Summary of TMA4120

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Contents

1 LaPlace transform

Corollary/intuition ?

Differentiation

Integration

Convolution

S-shifting

T-shifing

1.1 Table of transforms

1.2 System of ODE's

General approach

- First step

2 Fouries analysis

2.1 Fourier series

For a function to be represented by a fourier series it must be periodic.

Euler formulas

Even function $f(x) = f(-x)$. Even functions reduce to a Fourier cosine series.

Odd function $f(x) = -f(-x)$ Odd functions reduce to Fourier sine series.

If $f(x)$ i given for $0 \leq x \leq L$ only, $f(x)$ has two half range expansions of period $2L$, namely, a cosine and sine series.

2.2 Fourier integral

Fourier sine & cosine integral

2.3 Fourier transform

Fourier transform

Inverse Fourier transform

Derivative

Convolution

Parseval's identity

2.4 DFT and FFT

Discrete Fourier Transform(DFT) ant Fast Fourier Transform(FFT).

2.5 Table of Fourier transforms

3 Partial differential equations

Basic concept is this. Classifying PDE's.

Fundamental theorem of superposition

3.1 Wave equation

General approach

3.1.1 D'Alamebert's solution

3.2 Heat equation

3.3 Laplace's equation

Solving Two-dimensional heat problems

Dirichlet problem

Neumann problem

3.4 Modeling very long rods

Heat equation solved with Fourier integrals and Fourier transform

4 Complex numbers & functions

4.1 Complex numbers

Complex conjugate

Euler formula Hyperbolic functions formula

4.2 Complex function

A complex function is said to have a limit if... epsilon delta proof + corollary.

Entire functions Functions that are analytic on the entire complex plane.

4.3 Analytic function

You might remember conservative vector fields from Calculus 2, well analytic functions are pretty much just like them. Any line integral over a simply closed curve will be 0.

Definition

Cauchy-Riemann equations

4.4 Laplaces's equation

All analytic functions are solution to Lapaces's equation. Lapaces's equation is the most important PDE in physics.

$$(\nabla)^2 u = 0 = u_{xx} + u_{yy} \tag{1}$$

Harmonic functions Solutions of Laplaces's equation having continuous second order derivatives are called harmonic functions.

5 Complex integration

5.1 Basic properties

Complex line integral formula

Dependence of path

ML-inequality Helps with estimating complex line integrals

5.2 Cauchy's integral theorem

5.3 Principle of deformation of path

Multiply connected domains

5.4 Cauchy's integral formula

Corollary

5.5 Derivatives of analytic functions

Corollary, this means that analytic functions have derivatives of all orders. Pretty crazy right?

6 Power series and Taylor series

General formula

General, it's pretty much the same as for real series. All of the old tests for convergence still hold.

Both real part and complex part must converge, theorem 2
Absolute convergence

Ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \tag{2}$$

Comparison test Theorem 5

Root test

6.1 Power series

A power series with a non zero radius of convergence R represents an analytic function at every point interior to its radius of convergence.

6.2 Radius of convergence

Cauchy Hadmard formula

6.3 Operations on power series

differentiation and integration

6.4 Taylor and Maclaurin series

Every analytic function can be represented by a power series, called Taylor series.

To develop a series in negative terms you must extract a z and treat $\frac{1}{z}$ as a new complex number.

$$\frac{1}{1-z} = \frac{-1}{z(1-z^{-1})} \tag{3}$$

Remember usefull algebraic manipulation for developing a series around a point. (7) p.691.

6.5 Usefull series

Geometric series