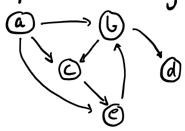
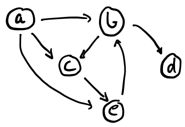
Multiple choice round 4

1. 

Determine the relation R.

$R = \{(a,b),(a,c),(a,e),(b,c),(b,d),(c,e),(e,b)\}$

2. 

Which statement is *false*?

$Ran(R) = \{a,b\}$ is false.

3. Let $R = \{(1,2),(3,4),(2,1),(4,1),(3,2)\}$ be a relation on $\{1,2,3,4\}$.
What is M_R?

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

4. Let $R = \{(1,2),(2,2),(3,4),(4,3),(2,1),(4,1),(3,2)\}$ be a relation on $\{1,2,3,4\}$.
What is R^2?

$$R^2 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(3,3),(4,2),(4,4)\}$$

5. Let $R = \{(1,1),(1,4),(2,1),(2,3),(3,1),(3,3),(3,4),(4,2)\}$ be a relation on $\{1,2,3,4\}$.
Which statement is true?

R is antisymmetric: when $a \neq b$, then either $a \not R b$ or $b \not R a$.

6. Let R be given by $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Which statement is *true*?

R is reflexive and antisymmetric.

7. Let $R = \{(1,1),(1,4),(2,2),(2,3),(3,2),(3,3),(4,4),(4,1)\}$ be a relation on $\{1,2,3,4\}$.
Which statement is true?

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$ Reflexive, symmetric and transitive $\Rightarrow$ equivalence relation.

8. R and S on $\{1,2,3\}$, where $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.
Matrix of $\bar{R} \cap S$?

$$M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{so} \quad M_{\bar{R} \cap S} = M_{\bar{R}} \wedge M_S = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

9. Which definition gives an equivalence relation on $\mathbb{Z}$?

$$x R y \iff x^2 - y = y^2 - x$$

4.4.1 $R = \{(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4)\}$

$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Reflexive, symmetric and transitive, since $(M_R)^2_{\odot} = M_R$.

Thus R is an equivalence relation. Portition $\{\{1,2\},\{3,4\}\}$.

4.4.2 $M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, *irreflexive, asymmetric and antisymmetric.*
Since $R^2 \subseteq R$, we have that R is transitive.

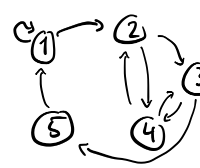
4.4.13 $A = \mathbb{Z}$; $aRb \iff a \leq b+1$.

For $a \in \mathbb{Z}$: $a \leq a+1$ so aRa and R is reflexive.
Take 0 and 1, then $0 \leq 1+1$ and $1 \leq 0+1$ so not antisymmetric, and so not asymmetric.
Though for 0 and 2, then $0 \leq 2+1$ and $2 > 0+1$ so not symmetric either.
Also $2 \leq 1+1$ and $1 \leq 0+1$, but $2 > 0+1$, so R is not transitive.

4.1.32 All partitions of $\{a,b,c,d\}$.

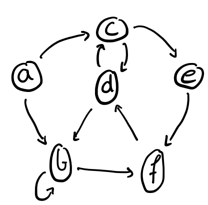
B.{{a},{b},{c},{d}}, {{a},{b,c,d}}, {{b}, {a,c,d}}, {{c},{{a,b,d}}, {{d}, {a,b,c}}, {{a,b},{c,d}}, {{a,c},{b,d}}, {{a,d}, {b,c}},
{{a,b,c,d}}, {{a},{b},{c,d}}, {{a}, {c}, {b,d}}, {{a}, {d}, {b, c}}, {{b}, {c}, {a,d}}, {{b}, {d}, {a,c}}, {{c}, {d}, {a,b}}

4.2.24 $A = \{a,b,c,d,e\}$ $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.



$R = \{(1,1),(1,2),(2,3),(2,4),(3,4),(3,5),(4,2),(4,3),(5,1)\}$

4.3.9–16.



9. Paths of length 1:
ab, ac, bb, bf, cd, ce,
db, dc, ef, fd.

10. Paths of length 2: abb, abf, acd, ace, bbb, bbf, bfd,
cdb, cdc, cef,
dbb, dbf, dcd, dce, efd, fdb, fdc

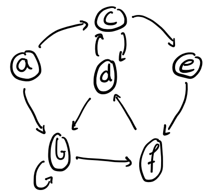
11. Paths of length 3: abbb, abbf, abfd, acdb, acdc, acef,
bbbb, bbbf, bbfd, bfdb, bfdc, cdbb,
cdbf, cdcd, cdcf, cefd, dbbb, dbbf,
dbfd, dcdb, dcdc, dcef, efdb, efdc,
fdbb, fdbf, fdcd, fdce

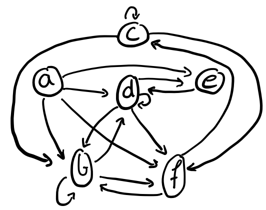
12. Cycle from c: cdc

13. Cycle from d: dcd

14. Cycle from a: none exist

15. Draw R^2





16. Find M_{R^2}.

$$M_{R^2} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^2_{\odot} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

It checks out wrt. the digraph.

4.4.6 $R = A \times A$ with $A = \{1,2,3,4\}$.
R is reflexive, symmetric and transitive.

4.4.14 $A = \mathbb{Z}_+$; $aRb \iff |a-b| \leq 2$.
R is reflexive and symmetric. Not antisymmetric by $0R1$ and $1R0$.
R is not transitive, since $0R2$ and $2R4$, but $0 \not R 4$.

4.4.18 $A = \mathbb{R}$; $aRb \iff a^2+b^2 = 4$.
Not reflexive $0 \not R 0$, and not irreflexive $\sqrt{2} R \sqrt{2}$.
R is symmetric, so not asymmetric.
Not antisymmetric: $\sqrt{2} R \sqrt{2}$ and $\sqrt{2} R -\sqrt{2}$.
Not transitive: $0R2$ and $2R0$, but $0 \not R 0$.

4.4.19 $A = \mathbb{Z}_+$; $aRb \iff \gcd(a,b) = 1$.

Not reflexive $p \not R p$.
Not irreflexive $1R1$.
R is symmetric, so not asymmetric.
Not antisymmetric $1R2$ and $2R1$.
Not transitive $2R1$ and $1R2$, but $2 \not R 2$.

4.7.1 $\quad A = B = \{1,2,3\}$, $R = \{(1,1),(1,2),(2,3),(3,1)\}$,
$S = \{(2,1),(3,1),(3,2),(3,3)\}$.

(a) $\overline{R} = \{(1,3),(2,1),(2,2),(3,2),(3,3)\}$.

(b) $R \cap S = \{(3,1)\}$.

(c) $R \cup S = \{(1,1),(1,2),(2,3),(3,1),(2,1),(3,2),(3,3)\}$.

4.7.4 $\quad A =$ a set of people. Let $aRb \iff a$ is older than b. Let $aSb \iff a$ is a brother of b.
Describe $R \cap S$.

$a(R \cap S)b \iff a$ is an older brother of b.

4.7.22 $\quad$ Let $A = \{1,2,3,4\}$. Let $R = \{(1,1),(1,2),(2,3),(2,4),(3,4),(4,1),(4,2)\}$
$S = \{(3,1),(4,4),(2,3),(2,4),(1,1),(1,4)\}$

(a) Is $(1,3) \in R \circ R$?

$(1,2),(2,3) \in R$ so yes.

(b) Is $(4,3) \in S \circ R$?

$(4,2) \in R$ and $(2,3) \in S$.

(c) Is $(1,1) \in R \circ S$?

$(1,1) \in S$ and $(1,1) \in R$.

(d) Compute $R \circ R$:

$$\begin{bmatrix} 1&1&0&0 \\ 0&0&1&1 \\ 0&0&0&1 \\ 1&1&0&0 \end{bmatrix}^2 = \begin{bmatrix} 1&1&1&1 \\ 1&1&0&1 \\ 1&1&0&0 \\ 1&1&1&1 \end{bmatrix}$$

$R \circ R = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,4),(3,1),(3,2),(4,1),(4,2),(4,3),(4,4)\}$.

(e) Compute $S \circ R$:

$$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1&1&0&0 \\ 0&0&1&1 \\ 0&0&0&1 \\ 1&1&0&0 \end{bmatrix} \odot \begin{bmatrix} 1&0&0&1 \\ 0&0&1&1 \\ 1&0&0&0 \\ 0&0&0&1 \end{bmatrix} = \begin{bmatrix} 1&0&1&1 \\ 1&0&0&1 \\ 1&0&0&1 \\ 1&0&1&1 \end{bmatrix}$$

(f) Compute $R \circ S$:

$$M_{R \circ S} = M_S \odot M_R = \begin{bmatrix} 1&0&0&1 \\ 0&0&1&1 \\ 1&0&0&0 \\ 0&0&0&1 \end{bmatrix} \odot \begin{bmatrix} 1&1&0&0 \\ 0&0&1&1 \\ 0&0&0&1 \\ 1&1&0&0 \end{bmatrix} = \begin{bmatrix} 1&1&0&0 \\ 1&1&0&1 \\ 1&1&0&0 \\ 1&1&0&0 \end{bmatrix}$$

(g) Compute $S \circ S$:

$$\begin{bmatrix} 1&0&0&1 \\ 0&0&1&1 \\ 1&0&0&0 \\ 0&0&0&1 \end{bmatrix}^2 = \begin{bmatrix} 1&0&0&1 \\ 1&0&0&1 \\ 1&0&0&1 \\ 0&0&0&1 \end{bmatrix}$$