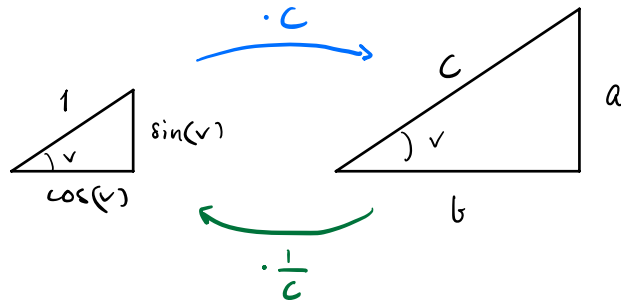


Trig, powers, exponentials and inverse functions

Ex1.
$$\frac{2^{-1} \cdot 2^4 \cdot (2^3)^{-2}}{2^{-5}} = \frac{2^4 \cdot 2^5}{2 \cdot 2^6} = 2^2 = 4$$

Ex2.
a)

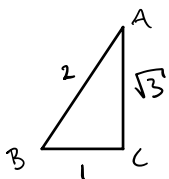


Proportional:
dividing the arbitrary
triangle by c in
turn yield

$$\cos(v) = \frac{b}{c} \quad \text{and} \quad \sin(v) = \frac{a}{c}$$

A scaling factor c or $\frac{1}{c}$ always exist for
similar objects.

b)



The angles of the triangle are well known,
but let's scale by $\frac{1}{2}$, so we have:



These coordinates yield

$$\frac{\pi}{2}, \frac{\pi}{3} \quad \text{and} \quad \frac{\pi}{6}.$$

c)

We have $\cos^2(v) + \sin^2(v) = 1^2$, which follows from
the Pythagorean theorem, i.e. hypotenuse has
length 1, so the sides are by definition
 $\cos(v)$ and $\sin(v)$.

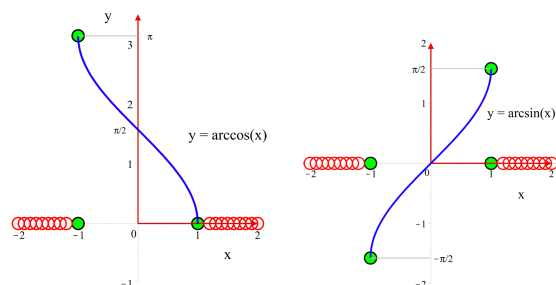
Ex 3.

a) Compute (Remember domains! Figure 3.9)

$$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

$$\arcsin(1) = \frac{\pi}{2}$$



Let $A = \{x \in \mathbb{R} \mid x \in [0, 2\pi]\}$ and $B = \{x \in \mathbb{R} \mid x \in [-\pi, \pi]\}$.

b) Solve for sets A, B and \mathbb{R} .

$$\cos(x) = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{5\pi}{3}, \quad x \in A.$$

$$x = \frac{\pi}{3} \vee x = -\frac{\pi}{3}, \quad x \in B.$$

$$x = \frac{\pi}{3} + 2p\pi \vee x = -\frac{\pi}{3} + 2p\pi, \quad p \in \mathbb{Z}, x \in \mathbb{R}.$$

c)

$$\sin(x) = -\frac{\sqrt{3}}{2} \Leftrightarrow x = \frac{4\pi}{3} \vee x = \frac{5\pi}{3}, \quad x \in A.$$

$$x = -\frac{2\pi}{3} \vee x = -\frac{\pi}{3}, \quad x \in B.$$

$$x = \frac{4\pi}{3} + 2p\pi \vee x = \frac{5\pi}{3} + 2p\pi, \quad p \in \mathbb{Z}, x \in \mathbb{R}.$$

d)

$$e^{i \cdot v} = \frac{1}{2} - \frac{\sqrt{3}}{2} i \quad \text{for sets A and B.}$$

$$\Leftrightarrow v = \frac{5\pi}{3}, \quad v \in A$$

$$v = -\frac{\pi}{3}, \quad v \in B.$$

Ex 4. Write as a power.

$$\begin{aligned} 3^2 \cdot 3^3 &= 3^5, & (5^8)^{-2} &= 5^{-16}, \\ 3^2 \cdot 3^{-5} &= 3^{-3}, & \frac{4^{1,3}}{4^{2,3}} &= 4^{-1}, \\ \left(\frac{1}{2}\right)^5 \cdot 6^5 &= 3^5, & \frac{5^3}{0,5^3} &= 10^3. \end{aligned}$$

Ex 5. Let $z = 1+i$ and compute

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin(v) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos(v) \Rightarrow \text{Arg}(z) = \frac{\pi}{4}.$$

Use the result to determine moduli and Arg for:

z^2 , z^5 , z^8 and z^{-10} .

$$|z^2| = |z|^2 = \sqrt{2}^2 = 2, \quad \text{Arg}(z^2) = \text{Arg}(z) + \text{Arg}(z) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}.$$

$$|z^5| = \sqrt{2}^5 = 4\sqrt{2}, \quad \text{Arg}(z^5) = \left[\frac{5\pi}{4} \right] = -\frac{3\pi}{4}$$

$$|z^8| = \sqrt{2}^8 = 16, \quad \text{Arg}(z^8) = \left[\frac{8\pi}{4} \right] = [2\pi] = 0$$

$$|z^{-10}| = \sqrt{2}^{-10} = \frac{1}{32}, \quad \text{Arg}(z^{-10}) = \left[-10 \frac{\pi}{4} \right] = -\frac{\pi}{2}$$

Provide the rectangular form of the numbers above.

$$\begin{aligned} z^2 &= 2i, & z^5 &= 4\sqrt{2} \cdot \left(\cos\left(-\frac{3\pi}{4}\right) + i \cdot \sin\left(-\frac{3\pi}{4}\right) \right) \\ & & &= 4\sqrt{2} \cdot \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) = -4 - 4i, \end{aligned}$$

$$z^8 = 16,$$

$$z^{-10} = -\frac{1}{32}i.$$

Ex 6. Binomial equations

a) Solve the equations.

$$z^2 = -4 \Leftrightarrow z = \pm 2i$$

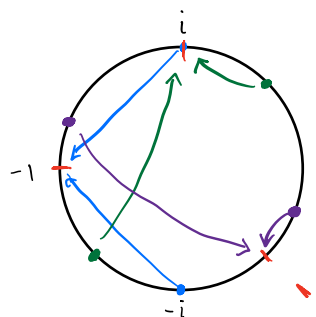
$$z^2 = i \Leftrightarrow z = \pm \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)$$

$$z^2 = 1 - i \Leftrightarrow$$

$$z = 2^{\frac{1}{4}} \left(\cos\left(\frac{\pi}{8}\right) - i \sin\left(\frac{\pi}{8}\right) \right) = 2^{\frac{1}{4}} \left(\frac{1}{2} \sqrt{2+\sqrt{2}} - i \cdot \frac{1}{2} \sqrt{2-\sqrt{2}} \right)$$

$$\vee z = 2^{\frac{1}{4}} \left(\cos\left(\frac{7\pi}{8}\right) + i \sin\left(\frac{7\pi}{8}\right) \right) \\ = 2^{\frac{1}{4}} \left(-\cos\left(\frac{\pi}{8}\right) + i \sin\left(\frac{\pi}{8}\right) \right)$$

sine is odd
and cosine is
even

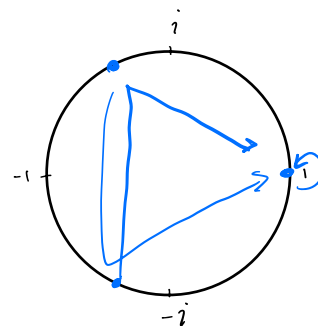
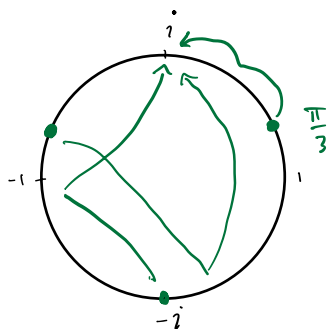


b)

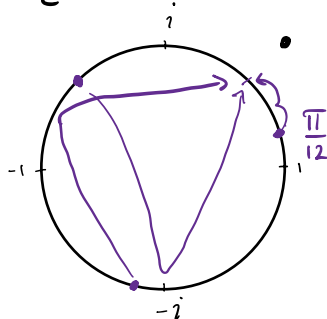
$$z^3 = 1 \Leftrightarrow z = 1 \vee z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$z^3 = -i \Leftrightarrow z = \frac{\sqrt{3}}{2} + \frac{1}{2} i \vee z = -\frac{\sqrt{3}}{2} + \frac{1}{2} i$$

$$\vee z = -i$$



$$z^3 = 1 + i \Leftrightarrow z = 2^{\frac{1}{6}} e^{\frac{\pi}{12} i} \vee z = 2^{\frac{1}{6}} e^{-\frac{7\pi}{12} i} \\ \vee z = 2^{\frac{1}{6}} e^{\frac{3\pi}{4} i}$$



$$|z^3| = \sqrt{2} = 2^{\frac{1}{2}} \Rightarrow |z| = 2^{\frac{1}{6}}$$

$$\left(\cos\left(\frac{\pi}{12}\right) = \frac{1}{2} \sqrt{2+\sqrt{3}}, \sin\left(\frac{\pi}{12}\right) = \frac{1}{2} \sqrt{2-\sqrt{3}} \right)$$

Ex 7.

a) Let $f(t) = b \cdot a^t$, $t \in \mathbb{R}$. Determine a assuming 20% growth.

Since 1 corresponds to no change, i.e. remain at 100%, then $a = 1,2$ is 20% growth. Sometimes $a = 1 + r$.

r is the rate

b) Let $f(t) = 2^t$ and $h(t) = 0,5^{2-t}$, $t \in \mathbb{R}$.

Determine growth rate and percent growth.

$$2 = 1 + r \Leftrightarrow r = 1, 100\%.$$

Let's simplify $0,5^{2-t} = \left(\frac{1}{2}\right)^2 \cdot \frac{1}{\left(\frac{1}{2}\right)^t} = \frac{1}{4} \cdot 2^t$.

Now h has the same growth as f .

c) State the base, and write on the form

$$x \mapsto e^{kx}, x \in \mathbb{R}.$$

— $2^x = e^{\ln(2)x}$

$$a = 2, k = \ln(2)$$

— $4^x = e^{\ln(4)x} = e^{2\ln(2)x}$

$$a = 4, k = \ln(4) = 2\ln(2)$$

— $\left(\frac{1}{4}\right)^x = e^{\ln\left(\frac{1}{4}\right)x} = e^{-\ln(4)x}$
 $= e^{-2\ln(2)x}$

$$a = \frac{1}{4}, k = \ln\left(\frac{1}{4}\right) = -2\ln(2).$$

