Mathematics in the modern World

Jhonie H. Castro

General Education

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Course Code: GE 3

Course Description: This course deals with the nature of mathematics, appreciation of its practical, intellectual, and aesthetic dimensions, and application of mathematical tools in daily life. The course begins with an introduction to the nature of mathematics as an exploration of patterns (in nature and the environment) and as an application of inductive and deductive reasoning. By exploring these topics, students are encouraged to go to beyond the typical understanding of mathematics as merely a bunch of formulas, but as a source of aesthetics in patterns of nature, for example and a rich language in itself (and of science) governed by logic and reasoning. The course the proceeds to survey ways in which mathematics provides a tool for understanding and dealing with various aspects of present-day living, such as managing personal finances, making social choices, appreciating geometric designs, understanding codes used in data transmission and security, and dividing limited resources fairly. These aspects will provide opportunities for actually doing mathematics in a broad range of exercises that bring out the various dimensions of mathematics as a way of knowing and test the students' understanding and capacity.

Course Intended Learning Outcomes (CILO):

At the end of the course, students should be able to:

- Discuss the nature of mathematics, what it is, how it is expressed, represented and used examine the different influences, factors, and forces that shape the self.
- Discuss the language and symbols of mathematics.
- Analyze codes and coding schemes used for identification, privacy, and security purposes

Course Requirements:

- **Class Standing - 60%**
- **Major Exams - 40%**

Periodic Grade 100%

PRELIM GRADE	=	60% Class standing + 40% (Prelim exam)
MIDTERM GRADE	=	30% (Prelim Grade) + (70% [60% Class standing + 40% (Midterm exam)])
FINAL GRADE	=	30% (Midterm Grade) + (70% [60% Class standing + 40% (Final exam)])

MODULE 1
MATHEMATICS IN OUR WORLD

Introduction

Mathematics is a useful way to think about nature and our world. The nature of mathematics underscores the exploration of patterns (in nature and the environment). Mathematics exists everywhere and it is applied in the most useful phenomenon. Even looking by just at the ordinary part of the house, the room and the street, mathematics is there. This is one subject thought as the sole objective language that people in the modern world understand each other.

The origin of mathematics can be traced to the history and significance of patterns and numbers. It deals with ideas translated to objects and concepts created by humans. They are invented to link the meaning of pattern which result experiences associated with the counting, sequences, and regularities.

Learning Outcomes

At the end of this module, students should be able to:

1. Identify patterns in nature and regularities in the world;
2. Articulate the importance of mathematics in one's life;
3. Argue about the nature of mathematics, what it is, how it is expressed, represented and used;
4. Discuss the role of mathematics in some disciplines; and
5. Express appreciation for mathematics as human endeavor.

Lesson 1. Patterns in Nature and the Regularities in the World

Patterns and counting are correlative. Counting happens when there is pattern. When there is counting, there is logic. Consequently, pattern in nature goes with logic or logical set-up. There are reasons behind a certain pattern. That's why, oftentimes, some people develop an understanding of patterns, relationships, and functions and use them to represent and explain real-world phenomena. Most people say that mathematics is the science behind patterns. Mathematics exists everywhere as patterns do in nature. Not only do patterns take many forms within the range of school mathematics, they are also a unifying mechanism.

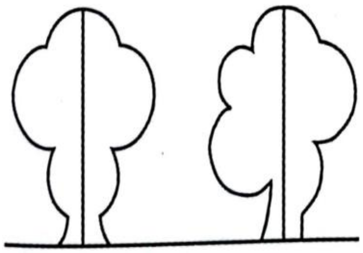
Number patterns-such as 2, 4, 6, 8-are familiar to us since they are among the patterns first learned in our younger years. As we advance, we experience number patterns again through the huge concept of functions in mathematics inside and outside school. But patterns are much broader and common anywhere anytime.

Patterns can be sequential, spatial, temporal, and even linguistic. The most basic pattern is the sequence of the dates in the calendar such as 1 to 30 being used month after month; the seven (7) days in a week i.e. Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday; the twelve (12) months i.e. January, February, March, April, May, June, July, August, September, October, November, December, and the regular holidays in a year i.e. New Year's Day, Valentine's Day, Holy Week, Labor Day, Independence Day, National Heroes Day, Ramadan, All Saints Day, Bonifacio Day, Christmas Day and Rizal Day. These are celebrated in the same sequence every year. All these phenomena create a repetition of names or events called regularity.

In this world, a regularity (Collins, 2018), is the fact that the same thing always happens in the same circumstances. While a pattern is a discernible regularity in the world or in a man-made design. As such, the elements of a pattern repeat in a predictable manner. Patterns in nature are visible regularities of form found in the natural world. These patterns recur in different contexts and can sometimes be modelled mathematically. Natural patterns include symmetries, trees, spirals, meanders, waves, foams, tessellations, cracks and stripes. A geometric pattern is a kind of pattern formed of geometric shapes and typically repeated like a wallpaper design. In Algebra, there are two common categories of patterns, the repeating pattern and the growing pattern.

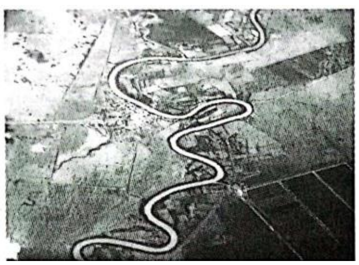
Regularity in the world states the fact that the same thing always happens in the same circumstances. According to **Ian Stewart** (1995), we live in a universe of patterns.

Some Examples of Patterns in Nature



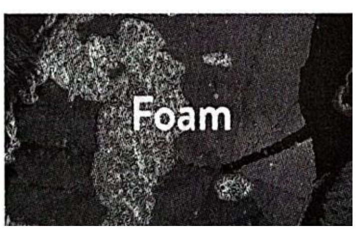
Symmetry means agreement in dimensions, due proportion and arrangement. In everyday language, it refers to a sense of harmonious and beautiful proportion and balance. In mathematics, "symmetry" means that an object is invariant to any of various transformations including reflection, rotation or scaling.

A **spiral** is a curve which emanates from a point, moving farther away as it revolves around the point. Cutaway of a nautilus shell shows the chambers arranged in an approximately logarithmic spiral



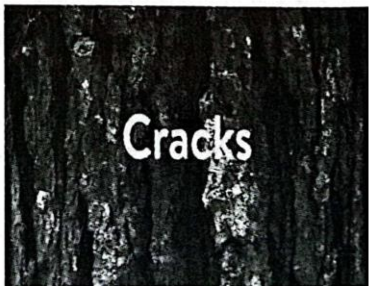
A **meander** is one of a series of regular sinuous curves, bends, loops, turns, or windings in the channel of a river, stream, or other watercourse. It is produced by a stream or river swinging from side to side as it flows across its floodplain or shifts its channel within a valley.

A **wave** is a disturbance that transfers energy through matter or space, with little or no associated mass transport. Waves consist of oscillations or vibrations of a physical medium or a field, around relatively fixed locations. Surface waves in water show water ripples.

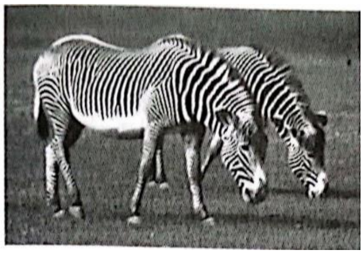


Foam is a substance formed by trapping pockets of gas in a liquid or solid. A bath sponge and the head on a glass of beer are examples of foams. In most foams, the volume of gas is large, with thin films of liquid or solid separating the regions of gas. Soap foams are also known as suds.

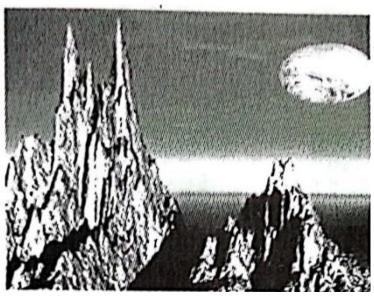
A **tessellation** of a flat surface is the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. In mathematics, tessellations can be generalized to higher dimensions and a variety of geometries.



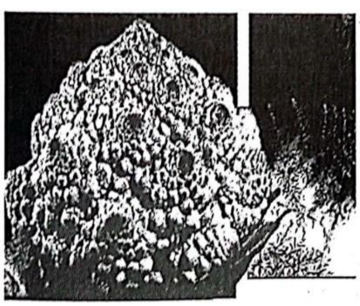
A **fracture or crack** is the separation of an object or material into two or more pieces under the action of stress. The fracture of a solid usually occurs due to the development of certain displacement discontinuity surfaces within the solid. If a displacement develops perpendicular to the surface of displacement, it is called a normal tensile crack or simply a crack; if a displacement develops tangentially to the surface of displacement, it is called a shear **crack,** slip band, or dislocation.



Stripes are made by a series of bands or strips, often of the same width and. color along the length.

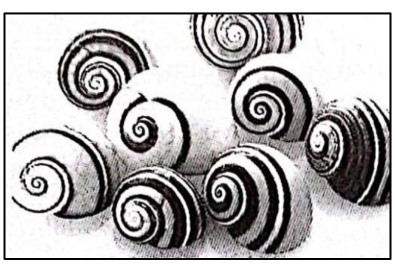


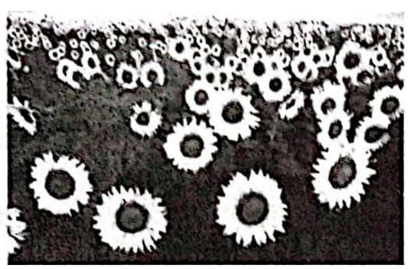
A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems - the pictures of chaos. Geometrically, they exist in between our familiar dimensions.

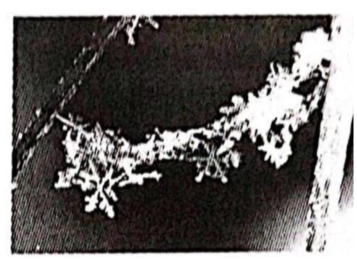


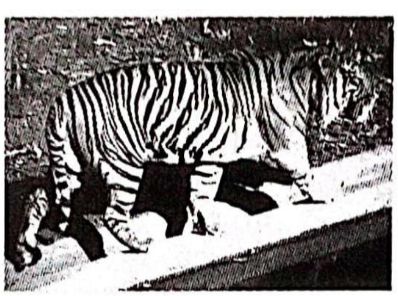
Affine Transformations - These are the processes of rotation, reflection and scaling. Many plant forms utilize these processes to generate their structure. In the case of Broccoli and Cauliflower heads, it can readily be seen that there is a type of pattern, which also shows some spiraling in the case of Broccoli.

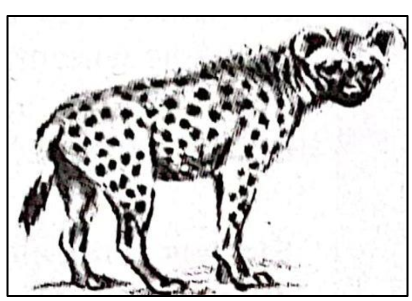
The following pictures show patterns and regularities in nature.

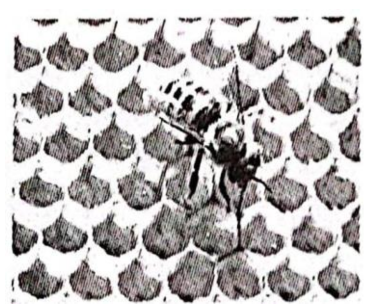












Fibonacci sequence

Another one in this world that involves pattern is the **Fibonacci number** (Grist, 2011). These numbers are nature's numbering system. They appear everywhere in nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees and even all of mankind.

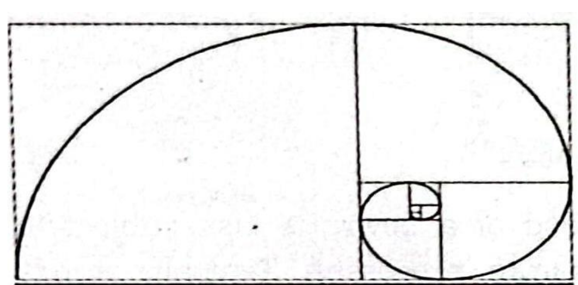
In Mathematics, the **Fibonacci numbers** are the numbers in the following integer sequence, called the **Fibonacci sequence**, and characterized by the fact that every number after the first two is the sum of the two preceding ones:

1,1,2,3,5,8,13,21,34,55,89,144, ...

The sequence $\mathbf{F_n}$ of Fibonacci numbers is defined by the recurrence relation:

$$\mathbf{F_n = F_{n-1} + F_{n-2}}$$

Also known as the Golden Ratio, its ubiquity and astounding functionality in nature suggests its importance as a fundamental characteristic of the universe. Leonardo Fibonacci came up with the sequence when calculating the ideal expansion pairs of rabbits over the course of one year. Today, its emergent patterns and ratios (phi = 1.61803...) can be seen from the microscale to the macroscale, and right through to biological systems and inanimate objects. While the Golden Ratio doesn't account for every structure or pattern in the universe, it's certainly a major player.



Importance of Mathematics in Life

According to Katie Kim (2015), Math is a subject that makes students either jump for joy or rip their hair out. However, math is inescapable as you become an adult in the real world. From calculating complicated algorithms to counting down the days till the next Game of Thrones episode, math is versatile and important, no matter how hard it is to admit. Before you decide to doze off in math class, consider this list of reasons why learning math is important to you and the world.

Example of Mathematics in real life application:

1. Restaurant Tipping
2. Netflix film viewing
3. Calculating Bills
4. Computing Test Scores
5. Tracking Career
6. Doing Exercise
7. Handling Money
8. Making Countdowns
9. Baking and Cooking
10. Surfing Internet

Lesson 2. Nature of Mathematics, Concept, and Applications

It is important to further discuss the nature of mathematics, what it is, how it is expressed, represented and used.

According to the **American Association for the Advancement of Science** (1990), Mathematics relies on both logic and creativity, and it is pursued both for a variety of practical purposes and for its intrinsic interest. For some people, and not only professional mathematicians, the essence of mathematics lies in its beauty and its intellectual challenge.

1. Patterns and Relationships

Mathematics is the science of patterns and relationships. As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. The abstractions can be anything from strings of numbers to geometric figures to sets of equations.

2. Mathematics, Science and Technology

Mathematics is abstract. Its function goes along well with Science and Technology. Because of its abstractness, mathematics is universal in a sense that other fields of human thought are not. It finds useful applications in business, industry, music, historical scholarship, politics, sports, medicine, agriculture, engineering, and the social and natural sciences.

3. Mathematical Inquiry

Normally, people are confronted with problems. In order to live at peace, these problems must be solved. Using mathematics to express ideas or to solve problems involves at least three phases: (1) representing some aspects of things abstractly, (2) manipulating the abstractions by rules of logic to find new relationships between them, and (3) seeing whether the new relationships say something useful about the original things.

4. Abstraction and Symbolic Representation

Mathematical thinking often begins with the process of abstraction-that is, noticing a similarity between two or more objects or events. Aspects that they have in common, whether concrete or hypothetical, can be represented by symbols such as numbers, letters, other marks, diagrams, geometrical constructions, or even words. Whole numbers are abstractions that represent the size of sets of things and events or the order of things within a set. The circle as a concept is an abstraction derived from human faces, flowers, wheels, or spreading ripples; the letter A may be an abstraction for the surface area of objects of any shape, for the acceleration of all moving objects, or for all objects having some specified property; the symbol +represents a process of addition, whether one is adding apples or oranges, hours, or miles per hour. Abstractions are made not only from concrete objects or processes; they can also, be made from other abstractions, such as kinds of numbers (the even numbers, for instance).

5. Manipulating Mathematical Statements

After abstractions have been made and symbolic representations of them have been selected, those symbols can be combined and recombined in various ways according to precisely defined rules. Typically, strings of symbols are combined into statements that express ideas or propositions.

For example, the symbol A for the area of any square may be used with the symbol s for the length of the square's side to form the proposition $\mathbf{A = s^2}$. This equation specifies how the area is related to the side-and also implies that it depends on nothing else.

6. Application

Mathematical processes can lead to a kind of model of a thing, from which insights can be gained about the thing itself. Any mathematical relationships arrived at by manipulating abstract statements may or may not convey something truthful about the thing being modeled.

For example, if 2 cups of water are added to 3 cups of water and the abstract mathematical operation $2+3 = 5$ is used to calculate the total, the correct answer is 5 cups of water. However, if 2 cups of sugar are added to 3 cups of hot tea and the same operation is used, 5 is an incorrect answer, for such an addition actually results in only slightly more than 4 cups of very sweet tea.

Sometimes common sense is enough to enable one to decide whether the results of the mathematics are appropriate.

Lesson 3. The Role of Mathematics in Some Disciplines

Mathematics is offered in any college course. It is found in every curriculum because its theories and applications are needed in any workplace. That's why students can't stay away from attending math classes. There has to be mathematics in the real world. This subject always brings life to any person or professional. Every second of the day needs mathematical knowledge and skills to perform academic activities and office routines. If ordinary people have to use math, then much more for students to know and master it so they will succeed in class in the school.

As posted by Angel Rathnabai (2014), Mathematics is not only number work or computation, but is more about forming generalization, seeing relationships, and developing logical thinking and reasoning.

Here are some main disciplines in which the role of Mathematics is widely accepted:

1. Mathematics in Physical Sciences

In Physics, every rule and principle take the mathematical form ultimately. Mathematics gives a final shape to the rules of physics. It presents them in a workable form. Mathematical calculations occur at every step in physics.

The units of measurement are employed to substances in physics as frequently as in mathematics. The Chare's law of expansion of gases is based upon mathematical calculations. Graduation of the stem of thermometer and then the conversion of scales is also a mathematical work. The concept is involved in Fluid Dynamics, Computational Fluid Dynamics, and Physical Oceanography.

2. Mathematics in Chemistry

Math is extremely important in physical chemistry especially in advanced topics such as quantum or statistical mechanics. Quantum relies heavily on group theory and linear algebra requires knowledge of mathematical/physical topics such as Hilbert spaces and Hamiltonian operators. Statistical mechanics relies heavily on probability theory. Other fields of chemistry also use a significant amount of math. For example, most modern IR and NMR spectroscopy machines use the Fourier transform to obtain spectra. Even biochemistry has important topics which rely heavily on math, such as binding theory and kinetics.

3. Mathematics in Biological Sciences

Biomathematics is a rich fertile field with open, challenging and fascination problems in the areas of mathematical genetics, mathematical ecology, mathematical neuron- physiology, development of computer software for special biological and medical problems, mathematical theory of epidemics, use of mathematical programming and reliability theory in biosciences and mathematical problems in biomechanics, bioengineering and bioelectronics.

4. Mathematics in Engineering and Technology

The use of mathematics in engineering is very well known. It is considered to be the foundation of engineering. Engineering deals with surveying, levelling, designing, estimating, construction etc., in all these processes, application of mathematics is very important. By the application of geometric principles to design and constructions, the durability of things constructed can be increased. With its help, results can often be verified in engineering.

Mathematics has played an important role in the development of mechanical, civil, aeronautical and chemical engineering through its contributions to mechanics of rigid bodies, hydro-dynamics, aero-dynamics, heat transfer, lubrication, turbulence, elasticity, and others.

5. Mathematics and Agriculture6. Mathematics and Economics

Agriculture as a science depends extensively on mathematics. It needs a direct application of mathematics, such as measurement of land or area, average investment and expenditure, average return or income, production per unit area, cost of labor, time and work, seed rate etc. Progress of the farm can be judged by drawing graphs of different items of production.

6. Mathematics and Economics

The level of mathematical literacy required for personal and social activities is continually increasing. Mastery of the fundamental processes is necessary for clear thinking. The social sciences are also beginning to draw heavily upon mathematics. Mathematical language and methods are used frequently in describing economic phenomena. According to Marshall - "The direct application of mathematical reasoning to the discovery of economic truths has recently rendered great services in the hand of master mathematicians. "Another important subject for economics is Game Theory. The whole economic situation is regarded as a game between consumers, distributors, and producers, each group trying to optimize its profits.

7. Mathematics and Psychology

The great educationist Herbart said, "It is not only possible, but necessary that mathematics be applied to psychology". Now, experimental psychology has become highly mathematical due to its concern with such factors as intelligence quotient, standard deviation, mean, median, mode, correlation coefficients and probable errors. Statistical analysis is the only reliable method of attacking social and psychological phenomena. Until mathematicians entered into the field of psychology, it was nothing but a flight of imagination.

8. Mathematics and Actuarial Science, Insurance and Finance

Actuaries use mathematics and statistics to make financial sense of the future. For example, if an organization embarks on a large project, an actuary may analyze the project, assess the financial risks involved, model the future financial outcomes and advise the organization on the decisions to be made. Much of their work is on pensions, ensuring funds stay solvent long into the future, when current workers have retired. They also work in insurance, setting premiums to match liabilities, areas of finance, from banking and trading on the stock market, to producing economic forecasts and making government policy.

9. Mathematics and Archaeology

Archaeologists use a variety of mathematical and statistical techniques to present the data from archaeological surveys and try to distinguish patterns in their results that shed light on past human behavior. Statistical measures are used during excavation to monitor which pits are most successful and decide on further excavation. Finds are analyzed using statistical and numerical methods to spot patterns in the way the archaeological record changes over time, and geographically within a site and across the country. Archaeologists also use statistics to test the reliability of their interpretations.

10. Mathematics and Logic

D' Alembert says, "Geometry is a practical logic, because in it, rules of reasoning are applied in the simplest and sensible manner". Pascal says, “Logic has borrowed the rules of geometry; the method of avoiding error is sought by everyone. The logicians profess to lead the way, the geometers alone reach it, and aside from their science there is no true demonstration". C.J.Keyser - "Symbolic logic is mathematics, mathematics is symbolic logic". The symbols and methods used in the investigation of the foundation of mathematics can be transferred to the study of logic. They help in the development and formulation of logical laws.

11. Mathematics in Music

Leibnitz, the great mathematician said, - “Music is a hidden exercise in arithmetic of a mind unconscious of dealing with numbers". Pythagoras said - “Where harmony is, there are numbers". Calculations are the root of all sorts of advancement in different disciplines. The rhythm that we find in all music notes is the result of innumerable permutations and combinations of SAPTSWAR. Music theorists often use mathematics to understand musical structure and communicate new ways of hearing music. This has led to musical applications of set theory, abstract algebra, and number theory. Music scholars have also used mathematics to understand musical scales, and some composers have incorporated the Golden ratio and Fibonacci numbers into their work.

12. Mathematics in Arts

"Mathematics and art are just two different languages that can be used to express the same ideas." It is considered that the universe is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures. The old Goethe Architecture was based on geometry. Even the Egyptian Pyramids, the greatest feat of human architecture and engineering, was based on mathematics. Artists who strive and seek to study nature must therefore first fully understand mathematics. Appreciation of rhythm, proportion, balance and symmetry postulates a mathematical mind.

13. Mathematics in Philosophy

The function of mathematics in the development of philosophical thought has been very aptly put by the great educationist Herbart, in his words, “The real finisher of our education is Philosophy, but it is the office of mathematics to ward off the dangers of philosophy." Mathematics occupies a central place between natural philosophy and mental philosophy. It was in their search of distinction between fact and fiction that Plato and other thinkers came under the influence of mathematics.

14. Mathematics in Social Networks

Graph theory, text analysis, multidimensional scaling and cluster analysis, and a variety of special models are some mathematical techniques used in analyzing data on a variety of social networks.

15. Mathematics in Political Science

In Mathematical Political Science, we analyze past election results to see changes in voting patterns and the influence of various factors on voting behavior, on switching of votes among political parties and mathematical models for conflict resolution. Here we make use of Game Theory.

16. Mathematics in Linguistics

The concepts of structure and transformation are as important for linguistic as they are for mathematics. Development of machine languages and comparison with natural and artificial language require a high degree of mathematical ability. Information theory, mathematical biology, mathematical psychology etc. are all needed in the study of Linguistics. Mathematics has had a great influence on research in literature. In deciding whether a given poem or essay could have been written by a particular poet or author, we can compare all the characteristics of the given composition with the characteristics of the poet or other works, of the author with the help of a computer.

17. Mathematics in Management

Mathematics in management is a great challenge to imaginative minds. It is not meant for the routine thinkers. Different mathematical models are being used to discuss management problems of hospitals, public health, pollution, educational planning and administration and similar other problems of social decisions. In order to apply mathematics to management, one must know the mathematical techniques and the conditions under which these techniques are applicable.

18. Mathematics in Computers

An important area of applications of mathematics is in the development of formal mathematical theories related to the development of computer science. Now most applications of mathematics to science and technology today are via computers. The foundation of computer science is based only on mathematics. It includes, logic, relations, functions, basic set theory, countability and counting arguments, proof techniques, mathematical induction, graph theory, combinatory, discrete probability, recursion, recurrence relations, and number theory, computer-oriented numerical analysis, Operation Research techniques, modern management techniques like Simulation, Monte Carlo program, Evaluation Research Technique,

Critical Path Method, Development of new Computer Languages, study of Artificial Intelligence, Development of Automata Theory etc. Cryptography is the practice and study of hiding information. In modern times cryptography is considered a branch of both mathematics and computer science and is affiliated closely with information theory, computer security and engineering. It is the mathematics behind cryptography that has enabled the e-commerce revolution and information age. Pattern Recognition is concerned with training computers to recognize pattern in noisy and complex situations. E.g. in recognizing signatures on bank cheques, in remote sensing etc.

19. Mathematics in Geography

Geography is nothing but a scientific and mathematical description of our earth in its universe. The dimension and magnitude of earth, its situation and position in the universe the formation of days and nights, lunar and solar eclipses, latitude and longitude, maximum and minimum rainfall, etc. are some of the numerous learning areas of geography which need the application of mathematics. The surveying instruments in geography have to be mathematically accurate. There are changes in the fertility of the soil, changes in the distribution of forests, changes in ecology which have to be mathematically determined, in order to exercise desirable control over them.

Indeed, mathematics exists everywhere in any program, course, or subject. It is something that we can never do away with. It is always a part of human endeavor. Mathematics is universal.

Lesson 4. Appreciating Mathematics as a Human Endeavor

In order to appreciate mathematics much better, every person should have the thorough understanding of the discipline as a human endeavor. Mathematics brings impact to the life a learner, worker, or an ordinary man in society. The influences of mathematics affect anyone for a lifetime. Mathematics works in the life of all professionals.

Mathematics is appreciated as human endeavor because all professionals and ordinary people apply its theories and concepts in the office, laboratory and marketplace. According to **Mark Karadimos** (2018), the following professions use Mathematics in their scope and field of work:

Accountants assist businesses by working on their taxes and planning for upcoming years. They work with tax codes and forms, use formulas for calculating interest, and spend a considerable amount of energy organizing paperwork.

Agriculturists determine the proper amounts of fertilizers, pesticides, and water to produce bountiful amounts of foods. They must be familiar with chemistry and mixture problems.

Architects design buildings for structural integrity and beauty. They must know how to calculate loads for finding acceptable materials in design which involve calculus.

Biologists study nature to act in concert with it since we are very closely tied to nature. They use proportions to count animals as well as use statistics/probability.

Chemists find ways to use chemicals to assist people in purifying water, dealing with waste management, researching superconductors, analyzing crime scenes, making food products and in working with biologists to study the human body.

Computer Programmers create complicated sets of instructions called programs/software to help us use computers to solve problems. They must have a strong sense of logic and have critical thinking and problem solving skills.

Engineers (Chemical, Civil, Electrical, Industrial, and Material) build products/structures/systems like automobiles, buildings, computers, machines, and planes, to name just a few examples. They cannot escape the frequent use of a variety of calculus. Geologists use mathematical models to find oil and study earthquakes.

Lawyers argue cases using complicated lines of reason. That skill is nurtured by high level math courses. They also spend a lot of time researching cases, which means learning relevant codes, laws and ordinances. Building cases demands a strong sense of language with specific emphasis on hypotheses and conclusions.

Managers maintain schedules, regulate worker performance, and analyze productivity.

Medical Doctors must understand the dynamic systems of the human body. They research illnesses, carefully administer the proper amounts of medicine, read charts/tables, and organize their workload and manage the duties nurses and technicians.

Meteorologists forecast the weather for agriculturists, pilots, vacationers,and those who are marine-dependent. They read maps, work with computer models, and understand the mathematical laws of physics.

Military Personnel carry out a variety of tasks ranging from aircraft maintenance to following detailed procedures. Tacticians utilize a branch of mathematics called linear programming.

Nurses carry out the detailed instructions doctors given them. They adjust intravenous drip rates, take vitals, dispense medicine, and even assist in operations.

Politicians help solve the social problems of our time by making complicated decisions within the confines of the law, public opinion, and (hopefully) budgetary restraints.

Salespeople typically work on commission and operate under a buy low,sell high profit model. Their job requires good interpersonal skills and the ability to estimate basic math problems without the need of paper/pencil.

Technicians repair and maintain the technical gadgets we depend on like computers, televisions, DVDs, cars, refrigerators. They always read measuring devices, referring to manuals, and diagnosing system problems.

Tradesmen (carpenters, electricians, mechanics, and plumbers) estimate job costs and use technical math skills specific to their field. They deal with slopes, areas, volumes, distances and must have an excellent foundation in math.

Assessment Task 1

A. From the ten (10) reasons why mathematics is important, state five (5) additional reasons with clear description of application. State disadvantage if a person does not know and understand mathematics.

Reasons	Setbacks
1.	1.
2.	2.
3.	3.
4.	4.
5.	5

B. Cite the mathematical application that you commonly do in each of the following stations and state your appreciation.

Stations	Application of and Appreciation for Mathematics
1.Market	1.
2.Bus/Jeepney	2.
3.Church	3.
4.Club meeting	4.
5.Clinic	5
6.Court	6.
7.Laboratory	7.
8.Birthday party	8.
9.Watching games	9.
10.Police Station	10.

Summary

Patterns and **counting** are correlative. Counting happens when there is pattern. When there is counting, there is logic. Consequently, pattern in nature goes with logic or logical set-up. There are reasons behind a certain pattern. That's why, oftentimes, some people develop an understanding of patterns, relationships, and functions and use them to represent and explain real-world phenomena. Most people say that mathematics is the science behind patterns. Mathematics exists everywhere as patterns do in nature. Not only do patterns take many forms within the range of school mathematics, they are also a unifying mechanism.

According to Katie Kim (2015), Math is a subject that makes students either jump for joy or rip their hair out. However, math is inescapable as you become an adult in the real world. From calculating complicated algorithms to counting down the days till the next Game of Thrones episode, math is versatile and important, no matter how hard it is to admit. Before you decide to doze off in math class, consider this list of reasons why learning math is important to you and the world.

It is important to further discuss the nature of mathematics, what it is, how it is expressed, represented and used.

According to the American Association for the Advancement of Science (1990), Mathematics relies on both logic and creativity, and it is pursued both for a variety of practical purposes and for its intrinsic interest. For some people, and not only professional mathematicians, the essence of mathematics lies in its beauty and its intellectual challenge.

Mathematics is offered in any college course. It is found in every curriculum because its theories and applications are needed in any workplace. That’s why students can't stay away from attending math classes. There has to be mathematics in the real world. This subject always brings life to any person or professional. Every second of the day needs mathematical knowledge and skills to perform academic activities and office routines. If ordinary people have to use math, then much more for students to know and master it so they will succeed in class in the school.

Reference

Mathematics in the Modern World (2019). Romeo M. Daligdig, EdD., Lorimar Publishing Inc. 2019, 10-B Boston Street, Brgy. Kaunlaran, Cubao, Quezon City, Metro Manila, Philippines.

MODULE 2
Mathematical Language and Symbols

Introduction

Mathematics is often described as the language of the universe, a precise and logical system through which we understand and describe the world around us. At the heart of this language are its symbols and notation, which allow complex ideas to be communicated clearly and efficiently.

Mathematical language is unique in its ability to express both abstract and concrete ideas through a combination of numbers, variables, operators, and other symbols. These symbols are not just shorthand for lengthy descriptions; they embody relationships, processes, and structures that are fundamental to mathematics.

In this module we will explore the foundational symbols and conventions of mathematical language, understanding how they are used to build the logical structures and proofs that underpin all of mathematics. Whether you're solving an equation or proving a theorem, mastery of this language is crucial to navigating the vast landscape of mathematical thought.

Learning Outcomes

At the end of this module, students should be able to:

1. Discuss the language, symbols and conventions of mathematics;
2. Explain the nature of mathematics as a language;
3. Perform operations on mathematical expressions correctly, its basic concepts and logic; and
4. Appreciate that mathematics is a useful language.

Lesson 1. The Language, Symbols, Syntax and Rules of Mathematics

The **language of mathematics** is the system used by mathematicians to communicate mathematical ideas among themselves. This language consists of a substrate of some natural language (for example English) using technical terms and grammatical conventions that are peculiar to mathematical discourse, supplemented by a highly specialized symbolic notation for mathematical formulas.

Mathematics as a language has **symbols** to express a formula or to represent a constant. It has **syntax** to make the expression well-formed to make the characters and symbols clear and valid that do not violate the **rules**. Mathematical symbols can designate numbers (constants), variables, operations, functions, brackets, punctuation, and grouping to help determine order of operations, and other aspects of logical syntax. A mathematical concept is independent of the symbol chosen to represent it. In short, **convention** dictates the meaning.

The **language of mathematics** makes it easy to express the kinds of symbols, syntax and rules that mathematicians like to do and characterized by the following:

a. precise (able to make very fine distinctions)

Example. The use of mathematical symbol is only done based on its meaning and purpose. Like + means add, - means subtract, x multiply and ÷ means divide.

b. concise (able to say things briefly)

Example: The long English sentence can be shortened using mathematical symbols. Eight plus two equals ten which means 8+2=10.

c. powerful (able to express complex thoughts with relative ease)

Example. The application of critical thinking and problem solving skill requires the comprehension, analysis and reasoning to obtain the correct solution.

Writing Mathematical Language as an Expression or a Sentence

In mathematics, an **expression** or **mathematical expression** is a finite combination of symbols that is well-formed according to rules that depend on the context. It is a correct arrangement of mathematical symbols used to represent a mathematical object of interest. An expression does not state a complete thought; it does not make sense to ask if an expression is true or false.

The most common expression types are **numbers**, **sets**, and **functions**. Numbers have lots of different names: for example, the expressions:

5 **2+3** **10/2** **(6-2)+1** **1+1+1+1+1,**

all look different, but are all just different names for the same number. This simple idea-that numbers have lots of different names-is extremely important in mathematics. The basic **syntax** for entering mathematical formulas or expressions in the system enables you to quickly enter expressions using 2-D notation. The most common mistake is to forget parentheses "( )". For example, the expression: **1/(x+1)** is different from **1/x+1** which the system interprets as **(1/x) + 1**.

Examples:

The use of expressions ranges from the **simple**:

8x-5 (linear polynomial)

$7x^2$+4x-10 (quadratic polynomial)

$\frac{x+2}{x^2+12}$ (rational fraction)

To the **Complex**:

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta}$$

For example, in the usual notation of arithmetic, the expression **1+2 x 3** is well-formed, but the following expression is not: **x4) x+,/y.**

On the other hand, a **mathematical sentence** is the analogue of an English sentence; it is a correct arrangement of mathematical symbols that states a complete thought. Sentences have verbs. In the mathematical sentence '3+4=7', the verb is '='.

A sentence can be (always) true, (always) false, or sometimes true/sometimes false. For example, the sentence '1+2=3' is true. The sentence '1+2=4' is false.

The sentence 'x=2' is sometimes true/sometimes false: it is true when x is 2, and false otherwise. The sentence 'x+3 = 3+x'is (always) true, no matter what number is chosen for x.

Mathematical Convention

A **mathematical convention** is a fact, name, notation, or usage which is generally agreed upon by mathematicians. For instance, the fact that one evaluates multiplication before addition in the expression (2 + 3) x 4 is merely conventional. There is nothing inherently significant about the order of operations. Mathematicians abide by conventions in order to allow other mathematicians to understand what they write without constantly having to redefine basic terms.

The following symbols are commonly used in the order of operations:

Symbol	Meaning	Example
+	add	3+7 = 10
−	subtract	5−2 = 3
×	multiply	4×3 = 12
÷	divide	20÷5 = 4
/	divide	20/5 = 4
( )	grouping symbols	2(a−3)
[ ]	grouping symbols	2[a−3(b+c)]
{ }	set symbols	{1, 2, 3}
π	pi	A = πr²
∞	infinity	∞ is endless
=	equals	1+1 = 2
≈	approximately equal to	π ≈ 3.14
≠	not equal to	π ≠ 2
< ≤	less than, less than or equal to	2 < 3
> ≥	greater than, greater than or equal to	5 > 1
√	square root ("radical")	√4 = 2
°	degrees	20°
∴	therefore	*a*=*b* ∴ *b*=*a*

Perform Operations on Mathematical Expressions Correctly

In simplifying mathematical expressions, the following order of operations is one critical point to observe. Order of operations is the hierarchy of mathematical operations. It is the set of rules that determines which operations should be done before or after others. Before, we used to have the **MDAS** that stands for Multiplication, Division, Addition and Subtraction. It was changed to use **PEMDAS** which means Parentheses, Exponents, Multiplication and Division and Addition and Subtraction. But now, most scientific calculators follow **BODMAS** that is Brackets, Order, Division and Multiplication, Addition and Subtraction.

The order of operations or BODMAS/PEMDAS is merely a set of rules that prioritize the sequence of operations starting from the most important to the least important.

Step 1: Do as much as you can to simplify everything inside the parenthesis first

Step 2: Simplify every exponential number in the numerical expression

Step 3: Multiply and divide whichever comes first, from left to right

Step 4: Add and subtract whichever comes first, from left to right

Examples:

1. Evaluate: (11-5)×2-3+1

Solution:

=6x2-3+1 (Remove the parenthesis)

=12-3+1 (Multiply)

=9+1 (Subtract)

=10 (Add)

2. Evaluate 10+2+12+2x3

Solution:

Using the PEMDAS rule, we need to evaluate the division and multiplication before subtraction and addition. It is recommended that you put in parentheses to remind yourself the order of operation.

From the given, 10+ +12+22×3

= (10÷2)+(12÷2×3)

= 5+18

= 2

3. Simplify: 4-3|4-2(6-3)|÷2

=4-314-2-2(6-3)|÷2

=4-3[4-2(3)]÷2=4-3[4-6]÷2

=4-3|-2|÷2

=4+6÷2

=4+3

=7

4. Simplify: $16 - 3(8-3)^2 \div 5$

Remember to simplify inside the parentheses before you square, because $(8-3)^2$ is not the same as 8^2-3^2.

$=16-3(8-3)^2\div5$

$=16-3(5)^2\div5$

$=16-3(25)\div5$

$=16-75\div5$

$=16-116-15$

$=1$

Lesson 2. The Four basic Concepts of Mathematics

1. Set.

A **set** is a collection of well-defined objects that contains no duplicates. The objects in the set are called the elements of the set. To describe a set, we use braces { }, and use capital letters to represent it.

Examples:

1. The books in the shelves in a library
2. The bank accounts in a bank.
3. The set of natural numbers N = {1, 2, 3, ... }.
4. The integer number; Z = {⋯,-3,-2,-1,0,1,2,3,⋯}.
5. The rational number is the set of quotients of integers Q- {p|q: p, q E Z and q*0

The three dots in enumerating the elements of the set are called ellipsis and indicate a continuing pattern. A finite set contains elements that can be counted and terminates at certain natural number, otherwise, it is infinite set.

Examples:

Set A = {2,4,6,8,10}

- the set of all even natural numbers less than or equal to 10. The order in which the elements are listed is not relevant: i.e., the set {2, 4, 6, 8,10} is the same as the set {8,4,2,10,6}.

There is exactly one set, the **empty set**, or **null set**, **Ø** or {}, which has no members at all. A set with only one member is called a **singleton** or a **singleton set**. ("singleton of a").

Specification of Sets

There are three main ways to specify a set:

(1) List Notation/Roster Method-by listing all its members

-list names of elements of a set, separate them by commas and enclose them in braces:

Examples:
1. {1,12,45},
2. {George Washington, Bill Clinton},
3. {a,b,d,m}.
4. "Three-dot abbreviation": {1, 2, ..., 100}

(2) Predicate Notation/Rule Method/Set-Builder Notation

-by stating a property of its elements. It has a property that the members of the set share (a condition or a predicate which holds for members of this set).

Examples:
1. {x|x is a natural number and x < 8) means "the set of all x such that x is a natural number and is less than 8"
2. {x|x is a letter of Russian alphabet}
3. {y|y is a student of UMass and y is older than 25}

(3) Recursive Rules

-by defining a set of rules which generates or defines its members

Examples.
1. the set E of even numbers greater than 3:
2. $4 \in E$
3. if $x \in E$, then "x+2" $\in E$
4. Nothing else belongs to E.

Equal Sets

Two sets are equal if they contain exactly the same elements.

Examples:
1. {3,8,9}.{3,8,9}={9,8,3}
2. {6,7,7,7,7}={6,7}
3. {1,3,5,7}≠ ,5≠{3, 3}

Equivalent Sets

Two sets are equivalent if they contain the same number of elements.

Example:

1. Which of the following sets are equivalent?

{ θ, €, £}, {o,L,0}, {1,4, 3}, {a,b,c},{±,,}

Solution: All of the given sets are equivalent. Note that no two of them are equal, but they all have the same number of elements.

Universal Set

A set that contains al the elements considered in a particular situation and denoted by U.

Example:

a. Suppose we list the digits only. Then, U = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, since U includes all the digits.
b. Suppose we consider the whole numbers. Then U = {0,1, 2,3,...} since U contains all whole numbers.

Subsets

A set A is called a subset of set B if every element of A is also an element of B."A is a subset of B is written as A ⊂ B.

Example:

a. A={7,,9} is a subset of B={6,9,7}
b. D={10,8,6} is a subset of G={10,8,6}

A **proper subset** is a subset that is not equal to the original set, otherwise **improper subset**.

Example: Given {3, 5, 7} then the proper subsets are {}, {5, 7}, {3, 5}, {3, 7}.

The improper subset is {3, 5, 7}.

Cardinality of the Set

It is the number of distinct elements belonging to a finite set. It is also called the **cardinal number** of the set A denoted by n(A) or card (A) and |A|.

Power Set

It is the family of all the subsets of A denoted by Power (A).

Example:

Given set A = {x, y}, the Power (A) = {{∅, {x}, {y}, {x, y}}or {x|x is a subset of A.}

Operation on Sets

Union is an operation for sets A and B in which a set is formed that consists of all the elements included in A or B or both denoted by U as A U B.

Examples:

1. Given U ={1,2,3,4,5,6,7,8,9}, A={1,3,5,7}, B={2,4,6,8} and C={1,2}, find the following:

a. $A \cup B$ b. $A \cup C$ c. $(A \cup B) \cup \{8\}$

Solution:

a. $A \cup B = \{1,2,3,4,5,6,7,8\}$
b. $A \cup C = \{1,2,3,5,7\}$
c. $(A \cup B) \cup \{8\} = \{1,2,3,4,5,6,7,8\}$

Intersection is the set containing all elements common to both A and B, denoted by n.

Example: Given U ={a,b,c,de}, A={c,d,e}, B={a,c,e} and C={a} and D={e}

Find the following:

a. $B \cap C$ b. $A \cap C$ c. $(A \cap B) \cap D$

Solutions:

a. BNC = {a} b. $A \cap C = \emptyset$ c. $A \cap B = \{c, e\}$, $(A \cap B) \cap D = \{e\}$

Complementation is an operation on a set that must be performed in reference to a universal set, denoted by A'.

Example. Given U ={a,b,c,d,e},A={c,d,e},,find A'.

Solution: A' = {a, b}

2. Relation

A relation is a rule that pairs each element in one set, called the domain, with one or more elements from n a second set called the range. It creates a set of ordered pairs.

Examples: Given:

Regular Holidays in the Philippines	Month and Date
1. New Year's Day	January 1
2. Labor Day	May 1
3. Independence Day	June 12
4. Bonifacio Day	November 30
5. Rizal Day	December 30

A clearer way to express a relation is to form a set of ordered pairs;

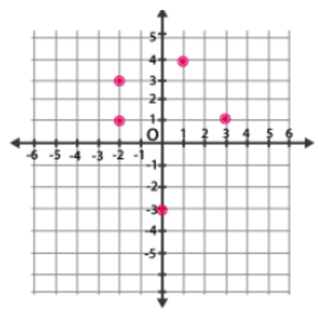
(New Year's Day, January 1), (Labor Day, May 1), (Independence Day, June 12), (Bonifacio Day, November 30), (Rizal Day, December 30). This set describes a **Relation**.

Relation Representation

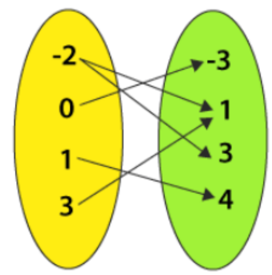
There are other ways too to write the relation, apart from set notation such as through tables, plotting it on XY- axis or through mapping diagram.

x	y
-2	1
-2	3
0	-3
1	4
3	1

Relation in table



Relation in graph



Relation in mapping diagram

3. Function

It is a rule that pairs each element in one set, called the domain, with exactly one element from a second set, called the range. This means that for each first coordinate, there is exactly one second coordinate or for every first element of x, there corresponds a unique second element y.

Remember: A one-to-one correspondence and many-to-one correspondence are called **Functions** while one-to-many correspondence is not.

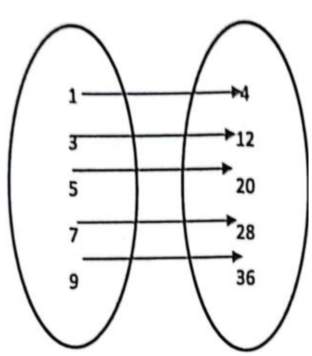
Examples: The function can be represented using the following:

1. **Table**. The perimeter of a square is four times the length of its side.

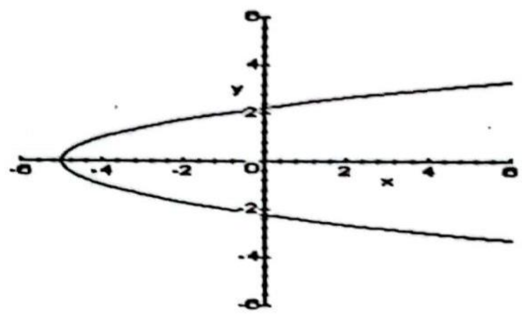
Side (S)	1	3	5	7	9
Perimeter (P)	4	12	20	28	36

2. **Ordered Pairs**: {(1,4), (3,12), (5,20), (7,28), (9,36)}

3. **Mapping**



4. **Graphing**. Using **vertical line Test**, that is, a set of points in the plane is the graph of a function if and only if no vertical line intersects the graph in more than one point. Below in figure A is not a function.



Lesson 3. Elementary Logic

According to David W. Kueker (2009), **logic** is simply defined as the analysis of methods of reasoning. In studying these methods, logic is interested in the form rather than the content of the argument. Mathematical Logic is, at least in its origins, the study of reasoning as used in mathematics. Mathematical reasoning is deductive- that is, it consists of drawing (correct) conclusions from given hypotheses. Thus the basic concept is that of a statement being a logical consequence of some other statements. In ordinary mathematical English the use of "therefore" customarily indicates that the following statement is a consequence of what comes before.

Examples:

1. All men are mortal. Luke is a man. Hence, Luke is mortal.
2. All dogs like fish. Cyber is a dog. Hence, Cyber likes fish.

Propositions and Connectives

A **Proposition** (or statement) is a sentence that is either true or false (without additional information).

The **logical connectives** are defined by truth tables (but have English language counterparts).

Logic	Math	English
Conjunction	$\wedge$	and

Disjunction	V	or (inclusive)
Negation	~	not
Conditional	→	if...then...
Biconditional	←	if and only if

A **denial** is a statement equivalent to the negation of a statement.

Examples:

1. The negation of P→Q is ~(P⇒Q).
2. A denia1 of P=Q is P is P ∧ ~ Q.

A **tautology** is a statement which is always true.

Examples:

- A V (B A C) → (A V B) A (A V C) Distributive Law
- ~(AVB)⇔~A∧~B
- P~(~P)

A **contradiction** is a statement which is always false.

Example: 1. (A V ~ A) → (B ^ ~B) a contradiction.

The **contrapositive** of the statement if P then Q is if ~Q then ~P. An implication and its contrapositive are logically equivalent, so one can always be used in place of the other.

A **predicate (open sentence)** is a sentence containing one or more variables which becomes a proposition upon replacement of the variables.

Assessment Task 2

Solve for the following.

1. Let $A=\{0,2,4,6,8\}$, $B=\{0,1,2,3,4)$ and $C=\{0,3,6,9\}$. What are
   a. $A \cup B \cup C$
   b. $A \cap B \cap C$?
2. Find the union of $A = \{2, 3, 4\}$ and $B = \{3, 4, 5\}$.
3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?
4. Find the union, intersection and the difference (A - B) of the following pairs of sets.
   a. A = The set of all letters of the word FEAST
      B = The set of all letters of the word TASTE
   b. $A = \{x: x \in W, 0 < x \leq 7\}$
      $B =\{x: x \in W, 4 < x < 9\}$
   c. $A = \{x \mid x \in N, x \text{ is a factor of } 12\}$
      $B = \{x \mid x \in N, x \text{ is a multiple of } 2, x < 12\}$
   d. $A = \{x: x \in I, -2 < x < 2\}$
      $B=\{x: x \in I,-1 < x < 4\}$
   e. $A = \{a, 1, m, n, p\}$
      $B = \{q, r, 1, a, s, n\}$
5. If $A = \{T, W, R\}$ and $B= (M, T, W\}$, what are
   a. $A \cup B$.
   b. $A \cap B$.
   c. $A-B$.
   d. $B -A$
6. You have no car, but in need of a car next week. Your pal, Peter, is too busy with work and study to go out, and so he can lend you his car Tuesday, Wednesday, and Thursday. Your pal Mary is crazy busy at the beginning of the week, but she has plans for the rest, so she can allow you to use her car Monday, Tuesday, and Wednesday. Peter will allow you to use his car those 3 days, and Mary will let you drive her car those 3 days, how many days do you have covered?

Summary

The **language of mathematics** is the system used by mathematicians to communicate mathematical ideas among themselves. This language consists of a substrate of some natural language (for example English) using technical terms and grammatical conventions that are peculiar to mathematical discourse, supplemented by a highly specialized symbolic notation for mathematical formulas.

Mathematics as a language has **symbols** to express a formula or to represent a constant. It has **syntax** to make the expression well-formed to make the characters and symbols clear and valid that do not violate the **rules**. Mathematical symbols can designate numbers (constants), variables, operations, functions, brackets, punctuation, and grouping to help determine order of operations, and other aspects of logical syntax. A mathematical concept is independent of the symbol chosen to represent it. In short, **convention** dictates the meaning.

In mathematics, an **expression** or **mathematical expression** is a finite combination of symbols that is well-formed according to rules that depend on the context. It is a correct arrangement of mathematical symbols used to represent a mathematical object of interest. An expression does not state a complete thought; it does not make sense to ask if an expression is true or false.

A **mathematical sentence** is the analogue of an English sentence; it is a correct arrangement of mathematical symbols that states a complete thought.

A **mathematical convention** is a fact, name, notation, or usage which is generally agreed upon by mathematicians.

A **set** is a collection of well-defined objects that contains no duplicates. The objects in the set are called the elements of the set. To describe a set, we use braces { }, and use capital letters to represent it.

A **relation** is a rule that pairs each element in one set, called the domain, with one or more elements from n a second set called the range. It creates a set of ordered pairs.

A **function** is a rule that pairs each element in one set, called the domain, with exactly one element from a second set, called the range. This means that for each first coordinate, there is exactly one second coordinate or for every first element of x, there corresponds a unique second element y.

A **logic** is simply defined as the analysis of methods of reasoning. In studying these methods, logic is interested in the form rather than the content of the argument. Mathematical Logic is, at least in its origins, the study of reasoning as used in mathematics.

Reference

Mathematics in the Modern World (2019). Romeo M. Daligdig, EdD., Lorimar Publishing Inc. 2019, 10-B Boston Street, Brgy. Kaunlaran, Cubao, Quezon City, Metro Manila, Philippines.

MODULE 3
Problem Solving and Reasoning

Introduction

Problem solving and reasoning are fundamental cognitive processes that enable individuals to navigate challenges, make decisions, and understand the world around them. These skills involve the ability to identify problems, analyze them from different perspectives, and develop effective strategies for resolution. Reasoning, which underpins problem solving, requires logical thinking, critical analysis, and the ability to draw inferences based on available information. Together, problem solving and reasoning are essential for academic success, professional development, and everyday life, as they empower individuals to approach complex situations with confidence and creativity.

Problem solving and reasoning are not only essential for personal and professional growth but are also critical components of innovation and progress in society. Problem solving involves a dynamic process that includes understanding the problem, generating possible solutions, evaluating these solutions, and implementing the best course of action. Reasoning, on the other hand, provides the intellectual foundation for this process by ensuring that decisions are based on sound principles and evidence.

Learning Outcomes

At the end of this module, students should be able to:

1. use different types of reasoning to justify statements and arguments made about mathematics and mathematical concepts;
2. write clear and logical proofs;
3. solve problems involving patterns and recreational problems following Polya's Steps; and
4. organize one's method and approaches for proving and solving problems.

Lesson 1. Reasoning

Mathematics is not just about numbers; much of it is problem solving and reasoning. Problem solving and reasoning are basically inseparable. The art of reasoning is very important in mathematics. This is the skill needed in exemplifying the critical thinking and problem solving ability. Logic and reasoning are very useful tools in decision making. People also do deductive reasoning extensively to show that certain **conjectures** are true as these follow the rules of logic. A conjecture is a conclusion made from observing data.

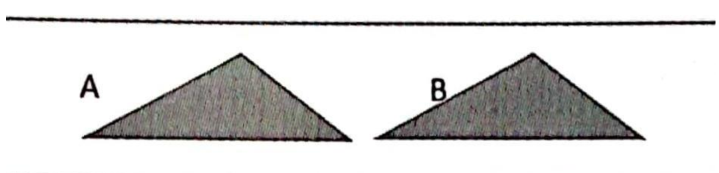
Reasoning is the practice of stating ideas clearly and precisely to arrive at a conclusion. In our life, we often make judgment and conclusion based on facts and observations. These are not always true. Thus, we have to know the different ways of arriving at accurate conclusions.

Kinds of Reasoning (Zuela, et. al, 2009):

1. **Intuition** is similar to guessing. It is also called reasoning by guessing or reasoning by common sense. It requires less mental activity. An intuition is the ability to acquire knowledge without proof, evidence, or conscious reasoning, or without understanding how the knowledge was acquired. Intuition is highly subjective. Different people think about problems in different ways. It is something that is known or understood without proof or evidence.

    Examples:

    1. In comparing two fractions, with the same numerator, one said he can subtract numerator from the denominator and the smaller difference is the larger fraction. Is this false intuition? Verify.

    Look at figure A and B. Which is bigger? You can give your answer by using intuition and without actually measuring.

    2. After the first meeting in her Statistics class, Mariah says,“I think I will like Statistics”.

2. **Analogy** is a form of reasoning in which other similarities are inferred from a particular similarity between two or things. It is reasoning by comparison.

   Examples.

   1. Tree is to leaf as flower is to petal.
   2. hammer : nail :: comb : hair
   3. Finding a Good Man is Like Finding a Needle in a Haystack -Dusty Springfield

3. **Inductive Reasoning** is the process of gathering specific information, usually through observation and measurement and then making a conjecture based on the gathered information.

   Inductive reasoning is a logical process in which generalizations are made based on specific observations or evidence. Unlike deductive reasoning, inductive reasoning does not guarantee absolute certainty; instead, it suggests likely conclusions. Inductive reasoning is used to infer patterns, make predictions, and generate hypotheses based on available information.

   Example:

   1. Find the sum.

$$1=1$$
$$1+3=4$$
$$1+3+5=9$$
$$1+3+5+7=16$$
$$1+3+5+7+9= \_\_\_\_$$
$$\underbrace{1+3+5+\cdots+(2n-1)}_{\text{n numbers}}= \_\_\_\_\_$$

   Solution:

   Take note that the sum is the square of the number of odd numbers being added.

$$\underbrace{1+3+5+7}_{\text{4 odd numbers}}=4^2=16$$

$$\underbrace{1+3+5+7+9}_{\text{5 odd numbers}}=5^2=25$$

   From this note, we can make this conjecture:

The sum of the first n odd numbers is n^2. Or, $1+3+5+\cdots+(2n-1)= n^2$

4. **Deductive Reasoning** is the process of showing that certain statements follow logically from agreed upon assumptions and proven facts.

   Deductive reasoning is a logical process in which specific conclusions are drawn from general principles, premises, or information. It involves applying well-established rules of logic to reach conclusions that are necessarily true if the premises are true. Deductive reasoning is a fundamental aspect of mathematical proofs, formal arguments, and structured problem solving.

Example:

1. Given: $4(3x-8) + 5 = x-5$. Solve the equation for x. Give reason for each step in the process.

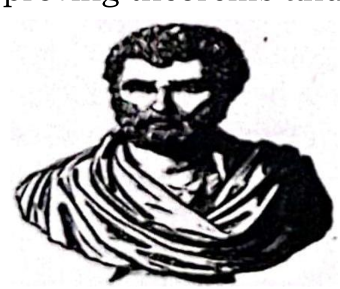
Solution:	$4(3x-8)+5 = 5=x-5$	
	$12x-32+5 = x-5$	Apply distributive property
	$12x-27 = x-5$	Combine similar terms
	$1x-27 = -5$	Apply subtraction property of equality
	$11x = 22$	Apply addition property of equality
	$X = 2$	Use division property of equality

2. Suppose that the given statement is true. Use deductive reasoning to give another statement that must be also true.

   All birds can fly.
   Tweetie is a bird.
   Solution: Tweetie can fly.

3. Every Filipino of age 18 and above can vote. Juan del Prado is a Filipino of age 24.Therefore, Juan del Prado can vote.

Deductive reasoning is used in formal geometric proofs and often resorted to in proving theorems and corollaries in Geometry.



Euclid (325 BCE- 265 BCE), the father of Geometry and the first Egyptian Mathematician who initiated a new way of thinking the study of geometry and introduced the method of proving a geometrical result by deductive reasoning based upon previously proved result and some self-evident specific assumptions called **axioms**.

If-then Statements and Converses

- **Conditional** - is a statement in mathematics that consists of a hypothesis and a conclusion. These statements are usually written in if-then form.
- **Hypothesis** - The hypothesis of a conditional states that the given facts are assumed as true. This is found in the "if" part of the conditional.
- **Conclusion** The conclusion of a conditional states what needs to be proven or established or true. This is found in the "then" part of conditional.
- **Converse** - A converse of a given conditional is formed when the "if" and "then" parts are reversed.
- **Biconditional** - A biconditional is a statement that combines a conditional and its converse with the phrase "if and only if" (abbreviated as “iff”).

Example

1. Transform the following conditionals to if-then statement and point out the hypothesis and the conclusion.

a. A segment has only one point.

b. Two lines intersect at only one point.

c. Vertical angles are congruent.

Solution:

A. If-then form: If a segment is given, then it has only one midpoint.

Hypothesis: A segment is given

Conclusion: The segment has only one point.

B. If-then form: If two lines intersect, then they intersect at only one point.

Hypothesis: Two lines intersect.

Conclusion: The two lines intersect at only one point.

C. If-then form: If two vertical angles are given, then they are congruent.

Hypothesis: Two vertical angles are congruent.

Conclusion: The two vertical angles are congruent.

Example

2. State the converse of the given conditionals.

a. If the sum of the measures of two angles is 180°, then they are supplementary.

a. If two segments are congruent, then they have equal lengths.

Solution:

A. If two angles are supplementary, then the sum of their measures is 180°.

B. If two segments have equal lengths, then they are congruent.

Example

a. A line is a bisector of a segment if it intersects the segment at its midpoint.

b. Three points are coplanar if they are contained in the same plane.

Solution:

A. **Conditional**: If a line is a bisector of a segment, then it intersects the segment at its midpoint.

Converse: If a line intersects a segment at its midpoint, then the line is a bisector of the segment.

B. **Conditional**: If three points are coplanar, then they are contained in the same plane.

Converse: If three points are contained in the same plane, then they Coplanar.

Mathematical Proofs

A proof is a sequence of true facts (statements) placed in a logical order.In proving, the following may be used as reasons:

- the given information (the hypothesis)
- definition and undefined terms
- algebraic properties
- postulates of geometry
- previously proven geometric conjectures (theorems)

Algebraic and Geometric Proofs

In order for us to prove properly and correctly, it is wise to remember and understand the necessary properties to be used in writing formal proofs:

Important Properties of Algebra

For real numbers w, x, y, and z:

Reflexive : x = x

Symmetric : if x = y then y = x.

Transitive : if x = y and y = z, then x = z

Substitution : if x + y = z and x = 3, then 3 + y = z

Distributive : x(y+z) = xy + xz.

Commutative Properties:

a. Addition : x + y = y + z
b. Multiplication : yz = zy

Associative Properties:

a. Addition : x+(y+z) = (x+y)+z
b. Multiplication : x(yz) = (xy)z

Addition Properties of Equality (APE)

a. If x = z, then x ± y = z ± y
b. If w = x and y = z, then w + y = x + z.

Multiplication Properties of Equality (MPE)

a. If x = z, then xy = yz or x/y = y/z
b. If w = x and y = z, then wy = xz or w/y = x/z

Example 1. Find the value of x in 2(x+1)=6x+4.

Proof:

Statements	Reasons
1. 2(x + 1) = 6x + 4	Given
2. 2x + 2 = 6x + 4	Distributive Property
3. 2x + 2 – 6x – 2 = 6x + 4 – 6x - 2	APE
4. -4x(-1/2) = 2(-1/2)	MPE
5. x = -1/2	Simplification

Geometric Properties

The following properties may be used to justify proof of some mathematical statements.

Reflexive Property (REF)

Given: $AB \cong CD$

Statement: $AB \cong AB$, $CD \cong CD$

Symmetric Property (SYM)

Given: $AB \cong CD$

Statement: $CD \cong AB$

Transitive Property (TRANS)

Given: $AB \cong CD$, $CD \cong EF$

Statement: $AB = EF$

Addition Property of Equality (APE)

Given 1: $AB \cong CD$

Statement 1: $AB \pm EF = CD \pm EF$

Given 2: $AB = CD$, $EF = GH$

Statement 2: $AB \pm EF = CD \pm GH$

Definition of Congruent Segments (DOCS)

Given 1. $AB \cong CD$

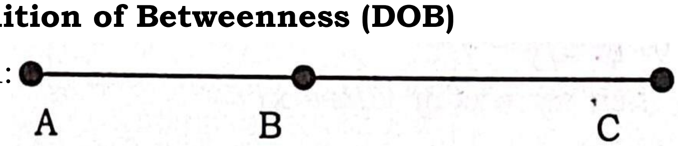
Statement 1. $AB = CD$

Given 2. $AB \cong CD$

Statement 2. $AB = CD$

Definition of Betweenness (DOB)

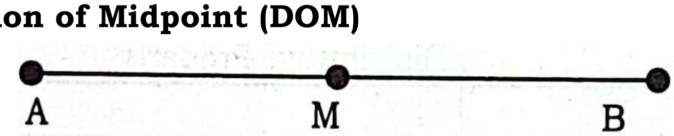
Given:



Statement: $AB + BC = AC$

Definition of Midpoint (DOM)

Given:



M is the midpoint of AB

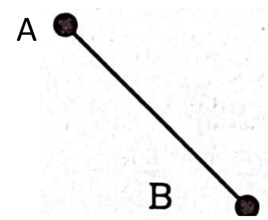
Statement: $AM \cong MB$

How to write proof?

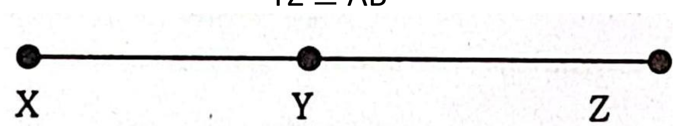
There are many ways on how to write proofs. We can have top-down or deductive reasoning or bottom-up or inductive reasoning. It can be formal or informal procedure.

Example: Prove the following using formal proof.

Given: Y is the midpoint of XZ.



YZ ≅ AB



Prove that Prove: XY ≅ AB

Statements	Reasons
1.Y is the midpoint of XZ	Given
2.XY ≅ YZ	Definition of Midpoint (DOM)
3.YZ ≅ AB	Given
4.XY ≅ AB	Transitive

Lesson 2. Polya's 4 Steps in Problem Solving

George Polya has had an important influence on problem solving in mathematics education. He stated that good problem solvers tend to forget the details and tend to focus on the structure of the problem, while poor problem solvers focus on the opposite. He designed the following:

4-Step Process:

1. **Understand the problem. (See)**

   Read and understand the problem. Identify what is the given information, known data or values and what is the unknown and to be solved as required by the problem.

   Consider the following questions:

   a. Can you restate the problem in your own words?

   b. Can you determine what is known about these types of problems?

   c. Is there missing information that if known would allow you to solve the problem?

d. Is there extraneous information that is not needed to solve the problem?
e. What is the goal?

2. **Devise a plan. (Plan)**

   Think of a way to solve the problem by setting up an equation, drawing a diagram, and making a chart that will help you find the unknown and the solution. To start devising a plan, try doing the following:

   a. Make a list of the known information.
   b. Make a list of information that is needed.
   c. Draw a diagram.
   d. Make an organized list that shows all the possibilities.
   e. Make a table or a chart.
   f. Work backwards.
   g. Try to solve similar but simpler problem
   h. Write an equation, as possible define what each variable represents
   i. Perform an experiment.
   j. Guess at a solution and then check the result.

3. **Carry out the plan. (Do)**

   Solve the equation you have set up and observe analytical rules and procedures until you arrive at the answer.

   a. Work carefully.
   b. Keep an accurate and neat record of all your attempts.
   c. Realize that some of your initial plans will not work and that you will have to devise another plan and modify your existing plan.

4. **Look back. (Check)**

   In order to validate the obtained value, you need to verify and check if the answer makes sense or correct based on the situation posed in the problem. Label your final correct answer.

   a. Ensure that the solution is consistent with the facts of the problem.

b. Interpret the solution within the context of the problem.
c. Ask yourself whether there are generalizations of the solution that you could apply to similar problems.

Example 1. A police station has 25 vehicles of motorcycles and cars. The total number of wheels is 70. Find the number of motorcycles and cars the station has.

Solution:

Step 1. Understand the problem.

Given: 25 vehicles

70 wheels

Required: The number of cars and the number of motorcycles.

Step 2. Devise a plan.

Let: x = the number of cars

y = the number of motorcycles

and x + y = 25 vehicles

4 wheels (x = cars) + 2 wheels (y = motorcycles) = 70 wheels

So, x + y = 25 vehicles and 4x + 2y = 70 wheels are the two equations formed based on the problem.

Step 3. Carry out the plan.

(1) x + y = 25

(2) 4x + 2y =70, solving two equations with two unknown using the process of elimination:

(1) -2(x + y = 25) → -2x - 2y = -50

(2) 4x + 2y =70 → 4x + 2y = 70

2x + 0 = 20

$$\frac{2x}{2} = \frac{20}{2}$$

x =10, since x denotes the number of cars

So, there are 10 cars. However, solving for y as the number of motorcycles is as follows:

Since x + y = 25, then 10 + y = 25, y = 25 - 10, finally y = 15, so there are 15 vehicles in the police station.

Step 4. Look back.

Therefore, there are 10 cars with 4 wheels and 15 motorcycles with 2 wheels. The total number of wheels is 70 wheels.

In this example, the use of Polya's 4-Step Strategy is very helpful in solving problem because one must read and understand properly the problem. Specify the given information and values and what to solve. Always think of drawing a pattern, setting up the table, working backward, or making lists and tables and design right away the needed equation and use other techniques in order to arrive at realistic and correct answer. Though, logical shortcuts can be employed in any problem.

Learning to solve problems is not a difficult task. It can be a huge fun and ultimately challenging. However, it requires you to think analytically; critically and creatively. Practice doing and solving is the tough secret why most students and professionals succeed in getting the problem solved and done to make the moment of solving more enjoyable, interesting and fulfilling.

The following problem solving strategies can be used:

- **Searching for Patterns**

The ability to recognize patterns is one important problem solving skill. It enables a person to see order or regularity in what takes place in our surroundings and so be able to make sense of what is going on.

Example 1. Find the next number in the sequence.

a. 5, 9, 13, 17, 21, 25, ...

b. 2, 6, 18, 54, 162, 486, ...

Solution a:

1. Understand the problem

Given: a. 5, 9, 13, 17, 21, 25, ...

Required: The next number in the sequence.

2. Devise a plan.

First term	→	5	Fourth term	→	17=13+4
Second term	→	9=5+4	Fifth term	→	21=17+4
Third term	→	13=9+4	Sixth term	→	25=21+4

3. Carry out the plan.

Sequence: 5 9 13 17 21 25. There is a common difference of 4.

4 4 4 4 4

Therefore, the next number in the sequence is 29.

4. Look back.

Answer.5, 9, 13, 17, 21, 25, 29

Solution b:

1. **Understand the problem.**

Given: b. 2 6 18 54 162 486

Required: The next number in the sequence.

2. Devise a plan.

First term	→	2	Fourth term	→	54=18×3
Second term	→	6=2=2×3	Fifth term	→	1162=54x3
Third term	→	18=6×3	Sixth term	→	486=162×3

3. **Carry out the plan.**

2 6 18 54 162 486

3 3 3 3 3

The common multiplier 3 is called the common ratio. The answer is 1,458.

4. Look Back.

2, 6, 18, 54, 162, 486, <u>1,458.</u>

- **Working Backward**

A strategy that starts at the end of the problem and works backward.

Example 1.

Anne has a certain amount of money in her bank account on Friday morning. During the day she wrote a check for Php24.50, made an ATM withdrawal of Php80 and deposited a check for Php235. At the end of the day, she saw that her balance was Php451.25.How much money did she have in the bank at the beginning of the day?

Solution:

1. Understand the problem

Given: Php 24.50 check, ATM withdrawal Php 80, check deposit Php 235

Required: Money she had in the bank at the beginning of the day.

2. Devise a plan

Start with 451.25. Subtract 235, add 80, and then add 24.50.

3. Carry out the plan

So, 451.25-235+80+24.50 = Php 320.75

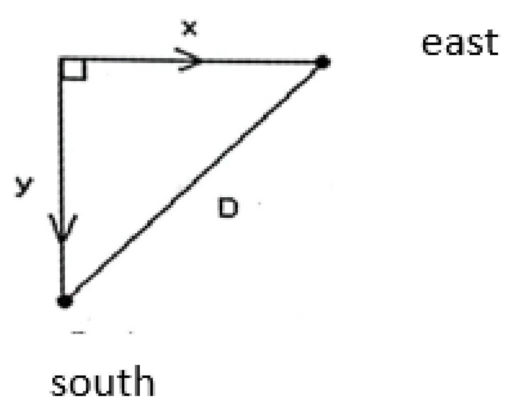
4. Look back.

Php 320.75 she had in the bank at the beginning of the day.

- **3. Drawing Pictures and Diagrams.**

A problem can be solved by means of a figure, a diagram, or a graph. It helps you visualize a problem, makes it easier for you to determine the relevant data and observe important connections and relationships.

Example 1. Two cars left, at 8:00 AM, from the same point, one traveling east at 50 mph and the other travelling south at 60 mph. At what time will they be 300 miles apart?



Solution:

1. Understand the problem.

Given: 8:00 AM, Car 1 with 50 mph east, Car 2 with 60 mph south, 300 miles apart

Required: The time when they will be 300 miles apart.

2. Devise a plan.

x= 50t and y = 60t

Since the two directions are at right angle, Pythagorean Theorem can be used to find distance D between the two cars as follows:

D=$\sqrt{(x^2+y^2)}$

3. Carry out the plan.

We now find the time at which D = 300 miles by solving

300=$\sqrt{(x^2+y^2)}$

Square both sides and substitute x and y by 50t and 60t respectively to obtain the equation $(50\,t)^2+(60\,t)^2=300^2$

Solve the above equations to obtain

t=3.84 hours (rounded to two decimal places) or 3 hours and 50 minutes (to the nearest minute)

4. Look back.

The two cars will be 300 miles apart at 8:00 AM + 3hrs and 50mins = 11:50 AM.

- **Making Lists and Tables**

The method “Make a Table" is helpful when solving problems involving numerical relationships. When data is organized in a table, it is easier to recognize patterns and relationships between numbers.

Example: An Algebra test consists of ten multiple choice questions. Ten points are given for each correct answer and three points are deducted for each incorrect answer. If Joshua did all questions and scored 48, how many incorrect answers did he have?

Solution:

1. Understand the problem.

Given: 10 questions answered, score = 48, 10 points for each correct answer, 3 points deducted for each incorrect answer.

Required: The number of correct answers

2. Devise a plan.

The number of correct answers (x) + number of incorrect answers (y) = 10.

Score = 10(x) - 3(y)

3. Carry out the plan.

Number of correct (x)	Number of incorrect(y)	Score = 10(x) - 3(y)
10	0	100
9	1	87
8	2	74
7	3	61
6	4	48
5	5	35
4	6	22
3	7	9

4. Look back.

For Joshua, x=65 and y=4, so, 6+4=10 items. Therefore, 10x6-(3x4) = 60-12 = 48.

Lesson 3. Other Mathematical Problems

Mathematical Problems Involving Patterns

Life is always confronted by problems. Some of these are no big deal because solutions can be easily seen like when the information and data provided show already a pattern where the solution shall start and proceed. This pattern serves as a guide in arriving at the correct and realistic value. Mathematics is an active human endeavor. To create mathematics, we need to solve problems. Pattern in many ways helps us solve problems fast and verifies the answers right away. However, some other patterns need ample time to be read and understood. Focus more on the differences between the numbers and discover the common value that rationalizes the sequence and denotes the logical order.

Recreational Problems Using Mathematics

Recreational mathematics is mathematics done for recreation or as a hobby which is intended to be fun. Typically it involves games or puzzles that relate to mathematics, although the term can cover other material. Typically, recreational mathematics involves general logical and lateral thinking skills, as opposed to advanced mathematical concepts, so that the average person is at least able to understand and appreciate a recreational problem and its solution. Recreational puzzles can also increase people's appreciation of mathematics as a whole.

Example 1: The Four 4's Puzzle.

Using exactly four of the digit “4” and any mathematical symbols you choose, for which natural numbers can you create a mathematical expression equal to that number? For example, 1 = 4 ÷ 4 + 4 – 4; 44 + 44 = 88; 4 ÷ 4 + 4 = 18; 44 – 4 ÷ 4 = 64. In these examples 4 is used in 4 times to get a number. Most probably, arithmetic symbols are used to signify the meaning and operations to be done.

Example 2: The Magic Square

A magic square is an arrangement of numbers in a square such that all rows, all columns, and both main diagonals sum to the same number, a number referred to as the magic constant. The square on the right is perhaps the best-known example of a magic square. Magic squares are a very well-known mathematical recreation, like:

6	1	8
7	5	3
2	9	4

Assessment Task 3

A. Solve the following problems using the 4 Steps of George Polya.

1. X, Y, Z collect stamps. They exchange stamps among themselves according to the following scheme: X gives Y as many stamps as Y has and Z as many stamps as Z has. After that, Y gives X and Z as many stamps as each then has, and then Z gives X and Y as many stamps as each has. If each finally has 64 stamps, with how many stamps does X start?
2. Mrs. Dizon withdrew 1/4 of her savings early in July and later deposited a total of Php. 1,500 on four separate days. If her bank statement showed a balance of Php 3,500 after four deposits, what was the balance immediately before her withdrawal?
3. Manny rode his bicycle 6 km east, 4 km west, and then 5 km east. How far is he from his starting point?
4. Two boats leave a river dock and travel in opposite directions. Boat X travels 8 km east, where its cargo is unloaded. Boat Y travels 6 km west, where its passengers disembark. At 2:00 PM, Boat X has traveled 4 km east of its first stop, and Boat B has traveled 3 km east form its stop. How far apart are the boats at 2:00 PM?
5. T, U, W, X, Y, and Z are points on a circle. Each of these points is connected to each other by a line segment. How many line segments are there?

Summary

Reasoning is the practice of stating ideas clearly and precisely to arrive at a conclusion. In our life, we often make judgment and conclusion based on facts and observations. These are not always true. Thus, we have to know the different ways of arriving at accurate conclusions.

Intuition is similar to guessing. It is also called reasoning by guessing or reasoning by common sense. It requires less mental activity. An intuition is the ability to acquire knowledge without proof, evidence, or conscious reasoning, or without understanding how the knowledge was acquired. Intuition is highly subjective. Different people think about problems in different ways. It is something that is known or understood without proof or evidence.

Analogy is a form of reasoning in which other similarities are inferred from a particular similarity between two or things. It is reasoning by comparison.

Inductive Reasoning is the process of gathering specific information, usually through observation and measurement and then making a conjecture based on the gathered information.

Deductive Reasoning is the process of showing that certain statements follow logically from agreed upon assumptions and proven facts.

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Distributive: x(y+z) = xy + xz.

Geometric Properties

The following properties may be used to justify proof of some mathematical statements.

Reflexive Property (REF)

Symmetric Property (SYM)

Transitive Property (TRANS)

Addition Property of Equality (APE)

Definition of Congruent Segments (DOCS)

4-Step Process:

6. **Understand the problem. (See)**
7. **Devise a plan. (Plan)**
8. **Carry out the plan. (Do)**
9. **Look back. (Check)**

The following problem solving strategies can be used:

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- **Working Backward**

A strategy that starts at the end of the problem and works backward.

- **Drawing Pictures and Diagrams.**

  A problem can be solved by means of a figure, a diagram, or a graph. It helps you visualize a problem, makes it easier for you to determine the relevant data and observe important connections and relationships.

- **Making Lists and Tables**

  The method “Make a Table" is helpful when solving problems involving numerical relationships. When data is organized in a table, it is easier to recognize patterns and relationships between numbers.

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