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How Stock Splits Affect Trading: A Microstructure Approach

David Easley, Maureen O'Hara, and Gideon Saar*

Abstract

Extending an empirical technique developed in Easley, Kiefer, and O'Hara (1996), (1997a), we examine different hypotheses about stock splits. In line with the trading range hypothesis, we find that stock splits attract uninformed traders. However, we also find that informed trading increases, resulting in no appreciable change in the information content of trades. Therefore, we do not find evidence consistent with the hypothesis that stock splits reduce information asymmetries. The optimal tick size hypothesis predicts that stock splits attract limit order trading and this enhances the execution quality of trades. While we find an increase in the number of executed limit orders, their effect is overshadowed by the increase in the costs of executing market orders due to the larger percentage spreads. On balance, the uninformed investors' overall trading costs rise after stock splits.

I. Introduction

Stock splits remain one of the most popular and least understood phenomena in equity markets. With the bull market of the nineties pushing stock prices to historic levels, stock splits have also soared, reaching a record level of 235 on the NYSE in 1997. The traditional wisdom is that stock splits are "good information"—that companies split their stocks when they are confident that earnings momentum will continue to push their stock's price upward. The positive stock price reaction accompanying the announcement of a split (e.g., Grinblatt, Masulis, and Titman (1984); Lamoureux and Poon (1987)) gives credence to this optimistic view. Yet, why a split per se is necessary is unclear since there is no bound limiting a stock's price level, and alternative signaling devices (such as dividend increases) are used extensively. Moreover, empirical research has documented a wide range of negative effects such as increased volatility, larger

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proportional spreads, and greater transaction costs following splits.¹ On balance, it remains a puzzle why companies ever split their shares.

A number of explanations for stock splits have been proposed in the literature. The trading range hypothesis (Copeland (1979)) argues that firms prefer to keep their stock price within a particular (lower) price range. This preference may be because of a specific clientele they wish to attract or a particular dispersion in ownership they wish to achieve, but in either case it reflects the view that greater liquidity for stocks may arise in certain price ranges than in others. The clientele preferring a lower price range is usually thought to be uninformed or small investors. Evidence of an enlarged ownership base and an increase in the number of small trades, particularly small buy orders submitted by individuals, after a split lends support to this hypothesis.²

But why should firms want to attract such a clientele? Different reasons have been put forward but none has received substantial support. One explanation is that small investors are good for market stability (Barker (1956); Stovall (1995)). Overwhelming evidence that return volatility increases after splits casts doubt on this explanation.³ Another explanation is that a self-serving management wants diffused ownership since small investors cannot exercise too much control (Powell and Baker (1993/1994)). Empirical research finds clear evidence, however, that institutional ownership increases, rather than decreases, after splits (see Maloney and Mulherin (1992); Powell and Baker (1993/1994)).

Yet another explanation receiving a great deal of attention in the literature is that this new, enlarged clientele provides better liquidity and thereby reduces the cost of trading (and investing) in the stock. Evidence for this explanation seems to be mixed. For example, while some papers that use volume to proxy for liquidity find that it decreases after a split, others report that it does not change. Another proxy of liquidity, the number of trades, was found to increase after splits. Proportional spreads and effective spreads have been found to increase, suggesting worsened liquidity. While the majority of studies find a reduction in liquidity following splits, the evidence seems sensitive to the proxy used and to the time horizon considered.⁴

A second hypothesis about stock splits involves the reduction of informational asymmetries. This argument has many variants, but most versions posit that splits reduce informational asymmetries either by directly signaling good information that previously was privately known or simply by attracting greater

¹Splitting also increases the fees companies pay to have their shares listed on exchanges. In particular, NYSE exchange fees are assessed on "shares outstanding" so that doubling shares in a stock split has a direct cost to the company.

²See Lamoureux and Poon (1987); Maloney and Mulherin (1992); Kryzanowski and Zhang (1996); Muscarella and Vetsuypens (1996); Angel, Brooks and Mathew (1997); Desai, Nimalendran, and Venkataraman (1998); Lipson (1999); and Schultz (2000).

³See Ohlson and Penman (1985); Dravid (1987); Lamoureux and Poon (1987); Dubofsky and French (1988); Sheikh (1989); Conroy, Harris, and Benet (1990); Dubofsky (1991); Angel, Brooks and Mathew (1997); Desai, Nimalendran, and Venkataraman (1998); Koski (1998); and Lipson (1999).

⁴See Copeland (1979); Lakonishok and Lev (1987); Lamoureux and Poon (1987); Conroy, Harris, and Benet (1990); Gray, Smith, and Whaley (1996); Kryzanowski and Zhang (1996); Muscarella and Vetsuypens (1996); Arnold and Lipson (1997); Desai, Nimalendran, and Venkataraman (1998); Lipson (1999); and Schultz (2000).

attention to the firm (see Grinblatt, Masulis, and Titman (1984); Brennan and Copeland (1988); Brennan and Hughes (1991)). The evidence on the reduction in the extent of information asymmetry following splits is also mixed. Brennan and Copeland (1988) find that the firm's choice of the number of shares outstanding is related to the abnormal announcement return, in line with their signaling model. Brennan and Hughes (1991) document a positive relation between the change in analyst following and the split factor that is consistent with the split attracting analysts' attention. But does this, in turn, reduce informational asymmetries? Evidence in Desai et al. (1998) suggests that it does not. Those authors use a spread decomposition procedure to show that the adverse selection component of the spread increases after a split. Thus, it is unclear whether the information environment of the firm following a split is characterized by higher or lower information asymmetry.⁵

Finally, a third hypothesis about stock splits suggests that splits arise to create an optimal tick size for stocks. This hypothesis (Harris (1996), Angel (1997)) argues that the increase in proportional spreads typically accompanying a split induces greater participation by liquidity providers. This can occur because some uninformed traders, previously not in the market, now find it profitable to supply liquidity via limit orders. Alternatively, liquidity can increase because some uninformed traders shift from using market orders to the now more attractive limit orders. In either case, the increased liquidity should enhance the overall execution of trades in the stock.⁶ There has been little empirical work to date examining this hypothesis. Angel (1997) provides evidence that limit orders on the NYSE are used less frequently in stocks with a smaller relative tick size. Arnold and Lipson (1997) find an increase in the number of executed limit orders and an increase in the proportion of executed limit orders relative to market orders following a split. Lipson (1999) uses NYSE system order data to show that the depth available in the limit order book at various split-adjusted distances from the mid-quote declines substantially after stock splits. However, he finds little evidence of a change in the execution costs when both market and limit orders are considered.

⁵Ikenberry, Rankine, and Stice (1996), p. 360 suggest another possible variant of the information asymmetry story by combining it with a trading range explanation. They argue that "if managers believe there are benefits from shares trading within some price range, yet also perceive that it is costly to trade below a lower limit, then the decision to split will be made conditional on the manager's expectations about future performance." They interpret the pre-split increase in prices (e.g., less than 3% of their sample firms have pre-split prices below the median for firms of similar size) as consistent with the firm wanting to return to a desired trading range, and the long-run excess return following the announcement period (e.g., 12.15% in the first three years) as consistent with the signaling of good news.

⁶Angel (1997) provides yet another explanation of stock splits that builds on Brennan and Hughes (1991). While, in their model, higher brokerage commissions prompt greater analyst coverage and intensified marketing efforts by brokers, Angel's idea is that the larger tick size provides incentives to the promoting firms through their market making activities. Hence, a higher tick size induces a wealth transfer from small investors who use market orders to the financial industry (and existing shareholders who benefit from the rise in the price of the stock that accompanies the marketing efforts). The overall trading costs of investors under this explanation can change in any direction (or not change at all) since they depend on the mix of market and limit orders investors use to execute their trading strategies. Anshuman and Kalay (1998) also note that setting an optimal tick size can cause informed traders to invest less in acquiring information and can thus lower the uninformed population's trading costs.

While all of these hypotheses—the trading range, the reduction of information asymmetry, and the optimal tick size—seem plausible, there is no consensus as yet on which, if any, is correct. This confusion remains, despite the extensive empirical analyses, for several reasons. One is simply that these theories are often quite broad, and their implications may be difficult to delineate. But even when specific hypotheses are distilled from the theories, the unobservability of informational asymmetries and of the composition of the trading population makes testing these hypotheses problematic.

In particular, most empirical analyses must rely on proxies that may be sufficiently noisy as to make interpretation of the results difficult. For example, using small trades to proxy for uninformed order flow ignores the ability of informed traders to split their orders. Using variance decomposition and spread decomposition procedures to derive proxies for information asymmetry relies on price (or quote) data, which in turn is affected by the split. Factors such as discreteness, clustering, and other institutional features can affect the estimates of these proxies, even if the underlying environment does not change. Similarly, while the tick size argument relies on greater participation of limit order traders, measuring the cost of trading has to take into account the strategies of traders who use both market and limit orders. Without knowledge of these strategies, an increase in the number of limit orders does not necessarily mean that the cost of trading declines.

In this paper, we evaluate the alternative hypotheses about stock splits by examining their implications for trading in the stocks. Each of these hypotheses envisions a different impact of splits on the underlying trading environment, and in this research we estimate these effects for a sample of splitting stocks. What underlies our ability to provide new and comprehensive results in a unified framework is our explicit use of a microstructure model. The model provides a framework for the statistical tests and aids in the interpretation of the results. We extend a technique developed in Easley, Kiefer, and O'Hara (1996), (1997a) and Easley, Kiefer, O'Hara, and Paperman (1996) to estimate the underlying parameters that define trade activity. These parameters include the rates of informed and uninformed trading, the probability of information events, and the propensity to execute trading strategies using limit orders.

Focusing our analysis on these trading parameters provides an effective way to differentiate between the hypothesized effects of particular theories, but it does so at the cost of simplifying away some of the non-trade related implications of these explanations. In particular, any signaling-related effects of splits may be immediately reflected in prices, and so would not be reflected in trades. Thus, our analysis is best viewed as providing evidence in support or against particular theories, and not as the definitive test of what causes stock splits.

We do find several important results. We show that uninformed trading increases following splits, and that there is a slightly increased tendency of uninformed buyers to execute trades using market orders. These effects are consistent with the entry of a new clientele and, thus, are supportive of the trading range hypothesis. We do not find any significant increase in liquidity, however, in part because we find that informed traders intensify their activity as well. We find no effects of splits on the probability of new information or on its direction and,

hence, we find little evidence to support this aspect of the informational asymmetry argument.

The increase in both uninformed and informed trading activity results in a very small decrease in the extent of the adverse selection problem. This finding contrasts with that of Desai, Nimalendran, and Venkataraman (1998) who, using a spread decomposition methodology, concluded that the adverse selection component of the spread increased following the split. Our trading model shows that the increase in relative spreads documented in the literature is not due to increased adverse selection, but rather to an increase in the underlying volatility of the stocks. These wider spreads are consistent with the premise of the optimal tick size hypothesis, but our finding that the overall trading costs of the uninformed traders increase after splits seems inconsistent with the enhanced liquidity that should follow according to this explanation. More specifically, we find that while limit order trading does increase, this increase is not sufficient to compensate the uninformed traders for the increase in the bid-ask spread and the more intense usage of market buy orders by uninformed traders. Therefore, we are able to show that a rise in the trading costs of uninformed traders can be consistent with both increased uninformed trading and increased limit order activity.

This paper is organized as follows. The next section describes our data set, the continuous time sequential trade model that we estimate, and the testing techniques. Section III details the implications of the different hypotheses about stock splits for the parameters of our model and presents the results. Section IV discusses the implications of our findings for the question of why firms split their stocks and offers concluding remarks.

II. Data and Methodology

A. Sample

Our basic sample is comprised of all NYSE common stocks that had two to one splits in 1995. We focus on NYSE stocks since the market microstructure model we use for the estimation describes pricing in a specialist-operated market like the NYSE. In addition, stocks that are traded on different exchanges may exhibit different patterns before and after splits (Dubofsky (1991)). By restricting our sample to NYSE stocks, we neutralize any nuisance effects introduced by the trading locale. We look only at two to one splits to make our sample as homogenous as possible. Using only one split factor, we can safely compare the firms in our sample to one another.⁷

The NYSE's Fact Book (1995) reports that there were 75 stocks with two to one splits in 1995. We supplemented information about announcement dates and ex-split dates from Standard & Poor's Stock Reports, Moody's Annual Dividend

⁷McNichols and Dravid (1990) argue that firms signal private information about future earnings by their choice of a split factor. This suggests that firms with different split ratios may have different motivations for splitting their stocks. While an interesting issue, these effects are unlikely to be reliably reflected in our trade parameters and, hence, we restrict our analysis to the most commonly used split factor.

Record, and the CRSP database.⁸ One firm that switched exchanges in the middle of the period was eliminated from the sample. Because a reasonable amount of trading is required for our estimation procedure to produce reliable parameters, we eliminated stocks from the sample that had entire days with no trading. Only one firm (with two series of stocks) had insufficient trading. Hence, the sample used in our empirical work consists of 72 stocks. Summary statistics about the firms in the sample are reported in Table 1. It is clear from the table that the sample is quite heterogeneous with regard to market capitalization, trading intensity, and prices. The changes between pre- and post-split periods in terms of the daily number of trades, the average trade size, and the proportional spreads are similar to those reported by others.

TABLE 1
Summary Statistics

Panel A.

| | Market Capitalization (millions) | Average Daily No. of Trades | | Average Size of Buy Orders | | Average Size of Sell Orders | |
|------------|----------------------------------|-----------------------------|------------|----------------------------|------------|-----------------------------|------------|
| | | Pre-Split | Post-Split | Pre-Split | Post-Split | Pre-Split | Post-Split |
| Mean | \$5,847 | 97.52 | 158.77 | \$81,480 | \$64,658 | \$82,553 | \$77,955 |
| Std. Dev. | \$9,366 | 109.47 | 198.82 | \$50,797 | \$44,295 | \$46,639 | \$47,378 |
| 100% (Max) | \$43,216 | 551.84 | 1155.29 | \$226,879 | \$202,282 | \$206,889 | \$204,982 |
| 75% (Q3) | \$6,753 | 126.17 | 185.80 | \$110,757 | \$87,384 | \$112,680 | \$105,589 |
| 50% (Med) | \$1,947 | 48.89 | 83.73 | \$75,894 | \$56,978 | \$81,971 | \$72,548 |
| 25% (Q1) | \$1,028 | 27.74 | 34.67 | \$39,325 | \$32,111 | \$43,498 | \$35,673 |
| 0% (Min) | \$245 | 6.02 | 7.56 | \$14,908 | \$10,891 | \$15,111 | \$13,426 |

Panel B.

| | Average Closing Price | | Average Dollar Opening Spread | | Average % Opening Spread | |
|------------|-----------------------|------------|-------------------------------|------------|--------------------------|------------|
| | Pre-Split | Post-Split | Pre-Split | Post-Split | Pre-Split | Post-Split |
| Mean | \$54.90 | \$34.28 | \$0.326 | \$0.289 | 0.68% | 0.99% |
| Std. Dev. | \$19.99 | \$14.72 | \$0.072 | \$0.049 | 0.29% | 0.39% |
| 100% (Max) | \$127.86 | \$82.06 | \$0.708 | \$0.394 | 1.71% | 2.14% |
| 75% (Q3) | \$68.03 | \$41.97 | \$0.358 | \$0.329 | 0.85% | 1.31% |
| 50% (Med) | \$47.67 | \$32.29 | \$0.321 | \$0.288 | 0.67% | 0.97% |
| 25% (Q1) | \$40.59 | \$23.57 | \$0.281 | \$0.249 | 0.44% | 0.64% |
| 0% (Min) | \$21.15 | \$11.67 | \$0.255 | \$0.189 | 0.20% | 0.31% |

This table contains summary statistics for the firms used in the study. The sample consists of 72 firms. To be included in the sample, a firm had to be listed on the NYSE and have a 2 to 1 stock split in 1995. The pre-split period consists of 45 days and ends 20 days before the announcement date of the split. The post-split period consists of 45 days and starts 20 days after the ex-date of the split. Panel A provides descriptive statistics on market capitalization and trading. The variables are calculated for each stock separately before the cross-sectional sample statistics are computed. Market capitalization of firms in the sample is taken from the CRSP database and is defined as their value at the end of 1995. Data on the average size of buy and sell orders and the daily number of trades is taken from the TAQ database. Only NYSE trades are considered. The classification into buy and sell orders is done using the Lee and Ready (1991) algorithm. Panel B provides descriptive statistics on prices and spreads. The variables are calculated for each stock separately before the cross-sectional sample statistics are computed. The opening spread is the difference between the bid and the ask of the opening quote on the NYSE. Proportional spreads are spreads divided by the mid-quote.

⁸Missing dates and a few conflicts were resolved using the actual news reports from the Dow Jones News Retrieval.

B. Estimation Windows

Ideally, we would like to compare a stock's trade process in two steady states: one before and one after the split. A split announcement may change the market's perception of a firm, dictating that the pre-split estimation period must precede the announcement date. Previous research shows an abnormal increase in trading activity beginning 10 days prior to the split announcement (Maloney and Mulherin (1992)), so we end the pre-split estimation period 20 days prior to the announcement day. In light of evidence that an abnormal imbalance of trades can last for about 10 days after the ex-date (Conrad and Conroy (1994)), our post-split estimation period begins 20 days after the ex-date.⁹ Prior work using maximum likelihood estimation of structural trading models (Easley, Kiefer, O'Hara, and Paperman (1996); and Easley, O'Hara, and Paperman (1998)) led us to choose 45 trading days for the length of both the pre- and post-split estimation periods.¹⁰

C. Data

Detailed trade data was obtained from the TAQ database. We use only NYSE transactions and quotes.¹¹ We exclude from our analysis the so-called "special" trades (e.g., irregular settlement periods, opening trades) and quotes (e.g., dissemination during trading halts), and we apply cleaning filters to check the trade and quote data for mistakes. Estimation of our model requires the daily number of market buy orders, market sell orders, limit buy orders, and limit sell orders. Information in the TAQ database does not specify whether the initiator of the trade bought or sold shares, nor does it identify which trades are limit orders. Hence, we use algorithms suggested by Lee and Ready (1991) and Greene (1997) to perform the relevant classifications.

The Lee and Ready (1991) algorithm is used to classify trades into buys and sells. Trades at prices above the midpoint of the bid-ask interval are classified as buys, and trades below the midpoint are classified as sells.¹² Trades that occur at the midpoint of the bid and ask are classified using the "tick test."¹³

The limit order (LO) algorithm from Greene (1997) is used to infer limit order execution.¹⁴ The LO algorithm uses the fact that NYSE rules require the specialist to update not just the prices but also the depths of his quotes to reflect

⁹Desai, Nimalendran, and Venkataraman (1998) also exclude from their tests the period starting 20 days before the announcement day and ending 20 days after the ex-date.

¹⁰If any split-induced effects are dissipated within the first 20 days, then our analysis will find no differences between the pre- and post-split parameters. However, such short-lived information effects would then seem to be very unlikely explanations for splitting the stock in the first place.

¹¹Our model is best viewed as describing the pricing problem of a specialist who extracts information from the order flow that she handles. In addition, previous research has shown that NYSE specialists in fact set over 90% of NYSE stock prices (Hasbrouck (1995) and Blume and Goldstein (1997)).

¹²We also adopt the suggestion made by Lee and Ready (1991) that if the prevailing quote is less than five seconds old at the time a trade was executed, the previous quote is used for the classification scheme.

¹³The tick test classifies trades executed at a price higher than the previous trade as buys, and trades executed at a lower price as sells. If the trade goes off at the same price as the last trade, then its price is compared to the previous most recent trade. This is continued until the trade is classified.

¹⁴The TAQ database contains only execution data and hence identification of limit order arrival to the specialist's book is impossible.

orders held in the limit order book. More specifically, the LO algorithm looks at the differences between two successive quotes. If the ask and bid remain the same but the depth on the ask decreases, the algorithm looks for a trade or trades that took place at the ask price after the first quote and before the second quote. It then classifies a portion of the trades equal to the difference in depths as having been executed against limit sell orders. If the ask price of the second quote is higher than that of the first quote, a portion of the intervening trades which were executed at the original ask price is said to have been executed against limit sell orders. A similar classification is applied to the bid side to identify the execution of limit buy orders.¹⁵ For a more detailed explanation of the algorithm, see Greene (1997). The two algorithms we use are consistent with each other in the sense that, if the LO algorithm identifies a trade as having been executed against a limit sell (buy) order, the trade will be classified by the Lee and Ready algorithm as a market buy (sell) order.¹⁶

D. Trading Model

In this section, we describe the sequential trade model used for the estimation. The mixed discrete and continuous time model extends Easley, Kiefer, and O'Hara (1996), (1997a) and Easley, Kiefer, O'Hara, and Paperman (1996) by admitting limit order use by uninformed traders.

1. Trade Process

Informed and uninformed individuals trade a single risky asset and money with a market maker over $i = 1, \dots, I$ discrete trading days. Within any trading day, time is continuous, and it is indexed by $t \in [0, T]$. The market maker stands ready to buy or sell one unit of the asset at her posted bid and ask prices at any time.¹⁷ Because she is competitive and risk neutral, these prices are set equal to the expected value of the asset conditional on her information at the time of trade and on the nature of the incoming order.

We define a private information event as the occurrence of a signal, either good information or bad information, about the future value of the asset that is not publicly observable. In effect, we define information events as private if they affect trading and public if they do not affect trading. Public information events may cause price changes, but little or no trade should be generated by a truly

¹⁵An implicit assumption that we make when using the algorithm is that limit orders are roughly the same size as market orders. Hence, each trade that is identified as having been executed against a limit order is counted as one limit order execution. The benefit we realize from using this algorithm in the present context is our ability to test the optimal tick size hypothesis directly by considering limit order execution before and after the stock split. This implementation of the algorithm implies that if the size of a typical market order changes, so does the size of a typical limit order. We believe that such an assumption is reasonable and by making it, we are able to use a very powerful tool in examining stock splits. We do not make a formal attempt in this paper to test this assumption.

¹⁶The LO algorithm also has the advantage that, in addition to limit orders, it probably reflects hidden limit orders and informal indications of interest given to specialists by floor brokers. These hidden limit orders influence the specialists' quoted prices and depths and, hence, are picked up by the LO algorithm.

¹⁷Since stock splits may affect the average trade size, we estimate our trade process model separately before and after a split. Trade size effects can be included and estimated via our technique, an issue we explicitly deal with in Easley, Kiefer, and O'Hara (1997b).

public information event. To the extent that seemingly public information events affect trade, they have a private component (such as understanding how to use a particular piece of information) and we classify them as private information events. The motivation for this distinction is that our empirical technique extracts information from trade data and so we cannot identify the events that we call public. We view information events as occurring, if at all, prior to the beginning of any trading day. Private information events are independently distributed across days, and they occur with probability α . These information events are good information with probability $1 - \delta$ or bad information with probability δ .

Let $(V_i)_{i=1}^I$ be the random variables representing the value of the asset at the end of trading days $i = 1, \dots, I$.¹⁸ We let the value of the asset conditional on good information on day i be \bar{V}_i , and the value of the asset conditional on bad information on day i be \underline{V}_i .¹⁹ The full information value of the asset is revealed after the end of trading every day. The value of the asset if there is no information on day i is denoted by V_i^* , where we define $V_i^* = \delta \underline{V}_i + (1 - \delta) \bar{V}_i$.²⁰

Orders to trade can arise from both informed traders (those who know the nature of the information event) and uninformed traders. A trader is allowed to send in one order at a time for a specified quantity of the stock. A trader may then continue to submit other orders throughout the trading day if he so desires.²¹ In actual markets, orders can be for varying sizes and, empirically, we look at the effects of splits on the average trade size. But, as Kyle (1985) notes, informed traders would be expected to choose the same trade sizes as uninformed traders, and so a change to the informed traders' strategy after a split (relative to the uninformed order flow) would be reflected in the arrival rate rather than the size of the trades.²²

We assume that uninformed traders can use either market or limit orders. A market order is an order to buy or sell a unit of the stock at the market maker's quoted price. A limit order is an order to buy or sell a unit of the stock if the market maker's price reaches a certain level. Traders use limit orders, in part, to avoid paying the bid-ask spread. For example, a trader who wants to buy the stock could enter a limit buy order at a price below the current ask to compete with the market maker for the next market sell order. Such a limit buy order may not execute as prices could move away from it but, if it does execute, the buyer

¹⁸These values will naturally be correlated. We do not make any specific assumptions about the correlations as they are not needed for our analysis.

¹⁹We assume that the random variables giving asset values are independent across firms. For this reason, the analysis in the text is done for a single firm. When we estimate the trading parameters of a firm, we do not use information on other firms to help in the estimation process.

²⁰This assumption is not required for our analysis (only $\bar{V}_i \geq V_i^* \geq \underline{V}_i$ is needed), however, the assumption makes the spread properties used in later sections more tractable.

²¹The structure of the sequential trade model naturally restricts a trader's ability to submit an infinite number of orders. This is because after a trader trades, he must essentially join the queue and wait for his turn to trade. Uninformed traders are less likely to trade again, while informed traders will continue to want to trade until the price has adjusted to the new information value. Thus, the order flow will be dominated by informed trade, and it is this preponderance of either buy or sell trades that reveals the private information to the market maker.

²²Easley, Kiefer, and O'Hara (1997b) explicitly allow for differential trade sizes and test for any preference across trade sizes by informed traders. They find only very weak trade size effects. As our analysis here separately estimates the pre- and post-split periods, we do not include multiple trade sizes.

pays a lower price than he would have paid with a market buy order. On any day without an information event, ε_B (ε_S) is the rate at which uninformed buy (sell) orders are executed. The fraction of uninformed limit buy (sell) orders is γ_B (γ_S), and the fraction of uninformed market buy (sell) orders is $1 - \gamma_B$ ($1 - \gamma_S$).²³

While the submission rate of uninformed limit orders is unrelated to the existence of any private information signal, their execution rates may depend on the underlying information because prices move in different directions on good-news, bad-news, and no-news days. On days with information events, we let β be the rate at which limit orders are executed (relative to the rate on no information days) when the orders are in the same direction as the information (e.g., buy orders on a good information day). Similarly, we let ρ be the rate at which limit orders are executed (relative to the rate on no information days) when the orders are in the opposite direction to the information (e.g., sell orders on a good information day).

While uninformed traders arrive every day, informed traders arrive only on days when there has been an information event. Informed traders are assumed to be risk neutral and competitive. A profit-maximizing informed trader buys the stock if he observes good information, and sells the stock upon observing bad information.²⁴ We assume that information arrives to one trader at a time, and the arrival of that trader to the market follows a Poisson process with rate μ .²⁵ An informed trader knows the direction in which the stock's price is eventually going to move. For example, on a good information day an informed trader expects quotes to rise on average over the day. If he uses a limit order in an attempt to buy the stock at a better price than the current ask, he would have to submit the order at a price below the current ask. But since prices will move upward on average, his order is unlikely to execute and this makes limit orders unappealing to informed traders. Hence, we assume that informed traders only use market orders.²⁶

The probability tree of the model is shown in Figure 1. Because days are independent, we can analyze the evolution of the market maker's beliefs separately on each day. Let $P(t) = (P_n(t), P_b(t), P_g(t))$ be the market maker's belief about the events "no information" (n), "bad information" (b), and "good information"

²³We model separately buying and selling to be able to test the different hypotheses about stock splits. For example, the trading range hypothesis postulates the entrance of a new clientele and hence will predict increased buying activity most likely using market orders.

²⁴We do not impose any restrictions on short selling. For sequential trade models with short sale constraints, see Diamond and Verrecchia (1987) and Saar (1999).

²⁵On any day, the Poisson processes used for the trading of uninformed and informed investors are assumed independent.

²⁶Glosten (1994) develops an example in which informed investors are allowed to use both limit and market orders. He shows that competition among informed traders and depreciation of the value of the private information due to public announcements will cause informed traders to use market orders. In a partial equilibrium setting, Angel (1994) and Harris (1998) also investigate the choice of informed traders between market and limit orders. Both papers find that informed investors will usually prefer market orders (especially if the informed traders are competitive and the horizon of the private information is short). Other theoretical papers that modeled both limit order trading and information asymmetry do not allow informed traders to use limit orders (e.g., Rock (1990) and Baruch (1999)). In Chakravarty and Holden (1995), informed traders can use both market and limit orders, but the market makers are not allowed to learn from limit orders. They show that informed traders use both types of orders when the true value of the stock is between the bid and the ask. In our model, the true value is never between the bid and the ask. Since we model many competing informed traders and the horizon of the private information is taken to be one day, it seems unlikely that informed traders will use limit orders.

(g) conditional on the history of trade prior to time t on day i . The expected value of the asset conditional on the history of trade prior to time t is thus

$$(1) \quad E[V_i|t] = P_n(t)V_i^* + P_b(t)\underline{V}_i + P_g(t)\bar{V}_i.$$

In our model, only uninformed traders use limit orders. So limit orders carry no information and their arrival does not affect quotes. Market orders are submitted by both informed and uninformed traders so their arrival does carry information to the market maker. At any time t , the zero expected profit bid price, $b(t)$, is the market maker's expected value of the asset conditional on the history prior to t and on the arrival of a market order to sell at t . Calculation shows that the bid at time t on day i is

$$(2) \quad b(t) = E[V_i|t] - \frac{\mu P_b(t)}{\varepsilon_S(1 - \gamma_S) + \mu P_b(t)} (E[V_i|t] - \underline{V}_i).$$

Similarly, the ask at time t , $a(t)$, is the market maker's expected value of the asset conditional on the history prior to t and on the arrival of a market order to buy at time t . Thus, the ask at time t on day i is

$$(3) \quad a(t) = E[V_i|t] + \frac{\mu P_g(t)}{\varepsilon_B(1 - \gamma_B) + \mu P_g(t)} (\bar{V}_i - E[V_i|t]).$$

When informed traders are present at the market ($\mu > 0$), the bid will be below $E[V_i|t]$ and the ask will be above $E[V_i|t]$. This spread arises since the market maker is setting prices to protect herself from expected losses to informed traders. The factors influencing the spread are easier to identify if we write the spread explicitly. Let $\Sigma(t) = a(t) - b(t)$ be the spread at time t . Then

$$(4) \quad \begin{aligned} \Sigma(t) = & \frac{\mu P_g(t)}{\varepsilon_B(1 - \gamma_B) + \mu P_g(t)} (\bar{V}_i - E[V_i|t]) \\ & + \frac{\mu P_b(t)}{\varepsilon_S(1 - \gamma_S) + \mu P_b(t)} (E[V_i|t] - \underline{V}_i). \end{aligned}$$

The spread at time t can be viewed in two parts. The first term is the probability that a market buy is information-based times the expected loss to an informed buyer, and the second is a symmetric term for sells. For example, the percentage spread of the opening quotes is

$$(5) \quad S^{\text{TH}} = \frac{\Sigma(0)}{V^*} = \mu \sqrt{\alpha \delta (1 - \delta)} \left[\frac{1}{\varepsilon_B(1 - \gamma_B) + \mu \alpha (1 - \delta)} + \frac{1}{\varepsilon_S(1 - \gamma_S) + \mu \alpha \delta} \right] \sigma_V,$$

where σ_V is the standard deviation of the percentage changes in the value of the asset. This standard deviation reflects the magnitude of the potential loss to the market maker from trading with informed traders. The remaining terms in the spread equation reflect the chances of encountering (or the risk of trading with) an informed trader.

FIGURE 1
Probability Tree of the Trading Model

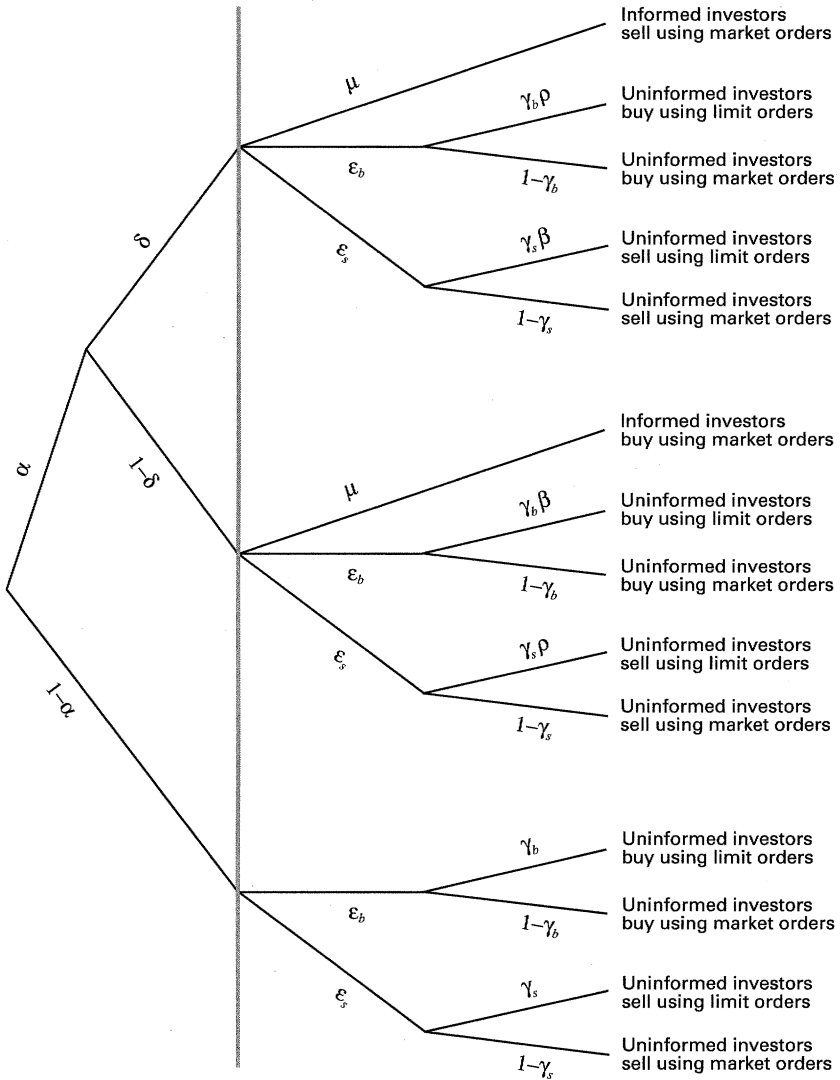


Figure 1 presents the probabilities associated with the events and outcomes of the trading model. To the left of the gray line are events that take place once a day before the beginning of trading. An information event occurs with probability α , and if it occurs, the information is bad (good) with probability $\delta(1 - \delta)$. To the right of the gray line are events that take place during the trading day. $\varepsilon_B(\varepsilon_S)$ is the rate at which uninformed buy (sell) orders are executed and $\gamma_B(\gamma_S)$ is the fraction of uninformed limit buy (sell) orders. On days with information events, β is the rate at which limit orders are executed when the orders are in the same direction as the information and ρ is the rate at which limit orders are executed when the orders are in the opposite direction to the information (relative to the rate on no information days). μ is the arrival rate of informed traders.

On each day, order arrival follows one of three Poisson processes. We do not know which process is operating on any day, but we do know that the data reflect the underlying information structure, with more market buys expected on days with good events, and more market sells on days with bad events. Similarly, on no-event days, there are no informed traders in the market, and so fewer market orders arrive. These rates and probabilities are determined by a mixture model in which the weights on the three possible components (i.e., the three branches of the tree reflecting no information, good information, and bad information) reflect their probability of occurrence in the data.

2. Likelihood Function

We first consider the likelihood of order arrival on a day of a known type. Suppose we consider the likelihood function on a bad information day. Market sell orders arrive at a rate $(\mu + \varepsilon_S(1 - \gamma_S))$, reflecting that both informed and uninformed traders will be selling. Market buy orders arrive at rate $\varepsilon_B(1 - \gamma_B)$, since only uninformed traders buy when there has been a bad information event. Finally, limit sell orders are executed at rate $\varepsilon_S\gamma_S\beta$ and limit buy orders are executed at rate $\varepsilon_B\gamma_B\rho$. The exact distribution of these statistics in our model is independent Poisson. Thus, the likelihood of observing any sequence of order executions that contains MS market sells, MB market buys, LS limit sells and LB limit buys on a bad-event day is given by

$$(6) \quad e^{-(\mu + \varepsilon_S(1 - \gamma_S))} \frac{(\mu + \varepsilon_S(1 - \gamma_S))^{\text{MS}}}{\text{MS}!} e^{-(\varepsilon_B(1 - \gamma_B))} \frac{[\varepsilon_B(1 - \gamma_B)]^{\text{MB}}}{\text{MB}!} \\ \times e^{-(\varepsilon_S\gamma_S\beta)} \frac{[\varepsilon_S\gamma_S\beta]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B\rho)} \frac{[\varepsilon_B\gamma_B\rho]^{\text{LB}}}{\text{LB}!}.$$

Similarly, on a no-event day, the likelihood of observing any sequence of orders that contains MS market sells, MB market buys, LS limit sells, and LB limit buys is given by

$$(7) \quad e^{-\varepsilon_S(1 - \gamma_S)} \frac{(\varepsilon_S(1 - \gamma_S))^{\text{MS}}}{\text{MS}!} e^{-(\varepsilon_B(1 - \gamma_B))} \frac{[\varepsilon_B(1 - \gamma_B)]^{\text{MB}}}{\text{MB}!} \\ \times e^{-(\varepsilon_S\gamma_S)} \frac{[\varepsilon_S\gamma_S]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B)} \frac{[\varepsilon_B\gamma_B]^{\text{LB}}}{\text{LB}!}.$$

Finally, on a good event day, this likelihood is

$$(8) \quad e^{-\varepsilon_S(1 - \gamma_S)} \frac{(\varepsilon_S(1 - \gamma_S))^{\text{MS}}}{\text{MS}!} e^{-(\mu + \varepsilon_B(1 - \gamma_B))} \frac{[\mu + \varepsilon_B(1 - \gamma_B)]^{\text{MB}}}{\text{MB}!} \\ \times e^{-(\varepsilon_S\gamma_S\rho)} \frac{[\varepsilon_S\gamma_S\rho]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B\beta)} \frac{[\varepsilon_B\gamma_B\beta]^{\text{LB}}}{\text{LB}!}.$$

It is evident from (6), (7), and (8) that (MB, MS, LB, LS) is a sufficient statistic for the data. Thus, to estimate the order arrival and execution rates of the buy and sell processes, we need only consider the total number of market buys,

market sells, executed limit buys, and executed limit sells on any day. The likelihood of observing (MB, MS, LB, LS) on a day of unknown type is the weighted average of (6), (7), and (8) using the probabilities of each type of day occurring. These probabilities of a no-event day, a bad-event day, and a good-event day are, respectively, given by $1 - \alpha$, $\alpha\delta$, and $\alpha(1 - \delta)$, and so the likelihood is

$$\begin{aligned}
 (9) \quad L((\text{MB}, \text{MS}, \text{LB}, \text{LS})|\theta) = & \\
 & (1 - \alpha)e^{-\varepsilon_S(1-\gamma_S)} \frac{(\varepsilon_S(1-\gamma_S))^{\text{MS}}}{\text{MS}!} e^{-\varepsilon_B(1-\gamma_B)} \frac{[\varepsilon_B(1-\gamma_B)]^{\text{MB}}}{\text{MB}!} \\
 & \times e^{-(\varepsilon_S\gamma_S)} \frac{[\varepsilon_S\gamma_S]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B)} \frac{[\varepsilon_B\gamma_B]^{\text{LB}}}{\text{LB}!} \\
 & + \alpha\delta e^{-(\mu+\varepsilon_S(1-\gamma_S))} \frac{(\mu+\varepsilon_S(1-\gamma_S))^{\text{MS}}}{\text{MS}!} e^{-(\varepsilon_B(1-\gamma_B))} \frac{[\varepsilon_B(1-\gamma_B)]^{\text{MB}}}{\text{MB}!} \\
 & \times e^{-(\varepsilon_S\gamma_S\beta)} \frac{[\varepsilon_S\gamma_S\beta]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B\rho)} \frac{[\varepsilon_B\gamma_B\rho]^{\text{LB}}}{\text{LB}!} \\
 & + \alpha(1-\delta)e^{-\varepsilon_S(1-\gamma_S)} \frac{(\varepsilon_S(1-\gamma_S))^{\text{MS}}}{\text{MS}!} e^{-(\mu+\varepsilon_B(1-\gamma_B))} \frac{[\mu+\varepsilon_B(1-\gamma_B)]^{\text{MB}}}{\text{MB}!} \\
 & \times e^{-(\varepsilon_S\gamma_S\rho)} \frac{[\varepsilon_S\gamma_S\rho]^{\text{LS}}}{\text{LS}!} e^{-(\varepsilon_B\gamma_B\beta)} \frac{[\varepsilon_B\gamma_B\beta]^{\text{LB}}}{\text{LB}!}.
 \end{aligned}$$

For any given day, the maximum likelihood estimate of the information event parameters α and δ will be either 0 or 1, reflecting that information events occur only once a day. Estimation of the information event parameters, therefore, requires data from multiple days. Because days are independent, the likelihood of observing the data $D = (\text{MB}_i, \text{MS}_i, \text{LB}_i, \text{LS}_i)_{i=1}^I$ over I days is just the product of the daily likelihoods,

$$(10) \quad L(D|\theta) = \prod_{i=1}^I L(\theta|\text{MB}_i, \text{MS}_i, \text{LB}_i, \text{LS}_i).$$

3. Maximum Likelihood Estimation

For each stock in the sample, we estimate the parameter vector θ separately for the pre- and post-split periods by maximizing the likelihood function in (10) conditional on the stock's trade data. The probability parameters α , δ , γ_B , and γ_S are restricted to (0,1) by a logit transform of the unrestricted parameters, while the parameters μ , ε_B , ε_S , β , and ρ are restricted to $(0, \infty)$ by a logarithmic transform. We then maximize over the unrestricted parameters using the quadratic hill-climbing algorithm GRADX from the GQOPT package. To insure that we, in fact, find a global maximum for each stock, we run the optimization routine starting from many different points in the parameter space. Standard errors for the economic parameter estimates are calculated from the asymptotic distribution of the transformed parameters using the delta method.²⁷

²⁷For a discussion of the delta method, see Goldberger (1991), p. 102.

E. Statistical Tests

We examine the influence of stock splits by looking at changes in the parameters of each individual stock (i.e., analysis of paired observations). To allow for a more meaningful cross-sectional comparison, we normalize the parameters $(\mu, \varepsilon_B, \varepsilon_S, \beta, \rho)$ by each stock's pre-split value. Hence, we compare percentage changes in these parameters instead of the raw differences. We do not apply this normalization to the probability parameters of the trading model $(\alpha, \delta, \gamma_B, \gamma_S)$ for two reasons. First, probabilities can be viewed as being normalized by definition and, hence, we need only compare their change in magnitude. Second, the pre-split probabilities for a few stocks are very close to zero (i.e., on the corner of the parameter space). Normalization by the pre-split value for these stocks would produce unusable observations.

To make cross-sectional statements about the tendency of parameters in the sample to change in one direction or the other, we use the non-parametric Wilcoxon signed-rank test (WT) and the Sign test (ST). These tests impose minimal structural assumptions on the cross-sectional characteristics of the parameters.²⁸ For each variable of interest, we report the mean and median change for the sample, along with the corresponding Wilcoxon test statistic and Sign statistic against the null hypothesis of no systematic change.

Because we use parameter estimates from a maximum likelihood estimation of the trading model, we need to take into account that these estimates may contain errors. A small positive change in a parameter of an individual stock may be indistinguishable from zero if the standard errors of the estimation are taken into consideration. The estimation of our model provides likelihood ratios that allow statistical comparisons across nested model specifications using χ^2 tests. To test the change in a specific parameter, we estimate the model while restricting the parameter to be the same across the two periods. We then compare the restricted and unrestricted models using a χ^2 test to determine if the change in the parameter was significant. Hence, to complement the non-parametric tests, we also report the number and direction of the individual changes that were found significant using the χ^2 tests. The multiple methods for examining the estimates should give the reader a sense of whether or not the variables of interest indeed change in a systematic fashion.

III. Results

A. Preliminaries

The various hypotheses about the effects of stock splits have implications for the parameters of our trade model. We now summarize the implications of the trading range hypothesis, the information asymmetry hypothesis, and the optimal

²⁸For the Wilcoxon signed-rank test, we need to assume that differences between pre- and post-split parameter values are symmetric. While parameter values are restricted in the estimation process to non-negative values, the differences are not restricted and are, therefore, good candidates for a symmetric distribution. The Wilcoxon signed-rank test provides a dimension absent from the Sign test in that it takes into account the magnitude of the change in addition to its direction.

tick size hypothesis for these model parameters. A listing of these effects is given in Table 2.

TABLE 2
Implications of Hypotheses about Stock Splits

| | ε_B | ε_S | γ_B | γ_S | μ | α | δ | β | ρ | PIN | S^{TH} | TC |
|-----------------------|-----------------|-----------------|------------|------------|-------|----------|----------|---------|--------|-----|----------|----|
| Trading Range | + | + | — | | | | | | | — | — | — |
| Information Asymmetry | | | | | | — | | | | — | — | — |
| Optimal Tick Size | + / 0 | + / 0 | + | + | | | | | + | | + | — |

This table presents the implications of the different hypotheses about stock splits for the parameters of the trading model. The notation “+” (“—”) should be read as a predicted increase (decrease) in the parameter. An empty cell means that the hypothesis does not have a clear implication concerning the parameter, and “+ / 0” means that two versions of the same hypothesis have different implications (increase or no change). $\varepsilon_B(\varepsilon_S)$ is the rate at which uninformed buy (sell) orders are executed and $\gamma_B(\gamma_S)$ is the fraction of uninformed limit buy (sell) orders. μ is the arrival rate of informed traders, α is the probability of an information event, and δ is the probability of bad information. On days with information events, β is the rate at which limit orders are executed when the orders are in the same direction as the information and ρ is the rate at which limit orders are executed when the orders are in the opposite direction to the information (relative to the rate on no information days). PIN is a function of the parameters that represents the probability of informed trade (equation (11)). S^{TH} is the theoretical opening spread expression that comes out of the model (equation (5)). TC is a measure of the total trading costs of the population of uninformed traders that is developed in Section III.

Perhaps the most straightforward of the explanations is the trading range hypothesis. According to this explanation, a firm splits its stock to change the ownership base of the stock by attracting an uninformed clientele that has a preference for a lower price range. So a stock split should, according to this hypothesis, raise ε_B and ε_S . The entrance of an eager clientele is generally assumed to take place using market orders, so γ_B should decrease. More uninformed traders would, all else equal, lower the probability that any market order was based on private information.²⁹ This is given in our model by

(11)
$$\text{PIN} = \frac{\alpha\mu}{\alpha\mu + \varepsilon_B(1 - \gamma_B) + \varepsilon_S(1 - \gamma_S)}$$

According to the trading range hypothesis, therefore, a split should cause PIN to fall.

In sequential trade models (e.g., Glosten and Milgrom (1985), Easley and O’Hara (1987)), the spread arises to protect the market maker from losses due to trading with informed traders, where the extent of these losses depends on the probability of encountering informed traders and on the variability of the value of the stock. This spread, therefore, provides a simple measure of trading costs in a stock. We derived the opening theoretical spread, S^{TH} , in equation (5). The trading range hypothesis predicts that a split should cause S^{TH} to fall.

Determining the effects of the information asymmetry hypothesis is more complex. According to this hypothesis, stock splits reduce the extent of private

²⁹The probability that a market order comes from an informed trader is the ratio of the arrival rate of informed orders to the arrival rate of all market orders.

information. In our model, private information depends upon the frequency of private information creation, α , and on the fraction of traders who know the new information, μ . If splits signal information that would be revealed only gradually over time, then a direct effect of the split will be to reduce α , at least over the short time horizon we examine in our study. A similar reduction in α would be expected if the split now attracts more analysts to the firm, effectively turning private information revealed by trading into public information revealed by analysts.

The influence of the split on μ is more intricate. If there are fewer informed traders, then *ceteris paribus*, we would expect μ to fall. However, with fewer informed traders, each may now find it optimal to increase his rate of trading, exerting an upward pressure on μ . Could the rise in μ be enough to offset the fall in α ? Not if the information asymmetry explanation is correct, suggesting that the most direct test of this explanation is to examine the composite variable PIN (which directly measures the overall probability of informed trading). A fall in adverse selection should also lead to a smaller spread, which is measured by S^{TH} . Thus, according to the information asymmetry hypothesis, a split should cause PIN and S^{TH} to decrease.³⁰

The third hypothesis envisions a very different effect of splits on trading. It is well known that the percentage spread, which is a very visible measure of transaction costs, increases after a stock split.³¹ The optimal tick size hypothesis (Angel (1997), Harris (1998)) claims that this increase in explicit costs serves investors by attracting liquidity providers (e.g., limit order traders), thereby enhancing liquidity. When markets are more liquid there is greater market depth, so the overall quality of execution (which includes both the price impact of the trade as well as the speed of its execution) is better despite the higher bid-ask spread.³² Thus, viewing trading costs more broadly, the split will lower the total cost of trading to the uninformed traders.³³ To examine this, we develop in the next section a measure of the overall trading costs incurred by the uninformed population (TC).

One implication of this hypothesis is that the number of executed uninformed limit buy and sell orders, $\gamma_B \varepsilon_B$ and $\gamma_S \varepsilon_S$, should increase. This can happen due to an increase in the total rate of uninformed trading, although such an increase is not necessary for the tick size hypothesis to hold: greater liquidity can arise from uninformed traders simply shifting from using market orders to using limit orders. Therefore, we should observe an increase in the fractions of total uninformed trades executed using limit buy and sell orders, γ_B and γ_S .

A final effect of this hypothesis is predicted by Harris (1996), p. 5. He argues that "traders will allow their orders to stand for longer, and they will cancel their

³⁰Note that while the trading range hypothesis also implies a decrease in PIN and S^{TH} , the mechanisms causing the decrease under the two hypotheses are different. In the trading range hypothesis, increased uninformed trading activity—holding constant the arrival of information—is driving down PIN and S^{TH} . Here, we expect information arrival to the market via the order flow to decrease—holding uninformed trading constant—and hence PIN and S^{TH} should decrease.

³¹Our sample is typical: the daily opening percentage spread increases by 49.48%.

³²For a discussion of the legal and economic aspects of best execution, see Macey and O'Hara (1997).

³³Note that, according to the "marketing" explanation mentioned in footnote 6, there is no predicted direction to the change in trading costs since splits are not aimed at enhancing liquidity but instead are meant to cause an upward price pressure.

orders less often, when the tick is large relative to the price.” Cancellation of limit orders in our model will affect the parameter ρ . This parameter is the rate at which limit orders are executed (relative to no information days) when the orders are in the opposite direction of the information. Thus, ρ measures how the execution of limit orders intensifies on days when they serve to absorb the informed market orders. If, as Harris hypothesizes, the larger tick causes more limit order traders to leave their orders in the book, then we would expect to find an increase in ρ .

B. Estimation Results

We now proceed to the estimation of the model and the determination of the pre- and post-split parameter values.³⁴ The small standard errors calculated from the asymptotic covariance matrix indicate that the parameters are estimated very precisely. Heuristically, intra-day data allows us to estimate the trading parameters of the model and inter-day data to estimate the information event parameters.³⁵ Thus, we would expect that the standard errors of the trading parameters ($\mu, \varepsilon_B, \varepsilon_S, \gamma_B, \gamma_S, \beta, \rho$) will be smaller than the standard errors of the information parameters, and this is what we find.

In particular, the standard errors of μ are on average about 8% of the parameter size in both pre- and post-split periods, while the standard errors of ε_B and ε_S are on average less than 4% of the parameter size in all periods. The standard errors of $\gamma_B, \gamma_S, \beta$, and ρ are on average between 4% and 10% of the parameter sizes. These very small standard errors mean that we can proceed with confidence to examine changes in the parameter estimates between the pre- and post-split periods. As for the information parameters, the standard errors of α are on average about 20% of the parameter size. This is still sufficiently accurate to warrant cross-sectional tests on the α estimates. The standard errors of δ are on average 40% of the parameter size. This is understandable, as this parameter is (heuristically) estimated from inter-day data using only days with information events. The results of tests concerning changes in δ should, therefore, be taken with caution.

As a final diagnostic check on the estimates, we consider the reasonableness of the information events' independence assumption.³⁶ Recall that the estimation assumes that information events are independent across days, and it is this independence that allows us to estimate the probabilities of good, bad, and no information days. We used a multiple-category runs test (see Moore (1978)) to test whether these information events (good, bad, and no information) are independent over time. For most stocks in the sample, the hypothesis of independence could not be rejected in both the pre- and post-split periods.³⁷

³⁴The parameter estimates and the standard errors are available from the authors upon request.

³⁵This is just intended to provide some intuition about how the estimation works. Of course, we actually use the entire data set to determine the joint parameter vector.

³⁶Maximum likelihood estimation of structural models does not provide an obvious goodness-of-fit measure such as the R^2 for regular linear (and non-linear in the parameters) models. However, the small standard errors of the parameter estimates and the analysis of the independence assumption testify to the fit and reasonableness of the specification.

³⁷The independence hypothesis was rejected for 14 stocks in the pre-split period and for 14 stocks (with some overlap) in the post-split period. We repeated the cross-sectional analysis in the paper

C. Empirical Findings

Table 3 summarizes our empirical findings. The trading range hypothesis predicts an increase in uninformed trade, and this is exactly what we find. The mean percentage change in the trading rate of uninformed buyers (ε_B) between the pre- and post-split periods is 66.71%, and is highly statistically significant. The trading rate of uninformed sellers (ε_S) increases by 58.70%. The χ^2 tests of individual stocks reveal that the changes in both ε_B and ε_S are statistically significant for almost all stocks. Examining changes in the execution rate of limit buy orders ($\gamma_{B\in B}$), we see that it increases by 86.83%. Similarly, the execution rate of limit sell orders ($\gamma_{S\in S}$) increases by 77.24%. These increases can be due either to the increase in overall uninformed trading following a split, or to a strategic shift by the uninformed traders from market orders to limit orders. Our limit order parameters γ_B and γ_S provide natural measures of the propensities of uninformed traders for executing trading strategies using limit orders. By examining the changes in γ_B and γ_S , we can control for the increase in the overall intensity of trading and focus on the uninformed traders' choice of order types.³⁸

The mean change in the magnitude of γ_B is -1.18% , but is only marginally significant. The χ^2 tests show that the decrease is significant for 25 stocks, while increases in the parameter are significant for 15 stocks. On a pre-split mean γ_B of 26.76%, a decrease of about 1% does not seem large, but it is in the direction predicted by the trading range hypothesis. The mean change in the magnitude of γ_S is 2.59%. These numbers provide very weak evidence of a change in the trading patterns of the uninformed buyers toward using more market orders and of the uninformed sellers toward using more limit orders. This divergence is at odds with the prediction of the optimal tick size hypothesis, which envisions greater propensity for executing trades using limit orders by both uninformed buyers and sellers.³⁹

By examining the parameters β and ρ , we can investigate the relationship between limit order execution and information in the market. The parameter β can be thought of as the multiplicative change in the likelihood of execution of a limit buy (sell) order when, instead of a day with no information, there is a good (bad) information day. In a similar fashion, ρ can be thought of as the multiplicative change in the likelihood of execution of a limit buy (sell) order when instead of a day with no information, there is a bad (good) information day. While β does not seem to change systematically, there is some evidence that ρ decreases after the split, with a mean percentage change of -6.89% . The χ^2 tests of changes in ρ are statistically significant for 24 of the 72 stocks: 18 that experience a decrease and six that experience an increase. The decrease in ρ after the split may suggest

using only the stocks that did not reject the independence hypothesis in both periods. The results were qualitatively similar to the results using the entire sample.

³⁸The parameters γ_B and γ_S are estimated while also controlling for information flows that may induce price volatility and, therefore, affect execution. This control is implemented by concurrently estimating the execution parameters β and ρ that relate limit order execution to the existence and direction of information in the market.

³⁹Note that the implications for γ_B of the trading range hypothesis and the optimal tick size hypothesis go in opposite directions. The evidence of a decrease in the parameter may be interpreted to mean that even if an optimal tick size effect exists, it is overwhelmed by the entrance of the eager clientele envisioned by the trading range hypothesis.

TABLE 3
Results

| Panel A. | | | | | | |
|---|-------------------|--------|----------------------|-------|----------------------------|-----------|
| | Percentage Change | | Non-Parametric Tests | | Significant χ^2 Tests | |
| | Mean | Median | Wilcoxon | Sign | Increases | Decreases |
| ε_B | 66.71% | 61.40% | 7.16 | 7.54 | 64 | 2 |
| ε_S | 58.70% | 47.64% | 6.74 | 5.66 | 59 | 3 |
| μ | 59.94% | 37.26% | 4.39 | 2.83 | 39 | 12 |
| β | -0.23% | -3.12% | -0.66 | -0.94 | 11 | 12 |
| ρ | -6.89% | -6.43% | -2.70 | -3.06 | 6 | 18 |
| S^{TH} | 42.66% | 26.67% | 4.33 | 3.94 | | |
| $\gamma_B \cdot \varepsilon_B$ | 86.83% | 53.20% | 6.63 | 5.85 | | |
| $\gamma_S \cdot \varepsilon_S$ | 77.24% | 59.58% | 7.00 | 7.31 | | |
| TC | 50.62% | 42.68% | 6.35 | 5.89 | | |
| Magnitude Change | | | | | | |
| α | 1.83% | 4.07% | 0.63 | 0.71 | 10 | 6 |
| δ | -0.05% | -2.25% | -0.28 | -0.47 | 7 | 10 |
| γ_B | -1.18% | -1.61% | -1.75 | -1.65 | 15 | 25 |
| γ_S | 2.59% | 2.59% | 2.92 | 2.59 | 27 | 12 |
| PIN | -1.04% | -1.04% | -2.03 | -2.36 | 3 | 10 |
| STD | 0.60% | 0.54% | 6.64 | 7.54 | | |
| Panel B. | | | | | | |
| | Pearson | | ASE | | Spearman | |
| $\% \Delta(\varepsilon_B + \varepsilon_S), \% \Delta \mu$ | 0.628 | | 0.093 | | 0.680 | |
| S^{TH}, S_0 (pre-split period) | 0.442 | | 0.116 | | 0.612 | |
| S^{TH}, S_0 (post-split period) | 0.549 | | 0.100 | | 0.585 | |

$\varepsilon_B(\varepsilon_S)$ is the rate at which uninformed buy (sell) orders are executed and $\gamma_B(\gamma_S)$ is the fraction of uninformed limit buy (sell) orders. $\gamma_B \varepsilon_B(\gamma_S \varepsilon_S)$ is, therefore, the rate at which uninformed limit orders are executed on no information days. On days with information events, β is the rate at which limit orders are executed when the orders are in the same direction as the information and ρ is the rate at which limit orders are executed when the orders are in the opposite direction to the information (relative to the rate on no information days). α is the probability of an information event occurring, $\delta(1 - \delta)$ is the probability that an information event is bad (good), and μ is the arrival rate of informed traders. PIN is a function of the parameters that represents the probability of informed trade (equation (11)). STD is the standard deviation of daily percentage opening to closing price changes. S^{TH} is the theoretical opening spread expression that comes out of the model (equation (5)). TC is a measure of the overall trading costs (per dollar) of the population of uninformed traders that is developed in Section III. Panel A presents the mean and the median of the percentage change (or magnitude change where appropriate) in the parameters. The Wilcoxon signed-rank statistic and the Sign statistic are calculated using the standard normal approximation. The critical values for a two-sided alternative are 1.96 and 1.64 for 5% and 10% significance levels, respectively. The 5% significance level is used in reporting the results of the χ^2 tests. Panel B presents Pearson and Spearman correlations (with asymptotic standard errors). The notation “% Δ ” represents percentage change. S_0 is the opening percentage spread (dollar spread over the midquote).

that limit order traders cancel their orders more often. When prices start moving against them, they remove the orders and there is less limit order execution against informed order flow after the stock split. The finding of a decrease in ρ does not support the prediction of the optimal tick size hypothesis as put forward by Harris (1996).

Our model also provides estimates of the information environment of stocks. The probability of an information event (α) increases by 1.83%, but this change is not statistically significant. The χ^2 tests of individual stocks are statistically significant for only 16 of the 72 stocks. With no predominant direction to the change in α , and with less than a quarter of the stocks having any significant change at

all, it seems that stock splits are not associated with a change in the frequency of information events. There is also no significant change in the parameter that describes the distribution of good and bad information events, δ . These findings cast serious doubt on the information asymmetry explanation for splits.

Turning to the informed traders, we find that their trading rate (μ) increases by 59.94%. This intensified informed trading is consistent with Kyle's (1985) prediction that informed investors adjust their trading in response to changes in the trading patterns of uninformed investors.⁴⁰ The Spearman correlation between the percentage change in uninformed trade ($\varepsilon_B + \varepsilon_S$) and the percentage change in informed trade (μ) is 0.680. This correlation suggests a relatively strong tendency of the informed and uninformed order flows to move together.

We can use the probability of informed trade (PIN), which is the execution rate of informed market orders divided by the overall rate of market orders, to represent the extent of the adverse selection problem. Both the trading range and information asymmetry hypotheses predict a fall in PIN after a split. The mean change in the magnitude of PIN is -1.04% , which seems rather small on a pre-split mean of 18.25% . In addition, the individual changes in PIN are statistically significant only for 13 of the 72 stocks. Of these stocks, 10 experience a decrease in PIN and three experience an increase. It is clear that evidence of a decrease in the adverse selection problem is, at best, very weak.⁴¹

Of related interest is the impact of a split on the cost of trading as measured by the opening theoretical spread, S^{TH} . Both the trading range and informational asymmetry hypotheses predict a decrease in S^{TH} , while the optimal tick size hypothesis predicts an increase. As equation (3) shows, calculating S^{TH} involves both our estimated trade parameters and the standard deviation of changes in the value of the stock, σ_V .⁴² Turning to the volatility calculation, we find that the increase in volatility noted in the literature holds true for our sample as well. The mean standard deviation of the daily open-to-close percentage change in price increases from 1.44% in the pre-split period to 2.04% in the post-split period. In other words, even when private information flow does not increase (no significant change in α), the split increases the volatility of stocks by increasing the

⁴⁰From Table 1, we see that the average trade size falls by about 20% after stock splits. This is most likely due to the entrance of a new uninformed clientele that uses smaller trades. The Kyle (1985) analysis suggests that informed traders will not try to use larger trade sizes to exploit their private information but instead will adjust their trading strategy as if to "hide" among the uninformed traders. They may, however, increase the frequency of their transactions. This is exactly what we find—the arrival rate of informed traders (μ) increases. The fact that we estimate the model separately before and after the split allows us to control for possible changes in trade size in the market while, at the same time, estimating the change to the strategy of the informed traders.

⁴¹This result contrasts with the findings of Desai, Nimalendran, and Venkataraman (1998) who show that the adverse selection component of the spread increases following stock splits. The difference may be due to their use of a spread decomposition procedure, which depends explicitly on price data. Since the split affects prices, the measure of adverse selection that comes out of that approach may be biased. Our model has the advantage that it is estimated solely from trade data, and so avoids this price-linked difficulty.

⁴²While our maximum likelihood procedure does not provide estimates of the true value process (\bar{V} and \underline{V}), we can use market price information to construct an empirical proxy for σ_V . As a proxy for the standard deviation of daily percentage changes in the value of the stock, we use the standard deviation of the percentage difference between the daily closing price and the daily opening price. The results are unchanged when we repeat the analysis with the midpoints of the opening and closing quotes instead of prices.

dispersion of their true values. As a result, the mean percentage change in the theoretical spread measure, S^{TH} , is 42.66%. Other researchers, noting the increase in spreads, have attributed this to greater adverse selection after splits. Our model-based analysis shows that this inference is incorrect and that the reason for the wider spreads is the increase in volatility.

It is interesting to compare the estimated theoretical spread with the true percentage spread in the market.⁴³ Such a comparison provides a simple, yet effective, check on the reasonableness of our empirical estimates. Because no quote information is used in the estimation of the theoretical spread, if the predicted spread is highly correlated with the actual spread then we can attach more confidence to the predictions arising from our model. The mean percentage changes in the actual opening spread, S_0 , and the theoretical opening spread, S^{TH} , are 49.48% and 42.66%, respectively. While not exact, the model does a good job of predicting actual spreads.⁴⁴ The Spearman correlation between the theoretical and actual opening percentage spreads is 0.612 in the pre-split period and 0.585 in the post-split period. We find this relatively high correlation encouraging as it testifies to a good “fit” between our model and the actual trading environment.

These spread results provide a partial estimate of trading costs, but to determine the effects of stock splits on the overall cost of trading we need a measure that accounts for limit order usage. We are specifically interested in the cost incurred by the uninformed population since it is hard to justify stock splits on the grounds of making the privately informed population better off.⁴⁵ In a market where uninformed traders use both market and limit orders, the cost of trading depends on the relative usage of the two types of orders. This is because, when an uninformed limit order executes against an uninformed market order, the aggregate trading cost to the uninformed population is zero. Only when an uninformed trader’s market order executes against the specialist or his limit order executes against an informed trader do uninformed investors as a group incur an execution cost. Therefore, to evaluate the cost to the population of uninformed traders we need to consider the cost of net uninformed trades (i.e., trades that were not executed against other uninformed traders).⁴⁶

⁴³Since quotes in our model are set by the market maker such that they reflect the adverse selection problem, we want to make sure that the spreads we use from the opening quotes are indeed set by the NYSE specialist. In other words, since a quote on the NYSE can be set by limit orders that were not monitored closely by those who submitted them, it need not reflect current information. Hence, we use the limit order algorithm to identify all the quotes that belong to limit orders (i.e., all the quotes that resulted in subsequent trades being categorized as limit order executions). We then calculate the opening percentage spread using the first quote of the day that is not identified as a limit order quote.

⁴⁴Actual spreads are affected by factors such as tick size rules and continuity requirements, which, naturally, are not incorporated in our model. Nonetheless, the closeness of the actual and theoretical spreads suggests that the model does incorporate the major factors influencing spreads.

⁴⁵One could claim that a decrease in the costs incurred by informed traders might intensify costly private information gathering and, therefore, bring hidden firm value to the market. In this “version” of the information asymmetry story, prices adjust to information only gradually (in contrast to the revelation of public information). Hence, this has the disadvantage that markets will not be as (informationally) efficient as they could be if the private information is revealed publicly. Furthermore, this story does not help with the problem of signal credibility since, if the information is credible enough to be purchased by some traders, it should, in general, be credible enough for the public.

⁴⁶Note that, since we are using data on limit order execution, we cannot measure the cost incurred by an uninformed trader who submits a limit order that fails to execute.

Our model provides the framework necessary to determine this cost. We define the Uninformed Trading Cost (UTC) as half the percentage spread times the absolute value of the difference between the value of uninformed market orders and the corresponding limit orders. Due to the different frequencies of execution of uninformed limit orders on days with different information contents, the costs must be weighted according to the distribution of the information events. Denote by T_B (T_S) the average size of a buy (sell) order. Then,

$$(12) \quad \text{UTC} = S_O/2 * \{ \alpha \delta [| \varepsilon_B(1 - \gamma_B) - \varepsilon_S \gamma_S \rho | T_B + | \varepsilon_S(1 - \gamma_S) - \varepsilon_B \gamma_B \rho | T_S] \\ + \alpha(1 - \delta) [| \varepsilon_B(1 - \gamma_B) - \varepsilon_S \gamma_S \rho | T_B + | \varepsilon_S(1 - \gamma_S) - \varepsilon_B \gamma_B \rho | T_S] \\ + (1 - \alpha) [| \varepsilon_B(1 - \gamma_B) - \varepsilon_S \gamma_S | T_B + | \varepsilon_S(1 - \gamma_S) - \varepsilon_B \gamma_B | T_S] \}.$$

Because uninformed trading increases in the market as a whole following a split, a more accurate measure of trading costs should be per dollar traded. Define the Uninformed Dollar Volume (UDV) as

$$(13) \quad \text{UDV} = \alpha \delta [(\varepsilon_B(1 - \gamma_B) + \varepsilon_S \gamma_S \rho) T_B + (\varepsilon_S(1 - \gamma_S) + \varepsilon_B \gamma_B \rho) T_S] \\ + \alpha(1 - \delta) [(\varepsilon_B(1 - \gamma_B) + \varepsilon_S \gamma_S \rho) T_B + (\varepsilon_S(1 - \gamma_S) + \varepsilon_B \gamma_B \rho) T_S] \\ + (1 - \alpha) [(\varepsilon_B(1 - \gamma_B) + \varepsilon_S \gamma_S) T_B + (\varepsilon_S(1 - \gamma_S) + \varepsilon_B \gamma_S) T_S].$$

The trading cost per dollar volume of the uninformed population (TC) is, therefore, equal to the trading costs divided by the dollar trading volume ($\text{TC} = \text{UTC} / \text{UDV}$).

The expression for the UTC shows that, if limit orders and market orders change after the split to better match each other, then the overall trading costs could go down despite the increase in spreads. The evidence, however, points in the other direction: the mean percentage change in TC is 50.62%, and this increase is highly statistically significant. Previous researchers have argued that investors are worse off following splits. Our analysis quantifies this effect and demonstrates why it arises.⁴⁷

IV. Discussion and Conclusions

In this research, we have investigated the effects of stock splits on the trading in a firm's shares. By estimating the parameters of a market microstructure sequential trade model, we were able to look at changes in the trading strategies of investors and in the information environment of stocks. These changes, in turn, can be linked back to the hypotheses on why firms split their shares. Using a structural model to extract from trade data information about unobservable attributes such as uninformed trading, the frequency of information events, and

⁴⁷One could argue, however, that trading against information does not represent a trading cost to the uninformed investor. Given his decision to trade, transaction costs (in the form of the bid-ask spread) can be minimized if the uninformed trader chooses a limit order to execute his trading desire rather than a market order. Hence, a second definition of uninformed trading costs can be formulated where only excess market orders over limit orders are penalized. Using this alternative definition of trading costs only reinforces our findings: the mean percentage change in per dollar uninformed costs using the alternative definition is 59.90% (Med = 52.75%, WT = 6.60, ST = 5.89).

adverse selection provides us with a set of tools different from those used to evaluate hypotheses on stock splits in the extant literature. Instead of using separate tests and proxies to assess the merits of each of the hypotheses, the model enabled us to obtain an integrated set of results with which to view all the hypotheses put forward in the literature as reasons for stock splits. This internal consistency also allowed us to demonstrate how an increase in trading costs can take place concurrently with an increase in uninformed trading and a more aggressive use of limit orders.

Our results are mildly supportive of the trading range hypothesis. There is an increase in the number of uninformed trades and a slight shift of uninformed buyers to execute their trades by using market orders, in-line with their being more “eager” to enter the market for the stock. This evidence is consistent with the entry of a new clientele. The increase in uninformed trading, however, is accompanied by an increase in informed trading, and so the extent of the adverse selection problem is not materially reduced. Increases in volatility without a substantial decrease in adverse selection cause our spread measure to increase, and the end result is worsened liquidity. While these results are inconsistent with the enhanced liquidity explanation of the trading range hypothesis, they are still consistent with the trading range idea. The new clientele may be willing to trade a stock with a higher bid-ask spread if they gain something else by adding the stock to their portfolios (e.g., increased diversification of their holdings).

As for the information asymmetry hypothesis, the parameter estimates suggest that the information environment of stocks does not change systematically after stock splits. The increase in informed trading after the split is also difficult to reconcile with a reduction in information asymmetry. While we report a slight decrease in the probability of informed trade, it is too small to offset the increased cost incurred by traders due to the increase in volatility. This increase in trading costs is inconsistent with an explanation of stock splits that implies a reduction in adverse selection costs.

Our findings can still be viewed as consistent with a simple signaling story. If the information revealed by the split was not known to privately informed traders (but perhaps only to insiders who are restricted from trading), then it may be plausible that the split would not change the structure of private information that we estimate. Furthermore, we are using only trade data for our estimation, and the adjustment of prices to public information need not be accompanied by trading. On the other hand, this simple signaling story does not explain why many new traders would buy the stock after the split. If the split benefits only existing shareholders, there should be no special incentive for a new clientele to enter.

The optimal tick size hypothesis would predict the increase in the intensity of limit order trading that we have found, but the rest of our results seem inconsistent with this explanation. The increase in limit order trading is not sufficient to compensate the uninformed population for the increase in the bid-ask spread and the more intense usage of market buy orders. These market orders face larger bid-ask spreads after the split, with the end result being that the uninformed population suffers from an increase in its overall trading cost. In addition, the basic intuition of the optimal tick size hypothesis is that the increased tick size after stock splits encourages liquidity provision by inhibiting “front running.” Hence,

uninformed traders should be able to decrease the monitoring of their limit orders after a split, which will result in fewer cancelled orders. We, on the other hand, find that the execution rate of limit orders on days in which informed traders (and prices) move against them decreases, evidence that is consistent with an increase in limit order cancellation.

Our results suggest that stock splits are not neutral events for a security. Both changes in the composition of the order flow and in the cost of trading accompany the decision to split. In general, these changes do not enhance the overall transactional efficiency of a stock. Splitting shares, however, does provide companies with a way to influence their ownership. If product demand is linked to ownership, then attracting a particular clientele may enhance long-run profitability even if it has short-run detrimental effects on liquidity.⁴⁸ Such effects are beyond our microstructure-based analysis here, but they may constitute a fruitful direction for future research.

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⁴⁸For example, companies such as McDonalds encourage small ownership holdings by facilitating transfers of single shares of the stock between parents (or other adults) and children. This could be consistent with developing a new generation of customers whose loyalty is enhanced by their ownership claim.

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