FYS4150 - Gaussian Elimination

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Abstract

We solve a ordinary differential equation by Gaussian elimination

1 Introduction

We want to solve the equation

$$-u''(x) = f(x)$$

for x in (0,1)

The equation can be rewritten as a system of linear equations. It turns out that the resulting matrix is tridiagonal and this simplifies matters greatly.

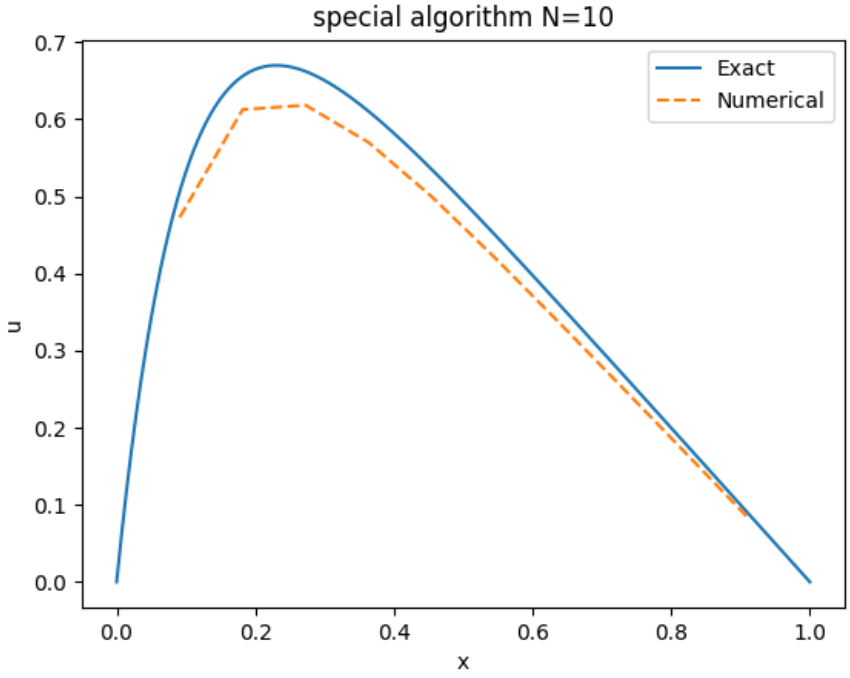
We can deal with this matrix equation by a special form of Gaussian elimination known as the Thomas Algorithm. This algorithm is very useful for a tridiagonal matrix because it is very easy to implement.

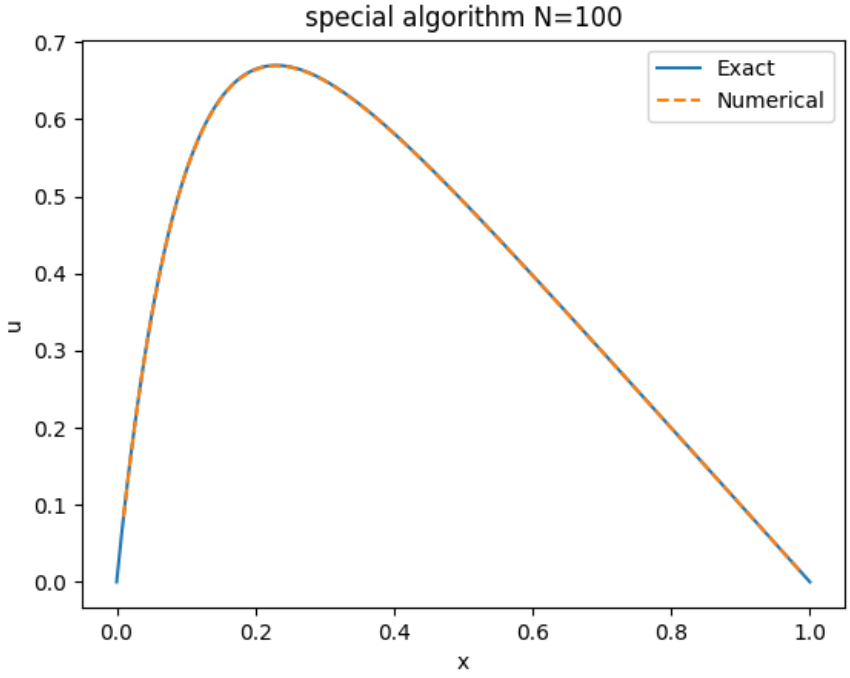
2 The Algorithm

The idea behind the algorithm is to perform Gaussian elimiantion to get rid of the elements on the lower diagonal. By doing this, the matrix is transformed into a upper triangular matrix and we can solve the resulting system by performing back-substitution.

3 Results

This method provides descent results. The plots show a comparison between the numerical approximation and the known exact solution. Cleary the results are improved upon by increasing the number of steps(decreasing step length).


special algorithm N=10
Exact
Numerical
u
x
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0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Exact
Numerical
u
x
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0.6
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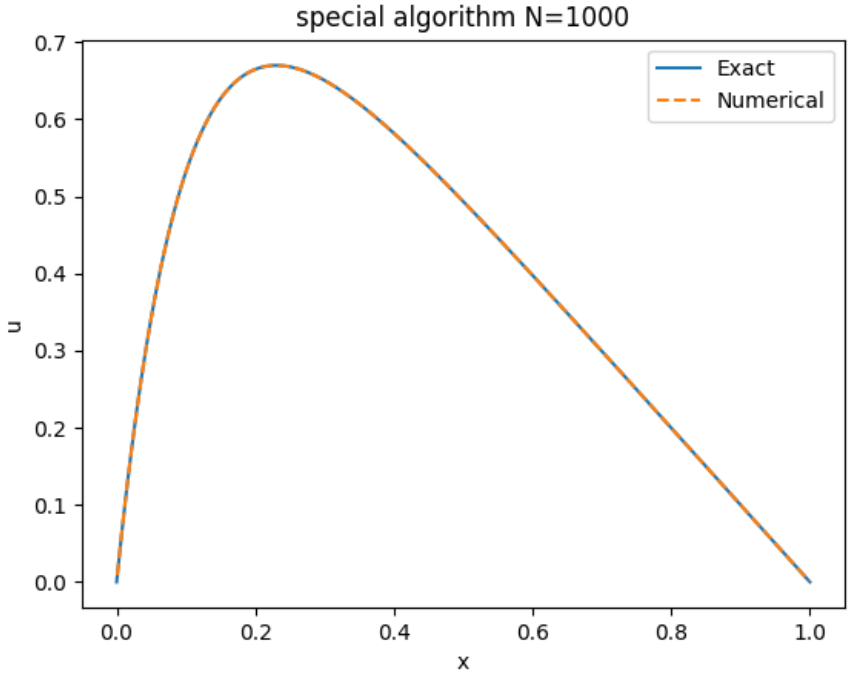
0.2
0.4
0.6
0.8
1.0


special algorithm N=1000
Exact
Numerical
u
x
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0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0