1.4.41 (a) Write in base 5.

(i) $29 = 5\cdot 5 + 4$, $5 = 1\cdot 5 + 0$, $1 = 0\cdot 5 + 1$

$29 = (104)_5$

(ii) $73 = 14\cdot 5 + 3$, $14 = 2\cdot 5 + 4$, $2 = 0\cdot 5 + 2$

$73 = (243)_5$

(iii) $215 = 43\cdot 5 + 0$, $43 = 8\cdot 5 + 3$, $8 = 1\cdot 5 + 3$, $1 = 0\cdot 5 + 1$

$215 = (1330)_5$

(iv) $732 = 146\cdot 5 + 2$, $146 = 29\cdot 5 + 1$, $29 = 5\cdot 5 + 4$, $5 = 1\cdot 5 + 0$, $1 = 0\cdot 5 + 1$

$732 = (10412)_5$

(b) Write in base 10.

(i) $(144)_5 = 4\cdot 5^0 + 4\cdot 5^1 + 1\cdot 5^2 = 4 + 20 + 25 = 49$

(ii) $(320)_5 = 0\cdot 5^0 + 2\cdot 5^1 + 3\cdot 5^2 = 0 + 10 + 75 = 85$

(iii) $(1242)_5 = 2\cdot 5^0 + 4\cdot 5^1 + 2\cdot 5^2 + 1\cdot 5^3 = 2 + 20 + 50 + 125 = 197$

(iv) $(11231)_5 = 1\cdot 5^0 + 3\cdot 5^1 + 2\cdot 5^2 + 1\cdot 5^3 + 1\cdot 5^4 = 1 + 15 + 50 + 125 + 625 = 816$

1.5.1 Let $A = \begin{bmatrix} 3 & -2 & 5 \\ 2 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ og $C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & -1 \\ 2 & 0 & 8 \end{bmatrix}$.

(a) $a_{12} = -2$, $a_{22} = 1$ and $a_{23} = 2$.

(b) $b_{11} = 3$ and $b_{31} = 4$.

(c) $c_{13} = 4$, $c_{23} = -1$ and $c_{33} = 8$.

(d) $c_{11} = 2$, $c_{22} = 6$ and $c_{33} = 8$.

1.5.5 $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 5 \\ 3 & 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}$,

$E = \begin{bmatrix} 3 & 2 & -1 \\ 5 & 4 & -3 \\ 0 & 1 & 2 \end{bmatrix}$ and $F = \begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix}$.

Compute

(a) $C + E = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 5 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -1 \\ 5 & 4 & -3 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 2 \\ 9 & 6 & 2 \\ 3 & 2 & 4 \end{bmatrix}$.

(b) $AB = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ -3 & 0 \end{bmatrix}$.

(c) $CB + F$: Note that C is 3×3 and B is 3×2, so CB is 2×3. Since F is 2×2 the sum $CB+F$ isn't defined.

(d) $AB + DF = \begin{bmatrix} 7 & 13 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 1 \\ -4 & 17 \end{bmatrix} = \begin{bmatrix} 21 & 14 \\ -7 & 17 \end{bmatrix}$.

1.5.9 (a) $A^T(D+F) = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix}^T \left(\begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix} \right) = \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -5 & 5 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 22 & 34 \\ 3 & 11 \\ -31 & 3 \end{bmatrix}.$

(b) $(BC)^T$: Since B is 3×2 and C is 3×3, then BC is not defined.
$C^T B^T$: Since C^T is 3×3 and B^T is 2×3, then $C^T B^T$ is not defined.

(c) $(B^T + A)C = \left(\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix} \right) \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 5 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 5 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 25 & 5 & 26 \\ 20 & -3 & 32 \end{bmatrix}.$

(d) $(D^T + E)F$: $D^T + E$ is not defined, as D^T is 2×2 and E is 3×3.

1.5.31 Compute $A \vee B$, $A \wedge B$ and $A \odot B$.

(a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

$A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $A \wedge B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

$A \vee B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $A \wedge B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $A \odot B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

$A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $A \wedge B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

1.5.33 Let $F = [f_{ij}]$ be a $p\times q$ matrix.

(a) Lower left corner element : f_{p1}
(b) Upper left corner element : f_{11}
(c) Lower right corner : f_{pq}
(d) Upper right corner: f_{1q}
(e) Generic element in row r of F^T : f_{ir}
(f) Generic element on the diagonal : f_{ii}

1.6.4 Determine closure wrt. operation.

(a) ($n \times n$ Boolean matrices, $\vee, \wedge, ^T$) meet

For Boolean matrices A and B, then $A \wedge B$ again Boolean, so the structure is closed wrt. meet.

(b) (prime numbers, $+$, $*$) addition.

We see that $2+2=4$ isn't prime, so the structure isn't closed under addition.

1.6.6 (a) Show that $\square$ is associative. $(a \square b) \square c = a \square (b \square c)$, $0 \square 0 = 0$, $1 \square 1 = 0$ and $0 \square 1 = 1$

$$(0 \square 0) \square 0 = 0 \square 0 = 0 \square (0 \square 0) \qquad (0 \square 1) \square 1 = 1 \square 1 = 0 \square (1 \square 1)$$
$$(0 \square 0) \square 1 = 0 \square 1 = 0 \square (0 \square 1) \qquad (1 \square 0) \square 1 = 1 \square 1 = 1 \square (0 \square 1)$$
$$(0 \square 1) \square 0 = 1 \square 0 = 0 \square (1 \square 0) \qquad (1 \square 1) \square 0 = 0 \square 0 = 1 \square (1 \square 0)$$
$$(1 \square 0) \square 0 = 1 \square 0 = 1 \square (0 \square 0) \qquad (1 \square 1) \square 1 = 0 \square 1 = 1 \square (1 \square 1)$$

Thus $\square$ is associative.

b) Show that ∇ is associative. $0 \nabla 0 = 0$, $0 \nabla 1 = 0$, $1 \nabla 0 = 0$ and $1 \nabla 1 = 1$.

$$(0 \nabla 0) \nabla 0 = 0 \nabla 0 = 0 \nabla (0 \nabla 0) \qquad (0 \nabla 1) \nabla 1 = 0 \nabla 1 = 0 \nabla (1 \nabla 1)$$
$$(0 \nabla 0) \nabla 1 = 0 \nabla 1 = 0 \nabla (0 \nabla 1) \qquad (1 \nabla 0) \nabla 1 = 0 \nabla 1 = 1 \nabla (0 \nabla 1)$$
$$(0 \nabla 1) \nabla 0 = 1 \nabla 0 = 0 \nabla (1 \nabla 0) \qquad (1 \nabla 1) \nabla 0 = 1 \nabla 0 = 1 \nabla (1 \nabla 0)$$
$$(1 \nabla 0) \nabla 0 = 1 \nabla 0 = 1 \nabla (0 \nabla 0) \qquad (1 \nabla 1) \nabla 1 = 1 \nabla 1 = 1 \nabla (1 \nabla 1)$$

Thus ∇ is associative.

2.1.7 (a) $2 < 3$ or 3 is a positive integer. Since $2 < 3$ the statement is true.

(b) $2 \geq 3$ or 3 is a positive integer. Since $3 \in \mathbb{Z}_+$ the statement is true.

(c) $2 < 3$ or 3 is not a positive integer. Since $2 < 3$ the statement is true.

(d) $2 \geq 3$ or 3 is not a positive integer. Neither statement is true, so the statement is false.

2.1.10 Which is the negation of "2 is even and -3 is negative".

(d) 2 is odd or -3 is not negative.

2.1.12 (a) $p \wedge \sim r$

(b) $q \vee p$

(c) $\sim p \wedge \sim q$

(d) $r \wedge q$

2.1.15 (a) $\forall x\ \exists y\ R(x,y)$

For every integer x, there is an integer y such that $x+y$ is even.

(b) $\exists x\ \forall y\ R(x,y)$

There is an integer x such that for every integer y, $x+y$ is even.

2.1.16 (a) $\forall x\ (\sim Q(x))$

No integer is prime.

(b) $\exists y\ (\sim P(y))$

An odd integer exists.

2.1.22 If $P(y) = 1+2+\cdots+y = 0$, then

(a) $P(1)$: $1=0$

(b) $P(5)$: $1+2+3+4+5=0$

(c) $P(k)$: $1+2+\cdots+k=0$

2.1.30 $(\sim p \vee q) \wedge \sim r$

p	q	r	$\sim p$	$\sim p \vee q$	$\sim r$	$(\sim p \vee q) \wedge \sim r$
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

2.1.37 Replace the guard $P(x)$ with $\sim P(x)$.

IF ($x \neq max$ and $y > 4$) THEN take action.

IF ($x = max$ or $y \leq 4$) THEN take action.

2.2.1 (a) $p \Rightarrow q$

(b) $r \Rightarrow p$

(c) $q \Rightarrow p$

(d) $\sim r \Rightarrow p$

2.2.10 (a) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Absurdity

(b) $q \vee (\sim q \wedge p)$

p	q	$\sim q$	$\sim q \wedge p$	$q \vee (\sim q \wedge p)$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	F

Contingency

2.2.11 (a) $p \Rightarrow (q \Rightarrow p)$

p	q	$q \Rightarrow p$	$p \Rightarrow (q \Rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Tautology

(b) $q \Rightarrow (q \Rightarrow p)$

p	q	$q \Rightarrow p$	$q \Rightarrow (q \Rightarrow p))$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

Contingency

2.2.21 p and q true, r,s and t false.

(a) $\sim(p \Rightarrow q)$ False

(b) $(\sim p) \Rightarrow r$ True

(c) $(p \Rightarrow s) \wedge (s \Rightarrow t)$ False

(d) $t \Rightarrow \sim q$ True

2.2.26 p: if the flood destroys my house or the fire destroys my house, then my insurance company will pay me.

p: $a \vee b \Rightarrow c$

(a) Converse: $c \Rightarrow a \vee b$. (i)

(b) Contrapositive: $\sim c \Rightarrow \sim a \wedge \sim b$. (iv)

2.2.27 $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

2.2.28 $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim p) \vee (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T