Avmyndan

Sum við 2D, so avmynda vit ein vektor $\underline{v} \in \mathbb{R}^3$

$$\underline{v} = v_1 \underline{e}_1 + v_2 \underline{e}_2 + v_3 \underline{e}_3,$$

við at fara úr $[\underline{e}_1, \underline{e}_2, \underline{e}_3]$ í $[\underline{a}_1, \underline{a}_2, \underline{a}_3]$. Vit fáa ein nýggjan vektor $\underline{v}'$

$$\underline{v}' = v_1 \underline{a}_1 + v_2 \underline{a}_2 + v_3 \underline{a}_3.$$

Sum lineera avmyndan $\underline{\underline{A}}$ er uppsetanin heilt einfalt

$$\underline{\underline{A}}\,\underline{v} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 a_{11} + v_2 a_{12} + v_3 a_{13} \\ v_1 a_{21} + v_2 a_{22} + v_3 a_{23} \\ v_1 a_{31} + v_2 a_{32} + v_3 a_{33} \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \\ v_3' \end{bmatrix} = \underline{v}'$$

Linear maps

Skalering er ein avmyndan

$$\underline{\underline{S}}\,\underline{v} = \begin{bmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} s_{11} v_1 \\ s_{22} v_2 \\ s_{33} v_3 \end{bmatrix}$$

Refleksión skal sum so megna at avmynda t.d. um $\underline{e}_2$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \mapsto \begin{bmatrix} v_1 \\ -v_2 \\ v_3 \end{bmatrix}, \quad \text{so} \quad \underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Umbýtan av koordinatásir framleiða spegling um 45°, so um $x_1 = x_3$ er matrican

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Shears eru enn fyri mesta partin um at flyta parallelt við koordinatásirnar. Sjónarmiðið kann takast frá $[\underline{e}_1, \underline{e}_2, \underline{e}_3]$, ella hvussu vit ynskja at manipulera $\underline{v}$. Bæði ganga út uppá tað sama.

Um ein dimensión skal flytast:

$$1. \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad 2. \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & b & 1 \end{bmatrix} \quad 3. \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

1. $\underline{e}_1 \mapsto \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ $\underline{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\underline{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ shear í $[\underline{e}_2, \underline{e}_3]$ planið
2. $\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\underline{e}_2 = \begin{bmatrix} a \\ 1 \\ b \end{bmatrix}$ $\underline{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ shear í $[\underline{e}_1, \underline{e}_3]$ planið
3. $\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\underline{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\underline{e}_3 = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ shear í $[\underline{e}_2, \underline{e}_3]$ planið

Val av a og b, so at vit enda á $\underline{e}_i$, $i=1,2,3$.

1. $a = -\frac{v_2}{v_1}$ og $b = -\frac{v_3}{v_1}$

2. $a = -\frac{v_1}{v_2}$ og $b = -\frac{v_3}{v_2}$

3. $a = -\frac{v_1}{v_3}$ og $b = -\frac{v_2}{v_3}$

Dømi Lat $\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ so er $\underline{\underline{A}} = \begin{bmatrix} 1 & -\frac{v_1}{v_2} & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{v_3}{v_2} & 1 \end{bmatrix}$ tað shear, sum sum er parallelt við $[\underline{e}_1, \underline{e}_3]$ og sendir $\underline{v}$ til $\begin{bmatrix} 0 \\ v_2 \\ 0 \end{bmatrix}$.

Tak $\underline{v} = \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}$ og $\underline{\underline{A}} = \begin{bmatrix} 1 & -\frac{4}{6} & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{3}{6} & 1 \end{bmatrix}$

$$\underline{\underline{A}}\ \underline{v} = \begin{bmatrix} 1 & -\frac{4}{6} & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{3}{6} & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\cdot 4 - \frac{4}{6}\cdot 6 + 0\cdot 3 \\ 0\cdot 4 + 1\cdot 6 + 0\cdot 3 \\ 0\cdot 4 - \frac{3}{6}\cdot 6 + 1\cdot 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

Shear eftir einstakar ásir hava fylgjandi skap

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & 1 \end{bmatrix}$$

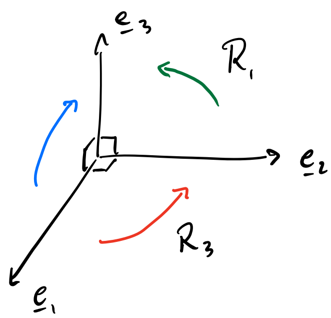
Her verður biðið um input frá tvey koordinat. Tað er møguligt at gera shear eftir eini ás við antin a ella b $=0$.

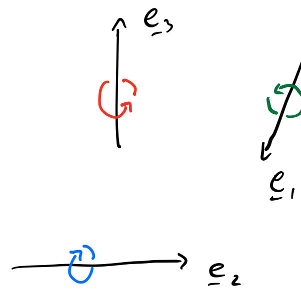
Rotatión spælir eftir tær somu reglurnar sum í 2D, men vit mugu halda eina dimensión í stað og rotera um hesa!
Rotera vit í $[\underline{e}_1, \underline{e}_2]$ flatanum, so skrivast hetta

$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ og vit endurnýta hesa til at

rotera í $[\underline{e}_1, \underline{e}_3]$ og $[\underline{e}_2, \underline{e}_3]$! Rotatiónin í $[\underline{e}_1, \underline{e}_2]$ er givin, men samstundis skal $\underline{e}_3$ vera í stað, altso rotera vit um $\underline{e}_3$.
Vit siga, at R_i er rotatión um $\underline{e}_i$, har $i=1,2,3$.

$$\underline{\underline{R}}_3 = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{\underline{R}}_2 = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}, \quad \underline{\underline{R}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$





Dømi Ein rotatión á 90° um $\underline{e}_2$ er í $[\underline{e}_1, \underline{e}_3]$ planinum, og lat okkum rotera $\underline{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

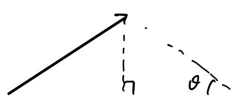
$$\underline{\underline{R}}_2 \underline{v} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}.$$

Akkurat 90° rotatión kann eisini samanberast við at gera tvørvektorin í roteraða planinum.

Sí R í (9.10) fyri rotatión av $\underline{v}$ um ein vektor $\underline{a}$ við vinkulin α.

Projektión kann sum áður gerast eftir koordinatásunum, men eisini eftir einum ávísum vektori.

Fløt projektión í ein flata kann t.d. skrivast $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.



Vit projektera ortogonalt, men vísa í kap. 10, hvussu vit gera oblique projektiónir.

Nú hava vit møguleikan at gera projektión í 1D ella 2D!
Vel ortonormal vektor $\underline{u}_1$ ella báðir $\underline{u}_1$ og $\underline{u}_2$.

Lat $\underline{A}_1 = \underline{u}_1$ og $\underline{\underline{A}}_2 = [\underline{u}_1 \ \ \underline{u}_2]$, so er

$\underline{\underline{P}}_1 = \underline{A}_1 \underline{A}_1^T = \underline{u}_1 \underline{u}_1^T$ projektión ortogonalt við $\underline{u}_1$, og

$\underline{\underline{P}}_2 = \underline{\underline{A}}_2 \underline{\underline{A}}_2^T = [\underline{u}_1 \ \ \underline{u}_2] \begin{bmatrix} \underline{u}_1^T \\ \underline{u}_2^T \end{bmatrix}$ ortogonal projektión á $[\underline{u}_1, \underline{u}_2]$.

Determinant Í 3D avmyndar $\underline{\underline{A}}$ $[\underline{e}_1, \underline{e}_2, \underline{e}_3]$ til $[\underline{a}_1, \underline{a}_2, \underline{a}_3]$ í rúminum. Tí skal determinanturin svara til eitt volumen mát. Forteknið vísir enn á um standard orienteringin broytist.

Givið $\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, so er determinanturin

$$|\underline{\underline{A}}| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

Hetta eitur cofactor expansion. Forteknið fylgir $(-1)^{i+j}$ hjá a_{ij}, og 2×2 matrican er cofaktorurin (við forteikn).

Sarrus Vit kunnu eisini brúka diagonalarnar

$$|\underline{\underline{A}}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{31} a_{22} a_{13} - a_{32} a_{23} a_{11} - a_{33} a_{21} a_{12}$$

Dømi 9.8' Her er cofaktor útvikling "skjótari"! Volumen, sum

$\underline{a}_1 = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$, $\underline{a}_2 = \begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix}$ og $\underline{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ útspenna.

$$|\underline{\underline{A}}| = \begin{vmatrix} 4 & -1 & 0 \\ 0 & 4 & 1 \\ 0 & 4 & 2 \end{vmatrix} = 4 \begin{vmatrix} 4 & 1 \\ 4 & 2 \end{vmatrix} = 4(4\cdot2 - 4\cdot1) = 16$$

Ovara tríkantsmatrix: $\begin{vmatrix} 4 & -1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 4 \begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} = 4 \cdot 4 \cdot 2 = 32$

Produkt av diagonalelementini! Hesi eru hóast alt eginvirðini.

$\underline{\underline{B}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 5 & 6 \end{bmatrix}$, nú er Sarrus eins skjótt

$$\begin{aligned} |\underline{\underline{B}}| &= 1\cdot2\cdot6 + 2\cdot1\cdot4 + 3\cdot3\cdot5 \\ &\quad - 4\cdot2\cdot3 - 5\cdot1\cdot1 - 6\cdot3\cdot2 \\ &= 12 + 8 + 45 - 24 - 5 - 36 \\ &= 0 \end{aligned}$$

$\underline{b}_3 = 2\underline{b}_2 - \underline{b}_1$

$$\begin{vmatrix} 4 & -1 & 0 \\ 0 & 4 & 1 \\ 0 & 4 & 2 \end{vmatrix} = 32 - 16 = 16.$$