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Informed Trading, Investment, and Welfare*

I.   Introduction

Our objective in this article is to set out a framework that allows an analytically rigorous discussion of the costs and benefits of stock market speculation by privately informed traders. In particular, we focus on the role of security prices in aggregating information and thereby influencing the allocation of risk, as well as the allocation of investment resources in the real sector. The effects studied in this article have been identified in the existing literature but have been analyzed individually, using a variety of different models. Our intended contribution is not to point out previously unknown effects but to carry out a complete welfare analysis in a single canonical model.

The standard framework for analyzing economic welfare is the Walrasian model with full information (the definitive treatment of full-information welfare economics is Graaff 1957). The competitive rational expectations equilibrium (REE) model is the generalization of this paradigm to asymmetric information. In order to prevent private information from being fully revealed in equilibrium, noise is added to the

This article studies the welfare economics of informed stock market trading. We analyze the effect of more informative prices on investment, given that this dependence will itself be reflected in equilibrium prices. While a higher incidence of informed speculation always increases firm value through a more informative trading process, the effect on agents' welfare depends on how revelation of information changes risk-sharing opportunities in the market. Greater revelation of information that agents wish to insure against reduces their hedging opportunities. On the other hand, early revelation of information that is uncorrelated with hedging needs allows agents to construct better hedges.

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model, for example, in the form of a random disturbance to the economy's excess demand, as in Grossman and Stiglitz (1980). The noise is usually interpreted as resulting from unmodeled, uninformed traders ("noise traders") trading for liquidity motives. This makes it impossible to work out a standard economic welfare analysis based on agents' utilities. Hence, although the noisy REE model is the natural extension of the Walrasian model to describe financial markets, unlike the Walrasian model, it has not been developed as a tool for welfare analysis.

While some papers have explicitly modeled the motives of noise traders, they are either not concerned with welfare issues or are restricted to settings in which information has no social value. The latter is true more generally of both the noisy rational expectations literature and the literature on market microstructure. Holmström and Tirole (1993) describe the situation thus: "There is a vast literature in finance devoted to the analysis of information flows in stock markets, including how completely and how fast information is incorporated into prices. But in almost no model is information socially useful" (p. 681). In contrast, in our model there are two possible benefits of information arising from two modifications to the standard setup. The first comes from the way we endogenize noise trading. We introduce a shock to traders' endowments that gives them a risk-sharing motive for trade as well as preventing the price from being fully revealing. This allows us to study the conditions under which more information is beneficial or harmful to agents who trade to insure their endowment shock. The effect of more informative prices on risk sharing depends on the correlation of the information that is revealed with agents' hedging needs. In general, an asset that is used as a hedging instrument will not be perfectly correlated with the initial risk exposure (in this case, the agents' endowments), that is, its value at the end of the holding period will be influenced by other risk factors. Early resolution of this extraneous uncertainty will help traders who want to hold the asset as a hedging instrument. We call this the "spanning effect." On the other hand, if speculators have information that is highly correlated with the hedgers' endowment shock and this becomes incorporated into the price, then hedgers will be worse off. This is the Hirshleifer effect: one cannot hedge a risk when the market price already reflects the realization of that risk (Hirshleifer 1971). In previous literature, the device of replacing a shock to asset demand by a shock to endowments was introduced by Diamond and Verrecchia (1981). The particular variant employed in this article can be found in Marín and Rahi (1999, 2000), who also discuss the Hirshleifer effect.

The second modification we make to the REE framework is to allow asset prices to affect corporate investment decisions. The setting we have in mind is that of the secondary market for shares, where outside investors possess private information that is pertinent to firms' investment policies. While an individual investor may have only a small piece of information about production possibilities beyond what managers already know, the pooled information of outside investors may be substantial. Hence, there is an important

role for stock prices in aggregating dispersed information that is useful for making production decisions. Indeed, it is this mechanism through which financial markets direct resource allocation elsewhere in the economy.

While the idea of stock prices influencing investment is straightforward, it introduces the complication that the prices themselves must reflect the changes in investment that are made in response to information revealed by them. This is the "feedback effect" described by Bresnahan, Milgrom, and Paul (1992, p. 213, n. 16): "We assume . . . there are no tricky gaming issues between management and the outsider traders. Suppose, for example, that the manager will withdraw the project if the stock market reaction is adequately adverse. Then the value of the security reflects this prospect." In this article, we model the feedback effect in equilibrium. Stock prices guide investment, and this dependence is incorporated into the equilibrium price formation process.

Several papers in the literature have been concerned with the effect on investment of contemporaneously determined stock prices. Leland's (1992) model of insider trading has the feature that production is responsive to the stock price, but the firm does not make any inferences from the price (in the usual sense of rational expectations). Henrotte (1992) analyzes the impact of security prices on a firm's output decisions, in the spirit of our feedback effect. However, his security is a futures contract on the firm's output, and hence, a change in firm value does not directly affect the security value. Boot and Thakor (1997) and Dow and Gorton (1997) do model the feedback effect fully. But the presence of exogenous liquidity traders in these models precludes a complete welfare analysis.

In Section II we set out a general model of a security market with agents who trade for informational and hedging motives. Apart from the feedback effect, this is a competitive rational expectations equilibrium model with a mass of risk-neutral uninformed agents. In other words, it differs from standard REE models such as Diamond and Verrecchia (1981) merely by the addition of risk-neutral uninformed agents who are analogous to the market maker in Kyle (1985). Likewise, it differs from Kyle-type microstructure models only insofar as it allows limit orders, as opposed to market orders, and assumes competitive price-taking behavior.[1] In Section III we consider a parametric specification of the model with one type of informed agent and two types of hedgers, with the equilibrium computed in Section IV. The feedback effect is analyzed in Section V. In Section VI we derive equilibrium ex ante expected utilities (proposition 3) and present comparative statics results (proposition 4). Section VII concludes. All proofs are in the appendix.

1. This article is about traders with superior private information and not specifically about insider traders. We interpret the term *private information* broadly to include any situation where an analyst or fund manager has a better insight than the market into general economic conditions, prospects for the interest rate term structure, the financial situation of an individual company, etc. In this context it is natural to think of informed traders as "informationally small." Hence, the assumption of competitive behavior.

II.   A General Model

We consider a firm, the value of whose productive assets is given by

$$v = f(k, y),$$

where k represents the level of investment and y is a random variable affecting profitability. The firm is managed so as to maximize its expected value.[2] We normalize the number of outstanding shares to one. In addition to these shares, a riskless bond is available for trade, which we take to be the numeraire, normalizing the interest rate to zero. There are n agents who trade to exploit superior information or to hedge their risk exposures. There is also a risk-neutral uninformed agent, agent 0, who can be thought of as a market maker. All agents are competitive price takers (i.e., each should be interpreted as a continuum of infinitesimal traders). Agent i ($i = 1, \ldots, n$) has a von Neumann-Morgenstern utility function U_i and a stochastic endowment e_i. He privately observes (s_i, x_i), where the signal s_i is correlated with the firm's profitability parameter y, and x_i, which is independent of s_i, parameterizes the agent's risk exposure to a random variable z. Taking an asset position t_i at the market price p leaves him with terminal wealth

$$w_i = e_i(x_i, z) + t_i(v - p). \tag{1}$$

Definition 1.   A rational expectations equilibrium is a price function $p(s_1, \ldots, s_n, x_1, \ldots, x_n)$, order flow $t(s_1, \ldots, s_n, x_1, \ldots, x_n)$, a trade t_i for each agent $i = 1, \ldots, n$, and an investment level k, such that

$$a) \quad t_i \in \arg\ \max E[U_i(w_i)|s_i, x_i, p, t], \quad (i = 1, \ldots, n),$$

$$b) \quad t = \sum_{i=1}^{n} t_i,$$

$$c) \quad p = E(v|p, t),$$

and

$$d) \quad k \in \arg\ \max E(v|p, t).$$

Agents know the price and order flow functions and learn from their observation of prices and order flows.[3] In particular, the firm is guided in its investment decisions by the information aggregated and conveyed by prices. Simultaneously, the price itself reflects this dependence. Since the market

2. This can be rationalized by assuming that the firm is originally owned by risk-neutral (uninformed) shareholders. This avoids the well-known problems of defining the objective of a firm in an incomplete markets economy.

3. Note that we allow uninformed traders to make inferences from the order flow in addition to the equilibrium price. As will be seen below in our discussion of the parametric model, this assumption is made for technical reasons to retain linearity in the CARA/Normal setting in the presence of the feedback effect.

maker is risk neutral and competitive, he determines the price through condition c and absorbs the aggregate trade of the other agents. This ensures market clearing.

One property of our model is worth commenting on: in equilibrium the expected trading profits of an uninformed agent are zero. Agent i is uninformed if s_i and x_i are degenerate random variables, but he nevertheless learns from prices and the order flow. His expected trading profit is

$$E[t_i(v - p)] = E[E(t_i(v - p)|p, t)]$$

$$= E[t_i(E(v|p, t) - p)]$$

$$= 0,$$

where we have used condition c in definition 1 and the fact that t_i is (p, t)-measurable. Note that in general t_i will be nonzero (as can be seen in the parametric model below).

It is interesting to contrast this "no-loss" result to results on the equilibrium profits of uninformed agents in the Kyle model. In Kyle (1985), noise traders lose money on average. In the variant proposed by Spiegel and Subrahmanyam (1992), noise traders are replaced by rational uninformed hedgers who also lose money on average. This feature of uninformed traders making losses has been identified with the presence of adverse selection in markets with asymmetrically informed agents. However, in our setting, which is very similar to the standard Kyle model, uninformed traders break even. The key difference is that in our model traders are allowed to use limit orders rather than market orders.[4] This means that traders can make inferences from prices and order flows and hence are at least as well informed as the market maker. Agents who have no private information about the asset value (degenerate s_i) but have some information on their own hedging needs (nondegenerate x_i) will in general be better informed than the market maker in equilibrium, once they combine their private information with the public signal (p, t). These agents may make positive or negative trading profits, depending on how strongly their hedging motive conflicts with their speculative motive. On the other hand, agents with degenerate signals (s_i, x_i) have exactly the same information as the market maker. As we have shown, these agents face actuarially fair prices, given their information, and just break even.

This does not mean that adverse selection is absent in our model, simply that losses suffered by uninformed agents are not a necessary consequence of adverse selection. Consider, for example, the case where $n = 1$ and both s_1 and x_1 are nondegenerate. In equilibrium, agent 1 is subject to adverse selection, since the market maker cannot distinguish between s_1 and x_1. Effectively, the different "types" of agent 1 are pooled together: the market maker cannot tell the difference between the agent wanting to buy because

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4. Note that the no-loss result does not depend on whether informed traders behave strategically or are competitive.

he received a good signal about the asset value and the agent wanting to buy because of a large hedging need. This adverse selection clearly remains if we introduce another trader, agent 2, with degenerate signals (s_2, x_2), even though this is an uninformed agent who makes no losses in equilibrium. In order to carry out a complete welfare analysis that includes both the feedback effect of stock prices on investment and the effect of asymmetric information on hedgers' utilities, we now study a parametric version of the model.

III.   A Parametric Model

In this section we consider specific forms for the functions and random variables of the model just described. The value of the firm is given by

$$ v = ky - \frac{c}{2}k^2, \tag{2} $$

where y denotes profitability per unit of investment, and c is a (positive) investment cost parameter. All traders are infinitesimal price-taking agents. There is a measure $q_S \in (0, \infty)$ of identical privately informed speculators who observe a signal s that is correlated with y. A speculator has no endowment. Taking an asset position t_S at the market price p leaves him with terminal wealth

$$ w_S = t_S(v - p). $$

In addition, there are two types of hedgers who are exposed to the random variable z. The risk exposure of a hedger of type 1 is itself random: his initial endowment is $e_1 = xz$ (where x is random). After privately observing x, he trades an amount t_1 that results in net wealth

$$ w_1 = xz + t_1(v - p). $$

A hedger of type 2 has a constant risk exposure with endowment $e_2 = z$ and trades t_2 to realize terminal wealth

$$ w_2 = z + t_2(v - p). $$

There is a measure $q_1 \in (0, \infty)$ of type 1 hedgers, and we normalize the mass of type 2 hedgers to be one. For convenience we will henceforth refer to an individual speculator as "the speculator" and likewise to a hedger of type i as "hedger i."

Agent i($i = S, 1, 2$) has constant absolute risk aversion r_i and has information $\mathcal{I}_i$, that is, $\mathcal{I}_S$ is the partition generated by observing (s, p, t), and similarly, $\mathcal{I}_1$ is induced by (x, p, t) and $\mathcal{I}_2$ by (p, t). All random variables are

joint normally distributed. Without loss of generality we can take $y = s + \epsilon$, where s is independent of ϵ. We assume that

$$
\begin{pmatrix} s \\ \epsilon \\ z \\ x \end{pmatrix} \sim N\left[0, \begin{pmatrix} V_s & 0 & V_{zs} & 0 \\ 0 & V_\epsilon & V_{z\epsilon} & 0 \\ V_{zs} & V_{z\epsilon} & V_z & 0 \\ 0 & 0 & 0 & V_x \end{pmatrix} \right].
$$

We use the following notational convention: for random variables g and h, $V_{gh} := \mathrm{Cov}\,(g, h)$. Also, ρ_{gh} denotes the correlation coefficient between g and h, and $\beta_{gh} := V_{gh}V_h^{-1}$ is the coefficient from the regression of g on h (the "beta" of g with respect to h).

In general, the risk z may be correlated with both s (the predictable component of y) and ϵ (the residual), and these correlations may be different. The magnitude of hedger 1's risk exposure, x, is independent of all other random variables. We assume that the covariance matrix above is positive definite, a necessary and sufficient condition for which is that all variances be strictly positive and $\rho_{zs}^2 + \rho_{z\epsilon}^2 < 1$. We also take V_{zs} to be nonnegative, which entails no loss of generality. Finally, to ensure that equilibria are not always fully revealing, we assume that $V_{z\epsilon}$ is nonzero.

As we shall see, the noise in this model that prevents equilibrium from being fully revealing arises from the trading of hedger 1. This agent trades a random amount that depends on his privately observed endowment shock x. The endowment shock could equally well be interpreted as a liquidity shock suffered by the agent resulting in a need to rebalance his portfolio. Unlike the usual noise-trader or liquidity-trader model, hedger 1 maximizes utility and makes inferences like any other rational trader.

The specification of hedger 1 and hedger 2 requires some comment. Why do we need both hedgers, and why is their risk exposure not symmetric? If we only had hedger 1, then, as we shall see below, in equilibrium there would be almost full revelation. The market maker would not be able to distinguish separately the trade of informed and uninformed, but each would be able to subtract his own demand from the total and infer the other's trade (and private information). Hence all traders, apart from the market maker, would be fully informed, and the equilibrium would be rather degenerate.

Given this, it would seem natural and more elegant to have two hedgers, both with different endowment shocks. However, the equilibrium for this model cannot be solved in closed form. Hence, the formulation we have chosen, which is the simplest one that admits a nondegenerate closed-form solution.

IV.   Equilibrium

We now proceed to compute the equilibrium. The market maker sets the price equal to his conditional expectation of the asset payoff given the order flow, that is, $p = E(v|p, t)$, where

$$t = q_s t_s + q_1 t_1 + t_2. \tag{3}$$

Agents observe the price and order flow. From this observation they can infer the firm's investment level k (k is (p, t)-measurable, since the firm has no private information). We see from equation (2) that

$$p = kE(s|p, t) - \frac{c}{2} k^2. \tag{4}$$

We look for a linear equilibrium where

$$E(s|p, t) = \lambda s + \mu x \tag{5}$$

for some parameters λ and μ that will be determined below. Note that it is clear from equations (4) and (5) that (provided λ and μ are both nonzero) the speculator and hedger 1 have the same information in equilibrium: $\mathcal{I}_s = \mathcal{I}_1$, which is the partition induced by knowing both s and x, while the firm and hedger 2 are unable to isolate s from x.

We can now apply the standard mean-variance certainty-equivalent analysis to the agent's optimization problem, since interim wealth is normally distributed conditional on his information. Agent i's expected utility is

$$E[-\exp(-r_i w_i)] = -E\Big[E[\exp(-r_i w_i)|\mathcal{I}_i]\Big]$$

$$= -E\left[\exp\left(-r_i\left[E(w_i|\mathcal{I}_i) - \frac{r_i}{2}\text{Var}(w_i|\mathcal{I}_i)\right]\right)\right]. \tag{6}$$

Let

$$\mathcal{E}_i := E(w_i|\mathcal{I}_i) - \frac{r_i}{2}\text{Var}(w_i|\mathcal{I}_i). \tag{7}$$

The agent's optimization problem reduces to choosing a position t_i to maximize $\mathcal{E}_i$, given his information. From the expression in equation (1) for w_i,

$$\mathcal{E}_i = E(e_i|\mathcal{I}_i) + t_i\Big[E(v|\mathcal{I}_i) - p\Big]$$

$$- \frac{r_i}{2}\Big[\text{Var}(e_i|\mathcal{I}_i) + t_i^2 \text{Var}(v|\mathcal{I}_i) + 2t_i \text{Cov}(v, e_i|\mathcal{I}_i)\Big]. \tag{8}$$

The optimal portfolio is therefore

$$t_i = \frac{E(v|\mathcal{I}_i) - p - r_i \operatorname{Cov}(v, e_i|\mathcal{I}_i)}{r_i \operatorname{Var}(v|\mathcal{I}_i)}. \tag{9}$$

PROPOSITION 1.　There exists a unique linear equilibrium. The price function is

$$p = \frac{1}{2c}(\lambda s + \mu x)^2,$$

the equilibrium investment is

$$k = \frac{1}{c}(\lambda s + \mu x),$$

the equilibrium holdings of the agents are given by

$$t_S = \frac{(1-\lambda)s - \mu x}{r_S k V_\epsilon},$$

$$t_1 = \frac{(1-\lambda)s - (\mu + r_1 V_{z\epsilon})x}{r_1 k V_\epsilon},$$

$$t_2 = -\frac{(1-\lambda)V_{zs} + V_{z\epsilon}}{k[(1-\lambda)V_s + V_\epsilon]},$$

and the order flow is

$$t = \frac{cq_1 V_{z\epsilon}^2 V_x}{q V_\epsilon V_s} - \frac{c[(1-\lambda)V_{zs} + V_{z\epsilon}]}{(\lambda s + \mu x)[(1-\lambda)V_s + V_\epsilon]},$$

where

$$\lambda = \frac{q^2 V_s}{V_{z\epsilon}^2 V_x + q^2 V_s},$$

$$\mu = -\frac{q V_{z\epsilon} V_s}{V_{z\epsilon}^2 V_x + q^2 V_s},$$

and

$$q := \frac{q_S r_S^{-1} + q_1 r_1^{-1}}{q_1}.$$

The numerator of q is the risk-tolerance-weighted average of the mass of traders who know the signal s (in equilibrium). It measures the intensity of informed trading. The mass of "random" hedgers q_1, on the other hand, is a measure of the intensity of "noise trading." Thus, q is the signal to noise ratio. Indeed, $\left|\frac{\lambda}{\mu}\right|$ is strictly increasing in q: a higher relative intensity of informed

trading makes the price more revealing. It is worth noting that λ is also strictly increasing in q and induces a bijection from $(0, \infty)$ to $(0, 1)$. This allows us to work with λ or q interchangeably—as q goes from zero to infinity, or equivalently λ goes from zero to one, the equilibrium goes from completely nonrevealing to fully revealing.

In this equilibrium, uninformed agents can infer $(\lambda s + \mu x)$ from the price and the order flow. If they were not allowed to condition on the order flow, they would be able to infer only the absolute value of $(\lambda s + \mu x)$ from the price, and linearity of the solution would be lost. This feature of the model arises from the feedback effect. Alternatively, we could assume that agents condition only on prices if there is an additional piece of public information that reveals the sign of $(\lambda s + \mu x)$.

V.   The Feedback Effect

From proposition 1 we see that the level of investment is more responsive to the share price the lower is the adjustment cost (measured by the parameter c). This feeds back into the equilibrium share price. The lower is c, the stronger is the feedback effect. We can easily calculate the equilibrium volatility of investment as well as the mean and variance of the share price.

PROPOSITION 2.   In equilibrium, the variance of the level of investment is

$$\text{Var}(k) = \frac{\lambda V_s}{c},$$

the mean and variance of the share price are, respectively,

$$E(p) = \frac{\lambda V_s}{2c}$$

and

$$\text{Var}(p) = \frac{\lambda^2 V_s^2}{2c^2},$$

and the expected value of the firm is

$$E(v) = \frac{\lambda V_s}{2c}.$$

Note that the expected value of the firm is equal to the expected share price (since $E(v) = E[E(v|p, t)] = E(p)$). With a greater intensity of informed trading and/or a lower cost of investment, both the average share price and the volatility of the share price are higher. Investment also is more volatile. The increased volatility here is beneficial from the point of view of the firm. It reflects a more efficient price that leads to a better corporate investment policy.

VI.   Welfare Analysis

We measure the welfare of agents in equilibrium in terms of their certainty-equivalent wealth. We denote this by U_i for agent i, and for convenience we refer to it as the agent's "payoff":

$$U_i := -\frac{1}{r_i}\ln\left[-EU_i(w_i)\right]$$

$$= -\frac{1}{r_i}\ln\left[E[\exp(-r_i w_i)]\right], \tag{10}$$

where expectations are taken over the ex ante distribution of wealth in equilibrium. Notice that, for agents S and 1, wealth is not normally distributed ex ante, and therefore certainty-equivalent wealth cannot be computed by the usual mean-variance formula.[5] In the expression for agent i's terminal wealth,

$$w_i = t_i(v - p) + e_i(x_i, z),$$

t_i is the ratio of two normals (for agents S and 1), v and p are both the product of two normals, while e_i is either zero (in the case of agent S) or the product of two normals (agent 1).

PROPOSITION 3.   The payoffs of the agents are

$$U_S = \frac{1}{2r_S}\ln\left[1 + (1 - \lambda)V_s V_\epsilon^{-1}\right]$$

$$U_1 = \frac{1}{2r_1}\ln\left[(1 - r_1^2 V_x V_z)[1 + (1 - \lambda)^2 V_s V_\epsilon^{-1}] \right.$$

$$\left. + (\mu + r_1[(1 - \lambda)V_{zs} + V_{z\epsilon}])^2 V_x V_\epsilon^{-1}\right]$$

$$U_2 = \frac{r_2}{2}\left[\frac{[(1 - \lambda)V_{zs} + V_{z\epsilon}]^2}{(1 - \lambda)V_s + V_\epsilon} - V_z\right].$$

We now wish to assess the welfare impact of changing q, the relative intensity of informed trading.

PROPOSITION 4.   The speculator's payoff U_S is decreasing, with respect to q, while the uninformed hedger's payoff U_2 is (*a*) decreasing if and only if $|\beta_{zs} - \beta_{z\epsilon}| \le \beta_{zs}$, (*b*) increasing if and only if $|\beta_{zs} - \beta_{z\epsilon}| \ge V_y V_\epsilon^{-1}\beta_{zs}$, and (*c*) strictly convex and attains a minimum if and only if $\beta_{zs} < |\beta_{zs} - \beta_{z\epsilon}| < V_y V_\epsilon^{-1}\beta_{zs}$.

Here we use the terms increasing and decreasing in the strict sense. Since λ is increasing in q, the statement regarding the speculator's payoff is immediate from proposition 3. The interpretation is straightforward: an individual

5. The welfare analysis in Leland (1992) is therefore inconsistent with expected utility: there is no von Neumann Morgenstern utility function that yields an objective that is linear in mean and variance of wealth for arbitrary distributions of wealth.

speculator's payoff U_S is decreasing in q, since a more revealing trading process means less favorable opportunities for speculative profit.

The comparative statics for the uninformed hedger are more subtle. Recall that β_{gh} is the regression coefficient from the regression of g on h. Whether hedger 2 prefers to be less or more informed in equilibrium depends on the relative size of the two betas, β_{zs} and $\beta_{z\epsilon}$. A bigger β_{zs} means a stronger Hirshleifer effect: observing a signal that is highly informative about endowments reduces risk-sharing opportunities in the market. On the other hand, the bigger is the magnitude of $\beta_{z\epsilon}$, the more desirable it is to obtain a good estimate of s so that the endowment risk associated with ϵ can be hedged more effectively. If $\beta_{z\epsilon}$ is very small relative to β_{zs} (case a), the Hirshleifer effect dominates and the hedger is worse off as informed trading increases and more information is revealed by the market. In case b the opposite is true: the hedger prefers more revelation to less, since the speculator's information resolves a lot of uncertainty regarding the asset payoff and not much regarding the endowment. In the intermediate case c, the hedger prefers the equilibrium to be either fully revealing or not revealing at all.

It has been observed that the typical daily pattern of trading volume in financial markets is U-shaped, with heavy trading in the morning and late afternoon and relatively little activity in the middle of the day. This is consistent with case c above: if prices are more revealing as the trading day progresses, uninformed hedgers would prefer to trade either at the open or the close.

VII.   Conclusions

In this article, we have presented a general model of a security market with agents who trade for informational and hedging motives. The model also incorporates the feedback effect of investment policy (as a function of the price) back onto price formation.

To analyze the welfare effects of informed trading, we use a parametric model where all agents are rational utility maximizers, and we compute explicit closed-form solutions for their equilibrium utility levels. We obtain a continuous parameterization of equilibrium with respect to the intensity of informed trading. A greater degree of informed trading reduces the returns to speculation. The effect on uninformed hedgers is not unambiguous: it depends on the whether the information being revealed is primarily about endowment risk (the Hirshleifer effect) or about extraneous uncertainty in the asset payoff (the spanning effect). From the point of view of investment efficiency, more informed trading is always beneficial, even though it entails higher volatility of the share price and of investment.

Appendix

LEMMA 1.   Suppose **A** is a symmetric $m \times m$ matrix, **b** is an m-vector, d is a scalar, and **w** is an m-dimensional normal variate: $\mathbf{w} \sim N(0, \Sigma)$, Σ positive definite. Then $E[\exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + d)]$ is well defined if and only if $(\mathbf{I} - 2\Sigma\mathbf{A})$ is positive definite and

$$E[\exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + d)] = |\mathbf{I} - 2\Sigma\mathbf{A}|^{-1/2} \exp\left[\frac{1}{2}\mathbf{b}^\top(\mathbf{I} - 2\Sigma\mathbf{A})^{-1}\Sigma\mathbf{b} + d\right].$$

This is a standard formula that we will use to compute ex ante expected utilities. For a proof see, for example, Marín and Rahi (1999).

Proof of proposition 1.   The firm solves the problem

$$\max_{k \in IR} \quad kE(s|p, t) - \frac{c}{2}k^2,$$

giving $k = c^{-1}E(s|p, t) = c^{-1}(\lambda s + \mu x)$, using (4). Also, from equations (4) and (5),

$$p = k(\lambda s + \mu x) - \frac{c}{2}k^2.$$

By substituting in the equilibrium k we obtain the desired expression for the price function.

For the speculator, using equations (2) and (9) and standard properties of the normal distribution (see, e.g., Anderson 1984), we get

$$t_S = \frac{E(v|s) - p}{r_S \mathrm{Var}(v|s)}$$

$$= \frac{ks - \frac{c}{2}k^2 - [k(\lambda s + \mu x) - \frac{c}{2}k^2]}{r_S k^2 V_\epsilon}$$

$$= \frac{(1 - \lambda)s - \mu x}{r_S k V_\epsilon}.$$

Similarly, for the hedgers,

$$t_1 = \frac{(1 - \lambda)s - (\mu + r_1 V_{z\epsilon})x}{r_1 k V_\epsilon},$$

$$t_2 = -\frac{\mathrm{Cov}(z, s|p, t) + V_{z\epsilon}}{k(\mathrm{Var}(s|p, t) + V_\epsilon)}. \tag{A1}$$

Substituting into equation (3), we can write the aggregate order flow as

$$t = t_2 + \frac{q_1}{k V_\epsilon}\tau,$$

where

$$\tau := q(1 - \lambda)s - (q\mu + V_{z\epsilon})x$$

and q is as defined in the statement of the proposition. We proceed under the assumption that observing prices and the order flow is equivalent to observing $(\lambda s + \mu x)$. As we shall see, this will turn out to be true in equilibrium. Then

$$E(s|p, t) = E(s|\lambda s + \mu x)$$

$$= \frac{\lambda V_s}{\lambda^2 V_s + \mu^2 V_x}(\lambda s + \mu x).$$

It follows from equation (5) that

$$\lambda^2 V_s + \mu^2 V_x = \lambda V_s. \qquad (A2)$$

We conjecture that τ is proportional to $(\lambda s + \mu x)$. Then

$$\frac{\lambda}{\mu} = -\frac{q(1 - \lambda)}{q\mu + V_{z\epsilon}}.$$

Cross-multiplying and simplifying, we get

$$\frac{\lambda}{\mu} = -\frac{q}{V_{z\epsilon}}. \qquad (A3)$$

Equations (A2) and (A3) can now be solved for λ and μ.

The conditional moments for hedger 2, who observes only the price and order flow, are equivalent to the moments conditional on $(\lambda s + \mu x)$. Using the standard properties of the normal distribution, together with equation (A2), we get:

$$\mathrm{Var}(s|p, t) = (1 - \lambda)V_s \qquad (A4)$$

$$\mathrm{Cov}(z, s \mid p, t) = (1 - \lambda)V_{zs}. \qquad (A5)$$

Substituting into equation (A1), we obtain the desired formula for t_2. The equilibrium order flow can now be readily computed. Q.E.D.

Proof of proposition 2.   The variance of k is immediate from the expression for k in proposition 1 and equation (A2). From the moment-generating function of the normal distribution, if $X \sim N(0, \sigma^2)$, then $E(X^2) = \sigma^2$ and $\mathrm{Var}(X^2) = 2\sigma^4$. Now we obtain the mean and variance of the share price by using the expression for the price function from proposition 1 and equation (A2). Finally, note that $E(v) = E[E(v|p, t)] = E(p)$. Q.E.D.

Proof of proposition 3.   From equations (6), (7), and (10),

$$U_i = -\frac{1}{r_i}\ln\big[E[\exp(-r_i\varepsilon_i)]\big].$$

Using equations (8) and (9), in equilibrium,

$$\mathcal{E}_i = E(e_i|\mathcal{I}_i) - \frac{r_i}{2}\text{Var}(e_i|\mathcal{I}_i) + t_i\Big[E(v|\mathcal{I}_i) - p - r_i\text{Cov}(v, e_i|\mathcal{I}_i)\Big]$$

$$- \frac{r_i}{2}t_i^2\text{Var}(v|\mathcal{I}_i) \tag{A6}$$

$$= E(e_i|\mathcal{I}_i) - \frac{r_i}{2}\text{Var}(e_i|\mathcal{I}_i) + \frac{r_i}{2}t_i^2\text{Var}(v|\mathcal{I}_i).$$

Setting $e_i = 0$ in equation (A6), substituting for the equilibrium holding of the speculator from proposition 1, and using lemma 1, we obtain

$$U_S = \frac{1}{2r_S}\ln\Big[1 + V_\epsilon^{-1}[(1 - \lambda)^2V_s + \mu^2V_x]\Big].$$

The formula for the speculator's payoff follows from equation (A2). Noting that hedger 1 has the same information in equilibrium as the speculator, we can derive U_1 by using equation (A6), proposition 1, and lemma 1. For hedger 2,

$$\mathcal{E}_2 = E(z|p, t) - \frac{r_2}{2}\text{Var}(z|p, t) + \frac{r_2}{2}t_2^2\text{Var}(v|p, t).$$

Analogous to equations (A4) and (A5), we get

$$E(z|p, t) = \frac{V_{zs}}{V_s}[\lambda s + \mu x]$$

$$\text{Var}(z|p, t) = V_z - \frac{V_{zs}^2}{V_s}\lambda$$

$$\text{Var}(v|p, t) = k^2[(1 - \lambda)V_s + V_\epsilon].$$

Substituting into the above expression for $\mathcal{E}_2$ and using proposition 1 and lemma 1, we obtain the formula for U_2. Q.E.D.

   Proof of proposition 4.   Note that λ is increasing in q. The comparative statics for $E(v)$ and U_s are immediate from proposition 3. From the expression for U_2 we see that if $V_{zs} = 0$, U_2 is increasing in q. This case is covered by item c in the proposition. Henceforth, we restrict V_{zs} to be strictly positive (note our convention that $V_{zs} \geq 0$). Differentiating U_2 with respect to λ, we obtain two critical points:

$$\lambda^* = 1 + \frac{V_\epsilon}{V_s}\frac{\beta_{z\epsilon}}{\beta_{zs}}$$

and

$$\lambda^{**} = 1 + \frac{V_\epsilon}{V_s}\Big(2 - \frac{\beta_{z\epsilon}}{\beta_{zs}}\Big).$$

Also we see that

$$\mathrm{sgn}\ \left[\frac{\partial^2 U_2}{(\partial\lambda)^2}\right]_{\lambda=\lambda^*} = -\mathrm{sgn}\ \left[\frac{\partial^2 U_2}{(\partial\lambda)^2}\right]_{\lambda=\lambda^{**}} = \mathrm{sgn}\ (\beta_{zs}-\beta_{z\epsilon}).$$

The comparative statics for U_2 can now be verified by considering each case in turn and restricting λ to the unit interval $(0,1)$. Q.E.D.

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