304. Finn yvirførslufunktiónina

$$y''' + 4y'' + 3y' = u' + 3u.$$

$$H(s) = \frac{s+3}{s^3+4s^2+3s}, \quad \text{har} \quad s \notin \{0, -1, -3\}.$$

$$\because \quad s^3 + 4s^2 + 3s = 0$$

$$\Leftrightarrow \quad s \cdot (s^2 + 4s + 3) = 0$$

$$\Leftrightarrow \quad s = 0 \quad \vee \quad s^2 + 4s + 3 = 0$$

$$\Leftrightarrow \quad s = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3}}{2} = \frac{-4 \pm 2}{2} = \begin{cases} -1 \\ -3 \end{cases}$$

Faktorisera vit nevnaran út frá røturnar, so fáa vit

$$H(s) = \frac{s+3}{s(s+1)(s+3)} = \frac{1}{s(s+1)}.$$

305. Finn yvirførslufunktiónina

$$\dddot{y} + 3\ddot{y} - 4y = \dot{u} + 2u.$$

$$H(s) = \frac{s+2}{s^3+3s^2-4}, \quad \text{har} \quad s \notin \{1, -2\}. \quad \therefore \quad H(s) = \frac{1}{(s-1)(s+2)} = \frac{1}{s^2+s-2}.$$

Finn stationeru svarini til

(a) $u(t) = e^{2t}$. $\quad y(t) = H(2) \cdot e^{2t} = \frac{1}{4} e^{2t}.$

(b) $u(t) = e^{2it}$. $\quad y(t) = H(2i)\, e^{2it} = \frac{1}{(2i)^2 + 2i - 2} e^{2it} = \frac{1}{2i - 6} e^{2it}.$

(c) $u(t) = \cos 2t$. $\quad y(t) = \mathrm{Re}\left(H(2i) \cdot e^{2it}\right) = \mathrm{Re}\left(\frac{-6-2i}{40} e^{2it}\right)$

$$= \mathrm{Re}\left(\frac{-3}{20} - \frac{1}{20} i \cdot \left(\cos(2t) + i \cdot \sin(2t)\right)\right) = -\frac{3}{20}\cos(2t) + \frac{1}{20}\sin(2t).$$

311. Skóða $y''(t) + 3y'(t) + 2y(t) = u'(t) + 2u(t).$

(a) Avger $H(s)$. $$H(s) = \frac{s+2}{s^2+3s+2}, \quad \text{har } s \notin \{-1,-2\}$$

$$\because \quad s^2+3s+2 = 0 \iff (s+1)(s+2) = 0 \iff s=-1 \vee s=-2.$$

$$\therefore \quad H(s) = \frac{1}{s+1}.$$

(b) Finn loysnina til $u(t) = e^t$.

$$y(t) = H(1)\cdot e^t = \frac{1}{2}e^t.$$

319. Skóða $$\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - 2y = u' - u.$$

(i) Avger $H(s)$. $$H(s) = \frac{s-1}{s^3+s^2-2}, \quad \text{har } s \notin \{1, -1 \pm i\}.$$

$$s^3+s^2-2 = 0 \iff (s-1)(s^2+2s+2) = 0$$
$$\iff (s-1)(s-(-1+i))(s-(-1-i)) = 0$$

$$\therefore \quad H(s) = \frac{1}{s^2+2s+2}$$

(ii) Finn loysnina til $u(t) = 1$. $\quad y(t) = H(0)\cdot 1 = \frac{1}{2}.$

(iii) Finn loysnina til $u(t) = e^{2t}$. $\quad y(t) = H(2)\cdot e^{2t} = \frac{1}{10}e^{2t}.$

(iv) Finn loysnina til $u(t) = \cos(t)$.

$$y(t) = \mathrm{Re}\left(H(i)\cdot e^{it}\right) = \mathrm{Re}\left(\frac{1}{1+2i}(\cos(t) + i\sin(t))\right)$$
$$= \mathrm{Re}\left(\frac{1-2i}{5}(\cos(t)+i\sin(t))\right)$$
$$= \frac{1}{5}\cos(t) + \frac{2}{5}\sin(t).$$

$$y(t) = |H(i)|\cos(t + \arg(H(i)))$$
$$= \frac{\sqrt{5}}{5}\cos(t - \arctan(2))$$

Betragt differentialligningen

$$\frac{d^4y}{dt^4}+6\frac{d^2y}{dt^2}+25y=u'+3u,$$

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hvor u er påvirkningen og y er svaret.

(i) Bestem overføringsfunktionen $H(s)$.
(ii) Bestem det stationære svar på påvirkningen $u(t)=e^{-t}$.
(iii) Bestem det stationære svar på påvirkningen $u(t)=e^t\cos t$.

(i) Funktiónin er givin á s.18 (1.20). Vit fáa tískil

$$H(s) = \frac{s+3}{s^4+6s^2+25},$$

har s ikki er ein loysn í homogenu differentiallíkningini.

$$\lambda^4 + 6\lambda^2 + 25 = 0 \quad , \quad \lambda^2 = \eta$$

$$\eta^2 + 6\eta + 25 = 0 \iff \eta = \frac{-6 \pm\sqrt{6^2-4\cdot1\cdot25}}{2\cdot1} = \frac{-6\pm 8i}{2} = -3\pm4i$$

$$\lambda = \pm\sqrt{-3\pm4i} = \pm(1\pm2i)$$ H(s) ikki def í $s=\pm(1\pm2i)$.

(ii) Setningur 1.24 gevur okkum loysnina

$$y(t) = H(-1)\cdot e^{-t} = \frac{-1+3}{(-1)^4+6(-1)^2+25}e^{-t} = \frac{2}{32}e^{-t} = \frac{1}{16}e^{-t}.$$

(iii) Legg til merkis, at $u(t)=e^t\cos(t) = \mathrm{Re}(e^{(1+i)t})$. Per setning 1.26(i), so er $\mathrm{Re}(e^{(1+i)t})$ stationert svar til $e^{(1+i)t}$.

$$y(t) = H(1+i)\cdot e^{(1+i)t} = \frac{1+i+3}{(1+i)^4+6(1+i)^2+25}e^{(1+i)t} = \frac{4+i}{-4+12i+25}e^{(1+i)t}$$

$$= \frac{(4+i)(21-12i)}{21^2+12^2}e^{(1+i)t} = \frac{96-27i}{585}e^{(1+i)t}$$

Vit fáa tískil

$$\mathrm{Re}\left(\frac{96-27i}{585}e^{(1+i)t}\right) = \mathrm{Re}\left(\frac{96-27i}{585}\left(e^t\cos(t)+i\,e^t\sin(t)\right)\right)$$

$$= \frac{32}{195}e^t\cos(t) + \frac{3}{65}e^t\sin(t).$$

Betragt differentialligningen

$$\frac{d^6x}{dt^6} - x = e^t \cos t.$$

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(i) Bestem samtlige komplekse løsninger til den tilsvarende homogene ligning.
(ii) Bestem samtlige reelle løsninger til den tilsvarende homogene ligning.
(iii) Bestem samtlige reelle løsninger til den inhomogene ligning.

Vink: (iii) $e^t \cos t = \mathrm{Re}\left(e^{(1+i)t}\right)$.

(i) Vit hava karakteristisku líkningina

$$\lambda^6 - 1 = 0 \iff \lambda^6 = 1 \iff \lambda = e^{i\frac{\pi n}{3}}, \quad \text{har } n = 1, \dots, 6.$$

Við setning 1.15(i) er loysnin

$$x(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{\frac{1+\sqrt{3}i}{2}t} + c_4 e^{-\frac{1+\sqrt{3}i}{2}t} + c_5 e^{-\frac{1-\sqrt{3}i}{2}t} + c_6 e^{\frac{1-\sqrt{3}i}{2}t},$$

har $c_1, \dots, c_6 \in \mathbb{C}$.

(ii) Við 1.15(ii) fáa vit

$$x_H(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + c_4 e^{\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) + c_5 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + c_6 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right),$$

har $c_1, \dots, c_6 \in \mathbb{R}$.

(iii) Vit hava, at $e^t \cos(t) = \mathrm{Re}\left(e^{(1+i)t}\right)$. Lat $u = e^{(1+i)t}$, so er

$$x^{(6)} - x = u \implies H(s) = \frac{1}{s^6 - 1}, \quad s \neq 1.$$

Av tí at $(1+i)^6 \neq 1$, so er $H(s)$ definerað í $s = 1+i$. Setningur 1.24 gevur okkum tí loysnina

$$x_1(t) = H(1+i)\, e^{(1+i)t} = \frac{1}{(1+i)^6 - 1} e^t \left(\cos(t) + i \sin(t)\right) = \frac{1}{-1+8i} e^t \left(\cos(t) + i\sin(t)\right)$$

$$= \frac{-1-8i}{65} e^t \left(\cos(t) + i \sin(t)\right)$$

$$= -\frac{1}{65} e^t \left(\cos(t) + i\sin(t)\right) - \frac{8i}{65} e^t \left(\cos(t) + i \sin(t)\right)$$

$$= \left(-\frac{1}{65}\cos(t) + \frac{8}{65}\sin(t)\right) e^t + i\left(-\frac{8}{65}\cos(t) - \frac{1}{65}\sin(t)\right) e^t$$

Setningur 1.26 letur okkum gera niðurstøðu uppá $\mathrm{Re}(u)$, sum er ynskta ávirkanin.

$$x_0(t) = -\frac{e^t \cos(t) + 8 e^t \sin(t)}{65}.$$

Allar reellar loysnir eru per setning 1.20 givnar við

$$x(t) = x_H(t) + x_0(t).$$