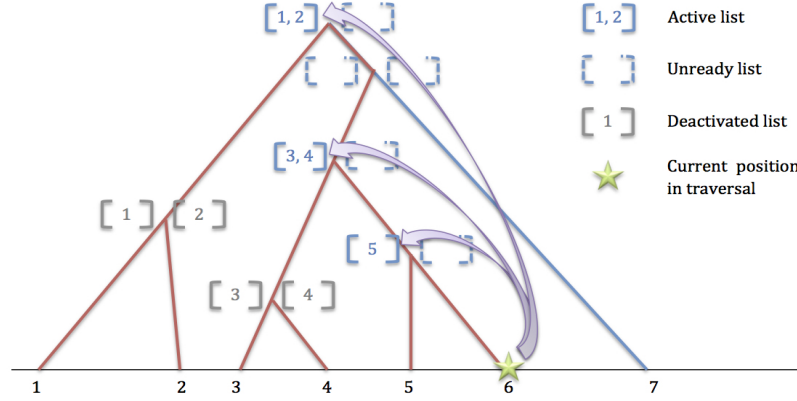


Fig. S1. The algorithm used for parsing a tree to get the all-pair tMRCA. We perform an in-order traversal of the tree using a stack data structure to maintain the lineage leading from the root to the currently traversed node, whose elements are lists representing all internal nodes along the lineage, containing either all the leaves from the left sub-tree of the corresponding internal node (when we are currently in its right sub-tree), or nothing (when we are currently in its left sub-tree), and the height of the internal nodes. When we get to a new leaf, we can report the tMRCA of this leaf with all previously traversed leaves according to the records in the stack. The traversal only needs linear time, but as we need to report a tMRCA for each pair of leaves, the complexity of the output is the overall time complexity, $O(n^2)$.

Algorithm S1 In-Order Traversal All-Pair-tMRCA Extraction

```

PARSELCA(T)
1  A = {}
2  Tokens = tokenize T into ["(" ")" " " numbers ";"]*
3  Leaves[] = [] // list for leaves
4  TMRCAs[] = [] // list for tmrca's
5  for token in Tokens
6      if token == "(" | ";"
7          pass
8      if token == "(" // new internal node
9          Leaves.push([ ]) // prepared for storing leaves under this internal node
10         TMRCAs.push(None) // not yet known the height of this internal node
11     if token == numbers
12         parse the numbers into leaf and tmrca
13         Leaves[-1].append(leaf) // Leaves[-1] is the last element of Leaves
14         if TMRCAs[-1] == None
15             TMRCAs[-1] = tmrca
16         for i in range(Leaves)
17             for leaf1 in Leaves[i]
18                 hash([leaf, leaf1]) to TMRCAs[i] in A
19     if token == ")"
20         if Leaves has reached it's bottom
21             pass
22         else
23             index = Leaves.last_index() // get the last element's index of Leaves
24             Leaves[index - 1].extend(Leaves[index])
25             Leaves.pop()
26             get tmrca from next numbers; move forward token's pointer
27             if TMRCAs[index - 1] == None
28                 TMRCAs[index - 1] = TMRCAs[index] + tmrca
29                 TMRCAs.pop()
30             else
31                 TMRCAs.pop()
32  return A
  
```

Algorithm S2 Hashing Based Block-Finding Algorithm (Part 1)

```

TREEPREPROCESS(tree1, n)
1  Repo1[] , Tempo[] = {}, {} // the final and temporary hashtable for tree1
2  List1[] = [] // in-order leaves
3  List2[] = [0]n // order look-up table
4  Stack[] = [] // cache the unprocessed internal nodes
5  L[] = tokenize tree1 into ["(" ")" "," numbers ";"]*
6  for token in L
7      if token == " , " | " ; "
8          pass
9      if token == "(" // a new internal node
10         Stack.push(None)
11     if token == numbers
12         parse the numbers into sample and tMRCa
13         List1.append(sample)
14         List2[sample - 1] = List1.index(sample) // sample begins from 1
15         if Stack[-1] == None
16             Stack[-1] = tMRCa
17             list = [tMRCa, List2[sample - 1], List2[sample - 1], None]
18             if tMRCa in Tempo // may be duplicated
19                 Tempo[tMRCa].append(list)
20             else
21                 Tempo[tMRCa] = [list]
22         else
23             Tempo[Stack[-1]][-1][3] = List2[sample - 1]
24     if token == ")"
25         if Stack has reached it's bottom
26             list = Tempo[Stack[-1]].pop()
27             if Stack[-1] in Repo1
28                 Repo1[Stack[-1]].append(list)
29             else
30                 Repo1[Stack[-1]] = [list]
31         else
32             get tMRCa from next numbers; move forward token's pointer
33             start, end = Tempo[Stack[-1]][-1][1], ...[3]
34             list = Tempo[Stack[-1]].pop()
35             if Stack[-1] in Repo1
36                 Repo1[Stack[-1]].append(list)
37             else
38                 Repo1[Stack[-1]] = [list]
39             tMRCa+ = Stack.pop()
40             if Stack[-1] == None
41                 Stack[-1] = tMRCa
42                 list = [tMRCa, start, end, None]
43                 if tMRCa in Tempo
44                     Tempo[tMRCa].append(list)
45                 else
46                     Tempo[tMRCa] = [list]
47             else
48                 Tempo[Stack[-1]][-1][3] = end
49  return Repo1, List1, List2

```

Algorithm S3 Hashing Based Block-Finding Algorithm (Part 2)

```

TREEPOSTPROCESS(tree2, Repo1, List1, List2, n)
1  Tempo[] = {} // the temporary hashtable for tree2
2  Result[] = {} // the final result hashtable
3  List3[] = [] // in-order leaves and their indices in List1
4  Stack[] = [] // cache the unprocessed internal nodes
5  L[] = tokenize tree2 into ["(" ")" " " " " numbers ";"]*
6  for token in L
7      if token == " " | ";"
8          pass
9      if token == "("
10         Stack.push(None)
11     if token == numbers
12         parse the numbers into sample and tMRCA
13         List3.append(List2[sample - 1]) // sample begins from 1
14         if Stack[-1] == None // come from left branch
15             Stack[-1] = tMRCA
16             start = List3.index(List2[sample - 1])
17             list = [tMRCA, start, start, None]
18             if tMRCA in Tempo // may be duplicated
19                 Tempo[tMRCA].append(list)
20             else
21                 Tempo[tMRCA] = [list]
22         else // come from right branch
23             Tempo[Stack[-1]][-1][3] = List3.index(List2[sample - 1])
24     if token == ")"
25         tMRCA = Stack[-1]
26         if tMRCA in Repo1
27             start, middle, end = Tempo[tMRCA][-1][1], ...[2], ...[3]
28             listleft = List3[start : middle]
29             listright = List3[middle + 1 : end]
30             INTERSECTION(tMRCA, listleft, listright, Repo1, List1, Result)
31         if Stack has reached its bottom
32             Tempo[Stack[-1]].pop()
33         else
34             get tMRCA from next numbers; move forward token's pointer
35             start, end = Tempo[Stack[-1]][-1][1], ...[3]
36             tMRCA+ = Stack.pop()
37             if Stack[-1] == None
38                 Stack[-1] = tMRCA
39                 list = [tMRCA, start, end, None]
40                 if tMRCA in Tempo
41                     Tempo[tMRCA].append(list)
42                 else
43                     Tempo[tMRCA] = [list]
44             else
45                 Tempo[Stack[-1]][-1][3] = end
46  return Result

```

Algorithm S4 Hashing Based Block-Finding Algorithm (Part 3)

INTERSECTION($tMRC A, list_{left}, list_{right}, Repo_1, List_1, Result$)

```

1   $list_{candidate} = Repo_1[tMRC A]$ 
2  for  $list$  in  $list_{candidate}$ 
    // we should greedily search evidence for unchanged pairs
3      for  $can_{left}, can_{right} = list_{left}, list_{right}$ 
          and  $can_{left}, can_{right} = list_{right}, list_{left}$ 
        // we should perform both intersection and reverse intersection
4           $left, right = [], []$ 
5          for  $leaf$  in  $can_{left}$ 
6              if  $leaf$  in range  $[list[1], list[2]]$ 
7                   $left.append(List_1[leaf])$ 
8          for  $leaf$  in  $can_{right}$ 
9              if  $leaf$  in range  $(list[2], list[3]]$ 
10                  $right.append(List_1[leaf])$ 
11          if  $left$  and  $right$  not empty
12              Hash  $(left \times right)$  to  $tMRC A$  in  $Result$ 
              //  $\times$  is cartesian product, and each pair will be hashed

```
