5.1.1 $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$.

Is R a function? If so determine its range.

(a) $R = \{(a,1), (b,2), (c,1), (d,2)\}$.

This is a function and $\text{Ran}(R) = \{1, 2\}$.

(b) $R = \{(a,1), (b,2), (a,2), (c,1), (d,2)\}$.

Not a function, since a maps to both 1 and 2.

5.1.10 Let $A = B = C = \mathbb{R}$ and $f: A \to B$, $g: B \to C$ be defined by

$$f(a) = a + 1 \quad \text{and} \quad g(b) = b^2 + 2.$$

(a) $(g \circ f)(-2) = g(-2+1) = g(-1) = (-1)^2 + 2 = 3.$

(b) $(f \circ g)(-2) = f((-2)^2 + 2) = f(6) = 6 + 1 = 7.$

(c) $(g \circ f)(x) = g(x+1) = (x+1)^2 + 2.$

(d) $(f \circ g)(x) = f(x^2+2) = x^2 + 2 + 1 = x^2 + 3.$

(e) $(f \circ f)(y) = f(y+1) = y + 1 + 1 = y + 2.$

(f) $(g \circ g)(y) = g(y^2+2) = (y^2+2)^2 + 2 = y^4 + 4y^2 + 6.$

5.1.13 Determine one to one, onto or both/neither.

(a) $A = B = \mathbb{Z}$; $f(a) = a - 1$.

If $f(a) = f(b)$ then $a - 1 = b - 1 \iff a = b$. Thus f is injective.

For any $b \in \mathbb{Z}$, there is $a = b + 1 \in \mathbb{Z}$ such that $f(a) = b$. Thus f is surjective.

(b) $A = \mathbb{R}$, $B = \{x \mid x \text{ is real and } x \geq 0\}$; $f(a) = |a|$.

Since $f(-1) = 1 = f(1)$, f is not injective.

For any $b \in B$ we have $a = b \in A$ such that $f(a) = b$, so f is surjective.

5.1.17 Let $f(n) = \max\{n, 50\}$ for $n \in \mathbb{Z}_+$. Determine 1-1/onto.

Both 1 and 2 map to 50, so f is not one to one.

Since $f(n) = 1$ has no solution, f is not onto.

5.1.19 Let $f: A \to B$ and $g: B \to A$. Verify that $g = f^{-1}$.

(a) $A = B = \mathbb{R}$; $f(a) = \frac{a+1}{2}$, $g(b) = 2b - 1$.

$$(f \circ g)(b) = f(2b-1) = \frac{2b - 1 + 1}{2} = b.$$

$$(g \circ f)(a) = g\left(\frac{a+1}{2}\right) = 2 \cdot \frac{a+1}{2} - 1 = a.$$

Hence $g = f^{-1}$.

(b) $A = \{x \in \mathbb{R} \mid x \geq 0\}$; $B = \{y \in \mathbb{R} \mid y \geq -1\}$;

$f(a) = a^2 - 1$, $g(b) = \sqrt{b+1}$.

$(f \circ g)(b) = f(\sqrt{b+1}) = \sqrt{b+1}^2 - 1 = b$.

$(g \circ f)(a) = g(a^2-1) = \sqrt{a^2 - 1 + 1} = a$.

So $g = f^{-1}$.

5.1.24 $A = \{1,2,3,4\} \xrightarrow{f} B = \{a,b,c,d\}$. Is f^{-1} a function?

$f = \{(1,a), (2,a), (3,c), (4,d)\}$

No, since 1 and 2 map to a, and so f is not one to one.

5.2.23 $Q(x)$: $\exists (y \in \mathbb{Z}_+)\ (xy = 60)$, evaluate:

(a) $Q(3)$ = true since $3 \cdot 20 = 60$.

(b) $Q(7)$ = false since $7 \nmid 60$.

(c) $Q(-6)$ = false since $\nexists y \in \mathbb{Z}_+ : -6 \mid 60$.

(d) $Q(15)$ = true since $15 \cdot 4 = 60$.

5.2.22 $P(x,y) = (x \vee y) \wedge \sim y$, evaluate:

(a) $P(T,T)$ = false.

(b) $P(F,T)$ = false.

(c) $P(T,F)$ = true.

5.2.25 Hashing function h takes first 3 digits and adds with last 4 digits then applies mod 59.

(a) $h(3759273) = 375 + 9273 \pmod{59}$

$= 9648 \pmod{59} = 31$.

(b) $h(7149021) = 714 + 9021 \pmod{59} = 0$.

(c) $h(5167249) = 516 + 7249 \pmod{59} = 36$.

5.2.24 Let h be as above. Assume 7500 records are to be stored.

(a) How many linked lists are required for storage? 59.

(b) If evenly distributed (approx), how many records will be stored by each linked list? $\frac{7500}{59}$