POS & LBS

EX01: Reference Frames

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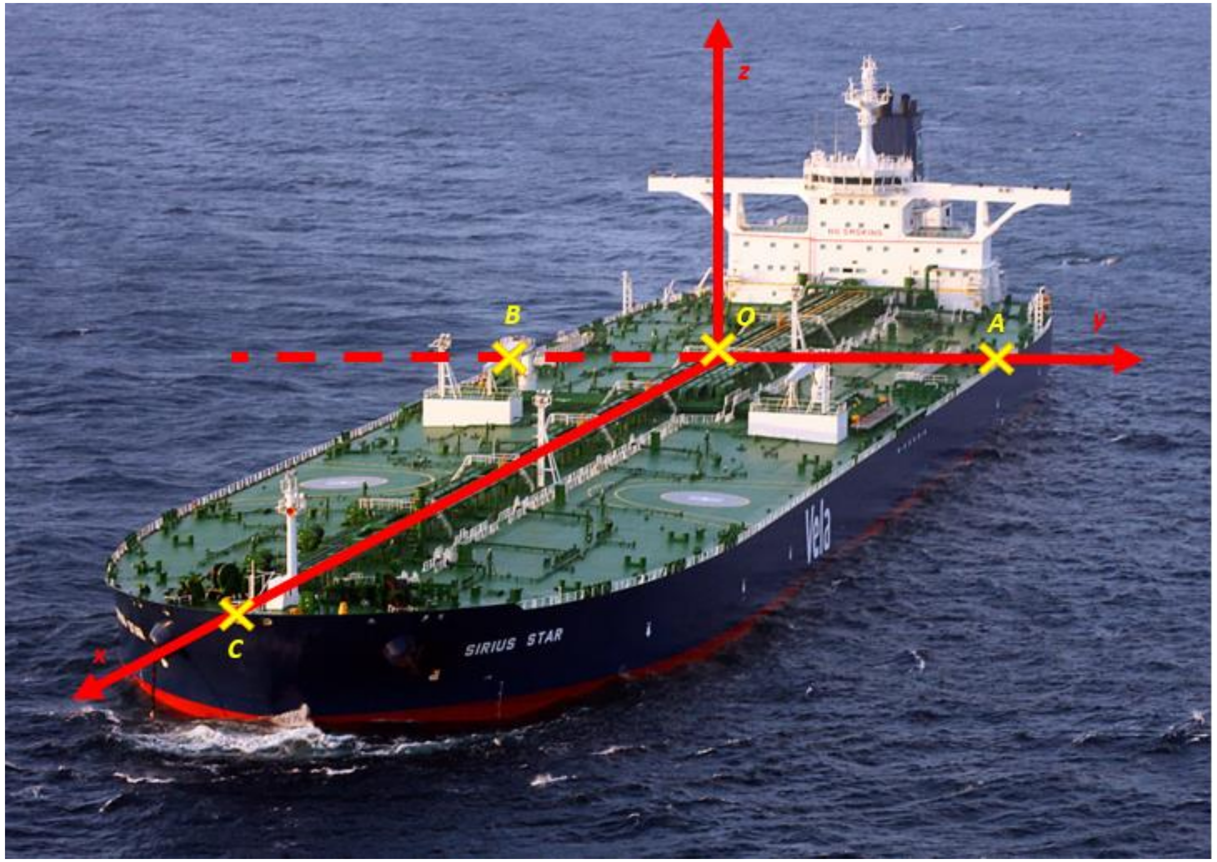
Exercise 1: Reference Frames

Container ship: 60m X 300m

Body frame:
- ✓ Origin O in ITRF
- ✓ A,B,C in BF

Deflection angle in O:
- ✓ Pitch α: X rotation (main axis) from BF to LC
- ✓ Roll β: Y rotation (secondary axis) from BF to LC
- ✓ Yaw γ: Z rotation to finally align BF to LC.



Exercise 1: Reference Frames
SUM UP

- ***Global reference system and frame***: establish a global frame for georeferencing
  - ITRS/ITRF
- ***Regional reference system and frame***: Earth crust is divided in plaques, each one moving: each point on Earth surface moves, given a region, global movements can be removed
  - ETRS/ETRF
- ***Local reference frame***: cartesian coordinates [East, North, Up]
  - Given an origin point *O* for the interest area, the coordinates of nearby points are expressed as East, North, Up (ellipsoidal) wrt the origin.
- ***Body Frame/Local level***: used foor navigation/surveying
  - In navigation X is oriented accoording to motion direction, Y is perpendicular to X in vehicle plane and Z is perpendicular to vehicle plane

From Boody Frame to Local Cartesian

$$\boldsymbol{x}_{LL}(P_i) = R_{LC\rightarrow BF}\boldsymbol{x}_{LC}(P_i)$$

$$R_{LC\rightarrow BF} = R_z(\gamma)\, R_y(\beta) R_x(\alpha)$$

$$\Downarrow$$

$$R_{BF\rightarrow LC} = (R_{LC\rightarrow BF})^T$$

$$\boldsymbol{x}_{LC}(P_i) = R_{BF\rightarrow LC}\boldsymbol{x}_{BF}(P_i)$$

$$\boldsymbol{R_x}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

$$\boldsymbol{R_y}(\beta) = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

$$\boldsymbol{R_z}(\gamma) = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 1: Reference Frames
From Local to Global Cartesian

$$\boldsymbol{x}_{LC}(P_i) = R_0 \Delta \boldsymbol{x}(P_i, P_0)$$

$$\Delta \boldsymbol{x} = (\boldsymbol{x}_{GC}(P_i) - (\boldsymbol{x}_{GC}(P_i))$$

$$R_{GC \rightarrow LC} = R_0$$

$$\Rightarrow \quad \boldsymbol{x}_{GC}(P_i) = \boldsymbol{x}_{GC}(P_0) + {R_0}^T \boldsymbol{x}_{LC}(P_i)$$

$$\mathbf{R}_0 = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\varphi_0 \cos\lambda_0 & -\sin\varphi_0 \sin\lambda_0 & \cos\varphi_0 \\ \cos\varphi_0 \cos\lambda_0 & \cos\varphi_0 \sin\lambda_0 & \sin\varphi_0 \end{bmatrix}$$

Where:
$\lambda_0 = geodetic\ longitude\ of\ O$
$\varphi_0 = geodetic\ latitude\ of\ O$

Exercise 1: Reference Frames
Cartesian ↔ Geodetic

From geodetic to cartesian:

$$X_P = (R_N + h_P)\cos\varphi_P \cos\lambda_P$$
$$Y_P = (R_N + h_P)\cos\varphi_P \sin\lambda_P$$
$$Z_P = [R_N(1-e^2) + h_P]\sin\varphi_P$$

Where R_N is the curvature radius:

$$R_N = \frac{a}{\sqrt{(1-e^2\sin^2\varphi_P)}}$$

From cartesian to geodetic:

$$e_b^2 = \frac{a^2 - b^2}{b^2}$$
$$r = \sqrt{X^2 + Y^2}$$
$$\psi = \arctan\left(\frac{Z}{r\sqrt{1-e^2}}\right)$$
$$R_N = \frac{a}{\sqrt{(1-e^2\sin^2\varphi_P)}}$$

$$\lambda = \arctan\left(\frac{Y}{X}\right)$$
$$\varphi = \arctan\frac{(Z + e_b^2 b\sin^3\psi)}{(r - e^2 a\cos^3\psi)}$$
$$h = \frac{r}{\cos\varphi} - R_N$$

$a = major\ axis = 6378137$
$e = eccentricity$

Exercise 1: Reference Frames
Covariance propagation

Starting point:

$$\boldsymbol{C_{BF}}(P_i) = \begin{bmatrix} \sigma_E^2 & 0 & 0 \\ 0 & \sigma_N^2 & 0 \\ 0 & 0 & \sigma_U^2 \end{bmatrix}$$

Covariances in LC:

$$\boldsymbol{C_{LC}}(P_i) = R_{BF\to LC}\ \boldsymbol{C_{LL}}(P_i)\ \ R_{BF\to LC}{}^{T}$$

Covariances in GC:

$$\boldsymbol{C_{GC}}(\Delta\boldsymbol{x}) = R_{LC\to GC}\ \boldsymbol{C_{LC}} R_{LC\to GC}{}^{T}$$
$$\boldsymbol{C_{GC}}(P_i) = \boldsymbol{C_{GC}}(P_O) + \boldsymbol{C_{GC}}(\Delta\boldsymbol{x})$$

Final covariance matrices in $[\varphi, \lambda, h]$

$$C_{[\varphi,\lambda,h]} = R_0(\varphi,\lambda,h)\boldsymbol{C_{GC}} R_0(\varphi,\lambda,h)^{\boldsymbol{T}}$$

Exercise 1: Reference Frames
Data

Origin in ITRF:

$$\boldsymbol{X}_O^{ITRF} = \begin{bmatrix} 44°23'24.000'' \\ 8°56'20.000'' \\ 70.00m \end{bmatrix}$$
$$\sigma_O = 10cm$$

A,B,C in Boody Frame with respect to origin O:

$$\boldsymbol{X}_A^{BF} = \begin{bmatrix} 0 \\ 30 \\ 0 \end{bmatrix} \quad \boldsymbol{X}_B^{BF} = \begin{bmatrix} 0 \\ -30 \\ 0 \end{bmatrix} \quad \boldsymbol{X}_C^{BF} = \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix}$$
$$\sigma_A = 2cm \quad \sigma_B = 2cm \quad \sigma_C = 2cm$$

Rotation angles BF/LC:

$$\gamma = 30°27'18''$$
$$\beta = 9.5''$$
$$\alpha = 10.23''$$

Exercise 1: Objective

Develop a script in matlab or python able to convert the coordinates of points A,B,C from the body frame to ITRF, then, convert ITRF coordinates in ETRF (note: from ITRF2014 to ETRF2014 on September 1st 2022). Propagate the given accuracies and provide the final accuracies in Lat,Lon,h in cm of A,B,C.

Prepare a pdf containing:

- Summarize the procedures implemented to achieve the objectives.
- Write the obtained results
- In the process of covariance propagation an approximation was done, do you remember where and why was it applied?

Exercise 1: Workflow

1. Import the given data in your working environment
2. Convert the coordinates of the origin from ITRF Geodetic to Global Cartesian (X, Y, Z)
3. Convert A,B,C from body frame to Local Cartesian
4. Convert A,B,C from Local Cartesian to ITRF Global Cartesian
5. Convert through EPN website ITRF GC coordinates of A,B,C to ETRF GC coordinates at epoch 1st September 2022 https://www.epncb.oma.be/_productsservices/coord_trans/index.php#results
6. Convert ETRF GC to Geodetic for A,B,C (note: latitude and longitude must be expressed in sexagesimal).
7. Propagate the accuracies from BF to LC
8. Propagate the accuracies from LC to GC
9. Save the following values on a separate txt file:
10. LC coordinates of points A,B,C
11. ITRF GC coordinates of A,B,C
12. ETRF Geodetic coordinates of A,B,C (note: latitude and longitude in sexagesimal)
13. Standard deviations of A,B,C in East, North,Up in cm
14. Final standard deviations of A,B,C in lat,lon,h in cm

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