

# Admissibility Over Winning: A New Approach to Reactive Synthesis in Robotics

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**Abstract**—Reactive synthesis is a framework for modeling and automatically synthesizing strategies in robotics, typically through computing a *winning* strategy in a 2-player game between the robot and the environment. Winning strategies, however, do not always exist, even in some simple cases. In such situations, it is still desirable for the robot to attempt its task rather than “giving up”. In this work, we explore the notion of admissibility to define strategies beyond winning, tailored specifically for robotic systems. We introduce an ordering of admissible strategies and define *admissibly rational strategies*, which aim to be winning and cooperative when possible, and non-violating and hopeful when necessary. We present an efficient synthesis algorithm and demonstrate that admissibly rational strategies produce desirable behaviors through case studies.

## I. INTRODUCTION

*Reactive synthesis* in robotics is the problem of automatically generating strategies for robotic systems to interact with a dynamic environment with the purpose of achieving a complex task expressed in a formal language. Its applications span various domains, from human-robot interaction [1]–[3] to autonomous navigation [4], [5], where robots face uncertainty and adversarial conditions. The interaction is modeled as a game between the robot and the environment, and the goal is to find a *winning strategy* for the robot which guarantees task achievement, regardless of the environment’s actions. However, in complex scenarios, such strategies may not always exist, leading to robot’s inability to take actions strategically. To address this, the concept of *best effort* [6] was introduced but appears too optimistic for robotics. Another approach, *admissibility* [7], [8], relaxes winning strategies, but existing work assumes rationality and known objectives for the environment, which is unrealistic for robotics. In this work, we aim to use the concept of admissibility to define strategies beyond winning, tailored specifically for robotic systems, and develop a reactive synthesis algorithm.

Consider the manipulation example in Fig. 1. The abstraction models a game between a robot (Sys player) and a human (Env player) interacting with objects in a shared workspace. The robot’s task is to move a box to a goal location, but it lacks a winning strategy because the human can always intervene. Despite this, we still want the robot to make its best attempt to perform the task rather than *giving up*, which is a limitation of existing reactive synthesis methods.

To approach this problem, we draw inspiration from admissible strategies [7], [8]. These are the strategies that are not dominated by others based on an ordering, such as action costs. A desirable property of admissible strategies

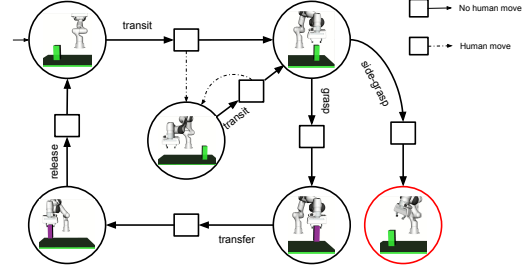


Fig. 1: Manipulation Domain Game [3]: Circle and Square states belong to the Sys and Env player, respectively.

is that they always exist. However, they are too broad, leading to large behavioral variations, not all of which are desirable for a given task. Our key insight is that, given a robotic task specification in *linear temporal logic over finite traces* [9], the objective in the game becomes reachability. This naturally partitions the state space into three regions: winning, pending, and losing. In the winning region, we define admissible strategies that guarantee task completion while seeking cooperation. In the pending region, we combine safety and admissibility to ensure that the task cannot be violated while trying to reach a goal state. For cases where violation avoidance cannot be guaranteed, we introduce the notion of hopeful-admissible strategies, which assume that the environment player does not always take the worst possible actions. By integrating these concepts, we construct the notion of *admissibly rational strategies* and show that they always exist. Then, we introduce a polynomial-time algorithm for synthesis of these strategies. Our experiments demonstrate that admissibly rational strategies produce desirable behaviors.

In short, our contributions are fourfold. (i) We introduce admissibly rational strategies for robotic reactive synthesis, which like admissible strategies, always exist, and unlike admissible strategies, can be synthesized using fixed-point based approaches. (ii) We provide a synthesis algorithm that is sound and complete. (iii) We illustrate the emergent behavior in two case studies, a robotic manipulation domain and Tic-Tac-Toe, demonstrating that a robot playing admissibly rational exemplifies human-like behavior. Finally, (iv) we open-source our implementation [10].

**Related Works:** Several works explore alternatives to winning strategies [6]–[8], [11]–[13]. In formal methods, [11] investigates alternative concepts to winning strategies in qualitative games. They use the notion of admissibility to define the notion of *best-effort* (BE). Works [6], [12] focus on worst-case complexity and propose efficient synthesis algorithms. The objective of the Sys player is to reach a

goal state and the payoff is binary - reach or not reach. In contrast, our work considers min-cost reachability games, where the robot aims to finish task with minimum total cost.

In [7], the notion of admissibility is considered in quantitative settings for normal-form games. The focus is on n-players, and the objective of each player is known a priori. Under the assumption that every player is playing admissibly, an admissible strategy is synthesized for all players. In [8], [13], [14], the notion of admissibility is explored for graph games with  $\omega$ -regular objectives. In contrast, in our settings, we focus on objectives that can be accomplished in a finite time. Importantly, we make no assumptions about the Env player's objectives or require them to play admissibly.

Closest to our work is [9]. They extend prior works [6], [12] on BE synthesis to nondeterministic planning domains. The objective for Sys player is boolean - reach the goal state in finite time. We focus on quantitative reachability objectives that can be accomplished in finite time. Our work builds on the theoretical results of recent work on admissibility for quantitative games [15]. They analyze and give necessary and sufficient conditions for admissible strategies to exist. We identify and formalize a class of admissible strategies that are suitable for robotics. We focus on strategies that mitigate optimism and are efficient to synthesize.

## II. PROBLEM FORMULATION

The aim of this work is to enable a robot to do its *best* to achieve a complex task in the face of human interventions using formal reasoning and game theory. In this section, we formulate this problem by first introducing quantitative games as abstractions of such interactions, and a temporal logic specification language that allows expressing tasks formally.

### A. Game Abstraction

**Definition 1** (2-player Quantitative Game). *A two-player turn-based quantitative game is a tuple  $\mathcal{G} = (V = V_s \cup V_e, v_0, A = A_s \cup A_e, \Delta, C, \Pi, L)$ , where*

- *$V$  is a finite set of states partitioned into Sys player states  $V_s$  and Env player states  $V_e$  such that  $V_s \cap V_e = \emptyset$ ,*
- *$v_0 \in V$  is the initial state,*
- *$A$  is the finite set of actions, where  $a_s \in A_s$  and  $a_e \in A_e$  are Sys and Env player actions, respectively,*
- *$\Delta : V \times A \rightarrow V$  is the transition function such that, for every  $(v_i, a_i) \in V_i \times A_i$ ,  $\Delta(v_i, a_i) \in V_j$ , where  $i, j \in \{s, e\}$  and  $i \neq j$ ,*
- *$C : V \times (A_s \cup A_e) \rightarrow \mathbb{N}^0$  is the cost function such that  $C(v_s, a_s) > 0$  and  $C(v_e, a_e) = 0$ ,*
- *$\Pi = \{\pi_0, \dots, \pi_n\}$  is the finite set of task-related propositions that are true or false, and*
- *$L : V \rightarrow 2^\Pi$  is the labeling function that maps each state  $v \in V$  to a set of atomic propositions  $L(v) \subseteq \Pi$  that are true in  $v$ .*

Every state in the game captures the current configuration of the robot and objects in the workspace. In Fig. 1, each state can be uniquely identified by the location of the box and the end effector status. The edges correspond to physically

viable actions from Sys player states. We note that such discrete abstractions are common in literature [2], [16]–[19] and can be constructed automatically for dynamical systems and robotic manipulators [20], [21].

Game  $\mathcal{G}$ , starting from the initial state  $v_0$ , is played in turns. We define a *play* to be a sequence of states  $P^{v_0} := v_0 v_1 v_2 \dots$ , where, for all  $i \geq 0$ ,  $v_i \in V$ . Further, we say a sequence is feasible if for every  $v_i$  in the play,  $\exists a_i \in A$  s.t.  $v_{i+1} = \Delta(v_i, a_i)$ . A play can be either infinite, denoted by  $P^{v_0} = v_0 v_1 \dots \in V^\omega$  or finite  $P^{v_0} = v_0 v_1 \dots v_n \in V^*$ . We generalize the notation by using  $P^v$  to denote plays that start from state  $v \in V$ . Finally, we assume game  $\mathcal{G}$  is non-blocking, i.e., there exists at least one action from every state.

For  $P^{v_0} = v_0 v_1 v_2 \dots$ , we define *trace*  $\rho$  to be the sequence of the labels of the states in  $P^{v_0}$ , i.e.,  $\rho := L(v_0)L(v_1)L(v_2) \dots$ . Intuitively, a trace captures the evolution of the game (robot's progress) with respect to a task. As game  $\mathcal{G}$  is played between two strategic agents, the choice of action at each state is given by their respective strategies.

**Definition 2** (Strategy). *A Sys player strategy  $\sigma : V^*V_s \rightarrow A_s$  is a function that maps a play that ends in  $V_s$  to an action in  $A_s$ . Similarly, an Env player strategy is  $\tau : V^*V_e \rightarrow A_e$ . The set of all Sys and Env player strategies are denoted by  $\Sigma$  and  $T$ , respectively.*

Strategies that depend only on the last state of plays are called *memoryless*, i.e., for every  $P^v \in V^*V_s$ ,  $\sigma(P^v) = \sigma(\text{last}(P^v))$ , where  $\text{last}(P^v) \in V_s$  is the last element of  $P^v$ . Strategies that are not memoryless are called *finite-memory*.

Given  $\sigma$  and  $\tau$ , a unique play  $P^{v_0}(\sigma, \tau) = v_0 v_1 \dots$  is induced such that, for every  $i \geq 0$ ,  $v_{i+1} = \Delta(v_i, a_i)$ , where  $a_i = \sigma(v_0 v_1 \dots v_i)$  if  $v_i \in V_s$ , otherwise  $a_i = \tau(v_0 v_1 \dots v_i)$ . We say  $\sigma$  is *compatible* with a play  $P^{v_0}$  if  $\exists \tau \in T$  such that  $P^{v_0} = P^{v_0}(\sigma, \tau)$ . For every play on  $\mathcal{G}$ , there is a cost (aka payoff) associated with that play.

**Definition 3** (Total-Payoff [3]). *Given strategies  $\sigma$  and  $\tau$ , the payoff of play  $P^{v_0}(\sigma, \tau)$  is  $\text{Val}(P^{v_0}(\sigma, \tau)) := \sum_{i=0}^{n-1} C(v_i, a_i)$ , where  $n = |P^{v_0}(\sigma, \tau)|$  is the length of  $P^{v_0}(\sigma, \tau)$ , and  $a_i = \sigma(v_0 v_1 \dots v_i)$  if  $v_i \in V_s$ , else  $a_i = \tau(v_0 v_1 \dots v_i)$ .*

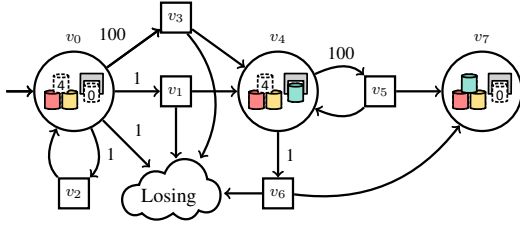
Note that payoff is finite for finite plays; otherwise it is infinite. Also, since the cost of human actions are always zero (see Def. 1),  $\text{Val}$  captures the cost (energy) spent by the robot.

### B. Robotic Tasks in LTL<sub>f</sub>

For tasks achievable in finite time, we use *Linear Temporal Logic over finite traces* (LTL<sub>f</sub>) [22], [23]. An LTL<sub>f</sub> formula  $\varphi$  is defined over atomic propositions in  $\Pi$  with the syntax:

$$\varphi := \pi \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc\varphi \mid \varphi \mathcal{U} \varphi$$

where  $\neg$  and  $\vee$  are the boolean *negation* and *disjunction* operators, and  $\bigcirc$  and  $\mathcal{U}$  are the temporal *next* and *until* operators, respectively. We can derive popular temporal operators *eventually* ( $F$ ) and *always* ( $G$ ) as  $F\varphi \equiv \text{true} \mathcal{U} \varphi$  and  $G\varphi \equiv \neg F\neg\varphi$ . The semantics of LTL<sub>f</sub> is defined over finite traces in  $(2^\Pi)^*$  (see [24] for more details). We say play



**Fig. 2:** Manipulation Domain: No path exists from Losing to  $v_i$ ;  $a_{ij} : v_i \rightarrow v_j$  and  $a_{iL} : v_i \rightarrow \text{Losing}$ . Actions  $a_{01}$  and  $a_{03}$ : transit-grasp blue box  $a_{02}$ : do-nothing, and  $a_{0L}$ : give-up.

$P^{v_0}(\sigma, \tau)$  satisfies formula  $\varphi$ , denoted by  $P^{v_0}(\sigma, \tau) \models \varphi$ , if the trace induced by  $P^{v_0}(\sigma, \tau)$  satisfies  $\varphi$ .

**Example 1.** Consider the following example from [25]: “Robot must put pitcher in the dishwasher with lid on, then either place it in drying-rack or dry it with the towel and return it to the original location.” It translates to formula:  $\varphi = F((p_{dw} \wedge p_{lid}) \wedge F(p_{dr} \vee p_{towel} \wedge \bigcirc F(p_{l0})))$ , where the atomic propositions are:  $p_{dw}$  (dishwasher),  $p_{dr}$  (lid on),  $p_{towel}$  (towel-dried),  $p_{l0}$  (original location).

Given task  $\varphi$ , the robot should choose a strategy that results in a  $\varphi$ -satisfying play. Strategy synthesis is the process of synthesizing a *winning* (enforcing) strategy  $\sigma$  such that  $P^{v_0}(\sigma, \tau) \models \varphi$  for every  $\tau \in T$ . However, such a strategy often does not exist, even in simple cases. Thus, we explore what strategies are apt when winning strategies do not exist.

### C. Beyond Winning Strategies: Admissible Strategies

While a winning strategy ensures task completion under every Env player strategy, such strategies do not always exist. Recall the example in Fig. 1, where no winning strategy exists but it is intuitive for the robot to at least try. To achieve a desirable behavior, we use the notion of *dominance*, which enables a ranking of strategies according to total-payoff.

**Definition 4** (Dominance [7], [26]). We say strategy  $\sigma_1$  very weakly dominates  $\sigma_2$ , denoted  $\sigma_1 \succeq \sigma_2$ , if  $\sigma_1$  always does as well as  $\sigma_2$ , i.e.,  $\text{Val}(P^v(\sigma_1, \tau)) \leq \text{Val}(P^v(\sigma_2, \tau))$  for all  $\tau \in T$ . Furthermore, strategy  $\sigma_1$  weakly dominates  $\sigma_2$ , denoted  $\sigma_1 \succ \sigma_2$ , if (i) it very weakly dominates  $\sigma_2$  ( $\sigma_1 \succeq \sigma_2$ ), and (ii) there exists at least one play under  $\sigma_1$  that does strictly better, i.e.,  $\exists \tau \in T$  s.t.  $\text{Val}(P^v(\sigma_1, \tau)) < \text{Val}(P^v(\sigma_2, \tau))$ .

Intuitively, playing a dominated strategy is sub-optimal as Sys player could have done better.

**Example 2.** In Fig. 1, say robot task is: grab a box. Let  $\sigma_1$  be transit from initial state and grasp and  $\sigma_2$  be same as  $\sigma_1$  except the robot grasps from side. Here  $\sigma_1 \succ \sigma_2$  as  $\sigma_1$  does as well as  $\sigma_2$  and sometimes better. Under  $\sigma_2$ ,  $\text{Val}(P^{v_0}(\sigma_2, \tau)) = \infty \forall \tau \in T$ . But,  $\exists \tau \in T$  such that  $\text{Val}(P^{v_0}(\sigma_1, \tau)) < \infty$  for  $\sigma_1$ .

We say a strategic agent must always play non-dominated (aka admissible) strategies.

**Definition 5** (Admissible Strategy [7]). A Sys player strategy  $\sigma_{\text{Adm}} \in \Sigma$  is called admissible (**Adm**) if it is not weakly dominated by any other Sys player strategy.

Admissible strategies are attractive as they always exist [14], [15]. Prior approaches that explore admissibility in game theory assume that both players have their respective objectives and are playing admissibly [7], [27]. We relax this strong assumption (i.e., knowledge of human’s objective) while still viewing human as a strategic agent. The human can behave adversarially, cooperatively, or somewhere in between. In our setting, we desire the robot to play admissibly and strategically choose actions towards accomplishing its task while still allowing room for cooperation whenever possible.

**Example 3** (Admissibly Rational). In Fig. 2, the robot is tasked with building an arch ( $v_7$ ). Let  $\sigma_1(\dots v_0) = a_{0L}$  and  $\sigma_2(\dots v_0) = a_{03}$ . Under  $\sigma_1$ , robot is not doing its best and hence is not admissible. While  $\sigma_2$  is admissible, it is not a strategic choice. A rational strategy would be  $\sigma_3(v_0) = a_{01}$  and  $\sigma_3(v_4) = a_{45}$ , i.e., commit to  $a_{01}$  and  $a_{45}$  as they ensure there always exists path to  $v_7$  and open for cooperation.

We define an admissible strategy as *rational* if it ensures task completion when possible, while actively seeking cooperation. In cases where task completion cannot be guaranteed, an *admissibly rational strategy* should at least ensure no violation to the task and the existence of a satisfying play. If even this is not possible, the robot should adopt a *hopeful* approach, which assumes that the Env player does not consistently take the worst actions. These are called *hopeless* strategies. Thus, playing hopefully means the robot assumes the Env player is not behaving hopelessly, and in response, the robot selects admissible strategies optimal under the worst-case scenario. In this work, we seek such a robot strategy.

**Problem 1.** Given robot game abstraction  $\mathcal{G}$  and an LTL<sub>f</sub> task formula  $\varphi$ , synthesize an admissibly rational strategy  $\sigma$ , which adheres to the following prioritized objectives: (O1) achieves  $\varphi$  while seeking cooperation, (O2) if (O1) is not possible, ensures no violation to  $\varphi$  (safety) with the possibility of achieving  $\varphi$ , and (O3) if neither (O1) nor (O2) are possible, optimize for the worst case payoff under the assumption that the Env player is not acting hopelessly.

Below, we define two notions of admissibility that formalize the desired characteristics of an admissibly rational strategy. We give synthesis algorithms to solve Problem 1, and show that the algorithms are sound and correct. All the proofs are in provided in the extended version [28].

## III. ADMISSIBLY-RATIONAL DFA GAMES

The standard strategy synthesis involves three steps: translation of the specification into a Deterministic Finite Automaton (DFA), composition of the abstraction with the automaton (DFA Game), and computation of a strategy. We first briefly show how to construct the DFA game and then focus on how to compute admissibly rational strategies on this game.

### A. Overview of DFA Games

For every LTL<sub>f</sub> formula  $\varphi$ , a DFA  $\mathcal{A}_\varphi$  can be constructed that accepts precisely the traces that satisfy  $\varphi$  [9]. DFA  $\mathcal{A}_\varphi$  is a tuple  $\mathcal{A}_\varphi = (q_0, Q, \Gamma, \delta_\varphi, Q_f)$ , where  $Q$  is a finite set of



states,  $q_0 \in Q$  is the initial state,  $\Gamma = 2^\Pi$  is a set of symbols,  $\delta_\varphi : Q \times \Gamma \rightarrow Q$  is the deterministic transition function, and  $Q_f \subseteq Q$  is a set of accepting states. A trace  $\rho \in \Gamma^*$  is a *finite* sequence of symbols in  $\Gamma$ . Trace  $\rho = \rho_1 \rho_2 \dots \rho_n$  induces a run  $q_0 q_1 \dots q_n$ , where  $q_{i+1} = \delta_\varphi(q_i, \rho_{i+1})$  for very  $0 \leq i < n$ . Trace  $\rho$  is accepted if the last state of its induced run is in  $Q_f$ . DFA  $\mathcal{A}_\varphi$  captures all possible traces that satisfy  $\varphi$ , called the *language* of  $\varphi$  or  $\mathcal{A}_\varphi$  [22], [24].

Composing  $\mathcal{A}_\varphi$  with  $\mathcal{G}$  results in a DFA Game that models all possible ways a robot can interact with the human while simultaneously capturing the status of the task completion.

**DFA Game:** A *DFA Game* is defined as  $\mathcal{P} = \mathcal{G} \otimes \mathcal{A}_\varphi$  where  $\otimes$  is the composition operator. The resulting product is a game  $\mathcal{P} = (S = V \times Q, s_0, A = A_s \cup A_e, \delta_\mathcal{P}, C, S_f)$ , where  $A$ ,  $A_s$  and  $A_e$  are the same as Def. 1.  $S$  is the set of states partitioned into **Sys**  $S_s = V_s \times Q$  and **Env**  $S_e = V_e \times Q$  states, and initial state  $s_0 = (v_0, \delta_\varphi(q_0, L(v_0)))$ .  $\delta_\mathcal{P} : S \times A \rightarrow S$  is the transition function such that, for  $s = (v, q)$  and  $s' = (v', q')$ ,  $s' = \delta_\mathcal{P}(s, a)$  if  $v' = \Delta(v, a)$  and  $q' = \delta_\varphi(q, L(v))$ . The cost function  $C$  is defined based on  $\mathcal{G}$  as  $C((v, q), a) = C(v, a)$ , and  $S_f = V \times Q_f$  is the set of accepting states.

The notions of strategy, play, and payoff extend to  $\mathcal{P}$  from  $\mathcal{G}$ . A play in  $\mathcal{P}$  terminates only when it reaches  $S_f$ . Since a feasible play on  $\mathcal{P}$  is constructed from  $\delta_\mathcal{P}$ , its projection onto DFA  $\mathcal{A}_\varphi$  is a valid run of  $\mathcal{A}_\varphi$ , and its projection onto  $\mathcal{G}$  is a feasible play. Therefore, the problem of computing a winning strategy for task  $\varphi$  on  $\mathcal{G}$  reduces to finding a **Sys** strategy on  $\mathcal{P}$  that guarantees every play reaches an accepting state in  $S_f$  under every **Env** strategy [20].

Next, we define two classical notions in game theory for a reachability objective and show how their synthesis is a central element in synthesizing admissibly rational strategies.

### B. Cooperative and Adversarial Games

As  $\text{LTL}_f$  specifications can be satisfied in finite time, all plays induced by a winning strategy  $\sigma$  are finite, and hence their corresponding payoff is finite, i.e.,  $\text{Val}(P^{s_0}(\sigma, \tau)) < \infty \forall \tau \in T$ . We define an optimal winning strategy as a winning strategy that achieves the best worst-case outcome.

**Definition 6** (Optimal Winning Strategy). A **Sys** player strategy  $\sigma^* \in \Sigma$  at state  $s \in S$  is *optimal winning* if it is (i) *winning*, i.e.,  $\text{Val}(P^s(\sigma^*, \tau)) < \infty$  for all  $\tau \in T$ , and (ii) *has the lowest worst-case payoff value*, i.e.,  $\sigma^* = \arg \min_{\sigma \in \Sigma} \max_{\tau \in T} \text{Val}(P^s(\sigma, \tau))$ . The value secured by  $\sigma^*$ , i.e.,  $\max_{\tau \in T} \text{Val}(P^s(\sigma^*, \tau))$ , is called the *worst-case optimal (WCO) value*.

In a cooperative game, the reachability objective of the **Sys** player and **Env** player are the same.

**Definition 7** (Optimal Cooperative Strategy). An optimal cooperative strategy  $\sigma^*$  from state  $s$  is an *optimal strategy when both players are playing cooperatively*, i.e.,  $\sigma^* = \arg \min_{\sigma \in \Sigma} \min_{\tau \in T} \text{Val}(P^s(\sigma, \tau))$ . The value  $\min_{\tau \in T} \text{Val}(P^s(\sigma^*, \tau))$  is called the *cooperative optimal (Co-Op) value*.

We denote by  $\text{cVal}^s$  and  $\text{aVal}^s$  the optimal cooperative and

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### Algorithm 1: Cooperative Game

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**Input :** DFA Game  $\mathcal{P}$   
**Output :**  $\text{cVal}, \Sigma_{ra}, \Sigma_{\text{Co-Op}}$   
1 **forall**  $s \in S$  **do**  $W(s), W'(s) \leftarrow \infty; \Sigma_{\text{Co-Op}}(s) \leftarrow \emptyset;$   
2 **forall**  $s \in S_f$  **do**  $W(s) \leftarrow 0;$   
3 **while**  $W' \neq W$  **do**  
4      $W' = W // s'$  is the successor state  
5      $W(s) = \min(C(s, a) + W'(s')) \quad \forall s \in S \setminus S_f$   
6      $\Sigma_{\text{Co-Op}}(s) =$   
        $\arg \min_a (C(s, a) + W'(s')) \quad \forall s \in S \setminus S_f$   
7 **forall**  $s \in S \setminus S_f$  **do**  
8     **if**  $W(s') < W(s)$  **then**  
9          $\Sigma_{ra}(s) = \Sigma_{ra}(s) \cup \{a : s' = \delta(s, a)\};$   
10 **return**  $\text{cVal} := W, \Sigma_{ra}, \Sigma_{\text{Co-Op}}$

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adversarial (worst-case) values for state  $s$ , respectively. For strategy  $\sigma$ , the cooperative and adversarial values are defined as  $\text{cVal}^s(\sigma) = \min_{\tau \in T} \text{Val}(P^s(\sigma, \tau))$  and  $\text{aVal}^s(\sigma) = \max_{\tau \in T} \text{Val}(P^s(\sigma, \tau))$ . The optimal state values naturally partitions  $S$  into three subsets of *winning region*  $S_{win}$ , *losing region*  $S_{los}$ , and *pending region*  $S_{pen}$ , where

$$S_i = \begin{cases} \{s \in S : \text{aVal}^s < \infty\}, & \text{if } i = \text{win} \\ \{s \in S : \text{aVal}^s = \text{cVal}^s = \infty\}, & \text{if } i = \text{los} \\ \{s \in S : \text{aVal}^s = \infty, \text{cVal}^s < \infty\}, & \text{if } i = \text{pen} \end{cases}$$

Note that these regions are disjoint sets such that their union is the set of all states  $S$ , and their pairwise intersection is empty. Next, we formally define strategies that capture the desirable characteristics for a strategy to be admissibly rational.

### C. Admissibly-Rational: Winning whenever possible

We know that a winning strategy has the desirable property of enforcing a visit to  $S_f$  from all states in the winning region. Unfortunately, not every winning strategy is admissible. From [15, Lemma 3], a subset of winning strategies, precisely Worst-case Cooperative Optimal (**WCo-Op**), are always admissible from the states in  $S_{win}$ . A strategy  $\sigma$  is **WCo-Op** if it is worst-case optimal and has the lowest  $\text{cVal}$  among all such strategies, i.e., if the **Env** player cooperates, this is the most optimal strategy among all winning strategies.

**Definition 8 (WCo-Op).** Strategy  $\sigma$  is *Worst-case Cooperative Optimal* if, for all **Sys** player states  $s \in S$ ,  $\text{aVal}^s(\sigma) = \text{aVal}^s < \infty$  and  $\text{cVal}^s(\sigma) = \min\{\text{cVal}^s(\sigma) \mid \sigma \in \Sigma_{\text{WCO}}\}$ , where  $\Sigma_{\text{WCO}}$  is the set of all **WCO** strategies with finite adversarial value.

Next, for the states that do not belong to the winning region, we synthesize an admissible strategy where **Sys** plays safely, i.e., guarantees no violation of  $\varphi$ .

### D. Admissibly-Rational: Safe whenever you can

At every **Sys** player state in the pending region  $S_{pen}$ , every action is admissible as shown by the following lemma.

**Lemma 1.** Given DFA game  $\mathcal{P}$ , every action  $a_s \in A_s$  at every state  $s \in S_{pen} \cap S_s$  belongs to a finite-memory admissible strategy if  $\delta_\mathcal{P}(s, a_s) \in S_{pen}$ .

*Proof.* We first show that any **Sys** player action from states in the pending region that the transition to **Env** player state in the losing region does *not* belong to an admissible strategy. Next, we will show that *all* **Sys** player actions that transition to states in the pending region belong to an admissible strategy. Finally, we show that all such actions belong to a *finite-memory* strategy.

By construction, for all states in the losing region  $cVal^s = aVal^s = \infty$  and in the pending region  $cVal^s < \infty$  and  $aVal^s = \infty$ . From [15, Thm. 3], the necessary and sufficient conditions for a strategy  $\sigma$  to be admissible is

$$(cVal^h(\sigma) < aVal^h) \vee \quad (1a)$$

$$(aVal^h = aVal^h(\sigma) = cVal^h(\sigma) = acVal^h). \quad (1b)$$

Here  $h$  is a finite prefix of a play  $P^s$ . The definitions for strategies,  $cVal$ , and  $aVal$ , are appropriately defined over history  $h$ . See [15] for full definitions.

For **Sys** player action  $a_s \in A_s$  from  $s \in S_{pen} \cap S_s$  such that  $s' = \delta_P(s, a_s)$  and  $s' \in S_{los}$ , the  $cVal^s \neq \infty$  and  $cVal^{s'} = \infty$ . Thus, it violates Eq. (1) as  $cVal^s \not< aVal^s$  and  $aVal^s = aVal^{s'} = cVal^{s'} \neq cVal^s$  for  $h = s$  and  $s' = \delta_P(s, \sigma(s))$ . Note that  $acVal^s$  is the minimum cooperative value from all worst-case optimal strategies from state  $s$  and is equal to  $cVal^s$  for states in the pending region. Thus, actions  $a_s$  that transition to  $s' \in S_{los}$  are not admissible.

Next, we consider **Sys** player action  $a_s \in A_s$  from  $s \in S_{pen} \cap S_s$  such that  $s' = \delta_P(s, a_s)$  and  $s' \in S_{pen}$ . For such actions  $cVal^s \neq \infty$  and  $cVal^{s'} \neq \infty$  and  $aVal^s = aVal^{s'} = \infty$ . Further, for any history  $h$  of  $P^{s_0} = s_0 s_1 s_2 \dots s_n$  such  $s_j \in S_{pen}$  and  $0 \leq j \leq n$  Eq. (1a),  $cVal^h(\sigma) < aVal^h$  where  $\sigma = \{a_s : s' = \delta_P(\text{last}(h), a_s) \wedge s' \in S_{pen}\}$ , is trivially satisfied, as  $cVal^h(\sigma) \neq \infty$  and  $aVal^h = \infty$ . Intuitively, under every action  $a_s$ , there exists a path to the goal state if the **Env** player cooperates and there always exists an **Env** player strategy that ensures a play never reaches a goal state. Thus, every action  $a_s$  belongs to admissible strategy as they satisfy Eq. (1a).

Finally, from [15, Corollary 1], we conclude that every action  $a_s$  belongs to a finite memory strategy as a memoryless strategies are not sufficient for admissibility. Thus, every action  $a_s \in A_s$  at every state  $s \in S_{pen} \cap S_s$  belongs to a finite-memory admissible strategy if  $\delta_P(s, a_s) \in S_{pen}$ .  $\square$

We say a strategy is *safe*, denoted by  $\sigma_{\text{Safe}}$ , if every play under  $\sigma$  visits only states in  $S_{pen} \cup S_{win}$ . Intuitively, such a strategy ensures that  $\varphi$  is not violated. For example, in Fig. 2,  $\sigma_{\text{Safe}}$  at  $v_0$  and  $v_4$  chooses  $a_{02}$  and  $a_{45}$ , respectively. However, these strategies do not guarantee the possibility of reaching  $S_f$ . To ensure the robot could progress towards  $S_f$  while being safe, we define safe admissible (**SAdm**) strategies.

**Definition 9 (SAdm).** For all  $s \in S$ , a strategy is *Safe Admissible*, denoted by  $\sigma_{\text{SAdm}}$ , if it is safe and admissible, and at least one play under  $\sigma_{\text{SAdm}}$  reaches  $S_f$ , i.e.,  $\exists \tau \in T$  and  $\exists s' \in P^s(\sigma_{\text{SAdm}}, \tau)$  s.t.  $s' \in S_f$ , and  $\forall \tau \in T$  and  $\forall s' \in P^s(\sigma_{\text{SAdm}}, \tau)$ ,  $s' \notin S_{los}$ .

---

## Algorithm 2: Admissibly Rational Synthesis

---

**Input :** DFA Game  $\mathcal{P}$   
**Output :**  $\Sigma_{\text{Adm-Rat}}$  strategy

```

1  $cVal, \Sigma_{ra}, \Sigma_{Co-Op} \leftarrow \text{CooperativeGame}(\mathcal{P})$ 
2  $\Sigma_{WCo} \leftarrow \text{AdversarialGame}(\mathcal{P})$ 
3  $S_{pen}, S_{win}, S_{los} \leftarrow \text{As per Sec. III-B}$ 
4 if  $s_0 \in S_{win}$  then return  $\Sigma_{WCo-Op} \leftarrow \text{Def. 8}$ ;
5 if  $s_0 \in S_{los}$  then return  $\Sigma$ ;
6  $\Sigma_{SAdm} \leftarrow \text{SafeAdmGame}(\mathcal{P}, \Sigma_{ra}, S_{los}, S_{pen})$ 
7 if  $\Sigma_{SAdm}(s) \neq \emptyset \ \forall s \in S_{pen}$  then return
    $\Sigma_{\text{Adm-Rat}} \leftarrow \Sigma_{WCo-Op} \circ \Sigma_{SAdm}$ ;
   /* HAdm strategies */
8  $\Sigma_{HAdm} \leftarrow \text{AdversarialGame}(\mathcal{P}^{\Sigma_{SAdm}}, T_{\text{HF}})$ 
9 return  $\Sigma_{\text{Adm-Rat}} \leftarrow \Sigma_{WCo-Op} \circ \Sigma_{SAdm} \circ \Sigma_{HAdm}$ 

```

---

The following lemma shows that, unlike admissible strategies, a **SAdm** strategy does not always exist.

**Lemma 2.** Given DFA Game  $\mathcal{P}$ , for all  $s \in S_{pen}$ , a *safe-admissible* strategy does not always exist.

*Proof.* It is sufficient to show a counterexample. Let us assume  $\sigma_{\text{SAdm}}$  always exists, i.e.,  $\sigma_{\text{SAdm}}(v_0) \neq \emptyset$ . We begin by noting that memoryless strategies are sufficient for safety games [29, Thm. 1]. In Fig. 2,  $\Sigma_{\text{Safe}}(v_0) = v_2$  and  $\Sigma_{\text{Safe}}(v_4) = v_5$  as under this strategy, there does not exist an **Env** player strategy  $\tau \in T$  such that the play  $P^{v_0}$  never reaches a losing region.

From Lemma 1, we know that every action from  $s \in S_{pen}$  belongs to a pending region. In Fig. 2, a strategy  $\sigma(v_0) = v_2$  is not admissible as under this strategy a play never reaches the goal state  $v_7$ , i.e.,  $cVal^{v_0}(\sigma) = aVal^{v_0}(\sigma) = \infty$  and  $aVal^{v_0} = \infty$ . For any strategy  $\sigma'(v_0) \neq \sigma(v_0)$ ,  $\sigma' \succ \sigma$  as there exists a play to goal state, i.e.,  $cVal^{v_0}(\sigma') < \infty$  and  $cVal^{v_0} \neq cVal^{v_0}(\sigma)$ . Thus,  $\sigma$  is not admissible. Thus,  $\sigma(v_0) \cap \Sigma_{\text{Safe}}(v_0) = \emptyset$

Hence, we have  $\sigma_{\text{SAdm}} = \sigma(v_0) \cap \Sigma_{\text{Safe}}(v_0) = \emptyset$  which violates our assumption. Thus, for all  $s \in S_{pen}$ , a safe-admissible strategy does not always exist.  $\square$

Consider the example in Fig. 2 again. While a  $\sigma_{\text{Safe}}$  exists, a **SAdm** strategy does not since at  $v_0$  the only safe action is  $a_{02}$  but it does not allow a path to  $S_f = \{v_7\}$ . The synthesis of **SAdm** strategies can be done in two stages (see Sec. IV).

### E. Admissibly-Rational: Hopeful with *Env* assumption

In cases that **SAdm** strategies do not exist, we are left with few choices. To ensure the robot still attempts to achieve its task, we synthesize worst-case optimal strategies by assuming that the **Env** player does *not* choose a hopeless strategy.

**Definition 10 (Hopeless strategy).** Given DFA game  $\mathcal{P}$ , for all  $s \in S$ , a strategy  $\tau_{\text{HL}} \in T$  is *hopeless* if under every  $\sigma \in \Sigma$ ,  $\exists s' \in P^s(\sigma, \tau_{\text{HL}})$  such that  $s' \in S_{los}$ .

An **Env** strategy that is not hopeless is called *hopeful*, denoted by  $\tau_{\text{HF}}$ .  $T_{\text{HF}}$  is the set of all such strategies.

**Algorithm 3: Safe Admissible Synthesis**


---

**Input** : DFA Game  $\mathcal{P}$ ,  $\Sigma_{ra}$ ,  $S_{los}$ ,  $S_{pen}$   
**Output** :  $\Sigma_{\text{Adm}}$  strategy

```

1  $\Sigma_{\text{Safe}} \leftarrow \text{SafetyGame}(\mathcal{P}, S \setminus S_{los})$ 
2 forall  $s \in S_{pen}$  do
3   if  $\Sigma_{\text{Safe}}(s) \cap \Sigma_{\text{MReach}}(s) \neq \emptyset$  then
4      $\Sigma_{\text{Adm}}(s) = \Sigma_{\text{Safe}}(s) \cap \Sigma_{\text{MReach}}(s)$ 
5 return  $\Sigma_{\text{Adm}}$ ,  $\Sigma_{\text{Adm}}$ 
  /* Safety Game over  $\mathcal{P}$  */
6 Function  $\text{SafetyGame}(\mathcal{P}, S_f)$ :
7    $X \leftarrow S_f$ ;  $X' \leftarrow \emptyset$ 
8   while  $X' = X$  do
9      $X' \leftarrow X$ 
10    forall  $s \in X$  do
11      if  $s \in S_s$  then
12        if  $\exists a : \delta(s, a) \in X$  then
13           $\Sigma_{\text{Safe}}(s) \leftarrow \{a : \delta(s, a) \in X\}$ 
14        else  $X \setminus \{s\}$ ;
15      else if  $\exists a : \delta_P(s, a) \notin X$  then  $X \setminus \{s\}$ ;
16 return  $\Sigma_{\text{Safe}}$ 

```

---

**Definition 11 (HAdm).** A strategy  $\sigma_{\text{HAdm}} \in \Sigma$  is hopeful admissible, if it is (i) admissible and (ii) worst-case optimal under hopeful *Env* strategies in  $T_{\text{HF}}$ , i.e.,  $\sigma_{\text{HAdm}} = \arg \min_{\sigma \in \Sigma_{\text{Adm}}} \max_{\tau \in T_{\text{HF}}} \text{Val}(P^s(\sigma, \tau))$ .

HAdm strategies are attractive because they always exist.

**Lemma 3.** Given DFA Game  $\mathcal{P}$ , a hopeful-admissible (HAdm) strategy always exists.

*Proof.* Using the results of [30, Thm. 1], we show that the value iteration algorithm on  $\mathcal{P}$  will always converge after finite iterations to a fixed point. Then, using [30, Corollary 18 and Proposition 19], we prove that witnessing strategies for WCO always exists.

[30, Corollary 18] is the proof for the existence of finite memory strategy for Sys player when the values of every state, after running the value iteration algorithm, are finite, i.e., not  $\pm\infty$ . When the weights in  $\mathcal{P}$  are non-negative, we can compute witnessing optimal memoryless strategies for WCO and Co-Op [31]. For states that belong to  $S_{los}$ , all valid actions from every Sys state belong to an optimal strategy  $\sigma_{\text{HAdm}}$ . If  $s_0 \in S_{win}$ , then an optimal memoryless strategy  $\sigma_{\text{HAdm}}$  exists which is WCO such that  $\text{aVal}^{s_0}(\sigma_{\text{HAdm}}) < \infty$ . We note that  $\tau_{\text{HL}} = \emptyset \implies T_{\text{HF}} \equiv T$ . From [15, Thm. 3], we have that  $\Sigma_{\text{Adm}}(s) \neq \emptyset \forall s \in S$  thus  $\sigma_{\text{HAdm}} = \arg \min_{\sigma \in \Sigma_{\text{Adm}}} \max_{\tau \in T} \text{Val}(P^s(\sigma, \tau)) \neq \emptyset$ .

If  $s \in S_{win}$ , then Sys player is the Sys player's winning region. From [15, Thm. 3], we know  $\Sigma_{\text{Adm}}$  can be classified into SC and WCo-Op as they are necessary and sufficient conditions. By definition, WCo-Op  $\neq$  WCO i.e., not all WCO strategies are always admissible [15, Lemma 3]. Thus,  $\Sigma_{\text{Adm}} \subseteq \Sigma$ . By construction, only WCo-Op strategies are worst-case optimal and admissible, i.e.,  $\Sigma_{\text{Adm}} \equiv \Sigma_{\text{WCo-Op}}$  in the Sys player's winning region. Thus,  $\Sigma_{\text{HAdm}} = \arg \min_{\sigma \in \Sigma} \max_{\tau \in T} \text{Val}(P^s(\sigma, \tau)) = \Sigma_{\text{WCo-Op}}$ .

If  $s \notin S_{win}$ , then Sys player is "trapped," and there does

not exist a winning strategy that enforces reaching a goal state. From such states,  $\text{aVal}^s(\sigma) = \infty \forall \sigma \in \Sigma$ . If  $s_0 \in S_{pen}$ , then an optimal memoryless strategy  $\sigma$  exists which is Co-Op such that  $\text{cVal}^{s_0}(\sigma) < \infty$ . If  $s \in S_{los}$  then  $\text{cVal}^s(\sigma) = \text{aVal}^s(\sigma) = \infty$  by definition and thus under no strategy for Sys and Env player can the plays ever reach a goal state. Thus, a worst-case optimal strategy for Sys player picks an action indifferently when it is not in the winning region.  $\square$

Thus, using the notion of WCo-Op, SAdm, and HAdm, we can construct an admissibly rational (Adm-Rat) strategy.

**Definition 12 (Adm-Rat).** For DFA Game  $\mathcal{P}$ , for all  $s \in S$ , an admissibly rational strategy Adm-Rat is defined as

$$\sigma_{\text{Adm-Rat}}(s) = \begin{cases} \sigma_{\text{WCo-Op}}(s) & \text{if } s \in S_{win} \\ \sigma_{\text{SAdm}}(s) & \text{if } s \in S_{pen} \wedge \\ & \text{cVal}^s(\sigma_{\text{SAdm}}) < \infty \\ \sigma_{\text{HAdm}}(s) & \text{otherwise.} \end{cases}$$

By definition, every action at every state in the losing region  $S_{los}$  is part of an admissible strategy and is worst-case optimal. From Lemma 3 and [30, Proposition 19], an Adm-Rat strategy always exists. Further, using [30, Thm. 1], we can show that optimal winning and cooperative strategies always exist and a witnessing strategy can always be synthesized.

**Corollary 1.** Adm-Rat always exists, and  $\sigma_{\text{Adm-Rat}}$  can always be computed.

#### IV. SYNTHESIS FRAMEWORK

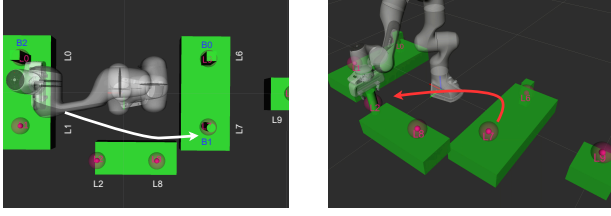
Here, we present an algorithm for synthesis of  $\sigma_{\text{Adm-Rat}}$ . The algorithm is based on four fixed-point computations, one for each  $\sigma_{\text{WCO}}$ ,  $\sigma_{\text{Co-Op}}$ ,  $\sigma_{\text{SAdm}}$ ,  $\sigma_{\text{HAdm}}$ . Specifically, Alg. 1 is a Value Iteration based algorithm for Cooperative Games, and Alg. 2 is for the computation of Adm-Rat strategies.

Alg. 1 computes optimal cVal and optimal strategy from every state in  $\mathcal{P}$  [30]. Lines 3-5 correspond to fixed-point computation when the Env player is cooperative. The While loop terminates when the values of the states have converged, i.e.,  $W' = W$ , where  $W$  is a state vector with optimal cVals for every state. For synthesis of (reachable) admissible strategies, under which at least one play can reach  $S_f$ , we iterate through every state  $s$  and choose actions corresponding to successor states  $s'$  such that  $\text{cVal}^{s'} < \text{cVal}^s$ . As  $C(s, a_s) > 0$ , at least one successor state always exists. Intuitively,  $\text{cVal}^{s'} < \text{cVal}^s$  implies that  $s'$  is "closer" to a goal state. Thus, any play induced by reachable admissible strategy (*ra*) gets closer to  $S_f$  at every step [32]. Thus, synthesis of *ra* strategies can be combined with cVal computation with minimal overhead.

In Alg. 2, we first play Cooperative and Adversarial Game and compute  $\sigma_{\text{WCO}}$  and  $\sigma_{\text{Co-Op}}$  along with  $\Sigma_{ra}$ . For the Adversarial Game,  $W(s)$  computation can be split into min and max operation at Sys and Env player states, respectively (See [30] for details). If  $s_0 \in S_{win}$ , we compute  $\sigma_{\text{WCo-Op}}$  as per Def. 8 and return it. If  $s_0 \in S_{los}$ , then all actions are admissibly rational and belong to  $\sigma_{\text{HAdm}}$ .

To compute  $\sigma_{\text{SAdm}}$ , we first compute  $\Sigma_{\text{Safe}}$  by playing a Safety game. Synthesis of  $\Sigma_{\text{Safe}}$  is a greatest fixed-point





**Fig. 3:** Initial setup (Left). There exists **HAdm** from  $s_0$  in white and **SAdm** in red after grasping. See suppl. video.

operation, for which efficient algorithms exist [24]. Note that  $\Sigma_{\text{Safe}}$  and  $\Sigma_{ra}$ , respectively, compute the the set of all Safe and reachable admissible actions at state  $s \in S_{\text{pen}} \cap S_s$ . Finally,  $\sigma_{\text{SAdm}}(s) = \Sigma_{ra}(s) \cap \Sigma_{\text{Safe}}(s)$  for all  $s \in S_{\text{pen}}$ . If  $\sigma_{\text{SAdm}}(s) = \emptyset$ , we compute  $\sigma_{\text{HAdm}}$ . We first compute the minimal set of Env player actions  $A_{\text{HL}}$  such that restricting the Env to  $a \in A_e \setminus A_{\text{HL}}$  gives a maximal  $T_{\text{HF}}$ . See the extended version for more details [28]. Further, we remove **Sys** player actions  $a_s$  s.t.  $\delta_{\mathcal{P}}(s, a_s) \in S_{\text{los}}$  (as they are not admissible) and play an adversarial game to compute  $\sigma_{\text{HAdm}}$ . We compose these strategies per Def. 12 to obtain  $\sigma_{\text{Adm-Rat}}$ .

**Theorem 1** (Sound and Complete). *Alg. 2 returns the set of all **Adm-Rat** strategies and has polynomial time complexity.*

*Proof.* Alg. 1 is the Value Iteration algorithm for computing optimal cVal and optimal  $\Sigma_{\text{Co-Op}}$  strategy from every state in  $\mathcal{P}$  [30]. Computation for Adversarial Game is similar to Alg. 1 Lines 3-5 where at Env states  $W(s)$  is defined as  $\max(\cdot)$ . Thus, Cooperative Game and Adversarial Game are Polynomial in  $|S|$  [30, Thm. 3]. For **WCo-Op**, we iterate through all states and compute  $\sigma_{\text{WCo-Op}}$  as per Def. 8. Safety Game in Alg. 3 is based on greatest fixed-point computation and the worst-case complexity is known to be polynomial in  $|S|$  [29, Thm. 1]. Further, in Alg. 3, Lines 2-4 terminate in polynomial time. Thus, Alg. 2 runs in polynomial time.

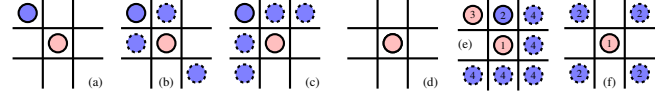
In Alg. 1,  $\arg\min$  in  $\Sigma_{\text{Co-Op}}$  computes the set of all actions for **Sys** player. The correctness and soundness of this algorithm is established in [30], [31]. The proof naturally applies to Adversarial Games where we modify the computation at Env player states. Alg. 3 compute the set of all safe strategies and soundness and correctness are established in [24], [29]. Computation of  $\Sigma_{ra}$  is the same as the computation of maximally permissive non-deferring strategies in [32]. [32, Thm. 4] proves the soundness and correctness. Thus, Alg. 1 computes the set of all **Co-Op** (and similarly **WCO**), and  $\Sigma_{ra}$ . Alg. 3 computes the set of all  $\Sigma_{\text{Safe}}$ . Thus, computation of  $\sigma_{\text{WCo-Op}}$  as per Def 8,  $\sigma_{\text{SAdm}}$  as per Def 9, and  $\sigma_{\text{HAdm}}$  as per Def 11 gives the set of all strategies. Thus, Alg. 2 computes the set of all **Adm-Rat** strategies.  $\square$

## V. EXPERIMENTS

We illustrate the emergent behavior when the robot plays admissibly rational on two case studies. Table I reports the size of  $\mathcal{G}$ ,  $\mathcal{P}$  and synthesis times for various strategies. Our synthesis tool is publicly available on Github [10]. See supplementary material for more experiments.

**TABLE I:** Abs. size & Syn. time (s): averaged over 10 runs.

	$ V $	$ \Delta $	$ S $	$ \delta_{\varphi} $	<b>WCo-Op</b>	<b>Co-Op</b>	<b>SAdm</b>	<b>HAdm</b>
PP Dom.	11272	47220	22545	94297	13.77	24.48	0.29	13.52
TTT( $\varphi_1$ )	5478	16167	5479	17038	5.63	6.13	0.07	5.63
TTT( $\varphi_2$ )	5748	16167	5479	17038	5.94	5.97	—	—



**Fig. 4:** Blue and Pink are **Sys** and **Env** actions. (a-d) current status, (b-e) and (c-f) illustrate  $\sigma_{\text{Adm-Rat}}$  for  $\varphi_1$ ,  $\varphi_2$ . (a-c): **Sys** plays first; (d-f): **Env** plays first. (e-f): The numbers indicate time step.

**Pick-and-Place Domain [3]:** Consider the manipulation domain in Fig. 3. There are three objects,  $B_0, B_1, B_2$ . The human can intervene and move objects within region  $R_1 = \{L_0, L_1, L_2\}$  and  $R_2 = \{L_6, L_7, L_8, L_9\}$ . Location  $L_9$  is out of robot’s reach. Once the human moves an object to  $L_9$ , it cannot be returned. The task is to dry  $B_1$  in location  $L_2$  or  $L_8$  then place  $B_1$  at  $L_6$  and  $B_0$  at  $L_7$  or  $B_1$  at  $L_1$  and  $B_2$  at  $L_0$ . The LTL<sub>f</sub> formula is  $\varphi = F((p_{18} \vee p_{12}) \wedge OF((p_{20} \wedge p_{11}) \vee (p_{16} \wedge p_{07})))$  where  $p_{ij}$  is  $B_i$  placed at  $L_j$ .

Using our tool, we synthesized a  $\sigma_{\text{Adm-Rat}}$ . Note that a winning strategy does not exist as the human can always intervene. The initial state belongs to the pending region and  $\sigma_{\text{SAdm}}(s_0) = \emptyset$  as the human has a strategy to move  $B_1$  to  $L_9$ . Hence, the robot initially plays hopefully ( $\sigma_{\text{Adm-Rat}}(s_0) = \sigma_{\text{HAdm}}(s_0)$ ) and transits to  $B_1$  and grasps it. Once the grasp is complete, a safe-admissible strategy exists (i.e.,  $\sigma_{\text{Adm-Rat}} = \sigma_{\text{SAdm}}$ ), and hence the robot can guarantee no violation to  $\varphi$  thereafter. Next, the robot moves  $B_1$  to  $L_2$ . This ensures that a path always exists to a goal state. From here on, the robot seeks an opportunity to complete the task. We note that  $\sigma_{\text{HAdm}}(s_0)$  also includes the action of going to  $B_0$ , which is optimistic that the human player will move  $B_1$  to  $L_8$ .

This case study shows that **Adm-Rat** strategies result in reasonable (and arguably human-like) behaviors when winning strategies do not exist. Specifically, **Adm-Rat** is most effective when a **SAdm** strategy exists. However, if **Adm-Rat** must resort to **HAdm** due to the absence of a **SAdm** strategy, overly optimistic behaviors may occur.

**Tic-Tac-Toe (TTT) from [33]:** Unlike the manipulator domain, TTT is naturally competitive. Additionally, TTT’s game arena is a tree-like structure. Thus, the **Sys** player cannot exploit cycles to “stay” in the pending region. We consider two specifications: “eventually win” ( $\varphi_1 = F \text{ win}$ ), and “eventually win or draw” ( $\varphi_2 = F(\text{win} \vee \text{draw})$ ).

$\varphi_1 = F \text{ win}$ . For this task, no winning strategy exists from  $s_0$ . Although Safe strategy exists, **SAdm** does not exist from any state since a “draw” state is a violation of  $\varphi_1$ . But,  $\sigma_{\text{HAdm}}$  from  $s_0$  exists. Fig. 4(a) is the game status when **Sys** goes first and (b) shows  $\sigma_{\text{HAdm}}$  actions. Under  $\sigma_{\text{HAdm}}$ , **Sys** is choosing **WCO** while being hopeful - the robot markers are *strategically* placed to win the game. The aVal of all states reachable under  $\sigma_{\text{HAdm}}$  in the hopeful game are finite, and thus  $\sigma$  is **WCO** and taking longer paths which eventually end in “draw” states. When **Env** goes first (Fig. 4(d)), **Sys** is overly optimistic. The aVal of some states reachable under

$\sigma_{\text{HAdm}}$  is  $\infty$ . Thus, for states with  $\text{aVal}^s = \infty$ ,  $\sigma_{\text{HAdm}}$  choose actions indifferently. In 4(e) Env places a marker in the middle. For Sys, almost all available moves are HAdm and cannot enforce reaching “win”. Thus, Sys is optimistic and hopes the Env will make a mistake.

$\varphi_2 = F(\text{win} \vee \text{draw})$ . For this task, a  $\sigma_{\text{WCo-Op}}$  strategy exists from the initial state. The Sys player under WCo-Op can always enforce at least a “draw” and particularly chooses actions that could win (Co-Op) as quickly as possible (Fig. 4(c)). A Winning strategy ensures always drawing and does not strategically choose to reach “win” states as these are not Co-Op. Thus, in Fig. 4(c), Sys does not place marker on the bottom right. When Env player goes first (Fig. 4(d)), Sys is trying to force Env to make a mistake so as to enforce drawing while the Co-Op attribute will try to win if possible. In Fig. 4(f), Sys places its marker in one of the corners. This allows room for error for Env. Thus, if Env makes mistakes, Sys wins the game else the game draws. The choice of the specification dictates what types of behavior Sys player exhibits and the level of optimism.

Specifications that are too strict ( $\varphi_1$ ) may exhibit either strategic or too optimistic behaviors. More flexible specifications ( $\varphi_2$ ) can lead to more balanced and realistic strategies. Thus, Adm-Rat captures nuanced strategic behaviors, such as forcing opponents into mistakes or playing defensively while looking for opportunities to win.

## VI. CONCLUSION

This paper explores what the robot should do when a winning strategy does not exist. We define admissibly rational strategies and provide algorithms for synthesis. Unlike the prior approach, synthesis can be reduced to classical fixed-point based computation. We discuss the emergent behavior in competitive and non-competitive games. Future work should explore mitigating optimism for the robot in the pending regions and explore symbolic variants for faster computation.

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