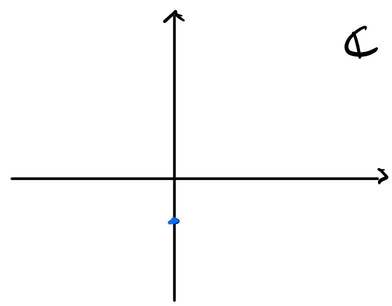
1. a)

$i^2 = -1 \qquad (-i)^2 = -1$

$i^3 = -i \qquad (-i)^3 = i$

$i^4 = 1 \qquad (-i)^4 = 1$

$i^5 = i$



$$(-i)^{-5} = \frac{1}{(-i)^5} = \frac{1}{-i} = i$$

b) + c)

$\mathrm{Re}(-5-7i) = -5$

$\mathrm{Im}(-5-7i) = -7$

d)

$7i - 5 = -5 + 7i$

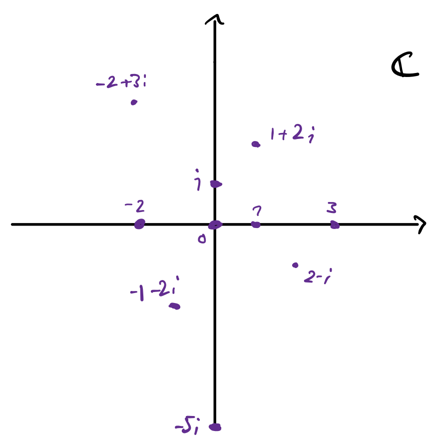
$i(7i-5) = -7 - 5i$

$i(7i-5)i = (-7-5i)i = 5 - 7i$

2. a) Kompleks: $-2,\ 0,\ i,\ 2-i,\ 1+2i,\ 1,\ -2+3i,\ -5i,\ 3,\ -1-2i$

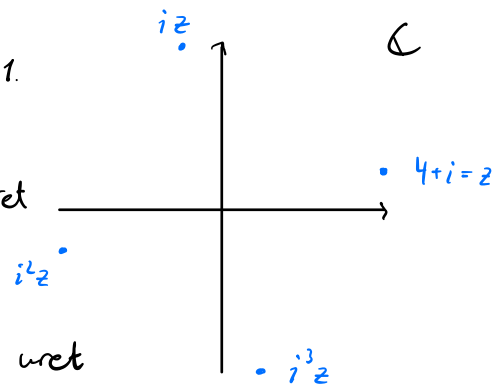
Reel: $-2,\ 0,\ 1,\ 3$

Imaginer: $i,\ -5i$



b) $z = 4 + i$

1.



2. Rotation mod uret
π rad
90 deg

3. Rotation med uret
— " —

3.a) 1. $(5+i)(1+9i) = 5 + 45i + i - 9 = -4 + 46i$

2. $i + i^2 + i^3 + i^4 = i - 1 - i + 1 = 0$

3. $$\frac{1}{1+3i} + \frac{1}{(1+3i)^2} = \frac{2+3i}{(1+3i)^2} = \frac{2+3i}{1+6i-9}$$

$$= \frac{(2+3i)(-8-6i)}{(-8+6i)(-8-6i)} = \frac{-16-12i-24i+18}{64+36}$$

$$= \frac{2-36i}{100} = \frac{1}{50} - \frac{9}{25}i$$

4. $$\frac{1}{(1+i)^4} = \frac{1}{2i \cdot 2i} = -\frac{1}{4}$$

5. $$\frac{5+i}{2-2i} = \frac{(5+i)(2+2i)}{8} = \frac{10+10i+2i-2}{8}$$

$$= \frac{8+12i}{8} = 1 + \frac{3}{2}i$$

6. $$\frac{3i}{4} = \frac{3}{4}i$$

$$\frac{i\,2}{4} = \frac{1}{2}i$$

b) $a, b \in \mathbb{R}$

1. $\dfrac{1}{a+ib} \neq \alpha + i\beta$, reelt divideret med komplekst tal.
ej formen $\alpha + i\beta$...

2. $$\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \underbrace{\frac{a}{a^2+b^2}}_{\alpha} - \underbrace{\frac{b}{a^2+b^2}}_{\beta} i$$

$$\operatorname{Re}\left(\frac{1}{a+ib}\right) = \frac{a}{a^2+b^2}, \quad \operatorname{Im}\left(\frac{1}{a+ib}\right) = -\frac{b}{a^2+b^2}$$

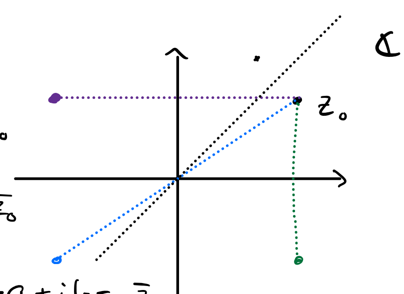
4.a) Lad $z = a+ib$ med $a, b \in \mathbb{R}$.

$$\bar{\bar{z}} = \overline{\overline{a+ib}} = \overline{a-ib} = a+ib$$

Lad $z_1 = z$ og $z_2 = c+id$ med $c, d \in \mathbb{R}$.

$$\overline{z_1 \cdot z_2} = \overline{(a+ib)(c+id)} = \overline{ac + i\,ad + i\,bc - bd}$$
$$= ac - bd - i(ad + bc)$$
$$\bar{z}_1 \cdot \bar{z}_2 = (a-ib)(c-id) = ac - i\,ad - i\,bc - bd$$
$$= ac - bd - i(ad + bc)$$

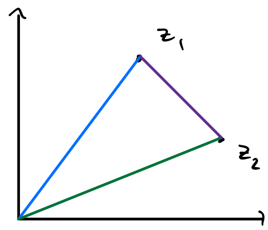
b) Lad $z_0 = a+ib \neq 0$.



1. Spejl om origo $-a-ib = -z_0$
2. om den reelle akse $a-ib = \bar{z}_0$
3. om den imaginære akse $-a+ib = -\bar{z}_0$
4. om vinkelhalveringslinjen $b + ia = \bar{z}_0 \cdot i$

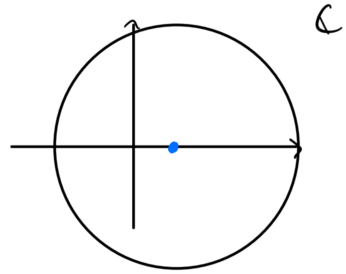
5. a) $z = a + ib \qquad |z| = \sqrt{a^2+b^2}$

b)



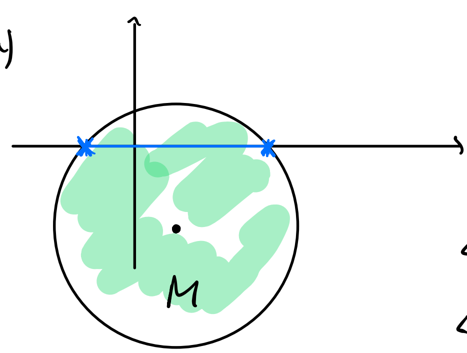
$$z_1 - z_2 = (a+ib) - (c+id)$$
$$= (a-c) + i(b-d)$$
$$|z_1 - z_2| = \sqrt{(a-c)^2 + (b-d)^2}$$

c) $\{ z \in \mathbb{C} \mid |z-1| = 3 \}$



6. $M = \{ z \in \mathbb{C} \mid |z - 1 + 2i| \leq 3 \}$
$= \{ z \in \mathbb{C} \mid |z - (1-2i)| \leq 3 \}$

a)



b) $|z - (1-2i)| = 3 \quad \wedge \quad z \in \mathbb{R}$

$$\sqrt{(z-1)^2 + 2^2} = 3$$
$$\Leftrightarrow \quad (z-1)^2 = 5$$
$$\Leftrightarrow \quad z = 1 \pm \sqrt{5}$$

$$z \in [1-\sqrt{5}, 1+\sqrt{5}] \subset \mathbb{R}.$$

7. $a = \frac{41}{42}$, $b = \frac{98}{99}$

a) $a < b$ $\qquad \frac{1}{42} > \frac{1}{99}$

b) $a = \frac{410}{420} < \frac{411}{420} < \frac{412}{420} < \frac{413}{420} < \frac{980}{990} = b$

c) Rationelle tal mellem a og b... uendelig mange.

8. Ordningsrelationen <.

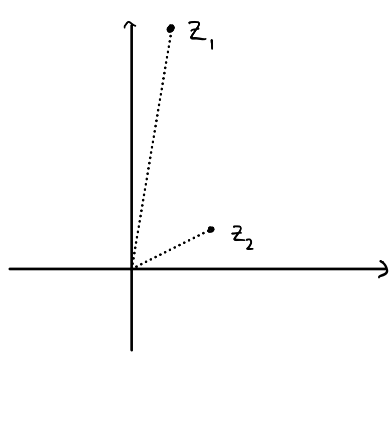
a) Afprøv påstandene.

1. $4<5$, $4 \not> 5$, $4 \neq 5$
2. $4<5 \wedge 5<6 \Rightarrow 4<6$
3. $4<5 \Rightarrow 4+2 < 5<2 \Leftrightarrow 6<7$
4. $4<5 \wedge 0<2 \Rightarrow 4 \cdot 2 < 5 \cdot 2 \Leftrightarrow 8 < 10$.

b) Ingen udvidning til $\mathbb{C}$.

En udvidelse vil bevare ordningen på de reelle tal, men da findes sammenligninger, hvorom der gælder

$z_1 < z_2$ i reel forstand, men $\text{Im}(z_1) >> \text{Im}(z_2)$



Såfremt modulus benyttes: $|z| < |w|$, så vil alle tal på samme cirkel omkring origo have samme værdi. Dette strider imod påstand 1.