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Journal of Financial Economics 79 (2006) 181–222

JOURNAL OF Financial ECONOMICS

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Crushed by a rational stampede: Strategic share dumping and shareholder insurrections ☆

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Received 13 June 2003; received in revised form 12 April 2004; accepted 28 September 2004
Available online 21 July 2005

Abstract

In this paper, we develop a dynamic model of institutional share dumping surrounding control events. Institutional investors sometimes dump shares, despite trading losses, in order to manipulate share prices and trigger activism by "relationship" investors. These institutional investors are motivated to trade not only by trading profits but also by a desire to protect the value of their inventory and to disguise the quality of their own information. Relationship investor profit from targeting firms both by improving firm performance and by generating private information.

JEL classification: G23; G32; G34

Keywords: Strategic trading; Ownership structure; Corporate governance

☆We would like to thank all the participants at the SFS/Northwestern Conference on Investments in Imperfect Capital Markets, session participants at the European Finance Association 2002 meetings and the Western Finance Association 2003 meetings, and seminar participants at McGill University, Southern Methodist University, Case Western Reserve University, and the Batterymarch seminar at MIT. Particular thanks are extended to Matthew Spiegel, for his discussion of the paper. The usual disclaimer applies.
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0304-405X/$ - see front matter
doi:10.1016/j.jfineco.2004.09.003

1. Introduction

Trade in the shares of a troubled firm is characterized both by institutional shareholder share dumping and share acquisition by relationship investors.[1] This pattern of trade in troubled firms cannot be completely explained without developing a dynamic model that incorporates both relationship investing and informed trading. This paper develops a dynamic model relating relationship investing and informed trading. In our analysis, one type of strategic investor, whom we refer to as the "institutional investor," is sometimes able to acquire private information about the quality of a firm's management.[2] Whether a given institutional investor really has private information, or rather, trades simply on market priors, is itself private information. Another type of strategic investor, whom we refer to as the "relationship investor," has no ex ante information endowment. However, in some circumstances a relationship investor can create value through activism. When management quality is good, activism does not create value; when management quality is bad, activism creates value. Institutional investors do not intervene in governance themselves, but instead follow the "Wall Street rule," expressing their preferences through trading. The problem for a given institutional investor is that, from the perspective of maximizing trading profits, she wants prices to be uninformative. However, uninformative prices may cause the relationship investor to abjure activism, which lowers the value of her portfolio. The relationship investor's problem is that he is never sure whether any given firm has poor management and thus needs a dose of activism. Direct communication between the institutional investor and the relationship investor is not incentive compatible: Because the institutional investor bears none of the costs of activism but she reaps buy-side profits from lowering the stock price, has an incentive to trigger activism by reporting adverse conditions even when they are absent. Thus, information transmission can occur through the observation of security prices.

In the first-period, the institutional investor and the liquidity investor trade and the marketmaker sets prices based on the pattern of their order flow. This first-period order flow, which reveals both net order quantities and trading volume, is observed by all agents, including the relationship investor. In the second period, the relationship investor, armed with the information from the order flow, then decides whether to trade and/or intervene in the second period. The institutional trader and the liquidity traders also decide the trades they will submit. The trading decisions of the institutional trader, relationship investor, and the liquidity trader determine second-period prices. Each first-period trade by an institutional investor has three effects; namely, it generates an immediate trading profit or loss, it affects the posterior beliefs of the relationship investor, and it affects the posterior beliefs, and

[1] See Parrino et al. (2003) for an empirical analysis of share dumping by institutional investors and see Webcomm.com (2002) and Byrnes and Laderman (1998) for examples of relationship investing in troubled firms following a stock price drop.

[2] We use "institutional investor" to refer to any investor who owns a stake sufficiently large that his trades affect prices and who lacks the ability to effectively intervene in the operation of the firm. Many pension funds, insurance companies, and large private investors could fall under this description.

thus future price determinations, of the marketmaker. When the institutional trader has superior information, she maximizes her immediate trading profits by trading on this information. Thus, the *trading profit incentive*, ceteris paribus, leads to the standard informed trading strategy—buy when endowed with good information, sell when endowed with bad information, and hold when uninformed.

Given this pattern of informed trade, selling leads the marketmaker to lower share prices. Lower share prices signal to the relationship investor that management is weak and thus activism may be profitable. Because the marketmaker cannot distinguish between informed and uninformed sales, even uninformed sales lower share prices. Thus, selling increases the intrinsic value of the uninformed institutional investor's post-trade portfolio. This *value conservation incentive* provides uninformed institutional investors an incentive to "dump" their shares. This dumping leads to "herding" equilibria in which the institution trades on the sell-side even when uninformed. In fact, lowering the ex ante precision of institutional information, by lowering the trading losses from dumping, can actually encourage uninformed institutional investors to dump. Thus, in contrast to the predictions of standard microstructure models, reductions in the ex ante quality of information can actually increase trading activity.

The third incentive for trade is *marketmaker belief manipulation*. Institutional traders have information on the quality of their own private information. Although rational expectations imply that, averaged across all liquidity trader actions, the marketmaker's assessment of the probability that the institutional investor is informed is correct, the marketmaker's assessment of actual information embedded in a given order flow may not be correct. For example, if an institutional trader sells when uninformed, the marketmaker will adjust the price of the security downward. However, the institutional investor knows that she has really not received any adverse information about the firm. Thus, the price reduction in the first period provides the institutional investor an incentive to buy back in the second period.

These three incentives, trading profit, value conservation, and belief manipulation, determine the structure of the equilibrium outcome. When the cost and benefits of activism are such that the likelihood of activism by the relationship investor is zero or nearly zero, the trading profit incentive dominates. The institutional investor buys with positive information, sells with negative information, and holds when uninformed. Although an uninformed institution realizes positive expected second-period profits from both "pump and dump"—driving the price up by a buy-side trade in the first-period and then selling the overvalued stock in the second period—and "dump and pump"—selling in the first period and then buying back the undervalued stock in the second period–the first period losses from these manipulative strategies always exceed second-period gains. Thus, the uninformed investor stays on the sidelines.

As the costs of relationship investing fall, and the relationship investor begins to make his presence felt, the balance of incentives shifts. Because the relationship investor's likelihood of activism is sensitive to the first-period stock price movements, the value conservation incentive provides the uninformed institutional investor with a strong incentive to dump shares in the first period to provoke activism. When this incentive becomes sufficiently strong, a herding equilibrium emerges in which the institution sells in the first period both when it has adverse information and when it is uninformed. Thus, our model predicts herd behavior by institutions on the sell side but not the buy side. This asymmetric herding prediction generally is consistent with the evidence of Wermers (1999), but inconsistent with a pure labor market-based theory of herding based on reputation protection.

Whether the uninformed institutional investor can augment her value conservation gains from first-period dumping through trading profits with second-period buying profits depends on rather subtle features of second-period market structure. When the relationship investor intervenes, unless his holdings are very large, he can only profit by following a random activism strategy, buying when intervening and selling otherwise. Playing such a random strategy exogenously generates private information for the relationship investor regarding his own actions. This private information creates adverse selection losses for the uninformed institution trying to exploit her own private information that she previously submitted an uninformed trade. When the activism probability is high, buy orders are highly informative, and thus engender large upward quote revisions. In this case, the uninformed shareholder who dumped in the first-period will hold in the second period. In contrast, when the activism probability is low, buy orders will not engender such large upward price revisions and the uninformed investor can profitably buy shares in the second period. Thus, some parametrization of our model supports dump-and-pump equilibria.

Our analysis of the interaction between institutional and relationship investors in a strategic trading framework produces a number of testable implications for researchers in market microstructure and corporate governance. These implications include:

- The likelihood of management shake-ups and governance changes is higher after institutional selling.
- Institutional investor herding will be concentrated on the sell side of the market.
- The likelihood of management shake-ups is not only affected by stock price drops; it is also correlated with trading volume.
- The price effect of institutional share dumping depends on the size of the institution's pre-trade portfolio positions.
- Institutional order flows will exhibit asymmetric time-series correlations, with the likelihood of buy orders following buy orders being higher than the likelihood of sell orders following sell orders.

Delineating the position of this paper within the financial economics literature is somewhat challenging. Because the paper models relationship traders who profit from information about their own activism, institutional traders who profit by exploiting information endowments, and the effect of informed trading on corporate behavior, the paper is a hybrid of the classic microstructure models of informed trade, models of investor activism, and models of the feedback effect of informed trading on corporate policy. First, consider our lineage on the classical microstructure side (see, e.g., Kyle, 1985; Glosten and Milgrom, 1985). In most respects, our model of informed trading is standard. However, one point of departure is that informed traders have private information about the precision of the signals they receive. In this respect, our work follows Gervais (2000). In fact, for exactly the same technical reasons as Gervais, we use the Glosten and Milgrom model of order flow rather than the Kyle model. Also like Gervais, our analysis shows that when traders have information about the precision of their own information, aggregate demand flow is not a sufficient statistic for order flow information. Moreover, because the Kyle model imposes a normal distribution structure on both the investors' private signals and asset values, it requires that the relationship investor contribution to firm value be additive. However, our result depends on the substitutability between the value produced by management and the relationship investor, i.e., the relationship investor's marginal contribution is higher when management quality is low. Without substitutability, the institutional investor has no incentive to deviate from the standard informed trading strategy of buying undervalued and selling overvalued shares. On another branch of the family tree–investor activism–our differences with the extant literature are more profound. Unlike the early literature on activist traders (see, e.g., Kyle and Vila, 1991; Bagnoli and Lipman, 1996), in which activist trading was a precursor to a hostile takeover offer, we focus on minority shareholder activists that attempt to change corporate policy rather than seize formal control.[3] Also, in contrast to Kyle and Vila, in our analysis non-value-improving investors (institutions) move first to affect the expectations of the value-increasing relationship investor. In fact, the central focus of our analysis is the interaction between different types of large strategic investors.

Our analysis is also related to the literature on the effect of informational trade on corporate investment policy. In this literature, management uses market order flow to gather information relevant to its investment decision. As shown by a number of researchers, in this setting, management has an incentive to encourage the production of information through informed trading (see, e.g., Subrahmanyam and Titman, 2001; Khanna and Sonti, 2004; Boot and Thakor, 1993) show that, because of complementarities, feedback effects from information can produce large moves in asset values. In contrast to both Khanna and Sonti and Subrahmanyam and Titman, our analysis features interaction not between informed traders and a nontrading agent, the manager, but rather between two different types of large, strategic trading agents, namely, relationship investors and informed institutional investors. Both types of agents may be willing to suppress information and sacrifice intrinsic value for the sake of trading profits. In addition, in Khanna and Sonti and Subrahmanyam and Titman, investment in new projects is positively correlated with firm quality, while in our model, investment in corporate activism is negatively correlated. Thus, our analysis highlights trade reversal strategies (such as dump and pump), rather than continuation strategies (such as momentum) highlighted in their work. Glostein and Guembel (2002) model an informed investor taking a short position in an asset and then trading in order to send an incorrect signal to

[3]This change in focus may have been motivated by legal and institutional changes that make hostile takeover offers more difficult (see, e.g., Lipin, 1998).

management regarding optimal investment policy. Similar to the other papers discussed above, their analysis considers trading by a single type of trader while we model the interaction between two types of strategic traders. Also, in contrast to both our analysis and the other literature, Goldstein and Guembel consider investors with short positions initiating trades that lower rather than increase value.

This paper is organized as follows. In Section 2, we lay out the basic features of our model. In Section 3, we present some basic results. In Section 4, we develop and analyze strategic trade, corporate activism, and price determination in the second period of the game. In Section 5, we analyze first-period trade and its effect on price behavior, trading strategies, and corporate governance in the second period of the game. In this section, we outline the conditions necessary for a share dumping equilibrium and the conditions for dumping institutions to recapture some trading profits by reversing earlier sell orders. Finally, in Section 6, we conclude the paper.

2. Basic framework

Consider a two-period, three-date economy, with time indexed by $t = 1, 2, 3$. Agents trade at dates 1 and 2. At date 3, the firm pays a liquidating dividend that equals either 1.0 or 0.0 per share. Strategic investors are of two types; specifically, an institutional investor, I, and a relationship investor, R. The institutional investor enters the game at date 1, has the option of trading at both dates 1 and 2, and has a nonnegative endowment of the firm's shares, N_I. The institutional investor is incapable of direct activism in the affairs of the firm. The relationship investor enters at date-2 and, at date-2, has the option of being active in the management of the firm and of trading anonymously. We denote the relationship investor activism policy by ι^R, with $\iota^R = \mathscr{A}$ representing activism, and $\iota^R = \mathscr{N}\mathscr{A}$ representing nonactivism. The relationship investor has a nonnegative endowment, N_R, of the firm's stock.

We assume that activism and management quality are perfect substitutes and activism imposes personal costs on the active relationship investor. In particular, the value of the firm depends on the activism decision of the relationship investor and the state of the world. There are two possible states of the world: Good (G) and Bad (B). Each of these states is equally likely ex ante. In state G, the value of a share of the firm's stock equals 1.0, regardless of the activism decision of the relationship investor. In state B, the value of the firm equals 0.0 unless the relationship investor is active, in which case the value of a share equals 1.0. Activism imposes a cost of $\phi \geqslant 0$ on the relationship investor. Table 1 below summarizes this information.

A number of different interpretations for this formal representation are possible. For example, one could imagine that there are two types of managers, a bad type who is prone to perk consumption and a good type who is immune to this temptation. Through activism, the relationship investor prevents a manager of the bad type from transferring firm value into perk consumption. Alternatively, one could view a bad manager as one unable to create value, and the relationship investor as being able to produce incontrovertible evidence of the quality of management, thereby inducing management turnover.

Table 1
States and payoffs

States of the world	Good (G)	Bad (B)
Prior probability	$\frac{1}{2}$	$\frac{1}{2}$
Value		
Nonactivism ($\mathcal{NA}$)	1.0	0.0
Activism ($\mathcal{A}$)	1.0	1.0

Table 2
Information structure

Signal (s)	H	L	U
Pr(s)	$\frac{\tau_0}{2}$	$\frac{\tau_0}{2}$	$1-\tau_0$
Pr (State $= G\|s$)	1	0	$\frac{1}{2}$
Pr (State $= B\|s$)	0	1	$\frac{1}{2}$

Table 3
Liquidity trader's order flow

Order	+1	0	−1
Probability	+1	0	−1

At the beginning of date 1, before trading, the institutional investor observes one of three signals, s, represented by H, L, and U. The high signal, H, is a perfect signal of state G; the low signal, L, is a perfect signal of state B; the uninformative signal, U, is completely uninformative. The prior probability of the institutional investor's receiving an informative signal is τ_0. Each of the informative signals is equally likely. Table 2 below summarizes this information structure.

Next, consider the trading environment. At dates 1 and 2, the liquidity trader submits an exogenous market order. We assume that the liquidity trader's order equals −1, 0, or +1 share with equal probability. This is summarized in Table 3 below.

Each trader submits a market order to a risk-neutral marketmaker in a competitive market. Orders arrive randomly; that is, each trader's order has an equal chance of arriving first at the marketmaker's desk. After the marketmaker has observed the order flow, the marketmaker sets a price. Because the marketmaker observes each individual order, any order by a strategic investor that does not mimic the one-share pattern favored by the liquidity trader would be identified by the marketmaker as a strategic trade; this would eliminate all profit from the investor's trade (and, on the sell-side, expose her to some adverse selection risk based on the other strategic investor's trading pattern). Thus, given the assumed one-share pattern of trade by the liquidity trader, we assume that strategic shareholders, if they decide to submit an order, submit a one-share order, either buy (+1) or sell (−1) one share. The trading environment is similar to that in Glosten and Milgrom (1985) in that the marketmaker observes a disaggregated order flow. It is also similar to Kyle (1985) in that the marketmaker processes all order flows for a given date at the same time. However, unlike in Kyle , the marketmaker observes volumes as well as net order flow. All our results, with the obvious exception of our prediction regarding volume, can also be obtained under Kyle's assumption that only net order is observed. We prefer our current setup because it permits predictions regarding volume.

We initiate our analysis by considering the prices set by the marketmaker. Because prices are determined by a competitive risk-neutral marketmaker, the price set equals the expected value of the security conditional on the marketmaker's information. At date 1, the order vector is represented by the ordered pair representing the trade of the institutional trader and the liquidity trader. For example, $[t_1^I, t_1^N] = [-1, +1]$ represents the date-1 sale of one share by the institutional investor and the purchase of one share by the liquidity trader. At date-2, an order vector is a triplet representing the orders of the institutional investor, the relationship investor, and the liquidity trader. For example, the vector $[t_2^I, t_2^R, t_2^N] = [+1, 0, -1]$ represents a date-2 trade vector consisting of a buy order from the institutional investor, no order from the relationship investor, and a sell order from the liquidity trader. Because markets are anonymous, the marketmaker cannot distinguish among order vectors that are permutation equivalent (that is, can be transformed into each other by a permutation of the indices of the vector). For example, the marketmaker cannot distinguish between the order vectors $[-1, +1]$ and $[+1, -1]$.

The relation of permutation equivalence partitions the set of order vectors into equivalence classes. We define an order flow as an equivalence class of order vectors. We represent an order flow by a list of trades enclosed in parentheses. For example, the order flow $(-1, -1, +1)$ represents the set of order vectors $\{[-1, -1, +1], [-1, +1, -1], [+1, -1, -1]\}$. When a trader places an order, she does not know the orders placed by the other traders. For this reason, she does not know the price at which her order will be filled. However, she knows the order she submits and the pricing function used by the marketmaker, and thus she knows what she will pay (or receive if she is selling), given the trades executed by the other traders. Thus, traders condition their expectations regarding the price they will pay on their trade by using their conjectures regarding the strategies used by the other traders. In equilibrium, each trader's conjectures are consistent with the (possibly random) strategies played by the other traders.

The terminal value of a share is a random variable V, equal to either 0.0 or 1.0, that depends on the information set of all the agents. Both the marketmaker and the relationship investor have no initial information endowment and use Bayes' rule to update their beliefs regarding the value of the firm. Thus, the equilibrium price set by the marketmaker in the first-period is a function of order flow. Given Bertrand competition, the price equals the expected value—the date-1 price that is a function of date-1 order flow, $P_1(o_1)$, is given by

$$P_1(o_1) = \mathrm{E}[V|o_1] = \sum_s \mathrm{E}[V|o_1, s]\,\mathrm{Pr}(s|o_1), \tag{1}$$

and the date-2 prices are conditioned on both the date-1 and the date-2 order flows, that is,

$$P_2(o_1, o_2) = \mathrm{E}[V|o_1, o_2] = \sum_s \mathrm{E}[V|o_1, o_2, s]\,\mathrm{Pr}(s|o_1, o_2). \tag{2}$$

At date 1, the institutional investor has private signal information, s. She is the only informed trader and thus does not update her beliefs about the value of the firm based on the first-period order flow. Thus, the expected buy and sell price of a share to the institutional investor at date 1, depends only on her private information signal, s, and her conjecture regarding the actions of the other agents. When the institutional investor submits an order flow at date 1, she is unsure of the order flow submitted by the liquidity trader. However, she knows whether she has submitted a buy or a sell order. Thus, if the institutional investor submits a buy (sell) order at date 1, her expected buy (sell) prices at date 1, which we denote by B_1^I (S_1^I), are given as

$$B_1^I = \sum_{o_1} P_1(o_1)\,\mathrm{Pr}(o_1|t_1^I = +1), \tag{3}$$

and

$$S_1^I = \sum_{o_1} P_1(o_1)\,\mathrm{Pr}(o_1|t_1^I = -1). \tag{4}$$

Date-2 expected buy or sell prices, B_2^I, S_2^I, are conditioned on date 1 information and thus are given as

$$B_2^I(o_1) = \sum_{o_2} P_2(o_1, o_2)\,\mathrm{Pr}(o_2|o_1, t_2^I = +1), \tag{5}$$

and

$$S_2^I(o_1) = \sum_{o_2} P_2(o_1, o_2)\,\mathrm{Pr}(o_2|o_1, t_2^I = -1). \tag{6}$$

The relationship investor trades only at date 2. His only private information consists of information regarding his own activism and trading strategy. His expectations regarding the realized order flow in the second-period depends on the first-period order flow, the equilibrium strategies of the agents, and his buy-sell decision. Thus, the second-period expected buy and sell prices faced by the relationship investor, B_2^R, S_2^R, are given as

$$B_2^R(o_1) = \sum_{o_2} P_2(o_1, o_2)\,\mathrm{Pr}(o_2|o_1, t_2^R = +1), \tag{7}$$

and

$$S_2^R(o_1) = \sum_{o_2} P_2(o_1, o_2)\,\mathrm{Pr}(o_2|o_1, t_2^R = -1). \tag{8}$$

The value of a share of the firm's stock depends on both the probability that the relationship investor is active and the state of the world. Thus, at date 1, the marketmaker and the institutional investor make a conjecture regarding the likelihood that the relationship investor will be active. The relationship investor observes date-1 order flow before he decides whether to be active. Thus, his possibly random activism strategy, $\tilde{\imath}^R(o_1)$, is a function of the order flow at date 1 because this order flow is observed before the relationship investor makes his activism decision. The value of a share as a function of the signal is determined as follows. First, let $x(o_1)$ represent the probability that the relationship investor follows the active activism strategy given the equilibrium strategy distribution; that is,

$$x(o_1) = \Pr(\imath^R(o_1) = \mathscr{A}|o_1). \tag{9}$$

Because the relationship investor's trading and activism strategies may be correlated, the date-2 order flow may provide information regarding the likelihood that the relationship investor has decided to be active. The marketmaker's pricing functions at date 1 and date 2 are given as

$$P_1(o_1) = x(o_1) + (1 - x(o_1))\left(\Pr(H|o_1) + \frac{1}{2}\Pr(U|o_1)\right), \tag{10}$$

and

$$\begin{aligned} P_2(o_1, o_2) = {} & \Pr(\imath^R(o_1) = \mathscr{A}|o_1, o_2) + (1 - \Pr(\imath^R(o_1) = \mathscr{A}|o_1, o_2)) \\ & \times \left(\Pr(H|o_1, o_2, \imath^R(o_1) = \mathscr{N}\mathscr{A}) + \frac{1}{2}\Pr(U|o_1, o_2, \imath^R(o_1) = \mathscr{N}\mathscr{A})\right). \end{aligned} \tag{11}$$

We can then define the value of a share conditioned on the institutional investor's signal as

$$\mathrm{E}[V|s, o_1] = \begin{cases} 1.0 & \text{if } s = H, \\ \frac{1}{2}(1 + x(o_1)) & \text{if } s = U, \\ x(o_1) & \text{if } s = L. \end{cases} \tag{12}$$

3. Basic results

In the context of our model, an equilibrium is a trading strategy for the institutional investor at dates 1 and 2, an activism/trading strategy for the relationship investor at date 2, and a set of beliefs for the relationship investor, the institutional investor, and the marketmaker that satisfy the conditions of sequential rationality and belief consistency. That is, strategies at each decision point are optimal given beliefs, and the agents' beliefs are consistent with actual strategies played by other agents. We initiate our analysis by characterizing some general features of candidate equilibrium strategy vectors. These characterizations will prove useful because they reduce the complexity of verifying the optimality of agent strategies by reducing the number of choices we have to consider when checking incentive compatibility conditions.

Lemma 1. (i) *If the institutional investor receives signal H, then, at both dates* 1 *and* 2, *her trading profits from selling* (*buying*) *are nonpositive* (*nonnegative*); (ii) *If the institutional investor receives signal L, then, at date*-1, *her trading profits from selling* (*buying*) *are nonnegative* (*nonpositive*).

Proof. See Appendix.

Lemma 2. (i) *If the relationship investor is active, his trading profit is nonnegative for buying the asset and nonpositive for selling the asset;* (ii) *If the relationship investor is not active, and the marketmaker believes that the strategy of the relationship investor consists of randomizing between only the two following strategies*—(a) *buying and being active*, (b) *selling and not being active—the relationship investor earns an expected nonnegative payoff from selling and nonpositive payoff from buying.*

Proof. See Appendix.

These results are helpful because they greatly reduce the number of incentive conditions that we have to check in the subsequent analysis. Perhaps what is even more noteworthy than what we can establish with such general arguments is what we cannot establish. For example, in general, an institutional investor who has received signal L may not earn a trading profit from selling at date 2. To see this, consider an equilibrium in which (a) the date-1 order flow does not rule out either the uninformed signal U, or low signal L, and (b) the relationship investor is randomizing between being active and buying and being nonactive and selling. Also, suppose that the institutional investor with the signal L places a sell order of -1 and the other two orders happen to be 0 and $+1$. Thus, the order flow is $(-1, 0, +1)$. As seen from the proofs of the lemmas given above, the selling profits of a given investor are proportional to the difference between a share's value given the incorrect and correct evaluations of the investor's action. In this case, if the marketmaker incorrectly believes that the institutional investor did not sell, he may attribute the sell order to the relationship investor and the buy order to the noise trader. Such an attribution, because it implies no activism, produces large downward revisions in the price. In contrast, the institutional investor, knowing that the -1 order comes from herself, and knowing further the equilibrium strategy of the relationship investor, would also know that the 0 order must have come from the liquidity trader and that the $+1$ order must have come from the relationship investor. Thus, she would know the true share value was 1.0. Hence, the sell trade produces a realized loss for some trading patterns of the other two traders, even when the institutional investor has adverse information. As we detail much more extensively in the following sections, the mixture of informed trading and relationship trading that occurs at date 2 restricts the trading profits of the institutional investor at date 2 by conflating her trade with relationship investor's trade, and thus increases the effective bid-ask spread that the institutional investor faces.

4. Strategic trading in the second period

The incentives of the parties in the second period are governed by the fact that trading profits and losses in the first period are sunk from the perspective of the second-period trading. Thus, the only channel for first-period actions to affect second-period trading is through their effect on the beliefs of (a) the relationship investor and the marketmaker regarding the private information of the institutional investor, and (b) the institutional investor and marketmaker regarding the activism policy of the relationship investor. In this section, we let τ represent the posterior probability (conditioned on date-1 order flow) that the marketmaker and the relationship investor assign to the institutional investor's being informed (that is, receiving signal H or L); we let p represent the probability that the marketmaker received signal H conditioned on the marketmaker's being informed. Also, let x represent the probability that the relationship investor buys and is active, and let $1-x$ represent the probability that the relationship investor sells and is nonactive. Initially, we treat x as an exogenous parameter. Later, we close the model by solving for the equilibrium value of x.

In this section we consider subgame equilibria of the second-period subgame. In order to avoid awkward locations, and because, in this section, we consider only second-period behavior, we shall call these subgame equilibria simply "equilibria." A *separating equilibrium* is an equilibrium in which the following trading strategies are followed: The relationship investor randomizes (perhaps trivially) between two strategies—(i) buying and being active, and (ii) selling and being nonactive. The institutional investor follows the strategy of always buying when her signal is H, selling when her signal is L, and holding when her signal is the uninformative signal, U. A *pumping equilibrium* is an equilibrium in which the following trading strategies are followed: The relationship investor randomizes between two strategies—(i) buying and being active, and (ii) selling and being nonactive. The institutional investor follows the strategy of always buying when her signal is H or U and selling when her signal is L. For some second-period beliefs, pumping equilibria exist. However, the separating equilibrium is the only pattern of second-period behavior that can be sustained as a subgame equilibrium for marketmaker's any belief that can be generated by first-period order flows.

4.1. Separating equilibria

To initiate our analysis, we must first determine the equilibrium second-period prices set by the marketmaker conditioned on second-period order flow. The set of prices is influenced both by the updating of beliefs engendered by the first-period order flow (captured by the τ, p, and x parameters) and by the observed second-period order flow. Given separation, there are nine possible order flows in the second period: $(+1,+1,+1)$, $(+1,+1,0)$, $(+1,+1,-1)$, $(+1,0,0)$, $(+1,0,-1)$, $(+1,-1,-1)$, $(0,0,-1)$, $(0,-1,-1)$, $(-1,-1,-1)$.

The price response to some of these order flows is easy to determine. For example, in the separating equilibrium we consider here, order flows with two or more buy orders can occur only if either the institutional investor received the high signal H, and/or the relationship investor intervened. Thus, two or more buy orders always generate a price of 1.0. Also, because the relationship investor trades with probability one in the second period, the order flow $(+1, 0, 0)$ reveals that the relationship investor must have bought and therefore intervened. Hence, this order flow also generates a price of 1.0. In contrast, the order flow $(-1, -1, -1)$ reveals that both the relationship and the institutional investors sold, producing a price of 0.0. The order flow $(-1, 0, 0)$ can only occur (again, because the relationship investor always trades) if the relationship investor sells and the institutional investor does not trade. This pattern of trade reveals that activism will not occur and that the institutional investor received signal U, which implies a share value of $\frac{1}{2}$. The remaining three order flows, $(-1, -1, 0)$, $(-1, -1, +1)$, and $(-1, 0, +1)$, require some computation to derive. To illustrate the process of deriving prices without (we hope) overtaxing the patience of the reader, we will derive only one of these prices: the price associated with the $(-1, -1, +1)$ order flow.

The order flow $(-1, -1, +1)$ can be generated in three ways: (i) the noise trader bought the asset while the institutional investor and the relationship investor sold; (ii) the relationship investor bought the asset while the other two sold; and, (iii) the institutional investor bought the asset while the others sold. The probability of occurrence of (i) is $\frac{1}{3}(1-x)(1-p)\tau$. This probability is the product of the probability that the noise trader buys, $\frac{1}{3}$, the probability that the relationship investor does not intervene, $(1-x)$, and the probability that the signal observed by the institutional investor is L, $(1-p)\tau$. Similarly, the probability of (ii) is $\frac{1}{3}x(1-p)\tau$ and of (iii) is $\frac{1}{3}(1-x)p\tau$. Using Bayes' rule we can compute the posterior probability that the relationship investor is active and intervened; $\Pr(\mathscr{A}|o_1, o_2)$ is computed to equal

$$\Pr(\mathscr{A}|o_1, o_2) = \frac{\frac{1}{3}x(1-p)\tau}{\frac{1}{3}(1-x)(1-p)\tau + \frac{1}{3}x(1-p)\tau + \frac{1}{3}(1-x)p\tau} = \frac{(1-p)x}{1-px}. \tag{13}$$

The posterior probability that the relationship investor did not intervene and the signal was H is

$$\Pr(H|o_1, o_2, \mathscr{N}\mathscr{A}) = \frac{\frac{1}{3}(1-x)p\tau}{\frac{1}{3}(1-x)(1-p)\tau + \frac{1}{3}(1-x)p\tau} = p, \tag{14}$$

and that the relationship investor did not intervene and the signal was L is

$$\Pr(L|o_1, o_2, \mathscr{N}\mathscr{A}) = \frac{\frac{1}{3}(1-x)(1-p)\tau}{\frac{1}{3}(1-x)(1-p)\tau + \frac{1}{3}(1-x)p\tau} = 1-p. \tag{15}$$

Using these posterior probabilities and Eq. (11) we can determine price by

$$P_2(o_1, o_2 = (-1, -1, +1)) = \frac{x(1-p)}{1-px} + \left(1 - \frac{x(1-p)}{1-px}\right)p = \frac{p + x - 2px}{1-px}. \tag{16}$$

Using similar calculations we can establish the prices associated with the remaining order flows. The results of these calculations are presented in Table 4.

Table 4
Second-period prices under separating equilibria

Order flow	Price
$(-1,-1,-1)$	0.0
$(-1,-1,0)$	$\frac{1-\tau}{2(1-p\tau)}$
$(-1,0,0)$	$\frac{1}{2}$
$(-1,-1,+1)$	$\frac{p+x-2px}{1-px}$
$(-1,0,+1)$	$\frac{1-(1-2p)\tau+x(1+\tau-4p\tau)}{2-2(1-p-x+2px)\tau}$
$(-1,+1,+1)$	1.0
$(+1,0,0)$	1.0
$(+1,+1,0)$	1.0
$(+1,+1,+1)$	1.0

Note that prices depend not only on aggregate order flow but also on trading volume, as can be seen by comparing the price associated with the $(-1,-1,+1)$ order flow with the price associated with the $(-1,0,0)$ order flow. Because only the institutional investor and the uninformed liquidity trader submit 0 orders, the order flow $(-1,0,0)$ reveals that the institution is uninformed and the relationship investor did not monitor, producing a share value of $\frac{1}{2}$. On the other hand, $(-1,-1,+1)$ order flow reveals only that the institutional investor is informed (either with H or L information). The value of the firm is 1.0 if the buy order is either from the institution or from the relationship investor and is 0.0 if this order is from the liquidity trader. Thus, when the ex ante likelihood of intervention, x, and/or good information, p, is high, the price associated with the high volume order flow $(-1,-1,+1)$ exceeds the price associated with the low volume order flow $(-1,0,0)$. The situation is reversed when the ex ante likelihood of both intervention and good information is low.

To complete our price specification, two more observations are required. First, for beliefs about the parameters x, p, and τ that generate singularity in the above expressions (that is, where the denominator is zero) we assign the prices using the limit produced by approaching the given singularity (assuming a uniform rate of convergence for all the parameters). Second, there is one order flow we have not considered, $(0,0,0)$. This pattern of order flow is off the equilibrium path as it can occur only if the relationship investor does not trade. In general, off-equilibrium beliefs are crucial for verifying the distance of equilibria. However, in our case, if the relationship investor deviates from his equilibrium decision of trading and decides not to trade (which is the only way this order flow can arise), the relationship investor's trading profits are zero and thus are independent of the marketmaker's belief assignments in any case. For this reason, the price we assign to this order flow is not crucial. For completeness, we assign the price of $\frac{1}{2}$ to the $(0,0,0)$ order flow.

To determine her second-period trading strategy the institutional investor will compare market prices with intrinsic values and maximize her trading profits. The intrinsic value of a share, as a function of the probability of activism of the relationship investor, x, and the institutional investor's private signal, s, is represented by V^I, and defined below:

$$V^I(s,x) = \begin{cases} 1.0 & \text{if } s = H, \\ \frac{1}{2}(1+x) & \text{if } s = U, \\ x & \text{if } s = L. \end{cases} \tag{17}$$

Because agents submit market orders and are not privy to the trading decisions of other agents, they do not know with certainty the price at which their order will be filled. However, in equilibrium, they correctly conjecture the price associated with each possible order flow and the probability distribution of the other agents' strategies. Moreover, they know whether their own trade is a buy or sell. Thus, each agent faces different expected buy and sell prices. These prices, in turn, are based on the marketmaker's conjectures regarding the agent's actions.

We compute the payoffs from these strategies below. For a fixed date-1 order flow, o_1, the date-2 price is a map from order flows to prices, $o_2 \to P_2(o_1, o_2)$.[4] The expected buy (sell) price faced by an agent is an expectation of this map, under the probability measure over second-period orders generated by conditioning on the fact that the agent is buying (selling). The buy and sell prices for the institutional investor are

$$\begin{aligned} B^I_{2,\text{sep}}(x,\tau,p) &= \sum_{o_2} P_2(o_1,o_2)\Pr(o_2|o_1, t^I_2 = +1) \\ &= 1 - \frac{(1-x)^2}{3}\left(\frac{(1-\tau)}{2(1-\tau(1-x-p+2px))} + \frac{(1-p)}{(1-px)}\right), \end{aligned} \tag{18}$$

and

$$\begin{aligned} S^I_{2,\text{sep}}(x,\tau,p) &= \sum_{o_2} P_2(o_1,o_2)\Pr(o_2|o_1, t^I_2 = -1) \\ &= \frac{2x+1}{3} - \frac{(1-x)}{3}\left(\frac{(1-\tau)x}{2(1-\tau(1-x-p+2px))}\right. \\ &\quad \left. + \frac{(1-p)}{(1-px)} - \frac{(1-\tau)}{2(1-p\tau)}\right). \end{aligned} \tag{19}$$

In order to establish the next result, Proposition 3, we need to show that regardless of the probability of monitoring selected by the relationship investor, x, and the beliefs of the marketmaker that represent his date-1 assessment of the information signal of the institutional investor, parameterized by τ and p, separating strategies are incentive compatible for the institutional investor. Thus, we need to show that buying is the most profitable trading strategy when the institutional investor observes H, holding is most profitable when the institutional investor observes U, and selling is most profitable when the institutional investor observes L. That is, we need to establish that the following inequalities hold for all parameter values, x, τ,

[4]The map is detailed in Table 4. The dependence on first-period order flow is reflected only through the beliefs τ, ρ, and x generated by the first-period order flow, o_1.

and p of our model:

$$V^I(H,x) - B^I_{2,\text{sep}}(x,\tau,p) \geqslant \max[0, S^I_{2,\text{sep}}(x,\tau,p) - V^I(H,x)], \tag{20}$$

$$0 \geqslant \max[V^I(U,x) - B^I_{2,\text{sep}}(x,\tau,p), S^I_{2,\text{sep}}(x,\tau,p) - V^I(U,x)], \tag{21}$$

and

$$S^I_{2,\text{sep}} - V^I(L,x) \geqslant \max[0, V^I(L,x) - B^I_{2,\text{sep}}(x,\tau,p)]. \tag{22}$$

Proposition 3 below formally establishes these inequalities. With Proposition 3 in hand, we can close the model by solving for the relationship investor's activism strategy without fear that the resulting strategy will render the candidate trading strategies of the institutional investor suboptimal.

Proposition 3. *The inequalities* 20, 21, *and* 22 *are always satisfied, that is, given any exogenously fixed probability* x *that the relationship investor buys and is active. If market prices are conditioned on* x *and on the institutional investor's following the separating strategy in the second-period of buying with positive information* (H), *holding with no information* (U), *and selling with adverse information* (L), *then the institutional investor's conjectured trading strategy is incentive compatible*.

Proof. See Appendix.

4.2. Pumping equilibria

In a second-period pumping equilibrium, an institutional investor with signal L sells, an investor with signal H buys, and the uninformed investor with signal U also buys, thus "pumping" up the second-period price. As in the separating case, we use Bayes rule to compute the prices associated with each on-the-equilibrium-path order flow. The results of the computations are provided below (Table 5).

In the pumping equilibrium, there are three off-equilibrium order flows: $(0,0,-1)$, $(0,0,0)$, and $(+1,0,0)$. Each of these order flows is reached in our calculations only if one of the parties deviates from his or her equilibrium strategy (of trading) to

Table 5
Second-period prices under pumping equilibria

Order flow	Price
$(-1,-1,-1)$	0.0
$(-1,-1,0)$	0.0
$(-1,-1,+1)$	$\frac{1-x-(1-2p-3x+4px)\tau}{2(1-x-(1-p-2(1-p)x)\tau)}$
$(-1,0,+1)$	$\frac{2(p+x-2px)+(1-p(2-4x)-3x)(1-\tau)}{2(p+x-2px)-2(1-p)(1-2x)(1-\tau)}$
$(-1,+1,+1)$	$\frac{1-\tau+2p\tau+x(1+(1-2p)\tau)}{2(1-(1-p)(1-x)\tau)}$
$(+1,+1,0)$	1.0
$(+1,+1,+1)$	1.0

holding. However, the trading profit from holding is zero regardless of the market's price response. Thus, these prices play no role in the analysis. However, for completeness we assign a price of $\frac{1}{2}$ to both $(0,0,-1)$ and $(0,0,0)$, and of 1.0 to $(+1,0,0)$. Again, as in the separating case, for choices of x, τ, and p that render the numerator and denominator both equal to zero, we set prices using the uniform limit approaching zero.

In equilibrium investors know their own trade: buy or sell and correctly conjecture the pricing operator used by the marketmaker. Thus, each agent faces different expected buy and sell prices. These prices, in turn, are based on the marketmaker's conjectures regarding the agent's actions. We compute the payoffs from these strategies below as we did in the separating case. The buy and sell prices for the institutional investor are presented below.

The buy price expected by the institutional investor in the second period is

$$\begin{aligned} B^I_{2,\text{pump}}(x,\tau,p) = {} & \frac{1}{6}\frac{(x+p\tau-px\tau)}{(1-\tau(1-p)(1-x))} + \frac{(p+x-2px)\tau}{6(1-\tau)+(p+x-2px)\tau} \\ & + \frac{(1-x)(2x+2p-1-3px)\tau}{6(1-x+x\tau-px\tau)} + \frac{3+2x}{6}, \end{aligned} \tag{23}$$

and the sell price expected by the institutional investor in period two is

$$\begin{aligned} S^I_{2,\text{pump}}(x,\tau,p) = {} & \frac{1+2x}{6} + \frac{x(p+x-2px)\tau}{6(1-x)(1-\tau)+(p+x-2px)\tau} \\ & + \frac{(2x+2p-1-3px)\tau}{6(1-x+x\tau-px\tau)} + \frac{x(x+p\tau-px\tau)}{6(1-\tau(1-p)(1-x))}. \end{aligned} \tag{24}$$

In contrast to the separating equilibrium case, not all parameters x, τ, and p support a pumping equilibrium. The binding constraint is the one ensuring that the uninformed institutional investor buys. The higher the ex ante likelihood that the institutional investor is informed, the higher the relative weight the marketmaker places on a buy order, represent real information rather than pumping. Similarly, the higher the ex ante likelihood of monitoring, the more weight the marketmaker places on a given buy order, suggesting activism. Thus, when x, p, and τ are high, the expected buy prices faced by the institutional investor are high. When buy prices are too high pumping is not incentive compatible, and thus the pumping equilibrium cannot be sustained. We let $PUMP$ represent the region of the parameter space that supports a pumping equilibrium, that is,

$$\begin{aligned} & PUMP = \{(x,\tau,p): \\ & V^I(H,x) - B^I_{2,\text{pump}}(x,\tau,p) \geqslant \max[0, S^I_{2,\text{pump}}(x,\tau,p) - V^I(H,x)], \\ & V^I(U,x) - B^I_{2,\text{pump}}(x,\tau,p) \geqslant \max[0, S^I_{2,\text{pump}}(x,\tau,p) - V^I(U,x)], \\ & S^I_{2,\text{pump}} - V^I(L,x) \geqslant \max[0, V^I(L,x) - B^I_{2,\text{pump}}(x,\tau,p)]\}. \end{aligned} \tag{25}$$

This region of the parameter space is graphically represented in Fig. 1.

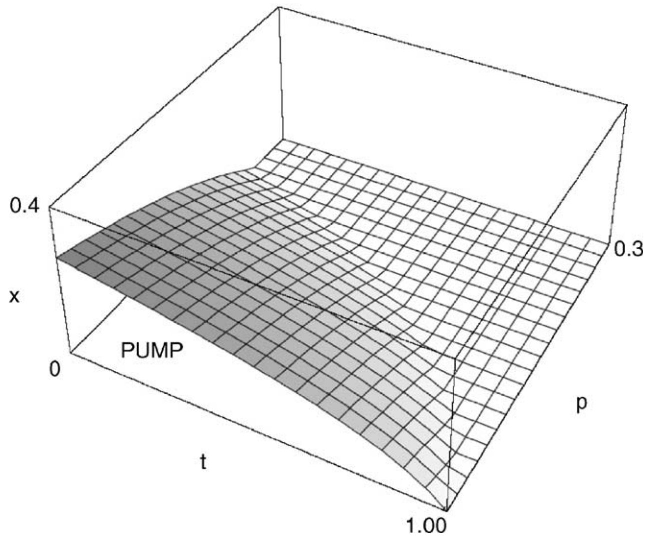

Fig. 1. It depicts the set of parameter values like the precision of the institutional investor's signal, τ, the likelihood of the institution having good information conditioned on being informed, p, and the likelihood of an active activism, x, that support the second period pumping equilibrium. The points in the $\tau - p - x$ cube that render the pumping strategy incentive compatible for the institutional investor are those points lying under the plotted surface. Parameter values outside the plot range are never consistent with a second-period pumping equilibrium.

4.3. Closing the model of second-period trade: the relationship investor's activism decision

The relationship investor can correlate his trading activity with his activism activity. Because the relationship investor's information about his activism decision is private at the time he trades, correlating his trading decision with his activism decision (that is, buying when he intervenes and selling when he does not intervene) can be profitable. As Lemma 2 shows, if the marketmaker conjectures that the relationship investor follows this strategy of correlation, then, in fact, the strategy is profitable. Thus, we can assume that when the relationship investor is active he buys, and when he is not active he sells. Thus, we need to consider only two strategies: The "active strategy" of being active and buying, and the "nonactive" strategy of being nonactive and selling.

The expected payoff from the active strategy is the sum of the trading profit from buying one unit of the stock and the value of the initial position less the cost of intervention,

$$(1 - B^R_{2,\mathrm{j}}) + N_R - \phi, \tag{26}$$

where $B^R_{2,\mathrm{j}}$ is the expected price at which the share of stock is purchased in equilibrium $j \in \{\text{sep}, \text{pump}\}$. Similarly, the expected payoff of the relationship investor from not intervening and selling is

$$\left(S^R_{2,\mathrm{j}} - \frac{1-\tau}{2} - p\tau\right) + \left(\frac{1-\tau}{2} + p\tau\right) N_R, \tag{27}$$

where $S^R_{2,\mathrm{j}}$ is the expected price at which the share of stock is sold in equilibrium $j \in \{\mathrm{sep}, \mathrm{pump}\}$. Let us define the net gain of the relationship investor, $\mathscr{G}^R(\cdot)$, as the difference between the payoff to intervening and not intervening:

$$\mathscr{G}^R_{\mathrm{j}}(x,\tau,p) = (1 - B^R_{2,\mathrm{j}}) - \left(S^R_{2,\mathrm{j}} - \frac{1-\tau}{2} - p\tau\right) + N_R\left(1 - \frac{1-\tau}{2} + p\tau\right) - \phi. \tag{28}$$

In the above, the expected sell profit is increasing in x, the likelihood of activism, and the expected buy profit is decreasing in x. Thus, at most one x equalizes the expected buy and sell payoffs, Eqs. (26) and (27), respectively. If no such x exists, then either the expected buy payoff exceeds the expected sell payoff even when $x = 1$, or the expected sell payoff exceeds the buy payoff even when $x = 0$. Thus, the equilibrium value of x, which we denote by $\mathscr{X}^*_{\mathrm{j}}$, is uniquely determined by the definition

$$\mathscr{X}^*_{\mathrm{j}}(\tau,p) = \begin{cases} 1 & \text{if } \mathscr{G}^R_{\mathrm{j}}(1,\tau,p) > 0, \\ 0 & \text{if } \mathscr{G}^R_{\mathrm{j}}(0,\tau,p) < 0, \\ \{x : \mathscr{G}^R_{\mathrm{j}}(x,\tau,p) = 0\} & \text{otherwise.} \end{cases} \tag{29}$$

The expected buy and sell price for the relationship investor under the separating and pumping equilibrium beliefs are

$$B^R_{2,\mathrm{sep}}(x,\tau,p) = 1 - \frac{(1-x)}{3}\left(\frac{(1-\tau)(1-p\tau)}{2(1-\tau(1-x-p+2px))} + \frac{(1-p)^2\tau}{(1-px)}\right), \tag{30}$$

$$\begin{aligned} S^R_{2,\mathrm{sep}}(x,\tau,p) = {} & \frac{2+2p\tau-\tau}{3} - \frac{(1-x)}{3}\left(\frac{(1-\tau+p\tau)(1-\tau)}{2(1-\tau(1-x-p+2px))}\right. \\ & \left. + \frac{(1-p)\tau}{(1-px)}\right), \end{aligned} \tag{31}$$

$$\begin{aligned} B^R_{2,\mathrm{pump}}(x,\tau,p) = {} & \frac{(1-p)\tau}{6}\left(\frac{(p+x-2px)\tau}{(1-x)(1-\tau)+(p+x-2px)\tau}\right. \\ & \left. + \frac{(2x+2p-1-3px)\tau}{(1-x+x\tau-px\tau)}\right) \\ & + \frac{2p\tau+5-2\tau}{6} + \frac{(x+p\tau-px\tau)}{6(1-\tau(1-p)(1-x))}, \end{aligned} \tag{32}$$

and

$$
\begin{aligned}
S^R_{2,\text{pump}}(x,\tau,p) = {} & \frac{(p\tau+1-\tau)}{6}\left(\frac{(x+p\tau-px\tau)}{(1-\tau(1-p)(1-x))}\right. \\
& \left. + \frac{(p+x-2px)\tau}{(1-x)(1-\tau)+(p+x-2px)\tau}\right) \\
& + \frac{(2x+2p-1-3px)\tau}{6(1-x+x\tau-px\tau)} + \frac{2p\tau-2\tau+3}{6},
\end{aligned} \tag{33}
$$

respectively.

4.3.1. Comparative statics

Before we move on to an analysis of the first-period, it is useful to develop a few of the comparative static properties of the second-period subgame equilibrium. These properties will be useful in explaining first-period behavior. We are interested in how the beliefs, generated by first-period trade, as well as the cost of activism and the relationship investor's initial holdings affect his activism probability. Because the institutional investor knows that her first-period trades affect the likelihood of activism through their effect on the relationship investor's inferences regarding firm value, these comparative static results are important for understanding our subsequent results. For expositional economy we restrict our comparative analysis to the separating equilibrium. As we see in Fig. 1, the pumping equilibrium can be sustained over only a limited range of parametric values, a range that is disjoint from the range over which we obtain our most interesting comparative static results. At the same time, it is readily apparent that our more straightforward comparative statics extend to the pumping equilibrium. Thus, a separate consideration of the pumping equilibrium seems unwarranted.

The relationship investor's activism strategy is determined by the relative profitability of active and nonactive strategies. The difference between the profitability of the two strategies for a given activism probability x can be decomposed into two components. The first component is net trading profit (that is, the difference between buy-side and sell-side trading profits), which we represent by $\mathsf{F}^R(\cdot)$ and define by

$$
\mathsf{F}^R(p,\tau,x) = (1-B^R_{2,\text{sep}}) - \left(S^R_{2,\text{sep}} - \left(\frac{1-\tau}{2}+p\tau\right)\right). \tag{34}
$$

The second component is net cost of activism, that is, the cost of activism netted against the appreciation in value of the relationship investor's initial stake in the firm. We represent this term by $\mathscr{C}^R(\cdot)$, which we define by

$$
\mathscr{C}^R(p,\tau,N_R,\phi) = \phi - N_R\left(1-\left(\frac{1-\tau}{2}+p\tau\right)\right). \tag{35}
$$

The net trading profit, F^R, is everywhere decreasing in x and crosses the horizontal axis at $x=\frac{1}{2}$. Thus, for all $x<\frac{1}{2}$ and all values of p and τ, $\mathsf{F}^R>0$; for all $x>\frac{1}{2}$, $\mathsf{F}^R<0$.

The function $\mathscr{C}^R$ is independent of x. Equilibrium values of x that lie between zero and one occur at the point at which F^R intersects $\mathscr{C}^R$.

We will say that the relationship investor is *biased against activism* if the net cost of activism $\mathscr{C}^R > 0$, *biased toward activism* if the net cost of activism $\mathscr{C}^R < 0$, and *activism neutral* if $\mathscr{C}^R = 0$. If an investor is biased against activism, then in order to be active at all, he must earn a positive net trading profit to counter his bias. Because net trading profit is only positive when $x < \frac{1}{2}$, this implies that an investor who is biased against activism always is active with a probability less than $\frac{1}{2}$. The converse holds for an investor biased in favor of activism. For future reference, we record this result below. Because net trading profit in the pumping equilibrium is decreasing and equals zero at $\frac{1}{2}$, this result also holds in the pumping equilibrium.

Lemma 4. *If the relationship investor is biased toward activism, then the relationship investor's equilibrium probability of monitoring,* $\mathscr{X}^*_{\text{sep}}$, *is always greater than or equal to* $\frac{1}{2}$. *If the relationship investor is biased against activism, then the relationship investor's equilibrium probability of being active,* $\mathscr{X}^*_{\text{sep}}$, *is always less than or equal to* $\frac{1}{2}$. *An activism-neutral investor always has probability* $\frac{1}{2}$ *of being active.*

The effects of the relationship investor's initial holdings and his cost of monitoring are straightforward. Decreases in the holdings of the relationship investor, N_R, or increases in the costs of activism, ϕ, shift the cost curve, $\mathscr{C}^R$, without affecting the net trading profit. Thus, such shifts lower the equilibrium level of activism. This result is recorded below.

Lemma 5. *If the initial holding of the relationship investor increases* (*decreases*), *the probability of activism increases* (*decreases*). *If the costs of activism increase* (*decrease*), *the equilibrium probability of activism decreases* (*increases*).

Fig. 2 depicts the effect of N_R and ϕ on the net trading profit function F^R and the net costs of activism function $\mathscr{C}^R$, and, thus, on the optimal activism strategy of the relationship investor. Note that, in both the pumping and separating cases, net trading profit is decreasing in the activism probability x, and, in both the pumping and separating cases, the net trading profit curve is unaffected by changes in N_R and ϕ. Thus, Lemma 5 can be extended to the pumping equilibrium as well.

In contrast to the comparative statics of holding and costs, the comparative statics for the effect of the relationship investor's beliefs on activism are somewhat subtle. Changes in beliefs affect both the net costs of activism and the net trading profits from activism. A higher likelihood of the good state leads to a smaller benefit of activism for the relationship investor's initial portfolio. Thus, increases in the likelihood of the good state, ceteris paribus, lower the equilibrium likelihood of activism. However, to sign the comparative static, the effect on net trading profits must also be considered. Because net trading profits equal zero if and only if $x_{\text{sep}} = \frac{1}{2}$, parameter shifts in the relationship investor's beliefs rotate the net profit curve, rather than shift its level. Moreover, net trading profits for the relationship investor depend not only on the level of the institutional investor's signal, but also on the precision of her signal. If we increase the level of the institutional investor's signal by raising p, but fix the precision of the signal by holding τ fixed, the net trading profit

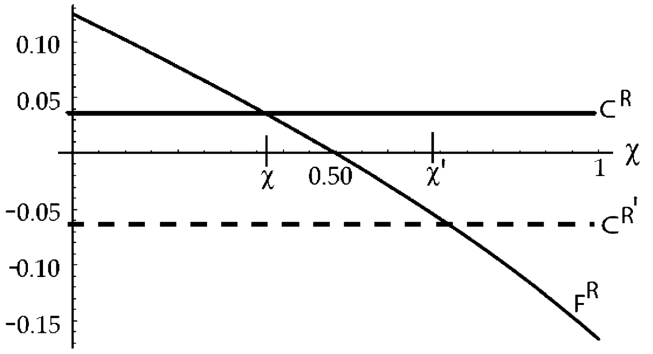

Fig. 2. It depicts the effect of changes in the net costs of activism on equilibrium activism probabilities. The relationship investor's net trading profit is represented by the F^R curve. We fix the net trading profit by fixing the likelihood that the institutional investor's informative signal is high, p, at 0.50 and fixing the likelihood of the institutional investor receiving an informative signal, τ, at 0.50. We fix the initial endowment of the relationship investor, N_R, at 0.20. The net cost of activism is varied by varying the direct cost of activism, ϕ between two levels: a high level ($\phi = 0.15$) and a low level ($\phi = 0.05$). The case where the net cost of activism is high is represented by the solid line C^R. The low cost case is represented by the dotted line $C^{R'}$. The equilibrium probability of activism, χ, is determined by the intersection of the net cost of activism and net trading profit curves.

line becomes flatter. This increased flatness makes the level of activism more sensitive to the net costs of trading. If we hold the expected signal level fixed and increase the precision of the institutional investor's signal by fixing p at $\frac{1}{2}$ and increasing τ, we also make the net profit curve flatter. Moreover, increasing precision while fixing signal quality leaves the net cost curve fixed. These observations lead to the following comparative statics illustrated in Fig. 3. Note that, at $p = \frac{1}{2}$, the pumping equilibrium fails to satisfy the institutional investor's incentive conditions and thus the result below does not extend to the pumping equilibrium.

Lemma 6. *When the institutional investor's likelihood of receiving a positive informative signal equals the likelihood of receiving a negative informative signal* (*that is*, $p = \frac{1}{2}$), *an increase in the likelihood that the institutional investor is informed*, τ, *increases the likelihood that the relationship investor who is biased toward activism will be active*, *and decreases the likelihood that a relationship investor biased against activism will be active.*

Proof. See Appendix.

Lemma 7. *If we fix the expected quality of the institutional investor's information by fixing* τ, *and increase the likelihood that the information is positive by increasing* p, *then a relationship investor biased against activism will be less active. The effect of an increase in* p *on the activism of a relationship investor biased towards activism is ambiguous*.

Proof. See Appendix.

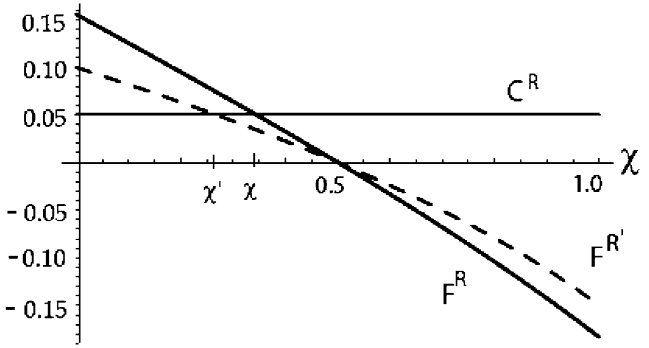

Fig. 3. It depicts the effect of changes in institutional trader information precision on relationship investor activism. The relationship investor's net cost of activism is represented by the C^R curve. We fix the net cost of activism by fixing his initial share endowment at $N_R = 0.10$, the cost of activism at $\phi = 0.10$, and the likelihood that firm value is high without activism at 0.50. The net trading profit from activism is varied by varying the precision of the institutional investor's signal from high ($\tau = 0.80$) to low ($\tau = 0.20$). Net trading profits under the low precision signal are represented by the solid line F^R and under the high precision signal by the dotted line $\mathsf{F}^{R'}$. The equilibrium probability of activism, χ, is determined by the intersection of the net cost of activism and net trading profit curves.

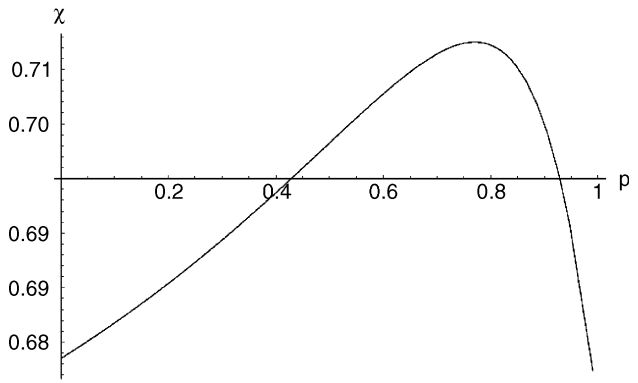

Fig. 4. It illustrates a case in which activism is not decreasing in the conditional likelihood of the good signal. In this figure horizontal axis represents the likelihood of the institution having good information (conditioned on being informed), p. The vertical axis represents the relationship investors equilibrium activism probability, χ. In the example, the relationship investors initial holding, N_R, equals 0.15 and the cost of activism, ϕ is 0.015; the precision of the institutional investor's signal is given by $\tau = 0.50$.

From an intrinsic value perspective, good information and activism are substitutes—the better the institutional investor's information, the less the expected marginal value of activism. Thus, one might expect that it is possible to extend Lemma 7 and prove that increases in p, the likelihood of signal H relative to signal L, always reduce the equilibrium level of activism. The following example, illustrated in Fig. 4, shows that this extension is not possible. In Fig. 4, the investor is biased toward activism, thus the hypothesis of Lemma 7 does not hold. As one can see, activism first increases and then decreases as the likelihood of good information, p, increases. Over the "paradoxical" range of values when p rises, the increase in p lowers the relationship investor's trading profits, both on the buy side and the sell side. However, because the p increases the intrinsic value of a share sold, but does not increase the intrinsic value of a share bought (because activism drives the share value to 1.0 regardless of p), sell-side profits fall faster than buy-side profits. To restore equality in the equilibrium conditions for randomization, buy-side profits must fall. The only way that can occur is for buying and activism to be better anticipated; that is, the likelihood of activism must rise. Although, for some range of model parameters, increases in p may increase the likelihood of activism, the range of values permitting such effects is restricted by Lemma 4, which implies that an increase in p sufficient to switch the relationship investor from being biased toward activism to being biased against activism, will always lower the likelihood of activism.

5. Strategic trading in the first period

We now turn our attention to the analysis of strategic institutional behavior in the first period. We consider patterns of first-period behavior that we find interesting. These involve herding in the first period, that is, trading by the uninformed institution even though such trading produces first-period trading losses. Note that while immediate trading profit rules decision making in the second period, in the first period, decisions are motivated also by their effect on activism and on marketmaker beliefs. The institution, by trading in the direction opposite that of her information endowment in the first period, can always manipulate future marketmaker beliefs at the cost of current trading losses. The next result shows that such strategies always generate expected total trading losses. In following such a strategy, an institution ends up buying at the expected ask price and selling at the expected bid price (or selling at the bid and buying at the ask). Such a strategy generates trading losses. This result is not crucial for establishing any of our subsequent results, but it is crucial for understanding these results. Below, we show that trading reversal strategies can be sustained in equilibrium. Specifically, Proposition 8 shows that in such an equilibrium, the trading profits from the trade reversal must be augmented by the strategy's positive effect on the intrinsic value of the firm's shares.

Proposition 8. *Assume that the marketmaker conjectures that the institutional investor will follow either the separating strategy or the pump strategy in the second period and the relationship investor is either active and buys or is nonactive and sells. Let* $o \to x_j(o)$ *represent any assignment of probability of the active strategy as a function of first-period order flow. Then total trading profits, over both date-*1 *and date-*2*, from buying at date-*1 *and selling at date-*2*, are nonpositive. Similarly, the total trading profits from selling at date-*1 *and buying at date-*2 *are nonpositive.*

Proof. See Appendix.

5.1. Herding equilibria

In this section we will construct herding equilibria with the following properties: In the first period, the institutional investor sells both when she receives unfavorable information (L) or no information (U) and buys when she observes favorable information. In the second period the institutional investor follows the "separating strategy," buying with favorable information, selling with unfavorable information, and otherwise holding.

The viability of this equilibrium rests on the interplay of the three incentives faced by the institutional investor: First-period trade by the institutional investor generates (a) first-period profit or loss, (b) second-period trading profits, and (c) changes in the value of the institution's portfolio through its effect on the likelihood of relationship investor activism. When an institutional investor receives the signal H, she knows activism is not required to preserve firm value and eliminates effect (c) from her calculations. Thus, after receipt of signal H she will maximize total trading profits across the two dates. Given that the marketmaker posts separating prices at date-2, we know from Proposition 3, that H will buy in the second period. By Lemma 1, selling at date-1 will generate nonpositive trading profits, which is clearly a suboptimal outcome. Thus, two viable candidate strategies exist for the institutional investor at date-1: buy at date-1 or hold at date-1. The institutional investor will prefer whichever strategy maximizes total trading profits. As the strategy of not trading at date-1 forgoes initial trading profits, it can be optimal only if by affecting marketmaker beliefs, it lowers the date-2 spread enough to compensate for the opportunity cost of lost date-1 profits. If the marketmaker conjectures that the institutional investor will buy at date-1, then deviating to a hold strategy will indeed lower the marketmaker's assessment of the likelihood that the institutional investor has the high signal. This may even lower the bid price at date-2. Thus, the viability of our equilibrium requires that we rule out cases in which this reduction in future bid prices is so large that the institution with positive information does not wish to trade in the first period.

Next, consider the incentives of the uninformed institution receiving signal U. In the second period separating case we consider here, the uninformed agent will not trade at date-2. Thus, only effects (a) and (c) influence her decision. In other words, the uninformed institutional investor trades off the effect of her trade on the value of her portfolio against first-period trading profits. The larger the institutional investor's share endowment, the more heavily portfolio considerations weigh against trading profits. Because share sales lower the relationship investor's inference regarding the value of the firm absent activism, they generally increase his propensity to intervene. Thus, on the one hand, the portfolio effect argues for selling. On the other hand, sales by the uninformed institutional investor generate trading losses. As we shall see, portfolio effects can dominate for moderately large initial holdings.

Next, consider the incentives of the institutional trader after receipt of an adverse signal, signal L. The portfolio incentive of L to sell at date 1 is stronger than under U. Trading effects are more subtle. On the one hand, in the first period, under signal L selling engenders a trading profit. It generates a loss under signal U. On the other hand, selling in the first period lowers second-period prices, which benefits the institution under signal U but not under signal L. Thus, the set of parameters must be constrained to ensure that this incentive not to sell in the first period is not too strong.

In the rest of this subsection we develop these equilibrium conditions and numerically identify regions of our parameter space that support herding equilibria. First, note that by using Bayes' rule, we can compute the effect of first-period trade on the probabilities of order flows, $\Pr(o_1)$, the beliefs of the marketmaker and the relationship investor, (τ^*, p^*), the prices the marketmaker sets, $P_{1,\text{herd}}$, and the equilibrium activism probabilities of the relationship investor, $\hat{x}^*_{\text{herd}}$. These computations are provided in Table 6 below. Note that $\hat{x}^*_{\text{herd}}(o_1) = \mathscr{X}^*_{\text{sep}}(\tau^*(o_1^N), p^*(o_1))$. These beliefs determine activism strategies, $\hat{x}^*_{\text{herd}}$, share valuations, $\bar{V}^I_{\text{herd}}$, and date-1 buy and sell prices, $B^I_{1,\text{herd}}$ and $S^I_{1,\text{herd}}$. These calculations are presented below as

$$B^I_{1,\text{herd}} = \frac{1}{6}(5 + \hat{x}^*_{\text{herd}}(-1,+1)) \tag{36}$$

and

$$S^I_{1,\text{herd}} = \frac{1}{6}(1 + \hat{x}^*_{\text{herd}}(-1,+1)) + \frac{1}{3(2-\tau_0)}(2 - 2\tau_0 + 2\hat{x}^*_{\text{herd}}(-1,0)). \tag{37}$$

In a herding equilibrium, the value of a share for each order flow, conditioned on the private information of the institutional investor, is given as

$$\hat{V}_{2,\text{herd}}(s, o_1) = V^I(s, \hat{x}^*_{\text{herd}}(o_1)). \tag{38}$$

In addition to determining the expected intrinsic value of the shares, the price of the shares determines the future bid and ask prices faced by the institutional investor at date-2. The expected bid and ask prices at date-2 in the separating equilibrium are determined as follows: The institutional investor knows the probability distribution over date-1 order flow given his date-1 order. Order flow, combined with equilibrium responses, generates an activism policy for the relationship investor, $\hat{x}^*_{\text{sep}}$, and beliefs of the marketmaker, (τ^*, p^*). These beliefs generate bid and ask prices in the second-period through the buy and sell functions $B^I_{2,\text{sep}}$ and $S^I_{2,\text{sep}}$ presented in Eqs. (18) and (19) of the previous section. These functions are defined as

$$\hat{B}^I_{2,\text{herd}}(o_1) = B^I_{2,\text{sep}}(\hat{x}^*_{\text{sep}}(o_1), \tau^*(o_1), p^*(o_1)), \tag{39}$$

Table 6
Prices and order flows under herding equilibrium

Order flow (o_1)	$\Pr(o_1)$	p^*	τ^*	$P_{1,\text{herd}}$
$(-1,0),(-1,-1)$	$\frac{2-\tau_0}{6}$	0	$\frac{\tau_0}{2-\tau_0}$	$\frac{1-\tau_0+\hat{x}^*_{\text{herd}}(-1,0)}{2-\tau_0}$
$(-1,+1)$	$\frac{1}{3}$	$\frac{1}{2}$	τ_0	$\frac{(1+\hat{x}^*_{\text{herd}}(-1,+1))}{2}$
$(+1,+1),(+1,0)$	$\frac{\tau_0}{6}$	1	1	1

and

$$\hat{S}^I_{2,\text{herd}}(o_1) = S^I_{2,\text{sep}}(\hat{x}^*_{\text{sep}}(o_1), \tau^*(o_1), p^*(o_1)). \tag{40}$$

The first-period payoff of the institutional investor from a first-period trade of t^I_1 given that she has received signal s is given by

$$\Pi^I_{1,\text{herd}}(s, t^I_1) = \begin{cases} -B^I_{1,\text{herd}} & t^I_1 = +1, \\ 0, & t^I_1 = 0, \\ S^I_{1,\text{herd}}, & t^I_1 = -1. \end{cases} \tag{41}$$

The second-period payoff to the institutional investor given that she has received signal s and first-period order flow o_1 is given by

$$\begin{aligned} \hat{\Pi}^I_{2,\text{herd}}(s, o_1) = {} & (N_I + t^I_1)\hat{V}^I_{2,\text{herd}}(s, o_1) \\ & + \begin{cases} 1 - \hat{B}^I_{2,\text{herd}}(o_1) & \text{if } s = H \\ 0 & \text{if } s = U \\ \hat{S}^I_{2,\text{herd}}(o_1) - \hat{V}^I_{\text{herd}}(L, o_1) & \text{if } s = L. \end{cases} \end{aligned} \tag{42}$$

The expected second-period payoff from a first-period trade of t^I_1 given that she has received signal s is given by taking expectations over the possible first-period order flows, using the fact that the liquidity trader's order is uniformly distributed over $\{-1, 0, +1\}$. Taking expectations yields

$$\bar{\Pi}^I_{2,\text{herd}}(s, t^I_1) = \frac{1}{3} \sum_{k \in \{-1,0,+1\}} \hat{\Pi}^I_{2,\text{herd}}(s, (t^I_1, k)). \tag{43}$$

The payoff to the institutional investor, given a first-period trade of t^I_1 and given that she has received signal s, is

$$\bar{\Pi}^I_{\text{herd}}(s, t^I_1) = \Pi^I_{1,\text{herd}}(s, t^I_1) + \bar{\Pi}^I_{2,\text{herd}}(s, t^I_1). \tag{44}$$

These definitions permit us to state the necessary and sufficient conditions for the existence of a herding equilibrium. As discussed earlier, the three constraints that determine the boundaries of the herding equilibrium outcome are that the institutional investor (a) prefers to sell rather than hold when she has adverse information; (b) prefers to sell rather than not to trade when she is uninformed; and (c) prefers to buy rather than hold when she has favorable information.

Proposition 9. *A herding equilibrium exists for a fixed specification of initial signal precision, τ_0, activism costs, ϕ, and initial share endowments, N_I and N_R, if and only if the following conditions are satisfied:*

$$\bar{\Pi}^I_{\text{herd}}(L, -1) = \max_{t^I_1 \in \{-1,0,+1\}} \bar{\Pi}^I_{\text{herd}}(L, t^I_1),$$

$$\bar{\Pi}^I_{\text{herd}}(U, -1) = \max_{t^I_1 \in \{-1,0,+1\}} \bar{\Pi}^I_{\text{herd}}(U, t^I_1),$$

and

$$\bar{\Pi}^I_{\text{herd}}(H,+1) = \max_{t_1^I \in \{-1,0,+1\}} \bar{\Pi}^I_{\text{herd}}(H, t_1^I).$$

Figs. 5 and 6 depict the region of the parameter space that supports the herding outcome. In Fig. 5, the precision of the institutional investor's information, τ_0, is fixed at a relatively high level of 0.90. The initial holding of the relationship investor, N_R, and the institution, N_I, are plotted on the vertical and horizontal axes, respectively. Fig. 6 is identical to Fig. 5 except that the ex ante precision of the institutional investor's information is only 0.10. The unshaded region represents the area that supports the herding outcome. From the graph it is clear that increases in the institutional investor's initial holdings favor herding. In contrast, the effect of relationship investor holdings is more subtle. If the relationship investor's holding is very small, then, as we argue in the comparative statics analysis of Section 4, activism is motivated by trading profits and thus is less sensitive to the relationship investor's assessment of the firm's condition. In this scenario, the ability of the institutional investor to trigger increased activism by selling is limited and herding is not favored. Similarly, if the relationship investor's holding is very large relative to activism costs, then the relationship investor will intervene as long as there is any probability that

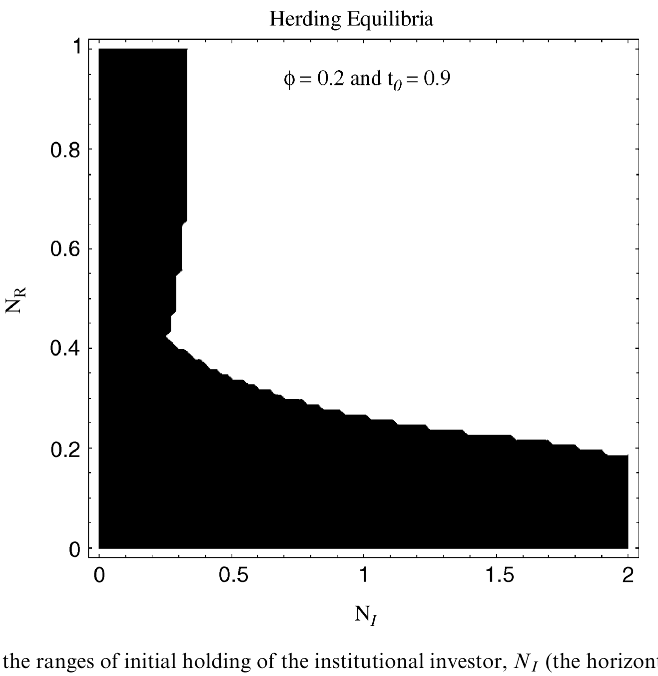

Fig. 5. It depicts the ranges of initial holding of the institutional investor, N_I (the horizontal axis) and the initial holding of the relationship investor, N_R (the vertical axis) for which the herding equilibrium exists. The unshaded region supports the equilibrium. We fix the costs of activism for the relationship investor at $\phi = 0.2$, and the precision of information of the institutional investor at $\tau_0 = 0.9$.

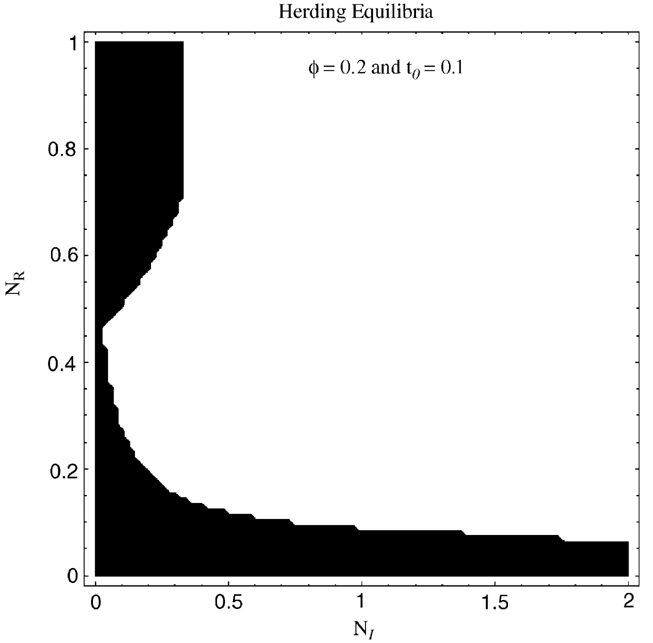

Fig. 6. It depicts the ranges of initial holding of the institutional investor, N_I (the horizontal axis) and the initial holding of the relationship investor, N_R (the vertical axis) for which the herding equilibrium exists. The unshaded region supports the equilibrium. We fix the costs of activism for the relationship investor at $\phi = 0.2$, and the precision of information of the institutional investor at $\tau_0 = 0.1$.

activism is beneficial. Again, this scenario does not favor herding. However, when the relationship investor's holding is moderate in size, that is, significant but not sufficient to motivate activism without trading profits, then activism is very sensitive to observed order flow and this effect favors herding. At the lower level of ex ante information precision depicted in Fig. 6, the trading losses from herding to uninformed traders are reduced. This effect increases the range of parameters that support the herding equilibrium. In fact, in this low ex ante information precision endowment scenario, herding can be supported by very small initial share endowments for the institutional investor at least when the relationship investor's endowment is of moderate size. This is the "best scenario" for herding. Thus, we expect institutional herding to be most pronounced when institutions have only a small likelihood of having valuable information about fundamentals and the relationship investors have a moderately large endowment of shares.

5.2. Dump and pump equilibria

Thus far we restrict our attention to equilibria characterized by separating behavior in the second period. In such equilibria the trading pattern in the second period matches the trader's information endowment: Buying is associated with the high signal, selling with the low signal, and holding with the uninformative signal. As we demonstrate in Proposition 3, such behavior in the second period always satisfies the second-period incentive compatibility and belief consistency conditions regardless of what information is produced by trade in the first-period and regardless of the activism strategy pursued by the relationship investor. However, separation by the institutional investor is not the only pattern of trade that can be supported in equilibrium. Different conjectured patterns of institutional trade, via Bayes' rule, generate different pricing operators, and thus a pattern of trade different from separating may also satisfy the incentive compatibility conditions for an equilibrium. A number of nonseparating patterns of trade merit investigation; however, the most interesting such pattern is that of trades that involve the institution's trading on different sides of the market at different dates; that is, the institution engages in "reverse direction trading." Proposition 8 shows that reverse direction trading cannot be profitable based on trading profits alone. However, since we know that endowment value can be increased by selling and thus triggering activism, it is reasonable to consider the issue of whether reverse direction trading that involves selling in the first period and buying in the second can be supported in equilibrium. Because an institutional investor always suffers a loss from buying in the second period if she has received the low information signal, and, because an institutional investor with a high signal knows the intrinsic value of the firm's shares is independent of activism levels, an investor with the high signal never sells in the first period and an investor with the low signal never buys in the second period. Thus, if we are to find reverse trading equilibria we need to consider reversals by the uninformed institution. Thus, in this section we will analyze the viability of "*dump and pump*" equilibria in which the institutional investor, when uninformed, sells in the first period and, after some first-period order flow, buys back in the second period. In these equilibria, the time-series correlation of trades varies with the direction of trade: Buy trades are followed by buy trades, but sell trades are likely to be reversed.

In the rest of this subsection we develop these equilibrium conditions and numerically identify regions of our parameter space that support dump and pump equilibria. We compute the probabilities of first-period order flows, $\Pr(o_1)$, beliefs of the marketmaker and the relationship investor, (τ^*, p^*), and the prices the marketmaker sets, $P_{1,\mathrm{d\&p}}$. These computations are provided in Table 7 below.

Table 7
Prices and order flows under dump and pump equilibrium

Order flow (o_1)	$\Pr(o_1)$	p^*	τ^*	$P_{1,\mathrm{d\&p}}$
$(-1,0),(-1,-1)$	$\frac{2-\tau_0}{6}$	0	$\frac{\tau_0}{2-\tau_0}$	$\frac{1-\tau_0+\hat{x}^*_{\mathrm{d\&p}}(-1,0)}{2-\tau_0}$
$(-1,+1)$	$\frac{1}{3}$	$\frac{1}{2}$	τ_0	$\frac{(1+\hat{x}^*_{\mathrm{d\&p}}(-1,+1))}{2}$
$(+1,+1),(+1,0)$	$\frac{\tau_0}{6}$	1	1	1

Note that

$$\hat{x}^*_{\mathrm{d\&p}}(o_1) = \begin{cases} \mathscr{X}^*_{\mathrm{pump}}(\tau^*(o_1), p^*(o_1)) & \text{if } o_1 = (-1,0) \text{ or } (-1,-1) \\ \mathscr{X}^*_{\mathrm{sep}}(\tau^*(o_1), p^*(o_1)) & \text{otherwise.} \end{cases} \tag{45}$$

Because first-period trading behavior of the institutional investor is the same in the herding and dump and pump equilibria, the inference about the institutional investor's private information based on order flow (τ^*, p^*), and the expected pattern of first-period order flow, Pr, is the same as in the previous section. Differences between herding and dump and pump cases arise from different anticipated levels of activism in the second period, which in turn depend on the differences in market liquidity in the second period engendered by differences in the trading pattern of the institutional investor. In addition to the first-period incentive compatibility conditions imposed on our analysis of herding equilibrium analyzed in the previous section, we have to impose the condition that in the second period, the pumping trading strategy is incentive compatible for the institutional investor. As shown in Fig. 1, the pumping strategy is never incentive compatible for the institutional investor when the marketmaker believes that $p > \frac{1}{4}$. Thus, if we are to find any pumping occurring in the second period, it will be after order flows that generate revised assessments of p that are less than $\frac{1}{4}$. As we see from Table 7, this observation implies that pumping can only occur after the two "down" order flows, $(-1,0)$ and $(-1,-1)$. After both of these order flows, the marketmaker revises p down to 0. Thus, in dump and pump equilibria, the institutional investor with information signal U sells in the first period and buys back in the second period whenever the realized first-period order flow is $(-1,0)$ or $(-1,-1)$. When the realized order flow is not equal to $(-1,0)$ or $(-1,-1)$, she holds in the second period. This pattern of trade implies the following equilibrium activism policies for the relationship investor.

$$\hat{x}^*_{\mathrm{d\&p}}(o_1) = \begin{cases} \mathscr{X}^*_{\mathrm{pump}}(\tau^*(o_1), p^*(o_1)) & \text{if } o_1 = (-1,0) \text{ or } (-1,-1) \\ \mathscr{X}^*_{\mathrm{sep}}(\tau^*(o_1), p^*(o_1)) & \text{otherwise.} \end{cases} \tag{46}$$

Beliefs determine activism strategies, $\hat{x}^*_{\mathrm{d\&p}}$, share valuations, $\bar{V}^I_{\mathrm{d\&p}}$, and first-period buy and sell prices, $B^I_{1,\mathrm{d\&p}}$ and $S^I_{1,\mathrm{d\&p}}$. These expressions are

$$B^I_{1,\mathrm{d\&p}} = \frac{1}{6}(5 + \hat{x}^*_{\mathrm{d\&p}}(-1,+1)) \tag{47}$$

and

$$S^I_{1,\mathrm{d\&p}} = \frac{1}{6}(1 + \hat{x}^*_{\mathrm{d\&p}}(-1,+1)) + \frac{1}{3(2-\tau_0)}(2 - 2\tau_0 + 2\hat{x}^*_{\mathrm{d\&p}}(-1,0)). \tag{48}$$

In the dump and pump equilibrium, the value of the share conditioned on order flow and the institutional investor's private information is given as

$$\hat{V}_{2,\mathrm{d\&p}}(s, o_1) = V^I(s, \hat{x}^*_{\mathrm{d\&p}}(o_1)). \tag{49}$$

In addition to determining the expected intrinsic value of the shares, the price of the shares determines expected future bid and ask prices faced by the institutional investor at date 2. The expected bid and ask prices at date 2 in the dump and pump equilibrium are determined as follows: The institutional investor knows the probability distribution over date-1 order flow given his date-1 order. Order flow, combined with equilibrium responses, generates an activism policy for the relationship investor, $\hat{x}^*_{\text{d\&p}}$, and beliefs of the marketmaker, (τ^*, p^*);

$$\hat{B}^I_{2,\text{d\&p}}(o_1) = \begin{cases} B^I_{2,\text{pool}}(\hat{x}^*_{\text{d\&p}}(o_1), \tau^*(o_1), p^*(o_1)) & \text{if } o_1 = (-1,0) \text{ or } (-1,-1) \\ B^I_{2,\text{sep}}(\hat{x}^*_{\text{d\&p}}(o_1), \tau^*(o_1), p^*(o_1)) & \text{otherwise;} \end{cases} \tag{50}$$

and

$$\hat{S}^I_{2,\text{d\&p}}(o_1) = \begin{cases} S^I_{2,\text{pool}}(\hat{x}^*_{\text{d\&p}}(o_1), \tau^*(o_1), p^*(o_1)) & \text{if } o_1 = (-1,0) \text{ or } (-1,-1) \\ S^I_{2,\text{sep}}(\hat{x}^*_{\text{d\&p}}(o_1), \tau^*(o_1), p^*(o_1)) & \text{otherwise.} \end{cases} \tag{51}$$

The first-period payoff of the institutional investor from a first-period trade of t^I_1, given that she has received signal s, is given by

$$\Pi^I_{1,\text{d\&p}}(s, t^I_1) = \begin{cases} -B^I_{1,\text{d\&p}}, & t^I_1 = +1, \\ 0, & t^I_1 = 0, \\ S^I_{1,\text{d\&p}}, & t^I_1 = -1. \end{cases} \tag{52}$$

The second-period payoff to the institutional investor, given that she has received signal s, and first-period order flow o_1, is given by

$$\begin{aligned} \hat{\Pi}^I_{2,\text{d\&p}}(s, o_1) = (N_I + t^I_1)\hat{V}^I_{2,\text{d\&p}}(s, o_1) \\ + \begin{cases} 1 - \hat{B}^I_{2,\text{d\&p}}(o_1) & \text{if } s = H, \\ 0 & \text{if } s = U \text{ and } o_1 \notin \{(-1,-1),(-1,0)\}, \\ \hat{V}^I_{\text{d\&p}}(U, o_1) & \\ \quad -\hat{B}^I_{2,\text{d\&p}}(o_1) & \text{if } s = U \text{ and } o_1 \in \{(-1,-1),(-1,0)\}, \\ \hat{S}^I_{2,\text{d\&p}}(o_1) & \\ \quad -\hat{V}^I_{\text{d\&p}}(L, o_1) & \text{if } s = L. \end{cases} \end{aligned} \tag{53}$$

The expected second-period payoff from a first-period trade of t_1^I, given that she has received signal s is given by

$$\bar{\Pi}^I_{2,\mathrm{d\&p}}(s,t_1^I) = \frac{1}{3}\sum_{k\in\{-1,0,+1\}} \hat{\Pi}^I_{2,\mathrm{d\&p}}(s,(t_1^I,k)). \tag{54}$$

The payoff to the institutional investor, given a first-period trade of t_1^I given that she has received signal s is

$$\bar{\Pi}^I_{\mathrm{d\&p}}(s,t_1^I) = \Pi^I_{1,\mathrm{d\&p}}(s,t_1^I) + \bar{\Pi}^I_{2,\mathrm{d\&p}}(s,t_1^I). \tag{55}$$

These definitions permit us to state the necessary and sufficient conditions for the existence of a dump and pump equilibrium. As discussed earlier, the three constraints that determine the boundaries of the dump and pump equilibrium outcome are that the institutional investor (a) prefers to sell rather than hold when she has adverse information, (b) prefers to sell rather than not to trade when she is uninformed, and (c) prefers to buy rather than hold when she receives favorable information.

Proposition 10. *A dump and pump equilibrium exists for a fixed specification of initial signal precision, τ_0, activism costs, ϕ, and initial share endowments, N_I and N_R, if and only if the following conditions are satisfied*:

$$\bar{\Pi}^I_{\mathrm{d\&p}}(L,-1) = \max_{t_1^I\in\{-1,0,+1\}} \bar{\Pi}^I_{\mathrm{d\&p}}(L,t_1^I),$$

$$\bar{\Pi}^I_{\mathrm{d\&p}}(U,-1) = \max_{t_1^I\in\{-1,0,+1\}} \bar{\Pi}^I_{\mathrm{d\&p}}(U,t_1^I),$$

$$\bar{\Pi}^I_{\mathrm{d\&p}}(H,+1) = \max_{t_1^I\in\{-1,0,+1\}} \bar{\Pi}^I_{\mathrm{d\&p}}(H,t_1^I),$$

$$(x^*(0,-1),\tau^*(0,-1),p^*(0,-1)) \in PUMP,$$

$$(x^*(-1,-1),\tau^*(-1,-1),p^*(-1,-1)) \in PUMP.$$

The parameter values supporting the dump and pump outcomes are illustrated in Fig. 7. In this figure, the horizontal axis represents the institutional investor's initial endowment of shares, N_I, and the vertical axis represents the relationship investor's cost of activism, ϕ. The ex ante information precision, τ_0, is fixed at 0.1 and the initial holdings of the relationship investor, N_R, are fixed at 0. As the figure shows, even with a zero share endowment for the relationship investor, costs must be fairly high to support the dump and pump outcome. Costs must be high enough to keep the relationship investor's activism probability sufficiently low to permit profitable second-period trading by the uninformed institutional investor. At the same time, the holdings of the institutional investor must be large enough to ensure that first-period trading losses from dumping caused by herding in the first period are covered by portfolio appreciation. Finally, it is worth noting that the dump and pump equilibrium supports a positive, albeit small, level of activism even under parameters that are so adverse that they would permit no activism in the second-period

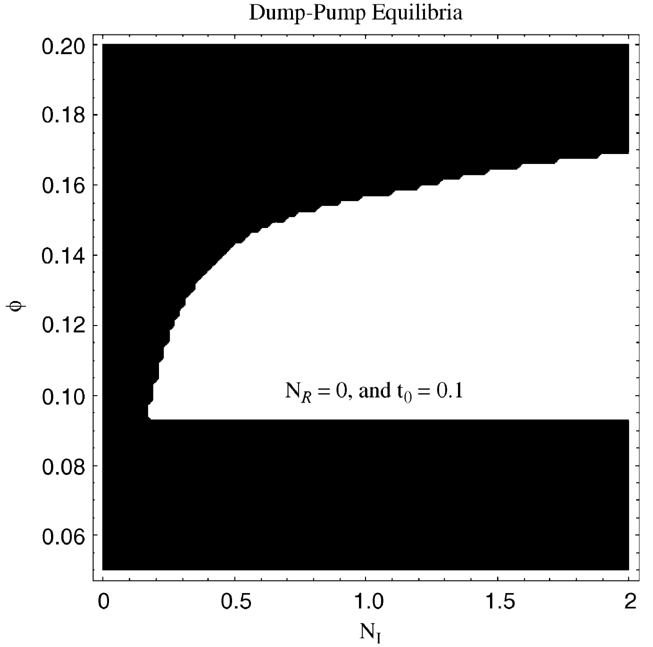

Fig. 7. It depicts the ranges of values of initial holding of the institutional investor, N_I (the horizontal axis) and the costs of activism for the relationship investor, ϕ (the vertical axis) for which the 'dump-pump' equilibrium exists (unshaded region). We fixed the relationship investor's initial holding, $N_R = 0$, and the precision of information of the institutional investor, $\tau_0 = 0.1$.

separating equilibrium. The reason for this is that the increased buy-side demand by the uninformed institutional investor in the second period lowers the expected buy-side prices faced by the relationship investor, and this encourages the relationship investor to adopt high activism strategies even though they have negligible endowments and face high costs. Thus, despite the rather disparaging appellation, the dump and pump manipulations by institutions increase intrinsic firm value through their effect on relationship investor behavior.

6. Conclusion

In this paper, we develop a dynamic model of institutional shareholder trading and activism. The model combines key elements of a rich model of market microstructure including volume-dependent quotes with a model of activism by outside relationship investors. The analysis shows that voice and exit, which may be substitutes at the level of an individual investor, are highly complementary at the aggregate level, with exiting investors attempting to provoke other activist investors into exercising voice, and activist investors looking to the trading patterns of exiting investors as a guidepost for their activism activity. This model explains the well-documented practice of institutional share dumping, that is, institutional investors selling shares in troubled firms. The model also has a number of implications for trading volume and corporate control, the portfolio positions of institutions, the market reaction to their trades, and the effect of strategic trading on governance.

Extending this research would produce even more insights into the interaction of governance and market microstructure. The analysis could be extended either by framing the relationship between shareholder activism and firm value in a more general fashion, analyzing the alternative information structures for the relationship investor, or by considering a more flexible model of liquidity trader behavior. First consider the relationship between activism and management quality. We assume that shareholder activism and good management are substitutes; that is, activists are not needed when management is strong. This model fits the pattern of disciplinary relationship investing usually observed in large firms. However, with smaller firms, relationship investors frequently adopt a developmental role. Such developmental efforts may well bear the greatest fruit when the targeted firm and its management have good prospects. In this case, activism and quality would be complements. Such complementarity could be developed in our framework, and modelling them it might produce an explanation for institutional pumping—institutions herding and buying into a small firm's shares, even when uninformed. This sort of herding phenomenon on the buy side, restricted to high-tech and small firms, has been documented by Wermers (1999).

In addition to extending our treatment of the value-to-activism relationship, we might also gain by extending our analysis of the information structures faced by the players. In our analysis, the relationship investor has the same information as the marketmaker; that is, he observes the entire pattern of order flow (absent, of course, the identity of the traders submitting each order). The relationship investor can use this information to structure his activism strategy. An interesting extension of our analysis would be to restrict the relationship investor's information set to a subset of the information in the order flow, say to observing only price or only price and aggregate demand. Such restrictions would affect the relationship investor's activism strategy both by increasing his exposure to trading losses with informed institutions and by changing the initial trading strategies of institutions, who would change their date-1 strategy of affecting the relationship investor's beliefs.

A third direction for future research would be to systematically vary the liquidity of markets across periods. Our analysis fixes market liquidity and assumes that liquidity demand is exogenous. In a model with variable and endogenous liquidity demand, date-1 institutional trades would affect not only the informativeness of prices but also, by releasing private information, second-period liquidity. Strategic institutions would factor these liquidity effects into their trading strategies. Increased liquidity can help or retard relationship investing activity depending on the cost of activism and the prior endowment of the relationship investor. Thus, such strategic manipulation of liquidity could provide another layer of incentives for the institutional investor.

Appendix A

A.1. Proof of Lemma 1

If the institutional investor has signal H, then the value of her share is equal to 1.0 regardless of the activism decision of the relationship investor. Because share prices are always between 0.0 and 1.0, assertion (i) of the lemma is clear. Next, consider (ii). First, note that the trading payoff to the institutional investor from buying 1 unit of the asset is

$$\mathrm{E}[V|L] - B^I. \tag{56}$$

To show that expected profits are nonpositive, we need only show that they are nonpositive for all realized date-1 order flows o_1 that occur with positive probability. To see this is sufficient, note that, by the law of iterated conditional expectations and because, given her information about firm quality, the institutional investor's information regarding her own order provides no information on firm value, we have that

$$\begin{aligned}\mathrm{E}[V|L] = \mathrm{E}[V|t_1^I = +1, L] &= \sum_{o_1} \mathrm{E}[V|t_1^I = +1, L, o_1]\,\mathrm{Pr}(o_1|t_1^I = +1, L)\\ &= \sum_{o_1} \mathrm{E}[V|t_1^I = +1, L, o_1]\,\mathrm{Pr}(o_1|t_1^I = +1).\end{aligned} \tag{57}$$

From Eqs. (57) and (3), it follows that it suffices to show the inequality for each order flow occurring with positive probability; that is, we need to show that

$$\mathrm{E}[V|L, o_1] \leqslant P_1(o_1). \tag{58}$$

From Eqs. (12), (10) it follows that Eq. (58) holds. Thus, we have established that the trading payoff from buying is nonpositive. Now note that using the same argument we can show that the trading payoff from selling is nonnegative. Thus, we have established our result.

A.2. Proof of Lemma 2

To prove (i) simply note that if the relationship investor is active, share value increases to 1.0, which is the upper bound on the share's value and thus of its price. Next, consider (ii). The expected profit to the relationship investor from selling a unit of stock at date-2 is given by the average of his payoffs over all possible second-period order flows. Thus, using the same argument as used in Lemma 1, we can show that for all realized second-period order flows, the relationship investor's conditional share value is less than the market price. By assumption, the relationship investor either uses the actions outlined in alternative (a), which we represent by the event **A**, or he follows actions defined by (b), which we represent by $-\mathbf{A}$.

Applying the principle of iterated expectations to Eq. (2) we obtain

$$P_2(o_1, o_2) = \mathrm{E}[V|o_2, o_1] = \Pr(\mathbf{A}|o_1, o_2)\mathrm{E}[V|o_2, o_1, \mathbf{A}] + \Pr(-\mathbf{A}|o_1, o_2)\mathrm{E}[V|o_2, o_1, -\mathbf{A}]. \tag{59}$$

The value of the share to the relationship investor conditioned on the order flow and the fact that he is not active is

$$\mathrm{E}[V|o_2, o_1, -\mathbf{A}]. \tag{60}$$

Thus, his trading profit from selling, under order flow (o_1, o_2), given by the difference between Eqs. (59) and (60), is

$$\Pr(\mathbf{A}|o_1, o_2)\,(\mathrm{E}[V|o_2, o_1, \mathbf{A}] - \mathrm{E}[V|o_2, o_1, -\mathbf{A}]). \tag{61}$$

By assumption, in event **A** the relationship investor must monitor, and thus $\mathrm{E}[V|o_2, o_1, \mathbf{A}] = 1$; thus, Eq. (61) equals

$$\Pr(\mathbf{A}|o_1, o_2)\,(1 - \mathrm{E}[V|o_2, o_1, -\mathbf{A}]) \geqslant 0. \tag{62}$$

Thus, as claimed, the selling payoff is nonnegative.

A.3. Proof of Proposition 3

The institutional investor receives signal U. We have to show that the expected buy price is greater than $B^I_{2,\mathrm{sep}} \geqslant \frac{1}{2}(1+x)$, the expected value of the asset to an uninformed institutional investor. To do this, use (18) to substitute for $B^I_{2,\mathrm{sep}}$, obtaining the inequality

$$1 - \frac{(1-x)^2}{3}\left(\frac{(1-\tau)}{2(1-\tau(1-x-p+2px))} + \frac{(1-p)}{(1-px)}\right) \geqslant \frac{1}{2}(1+x), \tag{63}$$

which gives

$$1 \leqslant \frac{3x + 5x\tau p + 12x^2\tau p^2 + 1 + 2\tau x^2 + 2\tau p^2 + 2p}{(1-\tau+\tau p(1-x)+\tau x(1-p))(1-xp)} - \frac{10\tau p^2 x + 5xp + \tau + \tau x + 9\tau x^2 p + x^2 p}{(1-\tau+\tau p(1-x)+\tau x(1-p))(1-xp)}. \tag{64}$$

Since the denominator $(1-\tau+\tau p(1-x)+\tau x(1-p))(1-xp) \geqslant 0$ for all values of all parameters, we cross-multiply and rearrange the above expression to obtain

$$0 \leqslant 2\tau p^2 + (2-\tau)p + (3 + 6\tau p - 9\tau p^2 - 4p - 2\tau)x + (10\tau p^2 - p - 8\tau p + 2\tau)x^2. \tag{65}$$

Eq. (65) is of the following quadratic functional form:

$$F(x) = A + Bx + Cx^2,$$

where $0 \leqslant x \leqslant 1$.

First we show the following result.

Quadratic result. A sufficient condition for the quadratic F to be nonnegative over the interval [0, 1] is that (a) $F(0)>0$, (b) $F(1)\geqslant 0$, and (c) $C>0 \Rightarrow A+\frac{1}{2}B\geqslant 0$.

Proof of Result. First note that if the antecedent in condition (c) fails to hold, then F is concave. By (a) and (b), F is positive at the endpoints of the interval [0, 1]. Thus, by concavity, it must be positive in the interior. Now suppose the antecedent condition holds. In this case, F is convex over the interval [0, 1]. This implies that it majorizes all of its support functions (linear functions that are both tangent to F and equal in value to F at some fixed value of $x \in [0,1]$). Let $f_0(x)$ be the support function for F at the point $x=0$ and let $f_1(x)$ be the support function for F at the point $x=1$. Then, we have that $F(x)\geqslant \max[f_0(x), f_1(x)]$. Because $f_1(x)=A+Bx$ and $f_0(x)=A+Bx+2C(x-\frac{1}{2})$, and because the minimum value of $\max[f_0(x), f_1(x)]$ occurs at $x=\frac{1}{2}$, we have that $F(x)\geqslant \max[f_0(x), f_1(x)]\geqslant \max[f_0(\frac{1}{2}), f_1(\frac{1}{2})]=A+\frac{1}{2}B$. Thus, the Quadratic result is established. Translating the three conditions of the result into the algebra used in this paper yields

$$\begin{array}{lll} \text{(a)} & A\geqslant 0, & 2\tau p^2+(2-\tau)p\geqslant 0, \\ \text{(b)} & A+B+C\geqslant 0, & 3(1-p)(1-p\tau)\geqslant 0, \\ \text{(c)} & C\geqslant 0 \Rightarrow A+\frac{1}{2}B\geqslant 0, & (1-p)+\frac{(1-\tau)}{2\tau p}+\frac{(1-\tau p^2)}{4\tau p}\geqslant 0. \end{array} \tag{66}$$

We need only show that $A+\frac{1}{2}B\geqslant 0$. A little algebra yields

$$\begin{aligned} A+\frac{1}{2}B &= (2\tau p^2+(2-\tau)p)+\frac{1}{2}(3+6\tau p-9\tau p^2-4p-2\tau) \\ &= 2\tau p(1-p)+(1-\tau)+\frac{(1-\tau p^2)}{2}. \end{aligned} \tag{67}$$

Because the right-hand side (RHS) of the above equation is nonnegative for all values of all the parameters of the model, we have established the result that the buy price is $B^I_{2,\text{sep}}>\frac{1}{2}(1+x)$; that is, that the uninformed institution will not buy.

Next, suppose that the institutional investor receives signal U. We have to show that the expected sell price $S^I_{2,\text{sep}}<\frac{1}{2}(1+x)$, the expected value of the asset to an uninformed institutional investor; that is, that the institution will not sell when uninformed.

$$\frac{2x+1}{3}-\frac{(1-x)}{3}\left(\frac{(1-\tau)x}{2(1-\tau(1-x-p+2px))}+\frac{(1-p)}{(1-px)}-\frac{(1-\tau)}{2(1-p\tau)}\right)\leqslant\frac{1}{2}(1+x). \tag{68}$$

Since the denominator $(\tau x(1-p)+\tau p(1-x)+1-\tau)(1-xp)(1-\tau p)\geqslant 0$ for all values of all parameters, we cross-multiply and rearrange to obtain

$$\begin{aligned} 0\leqslant{}& \tau p-\tau-2p-\tau^2+2+4\tau^2 p+2\tau x^2 p^3-5\tau^2 p^2 \\ &+(5\tau p^2+6\tau^2 p^2+\tau^2+1+\tau-8\tau p-3\tau^2 p^3-3\tau^2 p)x \\ &\times(\tau p^2+\tau p-p-2\tau^2 p^3+2\tau^2 p^2-\tau^2 p)x^2. \end{aligned} \tag{69}$$

Eq. (69) is of the form

$$F(x) = A + Bx + Cx^2, \tag{70}$$

where $0 \leqslant x \leqslant 1$. A sufficient condition for the function to be always nonnegative, $F(x) \geqslant 0$, is the Quadratic result established above.

$$\begin{array}{lll} \text{(a)} & A \geqslant 0 & (1-p)(1-\tau)(2(1-\tau p)+\tau) \\ & & +(1-p)\tau p \geqslant 0 \\ \text{(b)} & A+B+C \geqslant 0 & 3(1-p)(1-p\tau)^2 \geqslant 0, \\ \text{(c)} & C \geqslant 0 \Rightarrow A + \frac{1}{2}B \geqslant 0 & \text{(see Eq.(73) below),} \end{array} \tag{71}$$

where

$$A + \frac{1}{2}B = \tau p - \tau - 2p - \tau^2 + 2 + 4\tau^2 p + 2\tau x^2 p^3 - 5\tau^2 p^2 \\ + \frac{1}{2}(5\tau p^2 + 6\tau^2 p^2 + \tau^2 + 1 + \tau - 8\tau p - 3\tau^2 p^3 - 3\tau^2 p). \tag{72}$$

Upon simplification we obtain

$$0 \leqslant \frac{3}{2}(1-\tau)(1-\tau p)(1-p) + \tau(1-p)^2 + \frac{1}{2}(1-\tau^2 p^2)(1-p) \\ + \frac{1}{2}(\tau(1-\tau)(1-p) + (1-\tau)(1-\tau p)). \tag{73}$$

The RHS of Eq. (73) is nonnegative for all values of all parameters of the model. Thus we have established that the institutional investor will not sell when uninformed.

Next, we need to show that if investor receives signal L she will sell; that is, we have to show that the expected sell price $S^I_{2,\text{sep}} \geqslant x$, the expected value of the asset to an institutional investor with signal L. Expressed algebraically, this condition is given by

$$\frac{2x+1}{3} - \frac{(1-x)}{3}\left(\frac{(1-\tau)x}{2(1-\tau(1-x-p+2px))} + \frac{(1-p)}{(1-xp)} - \frac{(1-\tau)}{2(1-\tau p)}\right) \geqslant x, \tag{74}$$

which yields

$$2x \leqslant \frac{1 + 8\tau^2 p^3 x + 5\tau^2 xp + 10x^2\tau p^2 + \tau^2 x^2 + 2\tau^2 p^2 + 2\tau x^2 + x^2 + 3x^3\tau p^2 + 6x\tau p}{(\tau x(1-p) + \tau p(1-x) + 1 - \tau)(1-xp)(1-\tau p)} \\ - \frac{2\tau + 13\tau x^2 p + 5xp + 9x\tau^2 p^2 + \tau p + 4x^3\tau^2 p^3 + \tau^2 p + x^3 p - \tau^2 - 8\tau^2 x^2 p^2}{(\tau x(1-p) + \tau p(1-x) + 1 - \tau)(1-xp)(1-\tau p)} \\ - \frac{5\tau p^2 x + \tau^2 x^3 p + 2\tau^2 p^3 + 3t^2 x^2 p + 2\tau^2 x + 6x^2\tau^2 p^3 - 2p - 2x - 3\tau^2 x^3 p^2}{(\tau x(1-p) + \tau p(1-x) + 1 - \tau)(1-xp)(1-\tau p)}. \tag{75}$$

Since the denominator $(\tau x(1-p) + \tau p(1-x) + 1 - \tau)(1-xp)(1-\tau p) \geqslant 0$ for all values of all parameters, we cross-multiply and rearrange to obtain

$$0 \leqslant \tau p + (1-\tau)(2p(1-\tau p) + (1-\tau)) \\ + (4\tau p - p - 5\tau p^2 + 4\tau^2 p^3 - \tau^2 p(1+p))x. \tag{76}$$

Eq. (76) is of the following functional form:

$$F(x) = A + Bx, \tag{77}$$

where $0 \leqslant x \leqslant 1$. A sufficient condition for this linear function to be always positive, $F(x) > 0$, is that

$$F(0) \text{ is positive } A \geqslant 0, \quad \tau p + 2p(1-\tau)(1-\tau p) + (1-\tau)^2 \geqslant 0$$

$$F(1) \text{ is positive } A + B \geqslant 0 \, (1-\tau)^2 + (1-\tau p)^2 p + \tau p((3-\tau(p+2))(1-p)) \geqslant 0. \tag{78}$$

A.4. Proof of Lemma 6

On substituting $B^R_{2,\text{sep}}$, $S^R_{2,\text{sep}}$, and $p = \frac{1}{2}$ into the expression for $\mathsf{F}^R(p, \tau, x)$, we have

$$\mathsf{F}^R\left(p = \frac{1}{2}, \tau, x\right) = \frac{(1-\tau)(1-x)}{3} + \frac{\tau(1-x)}{2(2-x)} - \frac{1}{6}, \tag{79}$$

which is equal to zero for $x = \frac{1}{2}$. Differentiating with respect to τ we get

$$\frac{\partial \mathsf{F}^R(p = \frac{1}{2}, \tau, x)}{\partial \tau} = \frac{(1-x)(2x-1)}{6(2-x)}, \tag{80}$$

which is greater than zero if $x > \frac{1}{2}$ and less than zero if $x < \frac{1}{2}$. We also have

$$\mathscr{C}^R\left(p = \frac{1}{2}, \tau, N_R, \phi\right) = \phi - \frac{N_R}{2}. \tag{81}$$

We have that $\mathscr{X}^*_{\text{sep}}$ solves $\mathsf{F}^R(p = \frac{1}{2}, \tau, \mathscr{X}^*_{\text{sep}}) = \mathscr{C}^R(p = \frac{1}{2}, \tau, N_R, \phi)$. Differentiating both sides with respect to τ gives $\mathrm{d}\mathscr{X}^*_{\text{sep}}/\mathrm{d}\tau = (2\mathscr{X}^*_{\text{sep}} - 1)(1 - \mathscr{X}^*_{\text{sep}})(2 - \mathscr{X}^*_{\text{sep}})/(1-\tau)2(2 - \mathscr{X}^*_{\text{sep}})^2 + 3\tau$. For $\mathscr{C}^R(p = \frac{1}{2}, \tau, N_R, \phi) > 0$ we must have $\mathscr{X}^*_{\text{sep}} < \frac{1}{2}$ since $\partial \mathsf{F}^R(p, \tau, x)/\partial x < 0$, which gives $\mathrm{d}\mathscr{X}^*_{\text{sep}}/\mathrm{d}\tau < 0$. For $\mathscr{C}^R(p = \frac{1}{2}, \tau, N_R, \phi) < 0$ we must have $\mathscr{X}^*_{\text{sep}} > \frac{1}{2}$, since $\partial \mathsf{F}^R(p, \tau, x)/\partial x < 0$, which gives $\mathrm{d}\mathscr{X}^*_{\text{sep}}/\mathrm{d}\tau > 0$.

A.5. Proof of Lemma 7

We have

$$\mathsf{F}^R(p, \tau, x) = \frac{(1-\tau)(2-\tau)(1-x)}{6} \frac{1}{(1-\tau(1-x-p+2px))} + \frac{\tau(1-x)}{3} \frac{(1-p)(2-p)}{(1-px)} - \frac{1}{6}(1+\tau-2p\tau), \tag{82}$$

which is equal to zero for $x = \frac{1}{2}$. Differentiating with respect to p we get

$$\frac{\partial \mathsf{F}^R(p, \tau, x)}{\partial p} = \left(\frac{\tau(1-\tau)(2-\tau)(1-x)}{6(1-\tau(1-x-p+2px))^2} + \frac{\tau(1-p)(2-x-px)}{3(1-px)^2}\right)(2x-1), \tag{83}$$

which is greater than zero if $x > \frac{1}{2}$ and less than zero if $x < \frac{1}{2}$. We also have

$$\frac{\partial \mathscr{C}^R(p, \tau, N_R, \phi)}{\partial p} = \tau N_R > 0. \tag{84}$$

We have that $\mathcal{X}^*_{\text{sep}}$ solves $\mathsf{F}^R(p,\tau,\mathcal{X}^*_{\text{sep}}) = \mathscr{C}^R(p,\tau,N_R,\phi)$. Differentiating both sides with respect to p gives

$$\frac{\mathrm{d}\mathcal{X}^*_{\text{sep}}}{\mathrm{d}p} = \frac{\left(\frac{\tau(1-\tau)(2-\tau)(1-x)}{6(1-\tau(1-x-p+2px))^2} + \frac{\tau(1-p)(2-x-px)}{3(1-px)^2}\right)(2x-1) - \tau N_R}{\left(\frac{(1-\tau)(2-\tau)}{6}\frac{(1-\tau p)}{(1-\tau(1-x-p+2px))^2} + \frac{(1-p)^2(2-p)\tau}{3(1-px)^2}\right)}. \tag{85}$$

For $\mathscr{C}^R(p,\tau,N_R,\phi)>0$ we must have $\mathcal{X}^*_{\text{sep}}<\frac{1}{2}$ since $\frac{\partial \mathsf{F}^R(p,\tau,x)}{\partial x}<0$ which gives $\mathrm{d}\mathcal{X}^*_{\text{sep}}/\mathrm{d}p<0$.

A.6. Proof of Proposition 8

The prices posted by the marketmaker follow a martingale over time, that is, $P_1(o_1) = E_M[P_2(o_2)|o_1]$, where the subscript M represents expectations relative to the marketmaker's beliefs. Let $N^I_{2,\text{j}}$, $j=\{\text{sep},\text{pump}\}$ represent the marketmaker's expected price given that the institutional investor does not trade. Next note that conditional on the institution's trade decision, the marketmaker's and institution's beliefs are identical. Thus, the marketmaker's expectation of future prices given an institutional buy (sell) order is given by the institution's expectations, $B^I_{2,\text{j}}$ $(S^I_{2,\text{j}})$, $j=\{\text{sep},\text{pump}\}$, which were defined earlier in the paper. The martingale property implies that P_1 is a convex combination of $B^I_{2,\text{j}}$, $S^I_{2,\text{j}}$, and $N^I_{2,\text{j}}$. Thus, it follows that if $S^I_2<N^I_2<B^I_2$, then $B^I_2\geqslant P_1\geqslant S^I_2$; that is, the expected bid price is higher than the first-period price is higher than the expected ask price from the institutional investor's perspective. Thus, the institutional investor will not buy and sell since the profit to this strategy is $S^I_2 - P_1\leqslant 0$, nor will the institutional investor sell and then buy since the trading profit to this strategy is $P_1 - B^I_2\leqslant 0$.

Separating equilibrium case

To show $S^I_{2,\text{sep}}<N^I_{2,\text{sep}}<B^I_{2,\text{sep}}$, note that

$$B^I_{2,\text{sep}} = 1 - \frac{(1-x)^2}{3}\left(\frac{(1-\tau)}{2(1-\tau+p\tau+x\tau(1-2p))} + \frac{(1-p)}{(1-px)}\right), \tag{86}$$

$$N^I_{2,\text{sep}} = \frac{3x+3}{6} - \frac{(1-x)}{6}\left(\frac{(1-\tau)}{(1-\tau+p\tau+x\tau(1-2p))} - \frac{(1-\tau)}{(1-\tau p)}\right), \tag{87}$$

$$S^I_{2,\text{sep}} = \frac{1+2x}{3} + \frac{1-x}{3}\left(\frac{(1-\tau)}{2(1-p\tau)} - \frac{(1-\tau)x}{2(1-\tau+p\tau+x\tau(1-2p))} - \frac{(1-p)}{(1-px)}\right). \tag{88}$$

For $N^I_{2,\text{sep}}\geqslant S^I_{2,\text{sep}}$,

$$\frac{x+2}{1-x} + \frac{2(1-p)}{(1-px)} \geqslant \frac{(1-x)}{\left(1+\frac{p\tau(1-x)+x\tau(1-p)}{(1-\tau)}\right)}, \tag{89}$$

which is always true since the left-hand side (LHS)is greater than one and the RHS is less than one.

For $B^I_{2,\text{sep}} \geqslant N^I_{2,\text{sep}}$,

$$\frac{(1-\tau)x}{(1-\tau+p\tau(1-x)+x\tau(1-p))} \geqslant -\left(\frac{2(p+x-2px)}{(1-px)}+\frac{\tau(1-p)}{(1-\tau p)}\right), \tag{90}$$

which is always true since the LHS is greater than zero and the RHS is less than zero.

Pumping equilibrium case:

To show $S^I_{2,\text{pump}} < N^I_{2,\text{pump}} < B^I_{2,\text{pump}}$, note that for $N^I_{2,\text{pump}} \geqslant S^I_{2,\text{pump}}$,

$$\frac{(p+x-2px)\tau}{(1-x)(1-\tau)+(p+x-2px)\tau} \geqslant -\frac{x(1-\tau)}{(1-\tau(1-p)(1-x))}-\frac{(1+\tau-2p\tau)}{(1-x+x\tau-px\tau)}, \tag{91}$$

which is always true since the LHS is greater than zero and the RHS is less than zero.

For $B^I_{2,\text{pump}} \geqslant N^I_{2,\text{pump}}$,

$$\begin{aligned}&\frac{1-x}{6}\left(1+\frac{(2x+2p-1-3px)\tau}{(1-x+x\tau-px\tau)}\right)+\frac{1}{6}\frac{(x+p\tau-px\tau)}{(1-\tau(1-p)(1-x))}\\ &\quad\geqslant \frac{x}{6}\frac{(p+x-2px)\tau}{(1-x)(1-\tau)+(p+x-2px)\tau},\end{aligned} \tag{92}$$

which is always true since the first term on the LHS is greater than zero and the second term is greater than $1/x$ RHS.

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