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SIGNIFICANCE TESTS FOR EVENT STUDIES

by Dr. Simon Müller

*'Every number is guilty unless proven innocent.
(Rao, 1997: 152)*

An event study is oftentimes the first step in a sequence of analyses that aims at identifying the determinants of stock market responses to distinct event types. Event studies yield as an outcome abnormal returns (ARs), which are cumulated over time to cumulative abnormal returns (CARs) and then 'averaged' - in the case of so called sample studies - over several observations of identical events to AARs and CAARs - where the second 'A' stands for 'average'. These event study results are then oftentimes used as dependent variables in regression analyses.

Besides this type of analysis, event studies are traditionally also performed to specify if the abnormal effects pertaining to individual events or samples of events are significantly (http://en.wikipedia.org/wiki/Statistical_significance) different from zero, and thus not the result of pure chance. This assessment will be made by hypothesis testing (http://en.wikipedia.org/wiki/Statistical_hypothesis_testing).

Following general principles of inferential statistics, the *null hypothesis* (H_0) thus maintains that there are no abnormal returns within the event window, whereas the *alternative hypothesis* (H_1) suggests the presence of ARs within the event window. Formally, the testing framework reads as follows:

$$\begin{array}{ll|l} H_0 : \mu = 0 & & | \\ H_1 : \mu \neq 0 & & | \end{array}$$

Event studies may imply a hierarchy of calculations, with ARs being compounded to CARs, which can again be 'averaged' to CAAR in cross-sectional studies ('sample studies'). In long-run Event Studies instead of CAR the Buy-and-Hold abnormal return (BHAR) is widely used. BHAR can also be 'averaged' to ABHAR studies. There is a need for significance testing at each of these levels. μ in the above mentioned equations may thus represent ARs, CARs, BHARs, AARs, CAARs. We shortly revisit these six different forms of abnormal return calculations, as presented in the introduction (/event-study-methodology):



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$$\begin{aligned} AR_{i,t} &= R_{i,t} - E[R_{i,t} | \Omega_{i,t}] \\ AAR_t &= \frac{1}{N} \sum_{i=1}^N AR_{i,t} \\ CAR_i &= \left| \sum_{t=T_1+1}^{T_2} AR_{i,t} \right| \\ BHAR_i &= \left| \prod_{t=T_1+1}^{T_2} (1 + R_{i,t}) - \prod_{t=T_1+1}^{T_2} (1 + E[R_{i,t} | \Omega_{i,t}]) \right| \\ CAAR &= \frac{1}{N} \sum_{i=1}^N CAR_i \\ ABHAR &= \frac{1}{N} \sum_{i=1}^N BHAR_i \end{aligned}$$

For grouped observations, both along the firm or event dimension, we provide a precision-weighted CAAR, which offers a similar standardization as the Patell Test:

$$PWCAAR = \sum_{i=1}^N \sum_{t=T_1+1}^{T_2} \omega_i AR_{i,t}$$

where

$$\omega_i = \frac{\left(\sum_{t=T_1+1}^{T_2} S_{AR_{i,t}}^2 \right)^{-0.5}}{\sum_{i=1}^N \left(\sum_{t=T_1+1}^{T_2} S_{AR_{i,t}}^2 \right)^{-0.5}}$$

and $S_{AR_{i,t}}^2$ is the forecast-error corrected standard deviation defined in EQ ???

The literature on event study test statistics is very rich, as is the range of significance tests. Generally, significance tests can be grouped in parametric (http://en.wikipedia.org/wiki/Parametric_statistics) and nonparametric tests (http://en.wikipedia.org/wiki/Non-parametric_statistics) (NPTs). Parametric tests assume that individual firm's abnormal returns are normally distributed, whereas nonparametric tests do not rely on any such assumptions. In research, scholars commonly complement a parametric test with a nonparametric tests to verify that the research findings are not due to eg. an outlier (see Schipper and Smith (1983) for an example). Table 1 provides an overview and links to the formulas of the different test statistics.

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Table 1: 'Recommended' Significance Tests per Test Level

Null hypothesis tested	Parametric tests	Nonparametric tests
$H_0 : AR = 0$	AR Test	
$H_0 : AAR = 0$	Cross-Sectional Test , Time-Series Standard Deviation Test , Patell Test , Adjusted Patell Test , Standardized Cross-Sectional Test , Adjusted Standardized Cross-Sectional Test , and Skewness Corrected Test	Generalized Sign Test , Generalized Rank T Test , and Generalized Rank Z Test
$H_0 : CAR = 0$	CAR t-test	
$H_0 : CAAR = 0$	Cross-Sectional Test , Time-Series Standard Deviation Test , Patell Test , Adjusted Patell Test , Standardized Cross-Sectional Test , Adjusted Standardized Cross-Sectional Test , and Skewness Corrected Test	Generalized Sign Test , Generalized Rank T Test , and Generalized Rank Z Test
$H_0 : BHAR = 0$	BHAR Test	
$H_0 : ABHAR = 0$	ABHAR Test and Skewness Corrected Test	

N.B.: Most test statistics are calculated by our free abnormal return calculators (/instructions-axc)

Parametric test statistics ground on the classic t-test (<http://en.wikipedia.org/wiki/T-test>). Yet, scholars have further developed the test to correct for the t-test's prediction error. The most widely used of these 'scaled' tests are those developed by Patell (1976) and Boehmer, Musumeci and Poulsen (1991). Among the nonparametric tests, the rank-test of Corrado (1989), and the sign-based of Cowan (1992) are very popular. EST provides these test statistics (soon) in its analysis results reports.

Why different test statistics are needed

An informed choice of test statistic should be based on the research setting and the statistical issues the analyzed data holds. Specifically, event-date clustering poses a problem leading to (1) cross-sectional correlation of abnormal returns, and (2) distortions from event-induced volatility changes. Cross-sectional correlation arises when sample studies focus on (an) event(s) which happened for multiple firms at the same day(s). Event-induced volatility changes, instead, is a phenomenon common to many event types (e.g., M&A transactions) that becomes problematic when events are clustered. As consequence, both issues introduce a downward bias in the standard deviation and thus overstate the t-statistic, leading to an over-rejection of the null hypothesis.

Comparison of test statistics

There have been several attempts to address these statistical issues. Patell (1976, 1979), for example, tried to overcome the t-test's proneness to event-induced volatility by standardizing the event window's ARs. He used the dispersion of the estimation interval's ARs to limit the impact of stocks with high return standard deviations. Yet, the test too often rejects the true null hypothesis, particularly when samples are characterized by non-normal returns, low prices or little liquidity. Also, the test has been found to be still affected by event-induced volatility changes (Campbell and Wasley, 1993; Cowan and Sergeant, 1996; Maynes and Rumsey, 1993, Kolari and Pynnonen, 2010). Boehmer, Musumeci and Poulsen (1991) resolved this latter issue and developed a test statistic robust against volatility-changing events. Furthermore, the simulation study of Kolari and Pynnonen (2010) indicates an over-rejection of the null hypothesis for both the Patell and the BMP test, if cross-sectional correlation is ignored. Kolari and Pynnonen (2010) developed an adjusted version for both test statistics that accounts for cross-sectional correlation.

The nonparametric rank test (http://en.wikipedia.org/wiki/Rank_test) of Corrado and Zivney (1992) (RANK) applies re-standardized event window returns and has proven robust against induced volatility and cross-correlation. Sign tests (http://en.wikipedia.org/wiki/Sign_test) are another category of tests. One advantage the tests' authors stress over the common t-test is that they are apt to also identify small levels of abnormal returns. Moreover, scholars have recommend the used of nonparametric sign and rank tests for

applications that require robustness against non-normally distributed data. Past research (e.g. Fama, 1976) has argued that daily return distributions are more fat-tailed (exhibit very large skewness or kurtosis) than normal distributions, what suggests the use of nonparametric tests.

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Several authors have further advanced the sign and ranked tests pioneered by Cowan (1992) and Corrado and Zivney (1992). Campbell and Wasley (1993), for example, improved the RANK test by introducing an incremental bias into the standard error for longer CARs, creating the Campbell-Wasley test statistic (CUM-RANK). Another NPT is the generalized rank test (GRANK) test with a Student t-distribution with T-2 degrees of freedom (T is the number of observations). It seems that GRANK is one of the most powerful instruments for both shorter and longer CAR-windows.

The Cowan (1992) sign test (SIGN) is also used for testing CARs by comparing the share of positive ARs close to an event to the proportion from a no-hypothesis includes the possibility of asymmetric return distribution. Because this test considers only the sign of the difference between abnormal returns does not influence in any way its rejection rates. Thus, in the presence of induced volatility scholars recommend the use of BMP, GRANK, SIGN.

Most studies have shown that if the focus is only on single day ARs, the means of all tests stick close to zero. In the case of longer event windows, however, deviate from zero. Compared to their nonparametric counterparts, the Patell and the BMP-tests produce means that deviate quite fast from zero, whereas means of all tests gravitate towards zero. For longer event windows, academics recommend nonparametric over parametric tests.

Therefore, the main idea is that in case of longer event-windows, the conclusions for the tests power should be very carefully drawn because of the many cases of the null hypothesis. Overall, comparing the different test statistics yields the following insights (see Table 2 for further details):

- RESEARCH APPS: Generally, nonparametric tests tend to be more powerful than parametric tests
1. Parametric tests based on scaled abnormal returns perform better than those based on non-standardized returns
 3. The generalized rank test (GRANK) is one of the most powerful test for both shorter CAR-windows and longer periods

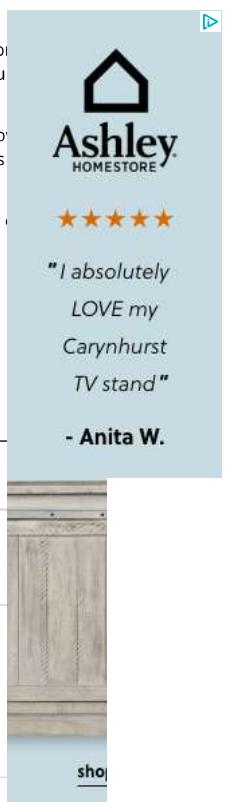


Table 2: Summary Overview of Main Test Statistics

#	Name [synonym]	Key Reference	ABBREVIATION in API RESULTS	Strengths	
Parametric Tests					
1	<u>T-test</u>			<ul style="list-style-type: none"> • Simplicity 	
2	<u>Cross-Sectional Test</u>		CSECT T		
3	<u>Time-Series Standard Deviation Test</u>		CDA T		
4	<u>Patell Test</u>	<u>Patell (1976)</u> (http://www.jstor.org/stable/2490543)	Patell Z	<ul style="list-style-type: none"> • Immune to the way in which ARs are distributed across the (cumulated) event window. 	<ul style="list-style-type: none"> • Prone to cross-sectional correlation and event-induced volatility.
5	<u>Adjusted Patell Test</u>	<u>Kolari and Pynnonen (2010)</u> (http://rfs.oxfordjournals.org/content/23/11/3996.abstract)	Adjusted Patell Z	<ul style="list-style-type: none"> • Same as Patell • Immune to cross-sectional correlation 	
6	<u>Standardized Cross-Sectional Test</u>	<u>Boehmer, Musumeci and Poulsen (1991)</u> (http://dx.doi.org/10.1016/0304-405X(91)90032-F)	StdCSECT Z	<ul style="list-style-type: none"> • Immune to the way in which ARs are distributed across the (cumulated) event window. • Accounts for event- 	<ul style="list-style-type: none"> • Prone to cross-sectional correlation.

#	Name [synonym]	Key Reference	ABBREVIATION in API RESULTS	Strengths	Weaknesses
7	<u>Adjusted Standardized Cross-Section Test</u>	Kolari and Pynnonen (2010). (http://rfs.oxfordjournals.org/content/23/11/3996.abstract)	Adjusted StdCsect Z	<ul style="list-style-type: none"> • Accounts for serial correlation. 	
8	<u>Skewness Corrected Test</u>	Hall (1992)	Skewness Corrected T	<ul style="list-style-type: none"> • Corrects the test statistics for skewed distributions. 	
9	<u>Jackknife Test</u>	Giacotto and Sfiridis (1996). (http://www.sciencedirect.com/science/article/pii/0148619596000197).	Jackknife T		

Nonparametric Tests

10	<u>Corrado Rank Test</u>	Corrado and Zivney (1992). (http://www.jstor.org/stable/2331331).	Rank Z		• Loses power for longer
11	<u>Generalized Rank Test</u>	Kolari and Pynnonen (2011). (http://www.sciencedirect.com/science/article/pii/S0927539811000624)	Generalized Rank T	Accounts for <ul style="list-style-type: none"> • cross-correlation of returns, • returns serial correlation • and event-induced volatility. 	
RESEARCH APPS	<u>Generalized Rank Test</u>	Kolari and Pynnonen (2011). (http://www.sciencedirect.com/science/article/pii/S0927539811000624)	Generalized Rank Z	see GRANKT	
13	<u>Sign Test</u>	Cowan (1992). (http://www.bus.iastate.edu/arnie/simnpar.pdf)	not available in API	• Accounts for skewness in security returns.	
14	<u>Cowan Generalized Sign Test</u>	Cowan (1992). (http://www.bus.iastate.edu/arnie/simnpar.pdf)	Generalized Sign Z		
15	<u>Wilcoxon signed-rank Test</u>	Wilcoxon (1945). (http://webspace.ship.edu/pgmarr/Geo441/Readings/Wilcoxon%201945%20-Individual%20Comparisons%20by%20Ranking%20Methods.pdf).		• Considers that both the sign and the magnitude of ARs are important.	

Insights about strengths and weaknesses were compiled from Kolari and Pynnonen (2011)

Formulas, acronyms, and the decision rule applicable to all test statistics

Let $L_1 = T_1 - T_0 + 1$ the estimation window length with T_0 as the 'earliest' day of the estimation window, and T_1 the 'latest' day of the estimation window day and $L_2 = T_2 - T_1$ the event window length with T_2 as the 'latest day' of the event window relative to the event day. Define M_i as the sample size (i.e. number of events / observations); S_{AR_i} represent the standard deviation as produced by the regression analysis over the estimation window according to the following formula



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$$S_{AR_i}^2 = \frac{1}{M_i - 2} \sum_{t=T_0}^{T_1} (AR_{i,t})^2$$

M_i refers to the number of non-missing (i.e., matched) returns. This standard deviation corresponds to the market model. For other models some adjustment need to be done.

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Parametric Test Statistics

[1] T test

Our API provides test statistics for single firms in each time point t . The Null is: $H_0 : AR_{i,t} = 0$

$$t_{AR_{i,t}} = \frac{AR_{i,t}}{S_{AR_i}},$$

where S_{AR_i} is the standard deviation of the abnormal returns in the estimation window,

$$S_{AR_i}^2 = \frac{1}{M_i - 2} \sum_{t=T_0}^{T_1} (AR_{i,t})^2.$$

Second, we provide t statistics of the cumulative abnormal returns for each firm. The t statistic und the Null $H_0 : CAR_i = 0$ is defined as

$$t_{CAR} = \frac{CAR_i}{S_{CAR}},$$

where

$$S_{CAR}^2 = L_2 S_{AR_i}^2.$$

[2] Cross-Sectional Test (Abbr.: Csect T)

A simple test for testing $H_0 : AAR = 0$ is given by

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$$t_{AAR_t} = \sqrt{N} \frac{AAR_t}{S_{AAR_t}},$$

where S_{AAR_t} is the standard deviation across firms at time t

$$S_{AAR_t}^2 = \frac{1}{N-1} \sum_{i=1}^N (AR_{i,t} - AAR_t)^2.$$

Test statistic for testing $H_0 : CAAR = 0$ is given by

$$t_{CAAR} = \sqrt{N} \frac{CAAR}{S_{CAAR}},$$

where S_{CAAR} is the standard deviation of the cumulative abnormal returns across the sample

$$S_{CAAR}^2 = \frac{1}{N-1} \sum_{i=1}^N (CAR_i - CAAR)^2.$$

Brown and Warner (1985) showed that the cross-sectional test is prone to event-induced volatility. Thus, the test has low power.

[3] Time-Series Standard Deviation or Crude Dependence Test (Abbr.: CDA T)

The time-series standard deviation test uses the entire sample for variance estimation. According to this construction, the time-series dependence test does not consider unequal variances across observations. We have for the variance estimation:

$$S_{AAR}^2 = \frac{1}{M-2} \sum_{t=T_0}^{T_1} (AAR_t - \overline{AAR})^2,$$

where $[T_0, T_1]$ is the estimation window and

$$\overline{AAR} = \frac{1}{M} \sum_{t=T_0}^{T_1} AAR_t,$$

Test statistic for testing $H_0 : AAR_t = 0$ is given by

$$t_{AAR_t} = \sqrt{N} \frac{AAR_t}{S_{AAR}},$$

Test statistic for testing $H_0 : CAAR = 0$ is given by

$$t_{CAAR} = \frac{CAAR}{\sqrt{T_2 - T_1} S_{AAR}}.$$

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[4] Patell or Standardized Residual Test (Abbr.: Patell Z)

The Patell test is a widely used test statistic in event studies. In the first step Patell (1976, 1979) suggested to standardize each $AR_{i,t}$ before calculating the test statistic by the forecast-error corrected standard deviation.

$$SAR_{i,t} = \frac{AR_{i,t}}{S_{AR_{i,t}}}$$

As the event-window abnormal returns are out-of-sample predictions, Patell adjusts the standard error by the forecast-error:

$$S_{AR_{i,t}}^2 = S_{AR_i}^2 \left(1 + \frac{1}{M_i} + \frac{(R_{m,t} - \bar{R}_m)^2}{\sum_{t=T_0}^{T_1} (R_{m,t} - \bar{R}_m)^2} \right)$$

with \bar{R}_m as the mean of the market returns in the estimation window. $SAR_{i,t}$ is distributed as a t-distribution with $M_i - 2$ degrees of freedom under the Null. Test statistic for testing $H_0 : AAR = 0$ is then given by

$$z_{Patell,t} = \frac{ASAR_t}{S_{ASAR_t}}$$

where $ASAR_t$ is the sum over the sample of the standardized abnormal returns

$$ASAR_t = \sum_{i=1}^N SAR_{i,t}$$

with expectation zero and variance

$$S_{ASAR_t}^2 = \sum_{i=1}^N \frac{M_i - 2}{M_i - 4}$$

Test statistic for testing $H_0 : CAAR = 0$ is given by

$$z_{Patell} = \frac{1}{\sqrt{N}} \sum_{i=1}^N \frac{CSAR_i}{S_{CSAR_i}}$$

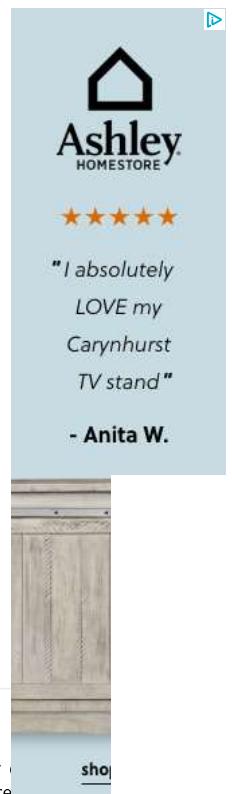
with $CSAR_i$ as the cumulative standardized abnormal returns

$$CSAR_i = \sum_{t=T_1+1}^{T_2} SAR_{i,t}$$

with expectation zero and variance

$$S_{CSAR_i}^2 = L_2 \frac{M_i - 2}{M_i - 4}$$

Under the assumption of cross-sectional independence and some other conditions (Patell, 1976), z_{Patell} is standard normal distribution.



[5] Kolari and Pynnönen adjusted Patell or Standardized Residual Test (Abbr.: Adjusted Patell Z)

Kolari and Pynnönen (<http://rfs.oxfordjournals.org/content/23/11/3996.abstract?etoc>) (2010) propose a modification to the Patell-test to account for abnormal returns. Using the standardized abnormal returns ($SAR_{i,t}$) defined as in (EQ: ??), and defining \bar{r} as the average of the sample cross-correlation period abnormal returns, the test statistic for $H_0 : AAR = 0$ of the adjusted Patell-test is

$$z_{Patell,t} = z_{Patell,t} \sqrt{\frac{1}{1 + (N - 1)\bar{r}}}$$

where $z_{Patell,t}$ is the Patell test statistic. It is easily seen that if the correlation \bar{r} is zero, the adjusted test statistic reduces to the original Patell test statistic. Assuming the square-root rule holds for the standard deviation of different return periods, this test can be used when considering Cumulated Abnormal Returns ($H_0 : CAAR = 0$):

$$z_{Patell} = z_{Patell} \sqrt{\frac{1}{1 + (N - 1)\bar{r}}}$$

[6] Standardized Cross-Sectional or BMP Test (Abbr.: StdCsect Z)

Similarly, Boehmer, Musumeci and Poulsen (1991) proposed a standardized cross-sectional method which is robust to the variance induced by the event. Test statistics on day t ($H_0 : AAR = 0$) in the event window is given by

$$z_{BMP,t} = \frac{ASAR_t}{\sqrt{N} S_{ASAR_t}}$$

with $ASAR_t$ defined as for Patell-test [2] and with standard deviation

$$S_{ASAR_t}^2 = \frac{1}{N - 1} \sum_{i=1}^N \left(SAR_{i,t} - \frac{1}{N} \sum_{l=1}^N SAR_{l,t} \right)^2$$

Furthermore, EST API provides the test statistic for testing $H_0 : CAAR = 0$ given by

$$z_{BMP} = \sqrt{N} \frac{\overline{SCAR}}{S_{\overline{SCAR}}},$$

where \overline{SCAR} is the averaged standardized cumulated abnormal returns across the N firms, with standard deviation

$$S_{\overline{SCAR}}^2 = \frac{1}{N-1} \sum_{i=1}^N \left(SCAR_i - \overline{SCAR} \right)^2,$$

$$\overline{SCAR} = \frac{1}{N} \sum_{i=1}^N SCAR_i$$

and $SCAR_i = \frac{CAR_i}{S_{CAR_i}}$. S_{CAR_i} is the forecast error corrected standard deviation from Mikkelsen and Partch (1988). The Mikkelsen and Partch correction adjusts for each firm the test statistic for serial correlation in the returns. The correction terms are

- Market Model:

$$S_{CAR_i}^2 = S_{AR_i}^2 \left(L_i + \frac{L_i^2}{M_i} + \frac{\left(\sum_{t=T_1+1}^{T_2} (R_{m,t} - \bar{R}_m) \right)^2}{\sum_{t=T_0}^{T_1} (R_{m,t} - \bar{R}_m)^2} \right)$$

- Comparison Period Mean Adjusted Model:

$$S_{CAR_i}^2 = S_{AR_i}^2 \left(L_i + \frac{L_i^2}{M_i} \right)$$

- Market Adjusted Model:

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$$S_{CAR_i}^2 = S_{AR_i}^2 L_i$$

where L_i is the count of non-missing return values in the event window and M_i is the count of non-missing return values in the estimation window for firm i , \bar{R}_m is the average of the sample market returns in the estimation window, see e.g. Patell Test.



[7] Kolari and Pynnönen Adjusted Standardized Cross-Sectional or BMP Test (Abbr.: Adjusted StdCsect Z)

Kolari and Pynnönen (<http://rfs.oxfordjournals.org/content/23/11/3996.abstract?etoc>) (2010) propose a modification to the BMP-test to account for autocorrelation in the abnormal returns. Using the standardized abnormal returns ($SAR_{i,t}$) defined as in the previous section, and defining \bar{r} as the average of the sample estimation period abnormal returns, the test statistic for $H_0 : AAR = 0$ of the adjusted BMP-test is

$$z_{BMP,t} = z_{BMP,t} \sqrt{\frac{1 - \bar{r}}{1 + (N - 1)\bar{r}}},$$

where $z_{BMP,t}$ is the BMP test statistic. It is easily seen that if the correlation \bar{r} is zero, the adjusted test statistic reduces to the original BMP test statistic. As the rule holds for the standard deviation of different return periods, this test can be used when considering Cumulated Abnormal Returns ($H_0 : CAAR = 0$).

$$z_{BMP} = z_{BMP} \sqrt{\frac{1 - \bar{r}}{1 + (N - 1)\bar{r}}}.$$

[8] Skewness Corrected Test (Abbr.: Skewness Corrected T)

The skewness-adjusted t-test, introduced by Hall 1992, corrects the cross-sectional t-test for skewed abnormal return distribution. This test is applicable for averaged abnormal return ($H_0 : AAR = 0$), the cumulative averaged abnormal return ($H_0 : CAAR = 0$), and the averaged buy-and-hold abnormal return ($H_0 : ABHAR = 0$). In the following, we are limited by the situation of cumulative averaged abnormal returns. First, let's revisit the cross-sectional standard deviation (unbiased by sample size):

$$S_{CAAR}^2 = \frac{1}{N-1} \sum_{i=1}^N (CAR_i - CAAR)^2$$

The skewness estimation (unbiased by sample size) is given by:

$$\gamma = \frac{N}{(N-2)(N-1)} \sum_{i=1}^N (CAR_i - CAAR)^3 S_{CAAR}^{-3}.$$

Furthermore, let

$$S = \frac{CAAR}{S_{CAAR}},$$

then the skewness adjusted test statistic for CAAR is given by

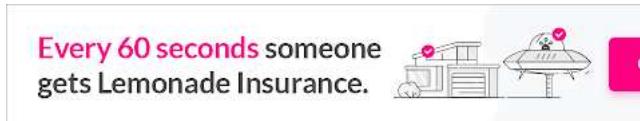
$$t_{skew} = \sqrt{N} \left(S + \frac{1}{3} \gamma S^2 + \frac{1}{27} \gamma^2 S^3 + \frac{1}{6N} \gamma \right),$$

which is asymptotically standard normal distributed. For a further discussion on skewness transformation we refer to Hall (1992) and for further discussion on unbiased estimation of the second and third moment we refer to Cramer (1961) or Rimoldini (2013).

[9] Jackknife Test (Abbr.: Jackknife T)

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Nonparametric Test Statistics

[10] Corrado Rank Test (Abbr.: Rank Z)

In a first step, the Corrado's (1989) rank test transforms abnormal returns into ranks. Ranking is done for all abnormal returns of both the event **and** the estimation period. If ranks are tied, the midrank is used. For adjusting on missing values Corrado and Zvyney (1992) suggested a standardization of the ranks by the number of non-missing values M_i plus 1

$$K_{i,t} = \frac{\text{rank}(AR_{i,t})}{1 + M_i + L_i}$$

where L_i refers to the number of non-missing (i.e., matched) returns in event window. The rank statistic for testing on a single day ($H_0 : AAR = 0$) is the

$$t_{rank,t} = \frac{\bar{K}_t - 0.5}{S_{\bar{K}}},$$

RESEARCH APPS where $\bar{K}_t = \frac{1}{N_t} \sum_{i=1}^{N_t} K_{i,t}$, N_t is the number of non-missing returns across firms, and

$$S_{\bar{K}}^2 = \frac{1}{L_1 + L_2} \sum_{t=T_0}^{T_2} \frac{N_t}{N} (\bar{K}_t - 0.5)^2$$



When analyzing a multiday event period, Campbell and Wasley (1993) ([http://dx.doi.org/10.1016/0304-405X\(93\)90025-7](http://dx.doi.org/10.1016/0304-405X(93)90025-7)) define the RANK-test considering the excess rank for the event window as follows ($H_0 : CAAR = 0$):

$$t_{rank} = \sqrt{L_2} \left(\frac{\bar{K}_{T_1, T_2} - 0.5}{S_{\bar{K}}} \right),$$

where $\bar{K}_{T_1, T_2} = \frac{1}{L_2} \sum_{t=T_1+1}^{T_2} \bar{K}_t$ is the mean rank across firms and time in event window. By adjusting the last day in the event window T_2 one can get a score definded by Campbell and Wasley (1993) ([http://dx.doi.org/10.1016/0304-405X\(93\)90025-7](http://dx.doi.org/10.1016/0304-405X(93)90025-7)).

Note 1: The adjustment for event induced variance as done by Campbell and Wasley (1993) ([http://dx.doi.org/10.1016/0304-405X\(93\)90025-7](http://dx.doi.org/10.1016/0304-405X(93)90025-7)) is omitted here implemented in a future version. In such a case, we recommend the GRANK-T or GRANK-Z test.



[11] Generalized Rank T Test (Abbr.: Generalized Rank T)

In the following steps we assume, for sake of simplicity, that there are no missing values in estimation or event window for each firm. In order to account for possible event-induced volatility, the GRANK test squeezes the whole event window into one observation, the so-called 'cumulative event day'. First, define the standardized cumulative abnormal returns of firm i in the event window

$$SCAR_i = \frac{CAR_i}{S_{CAR_i}},$$

where S_{CAR_i} is the standard deviation of the prediction errors in the cumulative abnormal returns of firm i , namely

$$S_{CAR_i}^2 = S_{AR_i}^2 \left(L + \frac{L_2}{L_1} + \frac{\sum_{t=T_1+1}^{T_2} (R_{m,t} - \bar{R}_m)^2}{\sum_{t=T_0}^{T_1} (R_{m,t} - \bar{R}_m)^2} \right).$$

The standardized CAR value $SCAR_i$ has an expectation of zero and approximately unit variance. To account for event-induced volatility S_{CAR_i} is re-standardized by the cross-sectional standard deviation

$$SCAR_i^* = \frac{SCAR_i}{S_{SCAR}},$$

where

$$S_{SCAR}^2 = \frac{1}{N-1} \sum_{i=1}^N (SCAR_i - \overline{SCAR})^2 \quad \text{and} \quad \overline{SCAR} = \frac{1}{N} \sum_{i=1}^N SCAR_i$$

By construction $SCAR_i^*$ has again an expectation of zero with unit variance. Now, let's define the generalized standardized abnormal returns ($GSAR$):

$$GSAR_{i,t} = \begin{cases} SCAR_i^* & \text{for } t \text{ in event window } SAR_{i,t} \\ 0 & \text{for } t \text{ in estimation window} \end{cases}$$

The CAR window is also considered as one time point, the other time points are considered GSAR is equal to the standardized abnormal returns. Define on this $L_1 + 1$ points the standardized ranks:

$$K_{i,t} = \frac{\text{rank}(GSAR_{i,t})}{L_1 + 2} - 0.5$$

Then the generalized rank t-statistic for testing $H_0 : CAAR = 0$ is defined as:

$$t_{grank} = Z \left(\frac{L_1 - 1}{L_1 - Z^2} \right)^{1/2}$$

with

$$Z = \frac{\overline{K}_0}{S_{\overline{K}}}$$

$t = 0$ indicates the cumulative event day, and

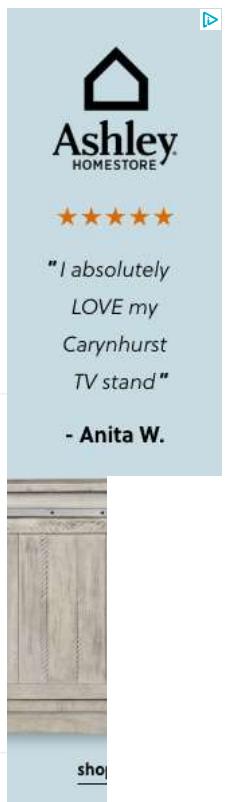
$$S_{\overline{K}}^2 = \frac{1}{L_1 + 1} \sum_{t \in CW} \frac{N_t}{N} \overline{K}_t^2$$

with CW representing the combined window consisting of estimation window and the cumulative event day, and

$$\overline{K}_t = \frac{1}{N_t} \sum_{i=1}^{N_t} K_{i,t}$$

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 t_{grank} is t-distributed with $L_1 - 1$ degrees of freedom.

Formulas testing on a single day ($H_0 : AAR = 0$) are straightforward from the ones shown above.



[12] Generalized Rank Z Test (Generalized Rank Z)

Using some facts about statistics on ranks, we get the standard deviation of \overline{K}_0 :

$$S_{\overline{K}_0}^2 = \frac{L_1}{12N(L_1 + 2)}$$

By this calculation, the following test statistic can be defined

$$z_{grank} = \frac{\overline{K}_0}{S_{\overline{K}_0}} = \sqrt{\frac{12N(L_1 + 2)}{L_1}} \overline{K}_0$$

which converges under null hypothesis quickly to the standard normal distribution as the firms N increase.

[13] Sign Test (Abbr.: not available in our API)

This sign test has been proposed by Cowan (1991) and builds on the ratio of positive cumulative abnormal returns \hat{p} present in the event window. Under the null hypothesis, this ratio should not significantly differ from 0.5.

$$t_{sign} = \sqrt{N} \left(\frac{\hat{p} - 0.5}{\sqrt{0.5(1 - 0.5)}} \right)$$

[14] Cowan Generalized Sign Test (Abbr.: Generalized Sign Z)

Under the Null Hypothesis of no abnormal returns, the number of stocks with positive abnormal cumulative returns (CAR) is expected to be in line with the fraction \hat{p} of positive CAR from the estimation period. When the number of positive CAR is significantly higher than the number expected from the estimated fraction, it is suggested to reject the Null Hypothesis.

The fraction \hat{p} is estimated as

$$\hat{p} = \frac{1}{N} \sum_{i=1}^N \frac{1}{L_1} \sum_{t=T_0}^{T_1} \varphi_{i,t}$$

where $\varphi_{i,t}$ is 1 if the sign is positive and 0 otherwise. The Generalized sign test statistic ($H_0 : CAAR = 0$) is

$$z_{gsign} = \frac{(w - N\hat{p})}{\sqrt{N\hat{p}(1 - \hat{p})}}$$

where w is the number of stocks with positive cumulative abnormal returns during the event period. For the test statistic, a normal approximation of the binomial distribution with the parameters \hat{p} and N is used.

Note 1: This test is based on the paper of Cowan, A. R. (1992) (<http://www.bus.iastate.edu/arnie/simnpar.pdf>).

Note 2: EST provides GSIGN test statistics also for single days ($H_0 : AAR = 0$) in the event time period.

Note 3: The GSIGN test is based on the traditional SIGN test where the null hypothesis assumes a binomial distribution with parameter $p = 0.5$ for the sign of the M cumulative abnormal returns.

Note 4: If M is small, the normal approximation is inaccurate for calculating the p-value, in such case we recommend to use the binomial distribution for calculating the p-value.

[15] Wilcoxon Test (Abbr.: Wilcoxon Z)

The Wilcoxon rank test can be regarded as an extension of the GSIGN test, since it considers both the sign and the magnitude of abnormal returns. This test assumes that none of the absolute values are equal and are non-zero. Let

$$W_t = \sum_{i=1}^N \text{rank}(A_{i,t})^+$$

where $\text{rank}(A_{i,t})$ is the positive rank of the absolute value of abnormal returns $A_{i,t}$ at time point t for firm i . The test statistic for testing ($H_0 : AAR = 0$) is then defined as

$$z_{\text{wilcoxon},t} = \frac{W - N(N-1)/4}{\sqrt{(N(N+1)(2N+1)/12)}}$$

for large N . In the case for testing the cumulative averaged abnormal return ($H_0 : CAAR = 0$), we add the CAAR value for each firm i to the abnormal return window and do the same calculations as for AAR.

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References and further studies

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