2023 CFA® Exam Prep

SchweserNotes™

Fixed Income, Derivatives, and Alternative Investments

LEVEL I BOOK 4

KAPLAN SCHWESER

Book 4: Fixed Income, Derivatives, and Alternative Investments

SchweserNotes™ 2023

Level I CFA®

SCHWESERNOTES™ 2023 LEVEL I CFA® BOOK 4: FIXED INCOME, DERIVATIVES, AND ALTERNATIVE INVESTMENTS

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READING 42

FIXED-INCOME SECURITIES: DEFINING ELEMENTS

EXAM FOCUS

Here your focus should be on learning the basic characteristics of debt securities and as much of the bond terminology as you can remember. Key items are the coupon structure of bonds and options embedded in bonds: call options, put options, and conversion (to common stock) options.

MODULE 42.1: BOND INDENTURES, REGULATION, AND TAXATION

Video covering this content is available online.

There are two important points about fixed-income securities that we will develop further along in the Fixed Income readings but may be helpful as you study this reading.

- The most common type of fixed-income security is a bond that promises to make a series of interest payments in fixed amounts and to repay the principal amount at maturity. When market interest rates (i.e., yields on bonds) *increase*, the value of such bonds *decreases* because the present value of a bond's promised cash flows decreases when a higher discount rate is used.
- Bonds are rated based on their relative probability of default (failure to make promised payments). Because investors prefer bonds with lower probability of default, bonds with lower credit quality must offer investors higher yields to compensate for the greater probability of default. Other things equal, a decrease in a bond's rating (an increased probability of default) will decrease the price of the bond, thus increasing its yield.

LOS 42.a: Describe basic features of a fixed-income security.

The features of a fixed-income security include specification of:

- The issuer of the bond.
- The maturity date of the bond.
- The par value (principal value to be repaid).
- Coupon rate and frequency.
- Currency in which payments will be made.

Issuers of Bonds

There are several types of entities that issue bonds when they borrow money, including:

- **Corporations.** Often corporate bonds are divided into those issued by financial companies and those issued by nonfinancial companies.
- **Sovereign national governments.** A prime example is U.S. Treasury bonds, but many countries issue sovereign bonds.
- **Non-sovereign governments.** Issued by government entities that are not national governments, such as the state of California or the city of Toronto.
- **Quasi-government entities.** Not a direct obligation of a country's government or central bank. An example is the Federal National Mortgage Association (Fannie Mae).
- **Supranational entities.** Issued by organizations that operate globally such as the World Bank, the European Investment Bank, and the International Monetary Fund (IMF).
- **Special purpose entities.** These are corporations set up to purchase financial assets and issue *asset-backed securities*, which are bonds backed by the cash flows from those assets.

Bond Maturity

The maturity date of a bond is the date on which the principal is to be repaid. Once a bond has been issued, the time remaining until maturity is referred to as the **term to maturity** or **tenor** of a bond.

When bonds are issued, their terms to maturity range from one day to 30 years or more. Both Disney and Coca-Cola have issued bonds with original maturities of 100 years. Bonds that have no maturity date are called **perpetual bonds**. They make periodic interest payments but do not promise to repay the principal amount.

Bonds with original maturities of one year or less are referred to as **money market securities**. Bonds with original maturities of more than one year are referred to as **capital market securities**.

Par Value

The **par value** of a bond is the **principal amount** that will be repaid at maturity. The par value is also referred to as the *face value*, *maturity value*, *redemption value*, or *principal value* of a bond. Bonds can have a par value of any amount, and their prices are quoted as a percentage of par. A bond with a par value of $1,000 quoted at 98 is selling for $980.

A bond that is selling for more than its par value is said to be trading at a **premium** to par; a bond that is selling at less than its par value is said to be trading at a **discount** to par; and a bond that is selling for exactly its par value is said to be trading **at par**.

Coupon Payments

The coupon rate on a bond is the annual percentage of its par value that will be paid to bondholders. Some bonds make coupon interest payments annually, while others make semiannual, quarterly, or monthly payments. A $1,000 par value semiannual-pay bond with a 5% coupon would pay 2.5% of $1,000, or $25, every six months. A bond with a fixed coupon rate is called a **plain vanilla** bond or a **conventional** bond.

Some bonds pay no interest prior to maturity and are called **zero-coupon bonds** or **pure discount bonds**. *Pure discount* refers to the fact that these bonds are sold at a discount to their par value and the interest is all paid at maturity when bondholders receive the par value. A 10-year, $1,000, zero-coupon bond yielding 7% would sell at about $500 initially and pay $1,000 at maturity. We discuss various other coupon structures later in this reading.

Currencies

Bonds are issued in many currencies. Sometimes borrowers from countries with volatile currencies issue bonds denominated in euros or U.S. dollars to make them more attractive to a wide range of investors. A **dual-currency bond** makes coupon interest payments in one currency and the principal repayment at maturity in another currency. A **currency option bond** gives bondholders a choice of which of two currencies they would like to receive their payments in.

LOS 42.b: Describe content of a bond indenture.

LOS 42.c: Compare affirmative and negative covenants and identify examples of each.

The legal contract between the bond issuer (borrower) and bondholders (lenders) is called a **trust deed**, and in the United States and Canada, it is also often referred to as the **bond indenture**. The indenture defines the obligations of and restrictions on the borrower and forms the basis for all future transactions between the bondholder and the issuer.

The provisions in the bond indenture are known as *covenants* and include both *negative covenants* (prohibitions on the borrower) and *affirmative covenants* (actions the borrower promises to perform).

Negative covenants include restrictions on asset sales (the company can't sell assets that have been pledged as collateral), negative pledge of collateral (the company can't claim that the same assets back several debt issues simultaneously), and restrictions on additional borrowings (the company can't borrow additional money unless certain financial conditions are met).

Negative covenants serve to protect the interests of bondholders and prevent the issuing firm from taking actions that would increase the risk of default. At the same time, the covenants must not be so restrictive that they prevent the firm from taking advantage of opportunities that arise or responding appropriately to changing business circumstances.

Affirmative covenants do not typically restrict the operating decisions of the issuer. Common affirmative covenants are to make timely interest and principal payments to bondholders, to insure and maintain assets, and to comply with applicable laws and regulations.

Two examples of affirmative covenants are cross-default and pari passu provisions. A **cross-default** clause states that if the issuer defaults on any other debt obligation, they will also be considered in default on this bond. A **pari passu** clause states that this bond will have the same priority of claims as the issuer's other senior debt issues.

LOS 42.d: Describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities.

Bonds are subject to different legal and regulatory requirements depending on where they are issued and traded. Bonds issued by a firm domiciled in a country and also traded in that country's currency are referred to as **domestic bonds**. Bonds issued by a firm incorporated in a foreign country that trade on the **national bond market** of another country in that country's currency are referred to as **foreign bonds**. Examples include bonds issued by foreign firms that trade in China and are denominated in yuan, which are called *panda bonds*, and bonds issued by firms incorporated outside the United States that trade in the United States and are denominated in U.S. dollars, which are called *Yankee bonds*.

Eurobonds are issued outside the jurisdiction of any one country and denominated in a currency different from the currency of the countries in which they are sold. They are subject to less regulation than domestic bonds in most jurisdictions and were initially introduced to avoid U.S. regulations. Eurobonds should not be confused with bonds denominated in euros or thought to originate in Europe, although they can be both. Eurobonds got the "euro" name because they were first introduced in Europe, and most are still traded by firms in European capitals. A bond issued by a Chinese firm that is denominated in yen and traded in markets outside Japan would fit the definition of a eurobond. Eurobonds that trade in the national bond market of a country other than the country that issues the currency the bond is denominated in, and in the eurobond market, are referred to as **global bonds**.

Eurobonds are referred to by the currency they are denominated in. Eurodollar bonds are denominated in U.S. dollars, and euroyen bonds are denominated in yen. At one time, the majority of eurobonds were issued in **bearer** form. Ownership of bearer bonds is evidenced simply by possessing the bonds, whereas ownership of **registered bonds** is recorded. Bearer bonds may be more attractive than registered bonds to those seeking to avoid taxes. As with most other bonds, eurobonds are now issued in registered form.

Other legal and regulatory issues addressed in a trust deed include:

- Legal information about the entity issuing the bond.
- Any assets (collateral) pledged to support repayment of the bond.
- Any additional features that increase the probability of repayment (credit enhancements).
- Covenants describing any actions the firm must take and any actions the firm is prohibited from taking.

Issuing entities

Bonds are issued by several types of legal entities, and bondholders must be aware of which entity has actually promised to make the interest and principal payments. Sovereign bonds are most often issued by the treasury of the issuing country.

Corporate bonds may be issued by a well-known corporation such as Microsoft, by a subsidiary of a company, or by a holding company that is the overall owner of several operating companies. Bondholders must pay attention to the specific entity issuing the bonds because the credit quality can differ among related entities.

Sometimes an entity is created solely for the purpose of owning specific assets and issuing bonds to provide the funds to purchase the assets. These entities are referred to as **special purpose entities (SPEs)** in the United States and special purpose vehicles (SPVs) in Europe.

Bonds issued by these entities are called **securitized bonds**. As an example, a firm could sell loans it has made to customers to an SPE that issues bonds to purchase the loans. The interest and principal payments on the loans are then used to make the interest and principal payments on the bonds.

Often, an SPE can issue bonds at a lower interest rate than bonds issued by the originating corporation. This is because the assets supporting the bonds are owned by the SPE and are used to make the payments to holders of the securitized bonds even if the company itself runs into financial trouble. For this reason, SPEs are called **bankruptcy remote vehicles** or entities.

Sources of repayment

Sovereign bonds are typically repaid by the tax receipts of the issuing country. Bonds issued by non-sovereign government entities are repaid by either general taxes, revenues of a specific project (e.g., an airport), or by special taxes or fees dedicated to bond repayment (e.g., a water district or sewer district).

Corporate bonds are generally repaid from cash generated by the firm's operations. As noted previously, securitized bonds are repaid from the cash flows of the financial assets owned by the SPE.

Collateral and credit enhancements

Unsecured bonds represent a claim to the overall assets and cash flows of the issuer. **Secured bonds** are backed by a claim to specific assets of a corporation, which reduces their risk of default and, consequently, the yield that investors require on the bonds. Assets pledged to support a bond issue (or any loan) are referred to as **collateral**.

Because they are backed by collateral, secured bonds are *senior* to unsecured bonds. Among unsecured bonds, two different issues may have different priority in the event of bankruptcy or liquidation of the issuing entity. The claim of senior unsecured debt is below (after) that of secured debt but ahead of *subordinated*, or junior, debt.

Sometimes secured debt is referred to by the type of collateral pledged. **Equipment trust certificates** are debt securities backed by equipment such as railroad cars and oil drilling rigs. **Collateral trust bonds** are backed by financial assets, such as stocks and (other) bonds. Be aware that while the term **debentures** refers to unsecured debt in the United States and elsewhere, in Great Britain and some other countries the term refers to bonds collateralized by specific assets.

The most common type of securitized bond is a **mortgage-backed security (MBS)**. The underlying assets are a pool of mortgages, and the interest and principal payments from the mortgages are used to pay the interest and principal on the MBS.

In some countries, especially European countries, financial companies issue **covered bonds**. Covered bonds are similar to asset-backed securities, but the underlying assets (the cover pool), although segregated, remain on the balance sheet of the issuing corporation (i.e., no SPE is created).

Credit enhancement can be either internal (built into the structure of a bond issue) or external (provided by a third party). One method of internal credit enhancement is *overcollateralization*, in which the collateral pledged has a value greater than the par value of the debt issued. One limitation of this method of credit enhancement is that the additional collateral is also the underlying assets, so when asset defaults are high, the value of the excess collateral declines in value.

Two other methods of internal credit enhancement are a *cash reserve fund* and an *excess spread account*. A cash reserve fund is cash set aside to make up for credit losses on the underlying assets. With an excess spread account, the yield promised on the bonds issued is less than the promised yield on the assets supporting the ABS. This gives some protection if the yield on the financial assets is less than anticipated. If the assets perform as anticipated, the excess cash flow from the collateral can be used to retire (pay off the principal on) some of the outstanding bonds.

Another method of internal credit enhancement is to divide a bond issue into *tranches* (French for *slices*) with different seniority of claims. Any losses due to poor performance of the assets supporting a securitized bond are first absorbed by the bonds with the lowest seniority, then the bonds with the next-lowest priority of claims. The most senior tranches in this structure can receive very high credit ratings because the probability is very low that losses will be so large that they cannot be absorbed by the subordinated tranches. The subordinated tranches must have higher yields to compensate investors for the additional risk of default. This is sometimes referred to as *waterfall structure* because available funds first go to the most senior tranche of bonds, then to the next-highest priority bonds, and so forth.

External credit enhancements include surety bonds, bank guarantees, and letters of credit from financial institutions. *Surety bonds* are issued by insurance companies and are a promise to make up any shortfall in the cash available to service the debt. *Bank guarantees* serve the same function. A *letter of credit* is a promise to lend money to the issuing entity if it does not have enough cash to make the promised payments on the covered debt. While all three of these external credit enhancements increase the credit quality of debt issues and decrease their yields, deterioration of the credit quality of the guarantor will also reduce the credit quality of the covered issue.

Taxation of Bond Income

Most often, the interest income paid to bondholders is taxed as ordinary income at the same rate as wage and salary income. The interest income from bonds issued by municipal governments in the United States, however, is most often exempt from national income tax and often from any state income tax in the state of issue.

When a bondholder sells a coupon bond prior to maturity, it may be at a gain or a loss relative to its purchase price. Such gains and losses are considered capital gains income (rather than ordinary taxable income). Capital gains are often taxed at a lower rate than ordinary income. Capital gains on the sale of an asset that has been owned for more than some minimum amount of time may be classified as *long-term* capital gains and taxed at an even lower rate.

Pure-discount bonds and other bonds sold at significant discounts to par when issued are termed **original issue discount (OID)** bonds. Because the gains over an OID bond's tenor as the price moves towards par value are really interest income, these bonds can generate a tax liability even when no cash interest payment has been made. In many tax jurisdictions, a portion of the discount from par at issuance is treated as taxable interest income each year. This tax treatment also allows that the tax basis of the OID bonds is increased each year by the amount of interest income recognized, so there is no additional capital gains tax liability at maturity.

Some tax jurisdictions provide a symmetric treatment for bonds issued at a premium to par, allowing part of the premium to be used to reduce the taxable portion of coupon interest payments.

MODULE QUIZ 42.1

1. A dual-currency bond pays coupon interest in a currency:
   A. of the bondholder's choice.
   B. other than the home currency of the issuer.
   C. other than the currency in which it repays principal.
2. A bond's indenture:
   A. contains its covenants.
   B. is the same as a debenture.
   C. relates only to its interest and principal payments.
3. A clause in a bond indenture that requires the borrower to perform a certain action is *most accurately* described as:
   A. a trust deed.
   B. a negative covenant.
   C. an affirmative covenant.
4. An investor buys a pure-discount bond, holds it to maturity, and receives its par value. For tax purposes, the increase in the bond's value is *most likely* to be treated as:
   A. a capital gain.
   B. interest income.
   C. tax-exempt income.

MODULE 42.2: BOND CASH FLOWS AND CONTINGENCIES

Video covering this content is available online.

LOS 42.e: Describe how cash flows of fixed-income securities are structured.

A typical bond has a **bullet** structure. Periodic interest payments (coupon payments) are made over the life of the bond, and the principal value is paid with the final interest payment at maturity. The interest payments are referred to as the bond's **coupons**. When the final payment includes a lump sum in addition to the final period's interest, it is referred to as a **balloon payment**.

Consider a $1,000 face value 5-year bond with an annual coupon rate of 5%. With a bullet structure, the bond's promised payments at the end of each year would be as follows.

Year	1	2	3	4	5
PMT	$50	$50	$50	$50	$1,050
Principal remaining	$1,000	$1,000	$1,000	$1,000	$0

A loan structure in which the periodic payments include both interest and some repayment of principal (the amount borrowed) is called an **amortizing loan**. If a bond (loan) is **fully amortizing**, this means the principal is fully paid off when the last periodic payment is made. Typically, automobile loans and home loans are fully amortizing loans. If the 5-year, 5% bond in the previous table had a fully amortizing structure rather than a bullet structure, the payments and remaining principal balance at each year-end would be as follows (final payment reflects rounding of previous payments).

Year	1	2	3	4	5
PMT	$230.97	$230.97	$230.97	$230.97	$230.98
Principal remaining	$819.03	$629.01	$429.49	$219.99	$0

A bond can also be structured to be **partially amortizing** so that there is a balloon payment at bond maturity, just as with a bullet structure. However, unlike a bullet structure, the final payment includes just the remaining unamortized principal amount rather than the full principal amount. In the following table, the final payment includes $200 to repay the remaining principal outstanding.

Year	1	2	3	4	5
PMT	$194.78	$194.78	$194.78	$194.78	$394.78
Principal remaining	$855.22	$703.20	$543.58	$375.98	$0

Sinking fund provisions provide for the repayment of principal through a series of payments over the life of the issue. For example, a 20-year issue with a face amount of $300 million may require that the issuer retire $20 million of the principal every year beginning in the sixth year.

Details of sinking fund provisions vary. There may be a period during which no sinking fund redemptions are made. The amount of bonds redeemed according to the sinking fund provision could decline each year or increase each year.

The price at which bonds are redeemed under a sinking fund provision is typically par but can be different from par. If the market price is less than the sinking fund redemption price, the issuer can satisfy the sinking fund provision by buying bonds in the open market with a par value equal to the amount of bonds that must be redeemed. This would be the case if interest rates had risen since issuance so that the bonds were trading below the sinking fund redemption price.

Sinking fund provisions offer both advantages and disadvantages to bondholders. On the plus side, bonds with a sinking fund provision have less credit risk because the periodic redemptions reduce the total amount of principal to be repaid at maturity. The presence of a sinking fund, however, can be a disadvantage to bondholders when interest rates fall.

This disadvantage to bondholders can be seen by considering the case where interest rates have fallen since bond issuance, so the bonds are trading at a price above the sinking fund redemption price. In this case, the bond trustee will select outstanding bonds for redemption randomly. A bondholder would suffer a loss if her bonds were selected to be redeemed at a price below the current market price. This means the bonds have more *reinvestment risk* because bondholders who have their bonds redeemed can only reinvest the funds at the new, lower yield (assuming they buy bonds of similar risk).

PROFESSOR'S NOTE

The concept of reinvestment risk is developed more in subsequent readings. It can be defined as the uncertainty about the interest to be earned on cash flows from a bond that are reinvested in other debt securities. In the case of a bond with a sinking fund, the greater probability of receiving the principal repayment prior to maturity increases the expected cash flows during the bond's life and, therefore, the uncertainty about interest income on reinvested funds.

There are several coupon structures besides a fixed-coupon structure, and we summarize the most important ones here.

Floating-Rate Notes

Some bonds pay periodic interest that depends on a current market rate of interest. These bonds are called **floating-rate notes (FRN)** or **floaters**. The market rate of interest is called the **market reference rate (MRR)**, and an FRN promises to pay the reference rate plus some interest margin. This added margin is typically expressed in **basis points**, which are hundredths of 1%. A 120 basis point margin is equivalent to 1.2%.

As an example, consider a floating-rate note that pays the London Interbank Offered Rate (LIBOR) plus a margin of 0.75% (75 basis points) annually. If 1-year LIBOR is 2.3% at the beginning of the year, the bond will pay 2.3% + 0.75% = 3.05% of its par value at the end of the year. The new 1-year rate at that time will determine the rate of interest paid at the end of the next year. Most floaters pay quarterly and are based on a quarterly (90-day) reference rate.

A floating-rate note may have a **cap**, which benefits the issuer by placing a limit on how high the coupon rate can rise. Often, FRNs with caps also have a **floor**, which benefits the bondholder by placing a minimum on the coupon rate (regardless of how low the reference rate falls). An **inverse floater** has a coupon rate that increases when the reference rate decreases and decreases when the reference rate increases.

Other Coupon Structures

Step-up coupon bonds are structured so that the coupon rate increases over time according to a predetermined schedule. Typically, step-up coupon bonds have a *call feature* that allows the firm to redeem the bond issue at a set price at each step-up date. If the new higher coupon rate is greater than what the market yield would be at the call price, the firm will call the bonds and retire them. This means if market yields rise, a bondholder may, in turn, get a higher coupon rate because the bonds are less likely to be called on the step-up date.

Yields could increase because an issuer's credit rating has fallen, in which case the higher step-up coupon rate simply compensates investors for greater credit risk. Aside from this, we can view step-up coupon bonds as having some protection against increases in market interest rates to the extent they are offset by increases in bond coupon rates.

A **credit-linked coupon bond** carries a provision stating that the coupon rate will go up by a certain amount if the credit rating of the issuer falls and go down if the credit rating of the issuer improves. While this offers some protection against a credit downgrade of the issuer, the higher required coupon payments may make the financial situation of the issuer worse and possibly increase the probability of default.

A **payment-in-kind (PIK) bond** allows the issuer to make the coupon payments by increasing the principal amount of the outstanding bonds, essentially paying bond interest with more bonds. Firms that issue PIK bonds typically do so because they anticipate that firm cash flows may be less than required to service the debt, often because of high levels of debt financing (leverage). These bonds typically have higher yields because of a lower perceived credit quality from cash flow shortfalls or simply because of the high leverage of the issuing firm.

With a **deferred coupon bond**, also called a **split coupon bond**, regular coupon payments do not begin until a period of time after issuance. These are issued by firms that anticipate cash flows will increase in the future to allow them to make coupon interest payments.

Deferred coupon bonds may be appropriate financing for a firm financing a large project that will not be completed and generating revenue for some period of time after bond issuance. Deferred coupon bonds may offer bondholders tax advantages in some jurisdictions. Zero-coupon bonds can be considered a type of deferred coupon bond.

An **index-linked bond** has coupon payments and/or a principal value that is based on a commodity index, an equity index, or some other published index number. **Inflation-linked bonds** (also called **linkers**) are the most common type of index-linked bonds. Their payments are based on the change in an inflation index, such as the Consumer Price Index (CPI) in the United States. Indexed bonds that will not pay less than their original par value at maturity, even when the index has decreased, are termed **principal protected bonds**.

The different structures of inflation-indexed bonds include the following:

- **Indexed-annuity bonds.** Fully amortizing bonds with the periodic payments directly adjusted for inflation or deflation.
- **Indexed zero-coupon bonds.** The payment at maturity is adjusted for inflation.
- **Interest-indexed bonds.** The coupon rate is adjusted for inflation while the principal value remains unchanged.
- **Capital-indexed bonds**. This is the most common structure. An example is U.S. Treasury Inflation Protected Securities (TIPS). The coupon rate remains constant, and the principal value of the bonds is increased by the rate of inflation (or decreased by deflation).

To better understand the structure of capital-indexed bonds, consider a bond with a par value of $1,000 at issuance, a 3% annual coupon rate paid semiannually, and a provision that the principal value will be adjusted for inflation (or deflation). If six months after issuance the reported inflation has been 1% over the period, the principal value of the bonds is increased by 1% from $1,000 to $1,010, and the six-month coupon of 1.5% is calculated as 1.5% of the new (adjusted) principal value of $1,010 (i.e., 1,010 × 1.5% = $15.15).

With this structure we can view the coupon rate of 3% as a real rate of interest. Unexpected inflation will not decrease the purchasing power of the coupon interest payments, and the principal value paid at maturity will have approximately the same purchasing power as the $1,000 par value did at bond issuance.

LOS 42.f: Describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender.

A **contingency provision** in a contract describes an action that may be taken if an event (the contingency) actually occurs. Contingency provisions in bond indentures are referred to as

embedded options, embedded in the sense that they are an integral part of the bond contract and are not a separate security. Some embedded options are exercisable at the option of the issuer of the bond and, therefore, are valuable to the issuer; others are exercisable at the option of the purchaser of the bond and, thus, have value to the bondholder.

Bonds that do not have contingency provisions are referred to as **straight** or **option-free** bonds.

A **call option** gives the *issuer* the right to redeem all or part of a bond issue at a specific price (call price) if they choose to. As an example of a call provision, consider a 6% 20-year bond issued at par on June 1, 2012, for which the indenture includes the following *call schedule*:

- The bonds can be redeemed by the issuer at 102% of par after June 1, 2017.
- The bonds can be redeemed by the issuer at 101% of par after June 1, 2020.
- The bonds can be redeemed by the issuer at 100% of par after June 1, 2022.

For the 5-year period from the issue date until June 2017, the bond is not callable. We say the bond has five years of *call protection*, or that the bond is *call protected* for five years. This 5-year period is also referred to as a *lockout period*, a *cushion*, or a *deferment period*.

June 1, 2017, is referred to as the *first call date*, and the *call price* is 102 (102% of par value) between that date and June 2020. The amount by which the call price is above par is referred to as the *call premium*. The call premium at the first call date in this example is 2%, or $20 per $1,000 bond. The call price declines to 101 (101% of par) after June 1, 2020. After, June 1, 2022, the bond is callable at par, and that date is referred to as the *first par call* date.

For a bond that is currently callable, the call price puts an upper limit on the value of the bond in the market.

A call option has value to the issuer because it gives the issuer the right to redeem the bond and issue a new bond (borrow) if the market yield on the bond declines. This could occur either because interest rates in general have decreased or because the credit quality of the bond has increased (default risk has decreased).

Consider a situation where the market yield on the previously discussed 6% 20-year bond has declined from 6% at issuance to 4% on June 1, 2017 (the first call date). If the bond did not have a call option, it would trade at approximately $1,224. With a call price of 102, the issuer can redeem the bonds at $1,020 each and borrow that amount at the current market yield of 4%, reducing the annual interest payment from $60 per bond to $40.80.

PROFESSOR'S NOTE

This is analogous to refinancing a home mortgage when mortgage rates fall in order to reduce the monthly payments.

The issuer will only choose to exercise the call option when it is to their advantage to do so. That is, they can reduce their interest expense by calling the bond and issuing new bonds at a lower yield. Bond buyers are disadvantaged by the call provision and have more reinvestment risk because their bonds will only be called (redeemed prior to maturity) when the proceeds can be reinvested only at a lower yield. For this reason, a callable bond must offer a higher yield (sell at a lower price) than an otherwise identical noncallable bond. The difference in price between a callable bond and an otherwise identical noncallable bond is equal to the value of the call option to the issuer.

There are three *styles of exercise* for callable bonds:

1. American style—the bonds can be called anytime after the first call date.
2. European style—the bonds can only be called on the call date specified.
3. Bermuda style—the bonds can be called on specified dates after the first call date, often on coupon payment dates.

Note that these are only style names and are not indicative of where the bonds are issued.

To avoid the higher interest rates required on callable bonds but still preserve the option to redeem bonds early when corporate or operating events require it, issuers introduced bonds with **make-whole** call provisions. With a make-whole bond, the call price is not fixed but includes a lump-sum payment based on the present value of the future coupons the bondholder will not receive if the bond is called early.

With a make-whole call provision, the calculated call price is unlikely to be lower than the market value of the bond. Therefore the issuer is unlikely to call the bond except when corporate circumstances, such as an acquisition or restructuring, require it. The make-whole provision does not put an upper limit on bond values when interest rates fall as does a regular call provision. The make-whole provision actually penalizes the issuer for calling the bond. The net effect is that the bond can be called if necessary, but it can also be issued at a lower yield than a bond with a traditional call provision.

Putable Bonds

A **put option** gives the *bondholder* the right to sell the bond back to the issuing company at a prespecified price, typically par. Bondholders are likely to exercise such a put option when the fair value of the bond is less than the put price because interest rates have risen or the credit quality of the issuer has fallen. Exercise styles used are similar to those we enumerated for callable bonds.

Unlike a call option, a put option has value to the bondholder because the choice of whether to exercise the option is the bondholder's. For this reason, a putable bond will sell at a higher price (offer a lower yield) compared to an otherwise identical option-free bond.

Convertible Bonds

Convertible bonds, typically issued with maturities of 5–10 years, give bondholders the option to exchange the bond for a specific number of shares of the issuing corporation's common stock. This gives bondholders the opportunity to profit from increases in the value of the common shares. Regardless of the price of the common shares, the value of a convertible bond will be at least equal to its bond value without the conversion option. Because the conversion option is valuable to bondholders, convertible bonds can be issued with lower yields compared to otherwise identical straight bonds.

Essentially, the owner of a convertible bond has the downside protection (compared to equity shares) of a bond, but at a reduced yield, and the upside opportunity of equity shares. For this reason convertible bonds are often referred to as a *hybrid security*—part debt and part equity.

To issuers, the advantages of issuing convertible bonds are a lower yield (interest cost) compared to straight bonds and the fact that debt financing is converted to equity financing when the bonds are converted to common shares. Some terms related to convertible bonds are:

- **Conversion price.** The price per share at which the bond (at its par value) may be converted to common stock.
- **Conversion ratio.** Equal to the par value of the bond divided by the conversion price. If a bond with a $1,000 par value has a conversion price of $40, its *conversion ratio* is 1,000 / 40 = 25 shares per bond.
- **Conversion value.** This is the market value of the shares that would be received upon conversion. A bond with a conversion ratio of 25 shares when the current market price of a common share is $50 would have a conversion value of 25 × 50 = $1,250.

Even if the share price increases to a level where the conversion value is significantly above the bond's par value, bondholders might not convert the bonds to common stock until they must because the interest yield on the bonds is higher than the dividend yield on the common shares received through conversion. For this reason, many convertible bonds have a call provision. Because the call price will be less than the conversion value of the shares, by exercising their call provision, the issuers can force bondholders to exercise their conversion option when the conversion value is significantly above the par value of the bonds.

Warrants

An alternative way to give bondholders an opportunity for additional returns when the firm's common shares increase in value is to include **warrants** with straight bonds when they are issued. Warrants give their holders the right to buy the firm's common shares at a given price over a given period of time. As an example, warrants that give their holders the right to buy shares for $40 will provide profits if the common shares increase in value above $40 prior to expiration of the warrants. For a young firm, issuing debt can be difficult because the downside (probability of firm failure) is significant, and the upside is limited to the promised debt payments. Including warrants, which are sometimes referred to as a "sweetener," makes the debt more attractive to investors because it adds potential upside profits if the common shares increase in value.

Contingent Convertible Bonds

Contingent convertible bonds (referred to as *CoCos*) are bonds that convert from debt to common equity automatically if a specific event occurs. This type of bond has been issued by some European banks. Banks must maintain specific levels of equity financing. If a bank's equity falls below the required level, they must somehow raise more equity financing to comply with regulations. CoCos are often structured so that if the bank's equity capital falls below a given level, they are automatically converted to common stock. This has the effect of decreasing the bank's debt liabilities and increasing its equity capital at the same time, which helps the bank to meet its minimum equity requirement.

MODULE QUIZ 42.2

1. A 10-year bond pays no interest for three years, then pays $229.25, followed by payments of $35 semiannually for seven years, and an additional $1,000 at maturity. This bond is:
    A. a step-up bond.
    B. a zero-coupon bond.

C. a deferred-coupon bond.

2. Which of the following statements is *most accurate* with regard to floating-rate issues that have caps and floors?
   A. A cap is an advantage to the bondholder, while a floor is an advantage to the issuer.
   B. A floor is an advantage to the bondholder, while a cap is an advantage to the issuer.
   C. A floor is an advantage to both the issuer and the bondholder, while a cap is a disadvantage to both the issuer and the bondholder.
3. Which of the following *most accurately* describes the maximum price for a currently callable bond?
   A. Its par value.
   B. The call price.
   C. The present value of its par value.

KEY CONCEPTS

LOS 42.a

Basic features of a fixed income security include the issuer, maturity date, par value, coupon rate, coupon frequency, and currency.

- Issuers include corporations, governments, quasi-government entities, and supranational entities.
- Bonds with original maturities of one year or less are money market securities. Bonds with original maturities of more than one year are capital market securities.
- Par value is the principal amount that will be repaid to bondholders at maturity. Bonds are trading at a premium if their market price is greater than par value or trading at a discount if their price is less than par value.
- Coupon rate is the percentage of par value that is paid annually as interest. Coupon frequency may be annual, semiannual, quarterly, or monthly. Zero-coupon bonds pay no coupon interest and are pure discount securities.
- Bonds may be issued in a single currency, dual currencies (one currency for interest and another for principal), or with a bondholder's choice of currency.

LOS 42.b

A bond indenture or trust deed is a contract between a bond issuer and the bondholders, which defines the bond's features and the issuer's obligations. An indenture specifies the entity issuing the bond, the source of funds for repayment, assets pledged as collateral, credit enhancements, and any covenants with which the issuer must comply.

LOS 42.c

Covenants are provisions of a bond indenture that protect the bondholders' interests. Negative covenants are restrictions on a bond issuer's operating decisions, such as prohibiting the issuer from issuing additional debt or selling the assets pledged as collateral. Affirmative covenants are administrative actions the issuer must perform, such as making the interest and principal payments on time.

LOS 42.d

Legal and regulatory matters that affect fixed income securities include the places where they are issued and traded, the issuing entities, sources of repayment, and collateral and credit enhancements.

- Domestic bonds trade in the issuer's home country and currency. Foreign bonds are from foreign issuers but denominated in the currency of the country where they trade. Eurobonds are issued outside the jurisdiction of any single country and denominated in a currency other than that of the countries in which they trade.
- Issuing entities may be a government or agency; a corporation, holding company, or subsidiary; or a special purpose entity.
- The source of repayment for sovereign bonds is the country's taxing authority. For non-sovereign government bonds, the sources may be taxing authority or revenues from a project. Corporate bonds are repaid with funds from the firm's operations. Securitized bonds are repaid with cash flows from a pool of financial assets.
- Bonds are secured if they are backed by specific collateral or unsecured if they represent an overall claim against the issuer's cash flows and assets.
- Credit enhancement may be internal (overcollateralization, excess spread, tranches with different priority of claims) or external (surety bonds, bank guarantees, letters of credit).

Interest income is typically taxed at the same rate as ordinary income, while gains or losses from selling a bond are taxed at the capital gains tax rate. However, the increase in value toward par of original issue discount bonds is considered interest income. In the United States, interest income from municipal bonds is usually tax-exempt at the national level and in the issuer's state.

LOS 42.e

A bond with a bullet structure pays coupon interest periodically and repays the entire principal value at maturity.

A bond with an amortizing structure repays part of its principal at each payment date. A fully amortizing structure makes equal payments throughout the bond's life. A partially amortizing structure has a balloon payment at maturity, which repays the remaining principal as a lump sum.

A sinking fund provision requires the issuer to retire a portion of a bond issue at specified times during the bonds' life.

Floating-rate notes have coupon rates that adjust based on a reference rate such as LIBOR.

Other coupon structures include step-up coupon notes, credit-linked coupon bonds, payment-in-kind bonds, deferred coupon bonds, and index-linked bonds.

LOS 42.f

Embedded options benefit the party who has the right to exercise them. Call options benefit the issuer, while put options and conversion options benefit the bondholder.

Call options allow the issuer to redeem bonds at a specified call price.

Put options allow the bondholder to sell bonds back to the issuer at a specified put price.

Conversion options allow the bondholder to exchange bonds for a specified number of shares of the issuer's common stock.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 42.1

1. **C** Dual-currency bonds pay coupon interest in one currency and principal in a different currency. These currencies may or may not include the home currency of the issuer. A currency option bond allows the bondholder to choose a currency in which to be paid. (LOS 42.a)
2. **A** An indenture is the contract between the company and its bondholders and contains the bond's covenants. (LOS 42.b)
3. **C** Affirmative covenants require the borrower to perform certain actions. Negative covenants restrict the borrower from performing certain actions. Trust deed is another name for a bond indenture. (LOS 42.c)
4. **B** Tax authorities typically treat the increase in value of a pure-discount bond toward par as interest income to the bondholder. In many jurisdictions this interest income is taxed periodically during the life of the bond even though the bondholder does not receive any cash until maturity. (LOS 42.d)

Module Quiz 42.2

1. **C** This pattern describes a deferred-coupon bond. The first payment of $229.25 is the value of the accrued coupon payments for the first three years. (LOS 42.e)
2. **B** A cap is a maximum on the coupon rate and is advantageous to the issuer. A floor is a minimum on the coupon rate and is, therefore, advantageous to the bondholder. (LOS 42.e)
3. **B** Whenever the price of the bond increases above the strike price stipulated on the call option, it will be optimal for the issuer to call the bond. Theoretically, the price of a currently callable bond should never rise above its call price. (LOS 42.f)

READING 43

FIXED-INCOME MARKETS: ISSUANCE, TRADING, AND FUNDING

EXAM FOCUS

This reading introduces many terms and definitions. Focus on different types of issuers, features of the various debt security structures, and why different sources of funds have different interest costs. Understand well the differences between fixed-rate and floating-rate debt and how rates are determined on floating-rate debt and for repurchase agreements.

MODULE 43.1: TYPES OF BONDS AND ISSUERS

Video covering this content is available online.

LOS 43.a: Describe classifications of global fixed-income markets.

Global bond markets can be classified by several bond characteristics, including type of issuer, credit quality, maturity, coupon, currency, geography, indexing, and taxable status.

Type of issuer. Common classifications are households, nonfinancial corporations, governments, and financial institutions. In developed markets, the largest issuers by total value of bonds outstanding in global markets are financial corporations and governments. In emerging markets, nonfinancial corporations are the largest issuers.

Credit quality. Standard & Poor's (S&P), Moody's, and Fitch all provide credit ratings on bonds. For S&P and Fitch, the highest bond ratings are AAA, AA, A, and BBB, and are considered *investment grade bonds*. The equivalent ratings by Moody's are Aaa through Baa3. Bonds BB+ or lower (Ba1 or lower) are termed high-yield, speculative, or "junk" bonds. Some institutions are prohibited from investing in bonds of less than investment grade.

Original maturities. Securities with original maturities of one year or less are classified as **money market securities**. Examples include U.S. Treasury bills, commercial paper (issued by corporations), and negotiable certificates of deposit, or CDs (issued by banks). Securities with original maturities greater than one year are referred to as **capital market securities**.

Coupon structure. Bonds are classified as either floating-rate or fixed-rate bonds, depending on whether their coupon interest payments are stated in the bond indenture or depend on the level of a short-term *market reference rate* determined over the life of the bond. Purchasing floating-rate debt is attractive to some institutions that have variable-rate sources of funds (liabilities), such as banks. This allows these institutions to avoid the balance sheet effects of interest rate increases that would increase the cost of funds but leave the interest income at a fixed rate. The value of fixed-rate bonds (assets) held would fall in the value, while the value of their liabilities would be much less affected.

Currency denomination. A bond's price and returns are determined by the interest rates in the bond's currency. The majority of bonds issued are denominated in either U.S. dollars or euros.

Geography. Bonds may be classified by the markets in which they are issued. Recall the discussion in the previous reading of domestic (or national) bond markets, foreign bonds, and eurobonds, and the differences among them. Bond markets may also be classified as **developed markets** or **emerging markets**. Emerging markets are countries whose capital markets are less well-established than those in developed markets. Emerging market bonds are typically viewed as riskier than developed market bonds and therefore have higher yields. In most emerging markets, and some developed markets, publicly traded debt securities are chiefly issued by governments. Investors who want exposure to private sector debt in these markets can obtain it indirectly by investing in financial institutions that lend to private sector borrowers.

Indexing. As discussed previously, the cash flows on some bonds are based on an index (**index-linked bonds**). Bonds with cash flows determined by inflation rates are referred to as inflation-indexed or inflation-linked bonds. Inflation-linked bonds are issued primarily by governments but also by some corporations of high credit quality.

Tax status. In various countries, some issuers may issue bonds that are exempt from income taxes. In the United States, these bonds can be issued by municipalities and are called **municipal bonds**, or **munis**.

LOS 43.b: Describe the use of interbank offered rates as reference rates in floating-rate debt.

Until recently, the most widely used reference rate for floating-rate bonds was the London Interbank Offered Rate (LIBOR). However, the fact that LIBOR is not based on actual transactions, and has been subject to manipulation by bankers reporting their expected interbank lending rates, has led to an effort to replace LIBOR with market-determined rates. It has been agreed that by the end of 2021, banks will no longer be required to report the estimated rates that are used to determine LIBOR. Thus, alternatives to LIBOR must be found for each of the various currencies involved. In the United States, the new rate will likely be the secured overnight financing rate (SOFR), which is based on the actual rates of repurchase (repo) transactions and reported daily by the Federal Reserve.

For floating-rate bonds, the market reference rate must match the frequency with which the coupon rate on the bond is reset. For example, a bond with a coupon rate that is reset twice each year would use a six-month MRR.

LOS 43.c: Describe mechanisms available for issuing bonds in primary markets.

Sales of newly issued bonds are referred to as **primary market** transactions. Newly issued bonds can be registered with securities regulators for sale to the public, a **public offering**, or sold only to qualified investors, a **private placement**.

A public offering of bonds in the primary market is typically done with the help of an investment bank. The investment bank has expertise in the various steps of a public offering, including:

- Determining funding needs.
- Structuring the debt security.
- Creating the bond indenture.
- Naming a bond trustee (a trust company or bank trust department).
- Registering the issue with securities regulators.
- Assessing demand and pricing the bonds given market conditions.
- Selling the bonds.

Bonds can be sold through an **underwritten offering** or a **best efforts offering**. In an underwritten offering, the entire bond issue is purchased from the issuing firm by the investment bank, termed the underwriter in this case. While smaller bond issues may be sold by a single investment bank, for larger issues, the *lead underwriter* heads a **syndicate** of investment banks who collectively establish the pricing of the issue and are responsible for selling the bonds to dealers, who in turn sell them to investors. The syndicate takes the risk that the bonds will not all be sold.

A new bond issue is publicized and dealers indicate their interest in buying the bonds, which provides information about appropriate pricing. Some bonds are traded on a *when issued* basis in what is called the **grey market**. Such trading prior to the offering date of the bonds provides additional information about the demand for and market clearing price (yield) for the new bond issue.

In a *best efforts* offering, the investment banks sell the bonds on a commission basis. Unlike an underwritten offering, the investment banks do not commit to purchase the whole issue (i.e., underwrite the issue).

Some bonds, especially government bonds, are sold through an auction.

PROFESSOR'S NOTE

Recall that auction procedures were explained in detail in the prerequisite readings for Economics.

U.S. Treasury securities are sold through single price auctions with the majority of purchases made by **primary dealers** that participate in purchases and sales of bonds with the Federal Reserve Bank of New York to facilitate the open market operations of the Fed. Individuals can purchase U.S. Treasury securities through the periodic auctions as well, but are a small part of the total.

In a **shelf registration**, a bond issue is registered with securities regulators in its aggregate value with a master prospectus. Bonds can then be issued over time when the issuer needs to raise funds. Because individual offerings under a shelf registration require less disclosure than a separate registration of a bond issue, only financially sound companies are granted this option. In some countries, bonds registered under a shelf registration can be sold only to qualified investors.

LOS 43.d: Describe secondary markets for bonds.

Secondary markets refer to the trading of previously issued bonds. While some government bonds and corporate bonds are traded on exchanges, the great majority of bond trading in the secondary market is made in the dealer, or over-the-counter, market. Dealers post bid (purchase) prices and ask or offer (selling) prices for various bond issues. The difference between the bid and ask prices is the dealer's spread. The average spread is often between 10 and 12 basis points but varies across individual bonds according to their liquidity and may be more than 50 basis points for an illiquid issue.[1]

Bond trades are cleared through a clearing system, just as equities trades are. Settlement (the exchange of bonds for cash) for government bonds is either the day of the trade (cash settlement) or the next business day (T + 1). Corporate bonds typically settle on T + 2 or T + 3, although in some markets it is longer.

One example of a secondary market transaction in bonds is a **tender offer**, in which an issuer offers to repurchase some of its outstanding bonds at a specified price. Typically, a tender offer involves bonds that are trading at a discount. For example, if a corporate bond is trading for 90% of par value, the company might offer to repurchase part of the issue for a higher price (say 93% of par value). This has advantages for both the issuer and the bondholders. The bondholders can receive a higher price for their bonds than they can currently obtain in the market, and the issuer can pay less than face value to retire the bonds.

LOS 43.e: Describe securities issued by sovereign governments.

National governments or their treasuries issue bonds backed by the taxing power of the government that are referred to as **sovereign bonds**. Bonds issued in the currency of the issuing government carry high credit ratings and are considered to be essentially free of default risk. Both a sovereign's ability to collect taxes and its ability to print the currency support these high credit ratings.

Sovereign nations also issue bonds denominated in currencies different from their own. Credit ratings are often higher for a sovereign's local currency bonds than for example, its euro or U.S. dollar-denominated bonds. This is because the national government cannot print the developed market currency and the developed market currency value of local currency tax collections is dependent on the exchange rate between the two currencies.

Trading is most active and prices most informative for the most recently issued government securities of a particular maturity. These issues are referred to as **on-the-run** bonds and also as **benchmark** bonds because the yields of other bonds are determined relative to the "benchmark" yields of sovereign bonds of similar maturities.

Sovereign governments issue fixed-rate, floating-rate, and inflation-indexed bonds.

LOS 43.f: Describe securities issued by non-sovereign governments, quasi-government entities, and supranational agencies.

Non-sovereign government bonds are issued by states, provinces, counties, and sometimes by entities created to fund and provide services such as for the construction of hospitals, airports, and other municipal services. Payments on the bonds may be supported by the revenues of a specific project, from general tax revenues, or from special taxes or fees dedicated to the repayment of project debt.

Non-sovereign bonds are typically of high credit quality, but sovereign bonds typically trade with lower yields (higher prices) because their credit risk is perceived to be less than that of non-sovereign bonds.

PROFESSOR'S NOTE

We will examine the credit quality of sovereign and non-sovereign government bonds in our reading on Fundamentals of Credit Analysis.

Agency or **quasi-government** bonds are issued by entities created by national governments for specific purposes such as financing small businesses or providing mortgage financing. In the United States, bonds are issued by government-sponsored enterprises (GSEs), such as the Federal National Mortgage Association and the Tennessee Valley Authority.

Some quasi-government bonds are backed by the national government, which gives them high credit quality. Even those not backed by the national government typically have high credit quality although their yields are marginally higher than those of sovereign bonds.

Supranational bonds are issued by supranational agencies, also known as *multilateral agencies*. Examples are the World Bank, the IMF, and the Asian Development Bank. Bonds issued by supranational agencies typically have high credit quality and can be very liquid, especially large issues of well-known entities.

MODULE QUIZ 43.1

1. LIBOR rates are determined:
    A. by countries' central banks.
    B. by money market regulators.
    C. in the interbank lending market.
2. In which type of primary market transaction does an investment bank sell bonds on a commission basis?
    A. Single-price auction.
    B. Best-efforts offering.
    C. Underwritten offering.
3. Secondary market bond transactions *most likely* take place:
    A. in dealer markets.
    B. in brokered markets.
    C. on organized exchanges.
4. Sovereign bonds are described as on-the-run when they:
    A. are the most recent issue in a specific maturity.
    B. have increased substantially in price since they were issued.
    C. receive greater-than-expected demand from auction bidders.
5. Bonds issued by the World Bank would *most likely* be:
    A. quasi-government bonds.
    B. global bonds.
    C. supranational bonds.

MODULE 43.2: CORPORATE DEBT AND FUNDING ALTERNATIVES

Video covering this content is available online.

LOS 43.g: Describe types of debt issued by corporations.

Bank Debt

Most corporations fund their businesses to some extent with bank loans. These are typically variable-rate loans. When the loan involves only one bank, it is referred to as a **bilateral loan**. In contrast, when a loan is funded by several banks, it is referred to as a **syndicated loan** and the group of banks is the syndicate. There is a secondary market in syndicated loan interests that are also securitized, creating bonds that are sold to investors.

Commercial Paper

For larger creditworthy corporations, funding costs can be reduced by issuing short-term debt securities referred to as **commercial paper**. For these firms, the interest cost of commercial paper is less than the interest on a bank loan. Commercial paper yields more than short-term sovereign debt because it has, on average, more credit risk and less liquidity.

Firms use commercial paper to fund working capital and as a temporary source of funds prior to issuing longer-term debt. Debt that is temporary until permanent financing can be secured is referred to as **bridge financing**.

Commercial paper is a short-term, unsecured debt instrument. In the United States, commercial paper is issued with maturities of 270 days or less, because debt securities with maturities of 270 days or less are exempt from SEC registration. Eurocommercial paper (ECP) is issued in several countries with maturities as long as 364 days. Commercial paper is issued with maturities as short as one day (overnight paper), with most issues maturing in about 90 days.

Commercial paper is often reissued or *rolled over* when it matures. The risk that a company will not be able to sell new commercial paper to replace maturing paper is termed *rollover risk*. The two important circumstances in which a company will face rollover difficulties are (1) there is a deterioration in a company's actual or perceived ability to repay the debt at maturity, which will significantly increase the required yield on the paper or lead to less-than-full subscription to a new issue, and (2) significant systemic financial distress, as was experienced in the 2008 financial crisis, that may "freeze" debt markets so that very little commercial paper can be sold at all.

In order to get an acceptable credit rating from the ratings services on their commercial paper, corporations maintain **backup lines of credit** with banks. These are sometimes referred to as *liquidity enhancement* or *backup liquidity lines*. The bank agrees to provide the funds when the paper matures, if needed, except in the case of a *material adverse change* (i.e., when the company's financial situation has deteriorated significantly).

Similar to U.S. T-bills, commercial paper in the United States is typically issued as a pure discount security, making a single payment equal to the face value at maturity. Prices are quoted as a percentage discount from face value. In contrast, ECP rates may be quoted as either a discount yield or an *add-on yield*, that is, the percentage interest paid at maturity in addition to the par value of the commercial paper. As an example, consider 240-day commercial paper with a holding period yield of 1.35%. If it is quoted with a discount yield, it will be issued at 100 / 1.0135 = 98.668 and pay 100 at maturity. If it is quoted with an add-on yield, it will be issued at 100 and pay 101.35 at maturity.

Corporate Bonds

In the previous reading, we discussed several features of corporate bonds. **Corporate bonds** are issued with various coupon structures and with both fixed-rate and floating-rate coupon payments. They may be secured by collateral or unsecured and may have call, put, or conversion provisions.

We also discussed a sinking fund provision as a way to reduce the credit risk of a bond by redeeming part of the bond issue periodically over a bond's life. An alternative to a sinking fund provision is to issue a **serial bond issue**. With a serial bond issue, bonds are issued with several maturity dates so that a portion of the issue is redeemed periodically. An important difference between a serial bond issue and an issue with a sinking fund is that with a serial bond issue, investors know at issuance when specific bonds will be redeemed. A bond issue that does not have a serial maturity structure is said to have a **term maturity structure** with all the bonds maturing on the same date.

In general, corporate bonds are referred to as short-term if they are issued with maturities of up to 5 years, medium-term when issued with maturities from 5 to 12 years, and long-term when maturities exceed 12 years.

Corporations issue debt securities called **medium-term notes (MTNs)**, which are not necessarily medium-term in maturity. MTNs are issued in various maturities, ranging from nine months to periods as long as 100 years. Issuers provide *maturity ranges* (e.g., 18 months to two years) for MTNs they wish to sell and provide yield quotes for those ranges. Investors interested in purchasing the notes make an offer to the issuer's agent, specifying the face value and an exact maturity within one of the ranges offered. The agent then confirms the issuer's willingness to sell those MTNs and effects the transaction.

MTNs can have fixed- or floating-rate coupons, but longer-maturity MTNs are typically fixed-rate bonds. Most MTNs, other than long-term MTNs, are issued by financial corporations and most buyers are financial institutions. MTNs can be structured to meet an institution's specifications. While custom bond issues have less liquidity, they provide slightly higher yields compared to an issuer's publicly traded bonds.

LOS 43.h: Describe structured financial instruments.

Structured financial instruments are securities designed to change the risk profile of an underlying debt security, often by combining a debt security with a derivative. Sometimes structured financial instruments redistribute risk. Examples of this type of structured instruments are asset-backed securities and collateralized debt obligations. Both of these types of structured securities are discussed in some detail in our review of asset-backed securities.

Here, we describe several other types of structured instruments with which candidates should be familiar.

1. **Yield enhancement instruments**

   A **credit-linked note (CLN)** has regular coupon payments, but its redemption value depends on whether a specific credit event occurs. If the credit event (e.g., a credit rating downgrade or default of a reference asset) does not occur, the CLN will be redeemed at its par value. If the credit event occurs, the CLN will make a lower redemption payment. Thus, the realized yield on a CLN will be lower if the credit event occurs. Purchasing a CLN can be viewed as buying a note and simultaneously selling a credit default swap (CDS), a derivative security. The buyer of a CDS makes periodic payments to the seller, who will make a payment to the buyer if a specified credit event occurs. The yield on a CLN is higher than it would be on the note alone, without the credit link. This extra yield compensates the buyer of the note (seller of the CDS) for taking on the credit risk of the reference asset, which is why we classify CLNs as a yield enhancement instrument.

2. **Capital protected instruments**

   A capital protected instrument offers a guarantee of a minimum value at maturity as well as some potential upside gain. An example is a security that promises to pay $1,000 at maturity plus a percentage of any gains on a specified stock index over the life of the security. Such a security could be created by combining a zero-coupon bond selling for $950 that matures at $1,000 in 1 year, with a 1-year call option on the reference stock index with a cost of $50. The total cost of the security is $1,000, and the minimum payoff at maturity (if the call option expires with a value of zero) is $1,000. If the call option has a positive value at maturity, the total payment at maturity is greater than $1,000. A structured financial instrument that promises the $1,000 payment at maturity under this structure is called a **guarantee certificate**, because the guaranteed payoff is equal to the initial cost of the structured security. Capital protected instruments that promise a payment at maturity less than the initial cost of the instrument offer less-than-full protection, but greater potential for upside gains because more calls can be purchased.

3. **Participation instruments**

   A participation instrument has payments that are based on the value of an underlying instrument, often a reference interest rate or equity index. Participation instruments do not offer capital protection. One example of a participation instrument is a floating-rate note. With a floating-rate note, the coupon payments are based on the value of a short-term interest rate, such as 90-day LIBOR (the reference rate). When the reference rate increases, the coupon payment increases. Because the coupon payments move with the reference rates on floating-rate securities, their market values remain relatively stable, even when interest rates change.

   Participation is often based on the performance of an equity price, an equity index value, or the price of another asset. Fixed-income portfolio managers who are only permitted to invest in "debt" securities can use participation instruments to gain exposure to returns on an equity index or asset price.

4. **Leveraged instruments**

   An **inverse floater** is an example of a leveraged instrument. An inverse floater has coupon payments that increase when a reference rate decreases and decrease when a reference rate increases, the opposite of coupon payments on a floating-rate note. A simple structure might promise to pay a coupon rate, C, equal to a specific rate minus a reference rate, for example, C = 6% - 180-day LIBOR. When 180-day LIBOR increases, the coupon rate on the inverse floater decreases.

Inverse floaters can also be structured with leverage so that the change in the coupon rate is some multiple of the change in the reference rate. As an example, consider a note with C = 6% - (1.2 × 90-day LIBOR) so that the coupon payment rate changes by 1.2 times the change in the reference rate. Such a floater is termed a **leveraged inverse floater**. When the multiplier on the reference rate is less than one, such as 7% – (0.5 × 180-day LIBOR), the instrument is termed a **deleveraged inverse floater**. In either case, a minimum or floor rate for the coupon rate, often 0%, is specified for the inverse floater.

LOS 43.i: Describe short-term funding alternatives available to banks.

Customer deposits (retail deposits) are a short-term funding source for banks. Checking accounts provide transactions services and immediate availability of funds but typically pay no interest. Money market mutual funds and savings accounts provide less liquidity or less transactions services, or both, and pay periodic interest.

In addition to funds from retail accounts, banks offer interest-bearing **certificates of deposit (CDs)** that mature on specific dates and are offered in a range of short-term maturities. Nonnegotiable CDs cannot be sold and withdrawal of funds often incurs a significant penalty.

Negotiable certificates of deposit can be sold. At the wholesale level, large denomination (typically more than $1 million) negotiable CDs are an important funding source for banks. They typically have maturities of one year or less and are traded in domestic bond markets as well as in the eurobond market.

Another source of short-term funding for banks is to borrow excess reserves from other banks in the **central bank funds market**. Banks in most countries must maintain a portion of their funds as reserves on deposit with the central bank. At any point in time, some banks may have more than the required amount of reserves on deposit, while others require more reserve deposits. In the market for central bank funds, banks with excess reserves lend them to other banks for periods of one day (overnight funds) and for longer periods up to a year (term funds). **Central bank funds rates** refer to rates for these transactions, which are strongly influenced by the effect of the central bank's open market operations on the money supply and availability of short-term funds.

In the United States, the central bank funds rate is called the Fed funds rate and this rate influences the interest rates of many short-term debt securities.

Other than reserves on deposit with the central bank, funds that are loaned by one bank to another are referred to as **interbank funds**. Interbank funds are loaned between banks for periods of one day to a year. These loans are unsecured and, as with many debt markets, liquidity may decrease severely during times of systemic financial distress.

LOS 43.j: Describe repurchase agreements (repos) and the risks associated with them.

A **repurchase (repo) agreement** is an arrangement by which one party sells a security to a counterparty with a commitment to buy it back at a later date at a specified (higher) price. The *repurchase price* is greater than the selling price and accounts for the interest charged by the buyer, who is, in effect, lending funds to the seller with the security as collateral. The interest rate implied by the two prices is called the *repo rate*, which is the annualized percentage difference between the two prices. A repurchase agreement for one day is called an *overnight repo* and an agreement covering a longer period is called a *term repo*. The interest cost of a repo is customarily less than the rate on bank loans or other short-term borrowing.

As an example, consider a firm that enters into a repo agreement to sell a 4%, 12-year bond with a par value of $1 million and a market value of $970,000 for $940,000 and to repurchase it 90 days later (the **repo date**) for $947,050.

The implicit interest rate for the 90-day loan period is 947,050 / 940,000 - 1 = 0.75% and the *repo rate* would be expressed as the equivalent annual rate.

The percentage difference between the market value and the amount loaned is called the **repo margin** or the **haircut**. In our example, it is 940,000 / 970,000 - 1 = –3.1%. This margin protects the lender in the event that the value of the security decreases over the term of the repo agreement.

The repo rate is:

- Higher, the longer the repo term.
- Lower, the higher the credit quality of the collateral security.
- Lower when the collateral security is delivered to the lender.
- Higher when the interest rates for alternative sources of funds are higher.

The repo margin is influenced by similar factors. The repo margin is:

- Higher, the longer the repo term.
- Lower, the higher the credit quality of the collateral security.
- Lower, the higher the credit quality of the borrower.
- Lower when the collateral security is in high demand or low supply.

The reason the supply and demand conditions for the collateral security affects pricing is that some lenders want to own a specific bond or type of bond as collateral. For a bond that is high demand, lenders must compete for bonds by offering lower repo lending rates.

Viewed from the standpoint of a bond dealer, a **reverse repo agreement** refers to taking the opposite side of a repurchase transaction, lending funds by buying the collateral security rather than selling the collateral security to borrow funds.

MODULE QUIZ 43.2

1. With which of the following features of a corporate bond issue does an investor *most likely* face the risk of redemption prior to maturity?
   A. Serial bonds.
   B. Sinking fund.
   C. Term maturity structure.
2. A financial instrument is structured such that cash flows to the security holder increase if a specified reference rate increases. This structured financial instrument is *best* described as:

A. a participation instrument.
B. a capital protected instrument.
C. a yield enhancement instrument.

3. Smith Bank lends Johnson Bank excess reserves on deposit with the central bank for a period of three months. Is this transaction said to occur in the interbank market?
   A. Yes.
   B. No, because the interbank market refers to loans for more than one year.
   C. No, because the interbank market does not include reserves at the central bank.
4. In a repurchase agreement, the percentage difference between the repurchase price and the amount borrowed is *most accurately* described as:
   A. the haircut.
   B. the repo rate.
   C. the repo margin.

KEY CONCEPTS

LOS 43.a

Global bond markets can be classified by the following:

- Type of issuer**:** Households, nonfinancial corporations, governments, financial institutions.
- Credit quality**:** Investment grade, noninvestment grade.
- Original maturity: Money market (one year or less), capital market (more than one year).
- Coupon**:** Fixed rate, floating rate.
- Currency and geography**:** Domestic, foreign, global, eurobond markets; developed, emerging markets.
- Other classifications**:** Indexing, taxable status.

LOS 43.b

Interbank lending rates, such as London Interbank Offered Rate (LIBOR), are frequently used as reference rates for floating-rate debt. An appropriate reference rate is one that matches a floating-rate note's currency and frequency of rate resets, such as six-month U.S. dollar LIBOR for a semiannual floating-rate note issued in U.S. dollars.

LOS 43.c

Bonds may be issued in the primary market through a public offering or a private placement.

A public offering using an investment bank may be underwritten, with the investment bank or syndicate purchasing the entire issue and selling the bonds to dealers; or on a best-efforts basis, in which the investment bank sells the bonds on commission. Public offerings may also take place through auctions, which is the method commonly used to issue government debt.

A private placement is the sale of an entire issue to a qualified investor or group of investors, which are typically large institutions.

LOS 43.d

Bonds that have been issued previously trade in secondary markets. While some bonds trade on exchanges, most are traded in dealer markets. Spreads between bid and ask prices are narrower for liquid issues and wider for less liquid issues.

Trade settlement is typically T + 2 or T + 3 for corporate bonds and either cash settlement or T + 1 for government bonds.

LOS 43.e

Sovereign bonds are issued by national governments and backed by their taxing power. Sovereign bonds may be denominated in the local currency or a foreign currency.

LOS 43.f

Non-sovereign government bonds are issued by governments below the national level, such as provinces or cities, and may be backed by taxing authority or revenues from a specific project.

Agency or quasi-government bonds are issued by government sponsored entities and may be explicitly or implicitly backed by the government.

Supranational bonds are issued by multilateral agencies that operate across national borders.

LOS 43.g

Debt issued by corporations includes bank debt, commercial paper, corporate bonds, and medium-term notes.

Bank debt includes bilateral loans from a single bank and syndicated loans from multiple banks.

Commercial paper is a money market instrument issued by corporations of high credit quality.

Corporate bonds may have a term maturity structure (all bonds in an issue mature at the same time) or a serial maturity structure (bonds in an issue mature on a predetermined schedule) and may have a sinking fund provision.

Medium-term notes are corporate issues that can be structured to meet the requirements of investors.

LOS 43.h

Structured financial instruments include asset-backed securities and collateralized debt securities as well as the following types:

- Yield enhancement instruments include credit linked notes, which are redeemed at an amount less than par value if a specified credit event occurs on a reference asset, or at par if it does not occur. The buyer receives a higher yield for bearing the credit risk of the reference asset.
- Capital protected instruments offer a guaranteed payment, which may be equal to the purchase price of the instrument, along with participation in any increase in the value of an equity, an index, or other asset.
- Participation instruments are debt securities with payments that depend on the returns on an asset or index, or depend on a reference interest rate. One example is a floating rate bond, which makes coupon payments that change with a short-term reference rate, such as LIBOR. Other participation instruments make coupon payments based on the returns on an index of equity securities or on some other asset.
- An inverse floater is a leveraged instrument that has a coupon rate that varies inversely with a specified reference interest rate, for example, 6% – (L × 180-day LIBOR). L is the leverage of the inverse floater. An inverse floater with L > 1, so that the coupon rate

changes by more than the reference rate, is termed a leveraged inverse floater. An inverse floater with L < 1 is a deleveraged floater.

LOS 43.i

Short-term funding alternatives available to banks include:

- Customer deposits, including checking accounts, savings accounts, and money market mutual funds.
- Negotiable CDs, which may be sold in the wholesale market.
- Central bank funds market. Banks may buy or sell excess reserves deposited with their central bank.
- Interbank funds. Banks make unsecured loans to one another for periods up to a year.

LOS 43.j

A repurchase agreement is a form of short-term collateralized borrowing in which one party sells a security to another party and agrees to buy it back at a predetermined future date and price. The repo rate is the implicit interest rate of a repurchase agreement. The repo margin, or haircut, is the difference between the amount borrowed and the value of the security.

Repurchase agreements are an important source of short-term financing for bond dealers. If a bond dealer is lending funds instead of borrowing, the agreement is known as a reverse repo.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 43.1

1. **C** LIBOR rates are determined in the market for interbank lending. (LOS 43.b)
2. **B** In a best-efforts offering, the investment bank or banks do not underwrite (i.e., purchase all of) a bond issue, but rather sell the bonds on a commission basis. Bonds sold by auction are offered directly to buyers by the issuer (typically a government). (LOS 43.c)
3. **A** The secondary market for bonds is primarily a dealer market in which dealers post bid and ask prices. (LOS 43.d)
4. **A** Sovereign bonds are described as *on-the-run* or *benchmark* when they represent the most recent issue in a specific maturity. (LOS 43.e)
5. **C** Bonds issued by the World Bank, which is a multilateral agency operating globally, are termed *supranational bonds*. (LOS 43.f)

Module Quiz 43.2

1. **B** With a sinking fund, the issuer must redeem part of the issue prior to maturity, but the specific bonds to be redeemed are not known. Serial bonds are issued with a schedule of maturities and each bond has a known maturity date. In an issue with a term maturity structure, all the bonds are scheduled to mature on the same date. (LOS 43.g)
2. **A** Floating-rate notes are an example of a participation instrument. (LOS 43.h)

3. **C** The interbank market refers to short-term borrowing and lending among banks of funds other than those on deposit at a central bank. Loans of reserves on deposit with a central bank are said to occur in the central bank funds market. (LOS 43.i)
4. **B** The repo rate is the percentage difference between the repurchase price and the amount borrowed. The repo margin or haircut is the percentage difference between the amount borrowed and the value of the collateral. (LOS 43.j)

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[1] Fixed Income Markets: Issuance, Trading, and Funding, Choudhry, M.; Mann, S.; and Whitmer, L.; in CFA Program 2023 Level I Curriculum, Volume 4 (CFA Institute, 2022).

READING 44

INTRODUCTION TO FIXED-INCOME VALUATION

EXAM FOCUS

The concepts introduced here are very important for understanding the factors that determine the value of debt securities and various yield measures. The relationships between yield to maturity, spot rates, and forward rates are core material and come up in many contexts throughout the CFA curriculum. Yield spread measures also have many applications. Note that while several of the required learning outcomes have the command word "calculate" in them, a good understanding of the underlying concepts is just as important for exam success on this material.

MODULE 44.1: BOND VALUATION AND YIELD TO MATURITY

Video covering this content is available online.

LOS 44.a: Calculate a bond's price given a market discount rate.

Calculating the Value of an Annual Coupon Bond

The value of a coupon bond can be calculated by summing the present values of all of the bond's promised cash flows. The market discount rate appropriate for discounting a bond's cash flows is called the bond's **yield-to-maturity (YTM)**. If we know a bond's yield-to-maturity, we can calculate its value, and if we know its value (market price), we can calculate its yield-to-maturity.

Consider a newly issued 10-year, $1,000 par value, 10% coupon, annual-pay bond. The coupon payments will be $100 at the end of each year the $1,000 par value will be paid at the end of year 10. First, let's value this bond assuming the appropriate discount rate is 10%. The present value of the bond's cash flows discounted at 10% is:

$$\frac{100}{1.1}+\frac{100}{1.1^2}+\frac{100}{1.1^3}+\ldots\ldots+\frac{100}{1.1^9}+\frac{1{,}100}{1.1^{10}} = 1{,}000$$

The calculator solution is:

N = 10; PMT = 100; FV = 1,000; I/Y = 10; CPT → PV = –1,000

where:
N = number of years
PMT = the *annual* coupon payment

I/Y = the *annual* discount rate

FV = the par value or selling price at the end of an assumed holding period

PROFESSOR'S NOTE

Take note of a couple of points here. The discount rate is entered as a whole number in percent, 10, not 0.10. The 10 coupon payments of $100 each are taken care of in the N = 10 and PMT = 100 entries. The principal repayment is in the FV = 1,000 entry. Lastly, note that the PV is negative; it will be the opposite sign to the sign of PMT and FV. The calculator is just "thinking" that to receive the payments and future value (to own the bond), you must pay the present value of the bond today (you must buy the bond). That's why the PV amount is negative; it is a cash outflow to a bond buyer.

Now let's value that same bond with a discount rate of 8%:

$$\frac{100}{1.08}+\frac{100}{1.08^2}+\frac{100}{1.08^3}+\dots\dots+\frac{100}{1.08^9}+\frac{1{,}100}{1.08^{10}} = 1{,}134.20$$

The calculator solution is:

N = 10; PMT = 100; FV = 1,000; I/Y = **8**; CPT → PV = –1,134.20

If the market discount rate for this bond were 8%, it would sell at a premium of $134.20 above its par value. ***When bond yields decrease, the present value of a bond's payments, its market value, increases.***

If we discount the bond's cash flows at 12%, the present value of the bond is:

$$\frac{100}{1.12}+\frac{100}{1.12^2}+\frac{100}{1.12^3}+\dots\dots+\frac{100}{1.12^9}+\frac{1{,}100}{1.12^{10}} = 887.00$$

The calculator solution is:

N = 10; PMT = 100; FV = 1,000; I/Y = **12**; CPT → PV = –887

If the market discount rate for this bond were 12%, it would sell at a discount of $113 to its par value. ***When bond yields increase, the present value of a bond's payments, its market value, decreases.***

PROFESSOR'S NOTE

It's worth noting here that a 2% decrease in yield-to-maturity increases the bond's value by more than a 2% increase in yield decreases the bond's value. This illustrates that the bond's price-yield relationship is convex, as we will explain in more detail in a later reading.

Calculating the value of a bond with semiannual coupon payments. Let's calculate the value of the same bond with semiannual payments.

Rather than $100 per year, the security will pay $50 every six months. With an annual YTM of 8%, we need to discount the coupon payments at 4% per period which results in a present value of:

$$\frac{50}{1.04}+\frac{50}{1.04^2}+\frac{50}{1.04^3}+\dots\dots+\frac{50}{1.04^{19}}+\frac{1{,}050}{1.04^{20}} = 1{,}135.90$$

The calculator solution is:

N = 20; PMT = 50; FV = 1,000; I/Y = 4; CPT → PV = −1,135.90

The value of a zero-coupon bond is simply the present value of the maturity payment. With a discount rate of 3% per period, a 5-period zero-coupon bond with a par value of $1,000 has a value of:

$$\frac{1{,}000}{1.03^5} = \$862.61$$

LOS 44.b: Identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity).

So far we have used a bond's cash flows and an assumed discount rate to calculate the value of the bond. We can also calculate the market discount rate given a bond's price in the market, because there is an inverse relationship between price and yield. For a 3-year, 8% annual coupon bond that is priced at 90.393, the market discount rate is:

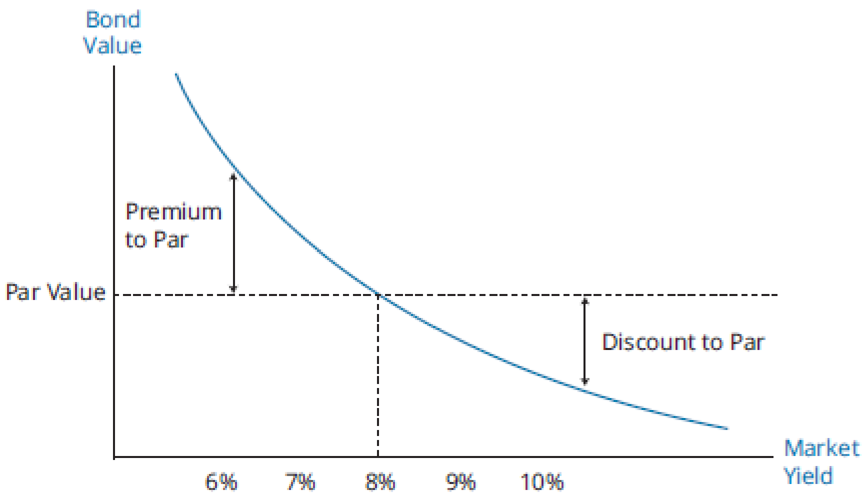
N = 3; PMT = 8; FV = 100; PV = −90.393; CPT → I/Y = 12%

We can summarize the relationships between price and yield as follows:

1. At a point in time, a decrease (increase) in a bond's YTM will increase (decrease) its price.
2. If a bond's coupon rate is greater than its YTM, its price will be at a premium to par value. If a bond's coupon rate is less than its YTM, its price will be at a discount to par value.
3. The percentage decrease in value when the YTM increases by a given amount is smaller than the increase in value when the YTM decreases by the same amount (the price-yield relationship is convex).
4. Other things equal, the price of a bond with a lower coupon rate is more sensitive to a change in yield than is the price of a bond with a higher coupon rate.
5. Other things equal, the price of a bond with a longer maturity is more sensitive to a change in yield than is the price of a bond with a shorter maturity.

Figure 44.1 illustrates the convex relationship between a bond's price and its yield-to-maturity:

Figure 44.1: Market Yield vs. Bond Value for an 8% Coupon Bond



Relationship Between Price and Maturity

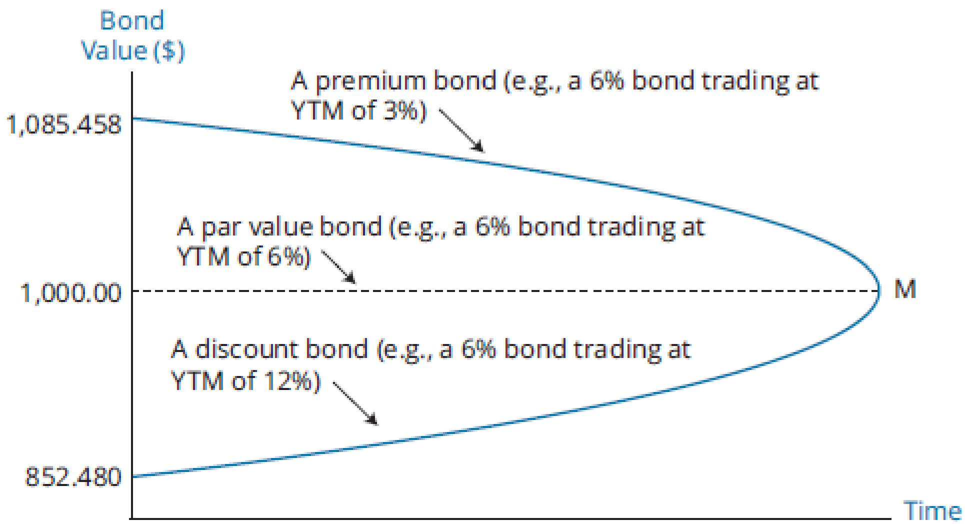
Prior to maturity, a bond can be selling at a significant discount or premium to par value. However, regardless of its required yield, the price will converge to par value as maturity approaches. Consider a bond with $1,000 par value and a 3-year life paying 6% semiannual coupons. The bond values corresponding to required yields of 3%, 6%, and 12% as the bond approaches maturity are presented in Figure 44.2.

Figure 44.2: Bond Values and the Passage of Time

Time to Maturity (in years)	YTM = 3%	YTM = 6%	YTM = 12%
3.0	$1,085.46	$1,000.00	$852.48
2.5	1,071.74	1,000.00	873.63
2.0	1,057.82	1,000.00	896.05
1.5	1,043.68	1,000.00	919.81
1.0	1,029.34	1,000.00	945.00
0.5	1,014.78	1,000.00	971.69
0.0	1,000.00	1,000.00	1,000.00

The change in value associated with the passage of time for the three bonds represented in Figure 44.2 is presented graphically in Figure 44.3. This convergence to par value at maturity is known as the **constant-yield price trajectory** because it shows how the bond's price would change as time passes if its yield-to-maturity remained constant.

Figure 44.3: Premium, Par, and Discount Bonds



MODULE QUIZ 44.1

1. A 20-year, 10% annual-pay bond has a par value of $1,000. What is the price of the bond if it has a yield-to-maturity of 15%?
   A. $685.14.
   B. $687.03.
   C. $828.39.
2. An analyst observes a 5-year, 10% semiannual-pay bond. The face amount is £1,000. The analyst believes that the yield-to-maturity on a semiannual bond basis should be 15%. Based on this yield estimate, the price of this bond would be:
   A. £828.40.
   B. £1,189.53.
   C. £1,193.04.
3. An analyst observes a 20-year, 8% option-free bond with semiannual coupons. The required yield-to-maturity on a semiannual bond basis was 8%, but suddenly it decreased to 7.25%. As a result, the price of this bond:
   A. increased.
   B. decreased.
   C. stayed the same.
4. A $1,000, 5%, 20-year annual-pay bond has a YTM of 6.5%. If the YTM remains unchanged, how much will the bond value increase over the next three years?
   A. $13.62.
   B. $13.78.
   C. $13.96.

MODULE 44.2: SPOT RATES AND ACCRUED INTEREST

Video covering this content is available online.

LOS 44.c: Define spot rates and calculate the price of a bond using spot rates.

The yield-to-maturity is calculated as if the discount rate for every bond cash flow is the same. In reality, discount rates depend on the time period in which the bond payment will be made. **Spot rates** are the market discount rates for a single payment to be received in the future. The discount rates for zero-coupon bonds are spot rates and we sometimes refer to spot rates as *zero-coupon rates* or simply *zero rates*.

In order to price a bond with spot rates, we sum the present values of the bond's payments, each discounted at the spot rate for the number of periods before it will be paid. The general equation for calculating a bond's value using spot rates (S_i) is:

$$\frac{CPN_1}{1 + S_1} + \frac{CPN_2}{(1 + S_2)^2} + \ldots\ldots + \frac{CPN_N + FV_N}{(1 + S_N)^N} = PV$$

> EXAMPLE: **Valuing a bond using spot rates**
>
> Given the following spot rates, calculate the value of a 3-year, 5% annual-coupon bond.
>
> Spot rates
>
> 1-year: 3%
>
> 2-year: 4%
>
> 3-year: 5%
>
> **Answer:**
>
> $$\frac{50}{1.03} + \frac{50}{(1.04)^2} + \frac{1{,}050}{(1.05)^3} = 48.54 + 46.23 + 907.03 = \$1{,}001.80$$

This price, calculated using spot rates, is sometimes called the *no-arbitrage price* of a bond because if a bond is priced differently there will be a profit opportunity from arbitrage among bonds.

Because the bond value is slightly greater than its par value, we know its YTM is slightly less than its coupon rate of 5%. Using the price of 1,001.80, we can calculate the YTM for this bond as:

N = 3; PMT = 50; FV = 1,000; PV = –1,001.80; CPT → I/Y = 4.93%

LOS 44.d: Describe and calculate the flat price, accrued interest, and the full price of a bond.

The coupon bond values we have calculated so far are calculated on the date a coupon is paid, as the present value of the remaining coupons. For most bond trades, the settlement date, which is when cash is exchanged for the bond, will fall between coupon payment dates. As time passes (and future coupon payment dates get closer), the value of the bond will increase.

The value of a bond between coupon dates can be calculated, using its current YTM, as the value of the bond on its last coupon date (PV) times (1 + YTM / # of coupon periods per year)$^{t/T}$, where t is the number of days since the last coupon payment, and T is the number of days in the coupon period. For a given settlement date, this value is referred to as the **full price** of the bond.

Let's work an example for a specific bond:

> EXAMPLE: **Calculating the full price of a bond**

A 5% bond makes coupon payments on June 15 and December 15 and is trading with a YTM of 4%. The bond is purchased and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price of the bond using actual days.

Step 1: Calculate the value of the bond on the last coupon date (coupons are semiannual, so we use 4 / 2 = 2% for the periodic discount rate):

N = 4; PMT = 25; FV = 1,000; I/Y = 2; CPT → PV = –1,019.04

Step 2: Adjust for the number of days since the last coupon payment:

Days between June 15 and December 15 = 183 days.

Days between June 15 and settlement on August 21 = 67 days.

Full price = $1{,}019.04 \times (1.02)^{67/183} = 1{,}026.46$.

The accrued interest since the last payment date can be calculated as the coupon payment times the portion of the coupon period that has passed between the last coupon payment date and the settlement date of the transaction. For the bond in the previous example, the accrued interest on the settlement date of August 21 is:

$25 (67 / 183) = $9.15

The full price (invoice price) minus the accrued interest is referred to as the **flat price** of the bond.

flat price = full price − accrued interest

So for the bond in our example, the flat price = 1,026.46 − 9.15 = 1,017.31.

The flat price of the bond is also referred to as the bond's **clean price** or **quoted price**, and the full price is also referred to as the **dirty price**.

Note that the flat price is not the present value of the bond on its last coupon payment date, 1,017.31 < 1,019.04.

So far, in calculating accrued interest, we used the actual number of days between coupon payments and the actual number of days between the last coupon date and the settlement date. This actual/actual method is used most often with government bonds. The 30/360 method is most often used for corporate bonds. This method assumes that there are 30 days in each month and 360 days in a year.

EXAMPLE: **Accrued interest**

An investor buys a $1,000 par value, 4% annual-pay bond that pays its coupons on May 15. The investor's buy order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.

Answer:

The annual coupon payment is 4% × $1,000 = $40.

Using the 30/360 method, interest is accrued for 30 − 15 = 15 days in May; 30 days each in June and July; and 10 days in August, or 15 + 30 + 30 + 10 = 85 days.

$$\text{accrued interest (30/360 method)} = \frac{85}{360} \times \$40 = \$9.44$$

Using the actual/actual method, interest is accrued for 31 − 15 = 16 days in May; 30 days in June; 31 days in July; and 10 days in August, or 16 + 30 + 31 + 10 = 87 days.

$$\text{accrued interest (actual/actual method)} = \frac{87}{365} \times \$40 = \$9.53$$

LOS 44.e: Describe matrix pricing.

Matrix pricing is a method of estimating the required yield-to-maturity (or price) of bonds that are currently not traded or infrequently traded. The procedure is to use the YTMs of traded bonds that have credit quality very close to that of a nontraded or infrequently traded bond and are similar in maturity and coupon, to estimate the required YTM.

EXAMPLE: **Pricing an illiquid bond**

Rob Phelps, CFA, is estimating the value of a nontraded 4% annual-pay, A+ rated bond that has three years remaining until maturity. He has obtained the following yields-to-maturity on similar corporate bonds:

- A+ rated, 2-year annual-pay, YTM = 4.3%
- A+ rated, 5-year annual-pay, YTM = 5.1%
- A+ rated, 5-year annual-pay, YTM = 5.3%

Estimate the value of the nontraded bond.

Answer:

Step 1: Take the average YTM of the 5-year bonds: (5.1 + 5.3) / 2 = 5.2%.

Step 2: Interpolate the 3-year YTM based on the 2-year and average 5-year YTMs:

$$4.3\% + (5.2\% - 4.3\%) \times [(3 \text{ years} - 2 \text{ years}) / (5 \text{ years} - 2 \text{ years})]$$
$$= 4.6\%$$

Step 3: Price the nontraded bond with a YTM of 4.6%:

N = 3; PMT = 40; FV = 1,000; I/Y = 4.6; CPT → PV = –983.54

The estimated value is $983.54 per $1,000 par value.

In using the averages in the preceding example, we have used simple *linear interpolation*. Because the maturity of the nontraded bond is three years, we estimate the YTM on the 3-year bond as the yield on the 2-year bond, plus one-third of the difference between the YTM of the 2-year bond and the average YTM of the 5-year bonds. Note that the difference in maturity between the 2-year bond and the 3-year bond is one year and the difference between the maturities of the 2-year and 5-year bonds is three years.

A variation of matrix pricing used for pricing new bond issues focuses on the spreads between bond yields and the yields of a benchmark bond of similar maturity that is essentially default risk free. Often the yields on Treasury bonds are used as benchmark yields for U.S. dollar-denominated corporate bonds. When estimating the YTM for the new issue bond, the appropriate spread to the yield of a Treasury bond of the same maturity is estimated and added to the yield of the benchmark issue.

EXAMPLE: **Estimating the spread for a new 6-year, A rated bond issue**

Consider the following market yields:

5-year, U.S. Treasury bond, YTM 1.48%

5-year, A rated corporate bond, YTM 2.64%

7-year, U.S. Treasury bond, YTM 2.15%

7-year, A rated corporate bond, YTM 3.55%

6-year U.S. Treasury bond, YTM 1.74%

Estimate the required yield on a newly issued 6-year, A rated corporate bond.

Answer:

1. Calculate the spreads to the benchmark (Treasury) yields.
   Spread on the 5-year corporate bond is 2.64 – 1.48 = 1.16%.
   Spread on the 7-year corporate bond is 3.55 – 2.15 = 1.40%.
2. Calculate the average spread because the 6-year bond is the midpoint of five and seven years:
   average spread = (1.16 + 1.40) / 2 = 1.28%
3. Add the average spread to the YTM of the 6-year Treasury (benchmark) bond.
   1.74 + 1.28 = 3.02%, which is our estimate of the YTM on the newly issued 6-year, A rated bond.

MODULE QUIZ 44.2

1. If spot rates are 3.2% for one year, 3.4% for two years, and 3.5% for three years, the price of a $100,000 face value, 3-year, annual-pay bond with a coupon rate of 4% is *closest* to:
   A. $101,420.
   B. $101,790.
   C. $108,230.
2. An investor paid a full price of $1,059.04 each for 100 bonds. The purchase was between coupon dates, and accrued interest was $23.54 per bond. What is each bond's flat price?
   A. $1,000.00.
   B. $1,035.50.
   C. $1,082.58.
3. Cathy Moran, CFA, is estimating a value for an infrequently traded bond with six years to maturity, an annual coupon of 7%, and a single-B credit rating. Moran obtains yields-to-maturity for more liquid bonds with the same credit rating:
   - 5% coupon, eight years to maturity, yielding 7.20%.
   - 6.5% coupon, five years to maturity, yielding 6.40%.

   The infrequently traded bond is *most likely* trading at:
   A. par value.
   B. a discount to par value.
   C. a premium to par value.

MODULE 44.3: YIELD MEASURES

Video covering this content is available online.

LOS 44.f: Calculate annual yield on a bond for varying compounding periods in a year.

LOS 44.g: Calculate and interpret yield measures for fixed-rate bonds and floating-rate notes.

Given a bond's price in the market, we can say that the YTM is the discount rate that makes the present value of a bond's cash flows equal to its price. For a 5-year, annual pay 7% bond that is priced in the market at $1,020.78, the YTM will satisfy the following equation:

$$\frac{70}{1+\text{YTM}}+\frac{70}{(1+\text{YTM})^2}+\frac{70}{(1+\text{YTM})^3}+\frac{70}{(1+\text{YTM})^4}+\frac{1,070}{(1+\text{YTM})^5}$$

$$= 1,020.78$$

We can calculate the YTM (discount rate) that satisfies this equality as:

N = 5; PMT = 70; FV = 1,000; PV = –1,020.78; CPT → I/Y = 6.5%

By convention, the YTM on a semiannual coupon bond is expressed as two times the semiannual discount rate. For a 5-year, semiannual pay 7% coupon bond, we can calculate the semiannual discount rate as YTM/2 and then double it to get the YTM expressed as an annual yield:

$$\frac{35}{1+\text{YTM}/2}+\frac{35}{(1+\text{YTM}/2)^2}+\frac{35}{(1+\text{YTM}/2)^3}+\ldots+\frac{35}{(1+\text{YTM}/2)^9}+\frac{1,035}{(1+\text{YTM}/2)^{10}}$$

$$= 1,020.78$$

N = 10; PMT = 35; FV = 1,000; PV = –1,020.78; CPT → I/Y = 3.253%

The YTM is 3.253 × 2 = 6.506%.

Yield Measures for Fixed-Rate Bonds

The number of bond coupon payments per year is referred to as the **periodicity** of a bond. A bond with a periodicity of 2 will have its yield to maturity quoted on a **semiannual bond basis**. For a given coupon rate, the greater the periodicity, the more compounding periods, and the greater the annual yield.

PROFESSOR'S NOTE

This is analogous to the relationship among a stated annual rate, the number of compounding periods per year, and the effective annual yield explained in Quantitative Methods.

In general, the annual (effective) yield for bond with its YTM stated for a periodicity of *n*, and *n* compounding periods per year, is:

$$\text{annual yield} = \left(1+\frac{\text{YTM}}{n}\right)^n - 1$$

EXAMPLE: **Effective annual yields**

What is the annual yield for a bond with a stated YTM of 10%:

1. When the periodicity of the bond is 2 (pays semiannually)?
2. When the periodicity of the bond is 4 (pays quarterly)?

Answer:

1. $\text{annual yield} = \left(1+\frac{0.10}{2}\right)^2 - 1 = 1.05^2 - 1 = 0.1025 = 10.25\%$

2. $\text{annual yield} = \left(1+\frac{0.10}{4}\right)^4 - 1 = 1.025^4 - 1 = 0.1038 = 10.38\%$

It may be necessary to adjust the quoted yield on a bond to make it comparable with the yield on a bond with a different periodicity. This is illustrated in the following example.

EXAMPLE: Adjusting yields for periodicity

An Atlas Corporation bond is quoted with a YTM of 4% on a semiannual bond basis. What yields should be used to compare it with a quarterly-pay bond and an annual-pay bond?

Answer:

The first thing to note is that 4% on a semiannual bond basis is an effective yield of 2% per 6-month period.

To compare this with the yield on an annual-pay bond, which is an effective annual yield, we need to calculate the effective annual yield on the semiannual coupon bond, which is $1.02^2 - 1 = 4.04\%$.

For the annual YTM on the quarterly-pay bond, we need to calculate the effective quarterly yield and multiply by four. The quarterly yield (yield per quarter) that is equivalent to a yield of 2% per six months is $1.02^{1/2} - 1 = 0.995\%$. The quoted annual rate for the equivalent yield on a quarterly bond basis is $4 \times 0.995 = 3.98\%$.

Note that we have shown that the effective annual yields are the same for:

- An annual coupon bond with a yield of 4.04% on an annual basis (periodicity of one).
- A semiannual coupon bond with a yield of 4.0% on a semiannual basis (periodicity of two).
- A quarterly coupon bond with a yield of 3.98% on quarterly basis (periodicity of four).

Bond yields calculated using the stated coupon payment dates are referred to as following the **street convention**. Because some coupon dates will fall on weekends and holidays, coupon payments will actually be made the next business day. The yield calculated using these actual coupon payment dates is referred to as the **true yield**. Some coupon payments will be made later when holidays and weekends are taken into account, so true yields will be slightly lower than street convention yields, if only by a few basis points.

When calculating spreads between government bond yields and the yield on a corporate bond, the corporate bond yield is often restated to its yield on actual/actual basis to match the day count convention used on government bonds (rather than the 30/360 day count convention used for calculating corporate bond yields).

Current yield (also called **income yield** or **running yield**) is simple to calculate, but offers limited information. This measure looks at just one source of return: *a bond's annual interest income*—it does not consider capital gains/losses or reinvestment income. The formula for the current yield is:

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}}$$

EXAMPLE: Computing current yield

Consider a 20-year, $1,000 par value, 6% *semiannual-pay* bond that is currently trading at a flat price of $802.07. Calculate the current yield.

Answer:

The *annual* cash coupon payments total:

annual cash coupon payment = par value × stated coupon rate
= $1,000 × 0.06 = $60

Because the bond is trading at $802.07, the current yield is:

$$\text{current yield} = \frac{60}{802.07} = 0.0748, \text{ or } 7.48\%$$

> Note that current yield is based on *annual* coupon interest so that it is the same for a semiannual-pay and annual-pay bond with the same coupon rate and price.

The current yield does not account for gains or losses as the bond's price moves toward its par value over time. A bond's **simple yield** takes a discount or premium into account by assuming that any discount or premium declines evenly over the remaining years to maturity. The sum of the annual coupon payment plus (minus) the straight-line amortization of a discount (premium) is divided by the flat price to get the simple yield.

> EXAMPLE: **Computing simple yield**
>
> A 3-year, 8% coupon, semiannual-pay bond is priced at 90.165. Calculate the simple yield.
>
> **Answer:**
>
> The discount from par value is 100 – 90.165 = 9.835. Annual straight-line amortization of the discount is 9.835 / 3 = 3.278.
>
> $$\text{simple yield} = \frac{8 + 3.278}{90.165} = 12.51\%$$

For a callable bond, an investor's yield will depend on whether and when the bond is called. The **yield-to-call** can be calculated for each possible call date and price. The lowest of yield-to-maturity and the various yields-to-call is termed the **yield-to-worst**. The following example illustrates these calculations.

> EXAMPLE: **Yield-to-call and yield-to-worst**
>
> Consider a 10-year, semiannual-pay 6% bond trading at 102 on January 1, 2014. The bond is callable according to the following schedule:
>
> Callable at 102 on or after January 1, 2019.
>
> Callable at 100 on or after January 1, 2022.
>
> Calculate the bond's YTM, yield-to-first call, yield-to-first par call, and yield-to-worst.
>
> **Answer:**
>
> The *yield-to-maturity* on the bond is calculated as:
>
> N = 20; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.867%
>
> 2 × 2.867 = 5.734% = YTM
>
> To calculate the *yield-to-first call*, we calculate the yield-to-maturity using the number of semiannual periods until the first call date (10) for *N* and the call price (1,020) for *FV*:
>
> N = 10; PMT = 30; FV = 1,020; PV = –1,020; CPT → I/Y = 2.941%
>
> 2 × 2.941 = 5.882% = yield-to-first call
>
> To calculate the *yield-to-first par call* (second call date), we calculate the yield-to-maturity using the number of semiannual periods until the first par call date (16) for *N* and the call price (1,000) for *FV*:
>
> N = 16; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.843%
>
> 2 × 2.843 = 5.686% = yield-to-first par call

The lowest yield, 5.686%, is realized if the bond is called at par on January 1, 2022, so the *yield-to-worst* is 5.686%.

The **option-adjusted yield** is calculated by adding the value of the call option to the bond's current flat price. The value of a callable bond is equal to the value of the bond if it did not have the call option, minus the value of the call option (because the issuer *owns* the call option).

The option-adjusted yield will be less than the yield-to-maturity for a callable bond because callable bonds have higher yields to compensate bondholders for the issuer's call option. The option-adjusted yield can be used to compare the yields of bonds with various embedded options to each other and to similar option-free bonds.

Floating-Rate Note Yields

The values of floating rate notes (FRNs) are more stable than those of fixed-rate debt of similar maturity because the coupon interest rates are reset periodically based on a reference rate. Recall that the coupon rate on a floating-rate note is the reference rate plus or minus a margin based on the credit risk of the bond relative to the credit risk of the reference rate instrument. The coupon rate for the next period is set using the current reference rate for the reset period, and the payment at the end of the period is based on this rate. For this reason, we say that interest is paid *in arrears*.

If an FRN is issued by a company that has more (less) credit risk than the financial institutions from which the MRR is derived, a margin is added to (subtracted from) the MRR. The liquidity of an FRN and its tax treatment can also affect the margin.

We call the margin used to calculate the bond coupon payments the **quoted margin** and we call the margin required to return the FRN to its par value the **required margin** (also called the **discount margin**). When the credit quality of an FRN is unchanged, the quoted margin is equal to the required margin and the FRN returns to its par value at each reset date when the next coupon payment is reset to the current market rate (plus or minus the appropriate margin).

If the credit quality of the issuer decreases, the quoted margin will be less than the required margin and the FRN will sell at a discount to its par value. If credit quality has improved, the quoted margin will be greater than the required margin and the FRN will sell at a premium to its par value.

A somewhat simplified way of calculating the value of an FRN on a reset date is to use the current reference rate plus the quoted margin to estimate the future cash flows for the FRN and to discount these future cash flows at the reference rate plus the required (discount) margin. More complex models produce better estimates of value.

EXAMPLE: **Valuation of a floating-rate note**

A $100,000 floating rate note is based on a 180-day MRR with a quoted margin of 120 basis points. On a reset date with 5 years remaining to maturity, the 180-day MRR is quoted as 3.0% (annualized) and the required rate of return (based on the issuer's current credit rating) is 4.5% (annualized). What is the market value of the floating rate note?

Answer:

The current annualized coupon rate on the note is 3.0% + 1.2% = 4.2%, so the next semiannual coupon payment will be 4.2% / 2 = 2.1% of face value. The required return in the market (discount margin) as an effective 180-day discount rate is 4.5% / 2 = 2.25%.

Using a face value of 100%, 10 coupon payments of 2.1%, and a discount rate per period of 2.25%, we can calculate the present value of the floating rate note as:

N = 10; I/Y = 2.25%; FV = 100; PMT = 2.1; CPT PV = 98.67

The current value of the note is 98.67% of its face value, or $98,670.

LOS 44.h: Calculate and interpret yield measures for money market instruments.

For money market securities, debt securities maturing in a year or less, yields are quoted based on various conventions. Some yield quotes are based on a 360-day year while others are based on a 365-day year. Some yield quotes are add-on yields and others are discount yields. Add-on yields are simply the interest to be earned on the amount paid or deposited. Discount yields are annualized current discounts from the face (maturity) values of money market securities.

Bank CDs and market reference rates are typically quoted as annualized add-on rates. U.S. Treasury bills are quoted as their annualized discount from face value, based on a 360-day year.

The relation between a security's yield quoted as an annualized add-on yield based on a 365-day year and its holding period yield (HPY) is:

Quoted add-on yield = HPY × 365/days to maturity

Consider a 100-day bank CD with an add-on yield (annualized) of 1.5%, based on a 365-day year. We can calculate the HPY of the CD as the quoted yield of 1.5% × 100/365 = 0.41%. The purchase of a $1,000 CD would provide a payment of $1,004.10 in 100 days.

The relation between a quoted discount and the actual discount based on a 360-day year is:

Quoted discount yield = discount on the security × 360/days to maturity

Consider a 180-day U.S. T-bill quoted at 2.2% (annualized) discount yield based on a 360-day year. The actual discount from face value on the T-bill is 180/360 × 2.2% = 1.1%. A $1,000 T-bill would be priced at (1 – 0.011) × 1,000 = $989. The HPY of the T-bill is 1,000/989 – 1 = 1.11%, slightly higher than its discount from face value of 1.1%.

An analyst should be able to convert the yield of a security calculated on one basis to its yield on another basis. Such adjustments allow us to compare the yields of two money market securities for which quoted yields are calculated differently. The following provides some examples of converting a yield to one based on a different convention.

EXAMPLE: **Money market yields**

1. A $1,000 90-day T-bill is priced with an annualized discount of 1.2%. Calculate its market price and its annualized add-on yield based on a 365-day year.
2. A $1 million negotiable CD with 120 days to maturity is quoted with an add-on yield of 1.4% based on a 365-day year. Calculate the payment at maturity for this CD and its bond equivalent yield.
3. A bank deposit for 100 days is quoted with an add-on yield of 1.5% based on a 360-day year. Calculate the bond equivalent yield and the yield on a semiannual bond basis.

Answer:

1. The discount from face value is 1.2% × 90 / 360 × 1,000 = $3 so the current price is 1,000 – 3 = $997.

   The equivalent add-on yield for 90 days is 3 / 997 = 0.3009%. The annualized add-on yield based on a 365-day year is 365 / 90 × 0.3009 = 1.2203%. This add-on yield based on a 365-day year is referred to as the **bond equivalent yield** for a money market security.

2. The add-on interest for the 120-day period is 120 / 365 × 1.4% = 0.4603%.

   At maturity, the CD will pay $1 million × (1 + 0.004603) = $1,004,603.

   The quoted yield on the CD is the bond equivalent yield because it is an add-on yield annualized based on a 365-day year.

3. Because the yield of 1.5% is an annualized yield calculated based on a 360-day year, the bond equivalent yield, which is based on a 365-day year, is:

   $$(365 / 360) \times 1.5\% = 1.5208\%$$

   We may want to compare the yield on a money market security to the YTM of a semiannual-pay bond. The method is to convert the money market security's holding period return to an effective semiannual yield, and then double it.

   Because the yield of 1.5% is calculated as the add-on yield for 100 days times 100 / 360, the 100-day holding period return is 1.5% × 100 / 360 = 0.4167%. The effective annual yield is $1.004167^{365/100} - 1$ = 1.5294%, the equivalent semiannual yield is $1.015294^{1/2} - 1$ = 0.7618%, and the annual yield on a semiannual bond basis is 2 × 0.7618% = 1.5236%.

   Because the periodicity of the money market security, 365 / 100, is greater than the periodicity of 2 for a semiannual-pay bond, the simple annual rate for the money market security, 1.5%, is less than the yield on a semiannual bond basis, which has a periodicity of 2.

MODULE QUIZ 44.3

1. A market rate of discount for a single payment to be made in the future is:
   A. a spot rate.
   B. a simple yield.
   C. a forward rate.
2. Based on semiannual compounding, what would the YTM be on a 15-year, zero-coupon, $1,000 par value bond that's currently trading at $331.40?
   A. 3.750%.
   B. 5.151%.
   C. 7.500%.
3. An analyst observes a Widget & Co. 7.125%, 4-year, semiannual-pay bond trading at 102.347% of par (where par is $1,000). The bond is callable at 101 in two years. What is the bond's yield-to-call?
   A. 3.167%.
   B. 5.664%.
   C. 6.334%.
4. A floating-rate note has a quoted margin of +50 basis points and a required margin of +75 basis points. On its next reset date, the price of the note will be:
   A. equal to par value.
   B. less than par value.
   C. greater than par value.
5. Which of the following money market yields is a bond-equivalent yield?
   A. Add-on yield based on a 365-day year.
   B. Discount yield based on a 360-day year.
   C. Discount yield based on a 365-day year.

MODULE 44.4: YIELD CURVES

Video covering this content is available online.

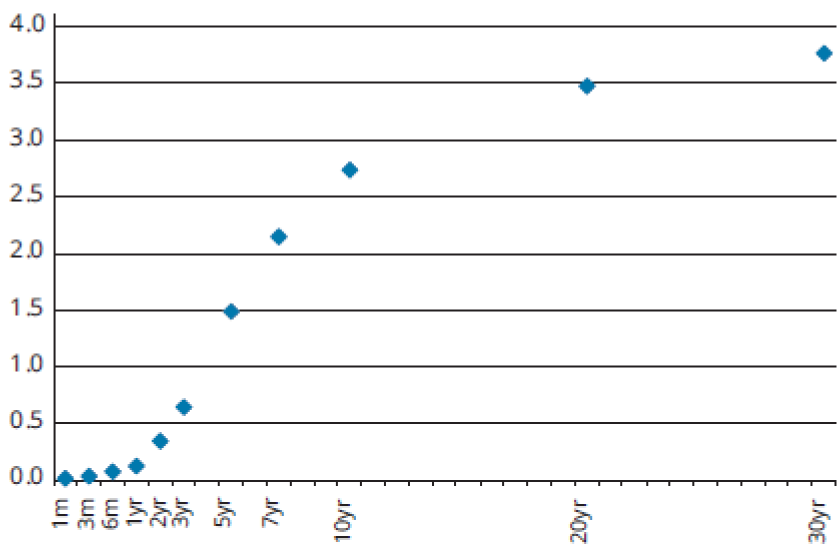
LOS 44.i: Define and compare the spot curve, yield curve on coupon bonds, par curve, and forward curve.

A **yield curve** shows yields by maturity. Yield curves are constructed for yields of various types and it's very important to understand exactly which yield is being shown. The **term structure** of interest rates refers to the yields at different maturities (terms) for like securities or interest rates. The yields on U.S. Treasury coupon bonds by maturity can be found at Treasury.gov, and several yield curves are available at Bloomberg.com.

The **spot rate yield curve** (spot curve) for U.S. Treasury bonds is also referred to as the *zero curve* (for zero-coupon) or *strip curve* (because zero-coupon U.S. Treasury bonds are also called *stripped Treasuries*). Recall that spot rates are the appropriate yields, and therefore appropriate discount rates, for single payments to be made in the future. Yields on zero-coupon government bonds are spot rates. Earlier in this reading, we calculated the value of a bond by discounting each separate payment by the spot rate corresponding to the time until the payment will be received. Spot rates are usually quoted on a semiannual bond basis, so they are directly comparable to YTMs quoted for coupon government bonds.

A **yield curve for coupon bonds** shows the YTMs for coupon bonds at various maturities. Yields are calculated for several maturities and yields for bonds with maturities between these are estimated by linear interpolation. Figure 44.4 shows a yield curve for coupon Treasury bonds constructed from yields on 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20 year, and 30-year maturities. Yields are expressed on a semiannual bond basis.

Figure 44.4: U.S. Treasury Yield Curve as of August 1, 2013



1 month	0.02	5 year	1.49
3 month	0.04	7 year	2.15
6 month	0.08	10 year	2.74
1 year	0.13	20 year	3.48
2 year	0.35	30 year	3.77
3 year	0.65		

Source: www.treasury.gov/resource-center

A **par bond yield curve**, or *par curve*, is not calculated from yields on actual bonds but is constructed from the spot curve. The yields reflect the coupon rate that a hypothetical bond at each maturity would need to have to be priced at par. Alternatively, they can be viewed as the YTM of a par bond at each maturity.

Consider a 3-year annual-pay bond and spot rates for one, two, and three years of S_1, S_2, and S_3. The following equation can be used to calculate the coupon rate necessary for the bond to be trading at par.

$$\frac{PMT}{1+S_1}+\frac{PMT}{(1+S_2)^2}+\frac{PMT+100}{(1+S_3)^3}=100$$

With spot rates of 1%, 2%, and 3%, a 3-year annual par bond will have a payment that will satisfy:

$\frac{PMT}{1.01}+\frac{PMT}{(1.02)^2}+\frac{PMT+100}{(1.03)^3}=100$, so the payment is 2.96 and the par bond, so the payment is 2.96 and the par bond

coupon rate is 2.96%.

Forward rates are yields for future periods. The rate of interest on a 1-year loan that would be made two years from now is a forward rate. A **forward yield curve** shows the future rates for bonds or money market securities for the same maturities for annual periods in the future. Typically, the forward curve would show the yields of 1-year securities for each future year, quoted on a semiannual bond basis.

LOS 44.j: Define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.

A forward rate is a borrowing/lending rate for a loan to be made at some future date. The notation used must identify both the length of the lending/borrowing period and when in the future the money will be loaned/borrowed. Thus, 1y1y is the rate for a 1-year loan one year from now; 2y1y is the rate for a 1-year loan to be made two years from now; 3y2y is the 2-year forward rate three years from now; and so on.

The Relationship Between Short-Term Forward Rates and Spot Rates

The idea here is that *borrowing for three years at the 3-year spot rate, or borrowing for one-year periods in three successive years, should have the same cost*. The S_i are the current spot rates for *i* periods.

This relation is illustrated as $(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$. Thus, $S_3 = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)]^{1/3} - 1$, which is the geometric mean return we covered in Quantitative Methods.

> EXAMPLE: **Computing spot rates from forward rates**
>
> If the current 1-year spot rate is 2%, the 1-year forward rate one year from today (*1y1y*) is 3%, and the 1-year forward rate two years from today (*2y1y*) is 4%, what is the 3-year spot rate?

Answer:

$$S_3 = [(1.02)(1.03)(1.04)]^{1/3} - 1 = 2.997\%$$

This can be interpreted to mean that a dollar compounded at 2.997% for three years would produce the same ending value as a dollar that earns compound interest of 2% the first year, 3% the next year, and 4% for the third year.

PROFESSOR'S NOTE

You can get a very good approximation of the 3-year spot rate with the simple average of the forward rates. In the previous example, we calculated 2.997% and the simple average of the three annual rates is:

$$\frac{2+3+4}{3} = 3\%.$$

Forward Rates Given Spot Rates

We can use the same relationships we use to calculate spot rates from forward rates to calculate forward rates from spot rates.

Our basic relation between forward rates and spot rates (for two periods) is:

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$$

This again tells us that an investment has the same expected yield (borrowing has the same expected cost) whether we invest (borrow) for two periods at the 2-period spot rate, S_2, or for one period at the current 1-year rate, S_1, and for the next period at the forward rate, 1y1y. Given two of these rates, we can solve for the other.

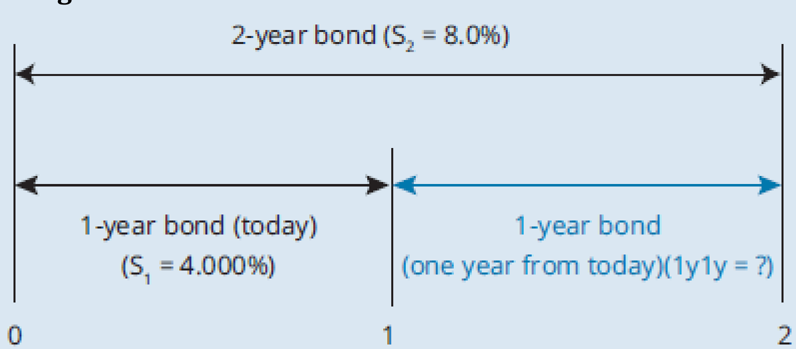
EXAMPLE: **Computing a forward rate from spot rates**

The 2-period spot rate, S_2, is 8%, and the 1-period spot rate, S_1, is 4%. Calculate the forward rate for one period, one period from now, *1y1y*.

Answer:

The following figure illustrates the problem.

Finding a Forward Rate



From our original equality, $(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, we can get

$$\frac{(1 + S_2)^2}{(1 + S_1)} = (1 + 1y1y)$$

Or, because we know that both choices have the same payoff in two years:

$$(1.08)^2 = (1.04)(1 + 1y1y)$$

$$(1 + 1y1y) = \frac{(1.08)^2}{(1.04)}$$

$$1y1y = \frac{(1.08)^2}{(1.04)} - 1 = \frac{1.1664}{1.04} - 1 = 12.154\%$$

In other words, investors are willing to accept 4.0% on the 1-year bond today (when they could get 8.0% on the 2-year bond today) only because they can get 12.154% on a 1-year bond one year from today. This future rate that can be locked in today is a forward rate.

Similarly, we can back other forward rates out of the spot rates. We know that:

$$(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$$

And that:

$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, so we can write $(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$

This last equation says that investing for three years at the 3-year spot rate should produce the same ending value as investing for two years at the 2-year spot rate, and then for a third year at *2y1y*, the 1-year forward rate, two years from now.

Solving for the forward rate, *2y1y*, we get:

$$\frac{(1 + S_3)^3}{(1 + S_2)^2} - 1 = 2y1y$$

EXAMPLE: **Forward rates from spot rates**

Let's extend the previous example to three periods. The current 1-year spot rate is 4.0%, the current 2-year spot rate is 8.0%, and the current 3-year spot rate is 12.0%. Calculate the 1-year forward rates one and two years from now.

Answer:

We know the following relation must hold:

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$$

We can use it to solve for the 1-year forward rate one year from now:

$$(1.08)^2 = (1.04)(1 + 1y1y), \text{ so } 1y1y = \frac{(1.08)^2}{(1.04)} - 1 = 12.154\%$$

We also know that the relations:

$$(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)\,(1 + 2y1y)$$

and, equivalently $(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$ must hold.

Substituting values for S_3 and S_2, we have:

$$(1.12)^3 = (1.08)^2 \times (1 + 2y1y)$$

so that the 1-year forward rate two years from now is:

$$2y1y = \frac{(1.12)^3}{(1.08)^2} - 1 = 20.45\%$$

We can check our results by calculating:

$S_3 = [(1.04)(1.12154)(1.2045)]^{1/3} - 1 = 12.00\%$

This may all seem a bit complicated, but the basic relation, that borrowing for successive periods at 1-period rates should have the same cost as borrowing at multiperiod spot rates, can be summed up as:

$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$ for two periods, and

$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$ for three periods.

PROFESSOR'S NOTE

Simple averages also give decent approximations for calculating forward rates from spot rates. In the preceding example, we had spot rates of 4% for one year and 8% for two years. Two years at 8% is 16%, so if the first-year rate is 4%, the second-year rate is close to 16 – 4 = 12% (actual is 12.154). Given a 2-year spot rate of 8% and a 3-year spot rate of 12%, we could approximate the 1-year forward rate from time two to time three as (3 × 12) – (2 × 8) = 20. That may be close enough (actual is 20.45) to answer a multiple-choice question and, in any case, serves as a good check to make sure the exact rate you calculate is reasonable.

We can also calculate implied forward rates for loans for more than one period. Given spot rates of: 1-year = 5%, 2-year = 6%, 3-year = 7%, and 4-year = 8%, we can calculate *2y2y*.

The implied forward rate on a 2-year loan two years from now, *2y2y*, is:

$$\left[\frac{(1 + S_4)^4}{(1 + S_2)^2}\right]^{1/2} - 1 = \left(\frac{1.08^4}{1.06^2}\right)^{1/2} - 1 = 10.04\%.$$

PROFESSOR'S NOTE

The approximation works for multiperiod forward rates as well.

The difference between four years at 8% (= 32%) and two years at 6% (= 12%) is 20%. Because that difference is for two years, we divide by two to get an annual rate of 10%, $\frac{(4 \times 8 - 6 \times 2)}{2} = 10$, which is very close to the exact solution of 10.04%.

Valuing a Bond Using Forward Rates

EXAMPLE: **Computing a bond value using forward rates**

The current 1-year rate, S_1, is 4%, the 1-year forward rate for lending from time = 1 to time = 2 is 1y1y = 5%, and the 1-year forward rate for lending from time = 2 to time = 3 is 2y1y = 6%. Value a 3-year annual-pay bond with a 5% coupon and a par value of $1,000.

Answer:

$$\text{bond value} = \frac{50}{1 + S_1} + \frac{50}{(1 + S_1)(1 + 1y1y)} + \frac{1{,}050}{(1 + S_1)(1 + 1y1y)(1 + 2y1y)}$$

$$= \frac{50}{1.04} + \frac{50}{(1.04)(1.05)} + \frac{1{,}050}{(1.04)(1.05)(1.06)} = \$1{,}000.98$$

PROFESSOR'S NOTE

If you think this looks a little like valuing a bond using spot rates, as we did for arbitrage-free valuation, you are correct. The discount factors are equivalent to spot rate discount factors.

If we have a semiannual coupon bond, the calculation methods are the same, but we would use the semiannual discount rate rather than the annualized rate and the number of periods would be the number of semiannual periods.

MODULE 44.5: YIELD SPREADS

Video covering this content is available online.

LOS 44.k: Compare, calculate, and interpret yield spread measures.

A **yield spread** is the difference between the yields of two different bonds. Yield spreads are typically quoted in basis points.

A yield spread relative to a benchmark bond is known as a **benchmark spread**. For example, if a 5-year corporate bond has a yield of 6.25% and its benchmark, the 5-year Treasury note, has a yield of 3.50%, the corporate bond has a benchmark spread of 625 − 350 = 275 basis points.

For fixed-coupon bonds, on-the-run government bond yields for the same or nearest maturity are frequently used as benchmarks. The benchmark may change during a bond's life. For a 5-year corporate bond, when issued, the benchmark spread is stated relative to a 5-year government bond yield, but two years later (when it has three years remaining to maturity) its benchmark spread will be stated relative to a 3-year government bond yield. A yield spread over a government bond is also known as a ***G*-spread**.

An alternative to using government bond yields as benchmarks is to use rates for interest rate swaps in the same currency and with the same tenor as a bond. Yield spreads relative to swap rates are known as **interpolated spreads** or ***I*-spreads**. *I*-spreads are frequently stated for bonds denominated in euros.

PROFESSOR'S NOTE

For bonds with tenors that do not match an on-the run government bond, yield spreads may be quoted relative to an "interpolated government bond yield." These are still *G*-spreads.

Yield spreads are useful for analyzing the factors that affect a bond's yield. If a corporate bond's yield increases from 6.25% to 6.50%, this may have been caused by factors that affect all bond yields (macroeconomic factors) or by firm-specific or industry-specific (microeconomic) factors. If a bond's yield increases but its yield spread remains the same, the yield on its benchmark must have also increased, which suggests macroeconomic factors caused bond yields in general to increase. However, if the yield spread increases, this suggests the increase in the bond's yield was caused by microeconomic factors such as credit risk or the issue's liquidity.

PROFESSOR'S NOTE

Recall from our discussion of the Fisher effect in Economics that an interest rate is composed of the real risk-free rate, the expected inflation rate, and a risk premium. We can think of macroeconomic factors as those that affect the real risk-free rate and expected inflation, and microeconomic factors as those that affect the credit and liquidity risk premium.

Zero-Volatility and Option-Adjusted Spreads

A disadvantage of *G*-spreads and *I*-spreads is that they are theoretically correct only if the spot yield curve is flat so that yields are approximately the same across maturities. Normally, however, the spot yield curve is upward-sloping (i.e., longer-term yields are higher than shorter-term yields).

A method for deriving a bond's yield spread to a benchmark spot yield curve that accounts for the shape of the yield curve is to add an equal amount to each benchmark spot rate and value the bond with those rates. When we find an amount which, when added to the benchmark spot rates, produces a value equal to the market price of the bond, we have the appropriate yield curve spread. A yield spread calculated this way is known as a **zero-volatility spread** or ***Z*-spread**.

EXAMPLE: **Zero-volatility spread**

The 1-, 2-, and 3-year spot rates on Treasuries are 4%, 8.167%, and 12.377%, respectively. Consider a 3-year, 9% annual coupon corporate bond trading at 89.464. The YTM is 13.50%, and the YTM of a 3-year Treasury is 12%. Compute the *G*-spread and the *Z*-spread of the corporate bond.

Answer:

The *G*-spread is:

$$\text{G-spread} = \text{YTM}_{\text{Bond}} - \text{YTM}_{\text{Treasury}} = 13.50 - 12.00 = 1.50\%.$$

To compute the *Z*-spread, set the present value of the bond's cash flows equal to today's market price. Discount each cash flow at the appropriate zero-coupon bond spot rate *plus* a fixed spread ZS. Solve for ZS in the following equation and you have the *Z*-spread:

$$89.464 = \frac{9}{(1.04 + \text{ZS})^1} + \frac{9}{(1.08167 + \text{ZS})^2} + \frac{109}{(1.12377 + \text{ZS})^3}$$

$$\Rightarrow \ \text{ZS} \ = \ 1.67\% \text{ or } 167 \text{ basis points}$$

Note that this spread is found by trial-and-error. In other words, pick a number "ZS," plug it into the right-hand side of the equation, and see if the result equals 89.464. If the right-hand side equals the left, then you have found the *Z*-spread. If not, adjust "ZS" in the appropriate direction and recalculate.

An **option-adjusted spread (OAS)** is used for bonds with embedded options. Loosely speaking, the option-adjusted spread takes the option yield component out of the *Z*-spread measure; the

OAS is the spread to the government spot rate curve that the bond would have if it were option-free.

If we calculate an OAS for a callable bond, it will be less than the bond's *Z*-spread. The difference is the extra yield required to compensate bondholders for the call option. That extra yield is the option value. Thus, we can write:

option value = Z-spread − OAS

OAS = Z-spread − option value

For example, if a callable bond has a *Z*-spread of 180 bp and the value of the call option is 60 bp, the bond's OAS is 180 − 60 = 120 bp.

MODULE QUIZ 44.4, 44.5

1. Which of the following yield curves is *least likely* to consist of observed yields in the market?
   A. Forward yield curve.
   B. Par bond yield curve.
   C. Coupon bond yield curve.
2. The 4-year spot rate is 9.45%, and the 3-year spot rate is 9.85%. What is the 1-year forward rate three years from today?
   A. 8.258%.
   B. 9.850%.
   C. 11.059%.
3. Given the following spot and forward rates:
   - Current 1-year spot rate is 5.5%.
   - One-year forward rate one year from today is 7.63%.
   - One-year forward rate two years from today is 12.18%.
   - One-year forward rate three years from today is 15.5%.

   The value of a 4-year, 10% annual-pay, $1,000 par value bond is *closest* to:
   A. $996.
   B. $1,009.
   C. $1,086.
4. A corporate bond is quoted at a spread of +235 basis points over an interpolated 12-year U.S. Treasury bond yield. This spread is:
   A. a *G*-spread.
   B. an *I*-spread.
   C. a *Z*-spread.

KEY CONCEPTS

LOS 44.a

The price of a bond is the present value of its future cash flows, discounted at the bond's yield-to-maturity.

For an annual-coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{(1+\text{YTM})} + \frac{\text{coupon}}{(1+\text{YTM})^2} + \ldots + \frac{\text{coupon} + \text{principal}}{(1+\text{YTM})^N}$$

For a semiannual-coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)^2} + \ldots + \frac{\text{coupon} + \text{principal}}{\left(1+\frac{\text{YTM}}{2}\right)^{N\times 2}}$$

LOS 44.b

A bond's price and YTM are inversely related. An increase in YTM decreases the price and a decrease in YTM increases the price.

A bond will be priced at a discount to par value if its coupon rate is less than its YTM, and at a premium to par value if its coupon rate is greater than its YTM.

Prices are more sensitive to changes in YTM for bonds with lower coupon rates and longer maturities, and less sensitive to changes in YTM for bonds with higher coupon rates and shorter maturities.

A bond's price moves toward par value as time passes and maturity approaches.

LOS 44.c

Spot rates are market discount rates for single payments to be made in the future.

The no-arbitrage price of a bond is calculated using (no-arbitrage) spot rates as follows:

$$\text{no-arbitrage price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_2)^2} + \cdots + \frac{\text{coupon} + \text{principal}}{(1+S_N)^N}$$

LOS 44.d

The full price of a bond includes interest accrued between coupon dates. The flat price of a bond is the full price minus accrued interest.

Accrued interest for a bond transaction is calculated as the coupon payment times the portion of the coupon period from the previous payment date to the settlement date.

Methods for determining the period of accrued interest include actual days (typically used for government bonds) or 30-day months and 360-day years (typically used for corporate bonds).

LOS 44.e

Matrix pricing is a method used to estimate the yield-to-maturity for bonds that are not traded or infrequently traded. The yield is estimated based on the yields of traded bonds with the same credit quality. If these traded bonds have different maturities than the bond being valued, linear interpolation is used to estimate the subject bond's yield.

LOS 44.f

The effective yield of a bond depends on its periodicity, or annual frequency of coupon payments. For an annual-pay bond the effective yield is equal to the yield-to-maturity. For bonds with greater periodicity, the effective yield is greater than the yield-to-maturity.

A YTM quoted on a semiannual bond basis is two times the semiannual discount rate.

LOS 44.g

Bond yields that follow street convention use the stated coupon payment dates. A true yield accounts for coupon payments that are delayed by weekends or holidays and may be slightly lower than a street convention yield.

Current yield is the ratio of a bond's annual coupon payments to its price. Simple yield adjusts current yield by using straight-line amortization of any discount or premium.

For a callable bond, a yield-to-call may be calculated using each of its call dates and prices. The lowest of these yields and YTM is a callable bond's yield-to-worst.

Floating rate notes have a *quoted margin* relative to a reference rate, typically LIBOR. The quoted margin is positive for issuers with more credit risk than the banks that quote LIBOR and may be negative for issuers that have less credit risk than loans to these banks. The *required margin* on a floating rate note may be greater than the quoted margin if credit quality has decreased, or less than the quoted margin if credit quality has increased.

LOS 44.h

For money market instruments, yields may be quoted on a discount basis or an add-on basis, and may use 360-day or 365-day years. A bond-equivalent yield is an add-on yield based on a 365-day year.

LOS 44.i

A yield curve shows the term structure of interest rates by displaying yields across different maturities.

The spot curve is a yield curve for single payments in the future, such as zero-coupon bonds or stripped Treasury bonds.

The par curve shows the coupon rates for bonds of various maturities that would result in bond prices equal to their par values.

A forward curve is a yield curve composed of forward rates, such as 1-year rates available at each year over a future period.

LOS 44.j

Forward rates are current lending/borrowing rates for short-term loans to be made in future periods.

A spot rate for a maturity of N periods is the geometric mean of forward rates over the N periods. The same relation can be used to solve for a forward rate given spot rates for two different periods.

To value a bond using forward rates, discount the cash flows at times 1 through N by the product of one plus each forward rate for periods 1 to N, and sum them.

For a 3-year annual-pay bond:

$$\text{price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_1)(1+1y1y)} + \frac{\text{coupon} + \text{principal}}{(1+S_1)(1+1y1y)(1+2y1y)}$$

LOS 44.k

A yield spread is the difference between a bond's yield and a benchmark yield or yield curve. If the benchmark is a government bond yield, the spread is known as a government spread or *G*-spread. If the benchmark is a swap rate, the spread is known as an interpolated spread or *I*-spread.

A zero-volatility spread or *Z*-spread is the percent spread that must be added to each spot rate on the benchmark yield curve to make the present value of a bond equal to its price.

An option-adjusted spread or OAS is used for bonds with embedded options. For a callable bond, the OAS is equal to the *Z*-spread minus the call option value in basis points.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 44.1

1. **B** N = 20; I/Y = 15; FV = 1,000; PMT = 100; CPT → PV = −$687.03. (LOS 44.a)
2. **A** N = 10; I/Y = 7.5; FV = 1,000; PMT = 50; CPT → PV = −$828.40. (LOS 44.a)
3. **A** The price-yield relationship is inverse. If the required yield decreases, the bond's price will increase, and vice versa. (LOS 44.b)
4. **A** With 20 years to maturity, the value of the bond with an annual-pay yield of 6.5% is N = 20, PMT = 50, FV = 1,000, I/Y = 6.5, CPT → PV = −834.72. With N = 17, CPT → PV = −848.34, so the value will increase $13.62. (LOS 44.a, 44.b)

Module Quiz 44.2

1. **A** $\text{bond value} = \frac{4{,}000}{1.032} + \frac{4{,}000}{(1.034)^2} + \frac{104{,}000}{(1.035)^3} = \$101{,}419.28$

   (LOS 44.c)
2. **B** The full price includes accrued interest, while the flat price does not. Therefore, the flat (or clean) price is 1,059.04 – 23.54 = $1,035.50.(LOS 44.d)
3. **C** Using linear interpolation, the yield on a bond with six years to maturity should be 6.40% + (1 / 3)(7.20% – 6.40%) = 6.67%. A bond with a 7% coupon and a yield of 6.67% is at a premium to par value. (LOS 44.e)

Module Quiz 44.3

1. **A** A spot rate is a discount rate for a single future payment. Simple yield is a measure of a bond's yield that accounts for coupon interest and assumes straight-line amortization of a discount or premium. A forward rate is an interest rate for a future period, such as a 3-month rate six months from today. (LOS 44.g)

   N = 30; FV = 1,000; PMT = 0; PV = –331.40; CPT → I/Y = 3.750 × 2 = 7.500%.
2. **C**

   Alternatively, $\left[\left(\frac{1{,}000}{331.4}\right)^{\frac{1}{30}} - 1\right] \times 2 = 7.5\%$

   (LOS 44.g)
3. **C** N = 4; FV = 1,010; PMT = 35.625; PV = −1,023.47; CPT → I/Y = 3.167 × 2 = 6.334%. (LOS 44.g)

4. **B** If the required margin is greater than the quoted margin, the credit quality of the issue has decreased and the price on the reset date will be less than par value. (LOS 44.g)

5. **A** An add-on yield based on a 365-day year is a bond-equivalent yield. (LOS 44.h)

Module Quiz 44.4, 44.5

1. **B** Par bond yield curves are based on the theoretical yields that would cause bonds at each maturity to be priced at par. Coupon bond yields and forward interest rates can be observed directly from market transactions. (Module 44.4, LOS 44.i)

2. **A**

$$(1.0945)^4 = (1.0985)^3 \times (1 + 3y1y)$$

$$3y1y = \frac{(1.0945)^4}{(1.0985)^3} - 1 = 8.258\%$$

Approximate forward rate = 4(9.45%) – 3(9.85%) = 8.25%. (Module 44.4, LOS 44.j)

3. **B**

$$\text{Bond value} = \frac{100}{1.055} + \frac{100}{(1.055)(1.0763)} + \frac{100}{(1.055)(1.0763)(1.1218)} + \frac{1{,}100}{(1.055)(1.0763)(1.1218)(1.155)} = 1{,}009.03$$

(Module 44.4, LOS 44.j)

4. **A** *G*-spreads are quoted relative to an actual or interpolated government bond yield. *I*-spreads are quoted relative to swap rates. *Z*-spreads are calculated based on the shape of the benchmark yield curve. (Module 44.5, LOS 44.k)

READING 45

INTRODUCTION TO ASSET-BACKED SECURITIES

EXAM FOCUS

In this reading we introduce asset-backed securities, describing their benefits, legal structure, and characteristics. Our primary focus is residential mortgage-backed securities (RMBS). Candidates should understand the characteristics of mortgage pass-through securities and how and why collateralized mortgage obligations are created from them. Be prepared to compare and contrast agency RMBS, nonagency RMBS, and commercial MBS. Finally, candidates should know why collateralized debt obligations are created and how they differ from the other securitized debt securities covered.

MODULE 45.1: STRUCTURE OF MORTGAGE-BACKED SECURITIES

Video covering this content is available online.

LOS 45.a: Explain benefits of securitization for economies and financial markets.

Securitization refers to a process by which financial assets (e.g., mortgages, accounts receivable, or automobile loans) are purchased by an entity that then issues securities supported by the cash flows from those financial assets. The primary benefits of the securitization of financial assets are (1) a reduction in funding costs for firms selling the financial assets to the securitizing entity and (2) an increase in the liquidity of the underlying financial assets.

Consider a bank that makes mortgage loans to home buyers and retains and services these loans (i.e., collects the mortgage payments and performs the necessary recordkeeping functions). To gain exposure to a bank's mortgage loans, investors traditionally could only choose among investing in bank deposits, bank debt securities, or the common equity of banks.

Compared to this traditional structure, with the bank serving the function of financial intermediary between borrowers and lenders, securitization can provide the following benefits:

- Securitization reduces intermediation costs, which results in lower funding costs for borrowers and higher risk-adjusted returns for lenders (investors).
- With securitization, the investors' legal claim to the mortgages or other loans is stronger than it is with only a general claim against the bank's overall assets.

- When a bank securitizes its loans, the securities are actively traded, which increases the liquidity of the bank's assets compared to holding the loans.
- By securitizing loans, banks are able to lend more than if they could only fund loans with bank assets. When a loan portfolio is securitized, the bank receives the proceeds, which can then be used to make more loans.
- Securitization has led to financial innovation that allows investors to invest in securities that better match their preferred risk, maturity, and return characteristics. As an example, an investor with a long investment horizon can invest in a portfolio of long-term mortgage loans rather than in only bank bonds, deposits, or equities. The investor can gain exposure to long-term mortgages without having the specialized resources and expertise necessary to provide loan origination and loan servicing functions.
- Securitization provides diversification and risk reduction compared to purchasing individual loans (whole loans).

LOS 45.b: Describe securitization, including the parties involved in the process and the roles they play.

We can illustrate the basic structure of a *securitization transaction* with this simplified, fictitious example of Fred Motor Company.

Fred Motor Company sells most of its cars on retail sales installment contracts (i.e., auto loans). The customers buy the automobiles, and Fred loans the customers the money for the purchase (i.e., Fred *originates* the loans) with the autos as collateral and receives principal and interest payments on the loans until they mature. The loans have maturities of 48 to 60 months at various interest rates. Fred is also the *servicer* of the loans (i.e., it collects principal and interest payments, sends out delinquency notices, and repossesses and disposes of the autos if the customers do not make timely payments).

Fred has 50,000 auto loans totaling $1 billion that it would like to remove from its balance sheet and use the proceeds to make more auto loans. It accomplishes this by selling the loan portfolio to a **special purpose entity (SPE)** called Auto Loan Trust for $1 billion (Fred is called the *seller* or *depositor*). The SPE, which is set up for the specific purpose of buying these auto loans and selling asset-backed securities (ABS), is referred to as the *trust* or the *issuer*. The SPE then sells ABS to investors. The loan portfolio is the collateral supporting the ABS because the cash flows from the loans are the source of the funds to make the promised payments to investors. An SPE is sometimes also called a special purpose vehicle (SPV). The SPE is a separate legal entity from Fred.

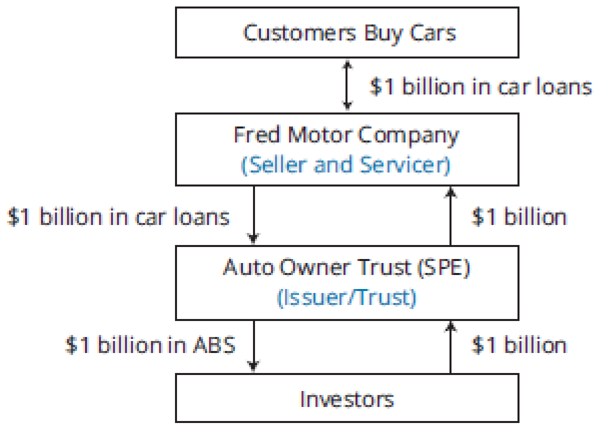
Let's review the parties to this transaction and their functions:

- The seller or depositor (Fred) originates the auto loans and sells the portfolio of loans to Auto Loan Trust, the SPE.
- The issuer/trust (Auto Loan Trust) is the SPE that buys the loans from the seller and issues ABS to investors.
- The servicer (Fred) services the loans.

- In this case, the seller and the servicer are the same entity (Fred Motor Company), but that is not always the case.

The structure of this securitization transaction is illustrated in Figure 45.1.

Figure 45.1: Structure of Fred Motor Company Asset Securitization



Subsequent to the initial transaction, the principal and interest payments on the original loans are allocated to pay servicing fees to the servicer and principal and interest payments to the owners of the ABS.

ABS are most commonly backed by automobile loans, credit card receivables, home equity loans, manufactured housing loans, student loans, Small Business Administration (SBA) loans, corporate loans, corporate bonds, emerging market bonds, and structured financial products. When the loans owned by the trust (SPE) are mortgages, we refer to the securities issued by the trust as **mortgage-backed securities (MBS)**.

Note that the SPE is a separate legal entity from Fred and the buyers of the ABS have no claim on other assets of Fred, only on the loans sold to the SPE.

If Fred had issued corporate bonds to raise the funds to make more auto loans, the bondholders would be subject to the financial risks of Fred. With the ABS structure, a decline in the financial position of Fred, its ability to make cash payments, or its bond rating do not affect the value of the claims of ABS owners to the cash flows from the trust collateral (loan portfolio) because it has been sold by Fred, which is now simply the servicer (not the owner) of the loans. The credit rating of the ABS securities may be higher than the credit rating of bonds issued by Fred, in which case the cost to fund the loans using the ABS structure is lower than if Fred funded additional loans by issuing corporate bonds.

LOS 45.c: Describe typical structures of securitizations, including credit tranching and time tranching.

Securitizations may involve a single class of ABS so the cash flows to the securities are the same for all security holders. They can also be structured with multiple classes of securities, each with a different claim to the cash flows of the underlying assets. The different classes are often referred to as **tranches**. With this structure, a particular risk of the ABS securities is redistributed across the tranches. Some bear more of the risk and others bear less of the risk. The total risk is unchanged, simply reapportioned.

With **credit tranching**, the ABS tranches will have different exposures to the risk of default of the assets underlying the ABS. With this structure, also called a **senior/subordinated structure**, the subordinated tranches absorb credit losses as they occur (up to their principal values). The level of protection for the senior tranche increases with the proportion of subordinated bonds in the structure.

Let's look at an example to illustrate how a senior/subordinated structure redistributes the credit risk compared to a single-class structure. Consider an ABS with the following bond classes:

Senior Tranche	$300,000,000
Subordinated Tranche A	$80,000,000
Subordinated Tranche B	$30,000,000
Total	$410,000,000

Tranche B is first to absorb any losses (and is termed the *first-loss tranche*) until they exceed $30 million in principal. Any losses from default of the underlying assets greater than $30 million, and up to $110 million, will be absorbed by Subordinated Tranche A. The Senior Tranche is protected from any credit losses of $110 million or less and therefore will have the highest credit rating and offer the lowest yield of the three bond classes. This structure is also called a **waterfall** structure because in liquidation, each subordinated tranche would receive only the "overflow" from the more senior tranche(s) if they are repaid their principal value in full.

With **time tranching**, the first (sequential) tranche receives all principal repayments from the underlying assets up to the principal value of the tranche. The second tranche would then receive all principal repayments from the underlying assets until the principal value of this tranche is paid off. There may be other tranches with sequential claims to remaining principal repayments. Time tranching reapportions prepayment risk—the risk of receiving principal payments either sooner than expected (contraction risk) or later than expected (extension risk). The first tranche receives all principal payments until it is paid off, and so has the greatest contraction risk. The second tranche receives no prepayments until the first tranche is paid in full, then receives all principal payments until the entire issue is paid off, and so has more extension risk and less contraction risk compared with the first tranche.

Both credit tranching and time tranching are often included in the same structure. More detail about time tranching and the related planned amortization/support tranche structure is included later in this review when we discuss the structures of mortgage-backed securities.

LOS 45.d: Describe types and characteristics of residential mortgage loans that are typically securitized.

A **residential mortgage loan** is a loan for which the collateral that underlies the loan is residential real estate. If the borrower defaults on the loan, the lender has a legal claim to the collateral property. One key characteristic of a mortgage loan is its **loan-to-value ratio (LTV)**, the percentage of the value of the collateral real estate that is loaned to the borrower. The lower the LTV, the higher the borrower's equity in the property.

For a lender, loans with lower LTVs are less risky because the borrower has more to lose in the event of default (so is less likely to default). Also, if the property value is high compared to the loan amount, the lender is more likely to recover the amount loaned if the borrower defaults and the lender repossesses and sells the property. In the United States, mortgages made to borrowers with good credit are termed *prime loans*. Mortgages to borrowers of lower credit quality, or that have a lower-priority claim to the collateral in event of default, are termed *subprime loans*.

Typical mortgage terms and structures differ across regions and countries. The key characteristics of mortgage loans include their maturity, the determination of interest charges, how the loan principal is amortized, the terms under which prepayments of loan principal are allowed, and the rights of the lender in the event of default by the borrower. We address each of the characteristics in more detail.

Maturity

The term of a mortgage loan is the time until the final loan payment is made. In the United States, mortgage loans typically have terms from 15 to 30 years. Terms are longer, 20 to 40 years, in many European countries and as long as 50 years in others. In Japan, mortgage loans may have terms of 100 years.

Interest Rate

A **fixed-rate mortgage** has an interest rate that is unchanged over the life of the mortgage.

An **adjustable-rate mortgage (ARM)**, also called a **variable-rate mortgage**, has an interest rate that can change over the life of the mortgage. An **index-referenced mortgage** has an interest rate that changes based on a market determined reference rate such as LIBOR or the one-year U.S. Treasury bill rate, although several other reference rates are used.

A mortgage loan may have an interest rate that is fixed for some initial period, but adjusted after that. If the loan becomes an adjustable-rate mortgage after the initial fixed-rate period it is called a *hybrid mortgage*. If the interest rate changes to a different fixed rate after the initial fixed-rate period it is called a *rollover* or *renegotiable mortgage*.

A **convertible mortgage** is one for which the initial interest rate terms, fixed or adjustable, can be changed at the option of the borrower, to adjustable or fixed, for the remaining loan period.

Amortization of Principal

With a **fully amortizing** loan, each payment includes both an interest payment and a repayment of some of the loan principal so there is no loan principal remaining after the last regular mortgage payment. When payments are fixed for the life of the loan, payments in the beginning of the loan term have a large interest component and a small principal repayment component, and payments at the end of the loan terms have a small interest component and large principal repayment component.

A loan is said to be **partially amortizing** when loan payments include some repayment of principal, but there is a lump sum of principal that remains to be paid at the end of the loan period which is called a *balloon payment*. With an **interest-only mortgage**, there is no principal repayment for either an initial period or the life of the loan. If no principal is paid for the life of the loan it is an *interest-only lifetime* mortgage and the balloon payment is the original loan principal amount. Other interest-only mortgages specify that payments are interest-only over some initial period, with partial or full amortization of principal after that.

Prepayment Provisions

A partial or full repayment of principal in excess of the scheduled principal repayments required by the mortgage is referred to as a **prepayment**. If a homeowner sells her home during the mortgage term (a common occurrence), repaying the remaining principal is required and is one type of prepayment. A homeowner who *refinances* her mortgage prepays the remaining principal amount using the proceeds of a new, lower interest rate loan. Some homeowners prepay by paying more than their scheduled payments in order to reduce the principal outstanding, reduce their interest charges, and eventually pay off their loans prior to maturity.

Some loans have no penalty for prepayment of principal while others have a **prepayment penalty**. A prepayment penalty is an additional payment that must be made if principal is prepaid during an initial period after loan origination or, for some mortgages, prepaid anytime during the life of the mortgage. A prepayment penalty benefits the lender by providing compensation when the loan is paid off early because market interest rates have decreased since the mortgage loan was made (i.e., loans are refinanced at a lower interest rate).

Foreclosure

Some mortgage loans are **nonrecourse loans**, which means the lender has no claim against the assets of the borrower except for the collateral property itself. When this is the case, if home values fall so the outstanding loan principal is greater than the home value, borrowers sometimes voluntarily return the property to the lender in what is called a *strategic default*.

Other mortgage loans are **recourse loans** under which the lender has a claim against the borrower for the amount by which the sale of a repossessed collateral property falls short of the principal outstanding on the loan. Understandably, borrowers are more likely to default on nonrecourse loans than on recourse loans. In Europe, most residential mortgages are recourse loans. In the United States, they are recourse loans in some states and nonrecourse in others.

LOS 45.e: Describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type.

LOS 45.f: Define prepayment risk and describe the prepayment risk of mortgage-backed securities.

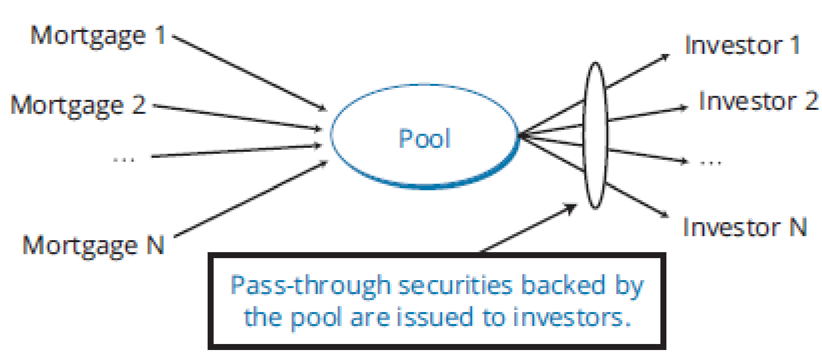
Residential mortgage-backed securities (RMBS) in the United States are termed **agency RMBS** or **nonagency RMBS**, depending on the issuer of the securities. Agency RMBS are issued by the Government National Mortgage Association (GNMA or Ginnie Mae), the Federal National Mortgage Association (Fannie Mae), and the Federal Home Loan Mortgage Corporation (Freddie Mac). Ginnie Mae securities are guaranteed by the GNMA and are considered to be backed by the full faith and credit of the U.S. government. Fannie Mae and Freddie Mac also guarantee the MBS they issue but are *government-sponsored enterprises (GSE)*. While they are not considered to be backed by the full faith and credit of the U.S. government, these securities are considered to have very high credit quality.

Agency RMBS are **mortgage pass-through securities**. Each mortgage pass-through security represents a claim on the cash flows from a pool of mortgages. Any number of mortgages may be used to form the pool, and any mortgage included in the pool is referred to as a **securitized mortgage**. The mortgages in the pool typically have different maturities and different mortgage rates. The **weighted average maturity (WAM)** of the pool is equal to the weighted average of the final maturities of all the mortgages in the pool, weighted by each mortgage's outstanding principal balance as a proportion of the total outstanding principal value of all the mortgages in the pool. The **weighted average coupon (WAC)** of the pool is the weighted average of the interest rates of all the mortgages in the pool. The investment characteristics of mortgage pass-through securities are a function of their cash flow features and the strength of the guarantee provided.

In order to be included in agency MBS pools, loans must meet certain criteria, including a minimum percentage down payment, a maximum LTV ratio, maximum size, minimum documentation required, and insurance purchased by the borrower. Loans that meet the standards for inclusion in agency MBS are called *conforming loans*. Loans that do not meet the standards are called *nonconforming* loans. Nonconforming mortgages can be securitized by private companies for *nonagency RMBS*.

Investors in mortgage pass-through securities receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that **pass-through rates** (i.e., the coupon rate on the MBS, also called its *net interest* or *net coupon*) are less than the mortgage rate of the underlying mortgages in the pool.

Figure 45.2: Mortgage Pass-Through Cash Flow



The timing of the cash flows to pass-through security holders does not exactly coincide with the cash flows generated by the pool. This is due to the delay between the time the mortgage service provider receives the mortgage payments and the time the cash flows are *passed through* to the security holders.

MODULE QUIZ 45.1

1. Economic benefits of securitization *least likely* include:
   A. reducing excessive lending by banks.
   B. reducing funding costs for firms that securitize assets.
   C. increasing the liquidity of the underlying financial assets.

2. In a securitization, the issuer of asset-backed securities is *best* described as:
    A. the SPE.
    B. the seller.
    C. the servicer.
3. A mortgage-backed security with a senior/subordinated structure is said to feature:
    A. time tranching.
    B. credit tranching.
    C. a pass-through structure.
4. A mortgage that has a balloon payment equal to the original loan principal is:
    A. a convertible mortgage.
    B. a fully amortizing mortgage.
    C. an interest-only lifetime mortgage.
5. Residential mortgages that may be included in agency RMBS are *least likely* required to have:
    A. a minimum loan-to-value ratio.
    B. insurance on the mortgaged property.
    C. a minimum percentage down payment.
6. The primary motivation for issuing collateralized mortgage obligations (CMOs) is to reduce:
    A. extension risk.
    B. funding costs.
    C. contraction risk.

MODULE 45.2: PREPAYMENT RISK AND NON-MORTGAGE-BACKED ABS

Video covering this content is available online.

Prepayment Risk

An important characteristic of pass-through securities is their **prepayment risk.** Because the mortgage loans used as collateral for agency MBS have no prepayment penalty, the MBS themselves have significant prepayment risk. Recall that prepayments are principal repayments in excess of the scheduled principal repayments for amortizing loans. The risk that prepayments will be slower than expected is called *extension risk* and the risk that prepayments will be more rapid than expected is called *contraction risk*.

Prepayments cause the timing and amount of cash flows from mortgage loans and MBS to be uncertain; rapid prepayment reduces the amount of principal outstanding on the loans supporting the MBS so the total interest paid over the life of the MBS is reduced. Because of this, it is necessary to make specific assumptions about prepayment rates in order to value mortgage pass-through securities. The single monthly mortality rate (SMM) is the percentage by which prepayments reduce the month-end principal balance, compared to what it would have been with only scheduled principal payments (with no prepayments). The **conditional prepayment rate (CPR)** is an annualized measure of prepayments. Prepayment rates depend on the weighted average coupon rate of the loan pool, current interest rates, and prior prepayments of principal.

The Public Securities Association (PSA) *prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages (becomes *seasoned*). The PSA benchmark is expressed as a monthly series of CPRs. If the prepayment rate (CPR) of an MBS is expected to be the same as the PSA standard benchmark CPR, we say the PSA is 100 (100% of the benchmark CPR). A pool of mortgages may have prepayment rates that are faster or slower than PSA 100, depending on the current level of interest rates and the coupon rate of the issue. A PSA of 50 means that prepayments are 50% of the PSA benchmark CPR, and a PSA of 130 means that prepayments are 130% of the PSA benchmark CPR.

Based on an assumption about the prepayment rate for an MBS, we can calculate its **weighted average life**, or simply average life, which is the expected number of years until all the loan principal is repaid. Because of prepayments, the average life of an MBS will be less than its weighted average maturity. During periods of falling interest rates, the refinancing of mortgage loans will accelerate prepayments and reduce the average life of an MBS. A high PSA, such as 400, will reduce the average life of an MBS to only 4.5 years, compared to an average life of about 11 years for an MBS with a PSA of 100.

Collateralized Mortgage Obligations

Collateralized mortgage obligations (CMO) are securities that are collateralized by RMBS. Each CMO has multiple bond classes (CMO tranches) that have different exposures to prepayment risk. The total prepayment risk of the underlying RMBS is not changed; the prepayment risk is simply reapportioned among the various CMO tranches.

Institutional investors have different tolerances for prepayment risk. Some are primarily concerned with extension risk while others may want to minimize exposure to contraction risk. By partitioning and distributing the cash flows generated by RMBS into different risk packages to better match investor preferences, CMOs increase the potential market for securitized mortgages and perhaps reduce funding costs as a result.

CMOs are securities backed by mortgage pass-through securities (i.e., they are securities secured by other securities). Interest and principal payments from the mortgage pass-through securities are allocated in a specific way to different bond classes called tranches, so that each tranche has a different claim against the cash flows of the mortgage pass-throughs. Each CMO tranche has a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

The primary CMO structures include sequential-pay tranches, planned amortization class tranches (PACs), support tranches, and floating-rate tranches.

Sequential Pay CMO

One way to reapportion the prepayment risk inherent in the underlying pass-through MBS is to separate the cash flows into tranches that are retired sequentially (i.e., create a **sequential pay CMO**). As an example of this structure, we consider a simple CMO with two tranches. Both tranches receive interest payments at a specified coupon rate, but all principal payments (both scheduled payments and prepayments) are paid to Tranche 1 (the *short tranche*) until its principal is paid off. Principal payments then flow to Tranche 2 until its principal is paid off.

Contraction and extension risk still exist with this structure, but they have been redistributed to some extent between the two tranches. The short tranche, which matures first, offers investors relatively more protection against extension risk. The other tranche provides relatively more protection against contraction risk. Let's expand this example with some specific numbers to illustrate how sequential pay structures work.

Consider the simplified CMO structure presented in Figure 45.3. Payments to the two sequential-pay tranches are made first to Tranche A and then to Tranche B.

Figure 45.3: Sequential Pay CMO Structure

CMO Structure		
Tranche	Outstanding Par Value	Coupon Rate
A	$200,000,000	8.50%
B	50,000,000	8.50%

Payments from the underlying collateral (which has a pass-through coupon rate of 8.5%) for the first five months, as well as months 183 through 187, are shown in Figure 45.4. These payments include scheduled payments plus estimated prepayments based on an assumed prepayment rate. (Note that some totals do not match due to rounding.)

Figure 45.4: CMO Projected Cash Flows

Month	Beginning Principal Balance	Principal Payment	Interest	Total Cash Flow = Principal Plus Interest
1	$250,000,000	$391,128	$1,770,833	$2,161,961
2	249,608,872	454,790	1,768,063	2,222,853
3	249,154,082	518,304	1,764,841	2,283,145
4	248,635,778	581,620	1,761,170	2,342,790
5	248,054,157	644,690	1,757,050	2,401,741
183	$51,491,678	$545,153	$364,733	$909,886
184	50,946,525	540,831	360,871	901,702
185	50,405,694	536,542	357,040	893,582
186	49,869,152	532,287	353,240	885,526
187	49,336,866	528,065	349,469	877,534

PROFESSOR'S NOTE

This example is provided as an illustration of how cash flows are allocated to sequential tranches. The LOS does not require you to do the calculations that underlie the numbers in Figure 45.4. The important point here is how the cash flows are allocated to each tranche.

Planned Amortization Class (PAC) CMO

Another CMO structure has one or more **planned amortization class (PAC)** tranches and **support tranches**. A PAC tranche is structured to make predictable payments, regardless of actual prepayments to the underlying MBS. The PAC tranches have both reduced contraction risk and reduced extension risk compared to the underlying MBS.

Reducing the prepayment risk of the PAC tranches is achieved by increasing the prepayment risk of the CMO's support tranches. If principal repayments are more rapid than expected, the support tranche receives the principal repayments in excess of those specifically allocated to the PAC tranches. Conversely, if the actual principal repayments are slower than expected, principal repayments to the support tranche are curtailed so the scheduled PAC payments can be made. The larger the support tranche(s) relative to the PAC tranches, the smaller the probability that the cash flows to the PAC tranches will differ from their scheduled payments.

For a given CMO structure there are limits to how fast or slow actual prepayment experience can be before the support tranches can no longer either provide or absorb prepayments in the amounts required to keep the PAC payments to their scheduled amounts. The upper and lower bounds on the actual prepayment rates for which the support tranches are sufficient to either provide or absorb actual prepayments in order to keep the PAC principal repayments on schedule are called the **initial PAC collar**.

A PAC may have an initial collar given as 100 – 300 PSA. This means the PAC will make its scheduled payments to investors unless actual prepayment experience is outside these bounds (i.e., above 300 PSA or below 100 PSA). If the prepayment rate is outside of these bounds so payments to a PAC tranche are either sooner or later than promised, the PAC tranche is referred to as a **broken PAC**.

Support tranches have both more contraction risk and more extension risk than the underlying MBS and have a higher promised interest rate than the PAC tranche.

As an example, Figure 45.5 shows the average life for a hypothetical structure that includes a PAC I tranche and a support tranche at various PSA speeds, assuming the PSA speed stays at that level for the entire life of the PAC tranche.

Figure 45.5: Average Life Variability of PAC I Tranche vs. Support Tranche

PSA Speed	PAC I Tranche		Support Tranche
0	13.2		24.0
50	8.8		21.1
100	6.5	↑	17.1
150	6.5		13.3
200	6.5	Initial Collar	10.4
250	6.5	↓	5.2
300	6.5		2.9
350	5.9		2.4
400	5.4		1.8
450	4.6		1.5
500	4.2		1.2

Figure 45.5 illustrates that the PAC I tranche has less prepayment risk than the support tranche because the variability of its average life is significantly lower.

- When prepayment speeds fall and prepayments decrease, the support tranche average life is significantly longer than the average life of the PAC I tranche. Thus, the support tranche has significantly more extension risk.
- When prepayment speeds rise and prepayments increase, the support tranche average life is much shorter than that of the PAC I tranche. Thus, the support tranche also has significantly more contraction risk.

- Within the initial PAC collar of 100 to 300 PSA, the average life of the PAC I tranche is constant at 6.5 years.

Nonagency RMBS

RMBS not issued by GNMA, Fannie Mae, or Freddie Mac are referred to as **nonagency RMBS**. They are not guaranteed by the government, so credit risk is an important consideration. The credit quality of a nonagency MBS depends on the credit quality of the borrowers as well as the characteristics of the loans, such as their LTV ratios. To be investment grade, most nonagency RMBS include some sort of **credit enhancement**. The level of credit enhancement is directly proportional to the credit rating desired by the issuer. Rating agencies determine the exact amount of credit enhancement necessary for an issue to hold a specific rating.

Credit tranching (subordination) is often used to enhance the credit quality of senior RMBS securities. A **shifting interest mechanism** is a method for addressing a decrease in the level of credit protection provided by junior tranches as prepayments or defaults occur in a senior/subordinated structure. If prepayments or credit losses decrease the credit enhancement of the senior securities, the shifting interest mechanism suspends payments to the subordinated securities for a period of time until the credit quality of the senior securities is restored.

LOS 45.g: Describe characteristics and risks of commercial mortgage-backed securities.

Commercial mortgage-backed securities (CMBS) are backed by income-producing real estate, typically in the form of:

- Apartments (multifamily).
- Warehouses (industrial use property).
- Shopping centers.
- Office buildings.
- Health care facilities.
- Senior housing.
- Hotel/resort properties.

An important difference between residential and commercial MBS is the obligations of the borrowers of the underlying loans. Residential MBS loans are repaid by homeowners; commercial MBS loans are repaid by real estate investors who, in turn, rely on tenants and customers to provide the cash flow to repay the mortgage loan. CMBS mortgages are structured as **nonrecourse loans**, meaning the lender can *only* look to the collateral as a means to repay a delinquent loan if the cash flows from the property are insufficient. In contrast, a residential mortgage lender with recourse can go back to the borrower personally in an attempt to collect any excess of the loan amount above the net proceeds from foreclosing on and selling the property.

For these reasons, the analysis of CMBS securities focuses on the credit risk of the property and not the credit risk of the borrower. The analysis of CMBS structures focuses on two key ratios to assess credit risk.

1. **Debt service coverage ratio** is a basic cash flow coverage ratio of the amount of cash flow from a commercial property available to make debt service payments compared to the required debt service cost.

   $$\text{debt-service coverage ratio} = \frac{\text{net operating income}}{\text{debt service}}$$

   Net operating income (NOI) is calculated after the deduction for real estate taxes but before any relevant income taxes. This ratio, which is typically between one and two, indicates greater protection to the lender when it is higher. Debt service coverage ratios below one indicate that the borrower is not generating sufficient cash flow to make the debt payments and is likely to default. Remember: *the higher the better* for this ratio from the perspective of the lender and the MBS investor.

2. **Loan-to-value ratio** compares the loan amount on the property to its current fair market or appraisal value.

   $$\text{loan-to-value ratio} = \frac{\text{current mortgage amount}}{\text{current appraised value}}$$

   The lower this ratio, the more protection the mortgage lender has in making the loan. Loan-to-value ratios determine the amount of collateral available, above the loan amount, to provide a cushion to the lender should the property be foreclosed on and sold. Remember: *the lower the better* for this ratio from the perspective of the lender and the MBS investor.

The basic **CMBS structure** is created to meet the risk and return needs of the CMBS investor. As with residential MBS securities, rating organizations such as S&P and Moody's assess the credit risk of each CMBS issue and determine the appropriate credit rating. Each CMBS is segregated into tranches. Losses due to default are first absorbed by the tranche with the lowest priority. Sometimes this most-junior tranche is not rated and is then referred to as the equity tranche, residual tranche, or first-loss tranche.

As with any fixed-rate security, call protection is valuable to the bondholder. In the case of MBS, call protection is equivalent to prepayment protection (i.e., restrictions on the early return of principal through prepayments). CMBS provide call protection in two ways: loan-level call protection provided by the terms of the individual mortgages and call protection provided by the CMBS structure.

There are several means of creating **loan-level call protection:**

- *Prepayment lockout.* For a specific period of time (typically two to five years), the borrower is prohibited from prepaying the mortgage loan.
- *Defeasance.* Should the borrower insist on making principal payments on the mortgage loan, the mortgage loan can be defeased. This is accomplished by using the prepaid principal to purchase a portfolio of government securities that is sufficient to make the remaining required payments on the CMBS. Given the high credit quality of government securities, defeased loans increase the credit quality of a CMBS loan pool.
- *Prepayment penalty points.* A penalty fee expressed in points may be charged to borrowers who prepay mortgage principal. Each point is 1% of the principal amount prepaid.

- *Yield maintenance charges.* The borrower is charged the amount of interest lost by the lender should the loan be prepaid. This *make whole* charge is designed to make lenders indifferent to prepayment, as cash flows are equivalent (at current market rates) whether the loan is prepaid or not.

With all loan call protection programs, any prepayment penalties received are distributed to the CMBS investors in a manner determined by the structure of the CMBS issue.

To create **CMBS-level call protection**, CMBS loan pools are segregated into tranches with a specific sequence of repayment. Those tranches with a higher priority will have a higher credit rating than lower priority tranches because loan defaults will first affect the lower tranches. A wide variety of features can be used to provide call protection to the more senior tranches of the CMBS.

Commercial mortgages are typically amortized over a period longer than the loan term; for example, payments for a 20-year commercial mortgage may be determined based on a 30-year amortization schedule. At the end of the loan term, the loan will still have principal outstanding that needs to be paid; this amount is called a **balloon payment**. If the borrower is unable to arrange refinancing to make this payment, the borrower is in default. This possibility is called balloon risk. The lender will be forced to extend the term of the loan during a workout period, during which time the borrower will be charged a higher interest rate. Because balloon risk entails extending the term of the loan, it is also referred to as extension risk for CMBS.

LOS 45.h: Describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and risks of each type.

In addition to those backed by mortgages, there are ABS that are backed by various types of financial assets including small business loans, accounts receivable, credit card receivables, automobile loans, home equity loans, and manufactured housing loans. Each of these types of ABS has different risk characteristics and their structures vary to some extent as well. Here we explain the characteristics of two types, ABS backed by automobile loans and ABS backed by credit card receivables. These two have an important difference in that automobile loans are fully amortizing while credit card receivables are nonamortizing.

Auto Loan ABS

Auto loan-backed securities are backed by loans for automobiles. Auto loans have maturities from 36 to 72 months. Issuers include the financial subsidiaries of auto manufacturers, commercial banks, credit unions, finance companies, and other small financial institutions.

The cash flow components of auto loan-backed securities include interest payments, scheduled principal payments, and prepayments. Auto loans prepay if the cars are sold, traded in, or repossessed. Prepayments also occur if the car is stolen or wrecked and the loan is paid off from insurance proceeds. Finally, the borrower may simply use excess cash to reduce or pay off the loan balance.

Automobile loan ABS all have some sort of credit enhancement to make them attractive to institutional investors. Many have a senior-subordinated structure, with a junior tranche that absorbs credit risk. One or more internal credit enhancement methods, a reserve account, an excess interest spread, or overcollateralization, is also often present in these structures. Just as with mortgages, prime loans refer to those made to borrowers with higher credit ratings and sub-prime loans refers to those made to borrowers with low credit ratings.

Credit Card ABS

Credit card receivable-backed securities are ABS backed by pools of credit card debt owed to banks, retailers, travel and entertainment companies, and other credit card issuers.

The cash flow to a pool of credit card receivables includes finance charges, annual fees, and principal repayments. Credit cards have periodic payment schedules, but because their balances are revolving (i.e., nonamortizing), the principal amount is maintained for a period of time. Interest on credit card ABS is paid periodically, but no principal is paid to the ABS holders during the **lockout period**, which may last from 18 months to 10 years after the ABS are created.

If the underlying credit card holders make principal payments during the lockout period, these payments are used to purchase additional credit card receivables, keeping the overall value of the receivables pool relatively constant. Once the lockout period ends, principal payments are passed through to security holders. Credit card ABS typically have an early (rapid) amortization provision that provides for earlier amortization of principal when it is necessary to preserve the credit quality of the securities.

Interest rates on credit card ABS are sometimes fixed but often they are floating. Interest payments may be monthly, quarterly, or for longer periods.

LOS 45.i: Describe collateralized debt obligations, including their cash flows and risks.

A **collateralized debt obligation (CDO)** is a structured security issued by an SPE for which the collateral is a pool of debt obligations. When the collateral securities are corporate and emerging market debt, they are called *collateralized bond obligations* (CBO). *Collateralized loan obligations* (CLO) are supported by a portfolio of leveraged bank loans. Unlike the ABS we have discussed, CDOs do not rely on interest payments from the collateral pool. CDOs have a **collateral manager** who buys and sells securities in the collateral pool in order to generate the cash to make the promised payments to investors.

Structured finance CDOs are those where the collateral is ABS, RMBS, other CDOs, and CMBS.

Synthetic CDOs are those where the collateral is a portfolio of credit default swaps on structured securities.

PROFESSOR'S NOTE

> Credit default swaps are derivative securities that decrease (increase) in value as the credit quality of their reference securities increases (decreases).

CDOs issue three classes of bonds (tranches): senior bonds, mezzanine bonds, and subordinated bonds (sometimes called the equity or residual tranche). The subordinated tranche has characteristics more similar to those of equity investments than bond investments. In creating a

CDO, the structure must be able to offer an attractive return on the subordinated tranche, after accounting for the required yields on the senior and mezzanine bond classes.

An investment in the equity or residual tranche can be viewed as a leveraged investment where borrowed funds (raised from selling the senior and mezzanine tranches) are used to purchase the debt securities in the CDO's collateral pool. To the extent the collateral manager meets his goal of earning returns in excess of borrowing costs (the promised return to CDO investors), these excess returns are paid to the CDO manager and the equity tranche.

The CDO structure typically is to issue a floating-rate senior tranche that is 70%–80% of the total and a smaller mezzanine tranche that pays a fixed rate of interest. If the securities in the collateral pool pay a fixed rate of interest, the collateral manager may enter into an interest rate swap that pays a floating rate of interest in exchange for a fixed rate of interest in order to make the collateral yield more closely match the funding costs in an environment of changing interest rates. The term *arbitrage CDO* is used for CDOs structured to earn returns from the spread between funding costs and portfolio returns.

The collateral manager may use interest earned on portfolio securities, cash from maturing portfolio securities, and cash from the sale of portfolio securities to cover the promised payments to holders of the CDOs senior and mezzanine bonds.

LOS 45.j: Describe characteristics and risks of covered bonds and how they differ from other asset-backed securities.

Covered bonds are similar to asset-backed securities, but the underlying assets (the cover pool), although segregated, remain on the balance sheet of the issuing corporation (i.e., no SPE is created). Covered bonds are issued primarily by European and Asian financial institutions.

Special legislation protects the assets in the cover pool in the event of firm insolvency (they are bankruptcy remote). In contrast to an SPE structure, however, covered bonds also provide bondholders with recourse to the issuing firm. This increases their credit quality, which results in covered bonds generally having lower yields than comparable ABS.

Unlike an ABS, in which the pool of assets is fixed at issuance, a covered bond requires the issuer to replace or augment nonperforming or prepaid assets in the cover pool so that it always provides for the covered bond's promised interest and principal payments. Covered bonds typically are not structured with credit tranching.

Covered bonds may have different provisions in case their issuer defaults. A **hard-bullet covered bond** is in default if the issuer fails to make a scheduled payment. A **soft-bullet covered bond** may postpone the originally scheduled maturity date by as much as a year. A **conditional pass-through covered bond** converts to a pass-through bond on the maturity date if any payments remain due.

MODULE QUIZ 45.2

1. The risk that mortgage prepayments will occur more slowly than expected is *best* characterized as:
   A. default risk.
   B. extension risk.
   C. contraction risk.

2. For investors in commercial mortgage-backed securities, balloon risk in commercial mortgages results in:
   A. call risk.
   B. extension risk.
   C. contraction risk.
3. During the lockout period of a credit card ABS:
   A. no new receivables are added to the pool.
   B. investors do not receive interest payments.
   C. investors do not receive principal payments.
4. A debt security that is collateralized by a pool of the sovereign debt of several developing countries is *most likely*:
   A. a CMBS.
   B. a CDO.
   C. a CMO.
5. A covered bond is *most likely* to feature:
   A. a fixed cover pool.
   B. recourse to the issuer.
   C. a special purpose entity.

KEY CONCEPTS

LOS 45.a

The primary benefits of the securitization of financial assets are:

- Reduce the funding costs for firms selling the financial assets to the securitizing entity.
- Increase the liquidity of the underlying financial assets.

LOS 45.b

Parties to a securitization are a seller of financial assets, a special purpose entity (SPE), and a servicer.

- The seller is the firm that is raising funds through the securitization.
- An SPE is an entity independent of the seller. The SPE buys financial assets from the seller and issues asset-backed securities (ABS) supported by these financial assets.
- The servicer carries out collections and other responsibilities related to the financial assets. The servicer may be the same entity as the seller but does not have to be.

The SPE may issue a single class of ABS or multiple classes with different priorities of claims to cash flows from the pool of financial assets.

LOS 45.c

Asset-backed securities (ABS) can be a single class of securities or multiple classes with differing claims to the cash flows from the underlying assets. Time tranching refers to classes that receive the principal payments from underlying securities sequentially as each prior tranche is repaid in full. With credit tranching, any credit losses are first absorbed by the tranche with the lowest priority, and after that by any other subordinated tranches, in order. Some structures have both time tranching and credit tranching.

LOS 45.d

Characteristics of residential mortgage loans include:

- Maturity.
- Interest rate: fixed-rate, adjustable-rate, or convertible.
- Amortization: full, partial, or interest-only.
- Prepayment penalties.
- Foreclosure provisions: recourse or nonrecourse.

The loan-to-value (LTV) ratio indicates the percentage of the value of the real estate collateral that is loaned. Lower LTVs indicate less credit risk.

LOS 45.e

Agency residential mortgage-backed securities (RMBS) are guaranteed and issued by GNMA, Fannie Mae, or Freddie Mac. Mortgages that back agency RMBS must be conforming loans that meet certain minimum credit quality standards. Nonagency RMBS are issued by private companies and may be backed by nonconforming mortgages.

Key characteristics of RMBS include:

- Pass-through rate, the coupon rate on the RMBS.
- Weighted average maturity (WAM) and weighted average coupon (WAC) of the underlying pool of mortgages.
- Conditional prepayment rate (CPR), which may be compared to the Public Securities Administration (PSA) benchmark for expected prepayment rates.

Nonagency RMBS typically include credit enhancement. External credit enhancement is a third-party guarantee. Internal credit enhancement includes reserve funds (cash or excess spread), overcollateralization, and senior/subordinated structures.

Collateralized mortgage obligations (CMOs) are collateralized by pools of residential MBS. CMOs are structured with tranches that have different exposures to prepayment risks.

In a sequential-pay CMO, all scheduled principal payments and prepayments are paid to each tranche in sequence until that tranche is paid off. The first tranche to be paid principal has the most contraction risk and the last tranche to be paid principal has the most extension risk.

A planned amortization class (PAC) CMO has PAC tranches that receive predictable cash flows as long as the prepayment rate remains within a predetermined range, and support tranches that have more contraction risk and more extension risk than the PAC tranches.

LOS 45.f

Prepayment risk refers to uncertainty about the timing of the principal cash flows from an ABS. Contraction risk is the risk that loan principal will be repaid more rapidly than expected, typically when interest rates have decreased. Extension risk is the risk that loan principal will be repaid more slowly than expected, typically when interest rates have increased.

LOS 45.g

Commercial mortgage-backed securities (CMBS) are backed by mortgages on income-producing real estate properties. Because commercial mortgages are nonrecourse loans, analysis of CMBS focuses on credit risk of the properties. CMBS are structured in tranches with credit losses absorbed by the lowest priority tranches in sequence.

Call (prepayment) protection in CMBS includes loan-level call protection such as prepayment lockout periods, defeasance, prepayment penalty points, and yield maintenance charges, and CMBS-level call protection provided by the lower-priority tranches.

LOS 45.h

Asset-backed securities may be backed by financial assets other than mortgages. Two examples are auto loan ABS and credit card ABS.

Auto loan ABS are backed by automobile loans, which are typically fully amortizing but with shorter maturities than residential mortgages. Prepayments result when autos are sold or traded in, stolen or wrecked and paid off from insurance proceeds, refinanced, or paid off from the borrower's excess cash.

Credit card ABS are backed by credit card receivables, which are revolving debt (nonamortizing). Credit card ABS typically have a lockout period during which only interest is paid to investors and principal payments on the receivables are used to purchase additional receivables.

LOS 45.i

Collateralized debt obligations (CDOs) are structured securities backed by a pool of debt obligations that is managed by a collateral manager. CDOs include:

- Collateralized bond obligations (CBOs) backed by corporate and emerging market debt.
- Collateralized loan obligations (CLOs) backed by leveraged bank loans.
- Structured finance CDOs backed by residential or commercial MBS, ABS, or other CLOs.
- Synthetic CDOs backed by credit default swaps on structured securities.

LOS 45.j

Covered bonds are similar to asset-backed securities, but instead of creating an SPE, the underlying assets remain on the balance sheet of the issuer. Covered bonds give bondholders recourse to the issuer as well as the asset pool, which increases the bonds' credit quality.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 45.1

1. **A** Banks that securitize loans they hold as assets receive cash with which they can make additional loans. The primary benefits of securitization to the economy include reducing firms' funding costs and increasing the liquidity of the financial assets that are securitized. (LOS 45.a)

2. **A** ABS are issued by a special purpose entity (SPE), which is an entity created for that specific purpose. In a securitization, the firm that is securitizing financial assets is described as the seller because it sells the assets to the SPE. The servicer is the entity that deals with collections on the securitized assets. (LOS 45.b)

3. **B** Senior and subordinated tranches are characteristics of a mortgage-backed security with credit tranching. (LOS 45.c)

4. **C** An interest-only lifetime mortgage includes no repayment of principal in its monthly payments so the balloon payment at maturity is equal to the original loan principal. A fully amortizing mortgage has no balloon payment at maturity. A convertible mortgage gives the borrower an option to change the loan from fixed-rate to adjustable-rate or from adjustable-rate to fixed-rate. (LOS 45.d)

5. **A** Conforming loans that may be securitized in agency RMBS have a *maximum* loan-to-value ratio, along with other requirements such as minimum percentage down payments and insurance on the mortgaged property. (LOS 45.e)

6. **B** Issuing CMOs may allow the issuer to raise funds at a lower cost by creating tranches that appeal to investors with different preferences for extension risk and contraction risk. CMOs do not reduce these risks compared to their pool of collateral; they only distribute the risks among the various CMO tranches. (LOS 45.e)

Module Quiz 45.2

1. **B** Extension risk is the risk that prepayments will be slower than expected. Contraction risk is the risk that prepayments will be faster than expected. (LOS 45.e, 45.f)

2. **B** Balloon risk is the possibility that a commercial mortgage borrower will not be able to refinance the principal that is due at the maturity date of the mortgage. This results in a default that is typically resolved by extending the term of the loan during a workout period. Thus, balloon risk is a source of extension risk for CMBS investors. (LOS 45.g)

3. **C** During the lockout period on a credit card receivables-backed ABS, no principal payments are made to investors. (LOS 45.h)

4. **B** A collateralized debt obligation (CDO) is backed by an underlying pool of debt securities, which may include emerging markets debt. Both collateralized mortgage obligations and commercial mortgage-backed securities are backed by mortgages only. (LOS 45.i)

5. **B** Covered bonds differ from ABS in that bondholders have recourse to the issuer as well as the cover pool. Covered bonds are not issued through special purpose entities. A covered bond issuer must maintain a dynamic cover pool, replacing any nonperforming or prepaid assets. (LOS 45.j)

READING 46

UNDERSTANDING FIXED-INCOME RISK AND RETURN

EXAM FOCUS

"Risk" in the title of this reading refers primarily to risk arising from uncertainty about future interest rates. Measurement of credit risk is addressed in the next reading. That said, there is a significant amount of testable material covered in this review. Calculations required by the learning outcomes include the sources of bond returns, three duration measures, money duration, the price value of a basis point, and approximate convexity. You must also be able to estimate a bond's price change for a given change in yield based on its duration and convexity. Important concepts include how bond characteristics affect interest rate risk, factors that affect a bond's reinvestment risk, and the interaction among price risk, reinvestment risk, and the investment horizon.

MODULE 46.1: SOURCES OF RETURNS, DURATION

Video covering this content is available online.

LOS 46.a: Calculate and interpret the sources of return from investing in a fixed-rate bond.

There are **three sources of returns** from investing in a fixed-rate bond:

1. Coupon and principal payments.
2. Interest earned on coupon payments that are reinvested over the investor's holding period for the bond.
3. Any capital gain or loss if the bond is sold prior to maturity.

We will assume that a bond makes all of its promised coupon and principal payments on time (i.e., we are not addressing credit risk). Additionally, we assume that the *interest rate earned on reinvested coupon payments is the same as the YTM on the bond*. There are five results to gain from the analysis presented here.

Given the assumptions just listed:

1. An investor who holds a fixed-rate bond to maturity will earn an annualized rate of return equal to the YTM of the bond when purchased.
2. An investor who sells a bond prior to maturity will earn a rate of return equal to the YTM at purchase if the YTM at sale has not changed since purchase.

3. If the market YTM for the bond, our assumed reinvestment rate, increases (decreases) after the bond is purchased but before the first coupon date, a buy-and-hold investor's realized return will be higher (lower) than the YTM of the bond when purchased.
4. If the market YTM for the bond, our assumed reinvestment rate, *increases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM at bond purchase if the bond is held for a *short* period.
5. If the market YTM for the bond, our assumed reinvestment rate, *decreases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM at bond purchase if the bond is held for a *long* period.

We will present mathematical examples to demonstrate each of these results as well as some intuition as to why these results must hold.

A bond investor's **annualized holding period rate of return** is calculated as the compound annual return earned from the bond over the investor's holding period. This is the compound rate of return that, based on the purchase price of the bond, would provide an amount at the time of the sale or maturity of the bond equal to the sum of coupon payments, sale or maturity value, and interest earned on reinvested coupons.

We will illustrate this calculation (and the first result listed earlier) with a 6% annual-pay three-year bond purchased at a YTM of 7% and held to maturity.

With an annual YTM of 7%, the bond's purchase price is $973.76.

N = 3; I/Y = 7; PMT = 60; FV = 1,000; CPT → PV = –973.76

At maturity, the investor will have received coupon income and reinvestment income equal to the future value of an annuity of three $60 coupon payments calculated with an interest rate equal to the bond's YTM. This amount is:

$60(1.07)^2 + 60(1.07) + 60 = \$192.89$

N = 3; I/Y = 7; PV = 0; PMT = 60; CPT → FV = –192.89

We can easily calculate the amount earned from reinvestment of the coupons as:

$192.89 - 3(60) = \$12.89$

Adding the maturity value of $1,000 to $192.89, we can calculate the investor's rate of return over the three-year holding period as $\left(\frac{1{,}192.89}{973.76}\right)^{\frac{1}{3}} - 1 = 7\%$ and demonstrate that $973.76 invested at a compound annual rate of 7% would return $1,192.89 after three years.

We can calculate an investor's rate of return on the same bond purchased at a YTM of 5%.

Price at purchase:

N = 3; I/Y = 5; FV = 1,000; PMT = 60; CPT → PV = –1,027.23

Coupons and reinvestment income:

$60(1.05)^2 + 60(1.05) + 60 = \$189.15$ or

N = 3; I/Y = 5; PV = 0; PMT = 60; CPT → FV = –189.15

Holding period return:

$$\left(\frac{1{,}189.15}{1{,}027.23}\right)^{\frac{1}{3}} - 1 = 5\%$$

With these examples, we have demonstrated our first result: that for a fixed-rate bond that does not default and has a reinvestment rate equal to the YTM, an investor who holds the bond until maturity will earn a rate of return equal to the YTM at purchase, regardless of whether the bond is purchased at a discount or a premium.

The intuition is straightforward. If the bond is selling at a discount, the YTM is greater than the coupon rate because together, the amortization of the discount and the higher assumed reinvestment rate on coupon income increase the bond's return. For a bond purchased at a premium, the YTM is less than the coupon rate because both the amortization of the premium and the reduction in interest earned on reinvestment of its cash flows decrease the bond's return.

Now let's examine the second result—that an investor who sells a bond prior to maturity will earn a rate of return equal to the YTM as long as the YTM has not changed since purchase. For such an investor, we call the time the bond will be held the investor's **investment horizon**. The value of a bond that is sold at a discount or premium to par will move to the par value of the bond by the maturity date. At dates between the purchase and the sale, the value of a bond at the same YTM as when it was purchased is its **carrying value** and reflects the amortization of the discount or premium since the bond was purchased.

PROFESSOR'S NOTE

Carrying value is a price along a bond's constant-yield price trajectory. We applied this concept in Financial Statement Analysis when we used the effective interest method to calculate the carrying value of a bond liability.

Capital gains or losses at the time a bond is sold are measured relative to this carrying value, as illustrated in the following example.

EXAMPLE: **Capital gain or loss on a bond**

An investor purchases a 20-year bond with a 5% semiannual coupon and a yield to maturity of 6%. Five years later the investor sells the bond for a price of 91.40. Determine whether the investor realizes a capital gain or loss, and calculate its amount.

Answer:

Any capital gain or loss is based on the bond's carrying value at the time of sale, when it has 15 years (30 semiannual periods) to maturity. The carrying value is calculated using the bond's YTM at the time the investor purchased it.

N = 30; I/Y = 3; PMT = 2.5; FV = 100; CPT → PV = –90.20

Because the selling price of 91.40 is greater than the carrying value of 90.20, the investor realizes a capital gain of 91.40 – 90.20 = 1.20 per 100 of face value.

Bonds held to maturity have no capital gain or loss. Bonds sold prior to maturity at the same YTM as at purchase will also have no capital gain or loss. Using the 6% three-year bond from our earlier examples, we can demonstrate this for an investor with a two-year holding period (investment horizon).

When the bond is purchased at a YTM of 7% (for $973.76), we have:

Price at sale (at end of year 2, YTM = 7%):

1,060 / 1.07 = 990.65 or

N = 1; I/Y = 7; FV = 1,000; PMT = 60; CPT → PV = –990.65

which is the carrying value of the bond.

Coupon interest and reinvestment income for two years:

60(1.07) + 60 = $124.20 or

N = 2; I/Y = 7; PV = 0; PMT = 60; CPT → FV = –124.20

Investor's annual compound rate of return over the two-year holding period is:

$$\left(\frac{124.20 + 990.65}{973.76}\right)^{\frac{1}{2}} - 1 = 7\%$$

This result can be demonstrated for the case where the bond is purchased at a YTM of 5% ($1,027.23) as well:

Price at sale (at end of year 2, YTM = 5%):

1,060 / 1.05 = 1,009.52 or

N = 1; I/Y = 5; FV = 1,000; PMT = 60; CPT → PV = –1,009.52

which is the carrying value of the bond.

Coupon interest and reinvestment income for two years:

60(1.05) + 60 = 123.00 or

N = 2; I/Y = 5; PV = 0; PMT = 60; CPT → FV = –123.00

Investor's annual compound rate of return over the two-year holding period is:

$$\left(\frac{123.00 + 1{,}009.52}{1{,}027.23}\right)^{\frac{1}{2}} - 1 = 5\%$$

For a bond investor with an investment horizon less than the bond's term to maturity, the annual holding period return will be equal to the YTM at purchase (under our assumptions), if the bond is sold at that YTM. The intuition here is that if a bond will have a rate of return equal to its YTM at maturity, which we showed, if we sell some of the remaining value of the bond discounted at that YTM, we will have earned that YTM up to the date of sale.

Now let's examine our third result: that if rates rise (fall) before the first coupon date, an investor who holds a bond to maturity will earn a rate of return greater (less) than the YTM at purchase.

Based on our previous result that an investor who holds a bond to maturity will earn a rate of return equal to the YTM at purchase if the reinvestment rate is also equal to the YTM at purchase, the intuition of the third result is straightforward. If the YTM, which is also the reinvestment rate for the bond, increases (decreases) after purchase, the return from coupon payments and reinvestment income will increase (decrease) as a result and increase (decrease) the investor's rate of return on the bond above (below) its YTM at purchase. The following calculations demonstrate these results for the three-year 6% bond in our previous examples.

For a three-year 6% bond purchased at par (YTM of 6%), first assume that the YTM and reinvestment rate increases to 7% after purchase but before the first coupon payment date. The bond's annualized holding period return is calculated as:

Coupons and reinvestment interest:

$60(1.07)^2 + 60(1.07) + 60 = \$192.89$

N = 3; I/Y = 7; PV = 0; PMT = 60; CPT → FV = −192.89

Investor's annual compound holding period return:

$$\left(\frac{1{,}192.89}{1{,}000}\right)^{\frac{1}{3}} - 1 = 6.06\%$$

which is greater than the 6% YTM at purchase.

If the YTM decreases to 5% after purchase but before the first coupon date, we have the following.

Coupons and reinvestment interest:

$60(1.05)^2 + 60(1.05) + 60 = \$189.15$

N = 3; I/Y = 5; PV = 0; PMT = 60; CPT → FV = −189.15

Investor's annual compound holding period return:

$$\left(\frac{1{,}189.15}{1{,}000}\right)^{\frac{1}{3}} - 1 = 5.94\%$$

which is less than the 6% YTM at purchase.

Note that in both cases, the investor's rate of return is between the YTM at purchase and the assumed reinvestment rate (the new YTM).

We now turn our attention to the fourth and fifth results concerning the effects of the length of an investor's holding period on the rate of return for a bond that experiences an increase or decrease in its YTM before the first coupon date.

We have already demonstrated that when the YTM increases (decreases) after purchase but before the first coupon date, an investor who holds the bond to maturity will earn a rate of return greater (less) than the YTM at purchase. Now, we examine the rate of return earned by an investor with an investment horizon (expected holding period) less than the term to maturity under the same circumstances.

Consider a three-year 6% bond purchased at par by an investor with a one-year investment horizon. If the YTM increases from 6% to 7% after purchase and the bond is sold after one year, the rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 7%:

N = 2; I/Y = 7; FV = 1,000; PMT = 60; CPT → PV = –981.92

There is no reinvestment income and only one coupon of $60 received so the holding period rate of return is simply:

$$\left(\frac{981.92 + 60}{1,000}\right) - 1 = 4.19\%$$

which is less than the YTM at purchase.

If the YTM *decreases* to 5% after purchase and the bond is sold at the end of one year, the investor's rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 5%:

N = 2; I/Y = 5; FV = 1,000; PMT = 60; CPT → PV = –1,018.59

And the holding period rate of return is simply:

$$\left(\frac{1,018.59 + 60}{1,000}\right) - 1 = 7.86\%$$

which is greater than the YTM at purchase.

The intuition of this result is based on the idea of a tradeoff between **market price risk** (the uncertainty about price due to uncertainty about market YTM) and **reinvestment risk** (uncertainty about the total of coupon payments and reinvestment income on those payments due to the uncertainty about future reinvestment rates).

Previously, we showed that for a bond held to maturity, the investor's rate of return increased with an increase in the bond's YTM and decreased with a decrease in the bond's YTM. For an investor who intends to hold a bond to maturity, there is no interest rate risk as we have defined it. Assuming no default, the bond's value at maturity is its par value regardless of interest rate changes so that the investor has only reinvestment risk. Her realized return will increase when interest earned on reinvested cash flows increases, and decrease when the reinvestment rate decreases.

For an investor with a short investment horizon, interest rate risk increases and reinvestment risk decreases. For the investor with a one-year investment horizon, there was no reinvestment risk because the bond was sold before any interest on coupon payments was earned. The investor had only market price risk so an increase in yield decreased the rate of return over the one-year holding period because the sale price is lower. Conversely, a decrease in yield increased the one-year holding period return to more than the YTM at purchase because the sale price is higher.

To summarize:

short investment horizon: market price risk > reinvestment risk

long investment horizon: reinvestment risk > market price risk

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LOS 46.b: Define, calculate, and interpret Macaulay, modified, and effective durations.

Macaulay Duration

Duration is used as a measure of a bond's interest rate risk or sensitivity of a bond's *full* price to a change in its yield. The measure was first introduced by Frederick Macaulay and his formulation is referred to as **Macaulay duration**.

A bond's (annual) Macaulay duration is calculated as the weighted average of the number of years until each of the bond's promised cash flows is to be paid, where the weights are the present values of each cash flow as a percentage of the bond's full value.

Consider a newly issued three-year 4% annual-pay bond with a yield to maturity of 5%. The present values of each of the bond's promised payments, discounted at 5%, and their weights in the calculation of Macaulay duration, are shown in the following table.

$C_1 = 40$	$PV_1 = 40 / 1.05$	$= 38.10$	$W_1 = 38.10 / 972.77$	$= 0.0392$
$C_2 = 40$	$PV_2 = 40 / 1.05^2$	$= 36.28$	$W_2 = 36.28 / 972.77$	$= 0.0373$
$C_3 = 1{,}040$	$PV_3 = 1{,}040 / 1.05^3$	$= \underline{898.39}$	$W_3 = 898.39 / 972.77$	$= \underline{0.9235}$
		972.77		1.0000

Note that the present values of all the promised cash flows sum to 972.77 (the full value of the bond) and the weights sum to 1.

Now that we have the weights, and because we know the time until each promised payment is to be made, we can calculate the Macaulay duration for this bond:

$$0.0392(1) + 0.0373(2) + 0.9235(3) = 2.884 \text{ years}$$

The Macaulay duration of a semiannual-pay bond can be calculated in the same way: as a weighted average of the number of *semiannual periods* until the cash flows are to be received. In this case, the result is the number of semiannual periods rather than years.

Because of the improved measures of interest rate risk described next, we say that Macaulay duration is the weighted-average time to the receipt of principal and interest payments, rather than our best estimate of interest rate sensitivity. Between coupon dates, the Macaulay duration of a coupon bond decreases with the passage of time and then goes back up significantly at each coupon payment date.

Modified Duration

Modified duration (ModDur) is calculated as Macaulay duration (MacDur) divided by one plus the bond's yield to maturity. For the bond in our earlier example, we have:

$$\text{ModDur} = 2.884 / 1.05 = 2.747$$

Modified duration provides an approximate percentage change in a bond's price for a 1% change in yield to maturity. The price change for a given change in yield to maturity can be calculated as:

$$\text{approximate percentage change in bond price} = -\text{ModDur} \times \Delta\text{YTM}$$

Based on a ModDur of 2.747, the price of the bond should fall by approximately 2.747 × 0.1% = 0.2747% in response to a 0.1% increase in YTM. The resulting price estimate of $970.098 is very close to the value of the bond calculated directly using a YTM of 5.1%, which is $970.100.

For an annual-pay bond, the general form of modified duration is:

$$ModDur = MacDur / (1 + YTM)$$

For a semiannual-pay bond with a YTM quoted on a semiannual bond basis:

$$ModDur_{SEMI} = MacDur_{SEMI} / (1 + YTM / 2)$$

This modified duration can be annualized (from semiannual periods to annual periods) by dividing by two, and then used as the approximate change in price for a 1% change in a bond's YTM.

Approximate Modified Duration

We can approximate modified duration directly using bond values for an increase in YTM and for a decrease in YTM of the same size.

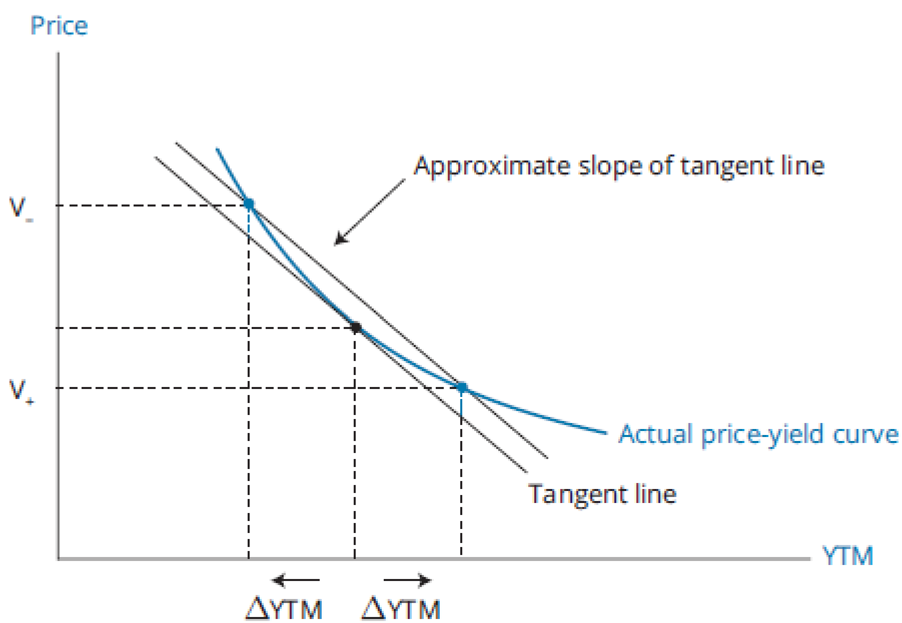
In Figure 46.1 we illustrate this method. The calculation of approximate modified duration is based on a given change in YTM. V_- is the price of the bond if YTM is *decreased* by ΔYTM and V_+ is the price of the bond if the YTM is *increased* by ΔYTM. Note that $V_- > V_+$. Because of the convexity of the price-yield relationship, the price increase (to V_-), for a given decrease in yield, is larger than the price decrease (to V_+).

$$\text{approximate modified duration} = \frac{V_- - V_+}{2 \times V_0 \times \Delta YTM}$$

The formula uses the average of the magnitudes of the price increase and the price decrease, which is why $V_- - V_+$ (in the numerator) is divided by 2 (in the denominator).

V_0, the current price of the bond, is in the denominator to convert this average price change to a percentage, and the ΔYTM term is in the denominator to scale the duration measure to a 1% change in yield by convention. Note that the ΔYTM term in the denominator must be entered as a decimal (rather than in a whole percentage) to properly scale the duration estimate.

Figure 46.1: Approximate Modified Duration



EXAMPLE: **Calculating approximate modified duration**

A bond is trading at a full price of 980. If its yield to maturity increases by 50 basis points, its price will decrease to 960. If its yield to maturity decreases by 50 basis points, its price will increase to 1,002. Calculate the approximate modified duration.

Answer:

The approximate modified duration is $\frac{1{,}002 - 960}{2 \times 980 \times 0.005} = 4.29$, and the approximate change in price for a 1% change in YTM is 4.29%.

Note that modified duration is a *linear estimate* of the relation between a bond's price and YTM, whereas the actual relation is convex, not linear. This means that the modified duration measure provides good estimates of bond prices for small changes in yield, but increasingly poor estimates for larger changes in yield as the effect of the curvature of the price-yield curve is more pronounced.

Effective Duration

So far, all of our duration measures have been calculated using the YTM and prices of straight (option-free) bonds. This is straightforward because both the future cash flows and their timing are known with certainty. This is not the case with bonds that have embedded options, such as a callable bond or a mortgage-backed bond.

We say mortgage-backed bonds have a *prepayment option*, which is similar to a call option on a corporate bond. The borrowers (people who take out mortgages) typically have the option to pay off the principal value of their loans, in whole or in part, at any time. These prepayments accelerate when interest rates fall significantly because borrowers can refinance their home loans at a lower rate and pay off the remaining principal owed on an existing loan.

Thus, the pricing of bonds with embedded put, call, or prepayment options begins with the benchmark yield curve, not simply the current YTM of the bond. The appropriate measure of interest rate sensitivity for these bonds is **effective duration**.

The calculation of effective duration is the same as the calculation of approximate modified duration with the change in YTM, Δy, replaced by Δcurve, the change in the benchmark yield curve used with a bond pricing model to generate V_- and V_+. The formula for calculating effective duration is:

$$\text{effective duration} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{curve}}$$

Another difference between calculating effective duration and the methods we have discussed so far is that the effects of changes in benchmark yields and changes in the yield spread for credit and liquidity risk are separated. Modified duration makes no distinction between changes in the benchmark yield and changes in the spread. Effective duration reflects only the sensitivity of the bond's value to changes in the benchmark yield curve. Changes in the credit spread are sometimes addressed with a separate "credit duration" measure.

Finally, note that unlike modified duration, effective duration does not necessarily provide better estimates of bond prices for smaller changes in yield. It may be the case that larger changes in yield produce more predictable prepayments or calls than small changes.

LOS 46.c: Explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options.

For bonds with embedded options, the future cash flows depend not only on future interest rates but also on the path that interest rates take over time (did they fall to a new level or rise to that level?). We must use effective duration to estimate the interest rate risk of these bonds. The effective duration measure must also be based on bond prices from a pricing model. The fact that bonds with embedded options have uncertain future cash flows means that our present value calculations for bond value based on YTM cannot be used.

MODULE QUIZ 46.1

1. The largest component of returns for a 7-year zero-coupon bond yielding 8% and held to maturity is:
   A. capital gains.
   B. interest income.
   C. reinvestment income.
2. An investor buys a 10-year bond with a 6.5% annual coupon and a YTM of 6%. Before the first coupon payment is made, the YTM for the bond decreases to 5.5%. Assuming coupon payments are reinvested at the YTM, the investor's return when the bond is held to maturity is:
   A. less than 6.0%.
   B. equal to 6.0%.
   C. greater than 6.0%.
3. Assuming coupon interest is reinvested at a bond's YTM, what is the interest portion of an 18-year, $1,000 par, 5% annual coupon bond's return if it is purchased at par and held to maturity?
   A. $576.95.
   B. $1,406.62.
   C. $1,476.95.
4. An investor buys a 15-year, £800,000, zero-coupon bond with an annual YTM of 7.3%. If she sells the bond after three years for £346,333 she will have:
   A. a capital gain.
   B. a capital loss.
   C. neither a capital gain nor a capital loss.
5. A 14% annual-pay coupon bond has six years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the approximate modified duration of the bond is *closest* to:
   A. 0.392.
   B. 3.888.
   C. 3.970.
6. Which of the following measures is *lowest* for a callable bond?
   A. Macaulay duration.
   B. Effective duration.
   C. Modified duration.
7. Effective duration is more appropriate than modified duration for estimating interest rate risk for bonds with embedded options because these bonds:
   A. tend to have greater credit risk than option-free bonds.
   B. exhibit high convexity that makes modified duration less accurate.
   C. have uncertain cash flows that depend on the path of interest rate changes.

MODULE 46.2: INTEREST RATE RISK AND MONEY DURATION

LOS 46.d: Define key rate duration and describe the use of key rate durations in measuring the sensitivity of bonds to changes in the shape of the benchmark yield curve.

Video covering this content is available online.

Recall that duration is an adequate measure of bond price risk only for parallel shifts in the yield curve. The impact of nonparallel shifts can be measured using a concept known as **key rate duration**. A key rate duration, also known as a **partial duration**, is defined as the sensitivity of the value of a bond or portfolio to changes in the spot rate for a specific maturity, holding other spot rates constant. A bond or portfolio will have a key rate duration for each maturity range on the spot rate curve. The sum of a bond's key rate durations equals its effective duration.

Key rate duration is particularly useful for measuring the effect of a nonparallel shift in the yield curve on a bond portfolio. We can use the key rate duration for each maturity to compute the effect on the portfolio of the interest rate change at that maturity. The effect on the overall portfolio is the sum of these individual effects.

LOS 46.e: Explain how a bond's maturity, coupon, and yield level affect its interest rate risk.

Other things equal, an *increase in a bond's maturity* will (usually) increase its interest rate risk. The present values of payments made further in the future are more sensitive to changes in the discount rate used to calculate present value than are the present values of payments made sooner.

We must say "usually" because there are instances where an increase in a discount coupon bond's maturity will decrease its Macaulay duration. For a discount bond, duration first increases with longer maturity and then decreases over a range of relatively long maturities until it approaches the duration of a perpetuity, which is (1 + YTM) / YTM.

Other things equal, an *increase in the coupon rate* of a bond will decrease its interest rate risk. For a given maturity and YTM, the duration of a zero-coupon bond will be greater than that of a coupon bond. Increasing the coupon rate means more of a bond's value will be from payments received sooner so that the value of the bond will be less sensitive to changes in yield.

Other things equal, an *increase (decrease) in a bond's YTM* will decrease (increase) its interest rate risk. To understand this, we can look to the convexity of the price-yield curve and use its slope as our proxy for interest rate risk. At lower yields, the price-yield curve has a steeper slope indicating that price is more sensitive to a given change in yield.

Adding either a put or a call provision will decrease a straight bond's interest rate risk as measured by effective duration. With a call provision, the value of the call increases as yields fall, so a decrease in yield will have less effect on the price of the bond, which is the price of a straight bond minus the value of the call option held by the issuer. With a put option, the bondholder's option to sell the bond back to the issuer at a set price reduces the negative impact of yield increases on price.

LOS 46.f: Calculate the duration of a portfolio and explain the limitations of portfolio duration.

There are two approaches to estimating the duration of a portfolio. The first is to calculate the weighted average number of periods until the portfolio's cash flows will be received. The second approach is to take a weighted average of the durations of the individual bonds in the portfolio.

The first approach is theoretically correct but not often used in practice. The yield measure for calculating portfolio duration with this approach is the **cash flow yield**, the IRR of the bond portfolio. This is inconsistent with duration capturing the relationship between YTM and price. This approach will not work for a portfolio that contains bonds with embedded options because the future cash flows are not known with certainty and depend on interest rate movements.

The second approach is typically used in practice. Using the durations of individual portfolio bonds makes it possible to calculate the duration for a portfolio that contains bonds with embedded options by using their effective durations. The weights for the calculation of portfolio duration under this approach are simply the full price of each bond as a proportion of the total portfolio value (using full prices). These proportions of total portfolio value are multiplied by the corresponding bond durations to get portfolio duration.

$$\text{portfolio duration} = W_1 D_1 + W_2 D_2 + \ldots + W_N D_N$$

where:

W_i = full price of bond *i* divided by the total value of the portfolio

D_i = the duration of bond *i*

N = the number of bonds in the portfolio

One limitation of this approach is that for portfolio duration to "make sense" the YTM of every bond in the portfolio must change by the same amount. Only with this assumption of a **parallel shift** in the yield curve is portfolio duration calculated with this approach consistent with the idea of the percentage change in portfolio value per 1% change in YTM.

We can think of the second approach as a practical approximation of the theoretically correct duration that the first approach describes. This approximation is less accurate when there is greater variation in yields among portfolio bonds, but is the same as the portfolio duration under the first approach when the yield curve is flat.

LOS 46.g: Calculate and interpret the money duration of a bond and price value of a basis point (PVBP).

The **money duration** of a bond position (also called *dollar duration*) is expressed in currency units.

money duration = annual modified duration × full price of bond position

Money duration is sometimes expressed as money duration per 100 of bond par value.

money duration per 100 units of par value = annual modified duration × full bond price per 100 of par value

Multiplying the money duration of a bond times a given change in YTM (as a decimal) will provide the change in bond value for that change in YTM.

EXAMPLE: **Money duration**

1. Calculate the money duration on a coupon date of a $2 million par value bond that has a modified duration of 7.42 and a full price of 101.32, expressed for the whole bond and per $100 of face value.
2. What will be the impact on the value of the bond of a 25 basis points increase in its YTM?

Answer:

1. The money duration for the bond is modified duration times the full value of the bond:

   7.42 × $2,000,000 × 101.32% = $15,035,888

   The money duration per $100 of par value is:

   7.42 × 101.32 = $751.79

   Or, $15,035,888 / ($2,000,000 / $100) = $751.79
2. $15,035,888 × 0.0025 = $37,589.72

   The bond value decreases by $37,589.72.

The **price value of a basis point** (PVBP) is the money change in the full price of a bond when its YTM changes by one basis point, or 0.01%. We can calculate the PVBP directly for a bond by calculating the average of the decrease in the full value of a bond when its YTM increases by one basis point and the increase in the full value of the bond when its YTM decreases by one basis point.

EXAMPLE: **Calculating the price value of a basis point**

A newly issued, 20-year, 6% annual-pay straight bond is priced at 101.39. Calculate the price value of a basis point for this bond assuming it has a par value of $1 million.

Answer:

First we need to find the YTM of the bond:

N = 20; PV = −101.39; PMT = 6; FV = 100; CPT→I/Y = 5.88

Now we need the values for the bond with YTMs of 5.89 and 5.87.

I/Y = 5.89; CPT → PV = −101.273 (V_+)

I/Y = 5.87; CPT → PV = −101.507 (V_-)

PVBP (per $100 of par value) = (101.507 − 101.273) / 2 = 0.117

For the $1 million par value bond, each 1 basis point change in the yield to maturity will change the bond's price by 0.117 × $1 million × 0.01 = $1,170.

MODULE QUIZ 46.2

1. A bond portfolio manager who wants to estimate the sensitivity of the portfolio's value to changes in the 5-year spot rate should use:
   A. a key rate duration.
   B. a Macaulay duration.
   C. an effective duration.
2. Which of the following three bonds (similar except for yield and maturity) has the *least* Macaulay duration? A bond with:

A. 5% yield and 10-year maturity.
B. 5% yield and 20-year maturity.
C. 6% yield and 10-year maturity.

3. Portfolio duration has limited usefulness as a measure of interest rate risk for a portfolio because it:
    A. assumes yield changes uniformly across all maturities.
    B. cannot be applied if the portfolio includes bonds with embedded options.
    C. is accurate only if the portfolio's internal rate of return is equal to its cash flow yield.

4. The current price of a $1,000, 7-year, 5.5% semiannual coupon bond is $1,029.23. The bond's price value of a basis point is *closest* to:
    A. $0.05.
    B. $0.60.
    C. $5.74.

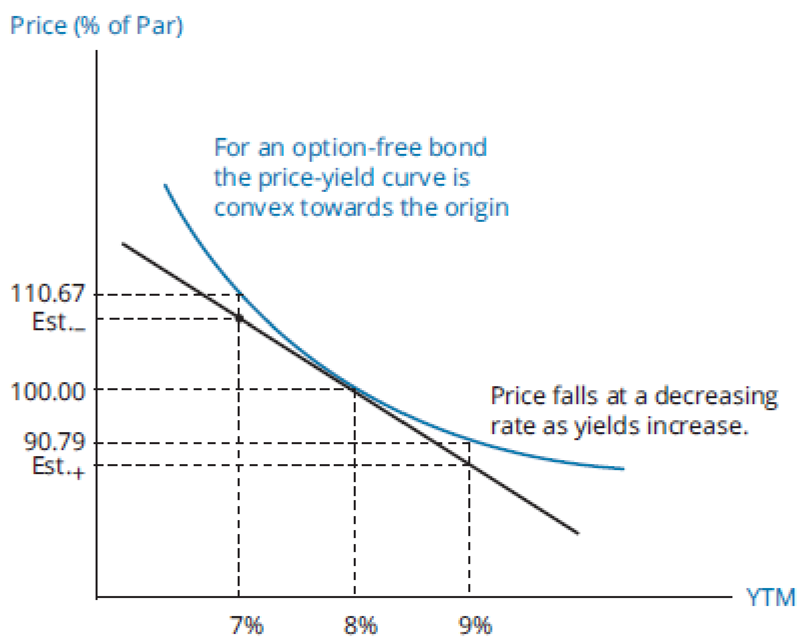
MODULE 46.3: CONVEXITY AND YIELD VOLATILITY

Video covering this content is available online.

LOS 46.h: Calculate and interpret approximate convexity and compare approximate and effective convexity.

Earlier we explained that modified duration is a linear approximation of the relationship between yield and price and that, because of the convexity of the true price-yield relation, duration-based estimates of a bond's full price for a given change in YTM will be increasingly different from actual prices. This is illustrated in Figure 46.2. Duration-based price estimates for a decrease and for an increase in YTM are shown as $Est._{-}$ and $Est._{+}$.

Figure 46.2: Price-Yield Curve for an Option-Free, 8%, 20-Year Bond



Estimates of the price impact of a change in yield based only on modified duration can be improved by introducing a second term based on the bond's convexity. **Convexity** is a measure of the curvature of the price-yield relation. The more curved it is, the greater the convexity adjustment to a duration-based estimate of the change in price for a given change in YTM.

A bond's convexity can be estimated as:

$$\text{approximate convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta YTM)^2 V_0}$$

where:

the variables are the same as those we used in calculating approximate modified duration

Effective convexity, like effective duration, must be used for bonds with embedded options.

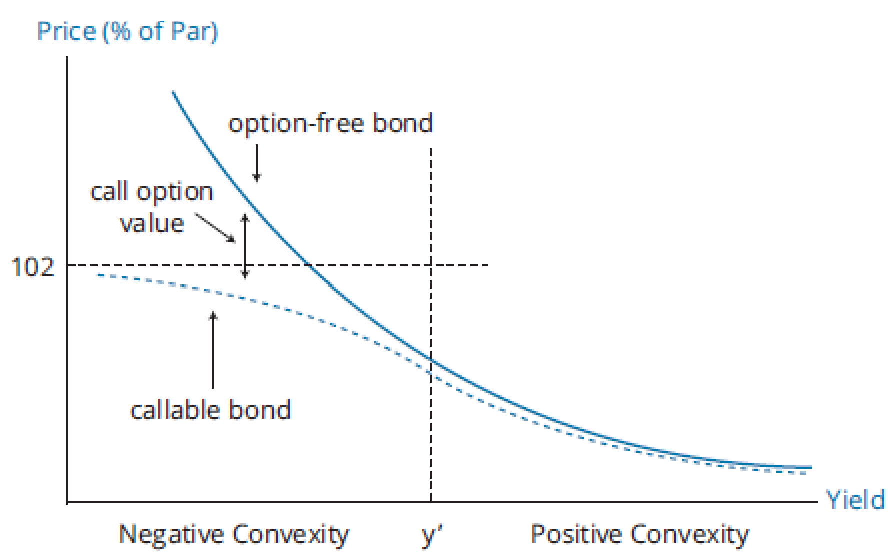
The calculation of effective convexity is the same as the calculation of approximate convexity, except that the change in the yield *curve*, rather than a change in the bond's YTM, is used.

$$\text{approximate effective convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta \text{curve})^2 V_0}$$

A bond's convexity is increased or decreased by the same bond characteristics that affect duration. A longer maturity, a lower coupon rate, or a lower yield to maturity will all increase convexity, and vice versa. For two bonds with equal duration, the one with cash flows that are more dispersed over time will have the greater convexity.

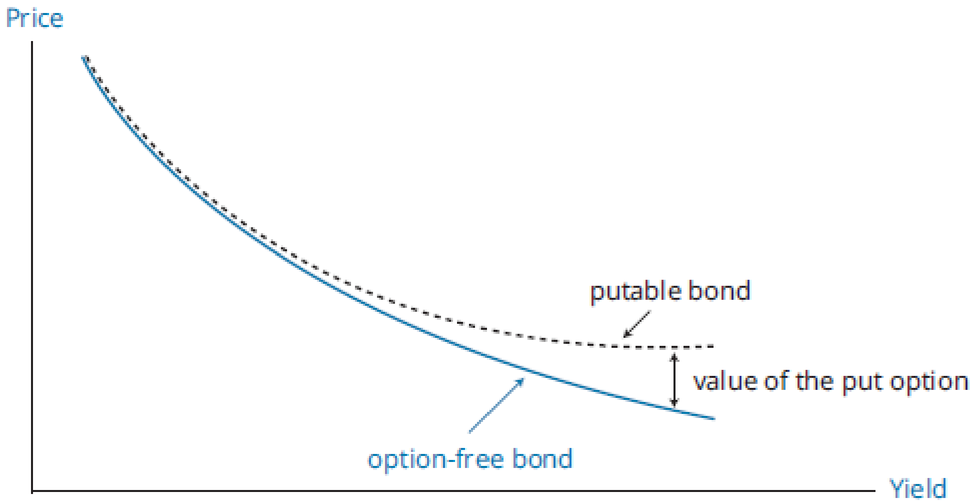
While the convexity of any option-free bond is positive, the convexity of a callable bond can be negative at low yields. This is because at low yields the call option becomes more valuable and the call price puts an effective limit on increases in bond value as shown in Figure 46.3. For a bond with negative convexity, the price increase from a decrease in YTM is *smaller* than the price decrease from an increase in YTM.

Figure 46.3: Price-Yield Function of a Callable vs. an Option-Free Bond



A putable bond has greater convexity than an otherwise identical option-free bond. In Figure 46.4 we illustrate the price-yield relation for a putable bond. At higher yields, the put becomes more valuable so that the value of the putable bond falls less than that of an option-free bond as yield increases.

Figure 46.4: Comparing the Price-Yield Curves for Option-Free and Putable Bonds



LOS 46.i: Calculate the percentage price change of a bond for a specified change in yield, given the bond's approximate duration and convexity.

By taking account of both a bond's duration (first-order effects) and convexity (second-order effects), we can improve an estimate of the effects of a change in yield on a bond's value, especially for larger changes in yield.

change in full bond price = −annual modified duration(ΔYTM) + ½ annual convexity$(\Delta YTM)^2$

EXAMPLE: Estimating price changes with duration and convexity

Consider an 8% bond with a full price of $908 and a YTM of 9%. Estimate the percentage change in the full price of the bond for a 30 basis point increase in YTM assuming the bond's duration is 9.42 and its convexity is 68.33.

Answer:

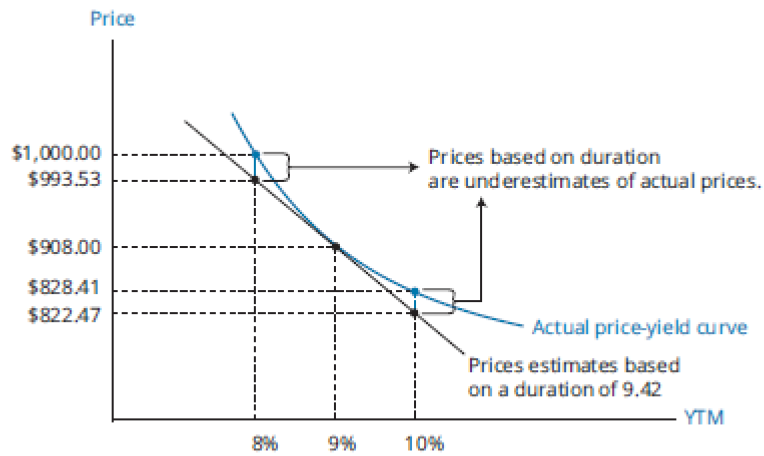
The duration effect is −9.42 × 0.003 = 0.02826 = −2.826%.

The convexity effect is 0.5 × 68.33 × $(0.003)^2$ = 0.000307 = 0.0307%.

The expected change in bond price is (−0.02826 + 0.000307) = −2.7953%.

Note that the convexity adjustment to the price change is the same for both an increase and a decrease in yield. As illustrated in Figure 46.5, the duration-only based estimate of the increase in price resulting from a decrease in yield is too low for a bond with positive convexity, and is improved by a positive adjustment for convexity. The duration-only based estimate of the decrease in price resulting from an increase in yield is larger than the actual decrease, so it's also improved by a positive adjustment for convexity.

Figure 46.5: Duration-Based Price Estimates vs. Actual Bond Prices



LOS 46.j: Describe how the term structure of yield volatility affects the interest rate risk of a bond.

The **term structure of yield volatility** refers to the relation between the volatility of bond yields and their times to maturity. We have seen that the sensitivity of a bond's price with respect to a *given* change in yield depends on its duration and convexity. From an investor's point of view, it's the volatility of a bond's price that is of concern. The volatility of a bond's price has two components: the sensitivity of the bond's price to a given change in yield and the volatility of the bond's yield.

In calculating duration and convexity, we implicitly assumed that the yield curve shifted in a parallel manner. In practice, this is often not the case. For example, changes in monetary policy may have more of an effect on short-term interest rates than on longer-term rates.

It could be the case that a shorter-term bond has more price volatility than a longer-term bond with a greater duration because of the greater volatility of the shorter-term yield.

LOS 46.k: Describe the relationships among a bond's holding period return, its duration, and the investment horizon.

Macaulay duration has an interesting application in matching a bond to an investor's investment horizon. When the investment horizon and the bond's Macaulay duration are matched, a parallel shift in the yield curve prior to the first coupon payment will not (or will minimally) affect the investor's horizon return.

Earlier, we illustrated the effect of a change in yield that occurs prior to the first coupon payment. Our results showed that for an investor with a short investment horizon (anticipated holding period), the market price risk of the bond outweighs its reinvestment risk. Because of this, an increase in yield prior to the first coupon date was shown to reduce the horizon yield for a short investment horizon and increase the horizon yield for a longer-term investment horizon. For a longer investment horizon, the increase in reinvestment income from the yield increase was greater than the decrease in the sale price of the bond.

For a decrease in yield, an investor with a short investment horizon will have a capital gain and only a small decrease in reinvestment income. An investor with a long horizon will be more affected by the decrease in reinvestment income and will have a horizon return that is less than the bond's original yield.

When the investment horizon just matches the Macaulay duration, the effect of a change in YTM on the sale price of a bond and on reinvestment income just offset each other. We can say that for such an investment, market price risk and reinvestment risk offset each other. The following example illustrates this result.

EXAMPLE: **Investment horizon yields**

Consider an eight-year, 8.5% bond priced at 89.52 to yield 10.5% to maturity. The Macaulay duration of the bond is 6. We can calculate the horizon yield for horizons of 3 years, 6 years, and 8 years, assuming the YTM falls to 9.5% prior to the first coupon date.

Answer:

Sale after 3 years

Bond price:

N = 5; PMT = 8.5; FV = 100; I/Y = 9.5; CPT → PV = 96.16

Coupons and interest on reinvested coupons:

N = 3; PMT = 8.5; PV = 0; I/Y = 9.5; CPT → FV = 28.00

Horizon return:

$$[(96.16 + 28.00) / 89.52]^{1/3} - 1 = 11.520\%$$

Sale after 6 years

Bond price:

N = 2; PMT = 8.5; FV = 100; I/Y = 9.5; CPT → PV = 98.25

Coupons and interest on reinvested coupons:

N = 6; PMT = 8.5; PV = 0; I/Y = 9.5; CPT → FV = 64.76

Horizon return:

$$[(98.25 + 64.76) / 89.52]^{1/6} - 1 = 10.505\%$$

Held to maturity, 8 years

Maturity value = 100

Coupons and interest on reinvested coupons:

N = 8; PMT = 8.5; PV = 0; I/Y = 9.5; CPT → FV = 95.46

Horizon return:

$$[(100 + 95.46) / 89.52]^{1/8} - 1 = 10.253\%$$

For an investment horizon equal to the bond's Macaulay duration of 6, the horizon return is equal to the original YTM of 10.5%. For a shorter three-year investment horizon, the price increase from a reduction in the YTM to 9.5% dominates the decrease in reinvestment income so the horizon return, 11.520%, is greater than the original YTM. For an investor who holds the bond to maturity, there is no price effect and the decrease in reinvestment income reduces the horizon return to 10.253%, less than the original YTM.

The difference between a bond's Macaulay duration and the bondholder's investment horizon is referred to as a **duration gap**. A positive duration gap (Macaulay duration greater than the investment horizon) exposes the investor to market price risk from increasing interest rates. A negative duration gap (Macaulay duration less than the investment horizon) exposes the investor to reinvestment risk from decreasing interest rates.

LOS 46.l: Explain how changes in credit spread and liquidity affect yield-to-maturity of a bond and how duration and convexity can be used to estimate the price effect of the changes.

The benchmark yield curve's interest rates have two components; the real rate of return and expected inflation. A bond's spread to the benchmark curve also has two components, a premium for credit risk and a premium for lack of liquidity relative to the benchmark securities.

Because we are treating the yields associated with each component as additive, a given increase or decrease in any of these components of yield will increase or decrease the bond's YTM by the same amount.

With a direct relationship between a bond's yield spread to the benchmark yield curve and its YTM, we can estimate the impact on a bond's value of a change in spread using the formula we introduced earlier for the price effects of a given change in YTM.

$$\%\Delta \text{ bond value} = -\text{duration}(\Delta\text{spread}) + \tfrac{1}{2}\text{ convexity}(\Delta\text{spread})^2$$

EXAMPLE: **Price effect of spread changes**

Consider a bond that is valued at $180,000 that has a duration of 8 and a convexity of 22. The bond's spread to the benchmark curve increases by 25 basis points due to a credit downgrade. What is the approximate change in the bond's market value?

Answer:

With Δspread = 0.0025 we have:

$(-8 \times 0.0025) + (0.5 \times 22 \times 0.0025^2) = -1.99\%$ and the bond's value will fall by approximately 1.99% × 180,000 = $3,588.

LOS 46.m: Describe the difference between empirical duration and analytical duration.

The duration measures we have introduced in this reading, based on mathematical analysis, are often referred to as **analytical durations**. A different approach is to estimate **empirical durations** using the historical relationship between benchmark yield changes and bond price changes.

When we estimate corporate bond durations based on a shift in the benchmark (government) yield curve, we implicitly assume that the credit spread for the corporate bond remains unchanged (i.e., changes in the benchmark yield curve and a bond's yield spread are uncorrelated). When this assumption is not justified, estimates of empirical duration, based on the actual relationship between changes in the benchmark yield curve and bond values, may be more appropriate.

An example of such a situation is an increase in market uncertainty during which investor demand shifts sharply toward bonds with low credit risk (a "flight to quality"). Yields on government bonds decrease, but credit spreads increase at the same time. As a result, government bond prices increase but corporate bond prices increase by less or possibly not at all. For a corporate bond portfolio, an estimate of empirical duration that accounts for this effect would be lower (i.e., less price response to a decrease in benchmark yields) than an estimate of analytical duration would indicate. An analytical estimate of the duration of a portfolio consisting primarily of government debt securities, in this case, would still be appropriate, while an empirically derived estimate of duration would be more appropriate for a portfolio comprising corporate bonds (risky credits).

MODULE QUIZ 46.3

1. A bond has a convexity of 114.6. The convexity effect, if the yield decreases by 110 basis points, is *closest* to:
   A. –1.673%.
   B. +0.693%.
   C. +1.673%.
2. The modified duration of a bond is 7.87. The approximate percentage change in price using duration only for a yield decrease of 110 basis points is *closest* to:
   A. –8.657%.
   B. +7.155%.
   C. +8.657%.
3. Assume a bond has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, the estimated percentage change in price for this bond, in response to a decline in yield of 200 basis points, is *closest* to:
   A. 19.05%.
   B. 22.95%.
   C. 24.89%.
4. Two bonds are similar in all respects except maturity. Can the shorter-maturity bond have greater interest rate risk than the longer-term bond?
   A. No, because the shorter-maturity bond will have a lower duration.
   B. Yes, because the shorter-maturity bond may have a higher duration.
   C. Yes, because short-term yields can be more volatile than long-term yields.
5. An investor with an investment horizon of six years buys a bond with a modified duration of 6.0. This investment has:
   A. no duration gap.
   B. a positive duration gap.
   C. a negative duration gap.
6. Which of the following *most accurately* describes the relationship between liquidity and yield spreads relative to benchmark government bond rates? All else being equal, bonds with:
   A. less liquidity have lower yield spreads.
   B. greater liquidity have higher yield spreads.
   C. less liquidity have higher yield spreads.

KEY CONCEPTS

LOS 46.a

Sources of return from a bond investment include:

- Coupon and principal payments.
- Reinvestment of coupon payments.

- Capital gain or loss if bond is sold before maturity.

Changes in yield to maturity produce market price risk (uncertainty about a bond's price) and reinvestment risk (uncertainty about income from reinvesting coupon payments). An increase (a decrease) in YTM decreases (increases) a bond's price but increases (decreases) its reinvestment income.

LOS 46.b

Macaulay duration is the weighted average number of coupon periods until a bond's scheduled cash flows.

Modified duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in its YTM.

$$\text{approximate modified duration} = \frac{V_{-} - V_{+}}{2 \times V_0 \times \Delta \text{YTM}}$$

Effective duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in the benchmark yield curve.

$$\text{effective duration} = \frac{V_{-} - V_{+}}{2 \times V_0 \times \Delta \text{curve}}$$

LOS 46.c

Effective duration is the appropriate measure of interest rate risk for bonds with embedded options because changes in interest rates may change their future cash flows. Pricing models are used to determine the prices that would result from a given size change in the benchmark yield curve.

LOS 46.d

Key rate duration is a measure of the price sensitivity of a bond or a bond portfolio to a change in the spot rate for a specific maturity. We can use the key rate durations of a bond or portfolio to estimate its price sensitivity to changes in the shape of the yield curve.

LOS 46.e

Holding other factors constant:

- Duration increases when maturity increases.
- Duration decreases when the coupon rate increases.
- Duration decreases when YTM increases.

LOS 46.f

There are two methods for calculating portfolio duration:

- Calculate the weighted average number of periods until cash flows will be received using the portfolio's IRR (its cash flow yield). This method is better theoretically but cannot be used for bonds with options.
- Calculate the weighted average of durations of bonds in the portfolio (the method most often used). Portfolio duration is the percentage change in portfolio value for a 1% change in yield, only for parallel shifts of the yield curve.

LOS 46.g

Money duration is stated in currency units and is sometimes expressed per 100 of bond value.

money duration = annual modified duration × full price of bond position

money duration per 100 units of par value =
annual modified duration × full bond price per 100 of par value

The price value of a basis point is the change in the value of a bond, expressed in currency units, for a change in YTM of one basis point, or 0.01%.

$$PVBP = [(V_- - V_+) / 2] \times \text{par value} \times 0.01$$

LOS 46.h

Convexity refers to the curvature of a bond's price-yield relationship.

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

Effective convexity is appropriate for bonds with embedded options:

$$\text{approximate effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta \text{curve})^2 V_0}$$

LOS 46.i

Given values for approximate annual modified duration and approximate annual convexity, the percentage change in the full price of a bond can be estimated as:

$$\%\Delta \text{ full bond price} = -\text{annual modified duration}(\Delta YTM) + \frac{1}{2}\text{annual convexity}(\Delta YTM)^2$$

LOS 46.j

The term structure of yield volatility refers to the relationship between maturity and yield volatility. Short-term yields may be more volatile than long-term yields. As a result, a short-term bond may have more price volatility than a longer-term bond with a higher duration.

LOS 46.k

Over a short investment horizon, a change in YTM affects market price more than it affects reinvestment income.

Over a long investment horizon, a change in YTM affects reinvestment income more than it affects market price.

Macaulay duration may be interpreted as the investment horizon for which a bond's market price risk and reinvestment risk just offset each other.

duration gap = Macaulay duration − investment horizon

LOS 46.l

A bond's yield spread to the benchmark curve includes a premium for credit risk and a premium for illiquidity.

Given values for duration and convexity, the effect on the value of a bond from a given change in its yield spread (Δspread) can be estimated as:

$\%\Delta \text{ bond value} = -\text{duration}(\Delta\text{spread}) + \frac{1}{2}\text{ convexity}(\Delta\text{spread})^2$

LOS 46.m

Macaulay, modified, and effective duration are examples of analytical duration. Empirical duration is estimated from historical data using models. Empirical duration may differ from analytical duration in interest rate environments where the assumptions underlying analytical duration may not hold, such as for credit-risky bonds in a flight-to-quality scenario.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 46.1

1. **B** The increase in value of a zero-coupon bond over its life is interest income. A zero-coupon bond has no reinvestment risk over its life. A bond held to maturity has no capital gain or loss. (LOS 46.a)

2. **A** The decrease in the YTM to 5.5% will decrease the reinvestment income over the life of the bond so that the investor will earn less than 6%, the YTM at purchase. (LOS 46.a)

3. **B** The interest portion of a bond's return is the sum of the coupon payments and interest earned from reinvesting coupon payments over the holding period.

    N = 18; PMT = 50 ; PV = 0; I/Y = 5%; CPT → FV = −1,406.62

    (LOS 46.a)

4. **A** The price of the bond after three years that will generate neither a capital gain nor a capital loss is the price if the YTM remains at 7.3%. After three years, the present value of the bond is $800{,}000 / 1.073^{12} = 343{,}473.57$, so she will have a capital gain relative to the bond's carrying value. (LOS 46.a)

5. **B** $V_- = 100.979$

    N = 6; PMT = 14.00; FV = 100; I/Y = 13.75; CPT → PV = −100.979

    $V_+ = 99.035$

    I/Y = 14.25; CPT → PV = $-99.035V_0 = 100.000$

    $\Delta y = 0.0025$

    $$\text{Approximate modified duration} = \frac{V_- - V_+}{2V_0\Delta YTM} = \frac{100.979 - 99.035}{2(100)(0.0025)} = 3.888$$

    (LOS 46.b)

6. **B** The interest rate sensitivity of a bond with an embedded call option will be less than that of an option-free bond. Effective duration takes the effect of the call option into account and will, therefore, be less than Macaulay or modified duration. (LOS 46.b)

7. **C** Because bonds with embedded options have cash flows that are uncertain and depend on future interest rates, effective duration must be used. (LOS 46.c)

Module Quiz 46.2

1. **A** Key rate duration refers to the sensitivity of a bond or portfolio value to a change in one specific spot rate. (LOS 46.d)
2. **C** Other things equal, Macaulay duration is less when yield is higher and when maturity is shorter. The bond with the highest yield and shortest maturity must have the lowest Macaulay duration. (LOS 46.e)
3. **A** Portfolio duration is limited as a measure of interest rate risk because it assumes parallel shifts in the yield curve; that is, the discount rate at each maturity changes by the same amount. Portfolio duration can be calculated using effective durations of bonds with embedded options. By definition, a portfolio's internal rate of return is equal to its cash flow yield. (LOS 46.f)
4. **B** PVBP = initial price – price if yield is changed by 1 basis point.

   First, we need to calculate the yield so we can calculate the price of the bond with a 1 basis point change in yield. Using a financial calculator: PV = –1,029.23; FV = 1,000; PMT = 27.5 = (0.055 × 1,000) / 2; N = 14 = 2 × 7 years; CPT → I/Y = 2.49998, multiplied by 2 = 4.99995, or 5.00%.

   Next, compute the price of the bond at a yield of 5.00% + 0.01%, or 5.01%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.505 (5.01 / 2); CPT → PV = $1,028.63.

   Finally, PVBP = $1,029.23 – $1,028.63 = $0.60. (LOS 46.g)

Module Quiz 46.3

1. **B** Convexity effect = 1/ 2 × convexity × $(\Delta YTM)^2$ = $(0.5)(114.6)(0.011)^2$ = 0.00693 = 0.693%

   (LOS 46.h)
2. **C** –7.87 × (–1.10%) = 8.657%

   (LOS 46.i)
3. **B** Total estimated price change = (duration effect + convexity effect) $\{[-10.5 \times (-0.02)] + [1/ 2 \times 97.3 \times (-0.02)^2]\} \times 100$ = 21.0% + 1.95% = 22.95%

   (LOS 46.i)
4. **C** In addition to its sensitivity to changes in yield (i.e., duration), a bond's interest rate risk includes the volatility of yields. A shorter-maturity bond may have more interest rate risk than an otherwise similar longer-maturity bond if short-term yields are more volatile than long-term yields. (LOS 46.j)
5. **B** Duration gap is Macaulay duration minus the investment horizon. Because modified duration equals Macaulay duration / (1 + YTM), Macaulay duration is greater than modified duration for any YTM greater than zero. Therefore, this bond has a Macaulay duration greater than six years and the investment has a positive duration gap. (LOS 46.k)
6. **C** The less liquidity a bond has, the higher its yield spread relative to its benchmark. This is because investors require a higher yield to compensate them for giving up liquidity. (LOS 46.l)

READING 47

FUNDAMENTALS OF CREDIT ANALYSIS

EXAM FOCUS

This reading introduces credit analysis, primarily for corporate bonds, but considerations for credit analysis of high yield, sovereign, and non-sovereign government bonds are also covered. Focus on credit ratings, credit spreads, and the impact on return when ratings and spreads change.

MODULE 47.1: CREDIT RISK AND BOND RATINGS

Video covering this content is available online.

LOS 47.a: Describe credit risk and credit-related risks affecting corporate bonds.

LOS 47.b: Describe default probability and loss severity as components of credit risk.

Credit risk is the risk associated with losses stemming from the failure of a borrower to make timely and full payments of interest or principal. Credit risk has two components: *default risk* and *loss severity*.

- **Default risk** is the probability that a borrower (bond issuer) fails to pay interest or repay principal when due.
- **Loss severity**, or *loss given default*, refers to the value a bond investor will lose if the issuer defaults. Loss severity can be stated as a monetary amount or as a percentage of a bond's value (principal and unpaid interest).

The **expected loss** is equal to the default risk multiplied by the loss severity. Expected loss can likewise be stated as a monetary value or as a percentage of a bond's value.

The **recovery rate** is the percentage of a bond's value an investor will receive if the issuer defaults. Loss severity as a percentage is equal to one minus the recovery rate.

Bonds with credit risk trade at higher yields than bonds thought to be free of credit risk. The difference in yield between a credit-risky bond and a credit-risk-free bond of similar maturity is called its **yield spread**. For example, if a 5-year corporate bond is trading at a spread of +250 basis points to Treasuries and the yield on 5-year Treasury notes is 4.0%, the yield on the corporate bond is 4.0% + 2.5% = 6.5%.

Bond prices are inversely related to spreads; a wider spread implies a lower bond price and a narrower spread implies a higher price. The size of the spread reflects the creditworthiness of the issuer and the liquidity of the market for its bonds. **Spread risk** is the possibility that a bond's spread will widen due to one or both of these factors.

- **Credit migration risk** or **downgrade risk** is the possibility that spreads will increase because the issuer has become less creditworthy. As we will see later in this reading, credit rating agencies assign ratings to bonds and issuers, and may upgrade or downgrade these ratings over time.
- **Market liquidity risk** is the risk of receiving less than market value when selling a bond and is reflected in the size of the bid-ask spreads. Market liquidity risk is greater for the bonds of less creditworthy issuers and for the bonds of smaller issuers with relatively little publicly traded debt.

LOS 47.c: Describe seniority rankings of corporate debt and explain the potential violation of the priority of claims in a bankruptcy proceeding.

Each category of debt from the same issuer is ranked according to a **priority of claims** in the event of a default. A bond's priority of claims to the issuer's assets and cash flows is referred to as its **seniority ranking**.

Debt can be either **secured debt** or **unsecured debt**. Secured debt is backed by collateral, while unsecured debt or *debentures* represent a general claim to the issuer's assets and cash flows. Secured debt has higher priority of claims than unsecured debt.

Secured debt can be further distinguished as *first lien* or *first mortgage* (where a specific asset is pledged), *senior secured*, or *junior secured* debt. Unsecured debt is further divided into *senior*, *junior*, and *subordinated* gradations. The highest rank of unsecured debt is senior unsecured. Subordinated debt ranks below other unsecured debt.

The general seniority rankings for debt repayment priority are the following:

- First lien/senior secured.
- Second lien/secured.
- Senior unsecured.
- Senior subordinated.
- Subordinated.
- Junior subordinated.

All debt within the same category is said to rank **pari passu**, or have same priority of claims. All senior secured debt holders, for example, are treated alike in a corporate bankruptcy.

Recovery rates are highest for debt with the highest priority of claims and decrease with each lower rank of seniority. The lower the seniority ranking of a bond, the higher its credit risk. Investors require a higher yield to accept a lower seniority ranking.

In the event of a default or reorganization, senior lenders have claims on the assets before junior lenders and equity holders. A strict priority of claims, however, is not always applied in practice. Although in theory the priority of claims is absolute, in many cases lower-priority debt holders (and even equity investors) may get paid even if senior debt holders are not paid in full.

Bankruptcies can be costly and take a long time to settle. During bankruptcy proceedings, the value of a company's assets could deteriorate due to loss of customers and key employees, while legal expenses mount. A bankruptcy reorganization plan is confirmed by a vote among all classes of investors with less than 100% recovery rate. To avoid unnecessary delays, negotiation and compromise among various claimholders may result in a reorganization plan that does not strictly conform to the original priority of claims. By such a vote or by order of the bankruptcy court, the final plan may differ from absolute priority.

LOS 47.d: Compare and contrast corporate issuer credit ratings and issue credit ratings and describe the rating agency practice of "notching".

Credit rating agencies assign ratings to categories of bonds with similar credit risk. Rating agencies rate both the issuer (i.e., the company issuing the bonds) and the debt issues, or the bonds themselves. Issuer credit ratings are called **corporate family ratings** (CFR), while issue-specific ratings are called **corporate credit ratings** (CCR). Issuer ratings are based on the overall creditworthiness of the company. The issuers are rated on their senior unsecured debt.

Figure 47.1 shows ratings scales used by Standard & Poor's, Moody's, and Fitch, three of the major credit rating agencies.

Figure 47.1: Credit Rating Categories

(a) Investment grade ratings		(b) Noninvestment grade ratings	
Moody's	Standard & Poor's, Fitch	Moody's	Standard & Poor's, Fitch
Aaa	AAA	Ba1	BB+
Aa1	AA+	Ba2	BB
Aa2	AA	Ba3	BB–
Aa3	AA–	B1	B+
A1	A+	B2	B
A2	A	B3	B–
A3	A–	Caa1	CCC+
Baa1	BBB+	Caa2	CCC
Baa2	BBB	Caa3	CCC–
Baa3	BBB–	Ca	CC
		C	C
		C	D

Triple A (AAA or Aaa) is the highest rating. Bonds with ratings of Baa3/BBB– or higher are considered **investment grade**. Bonds rated Ba1/BB+ or lower are considered **noninvestment grade** and are often called *high yield bonds* or *junk bonds*.

Bonds in default are rated D by Standard & Poor's and Fitch and are included in Moody's lowest rating category, C. When a company defaults on one of its several outstanding bonds, provisions in bond indentures may trigger default on the remaining issues as well. Such a provision is called a *cross default provision*.

The ratings of a firm's individual bonds can differ from its corporate (issuer) rating. The seniority and covenants (including collateral pledged) of an individual bond issue are the primary determinants of differences between an issuer's rating and the ratings of its individual bond issues. The assignment of individual issue ratings that are higher or lower than that of the issuer is referred to as **notching**.

Another example of a factor that rating agencies consider when notching an issue credit rating is **structural subordination**. In a holding company structure, both the parent company and the subsidiaries may have outstanding debt. A subsidiary's debt covenants may restrict the transfer of cash or assets "upstream" to the parent company before the subsidiary's debt is serviced. In such a case, even though the parent company's bonds are not junior to the subsidiary's bonds, the subsidiary's bonds have a priority claim to the subsidiary's cash flows. Thus the parent company's bonds are effectively subordinated to the subsidiary's bonds.

Notching is less common for highly-rated issuers than for lower-rated issuers. For firms with high overall credit ratings, differences in expected recovery rates among a firm's individual bonds are less important, so their bonds might not be notched at all. For firms with higher probabilities of default (lower ratings), differences in expected recovery rates among a firm's bonds are more significant. For this reason, notching is more likely for issues with lower creditworthiness in general. For a firm with speculative credit, its subordinated debt might be notched two ratings below its issuer rating.

LOS 47.e: Explain risks in relying on ratings from credit rating agencies.

Relying on ratings from credit rating agencies has some risks. Four specific risks are:

1. **Credit ratings are dynamic.** Credit ratings change over time. Rating agencies may update their default risk assessments during the life of a bond. Higher credit ratings tend to be more stable than lower credit ratings.
2. **Rating agencies are not perfect.** Ratings mistakes occur from time to time. During a past period, subprime mortgage securities were assigned much higher ratings than they deserved.
3. **Event risk is difficult to assess.** Risks that are specific to a company or industry are difficult to predict and incorporate into credit ratings. Litigation risk to tobacco companies is one example. Events that are difficult to anticipate, such as natural disasters, acquisitions, and equity buybacks using debt, are not easily captured in credit ratings.
4. **Credit ratings lag market pricing.** Market prices and credit spreads can change much faster than credit ratings. Additionally, two bonds with the same rating can trade at different yields. Market prices reflect expected losses, while credit ratings only assess default risk.

LOS 47.f: Explain the four Cs (Capacity, Collateral, Covenants, and Character) of traditional credit analysis.

A common way to categorize the key components of credit analysis is by the **four Cs of credit analysis**: capacity, collateral, covenants, and character.

Capacity

Capacity refers to a corporate borrower's ability repay its debt obligations on time. Analysis of capacity is similar to the process used in equity analysis. Capacity analysis entails three levels of assessment: (1) industry structure, (2) industry fundamentals, and (3) company fundamentals.

Industry structure

The first level of a credit analyst's assessment is industry structure. Industry structure can be described by Porter's five forces: threat of entry, power of suppliers, power of buyers, threat of substitution, and rivalry among existing competitors.

PROFESSOR'S NOTE

We describe industry analysis based on Porter's five forces in the readings on equity valuation.

Industry fundamentals

The next level of a credit analyst's assessment is industry fundamentals, including the influence of macroeconomic factors on an industry's growth prospects and profitability. Industry fundamentals evaluation focuses on:

- **Industry cyclicality.** Cyclical industries are sensitive to economic performance. Cyclical industries tend to have more volatile earnings, revenues, and cash flows, which make them more risky than noncyclical industries.
- **Industry growth prospects.** Creditworthiness is most questionable for the weaker companies in a slow-growing or declining industry.
- **Industry published statistics.** Industry statistics provided by rating agencies, investment banks, industry periodicals, and government agencies can be a source for industry performance and fundamentals.

Company fundamentals

The last level of credit analysts' assessment is company fundamentals. A corporate borrower should be assessed on:

- **Competitive position.** Market share changes over time and cost structure relative to peers are some of the factors to analyze.
- **Operating history.** The performance of the company over different phases of business cycle, trends in margins and revenues, and current management's tenure.
- **Management's strategy and execution.** This includes the soundness of the strategy, the ability to execute the strategy, and the effects of management's decisions on bondholders.
- **Ratios and ratio analysis.** As we will discuss later in this reading, leverage and coverage ratios are important tools for credit analysis.

Collateral

Collateral analysis is more important for less creditworthy companies. The market value of a company's assets can be difficult to observe directly. Issues to consider when assessing collateral values include:

- **Intangible assets.** Patents are considered high-quality intangible assets because they can be more easily sold to generate cash flows than other intangibles. Goodwill is not considered a high-quality intangible asset and is usually written down when company performance is poor.
- **Depreciation.** High depreciation expense relative to capital expenditures may signal that management is not investing sufficiently in the company. The quality of the company's assets may be poor, which may lead to reduced operating cash flow and potentially high loss severity.
- **Equity market capitalization.** A stock that trades below book value may indicate that company assets are of low quality.
- **Human and intellectual capital.** These are difficult to value, but a company may have intellectual property that can function as collateral.

Covenants

Covenants are the terms and conditions the borrowers and lenders agree to as part of a bond issue. Covenants protect lenders while leaving some operating flexibility to the borrowers to run the company. There are two types of covenants: (1) *affirmative covenants* and (2) *negative covenants*.

Affirmative covenants require the borrower to take certain actions, such as using the proceeds for the for the stated purpose; paying interest, principal, and taxes; carrying insurance on pledged assets; continuing in its current business activity; and following relevant laws and regulations. Affirmative covenants have, basically, administrative purposes.

Negative covenants restrict the borrower from taking certain actions that may reduce the value of the bondholders' claims. While affirmative covenants do not impose significant costs on the issuer (besides making the promised payments), negative covenants constrain the issuer's business activities and may thereby impose significant costs on the issuer. Examples of negative covenants include:

- Restrictions on the payment of dividends and share repurchases, for example, restricting distributions to shareholders to a certain percentage of net income.
- Restrictions on the amount of additional debt the borrower can issue, for example, setting a maximum debt-to-equity ratio or minimum interest coverage ratio.
- Restrictions on issuing any debt with a higher priority than the subject debt issue.
- Restrictions on pledging any collateral that is currently unencumbered as collateral for new borrowing.
- Restrictions on assets sales, for example, limiting asset sales to a certain percentage of total asset value.
- Restrictions on company investment, for example, requiring that a company not invest outside its current primary business activities.
- Restrictions on mergers and acquisitions.

Covenants that are overly restrictive of an issuer's operating activities may reduce the issuer's ability to repay; for example, prohibiting asset sales that could provide the cash to pay bond interest and principal. On the other hand, covenants create a legally binding contractual framework for repayment of the debt obligation, which reduces uncertainty for the debt holders. A careful credit analysis should include an assessment of whether the covenants protect the interests of the bondholders without unduly constraining the borrower's operating activities.

Character

Character refers to management's integrity and its commitment to repay the loan. Factors such as management's business qualifications and operating record are important for evaluating character. Character analysis includes an assessment of:

- **Soundness of strategy.** Management's ability to develop a sound strategy.
- **Track record.** Management's past performance in executing its strategy and operating the company without bankruptcies, restructurings, or other distress situations that led to additional borrowing.
- **Accounting policies and tax strategies.** Use of accounting policies and tax strategies that may be hiding problems, such as revenue recognition issues, frequent restatements, and frequently changing auditors.
- **Fraud and malfeasance record.** Any record of fraud or other legal and regulatory problems.
- **Prior treatment of bondholders.** Benefits to equity holders at the expense of debt holders, through actions such as debt-financed acquisitions and special dividends, especially if they led to credit rating downgrades.

MODULE QUIZ 47.1

1. The two components of credit risk are:
   A. default risk and yield spread.
   B. default risk and loss severity.
   C. loss severity and yield spread.
2. Expected loss can decrease with an increase in a bond's:
   A. default risk.
   B. loss severity.
   C. recovery rate.
3. Absolute priority of claims in a bankruptcy might be violated because:
   A. of the pari passu principle.
   B. creditors negotiate a different outcome.
   C. available funds must be distributed equally among creditors.
4. "Notching" is *best* described as a difference between:
   A. an issuer credit rating and an issue credit rating.
   B. a company credit rating and an industry average credit rating.
   C. an investment grade credit rating and a noninvestment grade credit rating.
5. Which of the following statements is *least likely* a limitation of relying on ratings from credit rating agencies?
   A. Credit ratings are dynamic.
   B. Firm-specific risks are difficult to rate.
   C. Credit ratings adjust quickly to changes in bond prices.
6. Ratio analysis is *most likely* used to assess a borrower's:
   A. capacity.
   B. character.

C. collateral.

MODULE 47.2: EVALUATING CREDIT QUALITY

Video covering this content is available online.

LOS 47.g: Calculate and interpret financial ratios used in credit analysis.

LOS 47.h: Evaluate the credit quality of a corporate bond issuer and a bond of that issuer, given key financial ratios of the issuer and the industry.

Ratio analysis is part of capacity analysis. Two primary categories of ratios for credit analysis are *leverage ratios* and *coverage ratios*. Credit analysts calculate company ratios to assess the viability of a company, to find trends over time, and to compare companies to industry averages and peers.

Profits and Cash Flows

Profits and cash flows are needed to service debt. Here we examine four profit and cash flow metrics commonly used in ratio analysis by credit analysts.

1. **Earnings before interest, taxes, depreciation, and amortization (EBITDA).** EBITDA is a commonly used measure that is calculated as operating income plus depreciation and amortization. A drawback to using this measure for credit analysis is that it does not adjust for capital expenditures and changes in working capital, which are necessary uses of funds for a going concern. Cash needed for these uses is not available to debt holders.
2. **Funds from operations (FFO).** Funds from operations are net income from continuing operations plus depreciation, amortization, deferred taxes, and noncash items. FFO is similar to cash flow from operations (CFO) except that FFO excludes changes in working capital.
3. **Free cash flow before dividends.** Free cash flow before dividends is net income plus depreciation and amortization minus capital expenditures minus increase in working capital. Free cash flow before dividends excludes nonrecurring items.
4. **Free cash flow after dividends.** This is free cash flow before dividends minus the dividends. If free cash flow after dividends is greater than zero, it represents cash that could pay down debt or accumulate on the balance sheet. Either outcome is a form of deleveraging, a positive indicator for creditworthiness.

Leverage Ratios

Analysts should adjust debt reported on the financial statements by including the firm's obligations such as underfunded pension plans (net pension liabilities) and off-balance-sheet liabilities such as operating leases.

The most common measures of leverage used by credit analysts are the debt-to-capital ratio, the debt-to-EBITDA ratio, the FFO-to-debt ratio, and the ratio of FCF after dividends to debt.

1. **Debt/capital.** Capital is the sum of total debt and shareholders' equity. The debt-to-capital ratio is the percentage of the capital structure financed by debt. A lower ratio indicates less credit risk. If the financial statements list high values for intangible assets

such as goodwill, an analyst should calculate a second debt-to-capital ratio adjusted for a write-down of these assets' after-tax value.

2. **Debt/EBITDA.** A higher ratio indicates higher leverage and higher credit risk. This ratio is more volatile for firms in cyclical industries or with high operating leverage because of their high variability of EBITDA.
3. **FFO/debt.** Because this ratio divides a cash flow measure by the value of debt, a higher ratio indicates lower credit risk.
4. **FCF after dividends/debt.** Greater values indicate a greater ability to service existing debt.

Coverage Ratios

Coverage ratios measure the borrower's ability to generate cash flows to meet interest payments. The two most commonly used are EBITDA-to-interest and EBIT-to-interest.

1. **EBITDA/interest expense.** A higher ratio indicates lower credit risk. This ratio is used more often than the EBIT-to-interest expense ratio. Because depreciation and amortization are still included as part of the cash flow measure, this ratio will be higher than the EBIT version.
2. **EBIT/interest expense.** A higher ratio indicates lower credit risk. This ratio is the more conservative measure because depreciation and amortization are subtracted from earnings.

Ratings agencies publish benchmark values for financial ratios that are associated with each ratings classification. Credit analysts can evaluate the potential for upgrades and downgrades based on subject company ratios relative to these benchmarks.

EXAMPLE: **Credit analysis based on ratios**

An analyst is assessing the credit quality of York, Inc. and Zale, Inc., relative to each other and their industry average. Selected financial information appears in the following table.

	York, Inc.	Zale, Inc.	Industry Average
Earnings before interest and taxes	$550,000	$2,250,000	$1,400,000
Funds from operations	$300,000	$850,000	$600,000
Interest expense	$40,000	$160,000	$100,000
Total debt	$1,900,000	$2,700,000	$2,600,000
Total capital	$4,000,000	$6,500,000	$6,000,000

Explain how the analyst should evaluate the relative creditworthiness of York and Zale.

Answer:

Leverage and coverage ratios based on these data are as follows:

EBIT / interest:

York: $550,000 / $40,000 = 13.8×

Zale: $2,250,000 / $160,000 = 14.1×

Industry average: $1,400,000 / $100,000 = 14.0×

Both York and Zale have interest coverage in line with their industry average.

FFO / total debt:

York: \$300,000 / \$1,900,000 = 15.8%

Zale: \$850,000 / \$2,700,000 = 31.5%

Industry average: \$600,000 / \$2,600,000 = 23.1%

Zale's funds from operations relative to its debt level are greater than the industry average, while York is generating less FFO relative to its debt level.

Total debt / total capital:

York: \$1,900,000 / \$4,000,000 = 47.5%

Zale: \$2,700,000 / \$6,500,000 = 41.5%

Industry average: \$2,600,000 / \$6,000,000 = 43.3%

York is more leveraged than Zale and the industry average. Based on these data, Zale appears to be more creditworthy than York.

LOS 47.i: Describe macroeconomic, market, and issuer-specific factors that influence the level and volatility of yield spreads.

We can think of the yield on an option-free corporate bond as the sum of the real risk-free interest rate, the expected inflation rate, a maturity premium, a liquidity premium, and a credit spread. All bond prices and yields are affected by changes in the first three of these components. The last two components are the yield spread:

yield spread = liquidity premium + credit spread

Yield spreads on corporate bonds are affected primarily by five interrelated factors:

1. **Credit cycle.** The market's perception of overall credit risk is cyclical. At the top of the credit cycle, the bond market perceives low credit risk and is generally bullish. Credit spreads narrow as the credit cycle improves. Credit spreads widen as the credit cycle deteriorates.
2. **Economic conditions.** Credit spreads narrow as the economy strengthens and investors expect firms' credit metrics to improve. Conversely, credit spreads widen as the economy weakens.
3. **Broker-dealer capital.** Because most bonds trade over the counter, investors need broker-dealers to provide market-making capital for bond markets to function. Yield spreads are narrower when broker-dealers provide sufficient capital but can widen when market-making capital becomes scarce.
4. **General market demand and supply.** Credit spreads narrow in times of high demand for bonds. Credit spreads widen in times of low demand for bonds. Excess supply conditions, such as large issuance in a short period of time, can lead to widening spreads.
5. **Issuer's financial performance.** Developments that are positive for the issuer's credit quality will narrow its yield spread, while developments that are negative for the issuer's credit quality will widen its credit spread.

Yield spreads on lower-quality issues tend to be more volatile than spreads on higher-quality issues.

LOS 47.j: Explain special considerations when evaluating the credit of high-yield, sovereign, and non-sovereign government debt issuers and issues.

High Yield Debt

High yield or *noninvestment grade* corporate bonds are rated below Baa3/BBB– by credit rating agencies. These bonds are also called *junk bonds* because of their higher perceived credit risk.

Reasons for noninvestment grade ratings may include:

- High leverage.
- Unproven operating history.
- Low or negative free cash flow.
- High sensitivity to business cycles.
- Low confidence in management.
- Unclear competitive advantages.
- Large off-balance-sheet liabilities.
- Industry in decline.

Because high yield bonds have higher default risk than investment grade bonds, credit analysts must pay more attention to loss severity. Special considerations for high yield bonds include their liquidity, financial projections, debt structure, corporate structure, and covenants.

Liquidity. Liquidity or availability of cash is critical for high yield issuers. High yield issuers have limited access to additional borrowings, and available funds tend to be more expensive for high yield issuers. Bad company-specific news and difficult financial market conditions can quickly dry up the liquidity of debt markets. Many high yield issuers are privately owned and cannot access public equity markets for needed funds.

Analysts focus on six sources of liquidity (in order of reliability):

1. Balance sheet cash.
2. Working capital.
3. Operating cash flow (CFO).
4. Bank credit.
5. Equity issued.
6. Sales of assets.

For a high yield issuer with few or unreliable sources of liquidity, significant amounts of debt coming due within a short time frame may indicate potential default. Running out of cash with no access to external financing to refinance or service existing debt is the primary reason why high yield issuers default. For high yield financial firms that are highly levered and depend on funding long-term assets with short-term liabilities, liquidity is critical.

Financial projections. Projecting future earnings and cash flows, including stress scenarios and accounting for changes in capital expenditures and working capital, are important for revealing potential vulnerabilities to the inability to meet debt payments.

Debt structure. High yield issuers' capital structures often include different types of debt with several levels of seniority and hence varying levels of potential loss severity. Capital structures typically include secured bank debt, second lien debt, senior unsecured debt, subordinated debt, and preferred stock. Some of these, especially subordinated debt, may be convertible to common shares.

A credit analyst will need to calculate leverage for each level of the debt structure when an issuer has multiple layers of debt with a variety of expected recovery rates.

High yield companies for which secured bank debt is a high proportion of the capital structure are said to be *top heavy* and have less capacity to borrow from banks in financially stressful periods. Companies that have top-heavy capital structures are more likely to default and have lower recovery rates for unsecured debt issues.

Corporate structure. Many high-yield companies use a holding company structure. A parent company receives dividends from the earnings of subsidiaries as its primary source of operating income. Because of structural subordination, subsidiaries' dividends paid upstream to a parent company are subordinate to interest payments. These dividends can be insufficient to pay the debt obligations of the parent, thus reducing the recovery rate for debt holders of the parent company.

Despite structural subordination, a parent company's credit rating may be superior to subsidiaries' ratings because the parent can benefit from having access to multiple cash flows from diverse subsidiaries.

Some complex corporate structures have intermediate holding companies that carry their own debt and do not own 100% of their subsidiaries' stock. These companies are typically a result of mergers, acquisitions, or leveraged buyouts.

Default of one subsidiary may not necessarily result in cross default. Analysts need to scrutinize bonds' indentures and other legal documents to fully understand the impact of complex corporate structures. To analyze these companies, analysts should calculate leverage ratios at each level of debt issuance and on a consolidated basis.

Covenants. Important covenants for high yield debt include:

- **Change of control put.** This covenant gives debt holders the right to require the issuer to buy back debt (typically for par value or a value slightly above par) in the event of an acquisition. For investment grade bonds, a change of control put typically applies only if an acquisition of the borrower results in a rating downgrade to below investment grade.
- **Restricted payments.** The covenant protects lenders by limiting the amount of cash that may be paid to equity holders.
- **Limitations on liens.** The covenant limits the amount of secured debt that a borrower can carry. Unsecured debt holders prefer the issuer to have less secured debt, which increases the recovery amount available to them in the event of default.

- **Restricted versus unrestricted subsidiaries.** Issuers can classify subsidiaries as restricted or unrestricted. Restricted subsidiaries' cash flows and assets can be used to service the debt of the parent holding company. This benefits creditors of holding companies because their debt is pari passu with the debt of restricted subsidiaries, rather than structurally subordinated. Restricted subsidiaries are typically the holding company's larger subsidiaries that have significant assets. Tax and regulatory issues can factor into the classification of subsidiary's restriction status. A subsidiary's restriction status is found in the bond indenture.

Bank covenants are often more restrictive than bond covenants, and when covenants are violated, banks can block additional loans until the violation is corrected. If a violation is not remedied, banks can trigger a default by accelerating the full repayment of a loan.

In terms of the factors that affect their return, high yield bonds may be viewed as a hybrid of investment grade bonds and equity. Compared to investment grade bonds, high yield bonds show greater price and spread volatility and are more highly correlated with the equity market.

High yield analysis can include some of the same techniques as equity market analysis, such as enterprise value. **Enterprise value** (EV) is equity market capitalization plus total debt minus excess cash. For high yield companies that are not publicly traded, comparable public company equity data can be used to estimate EV. Enterprise value analysis can indicate a firm's potential for additional leverage, or the potential credit damage that might result from a leveraged buyout. An analyst can compare firms based on the differences between their EV/EBITDA and debt/EBITDA ratios. Firms with a wider difference between these ratios have greater equity relative to their debt and therefore have less credit risk.

Sovereign Debt

Sovereign debt is issued by national governments. Sovereign credit analysis must assess both the government's ability to service debt and its willingness to do so. The assessment of willingness is important because bondholders usually have no legal recourse if a national government refuses to pay its debts.

A basic framework for evaluating and assigning a credit rating to sovereign debt includes five key areas:

1. **Institutional assessment** includes successful policymaking, minimal corruption, checks and balances among institutions, and a culture of honoring debts.
2. **Economic assessment** includes growth trends, income per capita, and diversity of sources for economic growth.
3. **External assessment** includes the country's foreign reserves, its external debt, and the status of its currency in international markets.
4. **Fiscal assessment** includes the government's willingness and ability to increase revenue or cut expenditures to ensure debt service, as well as trends in debt as a percentage of GDP.
5. **Monetary assessment** includes the ability to use monetary policy for domestic economic objectives (this might be lacking with exchange rate targeting or membership in a monetary union) and the credibility and effectiveness of monetary policy.

Credit rating agencies assign each national government two ratings: (1) a local currency debt rating and (2) a foreign currency debt rating. The ratings are assigned separately because defaults on foreign currency denominated debt have historically exceeded those on local currency debt. Foreign currency debt typically has a higher default rate and a lower credit rating because the government must purchase foreign currency in the open market to make interest and principal payments, which exposes it to the risk of significant local currency depreciation. In contrast, local currency debt can be repaid by raising taxes, controlling domestic spending, or simply printing more money. Ratings can differ as much as two notches for local and foreign currency bonds.

Sovereign defaults can be caused by events such as war, political instability, severe devaluation of the currency, or large declines in the prices of the country's export commodities. Access to debt markets can be difficult for sovereigns in bad economic times.

Non-Sovereign Government Bonds

Non-sovereign government debt is issued by local governments (cities, states, and counties) and quasi-governmental entities. **Municipal bonds** are a significant part of the overall U.S. bond market. Interest payments from municipal bonds are most often exempt from national income taxes. Default rates for municipal bonds are very low relative to general corporate bonds.

Most municipal bonds can be classified as *general obligation bonds* or *revenue bonds*. **General obligation** (GO) bonds are unsecured bonds backed by the full faith credit of the issuing governmental entity, which is to say they are supported by its taxing power. **Revenue bonds** are issued to finance specific projects, such as airports, toll bridges, hospitals, and power generation facilities.

Unlike sovereigns, municipalities cannot use monetary policy to service their debt and usually must balance their operating budgets. Municipal governments' ability to service their general obligation debt depends ultimately on the local economy (i.e., the tax base). Economic factors to assess in evaluating the creditworthiness of GO bonds include employment, trends in per capita income and per capita debt, tax base dimensions (depth, breadth, and stability), demographics, and ability to attract new jobs (location, infrastructure). Credit analysts must also observe revenue variability through economic cycles. Relying on highly variable taxes that are subject to economic cycles, such as capital gains and sales taxes, can signal higher credit risk. Municipalities may have long-term obligations such as underfunded pensions and post-retirement benefits. Inconsistent reporting requirements for municipalities are also an issue.

Revenue bonds often have higher credit risk than GO bonds because the project is the sole source of funds to service the debt. Analysis of revenue bonds combines analysis of the project, using techniques similar to those for analyzing corporate bonds, with analysis of the financing of the project.

MODULE QUIZ 47.2

1. Higher credit risk is indicated by a higher:
   - A. FFO/debt ratio.
   - B. debt/EBITDA ratio.
   - C. EBITDA/interest expense ratio.
2. Compared to other firms in the same industry, an issuer with a credit rating of AAA should have a lower:
   - A. FFO/debt ratio.
   - B. operating margin.
   - C. debt/capital ratio.

3. Credit spreads tend to widen as:
   A. the credit cycle improves.
   B. economic conditions worsen.
   C. broker-dealers become more willing to provide capital.
4. Compared to shorter duration bonds, longer duration bonds:
   A. have smaller bid-ask spreads.
   B. are less sensitive to credit spreads.
   C. have less certainty regarding future creditworthiness.
5. One key difference between sovereign bonds and municipal bonds is that sovereign issuers:
   A. can print money.
   B. have governmental taxing power.
   C. are affected by economic conditions.

KEY CONCEPTS

LOS 47.a

Credit risk refers to the possibility that a borrower fails to make the scheduled interest payments or return of principal.

Spread risk is the possibility that a bond loses value because its credit spread widens relative to its benchmark. Spread risk includes credit migration or downgrade risk and market liquidity risk.

LOS 47.b

Credit risk is composed of default risk, which is the probability of default, and loss severity, which is the portion of the value of a bond or loan a lender or investor will lose if the borrower defaults. The expected loss is the probability of default multiplied by the loss severity.

LOS 47.c

Corporate debt is ranked by seniority or priority of claims. Secured debt is a direct claim on specific firm assets and has priority over unsecured debt. Secured or unsecured debt may be further ranked as senior or subordinated. Priority of claims may be summarized as follows:

- First lien/senior secured.
- Second lien/secured.
- Senior unsecured.
- Senior subordinated.
- Subordinated.
- Junior subordinated.

LOS 47.d

Issuer credit ratings, or corporate family ratings, reflect a debt issuer’s overall creditworthiness and typically apply to a firm’s senior unsecured debt.

Issue credit ratings, or corporate credit ratings, reflect the credit risk of a specific debt issue. Notching refers to the practice of adjusting an issue credit rating upward or downward from the issuer credit rating to reflect the seniority, covenants, and possibly the expected recovery in the event of a default of a debt issue.

LOS 47.e

Lenders and bond investors should not rely exclusively on credit ratings from rating agencies for the following reasons:

- Credit ratings can change during the life of a debt issue.
- Rating agencies cannot always judge credit risk accurately.
- Firms are subject to risk of unforeseen events that credit ratings do not reflect.
- Market prices of bonds often adjust more rapidly than credit ratings.

LOS 47.f

Components of traditional credit analysis are known as the four Cs:

- Capacity: The borrower's ability to make timely payments on its debt.
- Collateral: The value of assets pledged against a debt issue or available to creditors if the issuer defaults.
- Covenants: Provisions of a bond issue that protect creditors by requiring or prohibiting actions by an issuer's management.
- Character: Assessment of an issuer's management, strategy, quality of earnings, and past treatment of bondholders.

LOS 47.g

Credit analysts use profitability, cash flow, and leverage and coverage ratios to assess debt issuers' capacity.

- Profitability refers to operating income and operating profit margin, with operating income typically defined as earnings before interest and taxes (EBIT).
- Cash flow may be measured as earnings before interest, taxes, depreciation, and amortization (EBITDA); funds from operations (FFO); free cash flow before dividends; or free cash flow after dividends.
- Leverage ratios include debt-to-capital, debt-to-EBITDA, and FFO-to-debt.
- Coverage ratios include EBIT-to-interest expense and EBITDA-to-interest expense.

LOS 47.h

Lower leverage, higher interest coverage, and greater free cash flow imply lower credit risk and a higher credit rating for a firm. When calculating leverage ratios, analysts should include in a firm's total debt its obligations such as underfunded pensions and off-balance-sheet financing.

For a specific debt issue, secured collateral implies lower credit risk compared to unsecured debt, and higher seniority implies lower credit risk compared to lower seniority.

LOS 47.i

Corporate bond yields comprise the real risk-free rate, expected inflation rate, credit spread, maturity premium, and liquidity premium. An issue's yield spread to its benchmark includes its credit spread and liquidity premium.

The level and volatility of yield spreads are affected by the credit and business cycles, availability of capital from broker-dealers, the supply and demand for debt issues, and the financial performance of the bond issuer. Yield spreads tend to narrow when the credit cycle is improving, the economy is expanding, and financial markets and investor demand for new debt issues are strong. Yield spreads tend to widen when the credit cycle, the economy, and financial markets are weakening, and in periods when the supply of new debt issues is heavy or broker-dealer capital is insufficient for market making.

LOS 47.j

High yield bonds are more likely to default than investment grade bonds, which increases the importance of estimating loss severity. Analysis of high yield debt should focus on liquidity, projected financial performance, the issuer's corporate and debt structures, and debt covenants.

Credit risk of sovereign debt includes the issuing country's ability and willingness to pay. Ability to pay is greater for debt issued in the country's own currency than for debt issued in a foreign currency. Willingness refers to the possibility that a country refuses to repay its debts.

Analysis of non-sovereign government debt is similar to analysis of sovereign debt, focusing on the strength of the local economy and its effect on tax revenues. Analysis of municipal revenue bonds is similar to analysis of corporate debt, focusing on the ability of a project to generate sufficient revenue to service the bonds.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 47.1

1. **B** Credit risk is composed of default risk and loss severity. Yield spreads reflect the credit risk of a borrower. (LOS 47.a)
2. **C** An increase in the recovery rate means that the loss severity has decreased, which decreases expected loss. (LOS 47.b)
3. **B** A negotiated bankruptcy settlement does not always follow the absolute priority of claims. (LOS 47.c)
4. **A** Notching refers to the credit rating agency practice of distinguishing between the credit rating of an issuer (generally for its senior unsecured debt) and the credit rating of particular debt issues from that issuer, which may differ from the issuer rating because of provisions such as seniority. (LOS 47.d)
5. **C** Bond prices and credit spreads change much faster than credit ratings. (LOS 47.e)
6. **A** Ratio analysis is used to assess a corporate borrower's capacity to repay its debt obligations on time. (LOS 47.f)

Module Quiz 47.2

1. **B** A higher debt/EBITDA ratio is sign of higher leverage and higher credit risk. Higher FFO/debt and EBITDA/interest expense ratios indicate lower credit risk. (LOS 47.g, 47.h)
2. **C** A low debt/capital ratio is an indicator of low leverage. An issuer rated AAA is likely to have a high operating margin and a high FFO/debt ratio compared to its industry group.

(LOS 47.g, 47.h)

3. **B** Credit spreads widen as economic conditions worsen. Spreads narrow as the credit cycle improves and as broker-dealers provide more capital to bond markets. (LOS 47.i)
4. **C** Longer duration bonds usually have longer maturities and carry more uncertainty of future creditworthiness. (LOS 47.i)
5. **A** Sovereign entities can print money to repay debt, while municipal borrowers cannot. Both sovereign and municipal entities have taxing powers, and both are affected by economic conditions. (LOS 47.j)

TOPIC QUIZ: FIXED INCOME

You have now finished the Fixed Income topic section. Please log into your Schweser online dashboard and take the Topic Quiz on Fixed Income. The Topic Quiz provides immediate feedback on how effective your study has been for this material. The number of questions on this quiz is approximately the number of questions for the topic on one-half of the actual Level I CFA exam. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow 1.5 minutes per question.

After you've completed this Topic Quiz, select "Performance Tracker" to view a breakdown of your score. Select "Compare with Others" to display how your score on the Topic Quiz compares to the scores of others who entered their answers.

READING 48

DERIVATIVE INSTRUMENT AND DERIVATIVE MARKET FEATURES

MODULE 48.1: DERIVATIVES MARKETS

LOS 48.a: Define a derivative and describe basic features of a derivative instrument.

A **derivative** is a security that *derives* its value from the value of another security or a variable (such as an interest rate or stock index value) at some specific future date. The security or variable that determines the value of a derivative security is referred to as the **underlying** for the derivative. The value of a derivative at a point in time is derived from the value of the underlying (asset or variable) on which the derivative contract is based.

A relatively simple example of a derivative is a **forward contract** that specifies the price at which one party agrees to buy or sell an underlying security at a specified future date. Consider a forward contract to buy 100 shares of Acme at $30 per share three months from now:

- Acme shares are the **underlying asset** for the forward contract.
- $30 is the **forward price** in the contract.
- The date of the future transaction, when the shares will be exchanged for cash, is referred to as the **settlement date** (maturity date) of the forward contract.
- 100 shares is the **contract size** of the forward contract.
- The forward price is set so that the forward contract has zero value to both parties at contract initiation; neither party pays at the initiation of the contract.

We can examine three outcomes for the price of Acme shares at settlement:

1. The **spot price**, or the market price of the underlying, is $30, equal to the forward price of $30.

   Neither party has profits or losses on the forward contract. Ignoring transactions costs, the party selling Acme shares could buy them back at $30 per share and the party buying the shares could sell them at $30 per share.

2. The spot price of Acme shares is $40, greater than the forward price of $30.

   The party buying 100 Acme shares for $3,000 at settlement can sell those shares at the spot price for $4,000, realizing a profit of $1,000 on the forward contract. The party that must deliver the Acme shares delivers shares with a market value of $4,000 and receives $3,000, realizing a $1,000 loss on the forward contract.

3. The spot price of Acme shares is $25 at settlement, less than the forward price of $30.

   The party buying 100 Acme shares for $3,000 at settlement can sell those shares at the market price of $25 to get $2,500 and realize a loss of $500 on the forward contract. The party that must deliver the

Acme shares delivers shares with a market value of $2,500 and receives $3,000, realizing a $500 gain on the forward contract.

To summarize, the buyer of the shares in a forward contract will have gains when the market price of the shares at settlement is greater than the forward price, and losses when the market price of the shares at settlement is less than the forward price. The party that must deliver the shares in a forward contract will have gains when the market price of the shares at settlement is less than the forward price, and losses when the market price of the shares at settlement is greater than the forward price. The gains of one party equal the losses of the other party at settlement.

We refer to the party that agrees to buy the underlying asset in a forward contract as the buyer of the forward. The buyer of the forward gains when the price of the underlying increases (and loses when it falls), similarly to a long position in the underlying. In this case we say the forward buyer has *long exposure* to the underlying, while the seller of the underlying has *short exposure* to the underlying, gaining when the price of the underlying decreases and losing when the value of the underlying increases.

In practice, a forward contract may be a **deliverable contract**. In our forward contract example, this means that the payment and the shares must be exchanged at the settlement date. A **cash-settled contract** specifies that only the gains and losses from the forward contract are exchanged at settlement. In our example above, with a share price of $25, cash settlement would require the buyer of Acme shares to pay $500 to the seller of Acme shares at settlement. Ignoring the transaction cost of buying and selling shares in the market, the gains and losses to the parties in our forward contract example are economically equivalent under the two alternative settlement methods.

We can view a derivatives contract as a way to transfer risk from one party to another. Consider a situation where the share seller in the forward contract owns 100 shares of Acme. She has existing risk because the future price of Acme shares is uncertain; the share price three months from now is a random variable. If she enters the forward contract from our example, she will receive $3,000 for her shares at settlement, regardless of their market price. This effectively transfers her existing Acme price risk to the buyer of the shares in the forward contract.

When a party to a derivative contract has an existing risk that is transferred to another party, we say that party has **hedged** (offset, reduced) their existing risk. If the risk of the forward contract exactly matches an existing risk, then the forward contract can be used to fully hedge the existing risk. If a derivative is used to reduce, but not entirely offset, an existing risk, we say the existing risk is **partially hedged**.

If the Acme share buyer in our example has no existing Acme price risk, she clearly increases her risk by entering into the forward contract. In this situation, the Acme share buyer is said to be **speculating** on the future price of Acme shares.

You may have realized our share seller could have achieved her goal of eliminating her Acme price risk by simply selling her shares (a **cash market transaction**). Derivatives have potential advantages over cash market transactions:

- Investors can gain exposure to a risk at low cost, effectively creating a highly leveraged investment in the underlying.

- Transaction costs for a derivatives position may be significantly lower than for the equivalent cash market trade.
- Initiating a derivatives position may have less impact on market prices of the underlying, relative to initiating an equivalent position in the underlying through a cash market transaction.

Underlying Assets and Variables

The underlying for a derivative is most often a stock or bond price, the level of a stock or bond index, or an interest rate. Here, we give examples of different underlying assets and variables for derivative contracts and the nature of the risks they transfer or modify. We present more details about derivatives based on these underlying assets in subsequent readings.

- A bond, for example a forward contract on a 30-year U.S. Treasury bond or other specific bond. The risk involved is the uncertainty about future bond prices.
- An index, for example the S&P 500 Index or the Citi Goldman Sachs Investment Grade Corporate Bond Index. The risk involved is the uncertainty about the future value of the index at a specific date. A portfolio manager can reduce the risk of a portfolio of large U.S. stocks for a period of time by selling a forward on the S&P 500 index. An investor can gain long exposure to a portfolio of high-grade corporate bonds, quickly and at low cost, by buying a forward on the index.
- A currency, for example British pounds (GBP). A U.S. manufacturer that expects a large payment in GBP in six months can offset the uncertainty about the USD value of this payment by selling a forward contract on the expected amount of GBP. A UK manufacturer that must make a large payment in USD in one year can offset the uncertainty about the GBP cost of the payment by buying a forward contract on the USD that is priced in GBP.
- An interest rate, for example the 1-year Treasury bill rate. Similar to derivatives based on bonds, except that a higher interest rate means gains for the buyer of an interest rate forward, whereas higher interest rates mean lower bond prices and losses for the buyer of a bond forward.
- Commodities, which are physical assets including **hard commodities** (typically mined or extracted, such as gold and oil) and **soft commodities** (typically grown, such as cotton, coffee, pork, and cattle). A farmer expecting a cotton crop in four months can reduce her cotton price risk by selling cotton forward. A utility that will require thousands of gallons of oil over the next year can reduce its oil price risk by buying oil forwards that settle at various times over the coming year.

PROFESSOR'S NOTE

It may help you to remember this common rule for hedging risk with futures, which are similar to forwards: "Do in the futures market what you must do in the future." A baking company that must buy wheat in the future should buy wheat futures (or forwards) to reduce the effects of the uncertainty about future wheat prices on their profits. A farmer who must sell wheat at harvest time should sell wheat futures (or forwards) to reduce price risk.

- Credit derivatives include credit default swaps (CDS), in which one party makes fixed periodic payments to another party, which will make a payment only if the underlying credit instrument (or portfolio of such credit securities) suffers a loss in value due to a default by the issuer (borrower of funds) of the credit instrument.

- Derivative contracts are also created with the weather (for farmers, energy producers, travel and tourism companies), cryptocurrencies, or longevity (for life insurers or annuity providers) as the underlying asset.

Along with forwards and futures, derivatives types that we will cover in the remainder of our derivatives readings are:

- *Options*: Put options give the buyer the right (but not the obligation) to sell the underlying for a specific price in the future. Call options give the buyer the right (but not the obligation) to buy the underlying for a specific price in the future.
- *Swaps*: In a simple interest rate swap, one party agrees to make periodic payments on a given amount at a fixed interest rate, and the other agrees to make periodic interest payments on the same given amount, but at an interest rate based on a future market reference rate (MRR). The resulting cash flows are equivalent to one party (the fixed-rate payer) borrowing at a fixed rate and using the proceeds to buy a floating-rate bond.

LOS 48.b: Describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets.

Exchange-Traded Derivatives

Centralized physical exchanges provide markets for futures contracts, some options contracts, and some other derivative contracts. The largest by volume of trades are the National Securities Exchange (India), the B3 market (Brazil), and the CME Group (the United States).

Exchange-traded derivatives are standardized and backed by a central clearinghouse. The exchange specifies the terms of each of the derivative contracts that will be traded and rules for trading on the exchange.

A **central clearinghouse (CCH)** essentially takes the opposite position to each side of a trade (called **novation**), guaranteeing the payments promised under the contract. The CCH requires deposits from both participants when a trade is initiated, and additional deposits for accounts that decline in value, to support its guarantee and minimize counterparty credit risk.

Exchange members (dealers or market makers) buy and sell derivatives at slightly different prices and primarily earn trading profits from the bid/ask spreads between buy and sell prices, rather than from holding (speculating on) specific derivatives positions, although they may hold such positions from time to time to meet customer needs.

The standardization of contracts allows exchange-traded derivatives to be more liquid and more transparent to market participants, compared to customized derivatives. A market participant who has taken a position in exchange-traded derivatives can easily exit that position by entering into a contract with a position opposite to their existing derivatives. Standardization of contracts reduces trading costs compared to customized derivative contracts.

Standardization also facilitates the clearing and settlement of trades. Clearing refers to executing the trade, recording the participants, and handling the exchange of any required payments. Settlement refers to the exchange of underlying assets or payments of the final amounts due at contract settlement (maturity).

Dealer (OTC) Markets for Derivatives

Forwards, most swaps, and some options are custom instruments created and traded by dealers in a market with no central location. Some dealer markets are quite structured (e.g., the Nasdaq market), while others are not. A dealer market with no central location is referred to as an over-the-counter (OTC) market. OTC markets are largely unregulated and less transparent than exchange markets. In OTC markets with no central clearinghouse, each side of a trade faces counterparty credit risk. Dealers (market makers) make derivatives trades with end users of derivatives and may also trade with each other to reduce their exposures to changes in prices of underlying assets.

OTC derivatives contracts can be customized to fit the needs of an end user regarding contract size, definition of the underlying, settlement date, whether the contract is deliverable of cash settled, and other relevant details. Users trying to gain or hedge a specific risk use OTC derivatives when a standardized derivative contract will not meet their needs (including a desire for privacy).

After the financial crisis of 2008, regulators worldwide instituted a **central clearing mandate** requiring that, for many swap trades, **a central counterparty (CCP)** takes on the counterparty credit risk of both sides of a trade, similar to the role of a central clearinghouse. As an example, multiple dealers record their swap trades on a **swap execution facility (SEF)**. When a dealer makes a swap trade, that information is sent to the SEF and the CCP replaces the trade with two trades, with the CCP as the counterparty to both of them, reducing counterparty risk. The downside of this structure is that counterparty risks are concentrated rather than distributed among financial intermediaries.

The following offers a summary of the primary differences between exchange-traded and OTC derivatives.

Exchange-traded derivatives are:

- Traded at a centralized location, an exchange.
- Traded by exchange members (market makers).
- Based on standardized contracts and have lower trading costs.
- Subject to the trading rules of the exchange (i.e., are more regulated).
- Backed by the central clearinghouse to minimize counterparty credit risk. They also require deposits by both parties at initiation, and additional deposits when a position decreases in value.
- More liquid.
- More transparent, as all transactions are known to the exchange and to regulators.

OTC derivatives (not subject to the central clearing mandate) are:

- Custom instruments.
- Less liquid and have higher transaction costs.
- Less transparent.

- Subject to counterparty risk.
- More difficult to clear and settle.
- Subject to higher trading costs.
- Not subject to requirements for the deposit of collateral.

Derivatives in dealer markets that are subject to the central clearing mandate have reduced counterparty risk, are subject to more disclosure of trades, and are easier to clear and settle, but are still customizable and are contracts with dealers or financial intermediaries.

MODULE QUIZ 48.1

1. Which of the following statements *most accurately* describes a derivative security? A derivative:
   A. always increases risk.
   B. has no expiration date.
   C. has a payoff based on an asset value or interest rate.
2. Which of the following statements about exchange-traded derivatives is *least accurate*? Exchange-traded derivatives:
   A. are liquid.
   B. are standardized contracts.
   C. carry significant default risk.

KEY CONCEPTS

LOS 48.a

A derivative is a security that derives its value from value of another security or variable at a specific future date. The security or variable that determines the value of a derivative security is referred to as the underlying.

Basic features of a derivative include the underlying, the price specified in the contract, the contract size, and the settlement date. The price is typically set so the contract has zero value at initiation to both parties. Contracts may be deliverable or cash-settled.

LOS 48.b

Exchange-traded derivatives are standardized and backed by a central clearinghouse that takes the opposite position to each side of a trade, guaranteeing the payments promised under the contract.

Over-the-counter (OTC) derivatives can be customized to fit the needs of the counterparties. OTC markets are largely unregulated and less transparent than exchange markets. Some OTC markets are subject to a central clearing mandate that reduces counterparty credit risk.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 48.1

1. **C** A derivative's value is derived from another asset or an interest rate.
   (LOS 48.a)

2. **C** Exchange-traded derivatives have relatively low default risk (counterparty credit risk) because the clearinghouse stands between the counterparties involved, in most contracts. (LOS 48.b)

READING 49

FORWARD COMMITMENT AND CONTINGENT CLAIM FEATURES AND INSTRUMENTS

MODULE 49.1: FORWARDS AND FUTURES

Video covering this content is available online.

LOS 49.a: Define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics.

Forward Contracts

In a **forward contract** between two parties, one party (the buyer) commits to buy and the other party (the seller) commits to sell a physical or financial asset at a specific price on a specific date (the settlement date) in the future.

The buyer has long exposure to the underlying asset in that he will make a profit on the forward if the price of the underlying at the settlement date exceeds the forward price, and have a loss if the price of the underlying at the settlement date is less than the forward price. The results are opposite for the seller of the forward, who has short exposure to the underlying asset.

Futures Contracts

A **futures contract** is quite similar to a forward contract but is standardized and exchange-traded. The primary ways in which forwards and futures differ are that futures trade in a liquid secondary market, are subject to greater regulation, and trade in markets with more disclosure (transparency). Futures are backed by a central clearinghouse and require daily cash settlement of gains and losses, so that counterparty credit risk is minimized.

On a futures exchange, **margin** is cash or other acceptable collaterals that both the buyer and seller must deposit. Unlike margin in bond or stock accounts, there is no loan involved and, consequently, there are no interest charges. This collateral provides protection for the clearinghouse. At the end of each trading day, the margin balance in a futures account is adjusted for any gains and losses in the value of the futures position based on the new settlement price, a process called the **mark-to-market** or marking-to-market. The settlement price is calculated as the average price of trades over a period at the end of the trading session.

Initial margin is the amount of cash or collateral that must be deposited in a futures account before a trade may be made. Initial margin per contract is relatively low and is approximately one day's maximum expected price fluctuation on the total value of the assets covered by the contract.

Maintenance margin is the minimum amount of margin that must be maintained in a futures account. If the margin balance in the account falls below the maintenance margin through daily mark to market from changes in the futures price, the account holder must deposit additional funds to bring the margin balance back up to the *initial* margin amount, or the exchange will close out the futures position. This is different from a margin call in an equity account, which requires investors only to bring the margin back up to the maintenance margin amount. Futures margin requirements are set by the exchange.

To illustrate the daily mark-to-market for futures, consider a contract for 100 ounces of gold that settles on May 15. The initial margin amount is $5,000 and the maintenance margin is $4,700.

On Day 0

- A buyer and seller make a trade at the end of the day at a price of $1,950 per ounce and both parties deposit the initial margin of $5,000 into their accounts.

On Day 1, the settlement price falls to $1,947.50. The seller has gains and the buyer has losses.

- The exchange will credit the seller's account for (1,950 – 1,947.50) × 100 = $250, increasing the margin balance to $5,250.
- The exchange will deduct (1,950 – 1,947.50) × 100 = $250 from the buyer's account, decreasing the margin balance to $4,750. Because $4,750 is more than the maintenance (minimum) margin amount of $4,700, no additional deposit is required.

On Day 2, the settlement price falls to $1,945. Again, the seller has gains and the buyer has losses.

- The exchange will credit the seller's account for (1,947.50 – 1,945) × 100 = $250, increasing their margin balance to $5,500.
- The exchange will deduct (1,947.50 – 1,945) × 100 = $250 from the buyer's account, decreasing the margin balance to $4,500. Because $4,500 is less than the maintenance (minimum) margin amount of $4,700, the buyer must deposit 5,000 – 4,500 = $500 into their margin account to return it to the initial margin amount of $5,000.
- At the end of Day 2, both parties have futures positions at the new settlement price of $1,945 per ounce.

Many futures contracts have **price limits**, which are exchange-imposed limits on how much each day's settlement price can change from the previous day's settlement price. Exchange members are prohibited from executing trades at prices outside these limits. If the equilibrium price at which traders would willingly trade is above the upper limit or below the lower limit, trades cannot take place. Some exchanges have **circuit breakers**; in this case, when a futures price reaches a limit price, trading is suspended for a short period.

MODULE QUIZ 49.1

1. Which type of contract always requires daily marking to market of gains and losses?
   A. Futures contracts only.
   B. Forward contracts only.
   C. Both futures and forward contracts.
2. Compared to a futures contract, an otherwise identical forward contract *most likely* has greater:

A. liquidity.
B. transparency.
C. counterparty risk.

MODULE 49.2: SWAPS AND OPTIONS

Video covering this content is available online.

Swaps

Swaps are agreements to exchange a series of payments on multiple settlement dates over a specified time period (e.g., quarterly payments for two years). At each settlement date, the two payments are netted so that only one net payment is made. The party with the greater liability at each settlement date pays the net difference to the other party.

Swaps trade in a dealer market and the parties are exposed to counterparty credit risk, unless the market has a central counterparty structure to reduce counterparty risk. In this case, margin deposits and mark-to-market payments may also be required to further reduce counterparty risk.

We can illustrate the basics of a swap with a simple fixed-for-floating interest rate swap for two years with quarterly interest payments based on a **notional principal** amount of $10 million. In such a swap, one party makes quarterly payments at a fixed rate of interest (the **swap rate**) and the other makes quarterly payments based on a floating **market reference rate**.

The swap rate is set so that the swap has zero value to each party at its inception. As expectations of future values of the market reference rate change over time, the value of the swap can become positive for one party and negative for the other party.

Consider an interest rate swap with a notional principal amount of $10 million, a fixed rate of 2%, and a floating rate of the 90-day secured overnight financing rate (SOFR). At each settlement date, the fixed-rate payment will be $10 million × 0.02/4 = $50,000. The floating-rate payment at the end of the first quarter will be based on 90-day SOFR at the initiation of the swap, so that both payments are known at the inception of the swap.

If, at the end of the first quarter, 90-day SOFR is 1.6%, the floating-rate payment at the second quarterly settlement date will be $10 million × 0.016 / 4 = $40,000. The fixed-rate payment is again $50,000, so at the end of the second quarter the fixed-rate payer will pay the net amount of $10,000 to the other party.

A company with 2-year floating-rate quarterly-pay note outstanding could enter such a swap as the fixed-rate payer, converting its floating-rate liability into a fixed-rate liability. It now makes fixed interest rate payments and can use the floating-rate payments from the counterparty to make the payments on its floating-rate debt. By entering into the swap, the company can hedge the interest rate risk (uncertainty about future quarterly rates) of their existing floating-rate liability.

As we will see in our reading on swap valuation, a swap can be constructed from a series of forward contracts in which the underlying is a floating rate and the forward price is a fixed rate. Each forward settles on one of the settlement dates of the swap. At each settlement date, the difference between the fixed and the floating rates would result in a net payment, just as with a swap. Often, interest rate forwards settle at the beginning of the quarter rather than the end; the cash flows are the present value equivalents of the end-of-quarter swap payments.

Credit Swaps

One type of swap that is structured a bit differently is a **credit default swap (CDS)**. With a CDS, the protection buyer makes fixed payments on the settlement dates and the protection seller pays only if the underlying (a reference security) has a **credit event**. This could be a bond default, a corporate bankruptcy, or an involuntary restructuring.

When a credit event occurs, the protection seller must pay an amount that offsets the loss in value of the reference security. The fixed payments represent the yield premium on the reference bonds that compensates bondholders for the expected loss from default, the probability of default times the expected loss in the event of default (or other credit event). The protection buyer is essentially paying the yield premium on the reference security for insurance against default.

The holder of a risky bond can hedge its default risk by entering a CDS as the protection buyer. The protection seller receives the default risk premium (credit spread) and takes on the risk of default, resulting in risk exposure similar to that of holding the reference bond.

Options

The two types of options of interest to us here are **put options** and **call options** on an underlying asset. We introduce them using option contracts for 100 shares of a stock as the underlying asset.

A put option gives the buyer the right (but not the obligation) to sell 100 shares at a specified price (the **exercise price**, also referred to as the **strike price**) for specified period of time, the **time to expiration**. The put seller (also called the *writer* of the option) takes on the obligation to purchase the 100 shares at the price specified in the option, if the put buyer exercises the option.

Note the "one-way" nature of options. If the exercise price of the puts is $25 at the expiration of the option, and the shares are trading at or above $25, the put holder will not exercise the option. There is no reason to exercise the put and sell shares at $25 when they can be sold for more than $25 in the market. This is the outcome for any stock price greater than or equal to $25. Regardless of whether the stock price at option expiration is $25 or $1,000, the put buyer lets the option expire, and the put seller keeps the proceeds from the sale.

If the stock price is below $25, the put buyer will exercise the option and the put seller must purchase 100 shares for $25 from the put buyer. On net, the put buyer essentially receives the difference between the stock price at expiration and $25 (times 100 shares).

A call option gives the buyer the right (but not the obligation) to buy 100 shares at a specified price (the exercise price) for a specified period of time. The call seller (writer) takes on the obligation to sell the 100 shares at the exercise price, if the call buyer exercises the option.

LOS 49.b: Determine the value at expiration and profit from a long or a short position in a call or put option.

Unlike forwards, futures, and swaps, options are sold at a price (they do not have zero value at initiation). The price of an option is also referred to as the **option premium**.

At expiration the payoff (value) of a call option to the owner is Max (0, S – X), where S is the price of the underlying at expiration and X is the exercise price of the call option. The Max ( ) function tells us that if S < X at expiration, the option value is zero, that is, it expires worthless and will not be exercised.

At expiration the payoff (value) of a put option to the owner is Max (0, X – S), where S is the price of the underlying at expiration and X is the exercise price of the put option. A put has a zero value at expiration unless X – S is positive.

For the buyer of a put or call option, the profit at expiration is simply the difference between the value (payoff) of the option at expiration and the premium the investor paid for the option.

Because the seller (writer) of an option receives the option premium, the profit to the option seller at expiration is the amount of the premium received minus the option payoff at expiration. The writer loses the payoff at expiration and will have a loss on the option if the payoff is greater than the premium received.

Note the risk exposures of call and put buyers and writers. The buyer of a put or call has no further obligation, so the maximum loss to the buyer is simply the amount they paid for the option. The writer of a call option has exposure to an unlimited loss because the maximum price of the underlying, S, is (theoretically) unlimited, so that the payoff S – X is unlimited. The payoff on a put option is X – S, so if the lower limit on S is zero, the maximum payoff on a put option is the exercise price, X.

Call Option Profits and Losses

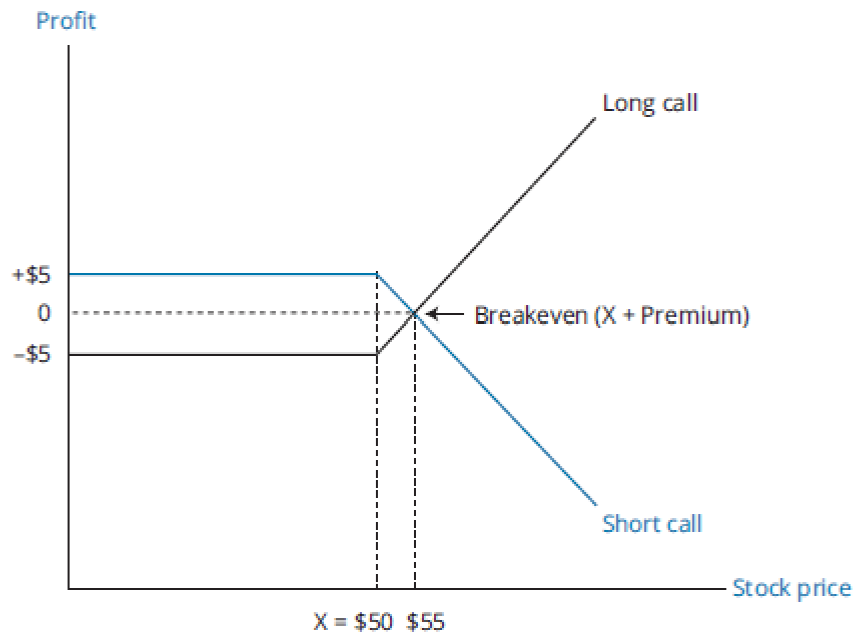
Consider a call option with a premium of $5 and an exercise price of $50. This means the buyer pays $5 to the writer. At expiration, if the price of the stock is less than or equal to the $50 exercise price, the option has zero value, the buyer of the option is out $5, and the writer of the option is ahead $5. When the stock's price exceeds $50, the option starts to gain (breakeven will come at $55, when the value of the stock equals the exercise price plus the option premium. Conversely, as the price of the stock moves upward, the seller of the option starts to lose (negative figures will start at $55, when the value of the stock equals the exercise price plus the option premium).

An illustration of the profit or loss at expiration for the buyer (long) and writer (short) of this call option, as a function of the stock price, is presented in Figure 49.1. This profit/loss diagram indicates the following:

- The maximum loss for the buyer of a call is the $5 premium (at any S ≤ $50).
- The breakeven point for the buyer and seller is the exercise price plus the premium (at S = $55).
- The profit potential to the buyer of the option is unlimited, and, conversely, the potential loss to the writer of the call option is unlimited.
- The call holder will exercise the option whenever the stock's price exceeds the exercise price at the expiration date.
- The greatest profit the writer can make is the $5 premium (at any S ≤ $50).

- The sum of the profits between the buyer and seller of the call option is always zero; thus, trading options is a *zero-sum game*. One party's profits equal the other party's losses.

Figure 49.1: Profit/Loss Diagram for a Call Option



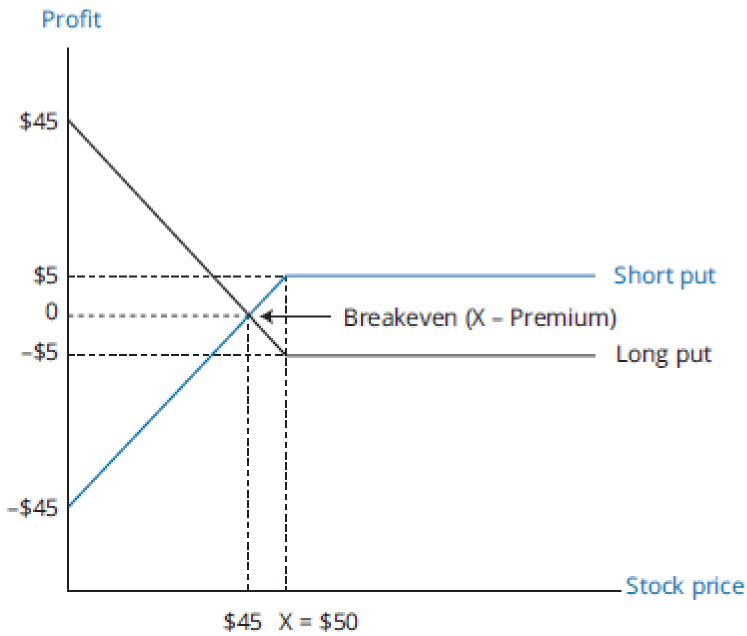
Put Option Profits and Losses

To examine the profits and losses associated with trading put options, consider a put option with a $5 premium and a $50 exercise price. The buyer pays $5 to the writer. When the price of the stock at expiration is greater than or equal to the $50 exercise price, the put has zero value. The buyer of the option has a loss of $5 and the writer of the option has a gain of $5. When the stock price is less than $50, the put option has a positive payoff. Breakeven will come at $45, when the value of the stock equals the exercise price less the option premium. At a stock price below $45, the put seller will have a loss.

Figure 49.2 shows the profit/loss diagram for the buyer (long) and seller (short) of the put option that we have been discussing. This profit/loss diagram illustrates that:

- The maximum loss for the buyer of the put is the $5 premium (at any S ≥ $50).
- The maximum gain to the buyer of the put is limited to the exercise price less the premium ($50 – $5 = $45). The potential loss to the writer of the put is the same amount.
- The breakeven price for the put buyer (seller) is the exercise price minus the option premium ($50 – $5 = $45).
- The maximum profit for the writer is the $5 premium (S ≥ $50).
- The profit (loss) of the put buyer will always equal the loss (profit) of the put writer.

Figure 49.2: Profit/Loss Diagram for a Put Option



EXAMPLE: Option profit calculations

Suppose that both a call option and a put option have been written on a stock with an exercise price of $40. The current stock price is $42, and the call and put premiums are $3 and $0.75, respectively.

Calculate the profit to the long and short positions for both the put and the call with an expiration day stock price of $35 and with a price at expiration of $43.

Answer:

Profit will be computed as ending option value – initial option cost.

Stock at $35:

- Long call: $0 – $3 = –$3. The option has no value, so the buyer loses the premium paid.
- Short call: $3 – $0 = $3. Because the option has no value, the call writer's gain equals the premium received.
- Long put: $5 – $0.75 = $4.25. The buyer paid $0.75 for an option that is now worth $5.
- Short put: $0.75 – $5 = –$4.25. The seller received $0.75 for writing the option, but the option will be exercised so the seller will lose $5 at expiration.

Stock at $43:

- Long call: –$3 + $3 = $0. The buyer paid $3 for the option, and it is now in the money by $3. Hence, the net profit is zero.
- Short call: $3 – $3 = $0. The seller received $3 for writing the option and now faces a –$3 valuation because the buyer will exercise the option, for a net profit of zero.
- Long put: –$0.75 – $0 = –$0.75. The buyer paid $0.75 for the put option and the option now has no value.
- Short put: $0.75 – $0 = $0.75. The seller received $0.75 for writing the option and it has zero value at expiration.

A buyer of puts or a seller of calls has short exposure to the underlying (will profit when the price of the underlying asset decreases). A buyer of calls or a seller of puts has long exposure to the underlying (will profit when the price of the underlying asset increases).

LOS 49.c: Contrast forward commitments with contingent claims.

A **forward commitment** is a legally binding promise to perform some action in the future. Forward commitments include forward contracts, futures contracts, and most swaps.

A **contingent claim** is a claim (to a payoff) that depends on a particular event. Options are contingent claims; the event is the price of the underlying being above or below the exercise price. Credit default swaps are also considered contingent claims because the payment by the protection seller depends on a credit event occurring.

MODULE QUIZ 49.2

1. Interest rate swaps are:
   A. highly regulated.
   B. equivalent to a series of forward contracts.
   C. contracts to exchange one asset for another.
2. A call option is:
   A. the right to sell at a specific price.
   B. the right to buy at a specific price.
   C. an obligation to buy at a certain price.
3. At expiration, the exercise value of a put option is:
   A. positive if the underlying asset price is less than the exercise price.
   B. zero only if the underlying asset price is equal to the exercise price.
   C. negative if the underlying asset price is greater than the exercise price.
4. At expiration, the exercise value of a call option is:
   A. the underlying asset price minus the exercise price.
   B. the greater of zero or the exercise price minus the underlying asset price.
   C. the greater of zero or the underlying asset price minus the exercise price.
5. An investor writes a put option with an exercise price of $40 when the stock price is $42. The option premium is $1. At expiration the stock price is $37. The investor will realize:
   A. a loss of $2.
   B. a loss of $3.
   C. a profit of $1.
6. Which of the following derivatives is a forward commitment?
   A. Stock option.
   B. Interest rate swap.
   C. Credit default swap.

KEY CONCEPTS

LOS 49.a

Forward contracts obligate one party to buy, and another to sell, a specific asset at a specific price at a specific time in the future.

Futures contracts are much like forward contracts, but are exchange-traded, liquid, and require daily settlement of any gains or losses.

A call option gives the holder the right, but not the obligation, to buy an asset at a specific price at some time in the future.

A put option gives the holder the right, but not the obligation, to sell an asset at a specific price at some time in the future.

In an interest rate swap, one party pays a fixed rate and the other party pays a floating rate, on a given amount of notional principal. Swaps are equivalent to a series of forward contracts based on a floating rate of interest.

A credit default swap is a contract in which the protection seller provides a payment if a specified credit event occurs.

LOS 49.b

Call option value at expiration is Max(0, underlying price minus exercise price) and profit or loss is Max(0, underlying price minus exercise price) minus the option cost (premium paid).

Put value at expiration is Max(0, exercise price minus underlying price) and profit or loss is Max(0, exercise price minus underlying price) minus the option cost.

A call buyer (call seller) benefits from an increase (decrease) in the value of the underlying asset.

A put buyer (put seller) benefits from a decrease (increase) in the value of the underlying asset.

LOS 49.c

A forward commitment is an obligation to buy or sell an asset or make a payment in the future. Forward contracts, futures contracts, and most swaps are forward commitments.

A contingent claim is a derivative that has a future payoff only if some future event takes place (e.g., asset price is greater than a specified price). Options and credit derivatives are contingent claims.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 49.1

1. **A** Futures contracts are marked to market daily. Forward contracts typically are not, but could be if there is central clearing party.
(LOS 49.a)

2. **C** Forward contracts involve counterparty risk; futures contracts trade through a clearinghouse. Because futures contracts trade on organized exchanges, they have greater liquidity and transparency than forward contracts. (LOS 49.a)

Module Quiz 49.2

1. **B** A swap is an agreement to buy or sell an underlying asset periodically over the life of the swap contract. It is equivalent to a series of forward contracts. (LOS 49.a)

2. **B** A call gives the owner the right to call an asset away (buy it) from the seller. (LOS 49.a)

3. **A** The exercise value of a put option is positive at expiration if the underlying asset price is less than the exercise price. Its exercise value is zero if the underlying asset price is greater than or equal to the exercise price. The exercise value of an option cannot be negative because the holder can allow it to expire unexercised. (LOS 49.b)

4. **C** If the underlying asset price is greater than the exercise price of a call option, the value of the option is equal to the difference. If the underlying asset price is less than the

exercise price, a call option expires with a value of zero. (LOS 49.b)

5. **A** Because the stock price at expiration is less than the exercise price, the buyer of the put option will exercise it against the writer. The writer will have to pay $40 for the stock and can only sell it for $37 in the market. However, the put writer collected the $1 premium for writing the option, which reduces the net loss to $2. (LOS 49.b)

6. **C** This type of custom contract is a forward commitment. (LOS 49.c)

READING 50

DERIVATIVE BENEFITS, RISKS, AND ISSUER AND INVESTOR USES

MODULE 50.1: USES, BENEFITS, AND RISKS OF DERIVATIVES

Video covering this content is available online.

LOS 50.a: Describe benefits and risks of derivative instruments.

Advantages of Derivatives

Derivative instruments offer several potential advantages over cash market transactions, including the following:

Ability to change risk allocation, transfer risk, and manage risk

We have discussed these benefits in our introduction to derivative contracts. Some examples of ways that risk exposures can be altered using derivatives, without any cash market securities transactions, are:

- A portfolio manager can increase or decrease exposure to the risk and return of a market index.
- A manufacturer can hedge the exchange rate risk of anticipated receipts or payments.
- The issuer of a floating-rate note can change that exposure to a fixed-rate obligation.

Derivative instruments can be used to create risk exposures that are not available in cash markets. Consider the following examples of changing an existing risk profile:

- The owner of common stock can buy puts that act as a floor on the sale price of their shares, reducing the downside risk of the stock by paying the cost of the puts.
- An investor can acquire the upside potential of an asset without taking on its downside risk by buying call options.

Information discovery

Derivatives prices and trading provide information that cash market transactions do not.

- Options prices depend on many things we can observe (interest rates, price of the underlying, time to expiration, and exercise price) and one we cannot, the expected future price volatility of the underlying. We can use values of the observable variables, together with current market prices of derivatives, to estimate the future price volatility of the underlying that market participants expect.

- Futures and forwards can be used to estimate expected prices of their underlying assets.
- Interest rate futures across maturities can be used to infer expected future interest rates and even the number of central bank interest rate changes over a future period.

Operational advantages

Compared to cash markets, derivatives markets have several operational advantages. Operational advantages of derivatives include greater ease of short selling, lower transaction costs, greater potential leverage, and greater liquidity.

- *Ease of short sales*. Taking a short position in an asset by selling a forward or a futures contract may be easy to do. Difficulty in borrowing an asset and restrictions on short sales may make short positions in underlying assets problematic or more expensive.
- *Lower transaction costs.* Transaction costs can be significantly lower with commodities derivatives, where transportation, storage, and insurance add costs to transactions in physical commodities. Entering a fixed-for-floating swap to change a floating-rate exposure to fixed rate is clearly less costly than retiring a floating-rate note and issuing a fixed-rate note.
- *Greater leverage*. The cash required to take a position in derivatives is typically much less than for an equivalent exposure in the cash markets.
- *Greater liquidity.* The low cash requirement for derivatives transactions makes very large transactions easier to handle.

Improved market efficiency

Low transaction costs, greater liquidity and leverage, and ease of short sales all make it less costly to exploit securities mispricing through derivatives transactions and improve the efficiency of market prices.

Risks of Derivatives

Implicit leverage

The implicit leverage in derivatives contracts gives them much more risk than their cash market equivalents. Just as we have shown regarding the leverage of an equity investment on margin, a lower cash requirement to enter a trade increases leverage. Futures margins, according to the CME Group, are typically in the 3% to 12% range, indicating leverage of 8:1 to 33:1. With required cash margin of 4%, a 1% decrease in the futures price decreases the cash margin by 25%.

A lack of transparency in derivatives contracts and securities that combine derivative and cash market exposures (structured securities) may lead to situations in which the purchasers do not well understand the risks of derivatives or securities with embedded derivatives.

Basis risk

Basis risk arises when the underlying of a derivative differs from a position being hedged with the derivative. For a manager with a portfolio of 50 large-cap U.S. stocks, selling a forward with the S&P 500 Index as the underlying (in an amount equal to the portfolio value) would hedge portfolio risk, but would not eliminate it because of the possibility that returns on the portfolio and returns on the index may differ over the life of the forward. Basis risk also arises in a situation where an investor's horizon and the settlement date of the hedging derivative differ, such as hedging the value of a corn harvest that will occur on September 15 by selling corn futures that settle on October 1. Again the hedge may be effective but will not be perfect, and the corn producer is said to have basis risk.

Liquidity risk

Derivative instruments have a special type of liquidity risk when the cash flows from a derivatives hedge do not match the cash flows of the investor positions. As an example, consider a farmer who sells wheat futures to hedge the value of her wheat harvest. If the future price of wheat increases, losses on the short position essentially offset the extra income from the higher price that will come at harvest (as intended with a hedge), but these losses may also cause the farmer to get margin calls during the life of the contract. If the farmer does not have the cash (liquidity) to meet the margin calls, the position will be closed out and the value of the hedge will be lost.

Counterparty credit risk

We have discussed counterparty credit risk previously. Here we note additionally that different derivatives and positions have important differences in the existence or amount of counterparty risk. The seller of an option faces no counterparty credit risk; once the seller receives the option premium there is no circumstance in which the seller will be owed more at settlement. On the other hand, the buyer of an option will be owed money at settlement if the option is in the money; thus, the buyer faces counterparty credit risk. In contrast, both the buyer and seller of a forward on an underlying asset may face counterparty credit risk.

In futures markets the deposit of initial margin, the daily mark-to-market, and the guarantee of the central clearinghouse all reduce counterparty risk. With forwards there may be no guarantees, or the terms of the forward contract may specify margin deposits, a periodic mark-to-market, and a central clearing party to mitigate credit risk.

Systemic risk

Widespread impact on financial markets and institutions may arise from excessive speculation using derivative instruments. Market regulators attempt to reduce systemic risk though regulation, for example the central clearing requirement for swap markets to reduce counterparty credit risk.

LOS 50.b: Compare the use of derivatives among issuers and investors.

Derivatives Use by Issuers

Corporate users of derivative instruments are considered issuers of derivatives. A non-financial corporation may have risks associated with changes in asset and liability values as well as earnings volatility from changes in various underlying securities or interest rates. Some examples are:

- A corporation may have income in a foreign currency and hedge the exchange rate risk with forwards to smooth earnings reported in their domestic currency.
- A corporation may use fair value reporting for its fixed-rate debt, and that value changes as interest rates change. By entering an interest rate swap as the floating-rate payer, the corporation has essentially converted the fixed-rate liability to a floating-rate liability that has much lower duration so that its balance sheet value is less sensitive to changes in interest rates.
- A corporation with a commodity-like product may carry its inventory at fair market value, leading to fluctuations in the value reported on the balance sheet over time as the market price of their product changes. By selling forward contracts on an underlying that matches well with their product, the firm will have gains or losses on the forwards that offset decreases or increases in reported inventory value. With the market value of the forward position also reported on the balance sheet, total assets will have less variation from changes in the market price of their product.

Accounting rules may permit **hedge accounting**. Hedge accounting allows firms to recognize the gains and losses of qualifying derivative hedges at the same time they recognize the corresponding changes in the values of assets or liabilities being hedged. Issuer hedges against the effects of a changing price or value of a derivative's underlying are classified by their purpose.

- A hedge of the domestic currency value of future receipts in a foreign currency using forwards is termed a **cash flow hedge**. A swap that converts a floating-rate liability to a fixed-rate liability is also considered a cash flow hedge (cash flows for interest payments are more certain).
- A **fair value hedge** is one that reduces (offsets) changes in the values of the firm's assets or liabilities. Our examples of a firm that uses derivatives to hedge against changes in the balance sheet value of its inventory, and a firm that uses an interest rate swap to decrease the volatility of debt values on its balance sheet, are considered fair value hedges.
- A **net investment hedge** is one that reduces the volatility of the value of the equity of a company's foreign subsidiary reported on its balance sheet. Foreign currency forwards or futures can be used to hedge changes in the reported value of the subsidiary's equity due to changes in exchange rates.

Derivatives Use by Investors

As we have seen, investors can hedge, modify, or increase their exposure to the risk of an underlying asset or interest rate with derivatives positions, either forward commitments or contingent claims. Some examples are:

- An investor can buy silver forwards to gain exposure to the price of silver, with no or low funds initially required.
- An investor can increase the duration of their bond portfolio by entering an interest rate swap as the floating-rate payer/fixed-rate receiver, which is similar to issuing floating-rate debt and buying a fixed-rate bond with the proceeds.
- An equity portfolio manager can modify their market risk exposure temporarily at low cost, increasing it by buying equity index futures or decreasing it by selling equity index

futures. Alternatively, the portfolio manager could decrease downside risk and preserve upside potential by buying puts on an equity index.

MODULE QUIZ 50.1

1. Which of the following *most* accurately describes a risk of derivative instruments?
   A. Derivatives make it easier for market participants to take short positions.
   B. The underlying of a derivative might not fully match a position being hedged.
   C. Volatility in underlying asset prices is implied by the prices of options on those assets.
2. Uses of derivatives by investors *most likely* include:
   A. hedging against price risk for inventory held.
   B. modifying the risk exposure of a securities portfolio.
   C. stabilizing the balance sheet value of a foreign subsidiary.

KEY CONCEPTS

LOS 50.a

Advantages of derivatives include the ability to change or transfer risk; information discovery about the expected prices or volatility of underlying assets or interest rates; operational advantages such as ease of short sales, low transaction costs, and greater leverage and liquidity; and improved market efficiency.

Risks of derivatives include implicit leverage, basis risk from inexact hedges, liquidity risk from required cash flows, counterparty credit risk, and systemic risk for financial markets.

LOS 50.b

Derivatives uses by issuers include managing risks associated with changes in asset and liability values as well as earnings volatility from changes in various underlying securities or interest rates.

Derivatives uses by investors include hedging, modifying, or increasing their exposure to the risk of an underlying asset or interest rate.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 50.1

1. **B** Basis risk arises when the underlying of a derivative differs from a position being hedged. Ease of taking short positions with derivatives compared to their underlying assets, and the information about implied volatility that is revealed by option prices, are two of the advantages of derivative instruments. (LOS 50.a)

2. **B** Modifying the risk exposure of a securities portfolio is an example of derivatives use by investors. Hedging against price risk for inventory and stabilizing the balance sheet value of a foreign subsidiary are examples of derivatives use by issuers. (LOS 50.b)

READING 51

ARBITRAGE, REPLICATION, AND THE COST OF CARRY IN PRICING DERIVATIVES

MODULE 51.1: ARBITRAGE, REPLICATION, AND CARRYING COSTS

Video covering this content is available online.

LOS 51.a: Explain how the concepts of arbitrage and replication are used in pricing derivatives.

In contrast to valuing risky assets as the (risk-adjusted) present value of expected future cash flows, the valuation of derivative securities is based on a **no-arbitrage condition**. *Arbitrage* refers to a transaction in which an investor purchases one asset or portfolio of assets at one price and simultaneously sells an asset or portfolio of assets that has the same future payoffs, regardless of future events, at a higher price, realizing a risk-free gain on the transaction.

While arbitrage opportunities may be rare, the reasoning is that when they do exist, they will be exploited rapidly. Therefore, we can use a no-arbitrage condition to determine the current value of a derivative, based on the known value of a portfolio of assets that has the same future payoffs as the derivative, regardless of future events. Because there are transaction costs of exploiting an arbitrage opportunity, small differences in price may persist when the arbitrage gain is less than the transaction costs of exploiting it.

We can illustrate no-arbitrage pricing with a 1-year forward contract, with a forward price of $F_0(1)$, on an Acme share that pays no dividends and is trading at a current price, S_0, of $30.

Consider two strategies to own an Acme share at t =1:

- *Portfolio 1*: Buy a pure discount bond with a yield of 5% that pays $F_0(1)$ at t = 1. The current cost of the bond is $F_0(1)/1.05$. Additionally, enter a forward contract on one Acme share at $F_0(1)$ as the buyer. The forward has a zero cost, so the cost of Portfolio 1 is $F_0(1)/1.05$.

  At t = 1 the bond pays $F_0(1)$, which will buy an Acme share at the forward price, so that the payoff on Portfolio 1 is the value of one share at t = 1, S_1.

- *Portfolio 2:* Buy a share of Acme at S_0 = 30 and hold it for one year. Cost at t = 0 is $30.

  At t = 1 the value of the Acme share is S_1 and this is the payoff for Portfolio 2.

The no-arbitrage condition (law of one price) requires that two portfolios with the same payoff in the future for any future value of Acme have the same cost today. Because our two portfolios have a payoff of S_1, they must have the same cost at t = 0 to prevent arbitrage. That is, $F_0(1)/1.05$ = \$30, so we can solve for the no-arbitrage forward price as $F_0(1)$ = 30(1.05) = 31.50.

To better understand the no-arbitrage condition, we will consider two situations in which the forward price is not at its no-arbitrage value: $F_0(1) > 31.50$ and $F_0(1) < 31.50$.

- If the forward contract price is 32 ($F_0(1) > 31.50$), the profitable arbitrage is to sell the forward (because the forward price is "too high") and buy a share of stock. At t = 1, deliver the share under the forward contract and receive 32, for a return of 32/30 – 1 = 6.67%, which is higher than the risk-free rate.

  We can also view this transaction as borrowing 30 at the risk-free rate (5%) to buy the Acme share at t = 0, and at t = 1 paying 31.50 to settle the loan. The share delivered under the forward has a contract price of 32, so the arbitrageur has an arbitrage profit of 32 – 31.50 = 0.50 with no risk and no initial cost.

- If the forward contract price is 31 ($F_0(1) < 31.50$), the profitable arbitrage is to buy the forward and sell short an Acme share at t = 0. The proceeds of the short sale, 30, can be invested at the risk-free rate to produce 30(1.05) = 31.50 at t = 1. The forward contract requires the purchase of a share of Acme for 31, which the investor can return to close out the short position. The profit to an arbitrageur is 31.50 – 31 = 0.50. With no cash investment at t = 0, the investor receives an arbitrage profit of 0.50 at t = 1.

When the forward price is "too high," the arbitrage is to sell the forward and buy the underlying asset. When the forward price is "too low," the arbitrage is to buy the forward and sell (short) the underlying asset. In either case, the actions of arbitrageurs will move the forward price toward its no-arbitrage level until arbitrage profits are no longer possible.

Replication refers to creating a portfolio with cash market transactions that has the same payoffs as a derivative for all possible future values of the underlying.

Our arbitrage example for Acme forwards will serve to illustrate replication.

A long forward on an Acme share can be replicated by borrowing 30 at 5% to purchase an Acme share, and repaying the loan on the settlement date of the forward. At settlement (t = 1), the payoff on the replication is S_1 – 30(1.05) =
S_1 – 31.50 (value of one share minus the repayment of the loan), the same as the payoff on a long forward at 31.50, for any value of Acme shares at settlement.

A short forward on an Acme share can be replicated by shorting an Acme share and investing the proceeds of 30 at 5%. At settlement the investor receives 31.50 from the investment of short sale proceeds, and must buy a share of Acme for S_1. The payoff on the replicating portfolio is 31.50 – S_1, the same as the payoff on a short forward at 31.50, for any value of Acme shares at settlement.

These replications allow us to calculate the no-arbitrage forward price of an asset, just as we did in our example using Acme shares. Because our replicating portfolio for a long forward has the same payoff as a long forward at time = T, the payoff at settlement on a portfolio that is long the replicating portfolio and short the forward must be zero to prevent arbitrage. For this strategy, when the forward is priced at its no-arbitrage value the payoff at time = T is:

$S_T - S_0(1 + Rf)^T - [S_T - F_0(T)] = 0$
so that $S_0(1 + Rf)^T + F_0(T) = 0$ and $F_0(T) = S_0(1 + Rf)^T$.

For a portfolio that is short the replicating portfolio and long the forward, the payoff at time T is:

$S_0(1 + Rf)^T - S_T + [S_T - F_0(T)] = 0$
so that $S_0(1 + Rf)^T - F_0(T) = 0$ and $F_0(T) = S_0(1 + Rf)^T$.

The forward price that will prevent arbitrage is $S_0(1 + Rf)^T$, just as we found in our example of a forward contract on an Acme share.

LOS 51.b: Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

When we derived the no-arbitrage forward price for an asset as $F_0(T) = S_0(1 + Rf)^T$, we assumed there were no benefits of holding the asset and no costs of holding the asset, other than the opportunity cost of the funds to purchase the asset (the risk-free rate of interest).

Any additional costs or benefits of holding the underlying asset must be accounted for in calculating the no-arbitrage forward price. There may be additional costs of owning an asset, especially with commodities, such as storage and insurance costs. For financial assets, these costs are very low and not significant.

There may also be monetary benefits to holding an asset, such as dividend payments for equities and interest payments for debt instruments. Holding commodities may have non-monetary benefits, referred to as **convenience yield**. If an asset is difficult to sell short in the market, owning it may convey benefits in circumstances where selling the asset is advantageous. For example, a shortage of the asset may drive prices up temporarily, making sale of the asset in the short term profitable.

We denote the present value of any costs of holding the asset from time 0 to settlement at time *T* (e.g., storage, insurance, spoilage) as PV_0(cost), and the present value of any cash flows from the asset or convenience yield over the holding period as PV_0(benefit).

Consider first a case where there are storage costs of holding the asset, but no benefits. For an asset with no costs or benefits of holding the asset, we established the no-arbitrage forward price as $S_0(1 + Rf)^T$, the cost of buying and holding the underlying asset until time T. When there are storage costs to hold the asset until time T, an arbitrageur must both buy the asset and pay the present value of storage costs at $t = 0$. This increases the no-arbitrage price of a 1-year forward to $[S_0 + PV_0(\text{cost})](1 + Rf)^T$. Here we see that *costs of holding an asset increase its no-arbitrage forward price*.

Next consider a case where holding the asset has benefits, but no costs. Returning to our example of a 1-year forward on a share of Acme stock trading at 30, now consider the costs of buying and holding an Acme share that pays a dividend of $1 during the life of the forward contract. In this case, an arbitrageur can now borrow the present value of the dividend (discounted at Rf), and repay that loan when the dividend is received. The cost to buy and hold Acme stock with an annual dividend of $1 is $[30 - PV_0(1)](1.05) = 30(1.05) - 1$. This illustrates that *benefits of holding an asset decrease its no-arbitrage forward price.*

The no-arbitrage price of a forward on an asset that has both costs and benefits of holding the asset is simply $[S_0 + PV_0 \text{ (costs)} - PV_0\text{(benefit)}](1 + Rf)^T$.

We can also describe these relationships when costs and benefits are expressed as continuously compounded rates of return. Recall from Quantitative Methods that given a stated annual rate of *r* with continuous compounding, the effective annual return is $e^r - 1$, and the relationships between present and future values of S for a 1-year period are $FV = Se^r$ and $PV = Se^{-r}$. For a period of T years, $FV = Se^{rT}$ and $PV = Se^{-rT}$. With continuous compounding the following relationships hold:

- With no costs or benefits of holding the underlying asset, the no-arbitrage price of a forward that settles at time T is S_0e^{rT}, where *r* is the stated annual risk-free rate with continuous compounding.
- With storage costs at a continuously compounded annual rate of *c*, the no-arbitrage forward price until time T is $S_0e^{(r+c)T}$.
- With benefits, such as a dividend yield, expressed at a continuously compounded annual rate of *b*, the no-arbitrage forward price is until time T is $S_0e^{(r+c-b)T}$.

EXAMPLE: **No-arbitrage price with continuous compounding**

Consider a stock index trading at 1,550 with a dividend yield of 1.3% (continuously compounded rate) when the risk-free rate is 3% (continuously compounded rate). Calculate the no-arbitrage 6-month forward price of the stock index.

Answer:

The no-arbitrage price of a long 6-month forward is $1{,}550 \times e^{(0.03 - 0.013)(0.5)} = 1{,}563.23$.

The **net cost of carry** (or simply **cost of carry** or **carry**) is the benefits of holding the asset minus the costs of holding the asset (including the opportunity cost of funds, Rf). When the benefits (cash flow yield or convenience yield) exceed the costs (including the opportunity cost of funds) of holding the asset, the forward price will be less than the spot price.

Forward Contracts on Currencies

Recall from Economics that we defined the no-arbitrage price of a forward on a currency as the forward price that satisfies the equality:

$$\text{forward exchange rate (p/b)} = \frac{1+\text{interest rate}_{\text{price currency}}}{1+\text{interest rate}_{\text{base currency}}} \times \text{spot exchange rate}$$

We can use this no-arbitrage forward rate to examine how an arbitrage profit can be made when the exchange rate in a forward contract is greater or less than the no-arbitrage forward exchange rate. The forward exchange rate depends on the spot exchange rate and the *difference* between the interest rates on the base and price currencies.

Consider a situation at $t = 0$ where the risk-free rate in euros is 3%, the risk-free rate in U.S. dollars is 2%, and the current USD/EUR exchange rate is 1.10. We will examine the arbitrage transactions that establish this relationship by looking at the trades for an investor based in the United States that seeks to profit from the higher interest rate on euros. The investor borrows 100 USD for one year at 2%, exchanges the USD for euros, invests the euros for one year at 3%, and then exchanges the resulting euros for USD. At the end of one year the arbitrageur will have $100/1.10 \times 1.03 = 93.64$ euros and owe $100(1.02) = 102$ USD.

As these transactions have no net cost, there should be no gain from this transaction relative to simply investing the USD for one year at 2%. If this is the case, the 93.64 euros should equal 102 USD. This is the case if the exchange rate at the end of the year is 102/93.64, which equals a USD/EUR exchange rate of 1.0893. This is the no-arbitrage forward rate. From the formula we saw in Economics we can arrive at the same solution by $1.10 \times (1.02/1.03) = 1.0893$.

If the arbitrageur has a forward contract to buy USD with a price of $1/1.0893 = 0.9180$ euros, he can exchange the 93.64 euros for $93.64/0.9180 = 102$ USD, which is the amount owed on the original loan of 100 USD. The depreciation of the euro in the forward price just offsets the higher euro interest, and the arbitrage transaction returns zero. With a forward exchange rate greater than 1.0893, the arbitrage would have a profit, and with a forward exchange rate less than 1.0893, an arbitrageur could profit from the opposite transactions.

If we convert the effective annual rates to equivalent stated annual rates with continuous compounding, we get $R_{USD} = \ln 1.02 = 1.98\%$ and $R_{EUR} = \ln 1.03 = 2.96\%$. In this case we can say: Forward exchange rate $= 1.10 \times e^{(0.0198-0.0296)} = 1.0893$.

MODULE QUIZ 51.1

1. Derivatives pricing models use the risk-free rate to discount future cash flows because these models:
   A. are based on portfolios with certain payoffs.
   B. assume that derivatives investors are risk-neutral.
   C. assume that risk can be eliminated by diversification.
2. Arbitrage prevents:
   A. market efficiency.
   B. earning returns higher than the risk-free rate of return.
   C. two assets with identical payoffs from selling at different prices.
3. The underlying asset of a derivative is *most likely* to have a convenience yield when the asset:
   A. is difficult to sell short.
   B. pays interest or dividends.
   C. must be stored and insured.
4. An investor can replicate a forward on a stock that pays no dividends by:
   A. selling the underlying short and investing the proceeds at the risk-free rate.
   B. buying the underlying in the spot market and holding it.
   C. borrowing at the risk-free rate to buy the underlying.
5. The forward price of a commodity will *most likely* be equal to the current spot price if the:
   A. convenience yield equals the storage costs as a percentage.
   B. convenience yield is equal to the risk-free rate plus storage costs as a percentage.
   C. risk-free rate equals the storage costs as a percentage minus the convenience yield.

KEY CONCEPTS

LOS 51.a

Valuation of derivative securities is based on a no-arbitrage condition. When the forward price is too high, the arbitrage is to sell the forward and buy the underlying asset. When the forward price is too low, the arbitrage is to buy the forward and sell short the underlying asset. Arbitrage will move the forward price toward its no-arbitrage level.

Replication refers to creating a portfolio with cash market transactions that has the same payoffs as a derivative for all possible future values of the underlying. Replication allows us to calculate the no-arbitrage forward price of an asset.

LOS 51.b

Assuming no costs or benefits of holding the underlying asset, the forward price that will prevent arbitrage is the spot price compounded at the risk-free rate over the time until expiration.

The cost of carry is the benefits of holding the asset minus the costs of holding the asset.

Greater costs of holding an asset increase its no-arbitrage forward price.

Greater benefits of holding an asset decrease its no-arbitrage forward price.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 51.1

1. **A** Derivatives pricing models use the risk-free rate to discount future cash flows because they are based on arbitrage relationships that are theoretically riskless. (LOS 51.a)

2. **C** Arbitrage forces two assets with the same expected future value to sell for the same current price. (LOS 51.a)

3. **A** Convenience yield refers to nonmonetary benefits from holding an asset. One example of convenience yield is the advantage of owning an asset that is difficult to sell short when it is perceived to be overvalued. Interest and dividends are monetary benefits. Storage and insurance are carrying costs. (LOS 51.b)

4. **C** Borrowing S_0 at Rf to buy the underlying asset at S_0 has a zero cost and pays the spot price of the underlying asset minus the loan repayment of at time = T of $S_0(1 + Rf)^T$, which is the same payoff as a long forward at $F_0 = S_0(1 + Rf)^T$, the no-arbitrage forward price. (LOS 51.a)

5. **B** When the opportunity cost of funds (Rf) and storage costs just offset the benefits of holding the commodity, the no-arbitrage forward price is equal to the current spot price of the underlying commodity. (LOS 51.b)

READING 52

PRICING AND VALUATION OF FORWARD CONTRACTS AND FOR AN UNDERLYING WITH VARYING MATURITIES

MODULE 52.1: FORWARD CONTRACT VALUATION

Video covering this content is available online.

LOS 52.a: Explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration.

Consider a forward contract that is initially priced at its no-arbitrage value of $F_0(T) = S_0(1 + Rf)^T$. At initiation, the value of such a forward is:
$V_0(T) = S_0 - F_0(T)\ (1 + Rf)^{-T} = 0$.

At any time during its life, the value of the forward contract to the buyer will be $V_t(T) = S_t - F_0(T)\ (1 + Rf)^{-(T-t)}$. This is simply the current spot price of the asset minus the present value of the forward contract price.

This value can be realized by selling the asset short at S_t and investing $F_0(T)\ (1 + Rf)^{-(T-t)}$ in a pure discount bond at Rf. These transactions end any exposure to the forward; at settlement, the proceeds of the bond will cover the cost of the asset at the forward price, and the asset can be delivered to cover the short position.

At expiration, time T, the value of a forward to the buyer is = $S_T - F_0(T)(1 + Rf)^{-(T-T)} = S_T - F_0(T)$. The long buys an asset valued at S_T for the forward contract price of $F_0(T)$, gaining if $S_T > F_0(T)$, losing if $S_T < F_0(T)$. If the forward buyer has a gain, the forward seller has an equal loss, and vice versa.

In the more general case, when there are costs and benefits of holding the underlying asset, the value of a forward to the buyer at time = $t < T$ is:

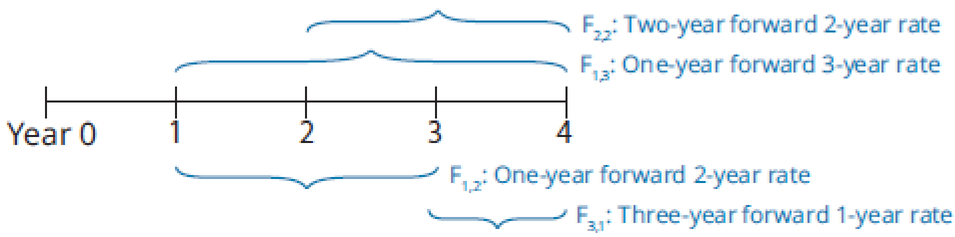
$$V_t(T) = [S_t + PV_t\ (costs) - PV_t(benefit)] - F_0(T)\ (1 + Rf)^{-(T-t)}$$

LOS 52.b: Explain how forward rates are determined for an underlying with a term structure and describe their uses.

Forward rates are yields for future periods. The rate of interest on a 1-year loan to be made two years from today is a forward rate.

The notation for forward rates must identify both the length of the loan period and how far in the future the money will be loaned (or borrowed). 1y1y or $F_{1,1}$ is the rate for a 1-year loan one year from now; 2y1y or $F_{2,1}$ is the rate for a 1-year loan to be made two years from now; the 2-year forward rate three years from today is 3y2y or $F_{3,2}$; and so on.

Exhibit 52.1: Forward Rates



For money market rates the notation is similar, with 3m6m denoting a 6-month rate three months in the future.

Recall that spot rates are zero-coupon rates. We will denote the YTM (with annual compounding) on a zero-coupon bond maturing in *n* years as Z_n.

An **implied forward rate** is the forward rate for which the following two strategies have the same yield over the total period:

- Investing from $t = 0$ to the forward date, and rolling over the proceeds for the period of the forward.
- Investing from $t = 0$ until the end of the forward period.

As an example, lending for two years at Z_2 would produce the same ending value as lending for one year at Z_1 and, at $t = 1$, lending the proceeds of that loan for one year at $F_{1,1}$. That is, $(1 + Z_2)^2 = (1 + Z_1)(1 + F_{1,1})$. When this condition holds, $F_{1,1}$ is the implied (no-arbitrage) forward rate.

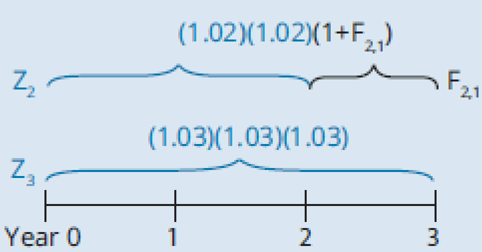
EXAMPLE: **Implied forward rate**

Consider two zero-coupon bonds, one that matures in two years and one that matures in three years, when $Z_2 = 2\%$ and $Z_3 = 3\%$. Calculate the implied 1-year forward rate two years from now, $F_{2,1}$.

Answer:

As illustrated in Exhibit 52.2, lending for three years at Z_3 should be equivalent to lending for two years at Z_2 and then for the third year at $F_{2,1}$.

Exhibit 52.2: Implied Forward Rate



Lending \$100 for two years at Z_2 (2%) results in a payment of $\$100(1.02)^2 = \$104.04$ at $t = 2$, while lending \$100 for three years at Z_3 (3%) results in a payment of $\$100(1.03)^3 = \$109.27$. The forward interest rate $F_{2,1}$ must be 109.27/104.04 – 1 = 5.03%, the implied forward rate from $t = 2$ to $t = 3$.

An example of an interest rate derivative is a **forward rate agreement (FRA)**, in which the fixed-rate payer (long) will pay the forward rate on a notional amount of principal at a future date, and the floating-rate payer will pay a future reference rate times that same amount of principal. In practice, only the net amount is exchanged.

Consider a 3-month forward on a 6-month MRR (F_{3m6m}) with a notional principal of \$1 million. At settlement in three months, the buyer receives (or pays) the present value of (realized 6-month MRR – 1%)/2 × \$1 million.

We divide by 2 because MRRs are typically annualized rates. We take the present value of the difference in interest because the settlement payment is at the beginning of the 6-month period, whereas the interest savings would be at the end of the period.

Assume that the current 3-month MRR is 1.0% and 9-month MRR is 1.2%. Adjusting for periodicity, the no-arbitrage condition for the value of F_{3m6m} is:

$$1 + 0.012\left(\frac{9}{12}\right) = \left[1 + 0.01\left(\frac{3}{12}\right)\right]\left[1 + F_{3m,6m}\left(\frac{6}{12}\right)\right]$$

The implied forward rate, F_{3m6m}, as an annualized rate, is:

$$F_{3m,6m} = \left[\frac{1 + 0.012\left(\frac{9}{12}\right)}{1 + 0.01\left(\frac{3}{12}\right)} - 1\right] \times \frac{12}{6} = 0.013$$

Now let's examine the payoff to the fixed-rate payer in an F_{3m6m} FRA with a notional principal of \$1 million when the 6-month MRR three months from now is 1.5%. Because the realized 6-month MRR is greater than the forward rate, the fixed-rate payer (floating-rate receiver) will have a gain.

The payment to the fixed-rate payer is the present value (discounted at 6-month MRR) of the interest differential between two 6-month loans, one at 1.3% and one at 1.5% (both annualized rates). The fixed-rate payer in the FRA receives:

$$\$1 \text{ million} \times \left(\frac{0.015 - 0.013}{2}\right)\left(\frac{1}{\frac{0.015}{2}}\right) = \$992.56.$$

FRAs are used primarily by financial institutions to manage the volatility of their interest-sensitive assets and liabilities. FRAs are also the building blocks of interest rate swaps over multiple periods. An FRA is equivalent to a single-period swap. Multiple-period swaps are used primarily by investors and issuers to manage interest rate risk.

MODULE QUIZ 52.1

1. Two parties agree to a forward contract to exchange 100 shares of a stock one year from now for \$72 per share. Immediately after they initiate the contract, the price of the underlying stock increases to \$74 per share. This share price increase represents a gain for:
   A. the buyer.
   B. the seller.

C. neither the buyer nor the seller.

2. The forward rate $F_{2,3}$ represents the interest rate on a loan for the period from:
   A. year 2 to year 3.
   B. year 2 to year 5.
   C. year 3 to year 5.
3. Given zero-coupon bond yields for 1, 2, and 3 years, an analyst can *least likely* derive an implied:
   A. 1-year forward 1-year rate.
   B. 2-year forward 1-year rate.
   C. 2-year forward 2-year rate.

KEY CONCEPTS

LOS 52.a

The value of a forward contract at initiation is zero.

During its life, the value of a forward contract to the buyer is the spot price of the asset minus the present value of the forward contract price, and the value to the seller is the present value of the forward contract price minus the spot price of the asset.

At expiration, the value of a forward contract to the buyer is the spot price of the asset minus the forward contract price, and the value to the seller is the forward contract price minus the spot price of the asset.

LOS 52.b

An implied forward rate is the forward rate for which the following two strategies have the same yield over the total period:

- Investing from $t = 0$ to the forward date, and rolling over the proceeds for the period of the forward.
- Investing from $t = 0$ until the end of the forward period.

In a forward rate agreement (FRA), the fixed-rate payer (long) will pay the forward rate on a notional amount of principal at a future date, and the floating-rate payer will pay a future reference rate times that same amount of principal. FRAs are used primarily by financial institutions to manage the volatility of their interest-sensitive assets and liabilities.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 52.1

1. **A** If the value of the underlying is greater than the forward price, this increases the value of the forward contract, which represents a gain for the buyer and a loss for the seller. (LOS 52.a)

2. **B** $F_{2,3}$ is the 2-year forward 3-year rate, covering a period that begins two years from now and extends for three years after that. (LOS 52.b)

3. **C** The forward rate $F_{2,2}$ extends four years into the future and cannot be derived using zero-coupon yields that only extend three years. From zero-coupon bond yields for 1, 2, and 3 years, we can derive implied forward rates $F_{1,1}$, $F_{1,2}$, and $F_{2,1}$. (LOS 52.b)

READING 53

PRICING AND VALUATION OF FUTURES CONTRACTS

MODULE 53.1: FUTURES VALUATION

Video covering this content is available online.

LOS 53.a: Compare the value and price of forward and futures contracts.

While the *price* of a forward contract is constant over its life when no mark-to-market gains or losses are paid, its *value* will fluctuate with changes in the value of the underlying. The payment at settlement of the forward reflects the difference between the (unchanged) forward price and the spot price of the underlying.

The price and value of a futures contract *both* change when daily mark-to-market gains and losses are settled. Consider a futures contract on 100 ounces of gold at $1,870 purchased on Day 0. The following illustrates the changes in contract price and value with daily mark-to-market payments.

Day 0	Price = Settlement Price of 1,870	MTM Value = 0
Day 1	Settlement Price = 1,875	MTM value = $500
	$500 addition to margin account	
	New futures price = 1,875	MTM value = 0
Day 2	Settlement price = 1,855	MTM value = –$2,000
	$2,000 deduction from margin account	
	New futures price = 1,855	MTM value = 0

The change in the futures price to the settlement price each day returns its value to zero. Prices of forward contracts for which mark-to-market gains and losses are settled daily will also be adjusted to the settlement price.

Interest rate futures contracts are available on many market reference rates. We may view these as exchange-traded equivalents to forward rate agreements. One key difference is that interest rate futures are quoted on a price basis. For a market reference rate from time A to time B, an interest rate futures price is stated as follows:

$$\text{futures price} = 100 - (100 \times MRR_{A,\,B-A})$$

For example, if the futures price for a 6-month rate six months from now is 97, then $MRR_{6m,\,6m}$ = 3%.

Like other futures contracts, interest rate futures are subject to daily mark-to-market. The **basis point value (BPV)** of an interest rate futures contract is defined as:

BPV = notional principal × period × 0.01%

If the contract in our example is based on notional principal of €1,000,000, its BPV is €1,000,000 × (0.0001 / 2) = €50. This means a one basis point change in the MRR will change the futures contract value by €50.

LOS 53.b: Explain why forward and futures prices differ.

For pricing, the most important distinction between futures and forwards is that with futures, mark-to market gains and losses are paid each day. Gains above initial margin can be withdrawn from a futures account and losses that reduce margin deposits below their maintenance level require payments into the account. Forwards most often have no mark-to-market cash flows, with gains or losses settled at contract expiration. Forwards typically do not require or provide funds in response to fluctuations in value during their lives.

If interest rates are constant or uncorrelated with futures prices over time, the prices of futures and forwards are the same. A positive correlation between interest rates and the futures price means that (for a long position) daily settlement provides funds (excess margin) when rates are high and they can earn more interest, and requires funds (margin deposits) when rates are low and opportunity cost of deposited funds is less. Because of this, futures are theoretically more attractive than forwards when interest rates and futures prices are positively correlated, and less attractive than forwards when interest rates and futures prices are negatively correlated.

Because of the short maturity of most forwards and the availability of funds at near risk-free rates, differences between equivalent forwards and futures are not observed in practice. Additionally, derivative dealers in some markets with central clearing are required to post margin and may require derivative investors to post mark-to-market margin payments as well.

A separate issue arises for interest rate forwards and futures settlement payments. Recall that the payoff on an interest rate forward is the present value (at the beginning of the forward period) of any interest savings (at the end of the forward period) from the difference between the realized MRR and the forward MRR. Because the realized MRR is the discount rate for calculating the payment for a given amount of future interest savings, the payment for an increase in the MRR will be less than the payment for an equal decrease in the MRR, as the following example will illustrate.

Consider a $1 million interest rate future on a 6-month MRR priced at 97.50 (an MRR of 2.5%) that settles six months from now. Each basis point change in the (annualized) MRR will change the value of the contract by 0.0001 × 6/12 × $1 million = $50. If the MRR at settlement is either 2.51% or 2.49%, the payoff on the future at the end of one year is either $50 higher or $50 lower than when the MRR at settlement is 2.5%.

Compare this result with the payoffs for an otherwise equivalent forward, F_{6m6m}, priced at 2.5%.

If the MRR at settlement is 2.51%, the long receives 50/(1 + 0.0251/2) = $49.3803.

If the MRR at settlement is 2.49%, the long must pay 50/(1 + 0.0249/2) = $49.3852.

The value of the forwards exhibit convexity. An increase in rates decreases the forward's value by less than a decrease in the interest rate increases the forward's value, just as we saw with bonds. Also just as with bonds, the convexity effect for the value of forwards increases for longer periods. The convexity of forwards is termed **convexity bias** and forwards and futures prices can be significantly different for longer-term interest rates.

MODULE QUIZ 53.1

1. For a forward contract on an asset that has no costs or benefits from holding it to have zero value at initiation, the arbitrage-free forward price must equal:
   A. the expected future spot price.
   B. the future value of the current spot price.
   C. the present value of the expected future spot price.
2. For a futures contract to be more attractive than an otherwise equivalent forward contract, interest rates must be:
   A. uncorrelated with futures prices.
   B. positively correlated with futures prices.
   C. negatively correlated with futures prices.

KEY CONCEPTS

LOS 53.a

For a forward contract on which no mark-to-market gains or losses are paid, the forward price is constant over its life, but the contract's value will fluctuate with changes in the value of the underlying.

For a futures contract, the price and value both change when daily mark-to-market gains and losses are settled. The change in the futures price to the settlement price each day returns its value to zero.

Unlike forward rate agreements, interest rate futures are quoted on a price basis:

$$\text{futures price} = 100 - (100 \times MRR_{A,\,B-A})$$

LOS 53.b

Because gains and losses on futures contracts are settled daily, prices of forwards and futures that have the same terms may be different if interest rates are correlated with futures prices. Futures are more valuable than forwards when interest rates and futures prices are positively correlated and less valuable when they are negatively correlated. If interest rates are constant or uncorrelated with futures prices, the prices of futures and forwards are the same.

Convexity bias can result in price differences between interest rate futures contracts and otherwise equivalent forward rate agreements.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 53.1

1. **B** For an asset with no holding costs or benefits, the forward price must equal the future value of the current spot price, compounded at the risk-free rate over the term of the forward contract, for the contract to have a value of zero at initiation. Otherwise an arbitrage opportunity would exist. (LOS 53.a)

2. **B** If interest rates are positively correlated with futures prices, interest earned on cash from daily settlement gains on futures contracts will be greater than the opportunity cost of interest on daily settlement losses, and a futures contract is more attractive than an otherwise equivalent forward contract that does not feature daily settlement. (LOS 53.b)

READING 54

PRICING AND VALUATION OF INTEREST RATES AND OTHER SWAPS

MODULE 54.1: SWAP VALUATION

LOS 54.a: Describe how swap contracts are similar to but different from a series of forward contracts.

Video covering this content is available online.

In a simple interest-rate swap, one party pays a floating rate and the other pays a fixed rate on a notional principal amount. Consider a 1-year swap with quarterly payments, one party paying a fixed rate and the other a floating rate equal to a 90-day market reference rate (MRR). At each payment date the difference between the swap fixed rate and the MRR is paid to the party that owes the least, that is, a net payment is made from one party to the other.

We can separate these payments into a known payment and three unknown payments that are equivalent to the payments on three forward rate agreements (FRAs). Let MRR_n represent the floating rate payment (based on the 90-day MRR) owed at the end of quarter *n* and *F* be the fixed payment owed at the end of each quarter. We can represent the swap payment to be received by the fixed-rate payer at the end of period *n* as $MRR_n - F$. We can replicate each of these payments to (or from) the fixed-rate payer in the swap with a forward contract, specifically a long position in an FRA with a contract rate equal to the swap fixed rate and a settlement value based on the 90-day MRR.

We illustrate this separation below for a 1-year fixed-for-floating swap with a fixed rate of *F* and floating-rate payments for period *n* of MRR_n. Note that if the fixed rate and MRR are quoted as annual rates, the payments will be $(MRR_n - F)$ times one-fourth of the notional principal.

First payment (90 days from now) = $MRR_1 - F$ which is known at time zero because the payment 90 days from now is based on the 90-day MRR at time zero and the swap fixed rate, *F*, both of which are known at the initiation of the swap.

Second payment (180 days from now) is equivalent to a long position in an FRA with contract rate *F* that settles in 180 days and pays $MRR_2 - F$.

Third payment (270 days from now) is equivalent to a long position in an FRA with contract rate *F* that settles in 270 days and pays $MRR_3 - F$.

Fourth payment (360 days from now) is equivalent to a long position in an FRA with contract rate *F* that settles in 360 days and pays $MRR_4 - F$.

Note that a forward on a 90-day MRR that settles 90 days from now, based on the 90-day MRR at that time, actually pays the present value of the difference between the fixed rate *F* and the

90-day MRR 90 days from now (times the notional principal amount). Thus, the forwards in our example actually pay on days 90, 180, and 270. However, the amounts paid are equivalent to the differences between the fixed-rate payment and floating-rate payment that are due when interest is actually due on days 180, 270, and 360, which are the amounts we used in the example.

Therefore, we can describe an interest-rate swap as equivalent to a series of forward contracts, specifically FRAs, each with a forward contract rate equal to the swap fixed rate. However, there is one important difference. Because the forward contract rates are all equal in the FRAs that are equivalent to the swap, these would not be zero-value forward contracts at the initiation of the swap. Recall that forward contracts are based on a contract rate for which the value of the forward contract at initiation is zero. There is no reason to suspect that the swap fixed rate results in a zero value forward contract for each of the future dates. Instead, a swap is most likely to consist of some forwards with positive values and some forwards with negative values. The sum of their values will equal zero at initiation.

Finding the swap fixed rate that gives the swap a zero value at initiation, which is also known as the **par swap rate**, is not difficult if we follow our principle of no-arbitrage pricing. The fixed rate payer in a swap can replicate that derivative position by borrowing at a fixed rate and lending the proceeds at a variable (floating) rate. For the swap in our example, borrowing at the fixed rate *F* and lending the proceeds at the 90-day MRR will produce the same cash flows as the swap. At each date, the payment due on the fixed-rate loan is F_n and the interest received on lending at the floating rate is MRR_n.

LOS 54.b: Contrast the value and price of swaps.

As with FRAs, the *price* of a swap is the fixed rate of interest specified in the swap contract (the par swap rate) and the *value* depends on how expected future floating rates change over time. At initiation, a swap has zero value because the present value of the fixed-rate payments equals the present value of the expected floating-rate payments.

We can solve for the no-arbitrage fixed rate, termed the **par swap rate**, from the following equality:

$$\frac{MRR_1}{1+S_1}+\frac{MRR_2}{(1+S_2)^2}+\frac{MRR_3}{(1+S_3)^3}+\frac{MRR_4}{(1+S_4)^4}=$$

$$\frac{F}{1+S_1}+\frac{F}{(1+S_2)^2}+\frac{F}{(1+S_3)^3}+\frac{F}{(1+S_4)^4}$$

where S_1 through S_4 are the current effective spot rates for 90, 180, 270, and 360 days, MRR_1 through MRR_4 are the forward 90-day rates implied by the spot rates, and *F* is the fixed rate payment.

Given the current spot rates (S_1 to S_4), we can calculate the implied (expected) forward rates (MRRs), and then solve for *F*, the fixed rate that will give the swap a value of zero.

An increase in expected future 90-day rates will produce an increase the value of the fixed-rate payer position in a swap, while a decrease in expected rates will decrease the value of that position. At any point in time, the value of the fixed-rate payer side of a swap will be the present value of the expected future floating-rate payments, minus the present value of the future fixed-rate payments. This calculation is based on the spot rates and implied future 90-day rates at that point in time and can be used for any required mark-to-market payments.

MODULE QUIZ 54.1

1. Which of the following is *most* similar to the floating-rate receiver position in a fixed-for-floating interest-rate swap?
   A. Buying a fixed-rate bond and a floating-rate note.
   B. Buying a floating-rate note and issuing a fixed-rate bond.
   C. Issuing a floating-rate note and buying a fixed-rate bond.
2. The price of a fixed-for-floating interest-rate swap:
   A. is specified in the swap contract.
   B. is paid at initiation by the floating-rate receiver.
   C. may increase or decrease during the life of the swap contract.

KEY CONCEPTS

LOS 54.a

In a simple interest-rate swap, one party pays a floating rate and the other pays a fixed rate on a notional principal amount. The first payment is known at initiation and the rest of the payments are unknown. The unknown payments are equivalent to the payments on FRAs. The par swap rate is the fixed rate at which the sum of the present values of these FRAs equals zero.

LOS 54.b

The price of a swap is the fixed rate of interest specified in the swap contract. The value depends on how expected future floating rates change over time. An increase in expected future short-term future rates will increase the value of the fixed-rate payer position in a swap, and a decrease in expected future rates will decrease the value of the fixed-rate payer position.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 54.1

1. **B** The floating-rate receiver (fixed-rate payer) in a fixed-for-floating interest-rate swap has a position similar to issuing a fixed-coupon bond and buying a floating-rate note. (LOS 54.a)

2. **A** The price of a fixed-for-floating interest-rate swap is defined as the fixed rate specified in the swap contract. Typically a swap will be priced such that it has a value of zero at initiation and neither party pays the other to enter the swap. (LOS 54.b)

READING 55

PRICING AND VALUATION OF OPTIONS

MODULE 55.1: OPTION VALUATION

Video covering this content is available online.

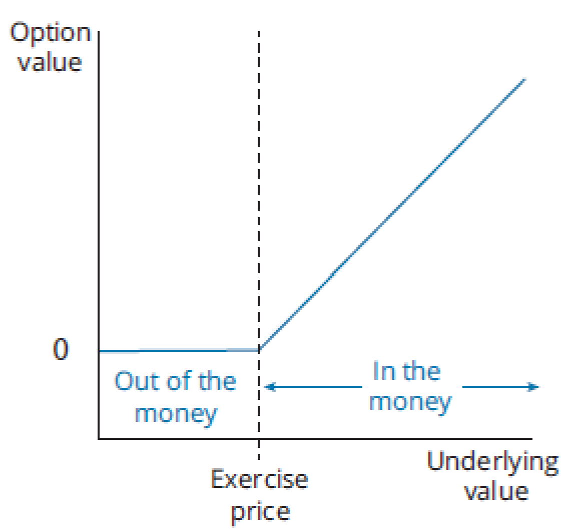
LOS 55.a: Explain the exercise value, moneyness, and time value of an option.

Moneyness refers to whether an option is *in the money* or *out of the money*. If immediate exercise of the option would generate a positive payoff, it is in the money. If immediate exercise would result in a loss (negative payoff), it is out of the money. When the current asset price equals the exercise price, exercise will generate neither a gain nor loss, and the option is *at the money*.

The following describes the conditions for a **call option** to be in, out of, or at the money. *S* is the price of the underlying asset and *X* is the exercise price of the option.

- *In-the-money call options*. If S − X > 0, a call option is in the money. S − X is the amount of the payoff a call holder would receive from immediate exercise, buying a share for *X* and selling it in the market for a greater price *S*.
- *Out-of-the-money call options.* If S − X < 0, a call option is out of the money.
- *At-the-money call options*. If S = X, a call option is said to be at the money.

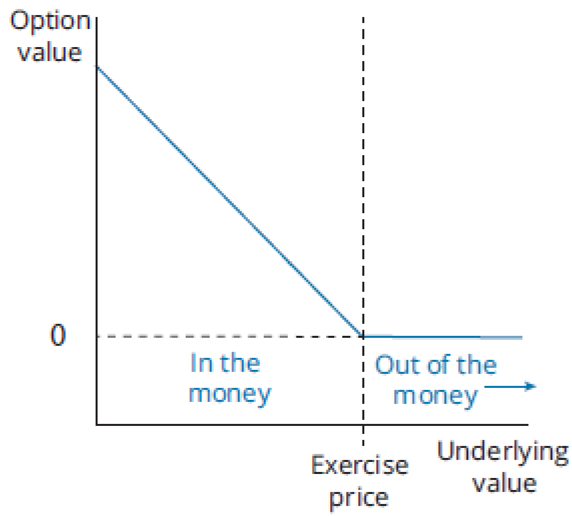
Figure 55.1: Call Option Moneyness



The following describes the conditions for a **put option** to be in, out of, or at the money.

- *In-the-money put options*. If X − S > 0, a put option is in the money. X − S is the amount of the payoff from immediate exercise, buying a share for *S* and exercising the put to receive *X* for the share.
- *Out-of-the-money put options*. When the stock's price is greater than the exercise price, a put option is said to be out of the money. If X − S < 0, a put option is out of the money.
- *At-the-money put options*. If S = X, a put option is said to be at the money.

Figure 55.2: Put Option Moneyness



EXAMPLE: Moneyness

Consider a July 40 call and a July 40 put, both on a stock that is currently selling for $37/share. Calculate how much these options are in or out of the money.

PROFESSOR'S NOTE

A July 40 call is a call option with an exercise price of $40 and an expiration date in July.

Answer:

The call is $3 out of the money because S − X = −$3.00. The put is $3 in the money because X − S = $3.00.

We define the **exercise value** (or **intrinsic value**) of an option as the maximum of zero and the amount that the option is in the money. That is, the exercise value is the amount an option is in the money, if it is in the money, or zero if the option is at or out of the money. The exercise value is the value of the option if exercised immediately.

Prior to expiration, an option has **time value** in addition to its exercise value. The time value of an option is the amount by which the **option premium** (price) exceeds the exercise value and is sometimes called the *speculative value* of the option. This relationship can be written as:

option premium = exercise value + time value

At any point during the life of an option, its value will typically be greater than its exercise value. This is because there is some probability that the underlying asset price will change in an amount that gives the option a positive payoff at expiration greater than the current exercise value. Recall that an option's exercise value (to a buyer) is the amount of the payoff at expiration and has a lower bound of zero.

When an option reaches expiration, there is no time remaining and the time value is zero. This means the value at expiration is either zero, if the option is at or out of the money, or its exercise value, if it is in the money.

LOS 55.b: Contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims.

To model forward commitments, we used no-arbitrage pricing based on an initial value of zero to both parties. With options, however, the initial values of options are positive; the buyer pays a premium (the option price) to the writer (seller). Another difference is that where forward commitments have essentially unlimited gains or losses for both parties (except to the extent that prices are constrained by zero), options are one-sided: Potential losses for the buyer, and potential gains for the writer, are limited to the premium paid. For these reasons, the no-arbitrage approach we use for pricing contingent claims is different from the model we use for forward commitments.

The following is some terminology that we will use to determine the minimum and maximum values for European options:

S_t = the price of the underlying stock at time t

X = the exercise price of the option

T–t = the time to expiration

c_t = the price of a European call at any time t prior to expiration at time = T

p_t = the price of a European put at any time t prior to expiration at time = T

Rf = the risk-free rate

Upper Bound for Call Options

The maximum value of a European call option at any time *t* is the time-*t* share price of the underlying stock. This makes sense because no one would pay more for the right to buy an asset than the asset's market value. It would be cheaper simply to buy the underlying asset. At time t = 0, the upper boundary condition for European call options is $c_0 \leq S_0$, and at any time t during a European call option's life, the upper boundary condition is $c_t \leq S_t$.

Upper Bound for Put Options

Logically the value of a put option cannot be more than its exercise price. This would be its exercise value if the underlying stock price goes to zero. However, because European puts cannot be exercised prior to expiration, their maximum value is the *present value* of the exercise price discounted at the risk-free rate. Even if the stock price goes to zero and is expected to stay at zero, the put buyer will not receive the intrinsic value, *X*, until the expiration date.

At time t = 0, the upper boundary condition can be expressed for European put options as:

$$p_0 \leq X\,(1 + Rf)^{-T}$$

At any time t during a European put option's life, the upper boundary condition is:

$$p_t \leq X\ (1 + Rf)^{-(T-t)}$$

Lower Bounds for Options

Theoretically, no option will sell for less than its intrinsic value and no option can take on a negative value. For European options, however, the lower bound is not so obvious because these options are not exercisable immediately.

To determine the lower bounds for European options, we can examine the value of a portfolio in which the option is combined with a long or short position in the stock and a pure discount bond.

For a *European call option*, construct the following portfolio:

- A long at-the-money European call option with exercise price *X,* expiring at time T.
- A long discount bond priced to yield the risk-free rate that pays *X* at option expiration.
- A short position in one share of the underlying stock priced at $S_0 = X$.

The current value of this portfolio is $c_0 - S_0 + X(1 + Rf)^{-T}$.

At expiration time T, this portfolio will pay $c_T - S_T + X$. That is, we will collect $c_T = \max[0, S_T - X]$ on the call option, pay S_T to cover our short stock position, and collect *X* from the maturing bond.

- If $S_T \geq X$, the call is in-the-money, and the portfolio will have a zero payoff because the call pays $S_T - X$, the bond pays $+X$, and we pay $-S_T$ to cover our short position. That is, the time t = T payoff is: $S_T - X + X - S_T = 0$.
- If $S_T < X$, the call is out-of-the-money, and the portfolio has a positive payoff equal to $X - S_T$ because the call value, c_T, is zero, we collect *X* on the bond, and pay $-S_T$ to cover the short position. So, the time t = T payoff is: $0 + X - S_T = X - S_T$.

No matter whether the option expires in-the-money, at-the-money, or out-of-the-money, the portfolio value will be equal to or greater than zero. We will never have to make a payment.

To prevent arbitrage, any portfolio that has no possibility of a negative payoff cannot have a negative value. Thus, we can state the value of the portfolio *at time* $t = 0$ as:

$$c_0 - S_0 + X(1 + Rf)^{-T} \geq 0$$

which allows us to conclude that:

$$c_0 \geq S_0 - X(1 + Rf)^{-T}$$

Combining this result with the earlier minimum on the call value of zero, we can write:

$$c_0 \geq \mathrm{Max}[0, S_0 - X(1 + Rf)^{-T}]$$

Note that $X(1 + Rf)^{-T}$ is the present value of a pure discount bond with a face value of *X*.

For a *European put option* we can derive the minimum value by forming the following portfolio at time $t = 0$:

- A long at-the-money European put option with exercise price X, expiring at T.
- A short position on a risk-free bond priced at $X(1 + Rf)^{-T}$, equivalent to borrowing $X(1 + Rf)^{-T}$.
- A long position in a share of the underlying stock priced at S_0.

At expiration time T, this portfolio will pay $p_T + S_T - X$. That is, we will collect $p_T = \text{Max}[0, X - S_T]$ on the put option, receive S_T from the stock, and pay X on the bond (loan).

- If $S_T > X$, the payoff will equal: $p_T + S_T - X = S_T - X$.
- If $S_T \leq X$, the payoff will be zero.

Again, a no-arbitrage argument can be made that the portfolio value must be zero or greater, because there are no negative payoffs to the portfolio.

At time $t = 0$, this condition can be written as:

$$p_0 + S_0 - X(1 + Rf)^{-T} \geq 0$$

and rearranged to state the minimum value for a European put option at time $t = 0$ as:

$$p_0 \geq X(1 + Rf)^{-T} - S_0$$

We have now established the minimum bound on the price of a European put option as:

$$p_0 \geq \text{Max}[0, X(1 + Rf)^{-T} - S_0]$$

Figure 55.1 summarizes what we now know regarding the boundary prices for European options at any time t prior to expiration at time $t = T$.

Figure 55.1: Lower and Upper Bounds for Options

Option	Minimum Value	Maximum Value
European call	$c_t \geq \text{Max}[0, S_t - X(1 + Rf)^{-(T-t)}]$	S_t
European put	$p_t \geq \text{Max}[0, X / X(1 + Rf)^{-(T-t)} - S_t]^{(T-t)}$	$X(1 + Rf)^{-(T-t)}$

PROFESSOR'S NOTE:

For the exam, know the price limits in Figure 55.1. You will not be asked to derive them, but you may be expected to use them.

LOS 55.c: Identify the factors that determine the value of an option and describe how each factor affects the value of an option.

There are six factors that determine option prices.

1. **Price of the underlying asset**. For call options, the higher the price of the underlying, the greater its exercise value and the higher the value of the option. Conversely, the lower the price of the underlying, the less its exercise value and the lower the value of the call option. In general, call option values increase when the value of the underlying asset increases.

   For put options this relationship is reversed. An increase in the price of the underlying reduces the value of a put option.

2. **The exercise price**. A higher exercise price decreases the values of call options and a lower exercise price increases the values of call options.

   A higher exercise price increases the values of put options and a lower exercise price decreases the values of put options.

3. **The risk-free rate of interest**. An increase in the risk-free rate will increase call option values, and a decrease in the risk-free rate will decrease call option values.

   An increase in the risk-free rate will decrease put option values, and a decrease in the risk-free rate will increase put option values.

   PROFESSOR'S NOTE

   One way to remember the effects of changes in the risk-free rate is to think about present values of the payments for calls and puts. These statements are strictly true only for in-the-money options, but it's a way to remember the relationships. The holder of a call option will pay in the future to exercise a call option and the present value of that payment is lower when the risk-free rate is higher, so a higher risk-free rate increases a call option's value. The holder of a put option will receive a payment in the future when the put is exercised and an increase in the risk-free rate decreases the present value of this payment, so a higher risk-free rate decreases a put option's value.

4. **Volatility of the underlying**. Volatility is what makes options valuable. If there were no volatility in the price of the underlying asset (its price remained constant), options would always be equal to their exercise values and time or speculative value would be zero. An increase in the volatility of the price of the underlying asset increases the values of both put and call options and a decrease in volatility of the price of the underlying decreases both put values and call values.

5. **Time to expiration**. Because volatility is expressed per unit of time, longer time to expiration effectively increases expected volatility and increases the value of a call option. Less time to expiration decreases the time value of a call option so that at expiration its value is simply its exercise value.

   For most put options, longer time to expiration will increase option values for the same reasons. For some European put options, however, extending the time to expiration can decrease the value of the put. In general, the deeper a put option is in the money, the higher the risk-free rate, and the longer the current time to expiration, the more likely that extending the option's time to expiration will decrease its value.

   To understand this possibility consider a put option at $20 on a stock with a value that has decreased to $1. The exercise value of the put is $19 so the upside is very limited, the downside (if the price of the underlying subsequently increases) is significant, and because no payment will be received until the expiration date, the current option value reflects the present value of any expected payment. Extending the time to expiration would decrease that present value. While overall we expect a longer time to expiration to increase the value of a European put option, in the case of a deep in-the-money put, a longer time to expiration could decrease its value.

6. **Costs and benefits of holding the asset**. If there are benefits of holding the underlying asset (dividend or interest payments on securities or a convenience yield on commodities), call values are decreased and put values are increased. The reason for this is most easily understood by considering cash benefits. When a stock pays a dividend, or a bond pays interest, this reduces the value of the asset. Decreases in the value of the underlying asset decrease call values and increase put values.

   Positive storage costs make it more costly to hold an asset. We can think of this as making a call option more valuable because call holders can have long exposure to the asset without paying the costs of actually owning the asset. Puts, on the other hand, are less valuable when storage costs are higher.

MODULE QUIZ 55.1

1. The price of an out-of-the-money option is:
   A. less than its time value.
   B. equal to its time value.
   C. greater than its time value.
2. The lower bound for the value of a European put option is:
   A. Max(0, S – X)
   B. Max[0, $X(1 + Rf)^{-(T-t)}$ – S]
   C. Max[0, S – $X(1 + Rf)^{-(T-t)}$]
3. A decrease in the risk-free rate of interest will:
   A. increase put and call option prices.
   B. decrease put option prices and increase call option prices.
   C. increase put option prices and decrease call option prices.

KEY CONCEPTS

LOS 55.a

If immediate exercise of an option would generate a positive payoff, the option is in the money. If immediate exercise would result in a negative payoff, the option is out of the money. An option's exercise value is the greater of zero or the amount it is in the money. Time value is the amount by which an option's price is greater than its exercise value. Time value is zero at expiration.

LOS 55.b

The approach for pricing contingent claims is different from the model for forward commitments because contingent claims have one-sided payoffs and values at initiation that are not equal to zero. A replication model for European options is based on the value of a portfolio in which the option is combined with a pure discount bond and a long or short position in the underlying.

LOS 55.c

Factors that determine the value of an option:

Increase in:	Effect on Call Option Values	Effect on Put Option Values
Price of underlying asset	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility of underlying asset	Increase	Increase
Time to expiration	Increase	Increase, except some European puts
Costs of holding underlying asset	Increase	Decrease
Benefits of holding underlying asset	Decrease	Increase

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 55.1

1. **B** Because an out-of-the-money option has an exercise value of zero, its price is its time value. (LOS 55.a)
2. **B** The lower bound for a European put ranges from zero to the present value of the exercise price less the current stock price, where the exercise price is discounted at the risk-free rate. (LOS 55.b)
3. **C** A decrease in the risk-free rate will decrease call option values and increase put option values. (LOS 55.c)

READING 56

OPTION REPLICATION USING PUT–CALL PARITY

MODULE 56.1: PUT–CALL PARITY

Video covering this content is available online.

LOS 56.a: Explain put–call parity for European options.

Our derivation of **put-call parity** for European options is based on the payoffs of two portfolio combinations: a fiduciary call and a protective put.

A *fiduciary call* is a combination of a call with exercise price *X* and a pure-discount, riskless bond that pays *X* at maturity (option expiration). The payoff for a fiduciary call at expiration is *X* when the call is out of the money, and X + (S − X) = S when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is (X − S) + S = X when the put is in the money, and *S* when the put is out of the money.

PROFESSOR'S NOTE

When working with put–call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to *X*.

If at expiration *S* is greater than or equal to *X*:

- The protective put pays *S* on the stock while the put expires worthless, so the payoff is *S*.
- The fiduciary call pays *X* on the bond portion while the call pays (S − X), so the payoff is X + (S − X) = S.

If at expiration *X* is greater than *S*:

- The protective put pays *S* on the stock while the put pays (X − S), so the payoff is S + (X − S) = X.
- The fiduciary call pays *X* on the bond portion while the call expires worthless, so the payoff is *X*.

In either case, the payoff on a protective put is the same as the payoff on a fiduciary call. Our no-arbitrage condition holds that portfolios with identical payoffs regardless of future conditions must sell for the same price to prevent arbitrage. We can express the put–call parity relationship as:

$$c + X(1 + Rf)^{-T} = S + p$$

Equivalencies for each of the individual securities in the put–call parity relationship can be expressed as:

$$S = c - p + X(1 + Rf)^{-T}$$
$$p = c - S + X(1 + Rf)^{-T}$$
$$c = S + p - X(1 + Rf)^{-T}$$
$$X(1 + Rf)^{-T} = S + p - c$$

Note that the options must be European style and the puts and calls must have the same exercise price and time to expiration for these relations to hold.

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the **synthetic** equivalents of the securities on the left. For example, to synthetically produce the payoff for a long position in a share of stock, use the following relationship:

$$S = c - p + X(1 + Rf)^{-T}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put–call parity relationship can be constructed in a similar manner.

PROFESSOR'S NOTE

After expressing the put–call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

EXAMPLE: **Call option valuation using put–call parity**

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. Estimate the price of the 3-month call option.

Answer:

Rearranging put–call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - \text{present value (X)}$$

$$\text{call} = \$1.50 + \$52 - \frac{\$50}{1.05^{0.25}} = \$4.11$$

This means that if a 3-month, \$50 call is available, it should be priced at (within transaction costs of) \$4.11 per share.

LOS 56.b: Explain put–call *forward* parity for European options.

Put–call-forward parity is derived with a forward contract rather than the underlying asset itself. Consider a forward contract on an asset at time *T* with a contract price of $F_0(T)$. At contract initiation the forward contract has zero value. At time *T*, when the forward contract settles, the long must purchase the asset for $F_0(T)$. The purchase (at time = 0) of a pure discount bond that will pay $F_0(T)$ at maturity (time = T) will cost $F_0(T)(1 + Rf)^{-T}$.

By purchasing such a pure discount bond and simultaneously taking a long position in the forward contract, an investor has created a synthetic asset. At time = T the proceeds of the bond are just sufficient to purchase the asset as required by the long forward position. Because there is no cost to enter into the forward contract, the total cost of the synthetic asset is the present value of the forward price, $F_0(T)(1 + Rf)^{-T}$.

The put–call-forward parity relationship is derived by substituting the synthetic asset for the underlying asset in the put–call parity relationship. Substituting $F_0(T)(1 + Rf)^{-T}$ for the asset price S_0 in $S + p = c + X(1 + Rf)^{-T}$ gives us:

$$F_0(T)(1 + Rf)^{-T} + p_0 = c_0 + X(1 + Rf)^{-T}$$

which is put–call-forward parity at time 0, the initiation of the forward contract, based on the principle of no arbitrage. By rearranging the terms, put–call forward parity can also be expressed as:

$$p_0 - c_0 = [X - F_0(T)](1 + Rf)^{-T}$$

Application of Options Theory to Corporate Finance

We can view the claims of a firm's equity holders and debt holders as a call option and a put option, respectively. Consider a firm that has a value of V_t at time = t and has issued debt in the form of a zero-coupon bond that will pay D at time = T. At time = T, if $V_T > D$ the equity holders receive $V_T - D$ and if $V_T < D$, the firm is insolvent and equity holders receive nothing. The payoff to the equity holders at time = T can be written as $\text{Max}(0, V_T - D)$ which is equivalent to a call option with the firm value as the underlying and an exercise price of D.

At time = T, if $V_T > D$ the debt holders receive D and if $V_T < D$, the firm is insolvent and debt holders receive V_T. The payoff to the debt holders at time = T can be written as $\text{Max}(V_T, D)$. This is equivalent to a portfolio that is long a risk-free bond that pays D at t = T, and short (has sold) a put option on the value of the firm, V_T, with an exercise price of D. If $V_T > D$ the portfolio pays D and the put expires worthless, and if $V_T < D$ the portfolio pays $D - (D - V_T) = V_T$ and the debtholders effectively pay $D - V_T$ on the short put position.

MODULE QUIZ 56.1

1. The put–call parity relationship for European options must hold because a protective put will have the same payoff as:
   A. a covered call.
   B. a fiduciary call.
   C. an uncovered call.
2. The put–call-forward parity relationship *least likely* includes:
   A. a risk-free bond.
   B. call and put options.
   C. the underlying asset.

KEY CONCEPTS

LOS 56.a

A fiduciary call (a call option and a risk-free zero-coupon bond that pays the strike price *X* at expiration) and a protective put (a share of stock and a put at *X*) have the same payoffs at expiration, so arbitrage will force these positions to have equal prices: $c + X(1 + Rf)^{-T} = S + p$. This establishes put–call parity for European options.

Based on the put–call parity relation, a synthetic security (stock, bond, call, or put) can be created by combining long and short positions in the other three securities.

$$c = S + p - X(1 + Rf)^{-T}$$
$$p = c - S + X(1 + Rf)^{-T}$$
$$S = c - p + X(1 + Rf)^{-T}$$
$$X(1 + Rf)^{-T} = S + p - c$$

LOS 56.b

Because we can replicate the payoff on an asset by lending the present value of the forward price at the risk-free rate and taking a long position in a forward, we can write put–call-forward parity as:

$$c_0 + X(1 + Rf)^{-T} = F_0(T)(1 + Rf)^{-T} + p_0$$

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 56.1

1. **B** Given call and put options on the same underlying asset with the same exercise price and expiration date, a protective put (underlying asset plus a put option) will have the same payoff as a fiduciary call (call option plus a risk-free bond that will pay the exercise price on the expiration date) regardless of the underlying asset price on the expiration date. (LOS 56.a)

2. **C** The put–call-forward parity relationship is $F_0(T)(1 + RFR)^{-T} + p_0 = c_0 + X(1 + Rf)^{-T}$, where $X(1 + Rf)^{-T}$ is a risk-free bond that pays the exercise price on the expiration date, and $F_0(T)$ is the forward price of the underlying asset. (LOS 56.b)

READING 57

VALUING A DERIVATIVE USING A ONE-PERIOD BINOMIAL MODEL

MODULE 57.1: BINOMIAL MODEL FOR OPTION VALUES

Video covering this content is available online.

LOS 57.a: Explain how to value a derivative using a one-period binomial model.

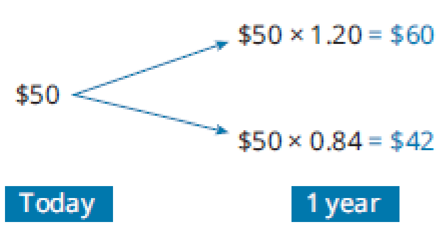
Recall from Quantitative Methods that a **binomial model** is based on the idea that, over the next period, some value will change to one of two possible values (binomial). To construct a one-period binomial model for pricing an option, we need:

- A value for the underlying at the beginning of the period.
- An exercise price for the option. The exercise price can be different from the value of the underlying. We assume the option expires one period from now.
- Returns that will result from an up-move and a down-move in the value of the underlying over one period.
- The risk-free rate over the period.

For now we do not need to consider the probabilities of an up-move or a down-move. Later in this reading we will examine one-period binomial models with risk-neutral probabilities.

As an example, we can model a call option with an exercise price of $55 on a stock that is currently valued (S_0) at $50. Let us assume that in one period the stock's value will either increase ((S_1^u)) to $60 or decrease ($(S_1^d)$) to $42. We state the return from an up-move (R^u) as $60/$50 = 1.20, and the return from a down-move (R^d) as $42/$50 = 0.84.

Figure 57.1: One-Period Binomial Tree



The call option will be in the money after an up-move or out of the money after a down-move. Its value at expiration after an up-move, c_1^u, is Max(0, $60 – $55) = $5. Its value after a down-move, c_1^d, is Max(0, $42 – $55) = 0.

Now we can use no-arbitrage pricing to determine the initial value of the call option (c_0). We do this by creating a portfolio of the option and the underlying stock, such that the portfolio will have the same value following either an up-move (V_1^u) or a down-move (V_1^d) in the stock. For our example, we would write the call option and buy a number of shares of the stock that we will denote as *h*. We must solve for the *h* that results in $V_1^u = V_1^d$.

- The initial value of our portfolio, V_0, is $hS_0 - c_0$ (remember we are short the call option).
- The portfolio value after an up-move, V_1^u, is $hS_1^u - c_1^u$.
- The portfolio value after a down-move, V_1^d, is $hS_1^d - c_1^d$.

In our example, $V_1^u = h(\$60) - \$5$, and $V_1^d = h(\$42) - 0$. Setting $V_1^u = V_1^d$ and solving for *h*, we get:

$h(\$60) - \$5 = h(\$42)$

$h(\$60) - h(\$42) = \$5$

$h = \$5 / (\$60 - \$42) = 0.278$

This result, the number of shares of the underlying we would buy for each call option we would write, is known as the **hedge ratio** for this option.

With $V_1^u = V_1^d$, the value of the portfolio after one period is known with certainty. This means we can say that either V_1^u or V_1^d must equal V_0 compounded at the risk-free rate for one period. In this example, $V_1^d = 0.278(\$42) = \$11.68$, or $V_1^u = 0.278(\$60) - \$5 = \$11.68$. Let us assume the risk-free rate over one period is 3%. Then $V_0 = \$11.68 / 1.03 = \$11.34$.

Now we can solve for the value of the call option, c_0. Recall that $V_0 = hS_0 - c_0$,
so $c_0 = hS_0 - V_0$. Here, $c_0 = 0.278(\$50) - \$11.34 = \$2.56$.

LOS 57.b: Describe the concept of risk neutrality in derivatives pricing.

Another approach to constructing a one-period binomial model involves risk-neutral probabilities of an up-move or a down-move. Consider a share of stock currently priced at \$30. The size of the possible price changes, and the probabilities of these changes occurring, are as follows:

R^u = up-move factor = 1.15

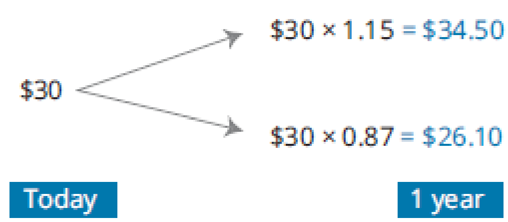
R^d = down-move factor = $\frac{1}{R^u} = \frac{1}{1.15} = 0.87$

π_U = risk-neutral probability of an up-move = 0.715

π_D = risk-neutral probability of a down-move = $1 - \pi_U = 1 - 0.715 = 0.285$

Note that the down-move factor is the reciprocal of the up-move factor, and the probability of an up-move is one minus the probability of a down-move. The one-period binomial tree for the stock is shown in Figure 57.2. The beginning stock value of \$30 is to the left, and to the right are the two possible end-of-period stock values, 30 × 1.15 = \$34.50 and 30 × 0.87 = \$26.10.

Figure 57.2: One-Period Binomial Tree



The risk-neutral probabilities of an up-move and a down-move are calculated from the sizes of the moves and the risk-free rate:

$$\pi_U = \text{risk-neutral probability of an up-move} = \frac{1 + Rf - R^d}{R^u - R^d}$$

$$\pi_D = \text{risk-neutral probability of a down-move} = 1 - \pi_U$$

where:

Rf = risk-free rate

R^u = size of an up-move

R^d = size of a down-move

PROFESSOR'S NOTE

These two probabilities are not the actual probabilities of the up- and down-moves. They are risk-neutral pseudo probabilities. The calculation of risk-neutral probabilities does not appear to be required for the Level I exam.

We can calculate the value of an option on the stock by:

- Calculating the payoffs of the option at expiration for the up-move and down-move prices.
- Calculating the expected payoff of the option in one year as the (risk-neutral) probability-weighted average of the up-move and down-move payoffs.
- Calculating the PV of the expected payoff by discounting at the risk-free rate.

EXAMPLE: **Calculating call option value with risk-neutral probabilities**

Use the binomial tree in Figure 57.2 to calculate the value today of a 1-year call option on a stock with an exercise price of $30. Assume the risk-free rate is 7%, the current value of the stock is $30, and the up-move factor is 1.15.

Answer:

First, we need to calculate the down-move factor and risk-neutral the probabilities of the up- and down-moves:

$$R^d = \text{size of down-move} = \frac{1}{R^u} = \frac{1}{1.15} = 0.87$$

$$\pi_U = \text{risk-neutral probability of an up-move} = \frac{1 + 0.07 - 0.87}{1.15 - 0.87} = 0.715$$

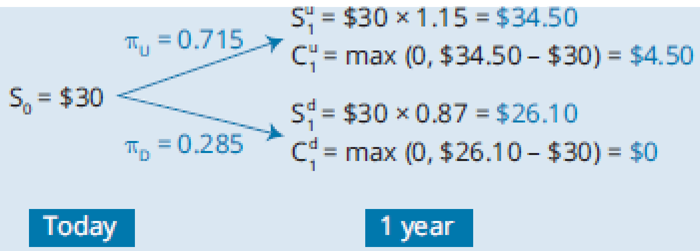
$$\pi_D = \text{risk-neutral probability of a down-move} = 1 - 0.715 = 0.285$$

Next, determine the payoffs on the option in each state. If the stock moves up to $34.50, a call option with an exercise price of $30 will pay $4.50. If the stock moves down to $26.10, the call option will expire worthless. The option payoffs are illustrated in the following figure.

Let the stock values for the up-move and down-move be S_1^u and S_1^d and for the call values, c_1^u and c_1^d.

One-Period Call Option With X = $30

The expected value of the option in one period is:



E(call option value in 1 year) = ($4.50 × 0.715) + ($0 × 0.285) = $3.22

The value of the option today, discounted at the risk-free rate of 7%, is:

$$C_0 = \frac{\$3.22}{1.07} = \$3.01$$

We can use the same basic framework to value a one-period put option. The only difference is that the payoff to the put option will be different from the call payoffs.

EXAMPLE: **Valuing a one-period put option on a stock**

Use the information in the previous example to calculate the value of a put option on the stock with an exercise price of $30.

Answer:

If the stock moves up to $34.50, a put option with an exercise price of $30 will expire worthless. If the stock moves down to $26.10, the put option will be worth $3.90.

The risk-neutral probabilities are 0.715 and 0.285 for an up- and down-move, respectively. The expected value of the put option in one period is:

E(put option value in 1 year) = ($0 × 0.715) + ($3.90 × 0.285) = $1.11

The value of the option today, discounted at the risk-free rate of 7%, is:

$$p_0 = \frac{\$1.11}{1.07} = \$1.04$$

In practice, we would construct a binomial model with many short periods and have many possible outcomes at expiration. However, the one-period model is sufficient to illustrate the concept and method.

Note that the actual probabilities of an up-move and a down-move do not enter directly into our calculation of option value. The size of the up-move and down-move, along with the risk-free rate, determines the risk-neutral probabilities we use to calculate the expected payoff at option expiration. Remember, the risk-neutral probabilities come from constructing a hedge that creates a certain payoff. Because their calculation is based on an arbitrage relationship, we can discount the expected payoff based on risk-neutral probabilities, using the risk-free rate.

MODULE QUIZ 57.1

1. To construct a one-period binomial model for valuing an option, are probabilities of an up-move or a down-move in the underlying price required?
    A. No.
    B. Yes, but they can be calculated from the returns on an up-move and a down-move.
    C. Yes, the model requires estimates for the actual probabilities of an up-move and a down-move.

2. In a one-period binomial model based on risk neutrality, the value of an option is *best* described as the present value of:
   A. a probability-weighted average of two possible outcomes.
   B. a probability-weighted average of a chosen number of possible outcomes.
   C. one of two possible outcomes based on a chosen size of increase or decrease.
3. A one-period binomial model for option pricing uses risk-neutral probabilities because:
   A. the model is based on a no-arbitrage relationship.
   B. they are unbiased estimators of the actual probabilities.
   C. the buyer can let an out-of-the-money option expire unexercised.

KEY CONCEPTS

LOS 57.a

A one-period binomial model for pricing an option requires the underlying asset's value at the beginning of the period, an exercise price for the option, the asset prices that will result from an up-move and a down-move, and the risk-free rate.

A portfolio of the underlying asset hedged with a position in an option can be created such that the portfolio has the same value for both an up-move and a down-move. Because the portfolio's value at the end of the period is certain, that value must be the portfolio's initial value compounded at the risk-free rate. The number of units of the underlying required to construct such portfolios is the hedge ratio.

LOS 57.b

To determine the value of an option using the concept of risk neutrality, we calculate its payoffs for both an up-move and a down-move, calculate the expected payoff as a weighted average using the risk-neutral probabilities of an up-move and a down-move, and discount this expected payoff for one period at the risk-free rate.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 57.1

1. **A** A one-period binomial model can be constructed based on replication and no-arbitrage pricing, without regard to the probabilities of an up-move or a down-move. (LOS 57.a)
2. **A** In a one-period binomial model based on risk-neutral probabilities, the value of an option is the present value of a probability-weighted average of two possible option payoffs at the end of a single period, during which the price of the underlying asset is assumed to move either up or down to specific values. (LOS 57.b)
3. **A** Because a one-period binomial model is based on a no-arbitrage relationship, we can discount the expected payoff at the risk-free rate. (LOS 57.b)

TOPIC QUIZ: DERIVATIVES

You have now finished the Derivatives topic section. Please log into your Schweser online dashboard and take the Topic Quiz on Derivatives. The Topic Quiz provides immediate feedback on how effective your study has been for this material. The number of questions on this quiz is approximately the number of questions for the topic on one-half of the actual Level I CFA exam. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow 1.5 minutes per question.

After you've completed this Topic Quiz, select "Performance Tracker" to view a breakdown of your score. Select "Compare with Others" to display how your score on the Topic Quiz compares to the scores of others who entered their answers.

READING 58

CATEGORIES, CHARACTERISTICS, AND COMPENSATION STRUCTURES OF ALTERNATIVE INVESTMENTS

MODULE 58.1: ALTERNATIVE INVESTMENT STRUCTURES

Video covering this content is available online.

LOS 58.a: Describe types and categories of alternative investments.

Alternative investments comprise various types of investments that do not fall under the heading of **traditional investments**, which refers to long-only investments in cash or publicly traded stocks and bonds.

Managers of alternative investment portfolios may use derivatives and leverage and short securities. Structures that employ such strategies but utilize publicly traded stocks and bonds are also termed alternative investments. Many types of real estate investment are considered alternatives to traditional investment as well.

Types of alternative investment structures include hedge funds, private equity funds, and various types of real estate investments. Alternative investments typically are actively managed and may include investments in commodities, infrastructure, and illiquid securities.

Fee structures for alternative investments are different from those of traditional investments, with higher management fees on average and often with additional incentive fees based on performance.

Compared with traditional investments, alternative investments typically exhibit several of the following characteristics:

- Less liquidity of assets held.
- More specialization by investment managers.
- Less regulation and transparency.
- More problematic and less available historical return and volatility data.
- Different legal issues and tax treatments.
- Relatively low correlations with returns of traditional investments.
- Relatively higher management fees and incentive fees based on performance.
- Restrictions on redemptions.
- Relatively more concentrated portfolios.

The perceived benefits of including alternative investments in portfolios are risk reduction from diversification (due to low correlations of alternative investments with traditional investments) and possible higher returns from holding illiquid securities and from markets for some alternative investments possibly being less efficient than those for traditional investments. Although correlations of returns on alternative investments with returns on traditional investments may be low on average, these correlations may increase significantly during periods of economic stress.

We will examine several categories of alternative investments in detail in our reading on Private Capital, Real Estate, Infrastructure, Natural Resources, and Hedge Funds. Here we introduce those categories:

1. **Hedge funds.** These funds may use leverage, hold long and short positions, use derivatives, and invest in illiquid assets. Managers of hedge funds use a great many different strategies in attempting to generate investment gains. They do not necessarily hedge risk as the name might imply.
2. **Private equity.** As the name suggests, private equity funds invest in the equity of companies that are not publicly traded, or in the equity of publicly traded firms that the funds intend to take private. Leveraged buyout (LBO) funds use borrowed money to purchase equity in established companies and comprise the majority of private equity investment funds. Venture capital funds invest in young, unproven companies at various early stages in their lives.
3. **Private debt.** Funds may make loans directly to companies, lend to early stage (venture) firms, or invest in the debt of firms that are financially distressed (struggling to make their debt payments or have entered into bankruptcy).
4. **Real estate.** Real estate investments include residential or commercial properties, as well as real estate–backed debt. These investments are held in a variety of structures, including full or leveraged ownership of individual properties, individual real estate–backed loans, private and publicly traded securities backed by pools of properties or mortgages, and limited partnerships.
5. **Commodities.** To gain exposure to changes in commodity prices, investors can own physical commodities, commodity derivatives, or the equity of commodity-producing firms. Some funds seek exposure to the returns on various commodity indices, often by holding derivatives contracts (futures) that are expected to track a specific commodity index.
6. **Farmland.** Agricultural land can produce income from leasing it out for farming or from raising crops or livestock for harvest and sale.
7. **Timberland.** Forested land is purchased or trees are planted for harvesting, which provides cash flows.
8. **Infrastructure.** Infrastructure refers to long-lived assets that provide public services. These include economic infrastructure assets, such as roads, airports, and utility grids, and social infrastructure assets, such as schools and hospitals. While often financed and constructed by governmental entities, infrastructure investments have more recently been undertaken by public–private partnerships, with each holding a significant stake in the infrastructure assets constructed. Various deal structures are employed, and the asset may revert to public ownership at some future date.

LOS 58.b: Describe characteristics of direct investment, co-investment, and fund investment methods for alternative investments.

Direct investing refers to an investor that purchases assets itself, rather than pooling its funds with others or using a specialized outside manager. Larger, more knowledgeable investors may purchase private companies or real estate on their own. For example, a sovereign wealth fund may have its own specialized managers to invest in real estate, agricultural land, or companies in the venture stage.

Direct investing has advantages in that there are no fees to outside managers and the investor has more choice and control over the investments made. Disadvantages include the possibility of less diversification across investments, higher minimum investment amounts, and greater investor expertise required to evaluate deals and do their own due diligence.

Fund investing refers to investing in a pool of assets along with other investors, using a fund manager that selects and manages a pool of investments according to an agreed-upon strategy. In this case, the individual investors do not control the selection of assets for investment or their subsequent management and sale. The manager typically receives a percentage of the investable funds (management fee), as well as a percentage of the investment gains (incentive fee).

The advantages of investing through a fund, compared with direct investment, include having the expertise of the fund manager, less involvement and expertise required of the investor, diversification across investments made by the fund, and lower minimum investment requirements. Disadvantages are the cost of fund manager fees and the possibility that the fund manager may perform poorly even when selected through careful due diligence.

With **co-investing**, an investor contributes to a pool of investment funds (as with fund investing) but also has the right to invest directly alongside the fund manager in some of the assets in which the manager invests. Co-investing can reduce overall fees, while benefiting from the manager's expertise, and provide the investor with more control over asset selection and management. Compared with fund investing, it requires greater expertise, due diligence, and involvement with asset management. Co-investment opportunities may be subject to **adverse selection** if fund managers choose to make the full investment through the fund for assets they are quite confident about, and offer co-investment opportunities on assets they are less confident about.

LOS 58.c: Describe investment and compensation structures commonly used in alternative investments.

Alternative investments are often structured as **limited partnerships**. In a limited partnership, the **general partner (GP)** is the fund manager and makes investment decisions. The **limited partners (LPs)** are the investors, who own a partnership share proportional to their investment. The limited partners typically have no say in how the fund is managed and no liability beyond their investment in the partnership. The general partner takes on the liabilities of the partnership, including the repayment of any partnership debt.

Limited partners commit to an investment amount and in some cases only contribute a portion of that initially, providing the remaining funds over time as required by the general partner (as fund investments are made). General partnerships are less regulated than publicly traded companies, and limited partnership shares are typically only available to accredited investors (those with sufficient wealth to bear the risk of the investment and enough investment sophistication to understand the risks involved).

The rules and operational details that govern a partnership are contained in the **limited partnership agreement**. Special terms that apply to one limited partner but not to others are stated in **side letters**.

Fee Structures

The total fees paid by investors in alternative investment funds often consist of a **management fee** and an **incentive fee** (or **performance fee**). The fund manager earns the management fee regardless of investment performance. Management fees are typically between 1% and 2%. For hedge funds, the management fees are calculated as a percentage of assets under management (AUM), typically the net asset value of the fund's investments. For private equity funds, the management fee is calculated as a percentage of **committed capital**, not capital invested. Committed capital is typically not all invested immediately but is "drawn down" (invested) as securities are identified and added to the portfolio. Committed capital is usually drawn down over three to five years, but the drawdown period is at the discretion of the fund manager. Committed capital that has not yet been drawn down is referred to as **dry powder**.

Incentive fees are a portion of profits on fund investments. Most often, the partnership agreement will specify a **hurdle rate** of return that must be met or exceeded before any incentive fees are paid. Hurdle rates can be defined in two ways: either "hard" or "soft." If a **soft hurdle rate** is met, incentive fees are a percentage of the total increase in the value of each partner's investment. With a **hard hurdle rate**, incentive fees are based only on gains above the hurdle rate.

As an example, consider a fund with a hurdle rate of 8% that has produced returns of 12% for the year so that it has exceeded the hurdle rate. We will use an incentive fee structure of 20% of gains. If the 8% is a soft hurdle rate, the incentive fee will be 20% of the gains of 12%, or 2.4%. If the 8% is a hard hurdle rate, the incentive fee will be 20% of the gains above the hurdle rate (12% – 8% = 4%), which would be 0.8%.

Typically, incentive fees are paid at the end of each year based on the increase in the value of fund investments, after management fees and other charges, which may include consulting and monitoring fees that are charged to individual portfolio companies.

A **catch-up clause** in a partnership agreement is based on a hurdle rate and is similar in its effect to a soft hurdle rate. Consider a fund with returns of 14%, a hurdle rate of 8%, and a 20% incentive fee. A catch-up clause would result in the first 8% of gains going to the limited partners and the next 2% going to the general partner, allowing the general partner to "catch up" to receiving 20% of the first 10% of gains. After the catch-up, further gains are split 80/20 between the limited partners and the general partner.

Another feature that is often included is a **high-water mark**. This means that no incentive fee is paid on gains that only offset prior losses. Thus, incentive fees are only paid to the extent that the current value of an investor's account is above the highest net-of-fees value previously recorded (at the end of a payment period). This feature ensures that investors will not be charged incentive fees twice on the same gains in their portfolio values. Because investors invest in a fund at different times, they each may have a different high-water mark value. Investors should check carefully whether incentive fees are calculated correctly.

A partnership's **waterfall** refers to the way in which payments are allocated to the GP and the LPs as profits and losses are realized on deals. With a **deal-by-deal waterfall** (or **American waterfall**), profits are distributed as each fund investment is sold and shared according to the partnership agreement. This favors the GP because incentive fees are paid before 100% of the limited partners' original investment plus the hurdle rate is returned to them. With a **whole-of-fund waterfall** (or **European waterfall**), the LPs receive all distributions until they have received 100% of their initial investment plus the hurdle rate (typically after all fund investments have been sold).

A **clawback provision** stipulates that if the GP accrues or receives incentive payments on gains that are subsequently reversed as the partnership exits deals, the LPs can recover previous (excess) incentive payments. With a deal-by-deal waterfall, it may be the case that successful deals are exited initially and losses are realized later. A clawback provision would allow the LPs to recover these incentive fees to the extent that the subsequent losses negate prior gains on which incentive fees had been paid.

MODULE QUIZ 58.1

1. Compared with managers of traditional investments, managers of alternative investments are likely to have fewer restrictions on:
   A. holding cash.
   B. buying stocks.
   C. using derivatives.
2. Compared with alternative investments, traditional investments tend to:
   A. be less liquid.
   B. be less regulated.
   C. require lower fees.
3. An investor that wants to gain exposure to alternative investments but does not have the in-house expertise to perform due diligence on individual deals is *most likely* to engage in:
   A. co-investing.
   B. fund investing.
   C. direct investing.
4. Management fees for a private capital fund are determined as a percentage of:
   A. invested capital.
   B. committed capital.
   C. assets under management.
5. For an investor in a private equity fund, the *least* advantageous of the following limited partnership terms is:
   A. a clawback provision.
   B. a European-style waterfall provision.
   C. an American-style waterfall provision.

KEY CONCEPTS

LOS 58.a

Alternative investments comprise various types of investments that do not fall under the heading of traditional investments. Categories of alternative investments include:

- Hedge funds.
- Private capital (private equity and private debt).
- Real estate.
- Natural resources (commodities, farmland, and timberland).
- Infrastructure.

LOS 58.b

Direct investing refers to purchasing assets independently, rather than pooling funds with others or using a specialized outside manager. Its advantages include not paying fees to outside managers and having control over which investments to make. Its disadvantages include high minimum investment amounts, possible lack of diversification, and the need to have expertise to evaluate deals.

Fund investing refers to pooling assets along with other investors, using a fund manager that selects and manages investments according to an agreed-upon strategy. Its advantages include having the expertise of the fund manager, less involvement and expertise required of the investor, diversification across investments, and lower minimum investment requirements. Its disadvantages include fund manager fees and the possibility that the fund manager may perform poorly.

Co-investing refers to fund investing that includes the right to invest additional capital directly alongside the fund manager. Its advantages include reduced overall fees, while benefiting from the manager's expertise, and more control over asset selection. Its disadvantages are that it requires greater expertise, due diligence, and involvement than fund investing.

LOS 58.c

Many alternative investments are structured as limited partnerships, in which the general partner is the fund manager and the limited partners are the investors. They are less regulated than publicly traded companies and typically only available to accredited investors. Limited partners may commit to an investment amount and, in some cases only, contribute a portion of that initially, providing the remaining funds over time as required by the general partner.

Fees in alternative investment funds often consist of a management fee and an incentive fee. For hedge funds, management fees are a percentage of assets under management. For private capital, management fees are a percentage of committed capital rather than capital invested.

Incentive fees are a portion of profits on fund investments. Typically, a hurdle rate must be exceeded before incentive fees are paid. With a soft hurdle rate, incentive fees are based on the total increase in the value of each partner's investment. With a hard hurdle rate, incentive fees are based only on gains above the hurdle rate. A high-water mark is a provision that no incentive fees are paid on gains that only offset prior losses.

A waterfall refers to the way payments are allocated to the general partner and the limited partners. With a deal-by-deal or American waterfall, profits are distributed as each fund investment is sold. With a whole-of-fund or European waterfall, the limited partners receive all distributions until they have received 100% of their initial investment plus the hurdle rate.

With a clawback provision, if the general partner receives incentive payments on gains that are subsequently reversed, the limited partners can recover excess incentive payments.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 58.1

1. **C** Traditional managers can hold cash and buy stocks but may be restricted from using derivatives. (LOS 58.a)
2. **C** Traditional investments typically require lower fees, are more regulated, and are more liquid than alternative investments. (LOS 58.a)
3. **B** With fund investing, due diligence on the fund's portfolio investments is a responsibility of the fund manager rather than the fund investors. Direct investing and co-investing require greater due diligence of individual deals on the part of the investor. (LOS 58.b)
4. **B** For a private capital fund, management fees are a percentage of committed capital rather than invested capital. For a hedge fund, management fees are a percentage of assets under management. (LOS 58.c)
5. **C** An American-style waterfall structure has a deal-by-deal calculation of incentive fees to the general partner. In this case, a successful deal where incentive fees are paid, followed by the sale of a holding that has losses in the same year, can result in incentive fees greater than those calculated with a European-style (whole-of-fund) waterfall. A clawback provision benefits the limited partner investors by allowing them to recover incentive fees paid earlier if the fund realizes losses later. A clawback provision coupled with an American-style waterfall will result in the same overall incentive fees as a European-style waterfall if the transactions occur in subsequent years. (LOS 58.c)

READING 59

PERFORMANCE CALCULATION AND APPRAISAL OF ALTERNATIVE INVESTMENTS

MODULE 59.1: PERFORMANCE APPRAISAL AND RETURN CALCULATIONS

Video covering this content is available online.

LOS 59.a: Describe issues in performance appraisal of alternative investments.

Alternative investments are typically exposed to some risks that unleveraged long-only traditional investments are not. These additional risks arise from the following:

- Lack of transparency.
- Illiquidity, including restrictions on and performance impact of redemptions.
- Complexity of positions and strategies employed.
- Use of derivatives.
- Use of securities that are marked to market.
- Use of leverage.
- Variety of manager strategies and areas of expertise.
- Cash drag from significant drawdown periods (e.g., with private equity).

Ideally, returns on alternative investments should be adjusted for these risks, although that may be difficult in practice. It is clear, however, that evaluating alternative investment returns (or expected returns) without considering these additional risks would be naïve and possibly quite misleading.

One commonly reported risk-adjusted return measure for alternative investments is the **Sharpe ratio**, which is calculated as excess return per unit of risk:

$$\text{Sharpe ratio} = \frac{\text{return} - \text{risk free rate}}{\text{standard deviation of returns}}$$

The biggest issue with using this statistic is that standard deviation may not be the appropriate measure of risk. Returns on alternative investments might not be normally distributed. When return skewness is significantly different from zero or return kurtosis is significantly different from three, standard deviation is not the most appropriate measure of risk, so Sharpe ratios are not the most appropriate measure of risk-adjusted returns. Additionally, if values used to calculate mean returns are based on model pricing (rather than market values), returns are smoothed relative to market-based return measures, so estimated standard deviation will understate the actual volatility of returns (uncertainty about actual asset values over time).

Some analysts prefer the **Sortino ratio** to the Sharpe ratio as a risk-adjusted return measure. The Sortino ratio substitutes downside deviation, which is based on only deviations below the mean (expected) return, for standard deviation in the Sharpe ratio. One limitation of both the Sharpe and Sortino ratios is that they do not take into account the diversification benefits from low correlations with returns to traditional investments.

The **Treynor ratio** uses beta (a measure of systematic risk that is based on return correlations) as the denominator. A lower beta (resulting from low correlation of returns with traditional assets) results in a higher Treynor ratio. A higher Sharpe, Sortino, or Treynor ratio indicates greater excess returns per unit of risk taken.

The **Calmar ratio** is computationally less complex. It is calculated as the average annual compound return divided by the maximum drawdown (decrease in value from peak to trough). Larger values are preferred, as again we are measuring returns relative to risk.

Lower correlations of returns among alternative investments in a portfolio are preferred to higher correlations because of diversification (risk-reduction) benefits. One caveat is that during periods of financial stress or crisis, these correlations, along with the correlation to returns of traditional investments, tend to increase toward one (that is, at such times asset prices all tend to decrease together).

Performance Appraisal for Private Capital and Real Estate

Both private capital and real estate investments are often characterized by inflows as well as outflows over an investment's life, and by cash returned to investors at various times over an investment's life. One simple measure of investment success is the **multiple of invested capital** (or **money multiple**)—the ratio of total capital returned plus the value of any remaining assets, to the total capital paid in over the life of the investment. Because this measure does not consider the timing of cash inflows and outflows, which can affect annual returns on invested capital significantly, it can be considered somewhat naïve. For this reason, internal rates of return, which are dependent on the timing of cash flows, are often used as performance measures for private equity and real estate investments. Comparing investments based on IRRs requires specific assumptions about both the cost of investment capital and the reinvestment returns that can be earned on cash distribution.

For funds investing in private equity or real estate where initial committed capital is drawn down over time as individual investments are made, if returns are calculated on committed capital, there is a fee drag on calculated returns. Typically, investments are made over a significant period of time and returns are staggered over subsequent periods. This results in a pattern of quite low returns on investment early in the fund's life and relatively higher returns over the final years. In real estate funds, significant investments in property improvements after acquisition may amplify this pattern of returns on investment.

Given the variability of cash flows over a fund's life and the importance of management decisions in the timing and magnitude of after-tax cash flows, an IRR over the life of a fund is the most appropriate measure of after-tax investment performance.

LOS 59.b: Calculate and interpret returns of alternative investments both before and after fees.

Before-fee returns on alternative investments are calculated the same way we calculate returns on any investment. The calculation of holding period returns and periodic rates of return, both with and without interim cash distributions, is described in Quantitative Methods and expanded on by application in other topic areas.

Calculating after-fee returns simply requires adjustment of the cash flows or values for the various fees involved, typically management and incentive fees. Some examples will illustrate the application of various fees and the relevant terminology.

EXAMPLE: **Hedge fund fees**

BJI Funds is a hedge fund with a value of $110 million at initiation. BJI Funds charges a 2% management fee based on assets under management at the beginning of the year and a 20% incentive fee with a 5% soft hurdle rate, and it uses a high-water mark. Incentive fees are calculated on gains net of management fees. The year-end values before fees are as follows:

- Year 1: $100.2 million
- Year 2: $119.0 million

Calculate the total fees and the investor's after-fee return for both years.

Answer:

Year 1:

Management fee: 110.0 million × 2% = $2.2 million

Gross value end of year (given): $100.2 million

Return net of management fees: $\frac{100.2 - 2.2}{110} - 1 = -10.9\%$

There is no incentive fee because the return after the management fee is less than the 5% hurdle rate.

Total fees: $2.2 million

Ending value net of fees: 100.2 million – $2.2 million = $98.0 million

Year 1 after-fees return: $\frac{98.0}{110.0} - 1 = -10.9\%$

Year 2:

Management fees: 98.0 million × 2% = $1.96 million

Year-end value net of management fees: $119.0 – $1.96 = $117.04 million

The high-water mark is $110 million.

Year 2 value net of management fees, above high-water mark: 117.04 million – 110.0 million = $7.04 million

Year 2 return net of management fees, above high-water mark: $\frac{7.04}{110} = 6.4\%$

Note that the incentive fee is calculated based on gains in value above $110 million because that is the high-water mark.

The incentive fee is calculated on the entire gain above the high-water mark because 6.4% is greater than the soft hurdle rate. If the 5% was a hard hurdle rate, the incentive fee would be calculated only on the gains more than 5% above the high-water mark.

Incentive fee: 7.04 × 0.20 = $1.41 million

Total fees: $1.96 million + $1.41 million = $3.37 million

Year 2 year-end value after fees: 119.0 – 3.37 = $115.63 million

Year 2 after-fee return: $\frac{115.63}{98.0} - 1 = 18.0\%$

EXAMPLE: **Fund-of-funds**

An investor makes a total investment of $60 million in a fund-of-funds that has a "1 and 10" fee structure, with management and incentive fees calculated independently based on year-end values. $40 million of the investment was allocated to the Alpha fund, and $20 million was allocated to the Beta fund. One year later, the value of the Alpha fund investment is $45 million and the value of the Beta fund investment is $28 million, both net of fund fees. Calculate the investor's return for the year net of fees.

Answer:

At year-end, the gross value of the investor's investment is $45 + $28 = $73 million.

The fund-of-funds management fee is 1% of $73 million, which is $0.73 million.

The investor's gain for the year before fund-of-funds fees is $73 – $60 = $13 million.

The fund-of-funds manager's incentive fee is 10% of $13 million, which is $1.3 million.

The year-end value of the investor's fund-of-funds investment is $73 – $0.73 – $1.3 = $70.97 million.

The investor's one-year return after fees is. $\frac{70.97}{60} - 1 = 18.3\%$

Note that the same investments made directly with the Alpha and Beta funds would have returned $\frac{73}{60} - 1 = 21.7\%$.

EXAMPLE: **Waterfall structure and clawback provision**

A private equity fund invests $100 million in a venture company that is sold for $130 million.

It also invests $100 million in an LBO that goes poorly and is liquidated for $80 million.

1. If the carried interest incentive fee for the GP is 20% and there is no clawback provision, what is the investor's return after incentive fees, assuming the investment outcomes are realized in the same year:
    a. under an American-style (deal-by-deal) waterfall structure?
    b. under a European-style (whole-of-fund) waterfall structure?
2. How would the answers be affected if the venture investment was sold in year 1 and the LBO investment was sold in year 2?
3. How would including a clawback provision affect investor returns calculated in question 1?

Answer:

1. Under an American-style (deal-by-deal) waterfall structure, an incentive fee of 20% × ($130 – $100) = $6 million would be paid on the venture investment. Because there is a loss on the LBO investment, no incentive fee is paid.

    Investor's return on investment: $\frac{130 + 80 - 6}{200} - 1 = 2\%$

    Under a European-style (whole-of-fund) waterfall structure, the gain for the period is 130 + 80 – 200 = $10 million and the incentive fee is 20% × 10 = $2 million.

    Investor's return on investment: $\frac{130 + 80 - 2}{200} - 1 = 4\%$

2. The European-style waterfall structure would have the same overall return as the American-style structure, as the incentive fee for the venture investment of $6 million would be paid in year 1 and no incentive fee would be received on the LBO investment.
3. With a clawback provision, after the LBO investment is sold, the incentive fee of $6 million paid on the venture investment is more than 20% of the return on the total investment. It is 60% of the total (net)

gain of $10 million. The investor could "claw back" $4 million of the $6 million paid as an incentive fee on the venture investment so that the total incentive fee is reduced to 20% of the $10 million gain.

Negotiated fee structures

Although "2 and 20" and "1 and 10" were at one time fairly standard fee structures for fund and fund-of-funds investments, these fee structures have been under competitive pressure. Investors making larger commitments can negotiate lower fees. There can also be a trade-off between liquidity provisions and fees. Investors can negotiate for lower fees or better liquidity (shorter lockups and notice periods). Hurdle rates, hard versus soft hurdles, and catch-up provisions may also be subject to negotiation.

Annual investor fees can also be **either-or fees**, the maximum of the management fee or the incentive fee. Under such a structure, with a 1% management fee and a 30% incentive fee, investor fees each year would be simply the management fee unless the calculated incentive fee is greater. Such a structure may also stipulate that the 1% management fees paid be subtracted from incentive fees when they are paid in a subsequent year.

Investors should be aware that other investors may receive terms that differ from those in the partnership agreement. Customized fee structures are contained in **side letters** to individual investors detailing how their terms differ from those in the standard offering documents.

Early investors in a fund may also receive lower fees or better liquidity terms as an incentive to invest at the inception of a fund. The investment interests of early investors that receive such relatively better terms are called **founder's shares**.

MODULE QUIZ 59.1

1. Standard deviation is *least likely* an appropriate measure of risk for:
   A. hedge funds.
   B. publicly traded REITs.
   C. exchange-traded funds.
2. A hedge fund has a return of 30% before fees in its first year. The fund has a management fee of 1.5% on end-of-year fund value and a 15% incentive fee, with an 8% hard hurdle rate on gains net of the management fee. The return after fees for an investor in this fund is *closest* to:
   A. 20.5%.
   B. 21.5%.
   C. 25.0%.
3. A private equity fund has a "2 and 20" fee structure with the incentive fee independent of management fees. The fund will sell a holding for a profit of 9%. The hurdle rate is specified as 8%. The provision that would result in an incentive fee of 1% is:
   A. a hard hurdle rate.
   B. a soft hurdle rate.
   C. a catch-up provision.

KEY CONCEPTS

LOS 59.a

Alternative investments are typically exposed to risks that traditional investments are not, including lack of transparency, illiquidity, complexity of positions and strategies, use of derivatives, securities that are marked to market, use of leverage, variety of manager strategies, and cash drag from significant drawdown periods. Return measures should be adjusted for these risks.

The Sharpe ratio (excess return per unit of risk) may not be appropriate because it measures risk as standard deviation, but alternative investment returns are unlikely to be normally distributed. Other measures available include the Sortino ratio (which uses downside deviations), the Treynor ratio (which uses beta as a risk measure), and the Calmar ratio (average annual compound return divided by the maximum drawdown).

Return measures for real estate and private capital include internal rates of return and the multiple of invested capital (ratio of total capital returned plus the value of remaining assets, to total capital paid in).

LOS 59.b

Before-fee returns on alternative investments are calculated just as we calculate fees on any investment. Calculating after-fee returns simply requires adjustment of the cash flows or values for the various fees involved, typically management and incentive fees.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 59.1

1. **A** Hedge funds may hold illiquid assets that may use estimated values to calculate returns. Risk as measured by standard deviation could be understated. For publicly traded securities, such as REITs and ETFs, standard definitions of risk are more applicable. (LOS 59.a)
2. **C** This is easiest to see with an assumed initial value. Assume 100 was invested.

   Year-end gross value = 130

   Management fee = 1.5% × 130 = 1.95

   Gains net of management fee = 30 – 1.95 = 28.05

   Hurdle gains = 8% × 100 = 8

   Incentive fee = 15% × (30 – 1.95 – 8) = 3.0075

   Total fees = 1.95 + 3.0075 = 4.9575

   Ending value after fees = 130 – 4.9575 = 125.0425

   Return after fees = 125.0425/100 – 1 = 25.0425%

   (LOS 59.b)
3. **C** With a catch-up provision, the limited partners get the first 8% of gross return and the general partner gets all returns above that to a maximum of 2%, and gains above that are shared 80% to the limited partners and 20% to the general partner.

   With a soft hurdle rate of 8%, the incentive fee would be 20% of 9%, or 1.8%.

   With a hard hurdle rate of 8%, the incentive fee would be 20% of (9% – 8%), or 0.2%.

   (LOS 59.b)

READING 60

PRIVATE CAPITAL, REAL ESTATE, INFRASTRUCTURE, NATURAL RESOURCES, AND HEDGE FUNDS

MODULE 60.1: PRIVATE EQUITY AND PRIVATE DEBT

Video covering this content is available online.

LOS 60.a: Explain investment characteristics of private equity.

Private capital refers to entities that provide financing to companies without issuing securities in the public markets. The two categories of private capital we will review here are private equity and private debt.

Private Equity

The majority of private equity funds invest either in private companies or public companies they intend to take private (leveraged buyout funds) or in early stage companies (venture capital funds). The companies in which a private equity fund invests are known as its **portfolio companies**. Two additional, but smaller, categories of private equity funds are distressed investment funds and developmental capital funds.

A private equity fund may also charge fees for arranging buyouts, for a deal that does not happen, or for handling asset divestitures after a buyout.

Private equity strategies

Leveraged buyouts (LBOs) are the most common type of private equity fund investment. "Leveraged" refers to the fact that the fund's purchase of the portfolio company is funded primarily by debt. When a private equity fund acquires a publicly traded company in an LBO, the company is said to be "going private."

Two types of LBOs are **management buyouts (MBOs)**, in which the existing management team is involved in the purchase, and **management buy-ins (MBIs)**, in which an external management team will replace the existing management team.

In an LBO, the private equity firm seeks to increase the value of the firm through some combination of new management, management incentives, restructuring, cost reduction, or revenue enhancement. Firms with high cash flow are attractive LBO candidates because their cash flow can be used to service and eventually pay down the debt taken on for acquisition.

Developmental capital or **minority equity investing** refers to the provision of capital for business growth or restructuring. The firms financed may be public or private. In the case of public companies, such financing is called **private investment in public equities (PIPEs)**.

Venture capital funds invest in companies in the early stages of their development. The investment often is in the form of equity but can be in convertible preferred shares or convertible debt. While the risk of start-up companies is often great, returns on successful companies can be very high. This is often the case when a company has grown to the point where it is sold (at least in part) to the public via an IPO.

Venture capital fund managers are actively involved in the development of their portfolio companies, often sitting on their boards or filling key management roles.

Categorization of venture capital investments is based on the company's stage of development. Terminology used to identify venture firm investment at different stages of the company's life includes the following:

1. The **formative stage** refers to investments made during a firm's earliest period and comprises three distinct phases.
   - **Angel investing** or **pre-seed capital** refers to investments made very early in a firm's life, often the idea stage, and the investment funds are used for business plans and assessing market potential. The funding source is usually individuals ("angels") rather than venture capital funds.
   - The **seed stage** or **seed capital** refers to investments made for product development, marketing, and market research. This is typically the stage during which venture capital funds make initial investments, through ordinary or convertible preferred shares.
   - The **early stage** or **start-up stage** refers to investments made to fund initial commercial production and sales.
2. **Later-stage investment** or **expansion venture capital** refers to the stage of development where a company already has production and sales and is operating as a commercial entity. Investment funds provided at this stage are typically used for expansion of production and/or increasing sales through an expanded marketing campaign.
3. **Mezzanine-stage financing** refers to capital provided to prepare the firm for an IPO. The term refers to the timing of the financing (between private company and public company) rather than the type of financing.

Private equity exit strategies

The average holding period for companies in private equity portfolios is five years. There are several primary methods of exiting an investment in a portfolio company:

1. **Trade sale.** Sell a portfolio company to a competitor or another strategic buyer.
2. **IPO.** Sell all or some shares of a portfolio company to the public.
3. **Recapitalization.** The company issues debt to fund a dividend distribution to equity holders (the fund). This is not an exit, in that the fund still controls the company, but is often a step toward an exit.

4. **Secondary sale.** Sell a portfolio company to another private equity firm or a group of investors.
5. **Write-off/liquidation.** Reassess and adjust to take losses from an unsuccessful outcome.

Private equity potential benefits and risks

There is evidence that over the last 20 years, returns on private equity funds have been higher on average than overall stock returns. Less-than-perfect correlation of private equity returns with traditional investment returns suggests that there may be portfolio diversification benefits from including private equity in portfolios.

The standard deviation of private equity returns has been higher than the standard deviation of equity index returns, suggesting greater risk. As with hedge fund returns data, private equity returns data may suffer from survivorship bias and backfill bias (both lead to overstated returns). Because portfolio companies are revalued infrequently, reported standard deviations of returns and correlations of returns with equity returns may both be biased downward.

LOS 60.b: Explain investment characteristics of private debt.

Private debt refers to lending to private entities. The terms used include the following:

- **Direct lending.** Loans made directly to a private entity without an intermediary. A **leveraged loan** refers to a loan made by a private debt fund using money borrowed from another source. That is, the fund's portfolio of loans is leveraged through the use of fund borrowing to magnify fund returns.
- **Venture debt.** Lending to venture firms (start-up or early stage firms that are not yet profitable). Venture debt is often convertible to the venture firm's common stock or combined with warrants granting the right to buy the venture firm's common stock at a given price for a specified period.
- **Mezzanine loans.** Private debt that is subordinated—that is, has a lower priority of claims than the borrower's existing (more senior) debt. May have special features such as conversion rights or warrants as compensation for the additional risk involved.
- **Distressed debt.** Purchasing the debt of firms in bankruptcy, in default on existing loans, or for which default seems imminent. Often, the fund becomes active in implementing a plan for the debtor firm to restructure its existing debt or make other changes that will result in an increase in the value of the acquired debt.

Private debt firms may also invest in a wide variety of other types of debt. Some private capital firms invest in both the debt and equity of their portfolio companies.

Risks and Returns of Private Capital

Given that private capital encompasses a variety of securities, from start-up equity to the senior debt of a mature company, the range of risks across the spectrum of private capital funds is very large. Additionally, investment in private equity or private debt funds may have quite limited liquidity.

Returns on private debt are necessarily higher than those on the debt of publicly traded companies because lack of liquidity increases the required returns on both private debt and private equity. Most often, both private equity and private debt funds are more active in the operations of the portfolio firms. Both venture capital and private equity firms often work as partners with a portfolio company's management, or as part of its management, to develop and implement a business strategy. Fund returns are quite dependent on the ability of the fund manager to both select and manage fund investments.

It is important for investors to understand the various risks that an investment in a given private capital fund entails and not to view returns above those of traditional debt and equity investments as a "free lunch." Diligence in manager selection is an important part of investment success.

MODULE QUIZ 60.1

1. In which stage of a firm's development is a venture capital fund *most likely* to make its initial investment?
   A. Start-up.
   B. Seed capital.
   C. Angel investing.
2. In a secondary sale, a private capital firm sells one of its portfolio companies to:
   A. the public.
   B. a competitor in its industry.
   C. another private capital fund.

MODULE 60.2: REAL ESTATE, INFRASTRUCTURE, AND NATURAL RESOURCES

Video covering this content is available online.

LOS 60.c: Explain investment characteristics of real estate.

Investment in **real estate** can provide income in the form of rents, as well as the potential for capital gains. Real estate as an asset class can provide diversification benefits to an investor's portfolio and a potential inflation hedge because rents and real estate values tend to increase with inflation. Real estate investments can be differentiated according to their underlying assets. Assets included under the heading of real estate investments include the following:

- Residential property—single-family homes.
- Commercial property—produces income (e.g., office buildings).
- Loans with residential or commercial property as collateral—mortgages ("whole loans"), construction loans.

Residential property is considered a direct investment in real estate. Some buyers pay cash, but most take on a mortgage (borrow) to purchase. The issuer (lender) of the mortgage has a direct investment in a whole loan and is said to "hold the mortgage." Issuers often sell the mortgages they originate, which are then pooled (securitized) as publicly traded mortgage-backed securities (MBS) that represent an indirect investment in the mortgage loan pool. Property purchased with a mortgage is called a leveraged investment, and the owner's equity is the property value minus the outstanding loan amount. Changes in property value over time, therefore, affect the property owner's equity in the property.

Commercial real estate properties generate income from rents. Homes purchased for rental income are considered investments in commercial property. Large properties (e.g., an office building) are a form of direct investment for institutions or wealthy individuals, either purchased for cash or leveraged (a mortgage loan is taken for a portion of the purchase price). Long-time horizons, illiquidity, the large size of investments needed, and the complexity of the investments make commercial real estate inappropriate for many investors. Commercial real estate properties can also be held by a limited partnership, in which the partners have limited liability and the general partner manages the investment and the properties, or by a real estate investment trust.

As with residential mortgages, whole loans (commercial property mortgages) are considered a direct investment, but loans can be pooled into commercial mortgage-backed securities (CMBS) that represent an indirect investment.

Real estate investment trusts (REITs) issue shares that trade publicly like shares of stock. REITs are often identified by the type of real estate assets they hold: mortgages, hotel properties, malls, office buildings, cell phone towers, or other commercial properties. Income is used to pay dividends. Typically, 90% of income must be distributed to shareholders to avoid taxes on this income that would have to be paid by the REIT before distribution to shareholders.

Potential Benefits and Risks of Real Estate

Real estate performance is measured by three different types of indices:

- An **appraisal index**, such as those prepared by the National Council of Real Estate Investment Fiduciaries (NCREIF), is based on periodic estimates of property values. Appraisal index returns are smoother than those based on actual sales and have the lowest standard deviation of returns of the various index methods.
- A **repeat sales index** is based on price changes for properties that have sold multiple times. The sample of properties sold and thus included in the index is not necessarily random and may not be representative of the broad spectrum of properties available (an example of sample selection bias).
- **REIT indices** are based on the actual trading prices of REIT shares, similar to equity indices.

Historically, REIT index returns and global equity returns have had a relatively strong correlation (on the order of 0.6) because business cycles affect REITs and global equities similarly. The correlation between global bond returns and REIT returns has been very low historically. In either case, diversification benefits can result from including real estate in an investor's portfolio. However, the method of index construction (e.g., appraisal or repeat sales indices) may be a factor in the low reported correlations, in which case actual diversification benefits may be less than expected.

Real Estate Investment Due Diligence

Property values fluctuate because of global and national economic factors, local market conditions, and interest rate levels. Other specific risks include variation in the abilities of managers to select and manage properties and changes in regulations. Decisions regarding selecting, financing, and managing real estate projects directly affect performance. The degree of leverage used in a real estate investment is important because leverage amplifies losses as well as gains.

Distressed-property investing has additional risk factors compared with investing in properties with sound financials and stable operating histories.

Real estate development has additional risk factors, including regulatory issues such as zoning, permitting, and environmental considerations or remediation, and economic changes and financing decisions over the development period. The possible inability to get long-term financing at the appropriate time for properties initially developed with temporary (short-term) financing presents another risk.

LOS 60.d: Explain investment characteristics of infrastructure.

Infrastructure investments include transportation assets, such as roads, airports, ports, and railways, as well as utility assets, such as gas distribution facilities, electric generation and distribution facilities, and waste disposal and treatment facilities. Other categories of infrastructure investments are communications (e.g., broadcast assets and cable systems) and social (e.g., prisons, schools, and healthcare facilities).

Investments in infrastructure assets that are already constructed are referred to as **brownfield investments**, and investments in infrastructure assets that are to be constructed are referred to as **greenfield investments**. In general, investing in brownfield investments provides stable cash flows and relatively high yields but offers little potential for growth. Investing in greenfield investments is subject to more uncertainty and may provide relatively lower yields but offers greater growth potential.

In addition to categorizing infrastructure investments by type or by whether or not construction of the assets is complete, they may be categorized by their geographic location.

Investment in infrastructure can be made by constructing the assets and either selling or leasing them to the government or by directly operating the assets. Alternatively, investment in infrastructure can be made by purchasing existing assets from the government to lease back to the government or operate directly. Infrastructure investments can also be made by a public–private partnership.

Infrastructure assets typically have a long life and are quite large in cost and scale, so direct investment in them has low liquidity. However, more-liquid investments backed by infrastructure assets are available through exchange-traded funds (ETFs), mutual funds, private equity funds, or master limited partnerships (MLPs). Publicly traded vehicles for investing in infrastructure are a small part of the overall universe of infrastructure investments and are relatively concentrated in a few categories of assets.

Investing in infrastructure assets can provide diversification benefits, but investors should be aware that they are often subject to regulatory risk, risk from financial leverage, and the possibility that cash flows will be less than expected. Investors who construct infrastructure assets have construction risk. When the assets are owned and operated by a private owner, operational risk must also be considered.

LOS 60.e: Explain investment characteristics of natural resources.

Commodities

While it is possible to invest directly in commodities such as grain and gold, the most commonly used instruments to gain exposure to commodity prices are derivatives. Commodities themselves are physical goods and thus incur costs for storage and transportation. Returns are based on price changes and not on income streams.

Futures, forwards, options, and swaps are all available forms of commodity derivatives. Futures trade on exchanges; some options trade on exchanges while others trade over the counter; and forwards and swaps are over-the-counter instruments originated by dealers. Futures and forwards are contractual obligations to buy or sell a commodity at a specified price and time. Options convey the right, but not the obligation, to buy or sell a commodity at a specified price and time.

Other methods of exposure to commodities include the following:

- Exchange-traded funds **(commodity ETFs)** are suitable for investors who are limited to buying equity shares. ETFs can invest in commodities or commodity futures and can track prices or indexes.
- **Managed futures funds**, such as commodity trading advisers (CTAs), are actively managed. Some managers concentrate on specific sectors (e.g., agricultural commodities), while others are more diversified. Managed futures funds can be structured as limited partnerships with fees like those of hedge funds (e.g., 2 and 20) and restrictions on the number, net worth, and liquidity of the investors. They can also be structured like mutual funds with shares that are publicly traded so that retail investors can also benefit from professional management. Additionally, such a structure allows a lower minimum investment and greater liquidity compared with a limited partnership structure.
- Specialized funds in specific commodity sectors can be organized under any of the structures we have discussed and focus on certain commodities, such as oil and gas, grains, precious metals, or industrial metals.

Potential benefits and risks of commodities

Returns on commodities over time have been lower than returns on global stocks or bonds. As with other investments, speculators can earn high returns over short periods when their expectations about short-term commodity price movements are correct and they act on them.

Historically, correlations of commodity returns with those of global equities and global bonds have been low, typically less than 0.2, so adding commodities to a traditional portfolio can provide diversification benefits. Because commodity prices tend to move with inflation rates, holding commodities can act as a hedge of inflation risk. To the extent that commodity prices move with inflation, the real return over time would be zero, although futures contracts may offer positive real returns.

Commodity prices and investments

Spot prices for commodities are a function of supply and demand. Demand is affected by the value of the commodity to end-users and by global economic conditions and cycles. Supply is affected by production and storage costs and existing inventories. Both supply and demand are affected by the purchases and sales of nonhedging investors (speculators).

For many commodities, supply is inelastic in the short run because of long lead times to alter production levels (e.g., drill oil wells, plant crops or decide to plant less of them). As a result, commodity prices can be volatile when demand changes significantly over the economic cycle. Production of some commodities, especially agricultural commodities, can be significantly affected by the weather, leading to high prices when production is low and low prices when production is high. Costs of extracting oil and minerals increase as more expensive methods or more remote areas are used. To estimate future needs, commodity producers analyze economic events, government policy, and forecasts of future supply. Investors analyze inventory levels, forecasts of production, changes in government policy, and expectations of economic growth in order to forecast commodity prices.

Commodity valuation

Wheat today and wheat six months from today are different products. Purchasing the commodity today will give the buyer the use of it if needed, while contracting for wheat to be delivered six months from today avoids storage costs and having cash tied up. An equation that considers these aspects is:

futures price ≈ spot price × (1 + risk-free rate) + storage costs − convenience yield

Convenience yield is the value of having the physical commodity for use over the period of the futures contract. If there is little or no convenience yield, futures prices will be higher than spot prices, a situation termed *contango*. When the convenience yield is high, futures prices will be less than spot prices, a situation referred to as *backwardation*.

Farmland and Timberland

Two additional assets under the heading of natural resources are timberland and farmland, for which one component of returns comes from sales of timber or agricultural products. Timberland returns also include price changes on the land, which depend on expectations of lumber prices and how much timber has been harvested. Farmland returns are based on land price changes, changes in farm commodity prices, and the quality and quantity of the crops produced.

While most agricultural crops must be harvested within a short period, timber is different in that the choice of when to harvest is based on current prices and expected growth rates. Because agricultural crops (including trees) consume carbon, they are attractive to investors with an ESG focus on climate change.

Risks of investing in farmland and timberland include low liquidity, high fixed costs of production, variable cash flows that depend on weather, and potential losses from natural disasters such as wildfires.

MODULE QUIZ 60.2

1. Direct commercial real estate ownership *least likely* requires investing in:
   A. large amounts.

B. illiquid assets.
C. a short time horizon.

2. Compared with purchasing commodities, long positions in commodity derivatives offer the benefit of:
   A. no storage costs.
   B. convenience yield.
   C. better correlation with spot prices.
3. Greenfield investments in infrastructure are *most accurately* described as investments in assets:
   A. that are operating profitably.
   B. that have not yet been constructed.
   C. related to environmental technology.

MODULE 60.3: HEDGE FUNDS

Video covering this content is available online.

LOS 60.f: Explain investment characteristics of hedge funds.

Hedge funds generally:

- Use leverage.
- Take both long and short positions.
- Use derivatives for speculation or hedging portfolio risk.

In addition to the structures for limited partnerships and types of fees paid to the general partner we have covered, hedge funds typically have restrictions on limited partner redemptions. A **lockup period** is the time after initial investment over which limited partners either cannot request redemptions or incur significant fees for redemptions (a **soft lockup**). A **notice period** (typically between 30 and 90 days) is the amount of time a fund has to fulfill a redemption request made after the lockup period has passed.

Hedge fund managers often incur significant transactions costs when they redeem shares. Redemption fees can offset these costs. Notice periods allow time for managers to reduce positions in an orderly manner. Redemptions often increase when hedge fund performance is poor over a period, and the costs of honoring redemptions may further decrease the value of the remaining partnership interests. This is an additional source of risk for hedge fund investors.

A **fund-of-funds** is an investment company that invests in hedge funds. Fund-of-funds investing can give investors diversification among hedge fund strategies, can provide expertise in selecting individual hedge funds, and can provide smaller investors with access to hedge funds in which they may not be able to invest directly.

Fund-of-funds managers charge an additional layer of fees beyond the fees charged by the individual hedge funds in the portfolio. Historically, these additional fees have been a 1% management fee and a 10% incentive fee. Because these fees to the fund-of-funds manager are on top of fees charged by the individual funds, they can significantly reduce investor net returns.

Recently, there has been market pressure to reduce hedge fund fees. Rather than the previous standard of 2 and 20, average hedge fund fees have fallen closer to 1.3% in management fees and 15% in incentive fees. Fund-of-funds fees have also fallen from 1 and 10 and some may charge only a management fee or a lower management fee combined with a reduced incentive fee.

Hedge Fund Strategies

Similar to categorizing alternative investments, classifying hedge funds can also be challenging. According to Hedge Fund Research, Inc., there are four main classifications of hedge fund strategies:

1. **Event-driven strategies** are typically based on a corporate restructuring or acquisition that creates profit opportunities for long or short positions in common equity, preferred equity, or debt of a specific corporation. Event-driven funds are typically long-biased.

   Subcategories are as follows:

   – **Merger arbitrage.** Buy the shares of a firm being acquired and sell short the firm making the acquisition. Although term "arbitrage" is used, such a strategy is not risk free because deal terms may change or an announced merger may not take place.

   – **Distressed/restructuring.** Buy the (undervalued) securities of firms in financial distress when analysis indicates that value will be increased by a successful restructuring; possibly short overvalued securities at the same time.

   – **Activist shareholder.** Buy sufficient equity shares to influence a company's policies, with the goal of increasing company value (e.g., by restructuring, change in strategy/management, or return of capital to equity holders).

   – **Special situations.** Invest in the securities of firms that are issuing or repurchasing securities, spinning off divisions, selling assets, or distributing capital.

2. **Relative value strategies** involve buying a security and selling short a related security, with the goal of profiting when a perceived pricing discrepancy between the two is resolved.

   – **Convertible arbitrage fixed income.** Exploit pricing discrepancies between convertible bonds and the common stock of the issuing companies and options on the common shares.

   – **Asset-backed fixed income.** Exploit pricing discrepancies among various MBS or asset-backed securities (ABS).

   – **General fixed income.** Exploit pricing discrepancies between fixed-income securities of various issuers and types.

   – **Volatility.** Exploit pricing discrepancies arising from differences between returns volatility implied by options prices and manager expectations of future volatility.

   – **Multistrategy.** Exploit pricing discrepancies among securities in asset classes different from those previously listed and across asset classes and markets.

3. **Macro strategies** are based on global economic trends and events and may involve long or short positions in equities, fixed income, currencies, or commodities. Managed futures funds may focus on trading commodity futures (these funds are known as **commodity trading advisers**, or CTAs) or incorporate financial futures.

4. **Equity hedge fund strategies** seek to profit from long or short positions in publicly traded equities and derivatives with equities as their underlying assets.

   – **Market neutral.** Use technical or fundamental analysis to select undervalued equities to be held long and to select overvalued equities to be sold short, in approximately equal amounts to profit from their relative price movements without exposure to market risk. Leverage may be used.

- **Fundamental long/short growth.** Use fundamental analysis to find high-growth companies. Identify and buy equities of companies that are expected to sustain relatively high rates of capital appreciation, and short equities of companies expected to have low or no revenue growth.
- **Fundamental value.** Buy equity shares that are believed to be undervalued based on fundamental analysis and sometimes short an index or companies believed to be overvalued. Exposures to value stocks and small-cap stocks often result.
- **Sector specific.** Identify opportunities within a sector, such as health care, biotech, technology, and financial services. Manager expertise within a specific sector is believed to lead to superior returns.
- **Short bias.** Employ technical and fundamental analysis and take predominantly short positions in overvalued equities, possibly with smaller long positions but with negative market exposure overall.

Hedge Fund Potential Benefits and Risks

Hedge fund returns have tended to be better than those of global equities in down equity markets and to lag the returns of global equities in up markets. Different hedge fund strategies have the best returns during different time periods. Statements about the performance and diversification benefits of hedge funds are problematic because of the great variety of strategies used. Less-than-perfect correlation with global equity returns may offer some diversification benefits, but correlations tend to increase during periods of financial crisis.

Characteristics of hedge fund indexes may bias returns and correlations with traditional investment returns. Because hedge funds might not be included in an index until they have been in existence for a given time period or until they reach a given size, index returns may exhibit **survivorship bias**. Funds that have been successful, so that they have stayed in business for multiple years or reached a specific level of assets under management, tend to be overrepresented in a hedge fund index, which biases returns upward. **Backfill bias** refers to the effect on historical index returns of adding fund returns for prior years to index returns when a fund is added to an index.

Model values and appraisal values are typically less volatile than market values. To the extent that funds use models or appraisals for asset valuation and return calculations, both standard deviations of fund returns and correlations of fund returns with those of traditional investments will be biased downward. Investors must understand these potential biases when using index returns to evaluate the risk and return characteristics of hedge funds.

MODULE QUIZ 60.3

1. An investor who chooses a fund-of-funds as an alternative to a single hedge fund is *most likely* to benefit from:
   A. lower fees.
   B. higher returns.
   C. more due diligence.
2. Diversification benefits from adding hedge funds to an equity portfolio may be limited because:
   A. correlations tend to increase during periods of financial crisis.
   B. hedge fund returns are less than perfectly correlated with global equities.

C. hedge funds tend to perform better when global equity prices are declining.

3. A hedge fund that operates as an activist shareholder is *most likely* engaging in:
   A. a macro strategy.
   B. a relative value strategy.
   C. an event-driven strategy.

KEY CONCEPTS

LOS 60.a

Private equity funds usually invest in the equity of private companies or companies wanting to become private, financing their assets with high levels of debt.

- Venture capital funds provide capital to companies early in their development. Stages of venture capital investing include the formative stage (composed of the angel investing, seed, and early stages), the later stage (expansion), and the mezzanine stage (prepare for IPO).
- Leveraged buyouts (LBOs) include management buyouts, in which the existing management team is involved in the purchase, and management buy-ins, in which an external management team replaces the existing management.
- Developmental capital or minority equity investing refers to providing capital for business growth or restructuring. The firms financed may be public or private. In the case of public companies, such financing is referred to as private investment in public equities.

Methods for exiting investments in portfolio companies include trade sale (sell to a competitor or another strategic buyer), IPO (sell some or all shares to investors), recapitalization (issue portfolio company debt), secondary sale (sell to another private equity firm or other investors), or write-off/liquidation.

LOS 60.b

Private debt refers to lending to private entities. Private debt investments include direct lending, venture debt, mezzanine loans, and distressed debt. Some private capital firms invest in both equity and debt.

LOS 60.c

Real estate as an asset class includes residential and commercial real estate, individual mortgages, and pools of mortgages or properties. It includes direct investment in single properties or loans, as well as indirect investment in limited partnerships, which are private securities, and mortgage-backed securities and real estate investment trusts, which are publicly traded.

Reasons to invest in real estate include potential long-term total returns, income from rent payments, diversification benefits, and hedging against inflation.

Real estate investment categories include residential properties, commercial real estate, real estate investment trusts, and mortgage-backed securities.

LOS 60.d

Infrastructure refers to long-lived assets that provide public services and are often built or operated by governments. Infrastructure investments may be classified as greenfield (assets to be built) or brownfield (existing assets).

Liquidity is low for direct investments in infrastructure because the assets are long-lived and tend to be large scale. However, some liquid investment vehicles exist that are backed by infrastructure assets.

LOS 60.e

Natural resource investments include commodities, farmland, and timberland.

Exposure to commodity prices is most commonly achieved with derivatives. Other methods of exposure include commodity ETFs, managed futures funds, and specialized funds in specific commodity sectors.

Commodity valuation is described by the following equation:

futures price ≈ spot price × (1 + risk-free rate) + storage costs − convenience yield

Returns on farmland and timberland include sales of the products raised on the land. Timberland returns also include price changes on the land, which depend on expected lumber prices and how much timber has been harvested. Farmland returns also depend on land price changes, changes in farm commodity prices, and the quality and quantity of the crops produced.

LOS 60.f

Hedge funds generally use leverage, take both long and short positions, and use derivatives. They typically have restrictions on limited partner redemptions, including a lockup period during which limited partners either cannot request redemptions or incur significant fees for redemptions and a notice period to fulfill a redemption request after the lockup period.

A fund-of-funds invests in other hedge funds. Its advantages are that it can give investors diversification among hedge fund strategies, provides expertise in selecting hedge funds, and provides access to hedge funds that are otherwise unavailable. Its disadvantage is an additional layer of fees beyond the fees charged by the hedge funds in the portfolio.

Event-driven strategies include merger arbitrage, distressed/restructuring, activist, and special situations.

Relative value strategies seek profits from unusual pricing issues.

Macro strategies are “top-down” strategies based on global economic trends.

Equity hedge strategies are “bottom-up” strategies that take long and short positions in equities and equity derivatives. Strategies include market neutral, fundamental growth, fundamental value, quantitative directional, short bias, and sector specific.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 60.1

1. **B** Venture capital funds typically make their initial investments during a firm’s seed stage for product development, marketing, and market research. At the angel investing stage, the

funding source is usually individuals rather than venture capital funds. The start-up stage or early stage follows the seed stage and refers to investments made to fund initial commercial production and sales. (LOS 60.a)

2. **C** In a secondary sale, a private capital firm sells one of its portfolio companies to another private capital fund or group of private investors. Selling a portfolio company to a competitor in its industry is known as a trade sale. Selling a portfolio company to the public requires an initial public offering. (LOS 60.a)

Module Quiz 60.2

1. **C** Commercial real estate ownership requires long-time horizons and purchasing illiquid assets that require large investment amounts.
(LOS 60.c)
2. **A** While commodity futures retain the risk and correlation characteristics of the underlying commodities, the investor does not incur storage costs. Derivatives cannot have higher correlation with spot prices than the commodity itself, as its price is the spot price. Convenience yield is a benefit of owning the actual commodities. (LOS 60.e)
3. **B** Greenfield investments refer to infrastructure assets that are yet to be constructed. (LOS 60.d)

Module Quiz 60.3

1. **C** A fund-of-funds manager is expected to provide more due diligence and better redemption terms. Funds-of-funds charge an additional layer of fees. Investing in funds-of-funds may provide more diversification, but may not necessarily provide higher returns. (LOS 60.f)
2. **A** Adding hedge funds to traditional portfolios may not provide the expected diversification to an equity portfolio because return correlations tend to increase during periods of financial crisis. (LOS 60.f)
3. **C** Activist shareholder strategies are a subcategory of event-driven strategies. (LOS 60.f)

TOPIC QUIZ: ALTERNATIVE INVESTMENTS

You have now finished the Alternative Investments topic section. Please log into your Schweser online dashboard and take the Topic Quiz on Alternative Investments. The Topic Quiz provides immediate feedback on how effective your study has been for this material. The number of questions on this quiz is approximately the number of questions for the topic on one-half of the actual Level I CFA exam. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow 1.5 minutes per question.

After you've completed this Topic Quiz, select "Performance Tracker" to view a breakdown of your score. Select "Compare with Others" to display how your score on the Topic Quiz compares with the scores of others who entered their answers.

APPENDIX

Rates, Returns, and Yields

A **holding period return (HPR)**, or holding period yield (HPY), can be for a period of any length and is simply the percentage increase in value over the period, which is calculated as:

HPR = ending value / beginning value – 1

1. If an investor puts $2,000 into an account and 565 days later it has grown in value to $2,700, the 565-day HPY is 2,700 / 2,000 – 1 = 35%.
2. If an investor buys a share of stock for $20/share, receives a $0.40 dividend, and sells the shares after nine months, the nine-month HPY is (22 + 0.40) / 20 – 1 = 12%.

An HPR for a given period is also the **effective yield** for that period.

An **effective annual yield** is the HPR for a one-year investment or the HPY for a different period converted to its annual equivalent yield.

3. If the six-month HPR is 2%, the effective annual yield is $1.02^2 - 1 = 4.040\%$.
4. If the 125-day HPR is 1.5%, the effective annual yield is $1.015^{365/125} - 1 = 4.443\%$.
5. If the two-year HPR (two-year effective rate) is 9%, the effective annual yield is $1.09^{1/2} - 1 = 4.4031\%$.

Compounding Frequency

Sometimes the “rate” on an investment is expressed as a **simple annual rate** (or *stated rate*)—the annual rate with no compounding of returns. The number of compounding periods per year is called the **periodicity** of the rate. For a periodicity of one, the stated rate and the effective annual rate are the same. When the periodicity is greater than one (more than one compounding period per year), the effective annual rate is the effective rate for the sub-periods, compounded for the number of sub-periods.

6. A bank CD has a stated annual rate of 6% with annual compounding (periodicity of 1); the effective annual rate is 6% and a $1,000 investment will return $1,000(1.06) = $1,060 at the end of one year.
7. A bank CD has a stated annual rate of 6% with semiannual compounding (periodicity of 2); the effective annual rate is $(1 + 0.06 / 2)^2 = 1.03^2 - 1 = 6.09\%$ and a $1,000 investment will return $1,000 (1.0609) = $1,060.90 at the end of one year.
8. A bank CD has a stated annual rate of 6% with quarterly compounding (periodicity of 4); the effective annual rate is $(1 + 0.06 / 4)^4 = 1.015^4 - 1 = 6.136\%$ and a $1,000 investment will return $1,000(1.06136) = $1,061.36 at the end of one year.

Note that increasing compounding frequency increases the effective annual yield for any given stated rate. In the limit, as compounding periods get shorter (more frequent), compounding is *continuous.* A stated rate of *r* %, with continuous compounding, results in an effective annual return of $e^r - 1$.

9. A bank CD has a stated annual rate of 6%, continuously compounded; its effective annual yield is $e^{0.06} - 1 = 6.184\%$ and a $1,000 investment will return $1,061.84 at the end of one year.

Bond Quotations and Terminology

The **stated (coupon) rate** on a bond is the total cash coupon payments made over one year as a percentage of face value.

10. A bond with a face value of $1,000 that pays a coupon of $50 once each year (an annual-pay bond) has a stated (coupon) rate of 50 / 1,000 = 5% and we say it has a periodicity of 1.
11. A bond with a face value of $1,000 that pays a coupon of $25 twice each year (a semiannual-pay bond) has a stated (coupon) rate of (25 + 25) / 1,000 = 5% and we say it has a periodicity of 2.
12. A bond with a face value of $1,000 that pays a coupon of $12.50(1.25%) four times each year (a quarterly-pay bond) has a coupon rate of (12.50 + 12.50 + 12.50 + 12.50) / 1,000 = 5% and we say it has a periodicity of 4.

The **current yield** on a bond is the stated (coupon) rate divided by the bond price as a percentage of face value or, alternatively, the sum of the coupon payments for one year divided by the bond price.

13. A bond with a stated coupon rate of 5% that is selling at 98.54% of face value has a current yield of 5 / 98.54 = 5.074%.
14. A bond that is trading at $1,058 and makes annual coupon payments that sum to $50 has a current yield of 50 / 1,058 = 4.726%.

The **yield to maturity** (YTM) of a bond, on an *annual basis*, is the effective annual yield and is used for bonds that pay an annual coupon. For bonds that pay coupons semiannually, we often quote the YTM on a *semiannual basis*, that is, two times the effective semiannual yield. To compare the yields of two bonds, we must calculate their YTMs on the same basis.

15. A bond with a YTM of 5% on a semiannual basis has a YTM on an annual basis (effective annual yield) of $(1 + 0.05 / 2)^2 - 1 = 5.0625\%$.
16. A bond with a YTM of 5% on an annual basis has a YTM on a semiannual basis of $(1.05^{1/2} - 1) \times 2 = 4.939\%$.

Internal Rate of Return (IRR)

The internal rate of return is the discount rate that makes the PV of a series of cash flows equal to zero. This calculation must be done with a financial calculator. We use the IRR for calculating the return on a capital project, the YTM on a bond, and the money weighted rate of return for a portfolio.

17. For the YTM of an annual-pay bond (YTM on an annual basis) on a coupon date with *N* years remaining until maturity, we calculate the annual IRR that satisfies:

$$-\text{bond price} + \frac{\text{coupon 1}}{1 + \text{IRR}} + \frac{\text{coupon 2}}{(1 + \text{IRR})^2} + \dots + \frac{\text{coupon N} + \text{face value}}{(1 + \text{IRR})^N} = 0$$

18. For the YTM of a semiannual-pay bond on a coupon date with *N* years remaining until maturity, we calculate the IRR that satisfies:

$$-\text{bond price} + \frac{\text{coupon 1}}{1+\frac{\text{IRR}}{2}} + \frac{\text{coupon 2}}{\left(1+\frac{\text{IRR}}{2}\right)^2} + \ldots + \frac{\text{coupon 2N} + \text{face value}}{\left(1+\frac{\text{IRR}}{2}\right)^{2N}} = 0$$

After solving for IRR / 2, which is the IRR for semiannual periods, we must multiply it by 2 to get the bond's YTM on a semiannual basis.

19. For a capital project, the (annual) IRR satisfies:

$$-\text{initial outlay} + \frac{CF_1}{1+\text{IRR}} + \frac{CF_2}{(1+\text{IRR})^2} + \ldots + \frac{CF_N}{(1+\text{IRR})^N} = 0$$

where annual cash flows (CF) can be positive or negative (when a future expenditure is required). Note that if the sign of the cash flows changes more than once, there may be more than one IRR that satisfies the equation.

Money Market Securities

For some money market securities, such as U.S. T-bills, price quotations are given on a bond discount (or simply discount) basis. The bond discount yield (BDY) is the percentage discount from face value of a T-bill, annualized based on a 360-day year, and is therefore not an effective yield but simply an annualized discount from face value.

20. A T-bill that will pay $1,000 at maturity in 180 days is selling for $984, a discount of 1 – 984 / 1,000 = 1.6%. The annualized discount is 1.6% × 360 / 180 = 3.2%.
21. A 120-day T-bill is quoted at a BDY of 2.83%, its price is [1 – (0.0283 × 120 / 360)] × 1,000 = $990.57. Its 120-day *holding period return* is 1,000 / 990.57 – 1 = 0.952%. Its *effective annual yield* is $(1{,}000 / 990.57)^{365/120} - 1 = 2.924\%$.

LIBOR (London Interbank Offered Rate) is an add-on rate quoted for several currencies and for several periods of one year or less, as an annualized rate.

22. HPY on a 30-day loan at a quoted LIBOR rate of 1.8% is 0.018 × 30 / 360 = 0.15% so the interest on a $10,000 loan is 10,000 × 0.0015 = $15.

A related yield is the **money market yield (MMY)**, which is HPY annualized based on a 360-day year.

23. A 120-day discount security with a maturity value of $1,000 that is priced at $995 has a money market yield of (1,000 / 995 – 1) × 360 / 120 = 1.5075%.

Forward rates are rates for a loan to be made in a future period. They are quoted based on the period of the loan. For loans of one year, we write 1y1y for a 1-year loan to be made one year from today and 2y1y for a 1-year loan to be made two years from today.

Spot rates are discount rates for single payments to be made in the future (such as for zero-coupon bonds).

24. Given a 3-year spot rate expressed as a compound annual rate (S_3) of 2%, a 3-year bond that makes a single payment of $1,000 in three years has a current value of $1{,}000 / (1 + 0.02)^3$ = $942.32.

An *N*-year spot rate is the geometric mean of the individual annual forward rates:

$$S_N = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)\ldots(1 + Ny1y)]^{1/N} - 1$$

and the annualized forward rate for M – N periods, *N* periods from now is:

$$NyMNy = \left[\frac{(1+S_M)^M}{(1+S_N)^N}\right]^{\frac{1}{M-N}} - 1$$

25. Given S_5 = 2.4% and S_7 = 2.6%, 5y2y = $[(1.026)^7 / (1.024)^5]^{1/2} - 1$ = 3.1017%, which is approximately equal to (7 × 2.6% – 5 × 2.4%) / 2 = 3.1%.

FORMULAS

Quoted add-on yield = HPY × 365/days to maturity

Quoted discount yield = discount on the security × 360/days to maturity

for an annual-coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{(1+\text{YTM})} + \frac{\text{coupon}}{(1+\text{YTM})^2} + \cdots + \frac{\text{coupon} + \text{principal}}{(1+\text{YTM})^N}$$

for a semiannual-coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)^2} + \cdots + \frac{\text{coupon} + \text{principal}}{\left(1+\frac{\text{YTM}}{2}\right)^{N\times 2}}$$

bond value using spot rates:

$$\text{no-arbitrage price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_2)^2} + \cdots + \frac{\text{coupon} + \text{principal}}{(1+S_N)^N}$$

full price between coupon payment dates:

(Bond value at last coupon date based on the current YTM) × $(1+ \text{YTM}/\#)^{t/T}$

where # is the number of coupon periods per year, *t* is the number of days from the last coupon payment date until the date the bond trade will settle, and *T* is the number of days in the coupon period.

flat price = full price − accrued interest

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}}$$

forward and spot rates: $(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$

option-adjusted spread: OAS = Z-spread − option value

$$\text{modified duration} = \frac{\text{Macaulay duration}}{1+\text{YTM}}$$

$$\text{approximate modified duration} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{YTM}}$$

$$\text{effective duration} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{curve}}$$

money duration = annual modified duration × full price of bond position

money duration per 100 units of par value =

annual modified duration × full bond price per 100 of par value

price value of a basis point: PVBP = $[(V_- - V_+) / 2]$ × par value × 0.01

$$\text{approximate convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta YTM)^2 V_0}$$

$$\text{approximate effective convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta \text{curve})^2 V_0}$$

$$\%\Delta \text{ full bond price} = -\text{annual modified duration}(\Delta YTM) + \frac{1}{2}\text{annual convexity}(\Delta YTM)^2$$

$$\%\Delta \text{ bond value} = -\text{duration}(\Delta\text{spread}) + \frac{1}{2}\text{convexity}(\Delta\text{spread})^2$$

duration gap = Macaulay duration − investment horizon

$$\text{return impact} \approx -\text{duration} \times \Delta\text{spread} + \frac{1}{2}\text{convexity} \times (\Delta\text{spread})^2$$

no-arbitrage forward price: $F_0(T) = S_0\,(1 + Rf)^T$

payoff to long forward at expiration = $S_T - F_0(T)$

value of forward at time *t*: $V_t(T) = [S_t + PV_t\,(\text{costs}) - PV_t(\text{benefit})] - F_0(T)\,(1 + Rf)^{-(T-t)}$

exercise value of a call = $\text{Max}[0, S - X]$

exercise value of a put = $\text{Max}[0, X - S]$

option value = exercise value + time value

put-call parity: $c + X(1 + Rf)^{-T} = S + p$

put-call-forward parity: $F_0\,(T)(1 + Rf)^{-T} + p_0 = c_0 + X(1 + Rf)^{-T}$

no-arbitrage forward price: $F_0(T) = S_0\,(1 + Rf)^T$

payoff to long forward at expiration = $S_T - F_0(T)$

value of forward at time *t*: $V_t(T) = [S_t + PV_t\,(\text{costs}) - PV_t(\text{benefit})] - F_0(T)\,(1 + Rf)^{-(T-t)}$

exercise value of a call = $\text{Max}[0, S - X]$

exercise value of a put = $\text{Max}[0, X - S]$

option value = exercise value + time value

put-call parity: $c + X(1 + Rf)^{-T} = S + p$

put-call-forward parity: $F_0\,(T)(1 + Rf)^{-T} + p_0 = c_0 + X(1 + Rf)^{-T}$

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