411.

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = 2x_1 - x_3 \\ \dot{x}_3 = -x_1 \end{cases} \qquad \dot{x} = Ax, \quad \text{har} \quad A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

(i) Finn fullkomuligu kompleksu loysnina.

$$\det(A - \lambda I) = (-\lambda)^3 - \lambda = -\lambda(\lambda^2 + 1) = 0 \iff \lambda = 0 \vee \lambda = \pm i.$$

$\lambda = 0$: $\quad A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = \pm i$: Vit nýtast bert eina vektorin. Við $-i$ fáa vit

$$\begin{bmatrix} i & 0 & 1 \\ -2 & i & -1 \\ -1 & 0 & i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0 \qquad \begin{matrix} v_1 = 1 \\ v_3 = -i \end{matrix} \implies v_2 = -1 - 2i, \quad \begin{array}{c} -i \\ \begin{bmatrix} 1 \\ -1+2i \\ -i \end{bmatrix} \end{array} \text{ og } \begin{array}{c} +i \\ \begin{bmatrix} 1 \\ -1-2i \\ i \end{bmatrix} \end{array}.$$

$$x(t) = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 e^{it} \begin{bmatrix} 1 \\ -1-2i \\ i \end{bmatrix} + c_3 e^{-it} \begin{bmatrix} 1 \\ -1+2i \\ -i \end{bmatrix}, \quad c_1, c_2, c_3 \in \mathbb{C}.$$

(ii)
$$x(t) = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \left(\cos(t) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} - \sin(t) \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right) + c_3 \left(\sin(t) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \cos(t) \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right), \quad c_1, c_2, c_3 \in \mathbb{R}.$$

414.

$$\dot{x} = \begin{bmatrix} 2 & 9 \\ -1 & -4 \end{bmatrix} x$$

(1) Finn λ og v. Fæst fullkomuliga loysnin?

$$\det(A - \lambda I) = (2-\lambda)(-4-\lambda) + 9 = \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2 = 0 \iff \lambda = -1, \quad p = 2.$$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \underline{0} \iff \begin{matrix} v_1 = -3 \\ v_2 = 1 \end{matrix}.$$

Hetta er ikki nóg mikið.

(2) Finn eina loysn $\quad x = ue^{-t} + vte^{-t}, \quad v \neq 0.$

Lat okkum seta $v = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$. Vit hava $\quad \dot{x} = (-u + v - vt)e^{-t}$

$$-u + v - vt = A(u + vt), \quad \text{sum væntað er } Av = -v.$$

$$\implies -u + v = Au \iff (A + I)u = v$$

$$\iff \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, \quad u = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\therefore \quad x(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} te^{-t}.$$

(3) Fullkomulig loysn: $\quad x(t) = c_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} e^{-t} + c_2 \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} te^{-t} \right), \quad c_1, c_2 \in \mathbb{R}.$

423 $\dot{x} = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$

(i) Avger fullkomuligu loysnina.

$$\det(A - \lambda I) = (1-\lambda)(-2-\lambda) = 0 \iff \lambda = 1 \lor \lambda = -2$$

$$\begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underline{0}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \underline{0}, \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}$$

$$x_h(t) = c_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

Lat okkum gita ein konstantan vektor $x_0 = \begin{pmatrix} a \\ b \end{pmatrix}$. Her er $\dot{x}_0 = 0$.

$$0 = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} \iff 0 = \begin{pmatrix} a+b \\ -2b \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$\iff \begin{cases} a + b = -1 \\ -2b = 4 \quad \Rightarrow b = -2 \end{cases} \Rightarrow a = 1 \quad \text{tvs} \quad x_0 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$x(t) = x_h(t) + x_0(t) = c_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

(ii) Finn loysnina har $x(0) = 0$.

$$x(0) = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} c_1 - \frac{1}{3} c_2 + 1 \\ c_2 - 2 \end{pmatrix}$$

$c_2 = 2$, so

$$c_1 - \frac{1}{3} \cdot 2 + 1 = 0 \iff c_1 = -\frac{1}{3}$$

$$x(t) = -\frac{1}{3} e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 e^{-2t} \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

421. Fullkomulig reel loysn hjá

(i) $\dot{x} = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

Fyrst finna vit loysnina hjá homogena systemið

$$\det(A - \lambda I) = \lambda \cdot (-1-\lambda) + 1 = -\lambda^2 - \lambda + 1 = 0$$

$$\iff \lambda = \frac{1 \pm \sqrt{1-4}}{-2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\begin{pmatrix} -\lambda & -1 \\ 1 & -1-\lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underline{0}, \qquad \begin{aligned} v_1 &= \frac{-1}{\lambda} \\ v_2 &= 1 \end{aligned} \qquad \begin{aligned} u_1 &= \frac{-1}{\bar\lambda} \\ u_2 &= 1 \end{aligned}$$

$$x(t) = c_1 e^{(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)t}\begin{pmatrix} \frac{-1}{-\frac{1}{2}+\frac{\sqrt{3}}{2}i} \\ 1 \end{pmatrix} + c_2 e^{(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)t}\begin{pmatrix} \frac{-1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i} \\ 1 \end{pmatrix}$$

Ivalcyst best at skalera: $\begin{pmatrix} \frac{-1}{\lambda} \\ 1 \end{pmatrix} \cdot (-2\lambda) = \begin{pmatrix} 2 \\ 1 \pm \sqrt{3}\, i \end{pmatrix}$ hjá $\lambda = -\frac{1}{2} \mp \frac{\sqrt{3}}{2} i$

$$x = c_1 \begin{pmatrix} 2\cos(\frac{\sqrt{3}}{2}t) \\ \cos(\frac{\sqrt{3}}{2}t) + \sqrt{3}\sin(\frac{\sqrt{3}}{2}t) \end{pmatrix} e^{-t/2} + c_2 \begin{pmatrix} 2\sin(\frac{\sqrt{3}}{2}t) \\ \sin(\frac{\sqrt{3}}{2}t) - \sqrt{3}\cos(\frac{\sqrt{3}}{2}t) \end{pmatrix} \quad c_1, c_2 \in \mathbb{R}.$$

Gita vit $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a\cos(t) + b\sin(t) \\ c\cos(t) + d\sin(t) \end{pmatrix} \Rightarrow \dot{x} = \begin{pmatrix} -a\sin(t) + b\cos(t) \\ -c\sin(t) + d\cos(t) \end{pmatrix}$

$$Ax + u = \begin{pmatrix} -c\cos(t) - d\sin(t) + \cos(t) \\ a\cos(t) + b\sin(t) - c\cos(t) - d\sin(t) + \sin(t) \end{pmatrix}$$

Her skal $-a = -d$, $b = 1-c$, $-c = b-d+1$ og $d = a-c$. Ein loysn er $a = 2$, $b = 1$, $c = 0$ og $d = 2$.

Partikuler loysn $x(t) = \begin{pmatrix} 2\cos(t) + \sin(t) \\ 2\sin(t) \end{pmatrix}$.

Fullkomulig loysn

$$x = c_1 \begin{pmatrix} 2\cos(\frac{\sqrt{3}}{2}t) \\ \cos(\frac{\sqrt{3}}{2}t) + \sqrt{3}\sin(\frac{\sqrt{3}}{2}t) \end{pmatrix} e^{-t/2} + c_2 \begin{pmatrix} 2\sin(\frac{\sqrt{3}}{2}t) \\ \sin(\frac{\sqrt{3}}{2}t) - \sqrt{3}\cos(\frac{\sqrt{3}}{2}t) \end{pmatrix} + \begin{pmatrix} 2\cos(t) + \sin(t) \\ 2\sin(t) \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

(ii) $\dot{x} = \begin{pmatrix} -3 & 4 \\ -2 & 1 \end{pmatrix} x + \begin{pmatrix} 2t \\ t \end{pmatrix}$

$$\det(A - \lambda I) = (-3-\lambda)(1-\lambda) + 8 = \lambda^2 + 2\lambda + 5 = 0 \iff \lambda = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

$\begin{pmatrix} -2-2i & 4 \\ -2 & 2-2i \end{pmatrix} v = \underline{0} \Rightarrow \begin{aligned} v_1 &= 1-i \\ v_2 &= 1 \end{aligned}$ hjá $\lambda = -1+2i$ og $\begin{pmatrix} 1+i \\ 1 \end{pmatrix}$ hjá $\lambda = -1-2i$.

$$x = c_1 \begin{pmatrix} \cos(2t) - \sin(2t) \\ \cos(2t) \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} \sin(2t) - \cos(2t) \\ \sin(2t) \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

Vit gita, at $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} at+b \\ ct+d \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$.

Vit fáa úr systeminum

$$\begin{pmatrix} -3 & 4 \\ -2 & 1 \end{pmatrix} x + \begin{pmatrix} 2t \\ t \end{pmatrix} = \begin{pmatrix} -3at - 3b + 4ct + 4d + 2t \\ -2at - 2b + ct + d + t \end{pmatrix}$$

$$= \begin{pmatrix} (-3a + 4c + 2)t + (-3b + 4d) \\ (-2a + c + 1)t + (-2b + d) \end{pmatrix} = \begin{pmatrix} 0 + a \\ 0 + c \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 & 4 & 0 \\ -1 & -3 & 0 & 4 \\ -2 & 0 & 1 & 0 \\ 0 & -2 & -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \\ 0 \end{pmatrix} \iff \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ \frac{6}{25} \\ -\frac{1}{5} \\ \frac{7}{25} \end{pmatrix}$$

$$x(t) = \begin{pmatrix} \frac{2}{5}t + \frac{6}{25} \\ -\frac{1}{5}t + \frac{7}{25} \end{pmatrix},$$ so vit fáa fullkomuligu loysnina

$$x = c_1 \begin{pmatrix} \cos(2t) - \sin(2t) \\ \cos(2t) \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} \sin(2t) - \cos(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} \frac{2}{5}t + \frac{6}{25} \\ -\frac{1}{5}t + \frac{7}{25} \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

409. $$\dot{x} = \begin{pmatrix} 5 & 3 \\ 9 & -1 \end{pmatrix} x$$

(i) Finn Φ. $$\det(A - \lambda I) = (5-\lambda)(-1-\lambda) - 27$$

$$= \lambda^2 - 4\lambda - 32 = (\lambda+4)(\lambda-8), \quad \lambda = -4 \vee \lambda = 8.$$

$$\lambda = -4: \quad \begin{pmatrix} 9 & 3 \\ 9 & 3 \end{pmatrix} v = \underline{0} \Rightarrow v = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\lambda = 8: \quad \begin{pmatrix} -3 & 3 \\ 9 & -9 \end{pmatrix} v = \underline{0} \Rightarrow v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x(t) = c_1 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{8t}, \quad c_1, c_2 \in \mathbb{R}$$

$$\Phi(t) = \begin{pmatrix} e^{-4t} & e^{8t} \\ -3e^{-4t} & e^{8t} \end{pmatrix}.$$

(ii) Finn loysnina hjá $\dot{x} = Ax + e^{4t}\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

$$\Phi(t)^{-1} = \frac{1}{4e^{4t}} \begin{pmatrix} e^{8t} & -e^{8t} \\ 3e^{-4t} & e^{-4t} \end{pmatrix}$$

Vit fáa nú

$$\Phi(t)^{-1} e^{4t} \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 8e^{8t} \\ 4e^{-4t} \end{pmatrix} = \begin{pmatrix} 2e^{8t} \\ e^{-4t} \end{pmatrix},$$

so

$$\int_0^t \Phi(\tau)^{-1} e^{4\tau} \begin{pmatrix} 3 \\ -5 \end{pmatrix} d\tau = \int_0^t \begin{pmatrix} 2e^{8t} \\ e^{-4t} \end{pmatrix} d\tau = \frac{1}{4} \begin{pmatrix} e^{8t} - 1 \\ 1 - e^{-4t} \end{pmatrix}.$$

Rokna vit saman fáa vit

$$\Phi(t)\int_0^t \Phi(\tau)^{-1} e^{4\tau}\begin{pmatrix}3\\-5\end{pmatrix} d\tau = \begin{pmatrix} e^{-4t} & e^{8t} \\ -3e^{-4t} & e^{8t}\end{pmatrix} \frac{1}{4}\begin{pmatrix} e^{8t}-1 \\ 1-e^{-4t}\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} e^{4t}-e^{-4t}+e^{8t}-e^{4t} \\ -3e^{4t}+3e^{-4t}+e^{8t}-e^{4t}\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -e^{-4t}+e^{8t} \\ -4e^{4t}+3e^{-4t}+e^{8t}\end{pmatrix}$$

$$= \begin{pmatrix}0\\-1\end{pmatrix}e^{4t} + \begin{pmatrix}-1\\3\end{pmatrix}e^{-4t} + \begin{pmatrix}1\\1\end{pmatrix}e^{8t}$$

Loysnin er givin við

$$x(t) = \begin{pmatrix}0\\-1\end{pmatrix}e^{4t} + c_1\begin{pmatrix}-1\\3\end{pmatrix}e^{-4t} + c_2\begin{pmatrix}1\\1\end{pmatrix}e^{8t}, \quad c_1, c_2 \in \mathbb{R}.$$

431. $$\dot{x} = \begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix} x + \begin{pmatrix}1\\1\end{pmatrix}$$

(i) Finn Φ. $\det(A-\lambda I) = (1-\lambda)(4-\lambda)-4 = \lambda^2-5\lambda = \lambda(\lambda-5) = 0 \iff \lambda=0 \vee \lambda=5.$

$\lambda=0$: $\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix} v = \underline{0} \Rightarrow v = \begin{pmatrix}2\\-1\end{pmatrix}$

$\lambda=5$: $\begin{pmatrix}-4 & 2\\ 2 & -1\end{pmatrix} u = \underline{0} \Rightarrow u = \begin{pmatrix}1\\2\end{pmatrix}$

$$x = c_1\begin{pmatrix}2\\-1\end{pmatrix} + c_2\begin{pmatrix}1\\2\end{pmatrix}e^{5t}, \quad c_1, c_2 \in \mathbb{R}$$

$$\Phi(t) = \begin{pmatrix}2 & e^{5t}\\ -1 & 2e^{5t}\end{pmatrix}.$$

(ii) Vís, at $x(t) = \underline{c}$ ikki kann vera ein loysn við at gita.

Tá $x = \begin{pmatrix}c_1\\c_2\end{pmatrix}$, so er $\dot{x} = \begin{pmatrix}0\\0\end{pmatrix}$. Men

$$\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix} x + \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}c_1+2c_2+1\\ 2c_1+4c_2+1\end{pmatrix} = \underline{0}$$ hevur onga loysn, parallelar linjur.

(iii) Finn fullkomuligu loysnina.

$$\Phi(t)^{-1} = \frac{1}{5e^{5t}}\begin{pmatrix}2e^{5t} & -e^{5t}\\ 1 & 2\end{pmatrix}.$$

$$\Phi(t)^{-1}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{5}\begin{pmatrix}1\\3e^{-5t}\end{pmatrix}$$

$$\int_0^t \Phi(\tau)^{-1}\begin{pmatrix}1\\1\end{pmatrix} d\tau = \frac{1}{5}\int_0^t \begin{pmatrix}1\\3e^{-5\tau}\end{pmatrix} d\tau = \frac{1}{5}\begin{pmatrix}t\\-\frac{3}{5}e^{-5t}+\frac{3}{5}\end{pmatrix}$$

$$= \begin{pmatrix}t/5\\-3/25\, e^{-5t} + 3/25\end{pmatrix}$$

$$\Phi(t)\int_0^t \Phi(\tau)^{-1}\begin{pmatrix}1\\1\end{pmatrix} d\tau = \begin{pmatrix}2 & e^{5t}\\-1 & 2e^{5t}\end{pmatrix}\begin{pmatrix}t/5\\-3/25\, e^{-5t} + 3/25\end{pmatrix}$$

$$= \begin{pmatrix}2t/5 - 3/25 + 3/25\, e^{5t}\\ -t/5 - 6/25 + 6/25\, e^{5t}\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}2t\\-t\end{pmatrix} - \frac{3}{25}\begin{pmatrix}1\\2\end{pmatrix} + \frac{3}{25}\begin{pmatrix}1\\2\end{pmatrix}e^{5t}.$$

Fullkomuliga loysnin er nú

$$\begin{pmatrix}x_1\\x_2\end{pmatrix} = \frac{1}{5}\begin{pmatrix}2t\\-t\end{pmatrix} - \frac{3}{10}\begin{pmatrix}0\\1\end{pmatrix} + c_1\begin{pmatrix}2\\-1\end{pmatrix} + c_2\begin{pmatrix}1\\2\end{pmatrix}e^{5t}, \quad c_1, c_2 \in \mathbb{R}.$$

(iv) Finn partikuleru loysnina við $x(0) = \begin{pmatrix}1\\0\end{pmatrix}$.

$$\begin{pmatrix}x_1(0)\\x_2(0)\end{pmatrix} = -\frac{3}{10}\begin{pmatrix}0\\1\end{pmatrix} + c_1\begin{pmatrix}2\\-1\end{pmatrix} + c_2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}1\\\frac{3}{10}\end{pmatrix} \iff \begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}\frac{17}{50}\\\frac{8}{25}\end{pmatrix}.$$

$$\begin{pmatrix}x_1\\x_2\end{pmatrix} = \frac{1}{5}\begin{pmatrix}2t\\-t\end{pmatrix} - \frac{3}{10}\begin{pmatrix}0\\1\end{pmatrix} + \frac{17}{50}\begin{pmatrix}2\\-1\end{pmatrix} + \frac{8}{25}\begin{pmatrix}1\\2\end{pmatrix}e^{5t}$$

$$= \frac{1}{5}\begin{pmatrix}2t\\-t\end{pmatrix} + \frac{1}{25}\begin{pmatrix}17\\-16\end{pmatrix} + \frac{8}{25}\begin{pmatrix}1\\2\end{pmatrix}e^{5t}.$$