Big Data Asset Pricing

Exercise 1: Beta-Dollar Neutral Portfolio Construction

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Statement: I certify with my signature that I have solved the exercise according to the Code of Professional Conduct and Ethics. For example, I have not plagiarized others, but, instead, solved the exercise myself (possibly with allowed collaboration with other students), and I have referenced my sources appropriately.

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The optimization problem is as follows:

$$\min_{x} \quad (x-y)^{'}(x-y)$$
$$s.t. \quad x^{'}\vec{1} = 0$$
$$x^{'}\beta = 0$$

where x is the vector of weights of the portfolio, y is the vector of weights of the benchmark. This problem would modify the benchmark portfolio in a way that the new portfolio is dollar and beta neutral. The first constraint is the dollar neutrality constraint and the second constraint is the beta neutrality constraint.

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$$\min_{x} \quad (x-y)^{'}(x-y)$$
$$s.t. \quad x^{'}B = 0$$

$$\Rightarrow \mathcal{L} = x^{'}x - x^{'}y - y^{'}x + y^{'}y + x^{'}B\lambda^{'}$$
$$F.O.C \Rightarrow 2x - y - y - B\lambda^{'} = 0 \Rightarrow x = y + \frac{1}{2}B\lambda^{'}$$

Now we can plug the solution for x in the constraint and solve for λ:

$$x^{'}B = 0$$
$$\Rightarrow (y + \frac{1}{2}B\lambda^{'})^{'}B = 0$$
$$\Rightarrow y^{'}B + \frac{1}{2}\lambda B^{'}B = 0$$
$$\Rightarrow \lambda = -2y^{'}B(B^{'}B)^{-1}$$
$$\Rightarrow x = y - B(BB^{'})^{-1}B^{'}y$$