Informed Speculation and Hedging in a Noncompetitive Securities Market

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We examine an adverse selection model of trading in which both informed and uninformed traders are rational, maximizing agents. Replacing the price inelastic "noise" or "liquidity" traders with strategic, utility-maximizing bedgers permits an explicit analysis of the uninformed traders' welfare, and demonstrates that several comparative statics obtained from the standard paradigm of Kyle (1984, 1985) are altered significantly upon endogenizing the trading motives of these agents. In contrast to extant models, market liquidity and price efficiency are both nonmonotonic in the number of uninformed bedgers in the market. Also, the welfare of bedgers monotonically decreases with the number of informed traders, despite greater competition between the informed.

The applied literature on financial markets with asymmetric information has made impressive advances in recent years. In no small measure, this is due to the model of Kyle (1984, 1985), which has now become a standard framework for analyzing strategic noisy rational expectations markets. Variants of this paradigm have been used to analyze a wide variety of issues, to derive comparative statics regarding market

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liquidity and the informational efficiency of prices, and to obtain implications for financial market regulation. While this model is both insightful and a powerful analytical tool, it does possess the shortcoming that "noise" or "liquidity" traders are assumed to trade exogenous quantities without regard to other parameters in the environment, and therefore to consistently suffer losses to traders with superior information. This assumption simplifies the analysis and prevents prices from becoming fully revealing, in addition to sustaining trade in a market with asymmetric information. The exogeneity associated with "liquidity" traders is also a characteristic of most other extant adverse selection models of trading [notable examples being Glosten and Milgrom (1985) and Easley and O'Hara (1987)]. Two substantive questions arise: (i) Given that the extensive applied research uses comparative statics obtained from these models, how robust is the analysis to endogenizing the motives of these noise or liquidity traders? (ii) What is the effect of changes in the economy's parameters on the welfare of these agents? The goal of this article is to address the above issues in the context of a model whose structure is similar to that of Kyle (1984), except that all traders act as maximizing agents. The analysis demonstrates that many conclusions in the theoretical microstructure literature are altered significantly upon endogenizing the trading motives of the noise traders.

The need to hedge endowment shocks provides a motive for risk-averse agents to trade, even when the market contains disparately informed individuals.² From a real-world perspective, the trades of large financial institutions account for a considerable proportion of the trading volume.³ In turn, the size of these trades makes them a significant source of short-run price movements. One can argue that, commonly, institutional trading does not arise from private information about specific securities. However, it is reasonable to expect these institutions to behave strategically (and in a utility-maximizing fashion), as opposed to the price-inelastic behavior of noise traders. To the extent that hedging significantly motivates the trades of these large institutions, it then becomes justifiable to model uninformed agents as strategic hedgers.

Applications and extensions of the Kyle (1984, 1985) model include Admati and Pfleiderer (1988a, 1988b), Foster and Viswanathan (1990, 1991), Caballé (1989), Caballé and Krishnan (1989), Bhushan (1991), Fishman and Hagerty (1989, 1992), Subrahmanyam (1991a, 1991b), Holden and Subrahmanyam (1992), Chowdhry and Nanda (1991), Paul (1991), and Kumar and Seppi (1990, 1991). For a discussion of regulatory issues based on the Kyle (1984, 1985) paradigm, see Kyle (1989) and Lindsey (1990).

² This is exactly the "liquidity" motive of the single trader in Glosten (1989). Hedging is also the trading motive of the uninformed "liquidity" traders in Grossman and Miller's (1988) model, which does not allow for asymmetric information.

³ See, for example, New York Stock Exchange (1990), p. 13.

With the above observations in mind, we replace the noise traders with risk-averse, strategic agents, who trade to hedge their endowment risk. We analyze the effect of the hedgers' trading strategies on market liquidity, informational efficiency, and the expected profits of informed agents. In addition, since the uninformed traders act as fully maximizing agents, we are able to explicitly study their welfare. Our model is thus characterized by three classes of agents. First, there are several risk-neutral informed traders with diverse information. Second, there is a market-maker who sets the price equal to the security's expected value, given the available public information and the order flow. Third, there are risk-averse traders endowed with nonzero levels of the asset, who enter the market for pure risk-sharing reasons. The model allows us to formulate a closed-form solution for the unique linear equilibrium, whose parameters are expressed in terms of the hedgers' risk aversion, their number, and the ex ante variability of their endowments. We show that a linear equilibrium exists only if the value received by the uninformed from hedging is sufficiently large, in contrast to the exogenous noise-trading literature, in which a linear equilibrium always exists so long as there is some finite amount of noise trading.4

Unlike existing models with exogenous noise trading, market liquidity in the present model may be nonmonotonic in the number of uninformed traders in the market. Specifically, when the risk aversion of the hedgers is large, adding an uninformed hedger to the market can, counterintuitively, reduce market liquidity. This occurs because the increased price variability due to the presence of a greater number of hedgers causes the hedgers to scale back their trading activity. As a result of the above effect, the *welfare* of each uninformed hedger may also be decreasing in the total number of hedgers in the market. This result contrasts with the positive relationship between noise traders' welfare (i.e., the inverse of their expected losses) and the amount of noise trading in extant models.

As the number of informed traders grows without bound in Kyletype models, market liquidity also becomes unboundedly large, as a result of increasing competition between informed traders. In the linear equilibrium of the present model, however, hedgers scale back their trades in response to an increase in the number of informed traders, so that increasing the number of informed traders toward some finite upper bound causes the market to become infinitely *illiq*-

⁴ Glosten (1989) and Bhattacharya and Spiegel (1991) have shown that if the degree of asymmetric information is too high, markets with maximizing agents may fail to produce *any* equilibria. We are, however, only able to show the nonexistence of the linear equilibrium in this article.

uid, and increasing the number of informed agents further causes the linear equilibrium to disappear.⁵

We also find that the welfare of the uninformed hedgers monotonically decreases in the number of informed traders, despite greater competition between the informed traders. This result obtains because the effect of increased price variability, because of the presence of a larger number of informed agents, always dominates the benefit of increased competition between the informed agents. Coupled with another result of the model that market liquidity may be nonmonotonic in the number of informed traders, this implies that using market liquidity as the sole measure of the well-being of market participants may be inappropriate.

In related literature, Laffont and Maskin (1990) and Bhattacharya and Spiegel (1991) also develop noncompetitive models of trading on private information in which all agents maximize a specific objective function. While every player in these models is a maximizing agent, they only analyze the case in which there is one strategic informed participant and a continuum of competitive uninformed participants. Our model allows for *multiple* informed and uninformed agents and both classes trade strategically. Also, these models use a Walrasian framework, making it difficult to determine whether the results differ from the rest of the literature because all agents are utility maximizers or because of the differences in the market-clearing mechanism. Glosten (1989) uses a market-maker framework without noise traders, but his model also has only one informed trader, and therefore cannot be used to address many of the issues analyzed in the present work.

In a recent paper, Dow and Gorton (1991) present a general equilibrium model with overlapping generations, in which liquidity-motivated trade is modeled by assuming that "young" agents invest for future consumption and "old" agents liquidate their portfolios, while some "middle-aged" agents become privately informed. In a rational expectations framework, they find that profitable informed trading reduces the welfare of all agents. In contrast, in the partial equilibrium framework of the present article, liquidity-motivated trade is generated because agents possess unhedged endowments of the single security. Also, unlike Dow and Gorton, we assume that informed and uninformed agents behave strategically. Our model presents a frame-

⁵ The issue of nonlinear equilibria is difficult to address under Kyle's (1984, 1985) structure. This being the case, we do not consider nonlinear strategies in this article.

⁶ This result also obtains in Kyle-type models with exogenous noise trading, so long as the informed observe sufficiently diverse signals.

work for welfare analysis in a market-maker framework, while their model uses the Walrasian framework.

Admati and Pfleiderer (1988b), Chowdhry and Nanda (1991), and Subrahmanyam (1991a) also have attempted to make the trading behavior of the noise traders strategic. They do this by allowing the noise traders to switch into whichever time periods or markets minimize their expected trading losses to the informed, while constraining them to trade a specific amount of the securities. In contrast to this "cost of trading," we consider the expected utility of the uninformed hedgers as the objective function that determines their strategic behavior.

A summary of the major results of this article is given in Table 1. The remainder of the article is organized as follows. In Section 1, we provide a closed-form solution for the unique linear equilibrium and discuss some comparative statics. In Sections 2 and 3, we analyze the expected profits of the informed traders and the welfare of the uninformed hedgers, respectively. In Section 4, we examine the viability of concentrated trading equilibria in our model; and in Section 5, we conclude with a recapitulation of the major results and briefly discuss some policy implications.

1. The Model

One risky security is traded in the model by three types of agents: k risk-neutral informed traders who possess diverse information about the fundamental value of the security, n risk-averse uninformed traders who trade for pure risk-sharing reasons and are referred to as "hedgers," and a competitive, risk-neutral market-maker.⁷ Trading takes place at time 0 and the security is liquidated at time 1. The liquidation value of the security S is given by

$$S = \bar{S} + \delta, \tag{1}$$

where \bar{S} is known to all traders at commencement of trading, and each informed trader i observes a realization of the random variable $\delta + \epsilon_i$. The random variables $\epsilon_1, \ldots, \epsilon_k$ are mutually independent and identically normally distributed, each having a mean of zero. Further, the random variable δ is also normally distributed with zero mean,

We have verified that equilibria can exist with risk-averse informed traders and risk-averse market-makers, but the conditions for equilibrium are considerably complicated and do not yield substantial additional insight. The conditions for equilibrium in the general case of a risk-averse market-maker and risk-averse informed traders are provided in Appendix B. See Subrahmanyam (1991b) for a model of imperfect competition and risk aversion under the Kyle (1984) structure with exogenous noise trading.

Table 1 The model's main results

Comparative static exercise or conclusion	Kyle-type framework	Present framework
A linear equilibrium exists	Yes	Sometimes
Adding informed traders with identical signals increases market liquidity	Yes	Sometimes
Adding informed traders with diverse signals increases market liquidity	Sometimes	Sometimes
Adding uninformed traders increases market liquidity	Yes	Sometimes
Increasing the ex ante variability of the security's liquida- tion value decreases market liquidity The informativeness of the price does not depend on the level of uninformed trading	Yes Yes	Sometimes
The informativeness of the price increases in the number of informed traders	Yes	Yes
Adding uninformed traders increases the expected profits per informed trader	Yes	Sometimes
Adding informed traders with identical signals increases the welfare per uninformed trader	Yes	Never
Adding informed traders with diverse signals increases the welfare per uninformed trader	Sometimes	Never
Adding uninformed traders increases the welfare per uninformed trader	Yes	Sometimes

and is independent of $\epsilon_1, \ldots, \epsilon_k$. We denote $var(\delta) = \psi$ and $var(\epsilon_i) = \phi$, $i = 1, \ldots, k$.

Each risk-averse hedger j has an endowment w_j of the security. The analysis assumes that all hedgers have negative exponential utility with a common risk-aversion coefficient A, and that w_j , j = 1,...,n, are independent of each other and independent of δ , $\epsilon_1, \ldots, \epsilon_k$. In addition, it is assumed that the w_j 's are identically normally distributed, with means of zero and variances of σ_w^2 .

The k informed traders and n hedgers submit orders to the market-maker, not knowing the market-clearing price when they do so. The market-maker observes the net order flow and then sets the price. The price is set to equal the expected fundamental value of the security, conditional on the order flow.

The analysis examines the unique linear (Nash) equilibrium of the model. Let us denote the total order flow by Q, and the price by P. Suppose that the market-maker uses a linear rule of the form

$$P = \bar{S} + \lambda Q,\tag{2}$$

where λ , the slope of the price schedule, is the usual inverse measure of market depth or liquidity. Further, suppose that informed traders use linear strategies of the form $\beta(\delta + \epsilon_i)$ and that all hedgers submit orders which are linear functions of their endowments, specifically,

of the form γw_i . The model yields closed-form solutions to the endogenous parameters λ , β , and γ . These solutions are presented in the following proposition.

Proposition 1. If

$$A^{2} n \sigma_{w}^{2} (\psi + 2\phi)^{2} > 4k(\psi + \phi), \tag{3}$$

then the unique linear equilibrium of our trading game is given by

$$\lambda = \frac{A\psi[k(\psi+\phi)]^{1/2}[(\psi+2\phi)^2 + \psi k\phi + [k(n-1)/n]\psi(\psi+\phi)]}{[(1+k)\psi+2\phi]^2 \{An^{1/2}\sigma_w(\psi+2\phi) - 2[k(\psi+\phi)]^{1/2}\}},$$
 (4)

$$\beta = \frac{[(1+k)\psi + 2\phi][An^{1/2}\sigma_w(\psi + 2\phi) - 2[k(\psi + \phi)]^{1/2}]}{A[k(\psi + \phi)]^{1/2}[(\psi + 2\phi)^2 + \psi k\phi + [k(n-1)/n]\psi(\psi + \phi)]},$$
 (5)

and

$$\gamma = \frac{(1+k)\psi + 2\phi}{n^{1/2}\sigma_w} \frac{An^{1/2}\sigma_w(\psi + 2\phi) - 2[k(\psi + \phi)]^{1/2}}{A[(\psi + 2\phi)^2 + \psi k\phi + [k(n-1)/n]\psi(\psi + \phi)]}.$$
 (6)

If condition (3) is not satisfied, there does not exist a linear equilibrium.

Proof. See Appendix A.

Condition (3) implies that a linear equilibrium exists only if the risk aversion of the hedgers, or the ex ante variability of their endowments, or their number, is sufficiently high, and the number of informed traders is sufficiently low. When (3) does not hold, the market-maker finds that linear price schedules cannot induce trading strategies that cause the price to equal the security's expected value given the aggregate order flow. The problem occurs because, when (3) fails to hold, the informed traders' activity overwhelms that of the hedgers, causing the market-maker to incur systematic losses. The restrictions imposed by Proposition 1 contrast with the results in a Kyle-type model, in which a linear equilibrium always exists.

In the remainder of the article, we discuss various comparative statics. Note that changing parameter values can cause the direction of the inequality in (3) to reverse, thus precluding the existence of a linear equilibrium. Since we confine ourselves to the linear strategy space in this article, the comparative statics presented in the remainder of the article should be interpreted as applying only to parameter spaces under which (3) holds.

1.1 Equilibrium analysis

The proposition below lists the comparative statics associated with λ . Following the proposition, we provide a discussion that illustrates the intuition behind our results.

Proposition 2. (i) λ is monotonically decreasing in A and σ_w^2

(ii) There exist nonempty sets of exogenous parameter values under which λ is nonmonotonic in n, k, ϕ , and ψ .

Proof. To prove the first statement, differentiate (4) with respect to the relevant variables. For a proof of the second statement, we offer the following examples.

Example 1: λ is nonmonotonic in n. Set k = 5, A = 2, $\sigma_w = 5$, $\phi = 0$, and $\psi = 1$. Then $\lambda(n = 1) = .022$, $\lambda(n = 2) = .045$, and $\lambda(n = 3) = .042$.

Example 2: λ is nonmonotonic in k. Set n = 5, A = 2, $\sigma_w = 2.5$, $\phi = 0$, and $\psi = 1$. Then $\lambda(k = 1) = .09804$, $\lambda(k = 6) = .09232$, and $\lambda(k = 10) = .09688$.

Example 3: λ is nonmonotonic in ϕ . Set n = 5, k = 10, A = 4, $\sigma_w = 2$, and $\psi = 1$. Then $\lambda(\phi = 0) = .081$, $\lambda(\phi = 2) = .086$, and $\lambda(\phi = 10) = .077$.

Example 4: λ is nonmonotonic in ψ . Set n = 5, k = 5, A = 2, $\sigma_w = 1.5$, and $\phi = 0$. Then $\lambda(\psi = 1) = .278$, $\lambda(\psi = 2) = .248$, and $\lambda(\psi = 3) = .261$.

Intuitively, if the parameters A and σ_w^2 are high, hedgers are willing to bear more expected losses to informed traders, and this serves to lower λ . An increase in n increases price variability, so that hedgers wish to scale down their trades. However, the potential adverse price impact of trades declines if there are more uninformed traders, so the uninformed wish to trade larger quantities. Now, if hedgers are highly risk averse, the first effect may dominate, so that λ may increase in n. This is confirmed in Figure 1, which shows that, under the stated parameter values, if A = 7, then λ increases in n for part of the range of n in the figure; whereas for A = 1.4, λ decreases in n.

Proposition 2 provides the counterintuitive result that having more uninformed traders is not necessarily desirable from the point of view of market liquidity (i.e., adding another risk-averse trader to the market can actually cause market liquidity to decline). In contrast, for both Kyle-type models and the model of Glosten and Milgrom (1985), the slopes of the price schedules (respectively, bid-ask spreads) are monotonically decreasing in the variance of noise trading (respectively, the proportion of uninformed traders in the market).

An increase in the number of informed traders k has three effects on λ . First, it increases competition among informed traders, and this

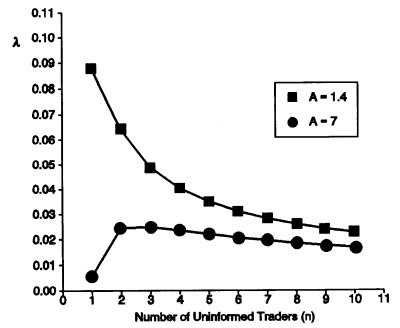


Figure 1 Equilibrium liquidity parameter λ versus number of uninformed hedgers n The parameter values used are k = 11, $\sigma_w = 5$, $\phi = 0$, and $\psi = 1$.

tends to increase market liquidity. Second, raising k increases the amount of information represented in the order flow, which tends to reduce liquidity. This results from the fact that when the informed traders observe diverse signals, adding another informed trader increases the informational content of the pool of private information in the market. If the informed traders all observe the same signal ($\phi = 0$), then this second influence disappears and only the competition effect operates. In fact, it is shown in Admati and Pfleiderer (1988b) that if informed traders all observe the same signal, λ declines in the number of informed traders (under exogenous noise trading).

The third effect of k on λ is unique to our model. As the number of informed traders increases, the hedgers become concerned about the increased adverse price impact of their trades and the increased perceived variability of the price. If they are sufficiently risk averse, their trading intensity is high, implying that adding another informed trader will not add much to the price variability. As a result, the increased competitiveness among the informed traders causes the market's liquidity to increase. Conversely, if the hedger's risk aversion is small, they do not trade large quantities. Therefore, adding another informed trader causes the price variability to rise so much that the uninformed hedgers trade even less intensely, leading to an increase

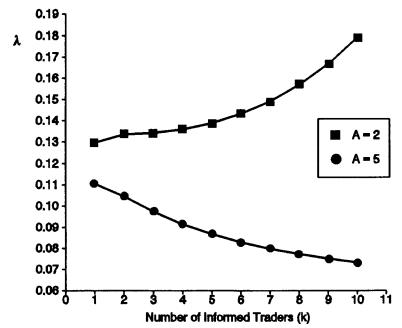


Figure 2 Equilibrium liquidity parameter λ versus number of informed traders k. The parameter values used are n = 5, $\sigma_v = 2$, $\phi = 0$, and $\psi = 1$.

in λ . These phenomena are illustrated in Figure 2, which shows that under the stated parameter values, if A = 5, λ decreases in k, while if A = 2, λ increases in k. [Note that $\phi = 0$, so that informed traders all observe the same signal; yet for A = 2, λ is increasing in k, in contrast to the model of Admati and Pfleiderer (1988b).]

The comparative statics associated with ϕ can be explained by noting that if the variance of noise in the informed traders' signals ϕ is reduced, their signals become more homogenous, increasing the competition between them and thereby decreasing λ . However, the decrease in ϕ also implies that their pool of information is more precise, and this serves to increase λ . These competing effects lead λ to be nonmonotonic in ϕ . A similar intuition can be given for the comparative statics associated with ψ .

While λ is nonmonotonic in n, k, ϕ , and ψ for small values of the model's parameters, the limiting results are unambiguous, as the following corollary demonstrates.

Corollary to Proposition 1. The market becomes infinitely liquid $(\lambda \text{ goes to } 0)$ as either ϕ or n go to infinity, and becomes infinitely illiquid $(\lambda \text{ goes to infinity})$ as ψ goes to infinity. For some finite k, say

 k^* , as k goes to k^* the market becomes infinitely illiquid (λ goes to infinity), and for $k > k^*$ a linear equilibrium does not exist.

Proof. For the statements regarding n, ϕ , or ψ , take the appropriate limits of (4). To see the proof for the statement regarding k, note that as k goes to some value k^* , the denominator of (4) goes to zero, while the numerator goes to a finite value. Also note that for $k > k^*$, condition (3) will not hold.

Notice that in a Kyle-type model with exogenous noise trading, increasing k eventually adds to market liquidity, and in the limit the market becomes infinitely liquid, because the competition effect eventually dominates. Here, however, the opposite happens. Part of the reason for this is that as the insiders compete with each other they add to the price uncertainty at which trades are conducted. As this added uncertainty increases with k, it eventually causes the uninformed to decrease their trading intensity, thereby reducing liquidity. In contrast, the results regarding n and ϕ closely parallel those of a typical noise-trading model. As ϕ gets large, the degree of informational asymmetry in the market decreases, allowing liquidity to increase. In a similar vein, increasing the number of uninformed traders eventually creates sufficient hedging demand, so that the market-maker can move λ toward zero.

An interesting possibility in our model is that a hedger may choose to more than offset his endowment by trading from an initial long position to a short position or vice versa (i.e., he may wish to "overhedge" himself). At first glance, this result seems counterintuitive because the hedger faces adverse selection in the pricing and, furthermore, moves the price when he trades. However, the phenomenon may occur in our market because the hedger faces two correlated sources of risk: both the fundamental value of the security S and the transaction price P are uncertain. To gain more intuition, observe that the variance of a hedger's payoff can be written as

$$var[(y_j + w_j)S - y_j P] = (y_j + w_j)^2 \psi + y_j^2 var(P) - 2y_j w_j cov(P, \delta) - 2y_j^2 cov(P, \delta),$$

where y_j and w_j denote the trade and the endowment of the hedger, respectively. Note that by taking on a larger position y_j , the hedger increases the weight placed on the last covariance term in the above expression. In other words, the hedger is able to better exploit the positive correlation between the price and the fundamental value of

⁸ We thank the referee for bringing this possibility to our attention.

the security by taking on a larger absolute position. Of course, in doing so, he also increases the weight on the price variance term and the adverse price impact of his trade. However, if $cov(P, \delta)$ is sufficiently large, it may benefit the hedger to hedge more than his endowment.

From (6) it can be shown that a hedger overhedges (i.e., $\gamma < -1$) whenever

$$kA^2\sigma_w^2\psi^2(\psi+\phi)/n[(1+k)\psi+2\phi]^2>4.$$
 (7)

Parameter values can easily be found under which the above condition holds. For example, if $\sigma_w = 2$, A = 5, k = 1, $\psi = 1$, and $\phi = 0$, then each hedger overhedges whenever there are less than a total of seven hedgers in the market. Ceteris paribus, the correlation between the price and the value tends to be large whenever the number of hedgers is small relative to the number of informed traders. In keeping with this observation, condition (7) indicates that the tendency for hedgers to overhedge themselves is strong when n is small relative to k.

1.2 Equilibrium informational efficiency

It is of interest to calculate the posterior variance of the security's terminal value $var(\delta | P) = var(\delta | Q)$, an inverse measure of the informational efficiency of the price. Now

$$\operatorname{var}(\delta | Q) = \operatorname{var}\left(\delta \left| k\beta\delta + \beta \sum_{i=1}^{k} \epsilon_{i} + \gamma \sum_{j=1}^{n} w_{j}\right)\right|$$
$$= \frac{\psi(k\beta^{2}\phi + n\gamma^{2}\sigma_{w}^{2})}{k\beta^{2}(k\psi + \phi) + n\gamma^{2}\sigma_{w}^{2}}.$$

Substituting for β and γ from (5) and (6), respectively, we have

$$\operatorname{var}(\delta | P) = \frac{\psi(\psi + 2\phi)}{(1+k)\psi + 2\phi}.$$

The expression for the posterior variance is startlingly simple and does not involve any of the parameters associated with the hedgers (i.e., A, σ_w^2 and n). This is because of the assumed risk neutrality of the informed traders, who scale their activity proportionately upward or downward in response to a change in any of the above variables. This result is identical to that in Kyle (1984, 1985), in which the uninformed (noise) demand is exogenous.

⁹ This invariance result would not obtain if informed traders (or market-makers) were risk averse [see Subrahmanyam (1991b)].

2. Expected Profits from Informed Trading

In this section, we analyze comparative statics associated with the expected profits per informed agent. Instead of directly calculating the (ex ante) profits of the informed traders, note that the unconditional expected losses of the hedgers are given by

$$E\left[(P-S)\sum_{j=1}^{n}\gamma w_{j}\right]=\lambda n\gamma^{2}\sigma_{w}^{2}.$$

The above expression must equal the aggregate expected profits of the informed traders, since the market-maker earns a zero expected profit. ¹⁰ Substituting for γ and λ from (6) and (4) and dividing by k, the expected profits of each informed trader, denoted by π , are given by

$$\pi = \psi \left[\frac{\psi + \phi}{k} \right]^{1/2} \frac{A n^{1/2} \sigma_w (\psi + 2\phi) - 2[k(\psi + \phi)]^{1/2}}{A[(\psi + 2\phi)^2 + \psi k\phi + k[(n-1)/n]\psi(\psi + \phi)]}.$$
 (8)

The following proposition provides comparative statics associated with the equilibrium expected profits of each informed trader.

Proposition 4. (i) The equilibrium expected profits per informed traders increase in A and σ_{w}^{2}

(ii) There exist nonempty sets of exogenous parameter values under which the equilibrium expected profits per informed trader can be nonmonotonic in n.

Proof. The proof of (i) is obtained by differentiating (8) with respect to the relevant variables. For proving (ii), the nonmonotonicity of the expected profits in n, consider the following parameter values: k = 3, A = 6, $\sigma_w = 2$, $\phi = 0$, and $\psi = 1$. Then $\pi(n = 1) = .821$, $\pi(n = 2) = .520$, and $\pi(n = 3) = .556$.

From Proposition 4, if the risk aversion of the hedgers or the variance of their endowments increases, the profits per informed trader increase. This is in keeping with the intuition provided earlier for the comparative statics associated with λ . The intuition for the non-monotonicity of the profits in n is identical to that provided for the effect of n on λ following Proposition 2. Again, result (ii) of Proposition 4 contrasts with that obtained in a model with exogenous noise trading.

At this point, it might appear possible to endogenize the number of informed traders by introducing a cost for acquiring information. However, since the linear equilibrium does not exist for all values of k, the number of informed traders, specifying the equilibria for all possible parameter spaces, becomes difficult when information acquisition is endogenous. Therefore, we do not endogenize information acquisition in our analysis.

3. Welfare of the Uninformed Hedgers

It is of considerable interest to examine the welfare of the uninformed in equilibrium, since microstructure literature has paid little attention to fully modeling the activities of such traders. From (A4) and (A5) of Appendix A, the hedgers' mean-variance utility function in terms of γ and λ is given by

$$M = \bar{S} w_{j} - w_{j}^{2} \left\{ \lambda \gamma^{2} + \frac{A}{2} \left[\frac{(\psi + 2\phi)\gamma}{(1+k)\psi + 2\phi} + 1 \right]^{2} \psi \right\} - w_{j}^{2} \frac{A}{2} \left\{ \gamma^{2} k \phi \left[\frac{\psi}{(1+k)\psi + \phi} \right]^{2} + \lambda^{2} \gamma^{4} \sigma_{w}^{2} (n-1) \right\}.$$
 (9)

The following proposition describes comparative statics results associated with the expression for *M*.

Proposition 5. (i) M is monotonically decreasing in k and ψ .

(ii) There exist nonempty sets of exogenous parameter values under which M is nonmonotonic in n and ϕ .

Proof. Define the quantity $EU \equiv (M - \bar{S}w_j)/w_j^2$. Substitute for λ from (4) and for γ from (6) into the expression for EU. After some straightforward but tedious algebraic simplification, it can be shown that

$$\operatorname{sign}\left[\frac{dEU}{dk}\right] = -\operatorname{sign}\left\{ (\psi + \phi)^{1/2} (\psi + 2\phi)^2 + \left[\psi \phi + \frac{n-1}{n} \psi(\psi + \phi)\right] \right\} \times \left[An^{1/2} \sigma_w (\psi + 2\phi) - \left[k(\psi + \phi)\right]^{1/2} \right\}, \tag{10}$$

and that the sign of $dEU/d\psi$ is unambiguously negative. From (3), the sign of dEU/dk is also necessarily negative in the linear equilibrium, so that dM/dk is necessarily negative, completing the proof of (i). For proving (ii), consider the following examples.

Example 1: Set
$$k = 5$$
, $A = 2$, $\sigma_w = 3$, $\phi = 0$, and $\psi = 1$. Then $EU(n = 1) = -0.7454$, $EU(n = 3) = -0.8685$, and $EU(n = 5) = -0.8667$. Example 2: Set $k = 50$, $A = 9$, $\sigma_w = 2.5$, $n = 1$, and $\psi = 1$. Then $EU(\phi = 0) = -2.828$, $EU(\phi = 1) = -4.028$, and $EU(\phi = 4) = -3.504$.

The result (i) of Proposition 4 implies that the welfare of the hedgers unambiguously decreases as the number of informed traders in the market increases. An increase in the number of informed traders increases the variability of the price, which reduces the effectiveness of the uninformed traders' hedging strategies. The offsetting effect,

that of intensified competition between informed traders, is always dominated by the price variability effect.

The intuition for the nonmonotonicity of M in n is as follows. As explained in the discussion following Proposition 2, an increase in n decreases the price impact of the hedgers' trades, but also increases the variability of the price. If the hedgers' risk aversion is low, the former effect dominates and increases the hedgers' welfare; whereas if their risk aversion is high, the latter effect dominates and decreases the hedgers' welfare. Figure 3 shows this result by plotting EU as a function of the number of uninformed traders n. Under the specified parameter values, for A = 2.5, EU increases in n, while for A = 6, EU decreases in n.

An increase in the number of hedgers is therefore not necessarily beneficial to the hedgers. This result is in contrast to existing models based on Kyle (1984, 1985), in which larger amounts of noise trading unambiguously decrease the expected losses per noise trader, since these losses monotonically decline in the liquidity of the market. Changes in ψ and ϕ also produce opposing forces, as explained in the discussion following Proposition 2, which explains why increasing their values induces ambiguous benefits to the hedgers.

In a Kyle-type model with exogenous noise trading, the welfare of the noise traders (i.e., the traders' expected loss to informed traders) is monotonic in the slope of the price function set by the market-maker. Figure 4 demonstrates that when the noise traders are modeled as maximizing agents, this monotonicity property is lost. From this figure it can be seen that, while the welfare of the hedgers is monotonic in k, market liquidity λ is nonmonotonic in k. In general, therefore, changes in λ do not translate directly into changes in the hedger's welfare. This indicates that examining changes in summary parameters such as market liquidity to measure changes in the well-being of market participants is a less than perfect substitute for an explicit welfare analysis.

4. On the Viability of Concentrated Trading Equilibria

Pagano (1989), Admati and Pfleiderer (1988b), and Subrahmanyam (1991a) present models where concentrated trading benefits the uninformed agents. We now show that this result may not obtain in extensions of our model to multiple markets or to an intertemporal setting and that, in fact, trading in a dispersed manner may make hedgers better off relative to concentrated trading. First, consider the case of multiple markets. Specifically, suppose there are two markets for the same security, whose liquidation payoffs are given by (1). Also suppose that market-makers in one market cannot observe the order

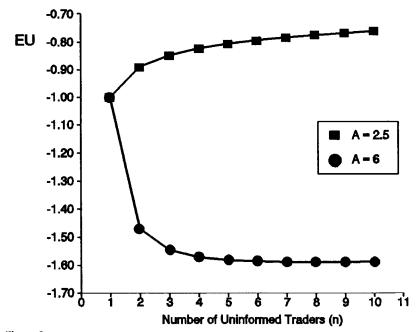


Figure 3 An affine transformation of the expected utility per hedger versus the number of hedgers. The parameter values used are k = 1, $\sigma_w = 1$, $\phi = 0$, and $\psi = 1$.

flow in the other market, and that there is one hedger and one informed trader in each market. Informed traders are constrained to trade in their respective markets, while the two hedgers have the discretion to choose between one of the two markets. Further, assume that $\psi = 1$, $\phi = 0$, and that both hedgers have a risk-aversion coefficient of 6 and independently distributed endowments, each with a mean of zero and a variance of unity. These parameter values correspond to those in Figure 3. Examining this figure, it is clear that it is an equilibrium for the two hedgers to trade in different markets and achieve an EU of -1, since if one of them deviated and traded on the same market as the other hedger, he would achieve an EU of -1.5. Further, a situation in which both hedgers trade on one market cannot be an equilibrium, since one hedger would find it advantageous to deviate and trade in the other market.¹¹

The result that hedgers may be better off trading in a dispersed fashion contrasts with Pagano (1989), who demonstrates that trading may occur on separate markets even though concentration in a single

We assume here that agents change their strategies in a consistent fashion in response to a deviation by a particular agent.

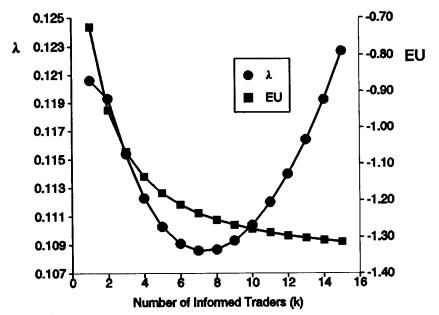


Figure 4
An affine transformation of the expected utility per hedger versus the number of informed traders

The parameter values used are A = 2.7, n = 5, $\sigma_w = 2$, $\phi = 0$, and $\psi = 1$.

market Pareto dominates trade on separate markets because of the "positive feedback" between the number of traders and market liquidity. By contrast, here, the "positive feedback" may not obtain; an increase in the number of hedgers may make hedgers worse off.

Consider now a two-period setting in which the liquidation value of the security is given by

$$S = \bar{S} + \delta_1 + \delta_2. \tag{11}$$

Each innovation δ_t is revealed after the tth round of trading. Further, δ_1 and δ_2 are independently normally distributed with a mean of zero and variances of ψ_1 and ψ_2 , respectively. There is one informed trader who learns precisely the realization of δ_1 (and trades in period 1), and one informed trader who learns precisely the realization of δ_2 (and trades in period 2). Now, suppose that two equally risk-averse hedgers receive independent and identically distributed endowment shocks with a mean of zero and a variance of σ_w^2 , before trading takes place at time 1.12 The question we now ask is whether a particular

¹² The assumption that hedgers do not receive any endowment shocks after the commencement of trading facilitates tractability by preserving the normality of the hedgers' payoffs, which enables us to apply the mean-variance framework to this intertemporal example.

hedger can be better off by maintaining an unhedged position between rounds 1 and 2 and trading in round 2, relative to a situation in which he trades in round 1 along with the other hedger. The answer to the above question is in the affirmative so long as ψ_1 is sufficiently low, so that the cost of maintaining an unhedged position is not too high. In fact, under the parameter values A=8, $\sigma_w=1$, $\psi_1=0.6$, and $\psi_2=1$, minor variations of the calculations in Sections 1 and 3 show that the unique linear equilibrium, up to a relabeling of the hedgers, consists of the two hedgers trading in different periods. Thus, in variants of our model, uninformed agents may not wish to concentrate their trading intertemporally.

5. Summary and Concluding Remarks

We have considered a model of trading that allows for strategic behavior on the part of both the informed and the uninformed traders. The uninformed trade for pure risk-sharing reasons, while the informed trade to acquire expected profits. Since the uninformed agents are not modeled as "noise" or "liquidity" traders but rather as rational, maximizing agents, we eliminate an exogeneity associated with most previous trading models. This modification also allows us to explicitly analyze the welfare of the uninformed agents.

In the article, we provide several conclusions that differ from those obtained in models with "noise" traders whose demands are unmodeled. For example, in this article: (i) market liquidity is nonmonotonic in the number of uninformed agents in the market; (ii) concentration does not necessarily benefit the uninformed agents, in the sense that adding another uninformed agent may decrease the expected utility per uninformed agent; (iii) increasing the number of uninformed agents may decrease the profits of the informed agents; and (iv) the welfare per uninformed agent monotonically decreases in the number of informed agents. Results (i) and (ii) indicate that the positive relationship between the number of uninformed agents and their welfare, which leads to "economies of concentration" in models with exogenous noise trading, may not obtain in a setting in which all traders are maximizing agents. Kyle (1984) finds that the expected profits of agents gathering costly information monotonically increase in the number of "noise" traders. Drawing on this result, he concludes in subsequent work that "a regulator seeking to increase the informativeness of the price should attempt to increase the informativeness of the price by attracting more noise traders, not by driving noise traders away" [Kyle (1989), p. 160]. In light of our result (iii), the above conclusion should be interpreted more cautiously. According to result (iv), policies designed to encourage competition between informed traders (e.g., by reducing the cost of acquiring information), presumably in order to increase market liquidity, have an unambiguously deleterious effect on the welfare of uninformed hedgers.

In a market-orders setting, our analysis formalizes the notion that the trading strategies of risk-averse agents are influenced significantly by the perceived volatility of prices in the market.¹³ The analysis also indicates that using market liquidity as the sole measure of the welfare of market participants may be misleading.

As a final remark, the results presented here indicate that comparative statics results can change significantly when the "noise" or "liquidity" traders are modeled as maximizing agents. Postulating the existence of noise traders with exogenous demands, therefore, is not solely a convenient modeling device designed to facilitate tractability. We have demonstrated that it is an assumption whose relaxation alters many of the basic results and, consequently, also influences policy implications derived from a market microstructure model.

Appendix A: Proof of Proposition 1

The informed traders maximize their individual profits in a Nash fashion. Suppose that each informed trader i conjectures that other informed traders will submit an order $\bar{\beta}(\delta + \epsilon_l)$, $l \neq i$, and that each hedger will submit an order γw_j , j = 1,...,n. Let x_i denote the order submitted by this informed trader. The profits of this informed trader are given by

$$(\bar{S} + \delta)x_i - x_i \left(\bar{S} + \lambda x_i + \lambda \sum_{l \neq i} x_l + \lambda \sum_{j=1}^n \gamma w_j\right).$$
 (A1)

Taking the expectation of the above expression conditional on $\delta + \epsilon_i$, differentiating the resulting expression with respect to x_i , and equating the result to zero yield

$$\frac{\psi}{\psi + \phi}(\delta + \epsilon_i) - 2\lambda x_i - \lambda(k - 1)\bar{\beta}\frac{\psi}{\psi + \phi}(\delta + \epsilon_i) = 0. \quad (A2)$$

Note from the above expression that it is required that the slope of the price schedule λ be positive to satisfy the second-order condition

¹³ A recent NYSE publication describes how the exchange has attempted to address the concern among market participants that stock prices are excessively volatile in the short term [see New York Stock Exchange (1990)].

¹⁴ It is straightforward to show, using symmetry arguments, that the unique linear equilibrium is one in which all informed traders use symmetric linear strategies, as do the hedgers.

of the informed trader. Solving (A2) for x_i , we have

$$x_i = (\delta + \epsilon_i)\psi(1 - \lambda(k-1)\bar{\beta})/2\lambda(\psi + \phi).$$

Let $x_i = \beta(\delta + \epsilon_i)$. We can solve for the Nash equilibrium by setting $\bar{\beta} = \beta$:

$$\beta = \psi/\lambda[(1+k)\psi + 2\phi]. \tag{A3}$$

Notice that the hedgers' strategies have no effect on the informed traders' positions for a given λ , since the hedgers' endowments have an ex ante mean of zero and are uncorrelated with the informational variables δ , and $\epsilon_i, \ldots, \epsilon_k$.

Let hedger j's order be denoted by y_j . Suppose that this trader conjectures that each informed trader i will submit an order $\beta(\delta + \epsilon_i)$, and that the other n-1 hedgers will submit orders $\overline{\gamma}w_p$, $p \neq j$. This hedger's payoff V_j is given by

$$V_{j} = (\bar{S} + \delta)(y_{j} + w_{j})$$

$$- y_{j} \left(\bar{S} + k\beta\lambda\delta + \beta\lambda \sum_{i=1}^{k} \epsilon_{i} + \lambda y_{j} + \lambda \sum_{p \neq i} \bar{\gamma} w_{p}\right). \quad (A4)$$

Conditional on this hedger's endowment w_j , V_j is normally distributed. By virtue of the assumption of exponential utility, we may maximize the following mean-variance function (denoted by M):

$$M = E(V_i | w_i) - .5A \operatorname{var}(V_i | w_i).$$
 (A5)

Substituting for V_j from (A4) into the above expression, differentiating with respect to y_j , and setting the resulting expression to zero yield

$$y_{j} = -\frac{A\psi(1-\lambda k\beta)w_{j}}{2\lambda + A[\psi(1-\lambda k\beta)^{2} + k\phi\lambda^{2}\beta^{2} + \lambda^{2}(n-1)\overline{\gamma}^{2}\sigma_{w}^{2}]}$$

Now suppose that $y_j = \gamma w_j$. Setting $\overline{\gamma} = \gamma$, we obtain the following cubic equation in γ :

$$A\lambda^{2}\gamma^{3}(n-1)\sigma_{w}^{2} + \gamma[Ak\phi\lambda^{2}\beta^{2} + A\psi(1-\lambda k\beta)^{2} + 2\lambda] + A\psi(1-\lambda k\beta) = 0.$$
(A6)

As the following analysis shows, the above cubic equation can be manipulated to produce closed-form solutions for the model's endogenous parameters— λ , β , and γ .

By the zero-profit condition and the fact that the order flow,

$$Q \equiv \sum_{i=1}^k x_i + \sum_{i=1}^n y_i,$$

is normally distributed, the slope of the price schedule λ is given by the linear projection of δ on Q, $\lambda = \text{cov}(\delta, Q)/\text{var}(Q)$. Substituting for the order-placement strategies of the informed traders from (A3) into the above formula for the linear projection, we have

$$k\psi^{2}(\psi + \phi) = n\gamma^{2}\sigma_{w}^{2}\lambda^{2}[(1 + k)\psi + 2\phi]^{2}.$$
 (A7)

Thus, it is necessary for equilibrium that $\lambda > 0$ (recall that a positive λ is required to satisfy the informed traders' second-order conditions) and that

$$\gamma = \pm [k\psi^2(\psi + \phi)]^{1/2}/n^{1/2}\lambda\sigma_w[(1 + k)\psi + 2\phi].$$
 (A8)

Since, from (A3), $1 - \lambda k\beta$ is positive, it is evident from (A6) that γ must be negative. Substituting for γ from (A8) (taking the negative sign) into (A6) yields a linear equation for γ , the solution of which yields (6). Substituting for γ into (A8) and solving for λ yields (4). Finally, substituting for λ into (A3) yields (5).

Appendix B

This appendix provides equilibrium conditions for the more general case of risk-averse informed traders and a risk-averse market-maker, without attempting to solve the resulting model. It will become evident that closed-form solutions are not possible for the general case.

B.1 Risk-averse informed traders

Suppose that risk-averse informed traders have exponential utility with a risk-aversion coefficient of A_I . The informed trader i's payoff is given by (A1). Conditional on the informed trader's signal $\delta + \epsilon_I$, his payoff is normally distributed. Denote his payoff as Pr. Then, maximizing the mean-variance function $E(\Pr) - (A_I/2) \text{var}(\Pr)$ with respect to x_I , and solving for the equilibrium by setting $x_I = \beta(\delta + \epsilon_I)$ and $\bar{\beta} = \beta$, we have

$$\beta = [1 - \lambda(k-1)\beta][\psi/(\psi+\phi)]$$

$$\times (2\lambda + A_I[1 - \lambda(k-1)\beta]^2[\phi\psi/(\psi+\phi)]$$

$$+ A_I\lambda^2\beta^2(k-1)\phi + A_I\lambda^2n\gamma^2\sigma_w^2)^{-1},$$
(B1)

which is a cubic equation in β for given λ and γ .

B.2 Risk-averse market-makers

The modeling of risk-averse market-makers follows Subrahmanyam (1991b). A single market-maker with exponential utility and a risk-aversion coefficient A_m is, for simplicity, constrained to take the entire order flow and, because of Bertrand competition, is constrained to

earn zero expected utility conditional on the order flow. 15 This implies that prices are set such that

$$E[Q(P-F)|Q] = (A_m/2) \text{var}[Q(P-F)|Q].$$
 (B2)

Prices are set according to the linear rule $P = \bar{S} + \lambda Q$. Substituting this rule into the left-hand side and right-hand side of (B2), and then substituting for the order flow Q in terms of β and γ , we have

$$\lambda = \frac{k\beta\psi + (A_m/2)\psi(k\beta^2\phi + n\gamma^2\sigma_w^2)}{k\beta^2(k\psi + \phi) + n\gamma^2\sigma_w^2}.$$
 (B3)

B.3 Equilibrium conditions

Note that the equation describing the optimal γ for each uninformed hedger in terms of λ and β remains unchanged from Equation (A6). Thus, the equilibrium is described by the system of three nonlinear equations—(B1), (B3), and (A6)—in three unknowns— β , γ , and λ . It is evident that closed-form solutions for the system are not obtainable.

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¹⁵ In general, market-makers would prefer to split the order flow among themselves if they are risk averse. The restriction that a single market-maker take the entire order flow, however, simplifies the characterization of the equilibrium considerably.

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