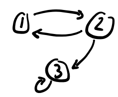
Vegleiðandi Multiple Choice round

1. Truth table $p \Rightarrow (\sim q \wedge r)$

Use $s \Rightarrow t$ is false if $T \Rightarrow F$, else true.

F F T F T T

2. $R = \{(1,2), (2,1), (2,3), (3,3)\}$



$R^{\infty} = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,3)\}$

3. ~"There exist an Island, where all fruits taste good".

On all islands, there exist a fruit which tastes bad.

4. License plates: AB456, AB612, CC043 2 from $\{A, \overset{10}{\ldots}, J\}$ and 3 from $\{0, 1, \overset{10}{\ldots}, 9\}$.

Letters can repeat, but not numbers.

Number of different plates?

$$10^2 \cdot {}_{10}P_3 = 10^2 \frac{10!}{7!} = 10^2 \cdot 10 \cdot 9 \cdot 8 = 72000.$$

5. Let $A, B \subseteq \mathbb{Z}$ be $A = \{2, 5, 12, 14, 19, 22\}$,

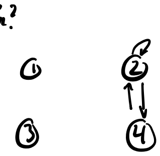
$B = \{n \in \mathbb{Z} \mid 3 | n\}$.

What is $A - B$?

$$A - B = \{2, 5, 14, 19, 22\}.$$

6. G on $\{1, 2, 3, 4\}$ defined by $xGy \Leftrightarrow gcd(x, y) = 2$.

Which represents G?



7. Lcm $(6, 34)$

$6 = 2 \cdot 3$

$34 = 2 \cdot 17$

$\Rightarrow 2 \cdot 3 \cdot 17 = 102$

8. Hundreds of balls: yellow, blue, green, purple and white.

Choose 4, order doesn't matter, how many combinations?

$${}_{5+4-1}C_4 = {}_8C_4$$

9. Contrapositive of $A \neq B \Rightarrow A \cap B = \emptyset$

$$A \cap B \neq \emptyset \Rightarrow A = B$$

10. 1000 in base 8.

$$1000 = 125 \cdot 8 + 0$$
$$125 = 15 \cdot 8 + 5$$
$$15 = 1 \cdot 8 + 7$$
$$1 = 0 \cdot 8 + 1$$

$$1000 = (1750)_8$$

11. R in $\mathbb{Z}_4$: $x R y \Leftrightarrow x \neq y$

How many of the properties: ref, irref, sym, asym, trans?

Irreflexive, symmetric.

12. Solve $a_n = 6a_{n-1} - 9a_{n-2}$, $a_1 = -3$, $a_2 = 9$

$$x^2 = 6x - 9 \Leftrightarrow x^2 - 6x + 9 = 0 \Leftrightarrow (x-3)^2 = 0$$
$$\Leftrightarrow x = 3$$

$a_n = u \cdot 3^n + v \cdot n \cdot 3^n$:

$$\begin{cases} 3u + 3v = -3 \\ 9u + 18v = 9 \end{cases} \Leftrightarrow \begin{cases} 9u + 9v = -9 \\ 9u + 18v = 9 \end{cases} \Rightarrow 9v = 18 \Leftrightarrow v = 2$$

$$\Rightarrow 3u + 3 \cdot 2 = -3$$
$$\Leftrightarrow 3u = -9$$
$$\Leftrightarrow u = -3$$

$$a_n = -3 \cdot 3^n + 2 \cdot n \cdot 3^n$$

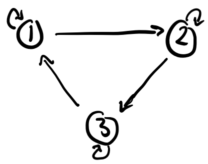
4.5.1 $A = \{a,b,c\}$ and $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Is R an equivalence relation?

Clearly reflexive and symmetric.

$$(M_R)^2_\odot = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \text{transitive}$$

So R is an equivalence relation.

4.5.4



$1R2 \quad 2 \not R 1 \qquad 1R2 \wedge 2R3$ but $3 \not R 1$

Reflexive, but not symmetric or transitive, so not an equivalence relation.

4.5.17 Let A and R be defined as in ex. 5. Compute A/R.

$A = \mathbb{Z}$ and $n \in \mathbb{Z}_+$. Let $R = \{(a,b) \in A \times A \mid a \equiv b \pmod n\}$.

Let $a \in A$, then

$$R(a) = [a]_n = a + \mathbb{Z}n = \{a + kn \mid k \in \mathbb{Z}\}$$
$$= \{\ldots, a-2n, a-n, a, a+n, a+2n, \ldots\}.$$

$$A/R = \{[0]_n, [1]_n, [2]_n, \ldots, [n-1]_n\}$$
$$= \{R(0), R(1), R(2), \ldots, R(n-1)\}.$$

4.5.18 Let $A = \{1,2,3,4\}$ and $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Compute A/R.

$$A/R = \{\{1\}, \{2,3,4\}\}.$$

5.4.21 Let $S = \{1,2,3,4,5\}$ and $A = S \times S$. Define R on A by

$$(a,b)\, R\, (a',b') \iff ab' = a'b \iff \frac{a}{b} = \frac{a'}{b'}$$

(a) Show that R is an equivalence relation.

For $(a,b) \in A$ then $ab = ab$, so reflexive. Notice $ab' = a'b \iff a'b = ab'$, so symmetric.

Let $ab' = a'b$ and $a'b'' = a''b'$, then $ab'' = \frac{a'b}{b'} b'' = \frac{a''b'b}{b'b''} b'' = a''b$.

Thus we have an equivalence relation.

(b) Compute A/R.

$A/R = \{\{(1,1),(2,2),(3,3),(4,4),(5,5)\}, \{(1,2),(2,4)\}, \{(1,3)\}, \{(1,4)\}, \{(1,5)\}, \{(2,1),(4,2)\}, \{(2,3)\}, \{(2,5)\},$
$\{(3,1)\}, \{(3,2)\}, \{(3,4)\}, \{3,5\}, \{(4,1)\}, \{(4,3)\}, \{4,5\}, \{(5,1)\}, \{(5,2)\}, \{(5,3)\}, \{(5,4)\}\}.$