Wavelets, fyrst og fremst hví hesi eru smart.

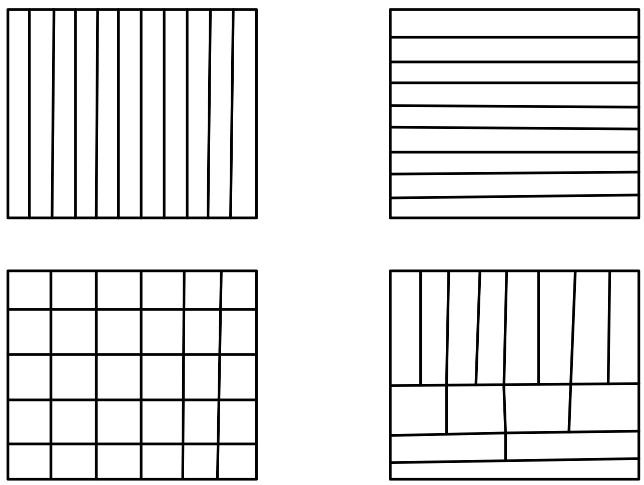
Heisenberg uncertainty principle. Time domain $\longleftrightarrow$ Frequency domain

$$\left(\int_{-\infty}^{\infty} x^2 \, |f(x)|^2 \, dx\right)\left(\int_{-\infty}^{\infty} w^2 \, |\hat{f}(w)|^2 \, dw\right) \geq \frac{1}{16\pi^2}$$

Resolution: 

Tíð Fourier transform

Perfect time resolution



Perfect frequency resolution

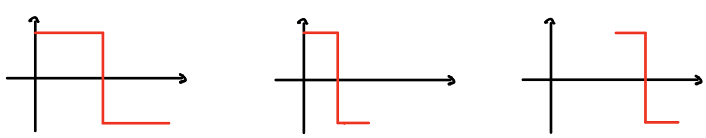
Spectrogram Multiresolution

Wavelets Við wavelets kunnu vit gera multiresolution analysis. Tankin er at gera stuttlivdar bylgjur og fáa upploysn í bæði tíð og frekvens.

Haar wavelettið kom fyrst í ~1910, men fyrst í 1980'ini kom rull á.

$$\text{Haar}: \quad \psi(x) = \begin{cases} 1, & 0 \leq x < \frac{1}{2}, \\ -1, & \frac{1}{2} \leq x < 1, \\ 0, & \text{annars.} \end{cases}$$

Her kallast ψ fyri móðurwavelet og $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$, $j,k \in \mathbb{Z}$, eru døttur wavelets.



Hesi wavelets ψ og $\psi_{j,k}$ eru eitt system, sum er basis fyri L^2, men harafturat eru nøgvir fyrimunir nú, tí bylgjurnar hava kompaktan stuðul. Wavelets yvirtaka nøgv av tí, sum vit annars brúka FFT til. Nøgdin av data sum er neyðug er nøgv minni og kann justerast alt eftir signalstyrkju.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

er ein ekvivalentur máti at skriva dótturwavelets.

Wavelet Transformatiónin er nú givin við innara produktinum

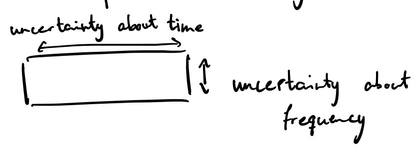
$$W_\psi(f)(a,b) = \langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t)\, \overline{\psi}_{a,b}(t)\, dt \,,$$

sum er kontinuert fyri $a,b \in \mathbb{R}$. Diskreta útgávan hevur $\psi_{j,k}(t) = \frac{1}{a^j} \psi\left(\frac{t-kb}{a^j}\right)$. Vit hava tískil

$$f(t) = \sum_{j,k=-\infty}^{\infty} \langle f(t), \psi_{j,k}(t) \rangle \, \psi_{j,k}(t) \,.$$

Fyrimunir og vansar: Scalogram hava ikki fulla upploysn. Kassarnir lýsa óvissu.

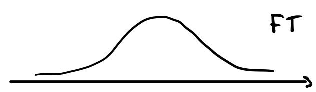
boxes represent uncertainty

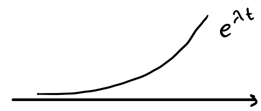


Wavelets eru stuttlivdar bylgjur. Per konstruktión hava tær kompakta stuðul, tvs. mongdin, har $\psi_{j,k} \neq 0$ er ein avmarkað og lukkað mongd $[a,b]$. Tað er av tí sama lættari í sjálvari nøgdini av útrokningum.

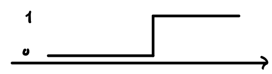
Laplace Transformatiónin

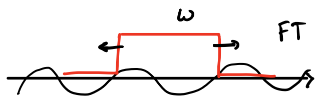
Ein generalisering av Fourier Transformatiónini, so at vit kunnu transformera funktiónir, sum ikki doyggja út. Tvs. nýtist ikki at vera $L^2(\mathbb{R})$.





Heaviside $H(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$





Laplace: PDE $\rightarrow$ ODE

ODE $\rightarrow$ alg. líkning

Folda $f(t)$ við $e^{-\gamma t} H(t)$, so at $f(t)\, e^{-\gamma t} H(t) \rightarrow 0$ tá $t \rightarrow \infty$.

$$F(t) = f(t)\, e^{-\gamma t} H(t) = \begin{cases} 0, & t<0 \\ f(t)\, e^{-\gamma t}, & t \geq 0 \end{cases}$$

Her er $e^{-\gamma t}$ ein "nóg" stabil dempandi funktión.

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} F(t)\, e^{-i\omega t}\, dt = \int_{0}^{\infty} f(t)\, e^{-\gamma t} e^{-i\omega t}\, dt$$

$$= \int_0^\infty f(t)\, e^{-(\gamma + i\omega)t}\, dt \quad , \quad s = \gamma + i\omega$$

$$= \int_0^\infty f(t)\, e^{st}\, dt.$$

Laplace Transformatiónin er ein eitt-síðað vektað Fourier Transformatión.

$$\bar{f}(s) = \int_0^\infty f(t)\, e^{st}\, dt$$

$\mathcal{L}$ verður eisini brúkt.

Inversa transformatiónin verður

$$f(t) = e^{\gamma t} F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\gamma t}\, \hat{F}(\omega)\, e^{i\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(s)\, e^{(\gamma + i\omega)t}\, d\omega \quad , \quad \begin{aligned} s &= \gamma + i\omega \\ ds &= i\, d\omega \Rightarrow d\omega = \tfrac{1}{i} ds \end{aligned}$$

$$= \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \bar{f}(s)\, e^{st}\, ds$$

Hetta er Laplace Transformatión parið, sum <u>er</u> ein Fourier Transformatión til funktiónir, sum ikki uppføra seg pent.