POS & LBS

EX02: GPS Orbits

AA: 2024/2025

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Exercise 2
GPS Almanacs

A set of quasi keplerian parameters to predict the orbit of a GPS satellite

Low accuracy, useful to investigate in advance (1 week / 1 day) the constellation visibility in a specific point / specific day

Orbit is considered as Keplerian
Rotation angles variations in time are expressed by a linear precession

Keplerian orbital elements

Parameter	Description	Unit
$\boldsymbol{A}$	Semi-major axis	m
$\boldsymbol{e}$	Eccentricity	$-$
$\boldsymbol{i_0}$	Inclination	rad
$\boldsymbol{\Omega_0}$	Right Ascension of Ascending Node	rad
$\boldsymbol{\omega}$	Argument of Perigee	rad
$\boldsymbol{M_0}$	True Anomaly	rad

Exercise 2
Orbit computation from almanacs

$$n = \sqrt{\frac{GM_E}{a^3}} \Rightarrow M(t) = M_0 + n(t - t_0)$$

$$\eta(t) = M(t) + e\sin(\eta(t))$$

$$\psi(t) = \tan^{-1}((\sqrt{1-e^2}\sin(\eta(t)))/(\cos(\eta(t)) - e))$$

$$r(t) = \frac{a(1-e^2)}{1+e\cos(\psi(t))} \cong a(1 - e\cos(\eta))$$

$$x(t) = r(t)\cos(\psi(t))$$

$$y(t) = r(t)\sin(\psi(t))$$

Nomenclature

n = mean motion

M = mean anomaly

η = eccentric anomaly

ψ = true anomaly

r = ORS radius

Satellite Position in Orbital Reference System!

Rotations are directly from Orbital Reference System to ITRF

$$\begin{bmatrix} X_S(t) \\ Y_S(t) \\ Z_S(t) \end{bmatrix}_{ITRF} = \mathbf{R}_3(-\Omega(t))\mathbf{R}_1(-i(t))\mathbf{R}_3(-\omega(t)) \begin{bmatrix} x_S(t) \\ y_S(t) \\ 0 \end{bmatrix}_{ORS}$$

Rotation from Orbital Reference System to International Terrestrial Reference System

where

$$\Omega(t) = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_E)(t - t_0)$$

$$\dot{\Omega}_E = 7.2921151467 \text{ x } 10^{-5} \text{ rad/sec}$$

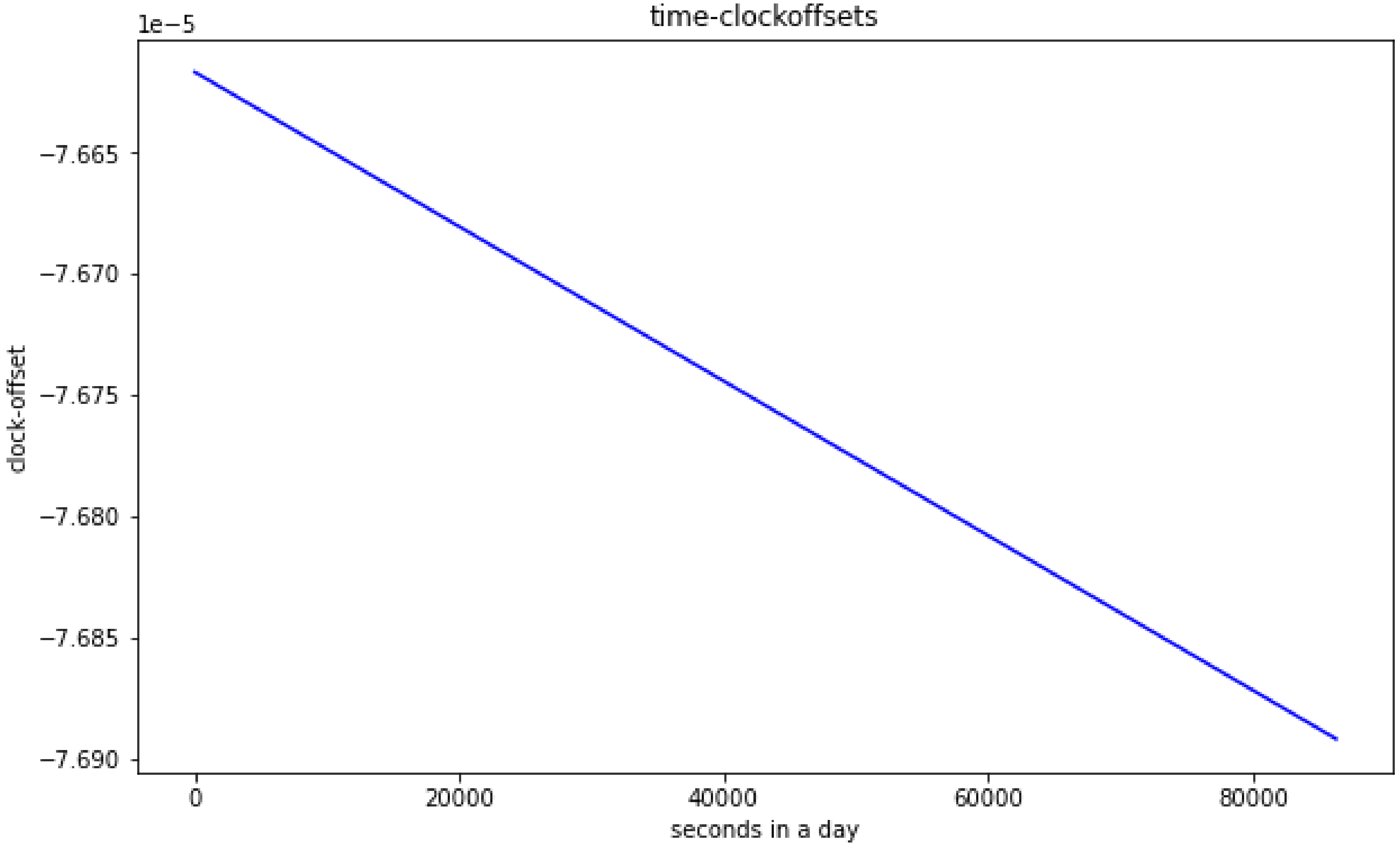
$\dot{\Omega}_E$: Earth rotation angular velocity

$$\omega(t) = \omega_0 + \dot{\omega}(t - t_0),$$

$$i(t) = i_0 + \dot{i}(t - t_0)$$

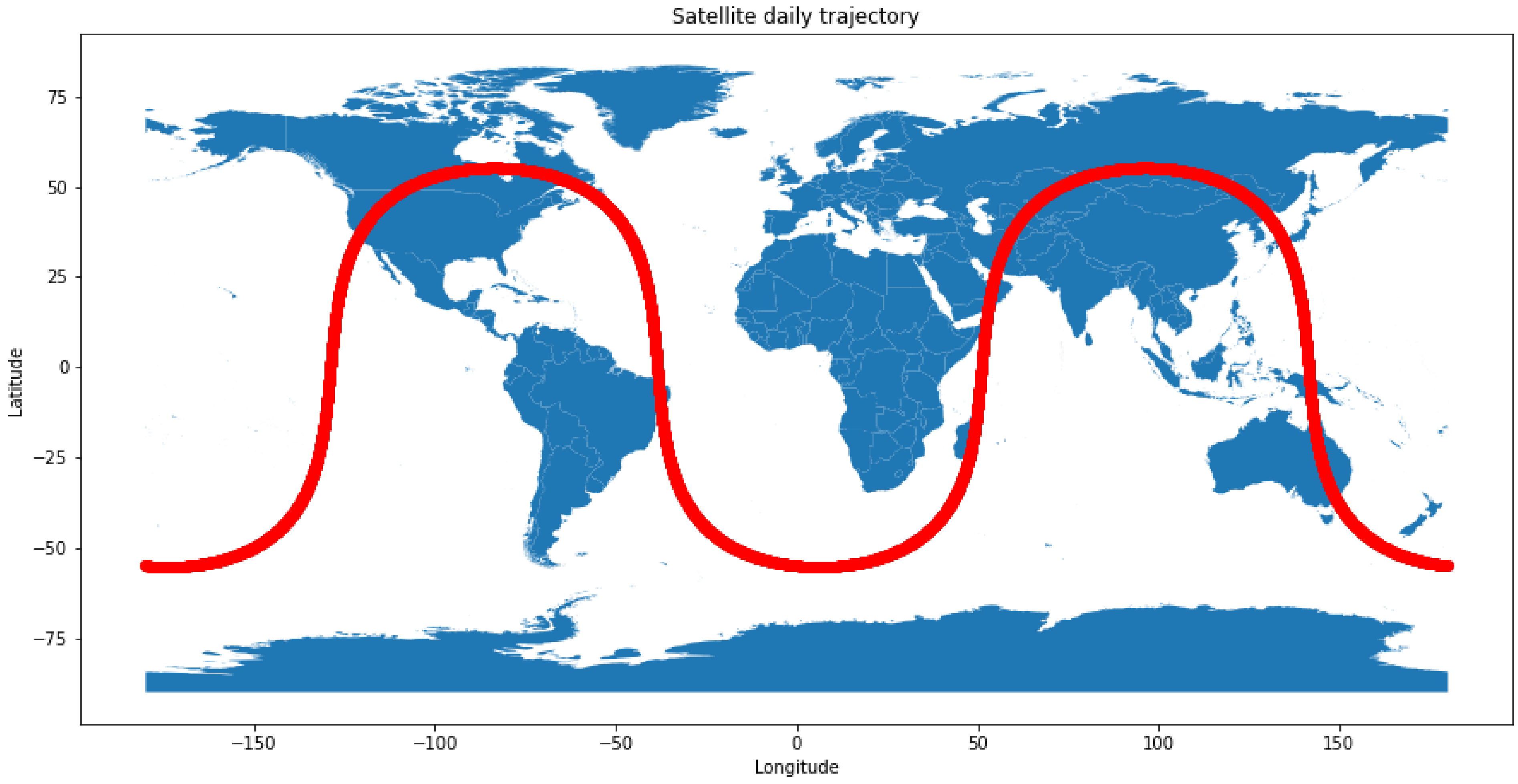
EXERCISE 2:
Results

Satellite clock offsets:



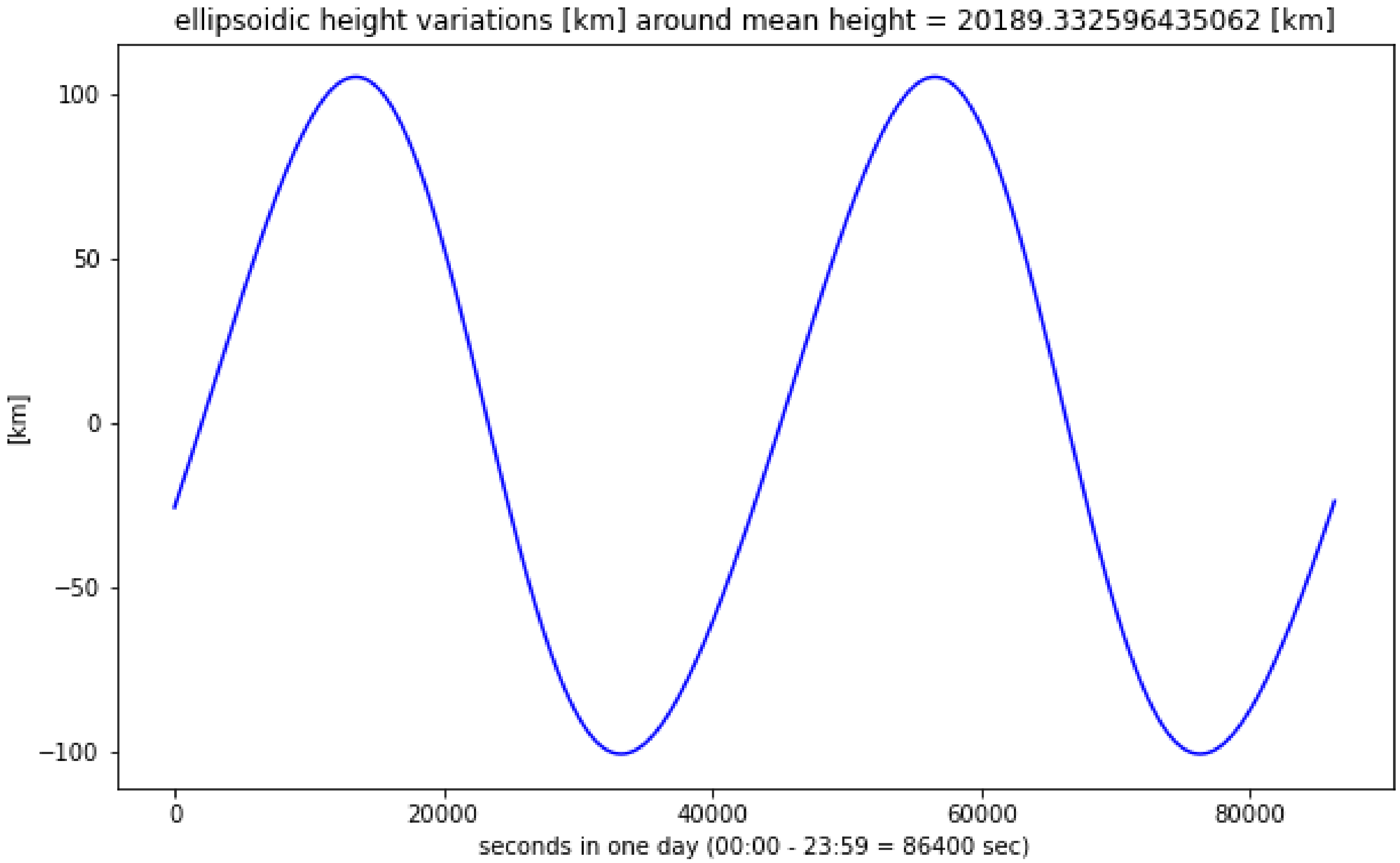
EXERCISE 2:
Results

Satellite orbits:



EXERCISE 2:
Results

Satellite height variation:



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