POS & LBS

EX06: Kalman Filtering

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Marianna Alghisi
marianna.alghisi@polimi.it

Exercise 6
Kalman Filter

- **Kalman filtering** is a mathematical method used to estimate the state of a system (like position, speed, or temperature) over time, even when the measurements are noisy or uncertain. It combines predictions from a model with actual measurements to produce a more accurate estimate.
- For example, it can help track a moving object like a car or a plane, even if the sensors tracking it are not perfect.

- Input data: 2D trajectory of a moving body [X, Y].
- Points of the trajectory are visibly affected by noise.
- We assume that the body is moving with costant velocity.

$$\Delta x = \Delta \mathrm{t}\, \dot{x}$$

1. State variable: X, Y position of the body and $\dot{X}, \dot{Y}$ velocities of the system.

$$\underline{x} = \begin{bmatrix} X(t) \\ Y(t) \\ \dot{X}(t) \\ \dot{Y}(t) \end{bmatrix}$$

2. Covariance error of the state initialization:

$$C_{error} = \begin{bmatrix} \sigma^2_{X_{err}} & 0 & 0 & 0 \\ 0 & \sigma^2_{Y_{err}} & 0 & 0 \\ 0 & 0 & \sigma^2_{\dot{x}_{err}} & 0 \\ 0 & 0 & 0 & \sigma^2_{\dot{Y}_{err}} \end{bmatrix}$$

Exercise 6
Kalman Filter – state model definition

1. State model:

$$\begin{cases} X(t+1) = X(t) + \Delta\dot{X}(t) + \epsilon \\ Y(t+1) = Y(t) + \Delta\dot{Y}(t) + \epsilon \\ \dot{X}(t+1) = \dot{X}(t) + \epsilon \\ \dot{Y}(t+1) = \dot{Y}(t) + \epsilon \end{cases}$$

2. State model can be wrote as matrix products:

$$\underline{x}(t+1) = T \times \underline{x}(t)$$

3. Where:

$$T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Model covariance

$$C_{model} = \begin{bmatrix} \sigma^2_{X_{mod}} & 0 & 0 & 0 \\ 0 & \sigma^2_{Y\_mod} & 0 & 0 \\ 0 & 0 & \sigma^2_{\dot{x}_{mod}} & 0 \\ 0 & 0 & 0 & \sigma^2_{\dot{Y}_{mod}} \end{bmatrix}$$

1. Observation model:

$$\begin{cases} \{X_0(t+1) = \ Y_0(t) + \Delta t * \dot{Y}(t) + \upsilon(t) \\ \{Y_0(t+1) = \ Y_0(t) + \Delta t * \dot{Y}(t) + \upsilon(t) \end{cases}$$

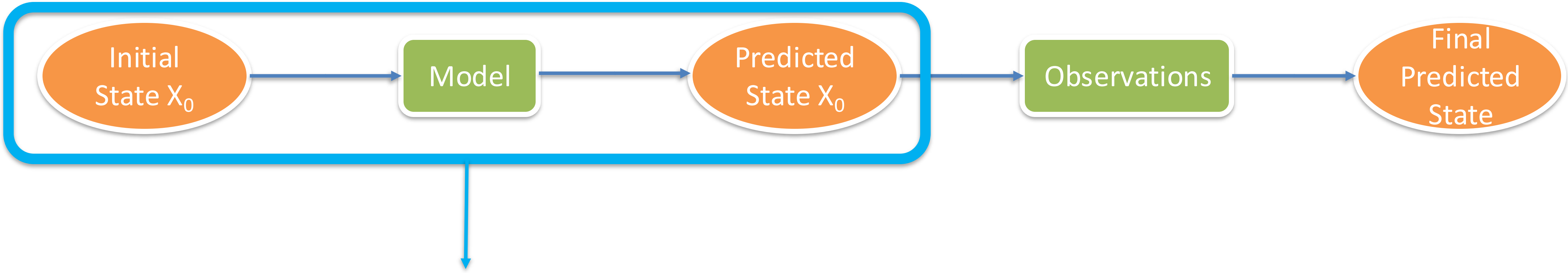
2. As matrices

$$\underline{y_0}(t) = A\underline{x}(t) + \underline{\upsilon}(t)$$

3. Observation covariance

$$\underset{[n\times n]}{C_{obs}} = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & \sigma_n^2 \end{bmatrix}$$

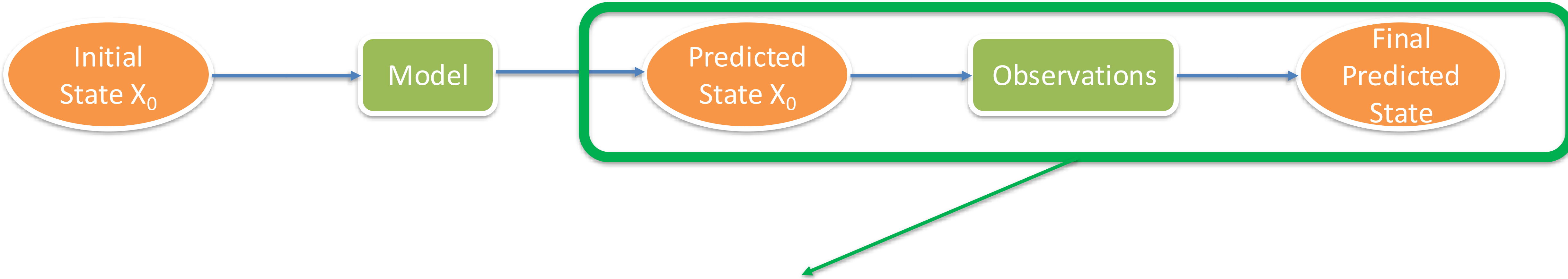
For t = epoch



1. ***State prediction according to the model:***

$$\tilde{x}(t+1) = T \times \hat{x}(t)$$

Exercise 6
Kalman filter - processing



Update the predicted state according to the available observations:

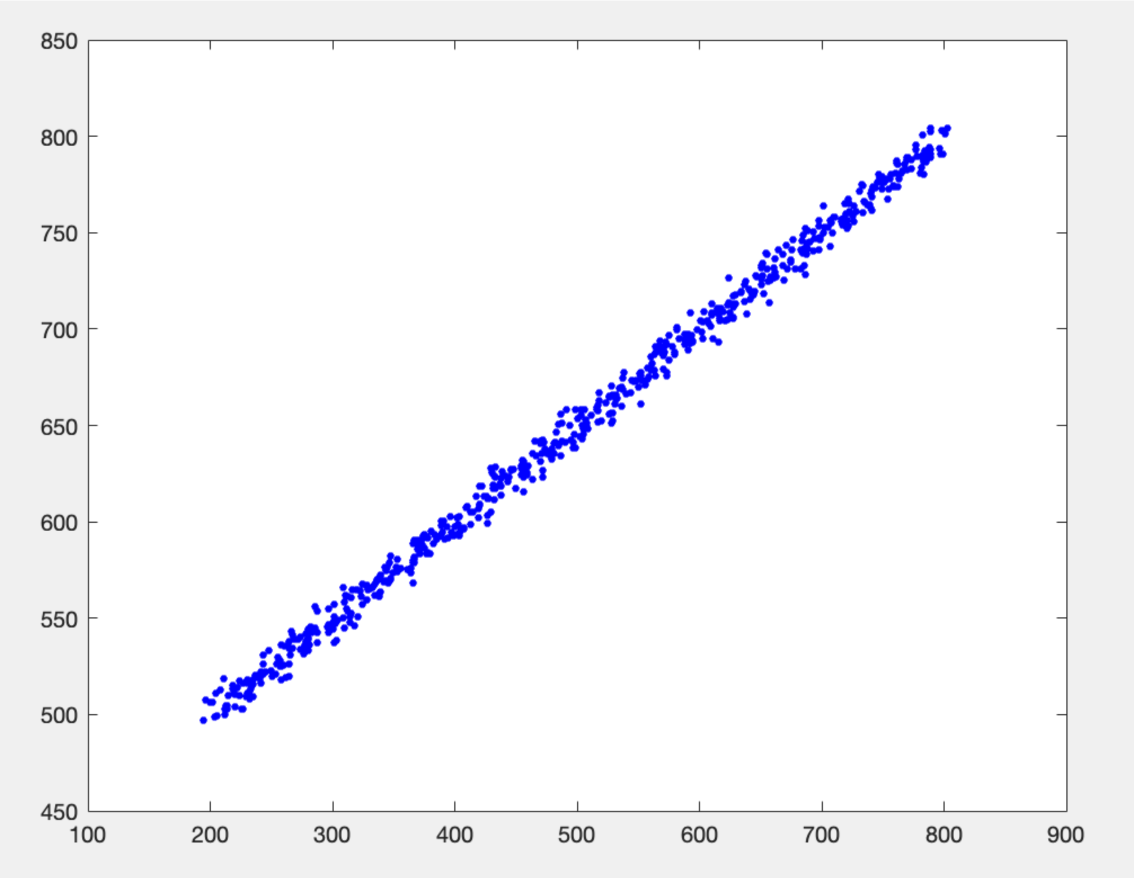
$$K = C_{model} + T * C_{error} * T'$$
$$G = K * A' \left(A * K * A' + C_{obs}\right)^{-1}$$
$$\hat{x} = (I - G * A) * \tilde{x} + G * y_0$$

Update error covariance:

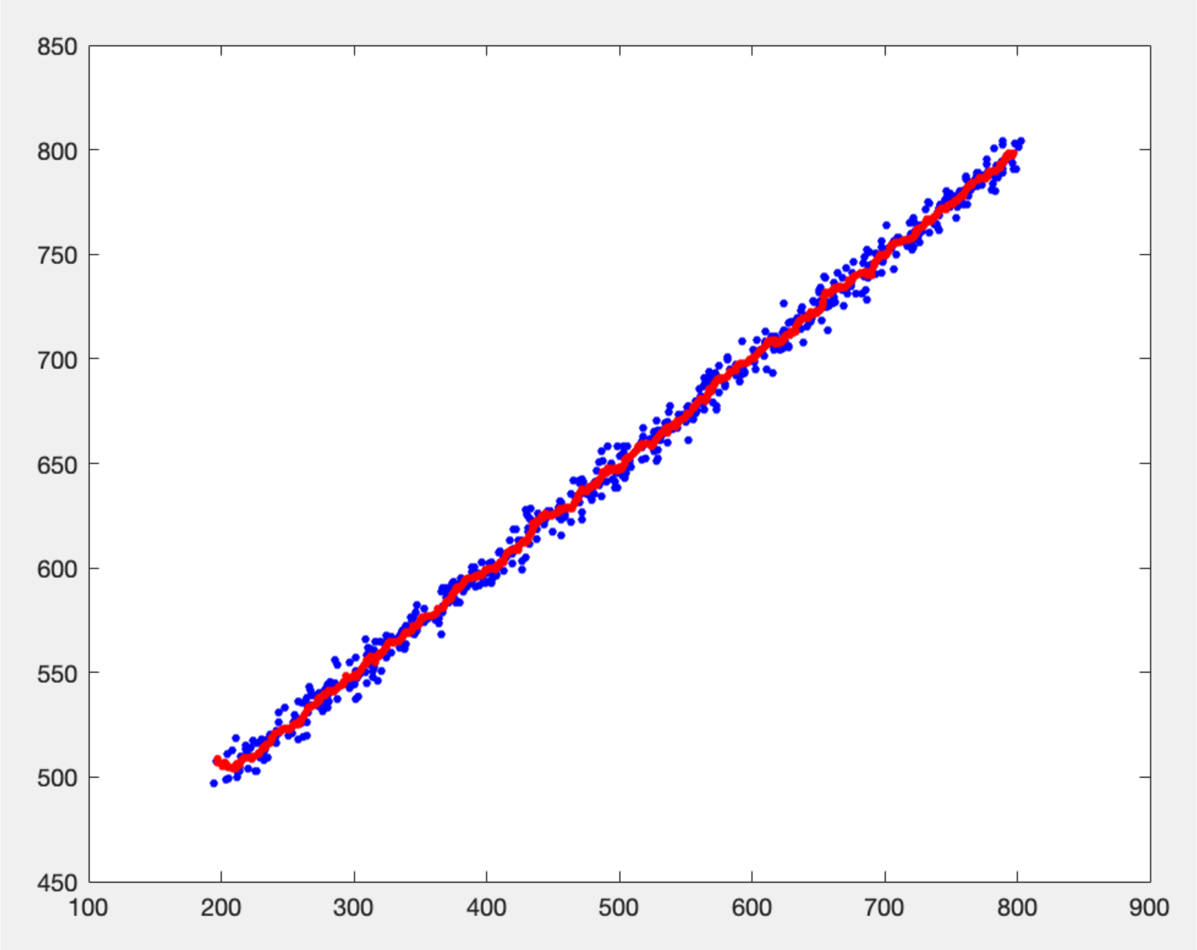
$$C_{error} = (I - G * A) * K$$

Exercise 6
Kalman Filter – data

- Model:
  - $\sigma_{xy} = 1m$
  - $\sigma_{\dot{x}\dot{y}} = 1m$
- Observations
  - $\sigma_{obs} = 25m$
- Initial covariance error
  - $\sigma_{xy} = 10m$
  - $\sigma_{\dot{x}\dot{y}} = 1m$



Exercise 3
Kalman Filter – Results



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