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Informed trading and the 'leakage' of information ☆

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This paper is dedicated to my teacher and friend, Karl Shell

Abstract

This paper examines the effect of 'leakage' of information, private information becoming available to uninformed traders at a later date, on information acquisition and revelation. Using a Shapley–Shubik market game framework it is shown that (a) if information acquisition by the informed traders is costless, this leads to faster revelation of information; (b) if information acquisition is costly, there may be no acquisition of information; (c) information leakage leads to a fall in value of information but does not affect the incentive for informed traders to sell the information.

JEL classification: D82; C72; G14; D51; D84

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1. Introduction

This paper examines the effect of 'leakage of information,' i.e., private information becoming available to uninformed agents, on strategic market trading in a dynamic context. In existing strategic models of trading under asymmetric information the set of informed traders, whether single or multiple, is held fixed over

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the different rounds of trading and the uninformed may try to infer information from the trades and equilibrium outcomes. In these models the informed can weigh the costs of the liquidity effect and the information effect and can control how much information is revealed through prices.[1] However, in many situations of interest, the informed cannot be sure that the information they have tried to keep private through uninformative trades may not become public at a later date, or the uninformed can become informed through mechanisms other than inferring information from equilibrium outcomes. We show that this phenomenon may make the prices incorporate the private information at a faster or slower rate depending on how the information acquisition is modelled, i.e., if is it costless or costly. In the latter case, one may end up in a situation similar to the Grossman–Stiglitz paradox. In addition, the incentives for the informed to sell their own private information to the uninformed are also examined when there is a potential for information leakage.

In modelling the phenomenon of strategic trading and information leakage, we use the strategic market game a la Shapley–Shubik [6,22,26–28] as the underlying game. Karl Shell has been a leading contributor to the study of this model. He has used it in the study of a variety of phenomena: sunspot equilibria [22,25], short-selling [24], non-equivalence between Arrow securities and contingent commodities in imperfectly competitive economies [23], and indeterminacy [26]. In the market game framework agents act strategically taking into account how their actions affect equilibrium prices and hence, account for their own actions revealing information. Thus, it is a nice model to study problems of information acquisition and revelation which is at the heart of market microstructure models in finance. As it is a strategic model, the problem of the informed agents acting 'schizophrenically' (see [12]), i.e., inferring information from prices while not taking into account the effect of the informativeness of their own trade, is absent. In the formulation of the game with asymmetric information we follow [6]. Thus, we do not allow agents to condition their trades on the information contained in current prices (see also [13,21]). If we are to think of trading taking place in real time, then this sequence of trading is the appropriate one. Adopting this formulation also avoids the problem associated with revealing rational expectations equilibria first pointed out in [4].

The model is one of batch trading where trades are cleared at one price. The allocation rule specifies that the allocation of goods or assets to a given trader is in proportion of the amount bid (as a fraction of total bids). The literature on strategic informed trading with batch trading (e.g., the market microstructure literature starting from [18] (also see [20] for a survey; [6,15])) focusses on the case where the informed, who could be one or more (for a static model see [19]), know they are the only informed traders and trade taking into account the information revelation through prices. There are two effects which they trade off—liquidity and informativeness. Depending on the formulation of the model, there can be fast revelation of prices (e.g. [8,14]) or slow revelation of prices (e.g. [18]). However, these

[1] Admati and Pfleiderer [2] have a model where the number of informed traders varies but the information is valuable only for one period so there is no intertemporal dimension to the decisions of the informed.

models abstract away from one important consideration—there is no change in the set of informed agents over time. This is not satisfactory as in any market the informed cannot be certain that they will continue to be the only agents who are informed—if someone leaked the information to one, how can they be sure that it will not be leaked to someone else in the future? This paper focusses on how this affects the behaviour of the informed agents.

There are three problems addressed in this paper. First, if the information is costless, how does the potential leakage of information affect incentives to use information at early stages of trading. In contrast to the intuition of [6] (in the market game framework) and [18] (in a model with a market maker), the informed traders will want to use their information quickly. Secondly, if information is costly, how does the potential leakage of information at a later date affect incentives to purchase information in the first place? In contrast to [15] traders may not want to purchase the information. Thus, one is back to the Grossman and Stiglitz paradox [11]. Thirdly, how does the potential of arrival of information affect the incentive of an informed agent to sell the information? One may suspect that as information leakage is likely to make the information less valuable in the future, there will be an added impetus to sell the information while it is still valuable. It turns out the decision to sell the information is not affected by information leakage even though the price it can be sold at is.[2]

There are several other formulations of strategic trading that have been advanced such as the models of [7,9,15,18], etc. We choose to work with the market game for three reasons. The first is tractability. The nice linear structure of the Kyle model [18] is no longer guaranteed once the market-maker has to update beliefs taking into account that the set of informed traders is evolving over time. The second is that we do not want to introduce noise traders. In our framework, every one is fully rational, except that some are informed and some are uninformed. Thus, if the price is non-revealing it is not due to noise or some configuration of parameters, but due to equilibrium behaviour. In addition, the welfare effects on all agents (see [29]) can be analysed. Thirdly, this framework is flexible enough that a wide variety of different configurations—costly and costless information, storage of commodities and assets [10], the role of short-selling [23], bankruptcy, etc., can be handled in the same model. However, in the current paper we do not allow traders to be on both sides of the market. This is common in market microstructure models. It also helps avoid the problems of indeterminacy in static and dynamic market games (see [10,26]). It will be interesting to see if allowing traders to be on both sides of the market can introduce market manipulation as well as excess volatility.

As the evidence finds that ‘buys’ move prices more than ‘sells’ (see [5,17]) we focus on the case of informed buyers and treat selling as non-information based. These papers also point out that when agents trade on short-lived information, they tend to prefer market orders and that in equity markets up to 90% of orders are market orders. In the market game formulation the bids of the agents are best interpreted as

[2] In contrast to [1], the informed agents want to sell information not due to insurance considerations.

market orders. The results derived can also help understand price increases before the information is announced in the market (see [3] for the case of tender-offers).

The outline of the paper is as follows. First, the general formulation of the game is outlined. Then, it is specialized to look at the effect of leakage on trading when information is costless; when the 'informed' have to pay to acquire it; and when they can sell it.

2. The model

We define a general market game following [6] with one essential difference: the evolution of the information partitions. There is the possibility that the information can leak to some uninformed traders so that at a later date they become exogenously informed. This is a game without storage of commodities.

There are $N = N^I + N^U < \infty$ traders indexed by n, and two periods, $t = 1, 2$. The n traders trade in both the periods and they consume at the end of trading. There are finitely many states of the world, $s \in S$, which are realized before trading in the first period and the state remains the same in the second period. Thus, information is long lived. For each agent $n \in N$, let $I^{n,1}$ be a partition of S representing the initial information of n. If the state of nature is s then each trader knows the set $I^{n,1}(s)$ in his information partition that contains s. The traders $n \in N^I$ are informed, i.e., $I^{n,1}(s) = \{s\}$ for $n \in N^I$. The traders $n \in N^U$ are uninformed. There are a finite number of commodities L in each state and period. The n traders have a utility function over their consumption set. The utility function, u^n, is twice continuously differentiable, concave, and strictly increasing. Each trader has an endowment $e^{n,t}$ in the two periods, and the endowment is measurable with respect to their information set.

The game can be thought of as an extensive form game. Nature moves by selecting a state s. In the first period, the $n \in N^I \cup N^U$ traders move simultaneously not knowing the realization of the state, but knowing the information partition they will face, i.e., they evaluate their expected utility. Thus, the informed can choose a strategy contingent on the realization of s, but the uninformed cannot. Nature selects the state s and trades are executed. A move for each player at each node is a $2(L-1)$-dimensional vector of bids and offers $z^{n,t} = (b_1^{n,t}, \dots, b_{L-1}^{n,t}, q_1^{n,t}, \dots, q_{L-1}^{n,t})$. The subscript denotes the commodity, the first argument of the superscript is the identity of the player, and the second argument is the period in which the trade is made, $t = 1, 2$. Each $b_l^{n,t}$ represents a quantity of the Lth good that is bid on commodity l, and $q_l^{n,t}$ represents the quantity of the lth good that is offered for sale. Given a vector of moves for each trader, the market adds the bids and offers for each commodity, $b_l^t(z) = \sum_n b_l^{n,t},\ i = 1, \dots, L-1$, and $q_l^t(z) = \sum_n q_l^{n,t},\ i = 1, \dots, L-1$. It sets $p_l^t(z) = \frac{b_l^t(z)}{q_l^t(z)}$. If there are no offers, the price is zero. The consumers receive net trades

$$y_l^{n,t} = \frac{b_l^{n,t}}{p_l^t} - q_l^n.$$

In addition,

$$y_L^{n,t} = \sum_{l=1}^{L-1} q_l^{n,t} p_l^t - \sum_{i=1}^{L-1} b_i^{n,t}.$$

As there is no inventorying, the final allocation of the players at node $s \in S$ is the net trade plus the endowment, i.e., $x_l^{n,t} = e_l^{n,t} + y_l^{n,t}$.

Each node $s \in S$ in period 1 leads to an infinity of successor nodes (s, z, M) where z is the collective play of the agents at node s, and M indexes the number of uninformed who have become informed. Thus, in node (s, z, M) there will be $N^I + M$ informed and $N^U - M$ will be uninformed. Denote the information set of the uninformed players $n \in N^U$ in time period 2 by $I^{n,2}$. These players refine the information observing prices, which may or may not convey any information, and there is a probability that they can become informed. Let ρ be the probability that one of these N^U players is leaked the information. The event of leakage of information is independent across the uninformed. Thus, the information set of player $n \in N^U$ with probability ρ is the same as that of the informed players, i.e., $I^{n,2} = s \vee p^1$ and with the complementary probability $1 - \rho$, $I^{n,2} = I^{n,1} \vee p^1$, where p^1 is the price realized in the first period of trading. As each of the N^U 'uninformed' traders can become exogenously informed before the second round of trading, the probability that M of them are exogenously informed is given by $\binom{N^U}{M} \rho^M (1-\rho)^{N^U - M}$. As the prices p^1 can be revealing, the number of traders who are informed in period 2 is to an extent endogenous. The rules of the second round are identical to those of the first round. The game without leakage of information is recovered when $\rho = 0$.

A strategy of a player n is to pick a move at each node in each period such that it is measurable with respect to his information set in that period. We also have the restrictions that $q_l^{n,t}(s) \leqslant e_l^{n,t}(s)$, for $l = 1, \ldots, L-1$, and $\sum_{j=1}^{L-1} b_j^{n,t}(s) \leqslant e_L^{n,t}(s)$, $t = 1, 2$. In other words, a trader cannot short sell a commodity in any period, and the total bids for commodities must be less than or equal to the endowment of the commodity money. We analyse the pure-strategy perfect Bayesian equilibria of this game.

3. Costless information

We look at an economy where there are two time periods $t = 1, 2$. There is only one perishable good (good 1) in each period in addition to the commodity money (good 2). There are two equiprobable states $s = 1, 2$ one of which is realized before trading in period 1. The state does not change in period 2. There is no inventorying. The first good is of lower value (quality) in state 1. There are three types of traders: sellers (who can be either informed or uninformed) and informed and uninformed buyers. There is one trader of each type.[3] Sellers have utility only for money, and have Q units of the good in each period. Thus, it does not matter if the sellers are thought to be informed or uninformed as they will always want to sell their entire endowment. The informed buyer ($n = I$) is exogenously informed in period 1.

Informed buyer (I) has the utility function:

$$U^I = [\tfrac{1}{2}\alpha(u_1(x_1^{I,1}) + w_1^{I,1}) + \tfrac{1}{2}\alpha(u_2(x_2^{I,1}) + w_2^{I,1})] + [\tfrac{1}{2}\beta(u_1(x_1^{I,2}) + w_1^{I,2}) + \tfrac{1}{2}\beta(u_2(x_2^{I,2}) + w_2^{I,2})],$$

where $x_s^{I,t}$ is the consumption of the good in state s in period t, and $w_s^{I,t}$ is the consumption of 'money' in state s and period t. In addition we restrict $\alpha, \beta > 0$.

The uninformed buyer ($n = U$) has identical preferences. The only way the two agents differ is in what information they may possess.

$$U^U = [\tfrac{1}{2}\alpha(u_1(x_1^{U,1}) + w_1^{U,1}) + \tfrac{1}{2}\alpha(u_2(x_2^{U,1}) + w_2^{U,1})] + [\tfrac{1}{2}\beta(u_1(x_1^{U,2}) + w_1^{U,2}) + \tfrac{1}{2}\beta(u_2(x_2^{U,2}) + w_2^{U,2})].$$

The buyers have zero holdings of the good, and M units of 'money' in each period. The preferences are restricted to be quasi-linear so that comparisons in terms of monetary costs can be made. This will prove to be important in some later results. The sub-utility functions $u_1(\cdot), u_2(\cdot)$, are defined on the non-negative orthant, are twice continuously differentiable, strictly monotonic, and strictly concave. To model the fact that the consumption good has less value in state 1, the following assumption is made.

Assumption 1. For any given $x > 0$,

1. $u_1(x) > u_2(x)$.
2. $u_1'(x) > u_2'(x)$.

The informed and uninformed differ only in the information they have. The informed will know the true state while the uninformed may not know the true state. While the uninformed in the first period cannot use the information which may be revealed by the prices, the uninformed in the second period can use information contained in past prices. There may be trivial equilibria where there is no trading in any period. These can be ruled out, for example by assuming that $\lim_{x \to 0} u_s'(x) \to \infty$ for $s = 1, 2$. We call an equilibrium where there is trade in each period an interior equilibrium. The uninformed do not have any intertemporal strategic choice. The informed do, since they choose when to use the information. In the last round of trading, they will always reveal the information. For the time being, we do not allow for the leakage of information to develop the benchmark case.

[3] This can be generalized to an equal finite, number of traders of each type.

Proposition 3.1. *If there is no leakage of information, in an interior equilibria, it must be the case that*

1. *In the second period,* $b_2^{I,2} < b_1^{I,2}$. *If prices are not revealing in the first period, then* $b_2^{I,2} < b^{U,2} < b_1^{I,2}$.
2. *In the second period, prices are higher in the first state as compared to the second, i.e.,* $p_1^2 > p_2^2$.

Proof. Look at the second period. There can be two situations. In the first, the uninformed have not learned from past prices, and in the second, the equilibrium prices are revealing in the first period. In either case, the informed will always choose to use their information in the second period as there is no gain from not using it.

The maximization problem for the informed in period 2 is

$$\operatorname*{Max}_{(b_1^{I,2}, b_2^{I,2})} \pi^{I,2} = \tfrac{1}{2}\beta(u_1(x_1^{I,2}) + w_1^{I,2}) + \tfrac{1}{2}\beta(u_2(x_2^{I,2}) + w_2^{I,2}).$$

This can be reduced to

$$\operatorname*{Max}_{(b_1^{I,2}, b_2^{I,2})} \pi^{I,2} = \frac{1}{2}\beta\left(u_1\left(\frac{b_1^{I,2}}{p_1^2}\right) + M - b_1^{I,2}\right) + \frac{1}{2}\beta\left(u_2\left(\frac{b_2^{I,2}}{p_2^2}\right) + M - b_2^{I,2}\right),$$

where $p_s^2 = \frac{b_s^{I,2} + b_S^{U,2}}{Q}$ and $b_s^{I,2}$ is the bid of the informed in period 2. The bid of the uninformed in period 2 is $b_s^{U,2}$, $s = 1, 2$. Note in the situation under consideration, $b_1^{U,2} = b_2^{U,2} = b^{U,2}$. We can derive the first-order conditions (which will be sufficient) as

$$\frac{1}{2}\beta\left(u_1'\left(\frac{b_1^{I,2}}{p_1^2}\right)\left(\frac{Qb^{U,2}}{(b_1^{I,2} + b^{U,2})^2}\right) - 1\right) = 0, \tag{1}$$

$$\frac{1}{2}\beta\left(u_2'\left(\frac{b_2^{I,2}}{p_2^2}\right)\left(\frac{Qb^{U,2}}{(b_2^{I,2} + b^{U,2})^2}\right) - 1\right) = 0. \tag{2}$$

These can be rearranged to obtain

$$u_1'\left(\frac{b_1^{I,2}}{p_1^2}\right)\left(\frac{b^{U,2}}{(b_1^{I,2} + b^{U,2})^2}\right) = u_2'\left(\frac{b_2^{I,2}}{p_2^2}\right)\left(\frac{b^{U,2}}{(b_2^{I,2} + b^{U,2})^2}\right). \tag{3}$$

Using assumptions on the sub-utility functions, this can be manipulated to show that $b_1^{I,2} > b_2^{I,2}$.

Now look at the maximization problem of the uninformed.

$$\operatorname*{Max}_{(b_1^{U,2}, b_2^{U,2})} \pi^{U,2} = \tfrac{1}{2}\beta(u_1(x_1^{U,2}) + w_1^{U,2}) + \tfrac{1}{2}\beta(u_2(x_2^{U,2}) + w_2^{U,2})$$

Again this can be transformed to:

$$\operatorname*{Max}_{(b_1^{U,2},b_2^{U,2})} \pi^{U,2} = \frac{1}{2}\beta\left(u_1\left(\frac{b_1^{U,2}}{p_1^2}\right) + M - b^{U,2}\right) + \frac{1}{2}\beta\left(u_2\left(\frac{b_2^{U,2}}{p_2^2}\right) + M - b^{U,2}\right).$$

Using $b_1^{U,2} = b_2^{U,2} = b^{U,2}$, the first order condition is

$$u_1'\left(\frac{b^{U,2}}{p_1^2}\right)\left(\frac{b_1^{I,2}}{(b_1^{I,2} + b^{U,2})^2}\right) + u_2'\left(\frac{b^{U,2}}{p_2^2}\right)\left(\frac{b_2^{I,2}}{(b_2^{I,2} + b^{U,2})^2}\right) = 2. \tag{4}$$

To solve for $b^{U,2}$, there can be three cases: either $b^{U,2} \leqslant b_2^{I,2}$, or $b^{U,2} \geqslant b_1^{I,2}$ or $b_2^{I,2} < b^{U,2} < b_1^{I,2}$. Looking at the first-order conditions, and using the properties of the sub-utility functions, one can rule out the first two cases.

If period 1 prices were revealing, the uninformed would be 'informed' as he would be able to infer the true state from the prices. In this case there are two 'informed.' Their first-order conditions reduce to

$$u_1'\left(\frac{b_1^{I,2}}{p_1^2}\right)\left(\frac{b_1^{I,2}}{(b_1^{I,2} + b_1^{I,2})^2}\right) = u_2'\left(\frac{b_2^{I,2}}{p_2^2}\right)\left(\frac{b_2^{I,2}}{(b_2^{I,2} + b_2^{I,2})^2}\right). \tag{5}$$

This reduces to

$$u_1'\left(\frac{b_1^{I,2}}{p_1^2}\right)\left(\frac{1}{4b_1^{I,2}}\right) = u_2'\left(\frac{b_2^{I,2}}{p_2^2}\right)\left(\frac{1}{4b_2^{I,2}}\right). \tag{6}$$

Using properties of the utility functions one can show that $b_1^{I,2} > b_2^{I,2}$.

From this it follows that in either outcome, $p_1^2 > p_2^2$. □

To analyse the game we proceed as follows. First, we examine the optimal behavior of the two buyers in the second period, depending on whether both are informed (whether the uninformed acquired the information from past prices) or one is informed and the other uninformed. We have shown that in the second period the informed will always choose to use the private information if it has remained as such. Then, we look at the behaviour of the two traders in the first period in isolation. The informed while trading (against the uninformed) has to weigh the payoff derived from first period trades against the informational impact of the trades. Thus, in the third step, the informed in the first period computes overall payoff of adopting a particular strategy taking into account the informational impact of trading in the first period, taking as given the optimal plays in the second period. The perfect Bayesian equilibrium will be that strategy which gives the highest equilibrium payoff to the informed taking into account the informational impact of the trades and the optimal response of the other player. The 'uninformed' do not have any intertemporal aspect to their strategy. In the second period, the 'uninformed' do perform Bayesian updating, but if the informed has chosen to adopt a non-revealing strategy, the priors will not be refined.

To choose the optimal strategy, the informed has to compare the gain from using the information in the first period rather than in the second, taking into account the informational impact of trades. If he uses the information in the first period then the prices will be revealing and in the second period he will be faced by the another informed agent. Whether he would rather use the information in the first period or the second would depend on the relative gains from waiting, i.e., on the parameters α, β. It will also depend on the payoffs when both are informed and when both are uninformed.

Proposition 3.2. *In any given period, the payoff to a buyer when both the traders are uninformed and when both are informed is the same.*

Proof. Consider the payoffs when both the traders are informed. Without any loss of generality, as we are looking at payoffs within a period, set $\alpha = \beta = 1$. Also suppress the time superscript. The superscript $n = 1, 2$ denotes buyer n. First consider the case when both are informed.The first-order conditions for the first player:

$$u_1'\left(\frac{b_1^1}{p_1}\right)\left(\frac{b_1^2}{(b_1^1 + b_1^2)^2}\right) = u_2'\left(\frac{b_2^1}{p_2}\right)\left(\frac{b_2^2}{(b_2^1 + b_2^2)^2}\right) = 1, \tag{7}$$

where b_s^n is the bid of player n in state s. The second player will have a similar first-order condition. We can replace (b_1^2, b_2^2) by (b_1^1, b_2^1) in the equation above to obtain

$$u_1'\left(\frac{Q}{2}\right)\left(\frac{b_1^1}{4(b_1^1)^2}\right) = u_2'\left(\frac{Q}{2}\right)\left(\frac{b_2^1}{4(b_2^1)^2}\right) = 1. \tag{8}$$

Hence, $b_1^n = \frac{u_1'(\frac{Q}{2})}{4}$ and $b_2^n = \frac{u_2'(\frac{Q}{2})}{4}$, for $n = 1, 2$.

Now consider the case when both are uninformed. The first-order condition for the first buyer is (using $b_1^n = b_2^n = b^n$):

$$\left(u_1'\left(\frac{b^1}{p_1}\right)\left(\frac{b^2}{(b^1 + b^2)^2}\right)\right) + \left(u_2'\left(\frac{b^1}{p_2}\right)\left(\frac{b^2}{(b^1 + b^2)^2}\right)\right) = 1. \tag{9}$$

In equilibrium, using symmetry of the players, this will reduce to

$$\frac{1}{2}\left(u_1'\left(\frac{Q}{2}\right)\left(\frac{1}{4(b^1)^2}\right)\right) + \frac{1}{2}\left(u_2'\left(\frac{Q}{2}\right)\left(\frac{1}{4(b^1)^2}\right)\right) = 1. \tag{10}$$

In this case $b^n = \frac{u_1'(\frac{Q}{2})}{8} + \frac{u_2'(\frac{Q}{2})}{8}$, for $n = 1, 2$. As the allocation of the consumption good is the same in both states, and the sum of the bids across the states are the same in the two cases, the payoffs to the two buyers will also be the same. $\square$

Note that the payoff to the informed player who acts as such will always be higher than that of the uninformed player. This can be seen as the informed bids more in the good state, and bids less in the bad state than the uninformed. Using this we can find the interior equilibrium.

Proposition 3.3. *In an interior equilibrium,*

1. *If $\beta > \alpha$, there will be no revelation of information in the first period. That is $b_1^{I,1} = b_2^{I,1}$, and $p_1^1 = p_2^1$.*
2. *If $\alpha > \beta$, prices will be revealing in the first period. That is $b_1^{I,1} > b_2^{I,1}$, and $p_1^1 > p_2^1$.*

Proof. To solve for the equilibrium first compute the payoffs in the second period for the two situations where there has been information revelation, and where there has been no revelation. Then look at the first period, and look at period 1 payoffs when the informed chooses to use his information, and when he chooses not to use his information. Unlike the second period, the informed player always faces an uninformed. Normalize $\alpha = 1$, and $\frac{\beta}{\alpha} = \delta$. Let the expected payoff in period 1 to the informed agent if he chooses to use information in period 1 be denoted as η; and the expected payoff when he chooses not to use information (i.e., acts as if uninformed) be denoted as μ. The expected payoff in period 2 when the informed faces an uninformed trader will be $\delta\eta$, and the expected payoff when he faces an 'informed' trader (as prices have revealed information) is denoted as $\delta\zeta$. Note, from Proposition 3.2, $\zeta = \mu$. The information will be used in period 1, if: $\eta + \delta\zeta > \mu + \delta\eta = \zeta + \delta\eta$. That is, when $\delta < 1$ or $\alpha > \beta$. □

So far, there is no leakage of information. Now suppose, that the uninformed can know the true state before trading opens in period 2 with probability ρ. The strategic choices within period 2 will not be affected, the only difference being that if prices were not revealing in period 1, then the informed would face an 'informed' with probability ρ, and with the complementary probability will still face an uninformed. In addition, suppose $\delta > 1$, so that in the absence of information leakage, the informed would wait to exploit information in period 2. In period 1, the informed faces an uninformed agent and he has to make a choice on whether to use the information in period 1 or in period 2, and is not certain what type of opponent he will face in period 2. This choice will be affected by the size of ρ.

Proposition 3.4. *Suppose information leaks out with probability $\rho > 0$, and that $\delta > 1$, then the informed will choose to reveal the information in period 1 if*

$$\frac{(\delta - 1)}{\delta} < \rho. \tag{11}$$

Proof. The informed would rather use information in period 1 than period 2 if

$$\eta + \delta\zeta > \mu + \delta(1 - \rho)\eta + \rho\delta\zeta.$$

Using $\zeta = \mu$, and the definition of δ this can be simplified to the desired inequality. $\square$

The intuition is that if there is a sufficiently high probability that the information will become worthless in the next period, then there will be an incentive to use it in this period, making the prices revealing at the first stage itself. The interesting feature is that the result does not depend on the form of the utility function but only on the discount parameter δ.

Example 3.5. Consider the economy above. Sellers have 20 units of the good in each period. Informed buyers (I) derive utility only from consuming in state 1 and the utility function is

$$u^I = [\tfrac{1}{2}(\alpha \log x_1^{I,1} + w_1^{I,1}) + \tfrac{1}{2} w_2^{I,1}] + [\tfrac{1}{2}(\beta \log x_1^{I,2} + w_1^{I,2}) + \tfrac{1}{2} w_2^{I,2}].$$

The uninformed (U) have the same preferences.

$$u^U = [\tfrac{1}{2}(\alpha \log x_1^{U,1} + w_1^{U,1}) + \tfrac{1}{2} w_2^{U,1}] + [\tfrac{1}{2}(\beta \log x_1^{U,2} + w_1^{U,2}) + \tfrac{1}{2} w_2^{U,2}].$$

The buyers have zero holdings of the good, and M units of 'money' in each period. Look at the benchmark economy where $\rho = 0$.

Consider the case that the uninformed have remained as such. In this case the informed will make zero bids in state 2 in period 2. The second period pay off to informed trader is

$$\pi^{I,2} = \frac{1}{2}\left(\beta \log \frac{b^{I,2}}{p_1^2} + M - b^{I,2}\right) + \frac{1}{2} M = \frac{1}{2}\left(\beta \log \frac{b^{I,2}}{p_1^2} - b^{I,2}\right) + M.$$

For the uninformed, the bidding is independent of state. Thus, set $b_1^{U,2} = b_2^{U,2} = b^{U,2}$.

$$\pi^{U,2} = \frac{1}{2}\left(\beta \log \frac{b^{U,2}}{p_1^2} + M - b^{U,2}\right) + \frac{1}{2}(M - b^{U,2}) = \frac{1}{2}\beta \log \frac{b^{U,2}}{p_1^2} - b^{U,2} + M.$$

Where $p_1^2 = \frac{b_1^{I,2} + b^{U,2}}{20}$ and $p_2^2 = \frac{b^{U,2}}{20}$.

The equilibrium bids can be derived for the two agents from the first-order conditions as:

1. For I, $\beta b^{U,2} = b_1^{I,2}(b^{U,2} + b_1^{I,2})$.
2. For U, $\beta b^{I,2} = 2b^{U,2}(b^{U,2} + b_1^{I,2})$.

Suppose, $\beta = 20$, then solve for $b_1^{I,2*} = 8.28, b_2^{I,2*} = 0$ and $b^{U,2*} = 5.84$. Given this, the allocations of the commodity are $x_1^{I,2*} = 11.728$, $x_1^{U,2*} = 8.272$, and $x_2^{I,2*} = 0$, $x_2^{U,2*} = 20$. The payoffs in period 2 are: $\pi^{I,2*} = 20.368$, and $\pi^{U,2*} = 15.286$.

Now consider the case where both agents are informed. In this case the payoff function to both is:

$$\pi^{n,2} = \frac{1}{2}\left(\beta \log \frac{b_1^{n,2}}{p_1^2} - b_1^{n,2}\right) + M.$$

The equilibrium bids can be computed as $b_1^{n,2*} = 10$, $b_2^{n,2*} = 0$, and the payoff to both is

$$\tfrac{1}{2}(20 \log \tfrac{10}{20} 20 - 10) + M.$$

For both traders this is equal to 18.026.

Now consider period 1 in isolation, ignoring the effect of trades on the revelation of information through prices. First, consider the case where the informed chooses to act as such. Then we are in a similar situation to the first case above. The payoff functions for the two traders will look similar except that the coefficient will be α rather than β. If we set $\alpha = 10$, then

1. $b_1^{I,1*} = 4.142$, $b_2^{I,1*} = 0$, and $\pi^{I,1*} = 10.234$.
2. $b_1^{U,1*} = b_2^{U,1*} = 2.928$, and $\pi^{U,1*} = 7.643$.

If the informed choose to act as uninformed in the first period, i.e., choose a strategy of bidding an equal positive amount in each state, then the payoff in the first period for the two traders will be

$$\pi^{n,1} = \frac{1}{2}\alpha \log \frac{b^1}{p_1^1} - b^1 + M.$$

This can be solved for $b_1^{n,1*} = b_2^{n,1*} = 2.5$, and $\pi^{n,1*} = \frac{1}{2} 10 \log \frac{2.5}{5} 20 - 2.5 + M = 9.013$.

Thus, payoff for the players given their configuration in period 1 is:

	Informed	Uninformed
Informed	NA	10.234,7.643
Uninformed	7.643, 10.234	9.013, 9.013

Similarly in period 2,

	Informed	Uninformed
Informed	18.026, 18.026	20.468, 15.286
Uninformed	15.286, 20.468	NA

Thus, informed can compute their payoff with disclosure of information (trading like informed in the first period), and payoff without disclosure (trading like an uninformed trader in the first period).

1. Payoff with disclosure $= 10.234 + 18.026 = 28.26$.
2. Payoff without disclosure $= 9.013 + 20.468 = 29.481$.

Thus, the unique equilibrium has the informed choosing not to disclose information in first period.

Now suppose that with probability ρ the uninformed can costlessly acquire information in period 2. The game otherwise is the same as before. To compute the equilibrium, all one has to do is take into account that if the informed did not disclose information through trades, with probability ρ in the next period the uninformed will be informed as well, and with probability $1-\rho$ will remain uninformed. Thus, the informed has to compute the payoff of non-disclosure and disclosure and choose the optimal strategy accordingly:

$$\text{Utility of non-disclosure} > \text{Utility of disclosure},$$

$$9.013 + (1-\rho)20.468 + \rho 18.026 > 10.234 + 18.026,$$

$$\frac{1.221}{2.442} > \rho,$$

$$0.5 > \rho.$$

4. Costly information

We maintain the structure and details of the game in Section 3, except that the information acquisition is changed. The 'informed' agent has the option to purchase the information before the first period of trading at a cost ϕ. Only they have the option to purchase the information. If the buyer chooses not to purchase the information before the first round of trading, the option lapses, and he does not have access to it before the second round. Thus, the acquisition of information is endogenous.

First consider the benchmark case without information leakage. The 'informed' buyer will find it worthwhile to purchase information if the cost of purchasing it is less than the gain from using it. If the information is purchased, it will never be used in the first round of trading if $\alpha < \beta$. Assume that this is the case, i.e., $\delta > 1$. The amount that he would be at most willing to pay is: Payoff when informed $-$ Payoff when not informed. This is equal to $(\mu + \delta\eta) - (\mu + \delta\mu) = \delta(\eta - \mu)$. Note, that this would also be the same amount he would be willing to pay if he had the option to purchase the information before the second round of trading.

Now suppose that with probability ρ the uninformed (only) can acquire information before the second round of trading. One can either interpret it as the information is leaked to them costlessly with probability ρ, or that with probability ρ the information is sold to them at a low cost such that they find it worthwhile to acquire the information if it is made available to them. From the viewpoint of the 'informed' what is important is that in the second period with probability ρ they face an informed rather than an uninformed buyer. The buyer who has foregone the option to purchase the information in the first period has no access to it. In this case would the informed purchase the information? We assume configurations such that the trader would wait to use information in the second period, and that the probability of leakage is not high enough to force revelation in the first period itself.

Proposition 4.1. *Suppose that $\delta > 1$; and $\frac{(\delta-1)}{\delta} > \rho$. Then there exists a ϕ' with $0 < \phi'$ such that if the cost of information is greater than ϕ', the 'informed' will not want to purchase the information.*

Proof. If the trader purchased the information then he will not want to use it in period 1 (as $\delta > 1$). The payoff from purchasing information and using it in period 2 is: $\mu + (1-\rho)\delta\eta + \rho\delta\zeta - \phi$ (note that $\zeta = \mu$). The payoff from not purchasing information is: $\mu + (1-\rho)\delta\mu + \rho\delta\kappa$, where the κ is the payoff of an uninformed trading against an informed. Thus, if $\mu + (1-\rho)\delta\eta + \rho\delta\zeta - \phi < \mu + (1-\rho)\delta\mu + \rho\delta\kappa$, then the trader will not find it worthwhile to purchase the information. This can be solved to obtain the threshold $\phi' \geqslant \rho\delta(\mu - \kappa) + (1-\rho)\delta(\eta - \mu) > 0$. □

We would like to know if $\phi' < \delta(\eta - \mu)$. Without further structure on preferences, this is difficult to check. However, the following example shows that a very small ρ and $\phi' < \delta(\eta - \mu)$ can be chosen.

Example 4.2. Consider the same economy as in Example 1. In this example, if $\phi < 29.481 - 27.039 = 2.442$, and there is no leakage of information, the buyers who have the option of purchasing the information will exercise it. In this scenario, they will also have the incentive not to use the information in the first round of trading. We also know from the previous example, that if $\rho < 0.5$, and if information was costless, the informed would not have disclosed information in period 1. Now, set $\rho = 0.05$, and $\phi = 2.4$. Will they still purchase the information when there is information leakage?

The utility from purchasing information and not disclosing it in the first round of trading is $9.013 + 0.95(20.468) + 0.05(18.026) - 2.4 = 26.9589$. The utility from purchasing and disclosing information in the first round of trading is $10.234 + 18.026 - 2.4 = 25.470$. If the buyer does not purchase the information in the first period then with a probability ρ he is the uninformed and the other buyer is informed, while with probability $1 - \rho$ both remain uninformed. The utility from not purchasing information: $9.013 + 0.95(18.026) + 0.05(15.286) = 27.024$.

Thus, even if there is a disadvantage to being the uninformed in the second period, its cost is outweighed by the cost of purchasing the information in the first place—as with probability ρ the information is worthless in the second period. The equilibrium has no purchase of information in the first period. The scenario is somewhat similar to the example of Grossman and Stiglitz [11] where no one purchases the information when there is a potential gain to purchasing it. With a very high probability (in this example 0.95) the markets will not be informationally efficient as no one will acquire the information.

What happens if the leakage of information is public in period 2? That is if the 'informed' had decided not to purchase information in the first period, and if information leaks, he also becomes informed and thus is not at a disadvantage.

Proposition 4.3. *Suppose that* $\delta > 1$; *and* $\frac{(\delta-1)}{\delta} > \rho$. *Then there exists a* ϕ'' *with* $0 < \phi''$ *such that if the cost of information is greater than* ϕ'', *the trader will not want to purchase the information.*

Proof. The payoff from purchasing information is: $\mu + (1-\rho)\delta\eta + \rho\delta\zeta - \phi$. The payoff from not purchasing information is: $\mu + \rho\delta\zeta + (1-\rho)\delta\mu = \mu + \delta\mu$. The trader will not purchase information if: $\mu + (1-\rho)\delta\eta + \rho\delta\zeta - \phi < \mu + \delta\mu$. This can be simplified to derive a threshold $\phi'' = \delta(1-\rho)(\eta - \mu)$. $\square$

Corollary 4.4. *The threshold cost of information* ϕ'' *is decreasing in the probability of information leakage* ρ.

5. Sale of information

So far the sale of information has not been modelled. One would want to know that if the person with access to information had a choice to sell the information whether he would be willing to sell it, and how the incentive to sell the information is affected by the potential leakage of the information. Admati and Pfleiderer [1] show that an 'informed' agent may wish to sell information if he is risk-averse (to permit better risk-sharing amongst the informed). Here the mechanism is slightly different.

We focus on the informed trader who has to decide whether to sell the information. The structure of the earlier game is modified to model this choice. First, look at the benchmark case of $\rho = 0$.

If the informed sells information at cost θ in period 1, the uninformed agent will immediately use the information, and the information will become public in period 2. Recognizing this, the informed will also want to use the information in the first stage. Thus, there will be two informed traders in the first period, and there will be no informational advantage in the second period as well. In the absence of information leakage, the maximum that can be charged for the information in period 1 is the loss to the uninformed from not possessing the information. That is, $(\zeta + \delta\zeta) - (\mu + \delta\kappa) = \delta(\mu - \kappa)$ if $\delta > 1$. He will want to sell the information if the payoff from selling information is higher than not selling it, i.e. if, $\zeta + \delta\zeta + \theta > \mu + \delta\eta$, or if $\delta(\mu - \kappa) > \delta(\eta - \mu)$. The information can also be sold in period 2 instead of period 1 if it has not been already used in period 1. The condition for the sale of information in this case is exactly the same as above so the informed is indifferent when to sell the information.

If we have $\delta < 1$, then it is in the interest of the informed to use the information in the first period. Thus, he will sell the information if $\zeta + \delta\zeta + \theta > \eta + \delta\mu$, or if $(\mu - \kappa) > (\eta - \mu)$. Thus, we have the following proposition.

Proposition 5.1. *The informed will sell the information to the uninformed if* $(\mu - \kappa) > (\eta - \mu)$.

Now suppose that information can leak out to the uninformed before trading in the second period. In this case the incentives to sell information may be affected. It turns out that in fact the condition to sell the information is not affected, but the price at which it can be sold is.

Proposition 5.2. *If there is leakage of information with probability* ρ, *then the informed will sell the information to the uninformed if* $(\mu - \kappa) > (\eta - \mu)$. *In this case, the maximal price that can be charged is* $\delta(1-\rho)(\mu-\kappa)$ *which is decreasing in* ρ. *The expected payoff of the informed is thus, also decreasing in* ρ.

Proof. First consider the case where $\delta > 1$. The payoff from selling information is: $\zeta + \delta\zeta + \theta$, and from not selling information is $\mu + \delta\rho\mu + \delta(1-\rho)\eta$. Information will be sold if $\theta > \delta(1-\rho)(\eta-\mu)$. The maximum amount the uninformed is willing to pay is $\theta = (\zeta + \delta\zeta) - (\mu + \rho\delta\mu + \delta(1-\rho)\kappa = \delta(1-\rho)(\mu-\kappa)$. Thus, the informed will find it worthwhile to sell information if $(\mu-\kappa) > (\eta-\mu)$. A similar argument applies to the case of $\delta < 1$. □

6. Conclusion

The efficient market hypothesis can be broken up into two parts [16]:

A. Prices reflect all available information.
B. Uninformed do not lose due to informational disadvantage.

We see from the examples above that in the case of costless information:

1. If there is no information leakage the informed may not disclose information in period, so that (A) and (B) fail.
2. However, if there is a potential for uninformed to acquire information at a later date, then prices are likely to be revealing so that (A) is true, but not (B).

Thus, the potential for information leakage forces early disclosure of information. In the case of costly information:

1. If there is no information leakage, then agents will want to purchase costly information, so that (A) and (B) fail.

2. However, if information can become available at a later date then no one may want to pay to acquire the information, so that (A) can fail, but (B) may not.

This paper points out that the efficiency of market prices will depend on what are the underlying parameters of the model, and obtaining a general statement is problematic. There are some issues that bear further investigation such as allowing for the inventorying of the goods so that they are more like assets, as well as allowing traders to be on both sides of the market.

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