POS & LBS

EX04: Point Positioning with Least Squares

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Exercise 4
Point Positioning LS

Point positioning in single epoch on single frequency code observations by least squares.

GOAL: *Estimate the receiver's position and receiver's clock offset by applying Least Squares Method to GNSS code observations.*

Code observation equation:

$$P_r^s = \rho_r^s + T_r^s + I_r^s + c * dt_r - c * dt^s$$
$$P_r^s = \rho_r^s + c * dt_r + b + \epsilon_r$$

System unknowns:

$$\underline{X} = \begin{bmatrix} x_r - \tilde{x}_r \\ y_r - \tilde{y}_r \\ z_r - \tilde{z}_r \\ c * dt_r \end{bmatrix}$$
$$\underline{\boldsymbol{x}}_r = \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix}$$

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Point Positioning LS

Linearized code observation equation:

$$P_r^s = e_r^s(\underline{\boldsymbol{x}}_r - \underline{\tilde{\boldsymbol{x}}}_r) + c * dt_r + b + \epsilon_r$$

$$dP_r^s = P_r^s - b = e_r^s(\underline{\boldsymbol{x}}_r - \underline{\tilde{\boldsymbol{x}}}_r) + c * dt_r + \epsilon_r$$

$$dP_r^s = A\underline{X} + \epsilon_r$$

Design matrix:

$$A = \begin{bmatrix} e_x^{s1} & e_y^{s1} & e_z^{s1} & 1 \\ \dots & \dots & \dots & \dots \\ e_x^{sn} & e_y^{sn} & e_z^{sn} & 1 \end{bmatrix}$$

$$e_r^s = \frac{\underline{\tilde{\boldsymbol{x}}}_r - \underline{\boldsymbol{x}}^s}{\underline{\tilde{\rho}}_r^s}$$

LS Solution:

$$N = A' * A$$

$$\underline{\hat{X}} = inv(N) * A' * dP_r^s$$

$$\underline{\hat{\boldsymbol{x}}}_r = \underline{\hat{X}}(1:3) + \underline{\tilde{\boldsymbol{x}}}_r$$

Exercise 4
Covariance and PDOP computation

$$\underline{v} = P_r^s - \hat{P}_r^s$$

$$\sigma_0^2 = \frac{\underline{v}' * \underline{v}}{m - n}$$

$$C_{xx} = \sigma_0^2 * \mathrm{N}^{-1}$$

$$Q_{geom} = N^{-1}(1:3,1:3)$$

$$Q = R_{GC2LC} * Q_{geom} * R_{GC2LC}^T$$

$$PDOP = \sqrt{Q(1,1)^2 + Q(2,2)^2 + Q(3,3)^2}$$

Exercise 4
Input data

- time_rx = epoch expressed in seconds with respect to the beginning of the GPS week.
- pr_C1 = vector containing the satellites’ pseudorange observations.
- sat_ids = vector containing the IDs of the in-view satellites.
- dtS = vector containing satellites’ clock offsets with respect GPS Time.
- xyz_sat = coordinates of the satellites.
- Ionoparams/alpha, beta = parameters of the Klobuchar model.

Provided functions:

- topocent = compute azimuth, elevation and distance given the satellite and receiver coordinates.
- iono_correction = computes the ionospheric effect according to Klobuchar model.
- tropo_correction = computes the tropospheric effect according to Saastamoinen model

Exercise 4
Possible Workflow

Expected output:

1. Coordinates of the receiver and its clock offset.
2. Variance covariance matrix of the coordinates.
3. PDOP

A possible workflow could be:

A. State initialization: receiver position = center of the Earth (0,0,0) and 0 clock error.

While i < iter max or delta > threshold

B. Compute corrections with the approx value of X.

C. Solve LS system and compute new X.

D. delta = $X_{prev} – X_{new}$

E. Compute variance covariance matrix of the estimates.

F. Rotate the variance covariance matrix and compute PDOP.

Exercise 4

Repeat the exercise by applying a cutoff of 5°

- Initialize the state X_0=X output from previous processing
- Compute new PDOP

Exercise 4
Results

The total number of iterations are: 6

The coordinates of the receiver are: [4407364.912   700838.638 4542060.029]

The clock offset of the receiver is: 6.326870e-04

PDOP value is 1.234678

Cut off Angle: 5

The total number of iterations (COA) are: 2

The coordinates of the receiver (COA) are: [4407346.602   700839.189 4542056.826]

The clock offset of the receiver (COA) is: 6.326547e-04

PDOP (COA) value is 1.097662

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