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Temporal Logic Control for Nonlinear Stochastic Systems Under Unknown Disturbances

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Abstract

In this paper, we present a novel framework to synthesize robust strategies for discrete-time nonlinear systems with random disturbances that are unknown, against temporal logic specifications. The proposed framework is data-driven and abstraction-based: leveraging observations of the system, our approach learns a high-confidence abstraction of the system in the form of an uncertain Markov decision process (UMDP). The uncertainty in the resulting UMDP is used to formally account for both the error in abstracting the system and for the uncertainty coming from the data. Critically, we show that for any given state-action pair in the resulting UMDP, the uncertainty in the transition probabilities can be represented as a convex polytope obtained by a two-layer state discretization and concentration inequalities. This allows us to obtain tighter uncertainty estimates compared to existing approaches, and guarantees efficiency, as we tailor a synthesis algorithm exploiting the structure of this UMDP. We empirically validate our approach on several case studies, showing substantially improved performance compared to the state-of-the-art.

Keywords: Data-Driven Control, Strategy Synthesis, Uncertain MDPs, Safe Autonomy

1. Introduction

The synthesis of safe strategies for stochastic systems is critical in ensuring *reliable* and *safe* operations in domains such as robotics, autonomous vehicles, and cyber-physical systems (Belta et al., 2007; Lavaei et al., 2022). A key challenge arises when the system dynamics include *unknown* random disturbances, making it difficult to account for uncertainties while guaranteeing performance against high-level complex specifications. Existing methods often assume known distributions for the disturbances or rely on abstractions with overly conservative uncertainty estimates, limiting their scalability and applicability to complex systems. This paper aims to address these gaps by presenting a novel framework to synthesize optimal strategies for nonlinear stochastic systems with unknown disturbances, ensuring both formal guarantees and computational efficiency.

Our framework employs a data-driven, abstraction-based approach to strategy synthesis for stochastic systems with unknown noise under *linear temporal logic over finite traces* (LTL_f) (De Giacomo and Vardi, 2013) specifications. Starting with data from the system's trajectories, we construct a high-confidence abstraction in the form of an uncertain Markov decision process (UMDP)

(Iyengar, 2005), a flexible model that captures complex uncertainties. Unlike existing methods relying on interval-based abstractions or conservative assumptions, our framework represents transition probability uncertainties as convex polytopes. These sets are derived through a novel two-layer discretization scheme and learning the support of the unknown disturbance. This leads to tighter uncertainty sets and less conservative results compared to existing methods. Exploiting this UMDP structure, we introduce a synthesis algorithm for LTL$_f$ specifications that simplifies the computation, reducing the complexity of standard UMDP linear programming approaches. By incorporating uncertainties from both abstraction errors and data limitations, our framework yields a strategy that is robust. Our empirical evaluations over various types of systems reveals the efficacy of this approach over existing methods, namely in data efficiency, tightness of results, and scalability.

The main contributions of this paper are fourfold: (i) a novel framework for synthesizing strategies for nonlinear stochastic systems under non-additive, unknown disturbances with LTL$_f$ specifications, (ii) a distribution-agnostic, data-driven construction of UMDP abstraction with a specific structure that reduces conservatism of existing abstraction-based techniques, (iii) a tailored synthesis algorithm for this UMDP abstraction that is both efficient and results in tight error bounds, and (iv) a series of case studies and benchmarks that show superiority of the framework over the state-of-the-art, with up to 3 orders of magnitude improvement in sample complexity and an order of magnitude reduction in computation time.

Related Work Abstractions of stochastic systems to finite Markov decision processes (MDPs) are powerful tools for controller synthesis on highly-complex systems under complex logic specifications (Lavaei et al., 2022). In particular, Interval MDPs (IMDPs) (Lahijanian et al., 2015; Givan et al., 2000) abstract systems by presenting uncertain transition probabilities within intervals, capturing the full range of system behaviors. For example, (Cauchi et al., 2019) efficiently abstracts linear systems with additive Gaussian noise, while (Skovbekk et al., 2023) extends this to nonlinear dynamics. Uncertain MDPs (UMDPs) (Iyengar, 2005; El Ghaoui and Nilim, 2005) generalize IMDPs by allowing transition probabilities to belong to more complex sets and have been used for strategy synthesis against specifications such as linear temporal logic (LTL) (Wolff et al., 2012). However, these abstractions typically require system models, which are often unavailable in practice.

To address model uncertainty, various methods leverage Gaussian processes (Jackson et al., 2021), neural networks (Adams et al., 2022), and ambiguity sets (Gracia et al., 2022), which are then abstracted as IMDPs or UMDPs. Statistical tools like the scenario approach have also been used to abstract stochastic (Badings et al., 2023b,a), non-deterministic (Kazemi et al., 2024), and deterministic systems (Coppola et al., 2022, 2023). Also, techniques such as super-martingales and barrier functions enable safety verification and control synthesis for general dynamics (Lechner et al., 2022; Badings et al., 2024; Mazouz et al., 2024). Nevertheless, all these works assume that the disturbance distribution is known.

When disturbance distributions are uncertain, some works combine IMDP abstractions with statistical tools like the scenario approach (Badings et al., 2023b,a; Schön et al., 2023), while others employ barrier certificates with the scenario approach for safety verification (Mathiesen et al., 2023). Another approach constructs Wasserstein ambiguity sets from data samples to abstract systems as UMDPs, ensuring high-confidence containment of unknown distributions (Gracia et al., 2022). However, these methods typically assume simple dynamics or additive noise. For general dynamics with unknown disturbance distributions, only a few works exist. (Salamati et al., 2021) uses barrier certificates for safety verification, and (Gracia et al., 2024) extends the ambiguity set approach for LTL control synthesis. Both assume that certain distribution-related properties, such as variance or support, are known—an assumption often unrealistic in practice, and also suffer from high sample complexity, especially under high-confidence requirements. Our work overcomes these limitations by removing assumptions about disturbance distributions and offering a data-efficient and scalable approach suitable for systems with general dynamics.

2. Problem Formulation

In this work the focus is on discrete-time stochastic systems given by

$$\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k, u_k, \boldsymbol{w}_k), \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ denotes the state at time $k \in \mathbb{N}_0$, $u_k \in U \subset \mathbb{R}^m$ is the control input chosen from a finite set U, and $\boldsymbol{w}_k \in W \subseteq \mathbb{R}^d$ is the disturbance. The latter is a sequence of independent and identically distributed (i.i.d.) random variables on the probability space $(W, \mathcal{B}(W), P)$, with $\mathcal{B}$ being the Borel σ-algebra on W, and where the support W and probability distribution P of $\boldsymbol{w}_k$ are *unknown*. The vector field (possibly nonlinear) $f : \mathbb{R}^n \times U \times W \to \mathbb{R}^n$ is assumed to be Lipschitz continuous on its third argument, uniformly for all values of its first argument on some set.

Assumption 1 *There exists a set $X \subset \mathbb{R}^n$, such that, for every $u \in U$, there exists a constant $L_u > 0$ such that, for all $x \in X$, $w, w' \in W$, it holds that $\|f(x,u,w) - f(x,u,w')\| \leqslant L_u\|w - w'\|$.*

In lieu of unknown W and P, we assume a dataset on the disturbance is available.

Assumption 2 *A set $\{\hat{\boldsymbol{w}}^{(i)}\}_{i=1}^N$ of N i.i.d. samples from P is available.*

Assumption 2 is commonly made in related work (Gracia et al., 2024) and can be practically satisfied through, e.g., observations of the state and control. The straightforward example is when f is affine in $\boldsymbol{w}_k$; otherwise, it suffices for f to be injective over only a subset of $\mathbb{R}^n$ as discussed in (Gracia et al., 2024). This condition is met by many practical systems, including those in our case studies.

Given $x_0, \ldots, x_K \in \mathbb{R}^n$, $u_0, \ldots, u_{K-1} \in U$, and $K \geqslant 0$, we denote a finite *trajectory* of System (1) by $\omega_x = x_0 \xrightarrow{u_0} \ldots \xrightarrow{u_{K-1}} x_K$. We let $|\omega_x|$ denote the length of ω_x, define Ω_x as the set of all trajectories with $|\omega_x| < \infty$ and denote by $\omega_x(k)$ the state of ω_x at time $k \in \{0, \ldots, |\omega_x|\}$. A *strategy* of System (1) is a function $\sigma_x : \Omega_x \to U$ that assigns a control u to each finite trajectory ω_x. Given $x \in \mathbb{R}^n$, $u \in U$, the *transition kernel* $T : \mathcal{B}(\mathbb{R}^n) \times \mathbb{R}^n \times U \to [0,1]$ of System (1) assigns the probability $T(B \mid x, u) = \int_W \mathbb{1}_B(f(x,u,w))P(dw)$, where the indicator function $\mathbb{1}_B(x) = 1$ if $x \in B$, and 0 otherwise, to each Borel set $B \in \mathcal{B}(\mathbb{R}^n)$. For a strategy σ_x and an initial condition $x_0 \in \mathbb{R}^n$, the transition kernel defines a unique probability measure $P^{\sigma_x}_{x_0}$ over the trajectories of System (1) (Bertsekas and Shreve, 1996). In this way, $P^{\sigma_x}_{x_0}[\boldsymbol{\omega}_x(k) \in X]$ denotes the probability that $\boldsymbol{x}_k$ belongs to the set $X \subseteq \mathbb{R}^n$ when following strategy σ_x from initial state x_0. In this work, we are interested in the temporal behavior of System (1) w.r.t. a bounded (*safe*) set $X \subset \mathbb{R}^n$ and a set of regions of interest R_{int}, with $r \subseteq X$ for all $r \in R_{\text{int}}$. We denote by $r_{\text{unsafe}} = \mathbb{R}^n \backslash X$ the unsafe region. We consider a set $AP := \{\mathfrak{p}_1, \ldots, \mathfrak{p}_{|AP|-1}, \mathfrak{p}_{\text{unsafe}}\}$ of *atomic propositions*, and associate a subset of atomic propositions to each region $r \in R_{\text{int}} \cup \{r_{\text{unsafe}}\}$. We define the *labeling function* $L : \mathbb{R}^n \to 2^{AP}$ as the function that maps each state $x \in \mathbb{R}^n$ to the atomic propositions that are true in the region where x lies, e.g., if we associate $\{\mathfrak{p}_1\}$ to region r_1, we conclude that $\mathfrak{p}_1$ is *true* at x, denoted $\mathfrak{p}_1 \equiv \top$, if $x \in r_1$. In consequence, each trajectory $\omega_x = x_0 \xrightarrow{u_0} \ldots \xrightarrow{u_{K-1}} x_K$ results in the (observation) *trace* $\rho = \rho_0 \ldots \rho_K$, where $\rho_k := L(x_k)$.

In order to formally characterize behaviors of System (1), we use *linear temporal logic over finite traces* (LTL$_f$) (De Giacomo and Vardi, 2013), which generalizes Boolean logic to temporal behaviors. An LTL$_f$ property φ is a logical formula defined over atomic proposition AP using Boolean connectives "negation" ($\neg$) and "conjunction" ($\wedge$), and the temporal operators "until" ($\mathcal{U}$) and "next" ($\bigcirc$). The syntax of formula φ is recursively defined as $\varphi := \top \mid \mathfrak{p} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \mathcal{U} \varphi_2$, where $\mathfrak{p} \in AP$ and φ_1, φ_2 are also LTL$_f$ formulas. The temporal operators "eventually" ($\Diamond$) and "globally" ($\Box$) are derived from the above syntax as $\Diamond\varphi := \top\mathcal{U}\varphi$ and $\Box\varphi := \neg\Diamond(\neg\varphi)$. LTL$_f$ formulae are semantically interpreted over finite traces (De Giacomo and Vardi, 2013). We say a trajectory ω_x satisfies a formula φ, i.e., $\omega_x \models \varphi$, if some prefix of its trace ρ satisfies φ.

Our goal is to synthesize a strategy for System (1) to ensure satisfaction of a given LTL$_f$ formula φ. However, note that (i) under a given strategy, the satisfaction of φ is probabilistic, and (ii) in our setting, the distribution of the disturbance is unknown. Hence, we aim to leverage data samples to generate a strategy that guarantees System (1) satisfies φ with high probability. Furthermore, note that the synthesized strategy must account for the learning gap due to the lack of knowledge of P.

Problem 1 *Consider stochastic System (1), a set $\{\hat{\mathbf{w}}^{(i)}\}_{i=1}^N$ of N i.i.d. samples from P, a bounded set $X \subset \mathbb{R}^n$ on which Assumption 1 holds, and an LTL$_f$ formula φ defined over the regions of interest R. Given a confidence level $1-\alpha \in (0,1)$, synthesize a strategy σ_x and a high probability bound function $\underline{p} : X \to [0,1]$ such that, with confidence at least $1-\alpha$, for every initial state $x_0 \in X$, σ_x guarantees that the probability that the paths $\omega_x \in \Omega_x$ satisfy φ while remaining in X is lower bounded by $\underline{p}(x_0)$, i.e., $P_{x_0}^{\sigma_x}[\omega_x \models \varphi \wedge \Box\neg\mathfrak{p}_{unsafe}] \geqslant \underline{p}(x_0)$.*

We emphasize that the noise distribution P is unknown, and no assumptions are imposed on it. Instead, since only samples are available, the probabilistic guarantees for the closed-loop system must hold with a *confidence*. This confidence is related to the probability that the N samples are representative of P and is interpreted in the frequentist sense: if the process of obtaining N samples from P and synthesizing the strategy is repeated infinitely many times, the condition $P_{x_0}^{\sigma_x}[\omega_x \models \varphi \wedge \Box\neg\mathfrak{p}_{\text{unsafe}}] \geqslant \underline{p}(x_0)$ for all $x_0 \in X$ holds in at least $1-\alpha$ of the cases.

Overview of the approach Given the uncountable nature of the state-space of System (1) and the unknown distribution P, solving Problem 1 exactly is infeasible. Therefore, we adopt an abstraction-based approach. This method provides a strategy along with a conservative, high-probability bound for every initial state. The abstraction is an uncertain Markov decision process (UMDP) constructed from a finite discretization of set X. We learn the transition relations between the discrete regions using System (1) and disturbance samples, capturing the system's behavior with confidence $1-\alpha$. Our UMDP construction is specifically designed to tightly capture the learning uncertainty. Then, we devise a strategy synthesis algorithm based on robust dynamic programming to (robustly) maximize the probability of satisfying φ on this UMDP. Next, we refine the obtained strategy to System (1) such that it guarantees the closed-loop system satisfies φ with a probability higher than the one obtained for the abstraction with confidence $1-\alpha$.

3. Preliminaries on Uncertain Markov Decision Processes

An uncertain MDP (UMDP), also known as a robust MDP, is a stochastic system that generalizes the MDP class by allowing its transition probability distributions to be uncertain, taking values from a set (Iyengar, 2005; El Ghaoui and Nilim, 2005; Wiesemann et al., 2013).

Definition 1 (Uncertain MDP) *A labeled uncertain Markov decision process (UMDP) $\mathcal{M}$ is a tuple $\mathcal{M} = (S, A, \Gamma, s_0, AP, L)$, where S and A are finite sets of states and actions, respectively, $s_0 \in S$ is the initial state, S and A are finite sets of states and actions, respectively, $s_0 \in S$ is the initial state, $\Gamma = \{\Gamma_{s,a} \subseteq \mathcal{P}(S) : s \in S, a \in A\}$, where $\mathcal{P}(S)$ is the set of probability distributions over S, and $\Gamma_{s,a}$ is a nonempty set of transition probability distributions for state $s \in S$ and action $a \in A$, AP is a finite set of atomic propositions, and $L : S \to 2^{AP}$ denotes the labeling function.*

A finite *path* of UMDP $\mathcal{M}$ is a sequence $\omega = s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} \ldots \xrightarrow{a_{K-1}} s_K$ of states $s_k \in S$ and actions $a_k \in A$ such that there exists $\gamma \in \Gamma_{s_k,a_k}$ with $\gamma(s_{k+1}) > 0$ for all $k \in \{0, \ldots, K-1\}$. We denote by Ω the set of all finite paths. Given a path $\omega \in \Omega$, $\omega(k) = s_k$ is the state of ω at time $k \in \{0, \ldots, K\}$, and we denote its last state by $\mathrm{last}(\omega)$. A *strategy* of a UMDP $\mathcal{M}$ is a function $\sigma : \Omega \to A$ that maps each finite path to the next action. We denote by Σ the set of all strategies of $\mathcal{M}$. Given path $\omega \in \Omega$ and $\sigma \in \Sigma$, the process evolves from $s_k = \mathrm{last}(\omega)$ under $a_k = \sigma(\omega)$ to the next state according to a probability distribution in Γ_{s_k,a_k}. An adversary is a function that chooses this distribution (Givan et al., 2000). Formally, an *adversary* is a function $\xi : S \times A \times \mathbb{N} \to \mathcal{P}(S)$ that maps each state s_k, action a_k, and time step $k \in \mathbb{N}$ to a transition probability distribution $\gamma \in \Gamma_{s,a}$, according to which s_{k+1} is distributed. We denote the set of all adversaries by Ξ. Given an initial condition $s_0 \in S$, a strategy $\sigma \in \Sigma$ and an adversary $\xi \in \Xi$, the UMDP collapses to a Markov chain with a unique probability distribution $Pr_{s_0}^{\sigma,\xi}$ over its paths.

4. Data-driven UMDP Abstraction

In this section, we introduce a construction of a UMDP $\mathcal{M}$, whose path probabilities are guaranteed to encompass the probabilities of System (1)'s trajectories with confidence $1 - \alpha$. We define the set of states S of $\mathcal{M}$ as follows. Let $R := \{r_1, \ldots, r_{|R|}\}$ be a finite partition of the continuous state-space $\mathbb{R}^n$ into non-overlapping, non-empty regions, which respects the regions of interest R_{int} and the safe set X, and such that $r \in \mathcal{B}(\mathbb{R}^n)$ for all $r \in R$. We let region $r_{|R|} := r_{\text{unsafe}}$ represent the unsafe set. We assign each region $r \in R$ to a state $s \in S$ in the abstraction $\mathcal{M}$ through the bijective map $J : R \to S$, which ensures that $J^{-1}(s) = r \in R$ is unique. For simplicity, we abuse the notation and also say $J(x) = s$ if $x \in r$ with $J(r) = s$. We define the action set A of $\mathcal{M}$ to be the finite control set U of System (1). Furthermore, and with a slight abuse of language, we denote by L the labeling function of $\mathcal{M}$, which maps each state $s \in S$ to the atomic propositions that hold at $x \in r = J^{-1}(s)$.

Next, we define the set of transition probability distributions of the abstraction. To that end, we begin by stating the following proposition, whose proof follows from (Badings et al., 2023a, Eq.12)[1], which gives uniform bounds in the probabilities that System (1) transitions from each point x in some region $r \in R$ to some region $\tilde{r} \subset \mathbb{R}^n$.

Proposition 2 *Given a region $r \in R$, an action $a \in A$ and a realization $w \in W$ of $\boldsymbol{w}$, denote by Reach$(r, a, w) := \{f(x, a, w) : x \in r\}$ the reachable set of r under a and w. Then, the probability of transitioning from each state $x \in r$ to region $\tilde{r} \in \mathcal{B}(\mathbb{R}^n)$ under action $a \in A$ is bounded by*

$$P(\{w \in W : \textit{Reach}(r, a, w) \subseteq \tilde{r}\}) \leqslant T(\tilde{r} \mid x, a) \leqslant P(\{w \in W : \textit{Reach}(r, a, w) \cap \tilde{r} \neq \varnothing\}) \quad (2)$$

Below, we use the samples of $\boldsymbol{w}$ to derive data-driven bounds that contain the ones in (2) and leverage these bounds to define the set of transition probability distributions for $\mathcal{M}$.

1. The measurability of the events in (2) is formally proved in the supplementary material (Gracia et al., 2025b).

4.1. Data-Driven Transition Probability Bounds

We now construct the sets $\Gamma_{s,a}$ of transition probability distributions of the abstraction by leveraging the samples from $\boldsymbol{w}$. Specifically, in our UMDP abstraction, the set $\Gamma_{s,a}$ for each state-action pair (s, a) is defined by: (i) interval bounds on the probability of transitioning to each state $s' \in S$, (ii) interval bounds on the probability of transitioning to a cluster of states in 2^S, and (iii) a bound on the probability of transitioning to states within the reachable set of the learned support of P. Notably, (ii) and (iii) distinguish our construction from prior work, which relies solely on (i). As a result, our UMDP incorporates additional constraints, leading to tighter uncertainty sets. This yields less conservative probabilistic guarantees, as shown in the case studies.

To derive the bounds in steps (i)-(iii), we use Proposition 2, samples from $\boldsymbol{w}$, and two well-known concentration inequalities. The proposition below enables us to compute bounds on transition probabilities between regions, which we later use to obtain bounds in (i)-(ii).

Proposition 3 *Consider the set $\{\hat{\boldsymbol{w}}^{(i)}\}_{i=1}^N$ of i.i.d. samples from $\boldsymbol{w}$. Pick $r \in R$, $a \in A$, $\tilde{r} \in \mathcal{B}(\mathbb{R}^n)$ and $\beta \in (0, 1)$, and let $\epsilon = \sqrt{\log(2/\beta)/(2N)}$. Then, with confidence at least $1 - \beta$ that, for all $x \in r$ we have*

$$T(\tilde{r} \mid x, a) \geqslant \underline{P}(r, a)(\tilde{r}) := \frac{1}{N}\Big|\{i \in \{1, \dots, N\} : \mathit{Reach}(r, a, \hat{\boldsymbol{w}}^{(i)}) \subseteq \tilde{r}\}\Big| - \epsilon \tag{3a}$$

$$T(\tilde{r} \mid x, a) \leqslant \overline{P}(r, a)(\tilde{r}) := \frac{1}{N}\Big|\{i \in \{1, \dots, N\} : \mathit{Reach}(r, a, \hat{\boldsymbol{w}}^{(i)}) \cap \tilde{r} \neq \varnothing\}\Big| + \epsilon. \tag{3b}$$

Proof Consider the lower bound in (2). Denote $E := \{w \in W : \mathrm{Reach}(r, a, w) \subseteq \tilde{r}\}$ and note that $T(\tilde{r} \mid x, a) \geqslant P(E) = \mathbb{E}_P[\mathbb{1}_E(\omega)]$ for all $x \in r$. Therefore applying Hoeffding's inequality to the random variable $\frac{1}{N}\sum_{i=1}^N \mathbb{1}_E(\hat{\boldsymbol{w}}^{(i)})$ yields $P^N[P(E) \geqslant \frac{1}{N}\sum_{i=1}^N \mathbb{1}_E(\hat{\boldsymbol{w}}^{(i)}) - \epsilon] \geqslant 1 - \beta/2$, with $\epsilon = \sqrt{\log(2/\beta)/(2N)}$. Thus, the first expression in (3) holds for all $x \in r$ with confidence $1 - \beta/2$. Employing a similar argument, we obtain that the second expression in (3) also holds for all $x \in r$ with confidence $1 - \beta/2$. Combining both results via the union bound, we obtain the result. ■

Remark 4 *The complexity of computing the bounds in (3) is proportional to N, which is typically high to obtain tight bounds. To reduce this complexity, we cluster the N samples from $\boldsymbol{w}$ into $N_c \ll N$ clusters, each with center c_j and diameter ϕ_j. Substituting the sets $\mathit{Reach}(r, a, \hat{\boldsymbol{w}}^{(j)})$ in (3) by $\{f(x, a, w) \in \mathbb{R}^n : x \in r, \|w - c_j\| \leqslant \phi_j/2)\}$, it is evident that Proposition 3 still holds, with relaxed bounds. Note that this clustering induces a partition on W, allowing to overapproximate the sets $\{f(x, a, w) \in \mathbb{R}^n : x \in r, \|w - c_j\| \leqslant \phi_j/2)\}$ as shown by Skovbekk et al. (2023).*

Next, we estimate the support of P in (iii). Including this information into $\mathcal{M}$ tightens the sets $\Gamma_{s,a}$ of transition probability distributions, thus yielding a less conservative abstraction.

Proposition 5 (Confidence Region (Tempo et al., 2013)) *Let $\hat{c} = \max\{\|\hat{\boldsymbol{w}}^{(1)}\|, \dots, \|\hat{\boldsymbol{w}}^{(N)}\|\}$. Then, for any $\epsilon_c, \beta_c > 0$ and $N \geqslant \log(1/\beta_c)/\log(1/(1 - \epsilon_c))$, it holds, with a confidence greater than $1 - \beta_c$ with respect to the random choice of $\{\hat{\boldsymbol{w}}^{(i)}\}_{i=1}^N$, that $P(\{w \in W : \|w\| \leqslant \hat{c}\}) \geqslant 1 - \epsilon_c$.*

We denote the learned confidence region for w by $\widehat{W} := \{w \in W : \|w\| \leqslant \hat{c}\}$, which contains at least $1 - \epsilon_c$ probability mass from P with a confidence greater than $1 - \beta_c$. We also define $\widehat{\mathrm{Post}}(s, a) := \{J(f(x, a, w)) \in S : x \in J^{-1}(s), w \in \widehat{W}\}$ for each $s \in S$, $a \in A$ to be the set of abstract states that can be reached from region $J^{-1}(s)$ and under some noise realization $w \in \widehat{W}$.

We now have all the components needed to formally define our abstraction class. Intuitively, the abstraction relies on a two-layer discretization: a fine one represented by S and a coarse one formed by clustering the elements of S (see Figure 1). Let $Q \subseteq 2^S$ represent this clustering, which is non-overlapping, i.e., $\bigcup_{q \in Q} q = S$ and $q \cap q' = \varnothing$ for all $q \neq q' \in Q$. This clustering is crucial for obtaining non-zero lower-bound transition probabilities in (3a), as $\text{Reach}(r, a, \hat{\boldsymbol{w}}^{(i)})$ often cannot be contained within a single small region but can be captured by a cluster of regions (see Figure 1 and the discussion in the supplementary material in (Gracia et al., 2025b)). Additionally, we leverage the learned support $\widehat{W}$ of the disturbance to impose the constrain that the successor state corresponding to a given state-action pair lies on some region with high probability. With this intuition, we formally define our abstraction as follows.

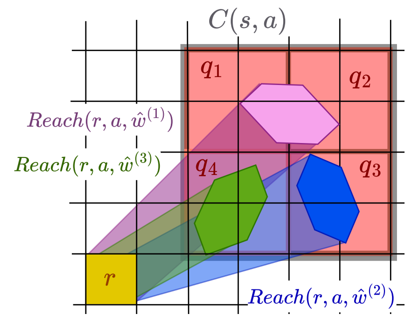

Figure 1: Illustration of the sets in Def. 1. $C(s, a) = \{q_1, q_2, q_3, q_4\}$, and each q_i contains 4 states. The probability that the successor state of $s = J(r)$ under action a will be in $C(s, a)$ is higher than $1 - \epsilon$.

Definition 6 (UMDP Abstraction) *Let $Q \subseteq 2^S$ be a non-overlapping clustering of S and $Q(s, a) \subseteq Q$ be the subset that covers $\widehat{Post}(s, a)$, i.e., $\widehat{Post}(s, a)$ is contained in $C(s, a) := \bigcup_{q \in Q(s,a)} q$. We define the UMDP abstraction of System (1) as $\mathcal{M} = (S, A, s_0, \Gamma, AP, L)$, with, $\forall s \in S$ and $\forall a \in A$,*

$$
\begin{aligned}
\Gamma_{s,a} := \Big\{ \gamma \in \mathcal{P}(S) : \underline{P}(r_s, a)(r_{s'}) \leqslant \gamma(s') \leqslant \overline{P}(r_s, a)(r_{s'}) \qquad \forall s' \in C(s, a), \\
\underline{P}(r_s, a)(r_{q'}) \leqslant \sum_{s' \in q'} \gamma(s') \leqslant \overline{P}(r_s, a)(r_{q'}) \ \ \forall q' \in Q(s, a), \sum_{s' \in C(s,a)} \gamma(s') \geqslant 1 - \epsilon_c, \Big\},
\end{aligned} \tag{4}
$$

where $\underline{P}, \overline{P}$ are defined in (3), $r_s = J^{-1}(s)$, and $r_q = \bigcup_{s' \in q} J^{-1}(s)$.

In Theorem 7, we establish that the UMDP $\mathcal{M}$ is a sound abstraction of System (1), i.e., that $\mathcal{M}$ captures all 1-step behaviors of System (1).

Theorem 7 (Soundness of UMDP Abstraction) *For all $s \in S$, $a \in A$, $x \in J^{-1}(s)$, define $\gamma_x \in \mathcal{P}(S)$ as $\gamma_x(s') := T(J^{-1}(s') \mid x, a)$ for all $s' \in S$. Then, $\gamma_x \in \Gamma_{s,a}$ with confidence of at least $1 - \alpha$, where $\alpha = \beta_c + \big(\sum_{s \in S, a \in A} |C(s, a)| + |Q(s, a)| \big) \beta$.*

The proof of Theorem 7 is provided in the supplementary material (Gracia et al., 2025b).

Corollary 8 *Given $\alpha \in (0, 1)$ and $\epsilon = \epsilon_c \in (0, 1)$, the sample complexity of obtaining a UMDP abstraction with confidence at least $1 - \alpha$ is $N = \max\big\{ \log(\frac{n_{learn}/\alpha}{2\epsilon^2}), \log(\frac{n_{learn}/\alpha}{\log(1/(1-\epsilon_c))}) \big\}$, with $n_{learn} = 1 + \sum_{s \in S, a \in A} |C(s, a)| + |Q(s, a)|$.*

5. Strategy Synthesis

Here, we focus on synthesizing a strategy for System (1) and provide a lower bound on the probability that the closed-loop system satisfies the LTL$_f$ formula φ. We first show that standard synthesis procedures for general UMDP abstractions from the literature (Wolff et al., 2012; Gracia et al., 2024, 2025a) also apply to our setting and then introduce a novel (tailored) algorithm that leverages the specific structure of our UMDP abstraction to reduce computational complexity.

Algorithm 1 2-layer O-maximization

Require: $\mathcal{M}, s \in S, a \in A, \underline{p}^k$
Ensure: γ

Sort $\text{Post}(s,a)$ according to $\{\underline{p}^k(s')\}_{s' \in \text{Post}(s,a)}$ in increasing order
$\gamma(s') \leftarrow \underline{P}(s,a)(s')$ for all $s' \in \text{Post}(s,a)$
$\gamma(s') \leftarrow 0$ for all $s' \notin \text{Post}(s,a)$
$\gamma(q') \leftarrow \sum_{s' \in q'} \gamma(s')$ for all $q' \in Q(s,a)$
$M \leftarrow 1 - \sum_{s' \in S} \gamma(s')$
for $q' \in Q(s,a)$ **do**
  $m \leftarrow \underline{P}(s,a)(q') - \gamma(q')$
  if $m > 0$ **then**
    for $s' \in q'$ **do**
      $g \leftarrow \min\{m, \overline{P}(s,a)(s') - \gamma(s')\}$
      $\gamma(s') \leftarrow \gamma(s') + g, \gamma(q') \leftarrow \gamma(q') + g$
      $m \leftarrow m - g, M \leftarrow M - g$
for $s' \in \text{Post}(s,a)$ **do**
  $q' \leftarrow$ get cluster q' such that $s' \in q'$
  if $q' \neq \varnothing$ **then**
    $m \leftarrow \min\{M, \overline{P}(s,a)(q') - \gamma(q'), \overline{P}(s,a)(s') - \gamma(s')\}$
    $\gamma(q') \leftarrow \gamma(q') + m$
  else
    $m \leftarrow \min\{M, \overline{P}(s,a)(s') - \gamma(s')\}$
  $\gamma(s') \leftarrow \gamma(s') + m, M \leftarrow M - m$

Strategy synthesis begins by translating φ into its equivalent deterministic finite automaton $\mathcal{A}_\varphi$ (De Giacomo and Vardi, 2013) and constructing the product $\mathcal{M}_\varphi = \mathcal{M} \otimes \mathcal{A}_\varphi$ (Wolff et al., 2012). A strategy σ^φ is then synthesized on $\mathcal{M}_\varphi$ via unbounded-horizon *robust dynamic programming* (RDP) with a reachability objective, as detailed in (Gracia et al., 2025a, Theorems 6.2, 6.6). σ_x robustly maximizes the probability of satisfying φ under adversarial choices of transition probabilities from the sets $\Gamma_{s,a}$. Finally, σ^φ is refined into a strategy σ_x for System (1). The following theorem, whose proof is given in the Supplementary Material, ensures that satisfaction probability bounds are preserved under this procedure, thus solving Problem 1.

Theorem 9 (Strategy Synthesis through Product UMDP) *Let σ^φ and $\underline{p}^\varphi$ be respectively the optimal strategy and the lower bound in the probability of satisfying φ obtained via RDP on $\mathcal{M}_\varphi$. Denote by S_0^φ the set of initial states of $\mathcal{M}_\varphi$ and by Lift $: S \to S_0^\varphi$ the function that maps states of $\mathcal{M}$ to S_0^φ. Furthermore, let σ_x be the strategy obtained by refining σ^φ to System (1). Then, with confidence $1-\alpha$, $Pr_x^{\sigma^\varphi}[\omega_x \models \varphi \wedge \mathcal{G}\neg \mathfrak{p}_u] \geqslant \underline{p}^\varphi(\textit{Lift}(s))$ for all $x \in X$, where $s = J(x)$.*

Tailored Synthesis Algorithm We introduce a synthesis algorithm tailored for UMDPs as such in Definition 6, which exploits their structure for greater efficiency. The algorithm draws inspirations from IMDP value iteration (Givan et al., 2000) to speed up the computation of the optimal adversaries in RDP, which is typically formulated a linear program in standard UMDPs. We note that this approach is applicable to our product UMDP $\mathcal{M}_\varphi$ because it retains the same structure of $\mathcal{M}$, as detailed in the supplementary material Gracia et al. (2025b). Therefore and, for simplicity, we describe the algorithm in the context of a reachability problem on $\mathcal{M}$ rather than on $\mathcal{M}_\varphi$.

The algorithm first streamlines the uncertainty set of $\mathcal{M}$. Observe that the sets $\Gamma_{s,a}$ of $\mathcal{M}$ can be simplified by discarding the last constraint in (4) and adjusting the transition probability upper bounds accordingly. Specifically, we increase $\overline{P}(s,a)(s_{|S|})$ and $\overline{P}(s,a)(q')$ for all $q' \in Q(s,a)$ with $s_{|S|} \in q'$ by ϵ_c, and set $\overline{P}(s,a)(s') = 0$ for all $s' \in C(s,a) \backslash \{s_{|S|}\}$ and $\overline{P}(s,a)(q') = 0$ for all $q' \in Q(s,a)$ with $s_{|S|} \notin q'$. As a result, the adversary can choose a distribution with a higher probability of transitioning to the unsafe region. It can be shown that the solution $\underline{p}$ obtained via RDP in this simplified UMDP is not greater than the one obtained for $\mathcal{M}$. We provide a formal proof in (Gracia et al., 2025b), and hereby assume that $\mathcal{M}$ is simplified as described. Then, on the modified $\mathcal{M}$, Alg. 1 computes the optimal adversary for each state-action pair (s,a) by extending the O-maximizing algorithm devised for IMDP value iteration (Givan et al., 2000) to $\mathcal{M}$: via a 2-layer O-maximizing logic, the algorithm efficiently allocates probability mass to states of $\mathcal{M}$ with the lowest value function while respecting the constraints in (4). It begins by ensuring that the lower bounds $\underline{P}(s,a)(s')$ are satisfied for all states s' (Lines 2-3), then proceeds to allocate mass to each cluster $q' \in Q(s,a)$ to meet the required lower bounds (Lines 4-12). The algorithm ensures that the total probability mass remains feasible by maintaining the constraints $\gamma(s') \leqslant \overline{P}(s,a)(s')$, $\sum_{s' \in q'} \gamma(s') \leqslant \overline{P}(s,a)(q')$ throughout the allocation (Lines 13-20). This allocation process guarantees that as much mass as possible is assigned to states with the smallest value function while ensuring $\gamma \in \Gamma_{s,a}$. Alg. 1 terminates once all mass is allocated.Note that RDP algorithm (Gracia et al., 2025a) calls Alg. 1 for every (s,a) and in every iteration until termination. The following theorem proves its correctness and runtime complexity.

Theorem 10 (Correctness of Algorithm 1) *Let $\underline{p}^k \in \mathbb{R}^{|S|}$ be the value function obtained after k iterations of RDP, $s \in S$ and $a \in A$. Define Post$(s,a) := \{s' \in S : \overline{P}(s,a)(s') > 0\}$. Then, for all $s \in S$, $a \in A$, the output γ of Algorithm 1 satisfies $\gamma \in \arg\min_{\gamma \in \Gamma_{s,a}} \sum_{s' \in S} \gamma(s')\underline{p}^k(s')$, and it has a computational complexity of $\mathcal{O}(|\mathit{Post}(s,a)| \log(|\mathit{Post}(s,a)|))$.*

The proof of Theorem 10 is provided in (Gracia et al., 2025b). Note that the computational complexity of solving the linear program in the theorem statement using a standard Simplex algorithm is of $\mathcal{O}(|\mathrm{Post}(s,a)|^3)$, highlighting the computational advantage of using Alg. 1.

Remark 11 *Note that classical IMDP abstractions use only the first constraint of Γ in (4), giving the adversary freedom to choose worse γ than if the abstraction were like our UMDPs. Consequently, IMDPs yield lower probabilistic guarantees, further motivating our abstraction choice.*

6. Case Studies

We now demonstrate empirically the effectiveness of our approach through 7 case studies. These include a nonlinear pendulum with non-additive disturbances, kinematic unicycle models with 2- and 3-D state-spaces and under nonlinear coulomb friction, a 2-D linear system with multiplicative noise, and a 4-D thermal regulation benchmark with multiplicative uncertainty. The considered specifications are reach-avoid (φ_1), the LTL$_f$ specification from (Gracia et al., 2024) (φ_2) and a 15-step safety specification (φ_3). For details of these setups, see (Gracia et al., 2025b).

We compare our approach against (Gracia et al., 2024), the only related work addressing the same problem, in Case Studies $1, 3,$ 5-6. 2 is a more challenging version of 1, where $\boldsymbol{w}$ is unbounded and the pendulum cannot swing up in one go due to control saturation. Similarly, 4 extends 3 with unbounded noise of larger variance and a smaller goal set. Note that (Gracia et al., 2024) relies on ambiguity set learning and cannot handle an unbounded $\boldsymbol{w}$. We also show results obtained using a naïve IMDP abstraction with and without learned support, to show tightness of our approach. Table 1 summarizes our results, highlighting the clear advantages of our approach over (Gracia et al., 2024). Our method significantly reduces sample complexity, often by orders of magnitude, and allows for smaller abstractions while achieving similar or tighter results, which also reduces abstraction time in most cases. For abstractions of the same size, our synthesis time is typically smaller, sometimes by an order of magnitude. Additionally, the table demonstrates that our approach produces tighter results (less error hence higher probabilistic guarantees) than using a naïve IMDP, showcasing the benefits of incorporating additional information into the definition of Γ, albeit with higher computational effort. We also compared the performance of Alg. 1 against the

Table 1: Benchmark results. "Approach" indicates the abstraction used: UMDP (Definition 1), "IMDP (Learn Support)" (a UMDP with only the first and third constraints in (4)), and "Naïve IMDP" (traditional IMDP). e_{avg} represents the average difference in satisfaction probabilities between the lower and upper bounds across all states. Time for abstraction and synthesis is given in minutes, and N_{cluster} denotes the number of noise samples after clustering.

System (Spec.)	Approach	$\lvert Q\rvert$	$\lvert A\rvert$	N	N_{cluster}	e_{avg}	Abstr. Time	Synth. Time
Pendulum (φ_1)	UMDP	10^4	5	5×10^3	47	0.552	1.796	3.643
Gracia et al. (2024)	UMDP	10^4	5	10^4	44	0.007	1.857	6.679
	UMDP	10^4	5	10^5	47	0.003	1.797	2.515
	Gracia et al. (2024)	4×10^4	5	10^6	49	0	5.273	61.167
Pendulum (φ_1)	UMDP	1.225×10^5	5	5×10^4	320	0.136	138.705	187.653
(Torque-Limited)	UMDP	6.25×10^4	5	10^5	175	0.082	40.602	70.773
	UMDP	4×10^4	5	10^5	172	0.279	25.751	42.321
	UMDP	4×10^4	5	10^6	179	0.058	26.694	39.667
	UMDP	4×10^4	5	10^7	191	0.033	27.852	27.066
	IMDP (Learn Support)	4×10^4	5	10^7	191	0.172	14.474	20.281
$3D$ Unicycle (φ_1)	UMDP	5.932×10^4	10	10^4	235	0.579	34.689	33.156
Gracia et al. (2024)	UMDP	5.932×10^4	10	10^5	325	0.267	45.610	36.18
	UMDP	5.932×10^4	10	10^6	358	0.156	52.733	33.751
	Gracia et al. (2024)	6.4×10^4	10	5×10^8	8869	0.447	457.431	43.342
$3D$ Unicycle (φ_1)	UMDP	7.401×10^4	10	10^5	269	0.404	48.057	75.164
(difficult)	UMDP	5.932×10^4	10	10^6	589	0.312	87.753	56.221
	UMDP	7.401×10^4	10	10^6	295	0.249	53.371	69.565
	IMDP (Learn Support)	7.401×10^4	10	10^6	295	0.6923	25.109	38.680
	Naïve IMDP	5.932×10^4	10	10^7	340	0.870	45.865	3.68
	UMDP	5.932×10^4	10	10^7	357	0.199	54.505	47.365
	UMDP	7.401×10^4	10	10^7	345	0.1848	62.510	63.437
Multiplicative	UMDP	3.6×10^3	1	4.652×10^3	231	0.316	0.294	0.302
noise (φ_1)	IMDP (Learn Support)	3.6×10^3	1	4.371×10^3	235	0.437	0.412	0.086
(Skovbekk et al., 2023)	UMDP	3.6×10^3	1	4.68×10^4	257	0.251	0.339	0.034
	UMDP	9.6×10^3	1	4.66×10^5	279	0.223	1.150	0.146
	IMDP	9.6×10^3	1	4.66×10^5	276	0.307	0.228	0.034
	Gracia et al. (2024)	10^4	1	4.66×10^5	1066	0.323	13.149	3.863
$2D$ Unicycle (φ_2)	UMDP	3.6×10^3	8	10^3	32	0.288	0.302	3.304
Gracia et al. (2024)	IMDP (Learn Support)	3.6×10^3	8	5×10^3	31	0.387	0.291	1.174
	UMDP	3.6×10^3	8	5×10^3	34	0.095	0.318	2.818
	UMDP	3.6×10^3	8	10^4	37	0.062	0.341	3.103
	UMDP	3.6×10^3	8	10^5	41	0.017	0.375	3.075
	UMDP	3.6×10^3	8	10^6	43	0.003	0.390	2.767
	UMDP	3.6×10^3	8	10^7	45	0.001	0.408	2.667
	Gracia et al. (2024)	3.6×10^3	8	10^7	46	0.03	0.106	3.043
4-Room Heating (φ_3)	UMDP	2.074×10^4	16	5×10^4	620	0.086	188.506	6.296
Abate et al. (2010)	UMDP	2.074×10^4	16	10^6	818	0.044	282.854	7.156
	IMDP (Learn Support)	2.074×10^4	16	10^6	818	0.069	204.963	3.296

linear programming solver *Linprog* on an abstraction with $|S| = 1600$ and $|A| = 8$, achieving the same guarantees but reducing the total synthesis time from 2880s to 60s, a reduction of $48\times$.

7. Conclusion

We propose an approach to synthesize strategies for nonlinear stochastic systems with unknown disturbances via abstractions to UMDPs. We also identify pitfalls in the use of naïve abstractions for nonlinear systems and present a synthesis algorithm tailored to our UMDP class. Our extensive case studies show the efficacy and advantages of our framework w.r.t. existing works. In future research we plan to increase the tightness of our results by including additional information into the UMDP and investigate overlapping clusters.

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