6.1.3 Determine whether R is a linear order.

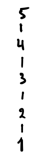
(a) $A = \mathbb{R}$ and $aRb \iff a \le b$.

For $a, b \in \mathbb{R}$ either $a \le b$ or $b \le a$, so $\le$ is a linear order on A.

(b) $A = \mathbb{R}$ and $aRb \iff a \ge b$.

For $a, b \in \mathbb{R}$ either $a \ge b$ or $b \ge a$, so $\ge$ is a linear order on A.

6.1.15 Determine the Hasse diagram given $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ on $A = \{1, 2, 3, 4, 5\}$.



6.1.20 Let $A = \mathbb{Z}_+ \times \mathbb{Z}_+$ have lexicographic order. Determine whether true or false.

(a) $(2, 12) \prec (5, 3)$ true.

(b) $(3, 6) \prec (3, 24)$ true.

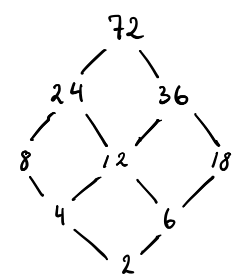
(c) $(4, 8) \prec (4, 6)$ false.

(d) $(15, 92) \prec (12, 3)$ false.

6.1.25 Describe how M_R determines if R is a partial order.

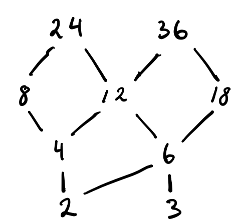
- Reflexive iff $a_{ii} = 1$ for all i.
- Antisymmetric iff $a_{ij} = 1 \Rightarrow a_{ji} = 0$ for all $i \neq j$.
- Transitive iff $M_R \odot M_R \le M_R$.

6.2.15 Determine greatest and least, if they exist.
$A = \{2, 4, 6, 8, 12, 18, 24, 36, 72\}$ with divisibility.



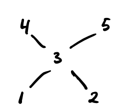
Here 2 is least, since $2 | a$ for all $a \in A$, and 72 is greatest as $a | 72$ for all $a \in A$.

6.2.16 $A = \{2, 3, 4, 6, 12, 18, 24, 36\}$ with divisibility.



Both 2 and 3 are minimal, and 24 and 36 are maximal. Thus there is no least or greatest element.

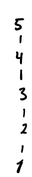
6.2.34 Construct the Hasse diagram using SORT on



$B = \{1, 2, 3, 4, 5\}$.

Apply SORT: $(B, \le)$

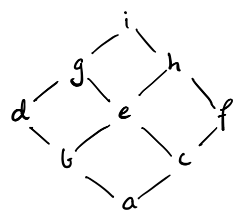
1 is minimal



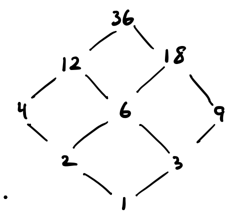
6.3.7 Is $A = \{2, 3, 6, 12, 24, 36, 72\}$ under divisibility a lattice?

$(A, |)$ is a lattice if $\{a,b\} \subseteq A$ has a least upper bound and greatest lower bound.
Thus this is not a lattice, as $\{2,3\}$ has no least upper bound.

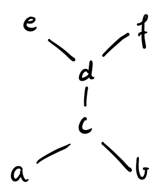
6.3.1 Does the Hasse diagram represent a lattice?



Yes see 6.2.15 for an example or the diagram on the right of D_{36}.



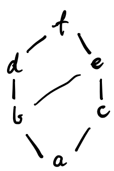
6.3.2



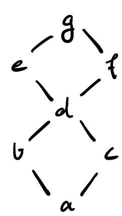
Note that $\{e,f\}$ has no upper bound, and $\{a,b\}$ has no lower bound, so this does not represent a lattice.

6.4.1-4

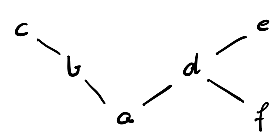
1.



2.



3.



4.

Determine whether the poset is a Boolean algebra.

A lattice is a Boolean algebra if it is isomorphic to $(B_n, \leq)$ for $n \in \mathbb{Z}_+$.
If $(A, \leq)$ were a Boolean algebra, then $|A| = 2^n$ for some $n \in \mathbb{Z}_+$. Thus none of the above is a Boolean algebra.