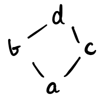
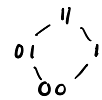
6.4.11 Are there any Boolean algebras with only 3 elements?

No, they must be isomorphic to B_n with 2^n elements.

6.4.8 

is isomorphic to B_2 

, and is therefore a Boolean algebra.

6.4.9 D_{385} : $385 = 5 \cdot 77 = 5 \cdot 7 \cdot 11$. Since no prime divides 385 more than once, then D_{385} is a Boolean algebra.

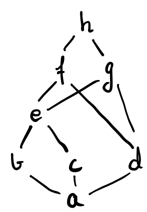
6.4.10 D_{60} : $60 = 2^2 \cdot 3 \cdot 5$, so D_{60} is not a Boolean algebra.

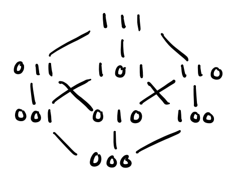
6.4.27 Let $A = \{a, b, c, d, e, f, g, h\}$ and R be defined by

$$M_R = \begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 0&1&0&0&1&1&1&1 \\ 0&0&1&0&1&1&1&1 \\ 0&0&0&1&0&1&1&1 \\ 0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}.$$

Prove or disprove (A,R) is a Boolean algebra.

If Boolean, then (A,R) is isomorphic to $(B_3, \leq)$, because there are $8 = 2^3$ elements.

Hasse diagram of (A,R) 

Hasse diagram of $(B_3, \leq)$ 

The diagrams are not isomorphic, so (A,R) is not a Boolean algebra.

6.5.3 Consider $p(x,y,z) = (x \wedge y') \vee (y \wedge (x' \vee y))$. If $B = \{0,1\}$ compute the truth table of $f: B_3 \to B$.

x	y	z	$P(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	1
1	0	0	1
0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	1

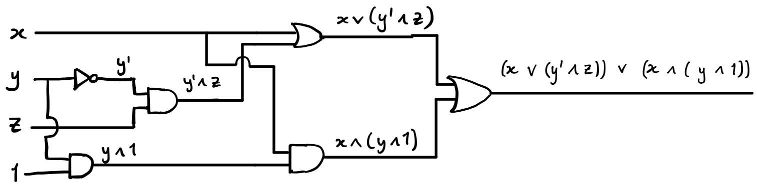
6.5.5 Show equivalence by Boolean arithmetic.

$(x \vee y) \wedge (x' \vee y)$; y

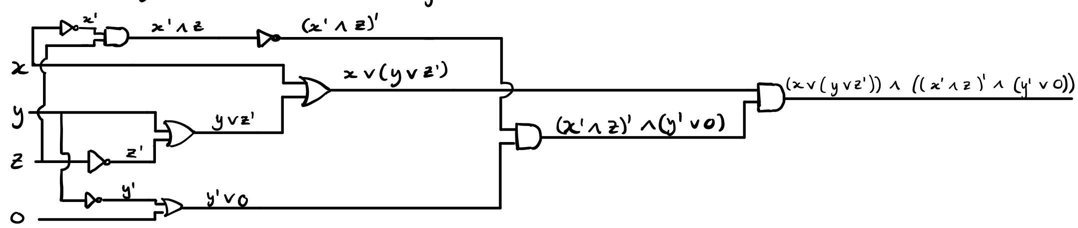
$$\begin{aligned}(x \vee y) \wedge (x' \vee y) &\overset{4}{=} (y \vee x) \wedge (y \vee x') \\ &\overset{10}{=} y \vee (x \wedge x') \\ &\overset{11}{=} y \vee 0 \\ &\overset{8}{=} y\end{aligned}$$

6.5.17 Logic diagram for f.

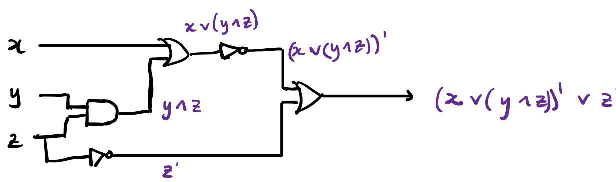
(a) $f(x,y,z) = (x \vee (y' \wedge z)) \vee (x \wedge (y \wedge 1))$.



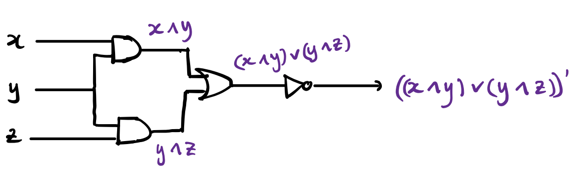
(b) $f(x,y,z) = (x \vee (y \vee z')) \wedge ((x' \wedge z)' \wedge (y' \vee 0))$.



6.5.18 Boolean function described by the diagram.



6.5.19 Boolean function described by the diagram.



6.6.9

$$\begin{array}{c|cc} & y' & y \\ \hline x' & \boxed{1} & 0 \\ x & 0 & \boxed{1} \end{array}$$

$(x'\wedge y')\vee(x\wedge y)$

6.6.14

$$\begin{array}{c|cccc} & \multicolumn{2}{c}{y'} & \multicolumn{2}{c}{y} \\ \hline x' & 1 & 1 & 1 & 1 \\ x & 0 & 0 & 1 & 0 \\ \hline & z' & \multicolumn{2}{c}{z} & z' \end{array}$$

$(x'\wedge y')\vee(x'\wedge y)\vee(y\wedge z)$

6.6.15

$$\begin{array}{cc|cccc|c} & & \multicolumn{2}{c}{z'} & \multicolumn{2}{c|}{z} & \\ \hline x' & & 0 & 0 & 1 & 1 & y' \\ x' & & 0 & 0 & 1 & 1 & y \\ x & & 1 & 0 & 0 & 1 & y \\ x & & 0 & 1 & 1 & 0 & y' \\ \hline & & w' & \multicolumn{2}{c}{w} & w' & \end{array}$$

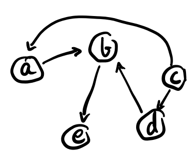
$(x'\wedge z)\vee(x\wedge w'\wedge y)\vee(x\wedge w\wedge y')$

7.1.1-2 Determine if R is a tree, and if so determine the root.

1. $A=\{a,b,c,d,e\}$ and $R=\{(a,d),(b,c),(c,a),(d,e)\}$.

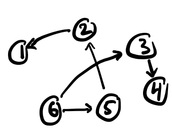
$b \to c \to a \to d \to e$ R is a tree and b is the root.

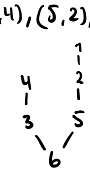
2. $A=$ -"- and $R=\{(a,b),(b,e),(c,d),(d,b),(c,a)\}$.



Two paths from c to b, so R is not a tree (Thm. 2(c)).

7.1.4 $A=\{1,\dots,6\}$ and $R=\{(2,1),(3,4),(5,2),(6,5),(6,3)\}$.





R is a tree, and 6 is the root.

7.1.28 Draw all possible unordered trees on $S=\{a,b,c\}$.