Parameter sensitivity of the 'Limits to Growth' world model

P. J. Vermeulen

Department of Applied Mathematics, University of Pretoria, Pretoria, South Africa

and D. C. J. de Jongh

National Research Institute for Mathematical Sciences, Council of Scientific and Industrial Research, PO Box 395, Pretoria, South Africa
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The mathematical model basic to the results of 'The Limits to Growth' is found to be very sensitive to small parameter variations. By changing three parameters by 10% each in 1975 the world population collapse predicted by the model is averted.

Introduction

The controversial book[1] 'The Limits to Growth' with its message of catastrophe to the world gave rise to much criticism. The most comprehensive work in this respect is the contribution of the Science Policy Research Unit of the University of Sussex[2]. The major submodels of the world model are analysed and pertinent discussions of dubious assumptions and approximations are provided.

Criticism of the model from a mathematical point of view is the main object of the present paper.

Sensitivity functions

Complete information regarding the mathematical structure of the world model is provided in the technical report[3] 'The Dynamics of Growth in a Finite World'. The model is presented as a simulation model, written in the simulation language DYNAMO and consisting of a large set of non-linear algebraic and difference equations. The model can also be described by a system of non-linear, first-order ordinary differential equations of the form:

$$\mathrm{d}x^i/\mathrm{d}t = f^i(x^j, p^s) \tag{1}$$

where the x^i denote 29 state variables and the p^s denote 38 parameters. This reformulation does not change the behaviour or operation of the model because the Meadows version is obtained from equation (1) on applying a simple numerical integration formula.

Using this representation of the model we have set up a system of 1102 simultaneous differential equations for the calculation of the sensitivity functions of the system:

$$U(I,J) = \sum_{M=1}^{29} D(I,M) \cdot U(M,J) + E(I,J) \tag{2}$$

and

$$U(I,J)\,(\text{at } t = 1900) = 0 \tag{3}$$

$$(I = 1, \ldots, 29\,; J = 1, \ldots, 38)$$

In the above equations the sensitivity functions $U(I,J)$ denote the partial derivatives $\partial x^i/\partial p^j$, the coefficients $D(I,M)$ denote the functions $\partial f^i/\partial x^m$ and the functions $E(I,J)$ denote $\partial f^i/\partial p^j$. We have also formulated expressions to calculate the normalized sensitivity functions:

$$N(I,J) = U(I,J)/(x^i/p^j) \tag{4}$$

The sets of equations (1), (2), (3) and (4) were solved simultaneously on an IBM 370/158 computer. The calculations were made in double precision using a variable step fourth order Runge–Kutta integration routine. The program listing (in FORTRAN and CSMP) is available from the authors on request.

The above approach to parameter sensitivity is well known in the engineering literature[4]. In the field of social systems it was first employed by Burns and Malone[5] for the analysis of Forrester's world model[6].

Inflation factors

The normalized sensitivity functions can be explained as follows. If a parameter, p^{12} say, is given an increase of 1% then the normalized sensitivity function $N(15, 12)$, say, gives the corresponding approximate percentage increase in the state variable x^{15} as a function of time. This approximation improves as the perturbation gets smaller. We therefore define an inflation factor as follows: if the parameter p^i is given an increment of 1 part in 10^n then the inflation factor $N(S,I)$ gives the corresponding increase of the state variable

Table 1 Maximum inflation factors

Parameters	Sector	State variables				
		Population	Capital	Resources	Agriculture	Pollution
Population						
	LEN	−6.89(2024)	−10.25(2092)	2.94(2022)	7.41(2100)	5.89(2100)
	RLT	11.40(2024)	12.98(2092)	−3.41(2022)	−9.22(2100)	7.30(2100)
	DCFSN	−11.34(2024)	−10.93(2092)	2.65(2022)	7.63(2100)	5.96(2100)
Capital						
	ICOR	−19.95(2024)	−30.76(2092)	11.69(2022)	23.29(2100)	19.65(2100)
	ALIC	10.92(2026)	17.65(2092)	−7.24(2022)	−12.79(2100)	−10.94(2100)
	FIOAC	−23.90(2024)	−37.48(2092)	13.78(2022)	28.22(2100)	19.69(2100)
Resources						
	NRUF	−2.75(2056)	4.13(2092)	−1.18(2022)	−3.58(2100)	−3.08(2100)
Agriculture						
	LYF	4.74(2024)	7.50(2092)	−2.88(2022)	−6.36(2100)	−5.27(2100)
	LFH	4.81(2024)	7.90(2092)	−2.91(2022)	−6.54(2100)	−5.40(2100)
	ILF	4.52(2024)	7.42(2092)	−2.69(2022)	−6.17(2100)	−5.08(2100)
Pollution						
	PPTD	−0.34(2084)	0.18(2100)	−0.068(2056)	0.39(2038)	2.38(2100)

x^s (as $N(S,I)$ parts in 10^n). It is now clear that an inflation factor of more than 1 indicates changes in the state variable which are bigger than the parameter change which caused them. The maximum inflation factor for every state variable relative to each parameter was obtained by means of the computer calculations. The so-called standard run or basic model of Meadows was employed for this investigation. The results considered most interesting are given in *Table 1.*

Table 1 illustrates the maximum inflation factors for state variables in the different subsectors relative to changes in the parameters shown. The times when these maxima occur are also noted (the Meadows model runs from 1900 to the year 2100). The most sensitive parameters in the model are those occurring in the equations for the capital sector, i.e. Industrial Capital Output Ratio (ICOR), Average Lifetime of Industrial Capital (ALIC) and Fraction of Industrial Output Allocated to Consumption (FIOAC). State variables in the capital sector are those most affected by changes in these parameters. The sensitive parameters in the population subsector are Life Expectancy Normal (LEN), Reproductive Lifetime (RLT) and Desired Completed Family Size Normal (DCFSN). The parameters in the agriculture subsector have the property that they influence the capital sector more than their own sector. These parameters are Land Yield Factor (LYF), Land Fraction Harvested (LFH) and Inherent Land Fertility (ILF). The parameters in the Resources and Pollution sectors are relatively insensitive.

If the perturbations are small enough, the inflation factors for the same time instants may be added. The fact that the maxima of the inflation factors for state variables in the capital sector all occur at the year 2092 made the following result possible. If the parameters ICOR and FIOAC are both increased by 1 part in 1000 and ALIC is decreased by 1 part in 1000 we find an inflation factor of more than (−)85 parts in 1000 in the capital subsector. If changes of the parameters in the population sector are included, an inflation factor of more than 100 can be reached. This result was verified by actually changing the parameters by hand in the original Meadows program. A decrease of more than 11% was observed in the state variable LUFD (Labour Utilization Fraction Delayed) in the year 2092 for a 0.1% change in each of the above six parameters.

The sensitivity analyses that are shown in the technical report[3] were not based on sensitivity functions. Meadows changed parameters by large amounts and in an intuitive manner. On the other hand, the inflation factors computed in the present study yield a systematic procedure for locating sensitive parameters in the model. Thus certain parameter sensitivities are demonstrated in this paper which have not been elicited by *ad hoc* methods[3].

Doom postponed?

From the complete set of graphs of the sensitivity functions one can pick out those parameters which should be changed in order to influence a state variable in a desired direction.

We have found that if the above three parameters in the capital sector are all given a 10% change in the year 1975, the disastrous population collapse, which is the main thesis of the Meadows study, is avoided. Furthermore, this result was confirmed by employing the standard run of the Meadows technical report and changing these parameters by hand. This result is in direct conflict with Meadows where he says[3]: '...... we have come to the conclusion that the standard behaviour mode of overshoot and decline exhibited by the model is remarkably insensitive to variations in the estimates of most system parameters'.

The parameter changes are as follows: FIOAC is given as 0.43 and changed to 0.473, which means that 47.3% of all

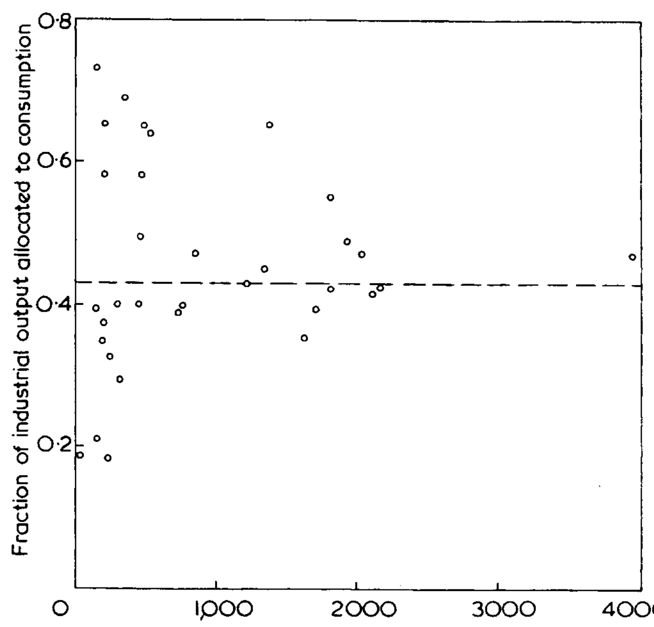

Figure 1 Evidence for the choice of 0.43 for FIOAC by Meadows *et al.*

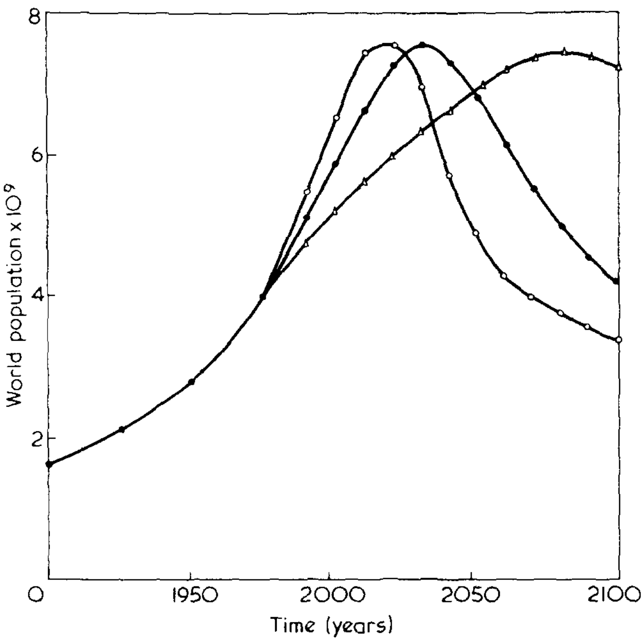

Figure 2 World population history from 1900 to 2100 for three simulation runs. •, Run 1: Meadows' standard run; △, run 2: Meadows' standard run except for three parameter changes in 1975 (ICOR and FIOAC increased by 10% and ALIC decreased by 10%); ○, run 3: Meadows' standard run except for three parameter changes in 1975 (ICOR and FIOAC decreased by 10% and ALIC increased by 10%)

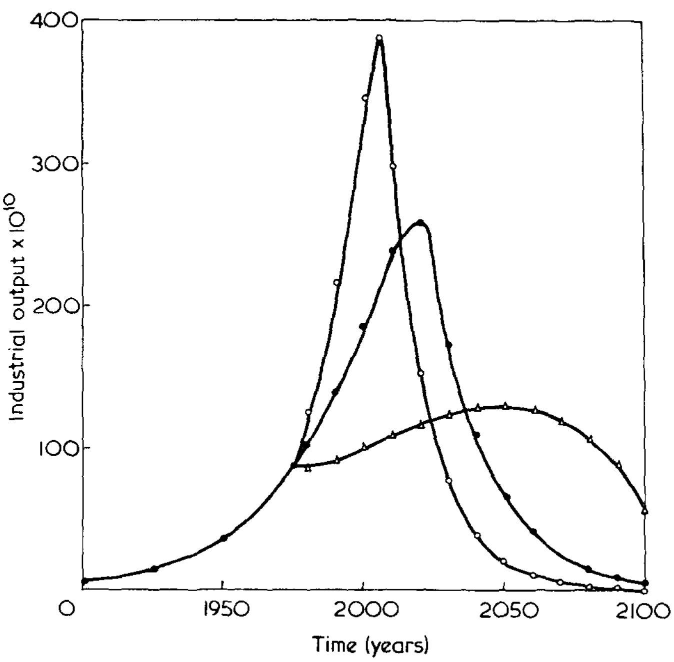

Figure 3 Total world industrial output history for runs 1 (•), 2 (△) and 3 (○)

industrial output is consumed. In the technical report the value of 0.43 for the standard run was inferred from the graph in *Figure 1*, where FIOAC was plotted against GNP per capita for 33 countries. The parameter ALIC is decreased from 14 years to 12.6 years. Meadows says[3]: 'Kuznets has estimated lifetimes of 50 years for building and construction materials and 10 years for producers' equipment. We would expect the average lifetime for all industrial capital to fall between these two figures'. The ICOR parameter is changed from 3 years to 3.3 years. The relationship between Industrial Output (IO) and Industrial Capital (IC) is:

$$IO = k \cdot IC/ICOR$$

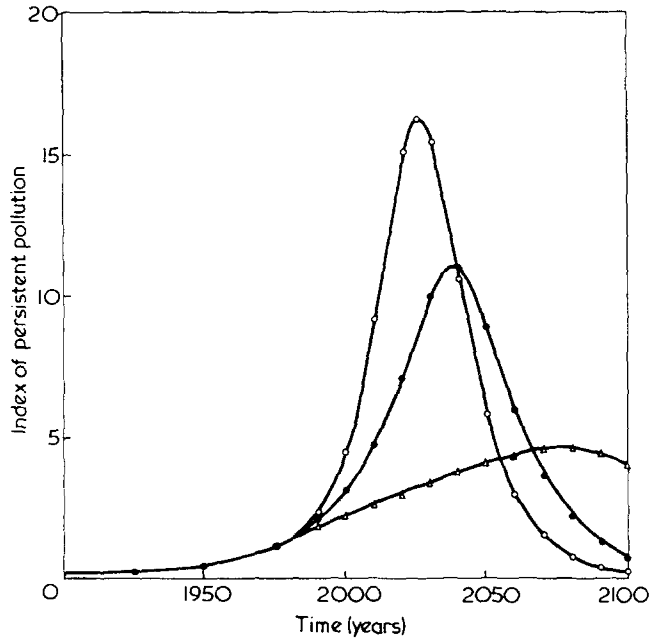

Figure 4 History of persistent pollution for runs 1 (•), 2 (△) and 3 (○)

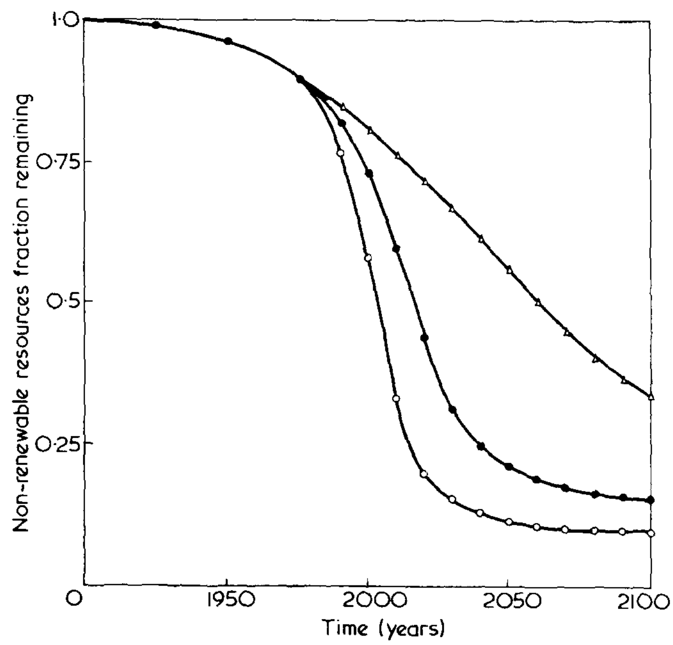

Figure 5 History of the depletion of non-renewable resources for runs 1 (•), 2 (△) and 3 (○)

By increasing ICOR by 10% we decrease the amount of IO (for the same amount of IC) by 9.1%. In two of his simulation runs Meadows decreased ICOR to 2 years and increased ICOR to 4 years[3].

At this stage it needs to be pointed out that the parameters of the standard run are assumed to have the same value for the 200 years from 1900 to 2100.

The net effect of the three changes above is to keep industrial output in check. This limitation causes pollution to be much less than in the normal run so that one of the main causes of the population collapse is avoided. Pertinent graphs for the Meadows standard run and our new run are given in *Figures 2, 3, 4* and *5*. On the same graphs are also shown the results of another run where we have changed the three parameters in 1975 by 10% in the opposite direction. The

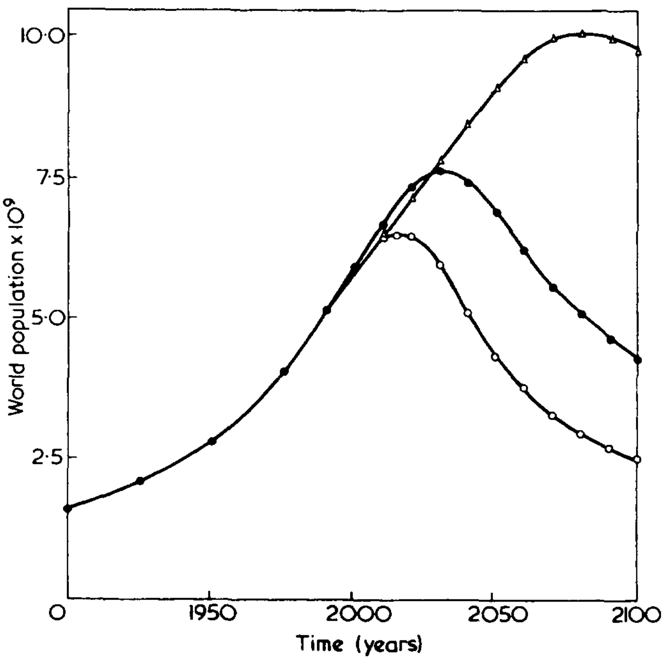

Figure 6 World population history from 1900 to 2100 for three simulation runs. •, Run 1: Meadows' standard run; △, run 4: the same model as for run 2 except for two parameter changes in 1975 (RLT decreased by 10% and DCFSN increased by 10%); ○, run 5: the same model as for run 3 except for two parameter changes in 1975 (RLT increased by 10% and DCFSN decreased by 10%)

outcome is a disaster of even greater proportions than in the Meadows standard run.

If we want to reach a total world population of nearly 10 billion people in the year 2100 instead of the 7.2 billion of our optimistic run, we need only change two additional parameters in the population sector by 10% in the year 1975. The changes are the following: the RLT of women is *decreased* from 30 years to 27 years and the DCFSN is increased from 4 children to 4.4 children. The population curves are shown in *Figure 6*. Once again the result for parameter changes in the opposite direction are also shown.

Conclusion

We have isolated sensitive pressure points in the Meadows world model. It is evident that the values of the economic parameters are very critical in the model. Changes of 10% in either direction can make a phenomenal difference in the simulation results. The question is whether the economic pressures are also the most influential in the real world.

In the last chapter of 'The Limits to Growth' very severe measures are suggested for every subsector in the world model in order to avoid the catastrophic population collapse of the standard run. In contrast, our results indicate that by combining three changes of 10% each in only the economic parameters, the population collapse can also be avoided. Our result is achieved with smaller pressures in appropriate directions. One speculates that the real world has so many pressure points and is so flexible that the correct small pressures on the correct parameters could cause desirable outcomes for the world's evolution. This conjecture demands that further concentrated and urgent research be undertaken.

Acknowledgements

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