Governance Under Common Ownership

Alex Edmans* **Doron Levit** **Devin Reilly**

LBS, CEPR, and ECGI Wharton and ECGI Analysis Group

Conventional wisdom is that diversification weakens governance by spreading investors too thinly. We show that, when investors own multiple firms ("common ownership"), governance through both voice and exit can strengthen—even if the firms are in unrelated industries. Under common ownership, informed investors have flexibility over which assets to sell upon a liquidity shock. They sell low-quality firms first, thereby increasing price informativeness. In a voice model, investors' incentives to monitor are stronger since "cutting and running" is less profitable. In an exit model, managers' incentives to work are stronger since the price impact of investor selling is greater. (*JEL*: D72, D82, D83, G34)

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Most theories of corporate governance consider a single firm. In reality, investors typically hold large stakes in many firms—shareholders own multiple blocks and banks lend large amounts to multiple borrowers. This paper analyzes the effect of common ownership on governance. Doing so is potentially complex, because governance can be undertaken through different mechanisms. Investors can govern through "voice"–direct intervention such as monitoring managers, suggesting a strategic change, or blocking a pet project. Alternatively, they can govern through "exit"–sell their securities if the manager shirks, reducing the price; ex ante, the threat of exit induces the manager to work.

While conventional wisdom is that common ownership weakens governance by spreading the investor too thinly, we show that it can strengthen governance. Moreover, the channel through which it does so is common to both voice and exit. If an investor owns multiple stakes, she has the choice of which stake to sell upon a liquidity shock. Thus, a sale is more likely to be driven by information than a shock, and transmits more information into prices. This greater price informativeness in turn enhances governance through both voice (since an investor who sells rather than monitors receives a lower sale price) and exit (since a manager who shirks and is sold suffers a lower stock price). Unlike other effects of common ownership, this effect continues to exist, and is in fact stronger, if the stakes are in unrelated firms.

We start with a pure-trading model in which firm value is exogenous and so governance is moot, to demonstrate most clearly the common channel that applies to both voice and exit. A large investor owns a portfolio of securities, such as a bank owning loans, a shareholder owning stock, or a holding company owning subsidiaries. She subsequently learns private information on firm value, which can be high or low. She may also suffer a privately observed liquidity shock that forces her to raise at least a given amount of funds, although she may choose to sell more, or to sell even absent a shock. Based on her private information and liquidity need, she retains, partially sells, or fully sells her stake. The security price is set by a market maker who observes the investor's trade but not firm value.

As a benchmark, we analyze separate ownership where the investor owns n units of a single security. If the firm turns out to be good (i.e., have high fundamental value) but the investor suffers a liquidity shock, she is forced to partially sell it. Thus, if the firm turns out to be bad (i.e., low-value), the investor sells it by the same amount, to disguise the sale as motivated by a shock. As a result, a bad firm does not receive too low a price, and a good firm does not always enjoy a high price, as it is sometimes sold and pooled with bad firms. Therefore, price informativeness is relatively low.

Under common ownership, the investor owns one unit of a security in each of n uncorrelated firms. Each firm's securities are traded by a separate market maker, who observes trading in only one firm. The key effect of common ownership is that it gives the investor a diversified portfolio of both good and bad firms, and thus the choice of which firms to sell upon a shock. If the shock is small, she can satisfy it by selling only bad firms. Then, being sold is not consistent with the firm being good and the sale being driven purely by a shock, and so fully reveals the firm as bad. For example, Warren Buffett's disposal in late 2014 of Exxon Mobil and ConocoPhillips but not Suncor Energy was viewed by the market as a negative signal on the sold companies in particular, rather than purely due to a shock (e.g., investment opportunities suddenly appearing in non-energy sectors). More broadly, Huang, Ringgenberg, and Zhang (2016) show that mutual funds sell their worst assets first upon a liquidity shock, Maksimovic and Phillips (2001) show that conglomerates tend to sell their least efficient plants, and Berndt and Gupta (2009) find that borrowers whose loans are sold in the secondary market underperform their peers.

In contrast, a good firm is retained even upon a shock, and thus receives a high price. Overall, price informativeness is higher under common ownership than under separate ownership, and decreases with the size of the liquidity shock. Intuitively, smaller shocks increase the investor's flexibility over which firms to sell upon a shock, and so being sold is a greater signal that the firm is bad. Note that the impact on price informativeness does not arise simply because common ownership gives the investor financial slack, that is, additional securities to sell upon a shock. We show that adding more firms to the investor's portfolio is critically different from adding liquid securities, such as Treasury bills, on which the investor has no private information.

The trading model is flexible and tractable, and can be embedded in a model of either exit or voice. Starting with the former, we endogenize firm value as depending on an effort decision by the firm's manager. If he works, the firm is good, else it is bad. The manager is concerned with both fundamental value and the short-term security price. If the security is equity, his price concerns can stem from termination threat, reputational considerations, or owning equity that vests in the short term; if it is debt, its price may affect his firm's reputation in debt markets and thus ability to raise future financing. The threat of selling, and thus suffering a low security price ex post, induces effort ex ante, as in Admati and Pfleiderer

(2009) and Edmans (2009).

Under separate ownership, effort incentives are low. If the manager works, the investor may suffer a shock and be forced to sell. Thus, the manager suffers a low stock price—the reward for working is low. If the manager shirks, his firm is sold, but he does not suffer too low a price, because the sale is also consistent with a shock—the punishment for shirking is also low. Under common ownership, the reward for working is higher, because the manager's firm need not be sold upon a shock. In addition, the punishment for shirking is now higher since being sold is more revealing of shirking. For both reasons, governance through exit is stronger. Intuitively, common ownership creates a tournament among the n managers, and both they and the market know that the investor will sell the worst performers first.

Moving to the voice application, firm value now depends on a monitoring decision undertaken by the investor. Monitoring incentives are low under separate ownership for two reasons. If the investor monitors, she may suffer a shock that forces her to sell prematurely, reducing her payoff to monitoring (Faure-Grimaud and Gromb 2004); if she does not monitor, she may sell ("cut and run"), which yields a relatively high price since the sale is also consistent with a shock (Kahn and Winton 1998; Maug 1998). Under common ownership, an investor does not have to sell a monitored firm even if she suffers a shock, increasing the payoff to monitoring, and suffers a low price if she cuts and runs. On the other hand, common ownership reduces the investor's stake in an individual firm, and thus the incentives to monitor. Despite common ownership spreading an investor more thinly, governance may still improve overall. We show that this will be the case if the number of firms in the investor's portfolio and the magnitude of the liquidity shock are sufficiently small.

The model has a number of implications. Starting with the trade-only model, the price decline upon a sale is stronger when an informed investor owns multiple securities. The "price" can refer to either the literal trading price or market perceptions of quality. For example, if a bank stops lending to a borrower, that borrower's perceived creditworthiness falls more if the bank had other borrowers it could have stopped lending to instead. Beyond security trading, a director's decision to quit a firm is a more negative signal if he serves on other boards; a conglomerate's decision to exit a business line is a more negative signal of industry prospects than if a focused firm scaled back its operations.

The full governance model shows that common ownership can strengthen both voice and exit. A bank's incentive to monitor a borrower, or a hedge fund's incentive to intervene, can rise if the investor owns multiple firms. A manager's incentive to work is stronger if his lender or main shareholder owns large positions in multiple firms. This result leads to an alternative interpretation of recent findings on the effects of common ownership. Azar, Schmalz, and Tecu (2017) document that common ownership leads to higher product prices, which they interpret as anti-competitive behavior. Governance of a given firm is weaker since the investor is conflicted by her holdings of competitor firms. This paper shows that higher product prices may instead result from improved governance, which can lead to superior product quality or more efficient pricing. Moreover, our analysis demonstrates that common ownership affects real outcomes even if the firms are unrelated—that is, if they are not product market competitors. In addition, a merger between investors (e.g., BGI and BlackRock) should improve governance, even if the investors do not have common holdings, and so the merger does not increase their stake in a given firm. Such mergers are sometimes used as a shock to anti-competitive behavior, but they may also be a shock to governance.

That common ownership may strengthen governance has the potential to justify why ownership structures in which shareholders own blocks in multiple firms can survive, even though they exacerbate the free-rider problem. Existing justifications are typically based on diversification of risk. While conventional wisdom might suggest that the diversification induced by risk concerns necessarily weakens governance, our model highlights an opposing force. Indeed, Kang, Luo, and Na (2018) find that institutional investors are more effective at governance the more blocks they have in other companies, controlling for portfolio size. Relatedly, while existing studies typically use the size of the largest blockholder or the number of blockholders as a measure of governance, our paper theoretically motivates a new measure—the number of other large stakes owned by its main shareholder or creditor.

We extend the model to cases in which the investor has less private information under common ownership, perhaps due to having a smaller stake in each firm (e.g., Van Nieuwerburgh and Veldkamp 2009, 2010; Kacperczyk, Van Nieuwerburgh, and Veldkamp 2014, 2016). We show that price informativeness can still be higher under common ownership. Even though the common owner has less information, a greater proportion of her information is transmitted into prices. In a second extension, we study index funds, whose assets under management have grown substantially over recent years and who also increasingly engage in monitoring (Appel, Gormley, and Keim 2016). Some commentators (e.g., Bhide 1993) argue that their inability to disproportionately sell bad firms increases monitoring incentives compared to an identical active fund with no trading restriction, since they cannot cut and run and are thus locked in to monitor. We show that this need not be the case. Intuitively, an unconstrained investor cannot commit not to sell the worst assets in her portfolio, leading to a severe price decline upon selling and thus a powerful commitment to monitor. We also show that the results are robust to having a single market maker for all securities, to the investor's initial portfolio (whether she is diversified or concentrated) being private information, to her information endowment (whether she is informed or informed) being private information, and to firms differing in their information asymmetry and thus liquidity.

This paper builds on a long-standing literature of governance through voice (e.g., Shleifer and Vishny 1986; Burkart, Gromb, and Panunzi 1997; Bolton and von Thadden 1998; Maug 1998; Kahn and Winton 1998; Pagano and Röell 1998; Faure-Grimaud and Gromb 2004) and a newer one on governance through exit (e.g., Admati and Pfleiderer 2009; Edmans 2009)—see Edmans (2014) and Edmans and Holderness (2017) for a survey of both literatures. While McCahery, Sautner, and Starks (2016) find that institutional investors use both governance mechanisms frequently, most theories analyze only one.[1] We identify a common channel through which common ownership can strengthen both mechanisms—increased price informativeness—and model both using a unified framework. In doing so, we also extend governance models to multiple firms, in contrast to most governance models, which consider a single firm.[2]

1 The Model

This section considers a pure trading model in which firm values are exogenous, to highlight the effect of common ownership on how an informed investor trades on private information, and in turn security prices and price informativeness. In Section 2.1 we endogenize firm value by allowing it to depend on effort by a manager in a model of governance through exit; in Section 2.2, it depends on intervention by the investor in a model of governance through voice. These models will demonstrate how the increased price informativeness, resulting from common

[1] Edmans and Manso (2011) and Levit (forthcoming) feature both mechanisms, but model each using quite different frameworks.

[2] One exception is the voice-only model of Admati, Pfleiderer, and Zechner (1994), which features no information asymmetry and instead focuses on the trade-off between risk sharing and the free-rider problem. Another is Diamond (1984), who shows that diversification reduces the deadweight loss required to incentivize the bank to repay its end investors. In his model, monitoring does not create value after a project is financed.

ownership, improves both governance mechanisms.

1.1 Setup

The model consists of a single investor who holds securities in one or many firms; these securities can be debt, equity, or any security monotonic in firm value. The investor corresponds to a large institution, such as a hedge fund, mutual fund, or bank, that holds sizable stakes in companies. These sizable stakes give her both the incentives and ability to engage in governance, as well as both the incentives to gather private information about firm value and the ability to do so, for example, via access to management.

We consider two versions of the model. The first is a preliminary benchmark of separate ownership, where the investor owns a continuum of units of a single security, of mass n; the results are identical if the benchmark is instead one unit in a continuum of n firms with perfectly correlated values. The second is the main model of common ownership, where she owns one unit in each of a continuum of firms, each indexed i, of mass n. (Online Appendix F.1 considers the case of two firms.) Note that, in both models, the investor owns the same number (n) of units and thus the same ex ante portfolio value. Whether the investor is concentrated or diversified is public information; Online Appendix F.3 analyzes the case in which it is her private information. Let z denote the investor's number of units in a given firm, that is, $z = n$ ($z = 1$) under separate (common) ownership. There are $m \geq n$ units of the security outstanding; the remaining $m - z$ units are owned by dispersed investors (households) who play no role. Thus, the number of securities held by investors who can engage in governance is the same in both models.

The model consists of three periods. At $t = 1$, Nature chooses the fundamental value of each security in firm i, $v_i \in \{\underline{v}, \overline{v}\}$, where $\overline{v} > \underline{v} > 0$ and $\Delta \equiv \overline{v} - \underline{v} > 0$. v_i are independently and identically distributed (i.i.d.) across firms, and $\tau \equiv \Pr[v_i = \overline{v}] \in (0, 1)$ is common knowledge. Due to the law of large numbers, the actual proportion of firms for which $v_i = \overline{v}$ is τ. The investor privately observes v_i under separate ownership and $\mathbf{v} \equiv [v_i]_{i=0}^{n}$ under common ownership (in Section 3.1, she has imperfect information under common ownership). We use "good" ("bad") firm to refer to a firm with $v_i = \overline{v}$ ($\underline{v}$), and for brevity we refer to v_i as the value of firm i, rather than the value of a security in firm i.

At $t = 2$, the investor is subject to a portfolio-wide liquidity shock $\theta \in \{0, L\}$, where $L > 0$

and $\Pr[\theta = L] = \beta \in (0, 1]$. The liquidity shock is any non-informational motive for trading; examples include withdrawals from end investors, an alternative investment opportunity, or an increase in capital requirements. The variable θ is privately observed by the investor and represents the dollar amount of funds that she must raise. If she cannot raise θ, she raises as much as possible. Formally, failing to raise θ imposes a cost $K > 0$ multiplied by the shortfall, which is sufficiently large to induce her to meet the liquidity need to the extent possible. The investor may raise more than θ dollars—that is, we allow for voluntary sales.

After observing the shock, the investor sells $x_i \in [0, z]$ units in firm i. We use "fully sold" to refer to firm i if $x_i = z$, and "partially sold" if $x_i \in (0, z)$. We assume that short selling is either costly or constrained, otherwise the investor's initial position would not matter; for simplicity, we model these costs or constraints by not allowing for short sales. Some investors (e.g., mutual funds) are constrained from short selling; for some assets, such as bank loans, short sales are generally not possible. The sold units x_i are purchased by the market maker for firm i, which can more generally refer to a competitive pool of buyers. Each firm has a separate market maker who is competitive and risk neutral, and observes only x_i and not x_j for $j \neq i$, θ, nor v_i; Online Appendix F.2 considers the case of a single market maker for all firms, who observes x_j, $j \neq i$. Each market maker sets the price $p_i(x_i)$ at $t = 2$ to equal the security's expected value, conditional on the observed trade x_i. We denote $\mathbf{p} \equiv [p_i(x_i)]_{i=0}^{n}$ and $\mathbf{x} \equiv [x_i]_{i=0}^{n}$.[3]

At $t = 3$, firm values are realized. The investor's utilities under separate and common ownership are respectively given by:

$$u_I(x_i, v_i, p_i(x_i), \theta) = x_i p_i(x_i) + (n - x_i) v_i - K \times \max\{0, \theta - x_i p_i(x_i)\}. \tag{1}$$

$$u_I(\mathbf{x}, \mathbf{v}, \mathbf{p}, \theta) = \int_0^n [x_i p_i(x_i) + (1 - x_i) v_i]\, di - K \times \max\{0, \theta - \int_0^n x_i p_i(x_i)\, di\}. \tag{2}$$

[3]Note that the investor has no incentive to buy additional securities, because such purchases would be fully revealed as stemming from information. If the investor had the possibility of receiving positive liquidity shocks, and were forced to buy shares that she already owns, she would be able to profit from positive information by disguising a purchase as being driven by a shock. Under separate ownership, a purchase is relatively uninformative because it could result from a shock. Under common ownership, a purchase is highly informative as, if the investor had received a liquidity shock, she could have bought other firms instead. This would reinforce the predictions of the core model. However, in reality the investor likely has the option to hold the inflow as cash (or purchase new assets) rather than being forced to buy more of her existing holdings, in which case she cannot disguise buying on positive information as being driven by a shock. This treatment is consistent with the seller's option to raise more than L and hold the excess as cash.

The equilibrium concept we use is Perfect Bayesian Equilibrium. Here, it involves: (i) A trading strategy by the investor that maximizes her expected utility u_I given each market maker's pricing rule and her private information on $\mathbf{v}$ (v_i) and (ii) a pricing rule by each market maker that allows him to break even in expectation, given the investor's strategy. Moreover, (iii) each market maker uses Bayes' rule to update his beliefs from the investor's trades, (iv) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium, and (v) the pricing function is monotonic, that is, $p_i(x_i)$ is weakly decreasing, holding constant x_j, $j \neq i$.[4] Since firms are ex ante identical, we focus on symmetric pure strategy equilibria,[5] in which each market maker uses a symmetric pricing function. We also assume that the investor does not sell a good firm if unshocked. This is intuitive since the price can never exceed the value of a good firm $\overline{v}$, but simplifies the analysis as we need not consider equilibria under which a good firm is partially sold, but still fully revealed as good, since bad firms are sold in greater volume. Price informativeness is exactly the same without this restriction.

1.2 Trade under separate ownership

Lemma 1 characterizes all equilibria under separate ownership.[6]

Lemma 1 (Separate ownership): An equilibrium under separate ownership always exists. In

[4]Focusing on weakly decreasing price functions imposes some restrictions on off-equilibrium prices, and thus the amounts sold in equilibrium. However, since these restrictions do not affect on-equilibrium prices, they generally do not affect real actions when introduced in Section 2. Weakly decreasing pricing functions are consistent with other microstructure theories (e.g., Kyle 1985) and empirical evidence (e.g., Gorton and Pennacchi 1995; Ivashina 2009).

[5]The results continue to hold if we allow for mixed strategies, although the proofs are much lengthier.

[6]While the prices on the equilibrium path are unique, the prices off-equilibrium are not. The pricing function in Equation (4) ensures monotonicity. A similar comment applies to subsequent pricing functions. In addition, equilibria can differ in their on-the-path trading volumes when $L/n > \underline{v}$. Specifically, if $L/n > \underline{v}$, then any $\overline{x}_{so} \in [\min\{\frac{n\underline{v}}{\overline{p}_{so}(\tau)}, n\}, \min\{\frac{L}{\overline{p}_{so}(\tau)}, n\}]$ can be an equilibrium. In those cases, we select $\overline{x}_{so} = \min\{\frac{L}{\overline{p}_{so}(\tau)}, n\}$. Intuitively, this selection implies that if there is an equilibrium in which the seller can meet her liquidity needs, then selected equilibria must have this property, and if such an equilibrium does not exist, then the selected equilibrium is the one that maximizes the seller's revenue. This selection can be justified using the Grossman and Perry (1986) criterion; moreover, it has no effect on price informativeness or any of our other main results, which apply when $L/n \leq \underline{v}$.

any equilibrium, the investor's trading strategy is:

$$x_{so}^{*}(v_i,\theta)=\begin{cases}0 & \textit{if } v_i=\overline{v} \textit{ and } \theta=0\\ \overline{x}_{so}=n\min\{\frac{L/n}{\overline{p}_{so}},1\} & \textit{otherwise}\end{cases}, \tag{3}$$

and prices are:

$$p_i^{*}(x)=\begin{cases}\overline{v} & \textit{if } x=0\\ \overline{p}_{so}=\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau} & \textit{if } x\in(0,\overline{x}_{so}],\\ \underline{v} & \textit{if } x>\overline{x}_{so}.\end{cases} \tag{4}$$

Equation (3) shows that, if the firm is good and the investor suffered a shock, she sells $\overline{x}_{so}$. This quantity is the minimum required to satisfy the shock: if it were greater, type $(\overline{v},L)$ would deviate and sell less, retaining more of a good firm and receiving no lower a price (since prices are non-increasing).[7] Thus, if the firm is bad, the investor sells the same amount ($\overline{x}_{so}$), to disguise the motive for her sale. The price of a sold security, $\overline{p}_{so}$, is relatively high as the market maker attaches a probability $\frac{\beta\tau}{\beta\tau+1-\tau}$ that the sale was of a good firm and due to a shock. Thus, price informativeness is relatively low under separate ownership.

1.3 Trade under common ownership

Lemma 2 characterizes all equilibria under common ownership.

Lemma 2 (Common ownership): An equilibrium under common ownership always exists.

(i) If $L/n\le\underline{v}(1-\tau)$, then in any equilibrium

$$x_{co}^{*}(v_i,\theta)=\begin{cases}0 & \textit{if } v_i=\overline{v}\\ \underline{x}(\theta)\in[\frac{\theta/n}{\underline{v}(1-\tau)},1] & \textit{if } v_i=\underline{v},\end{cases} \tag{5}$$

and prices are:

$$p_i^{*}(x_i)=\begin{cases}\underline{v}+\Delta\frac{\tau}{\tau+(1-\tau)(1-\beta)\cdot\mathbf{1}_{\underline{x}(0)=0}} & \textit{if } x_i=0\\ \underline{v} & \textit{if } x_i>0.\end{cases} \tag{6}$$

[7] We refer to the seller's type as (v_i,θ)—that is, a pair that indicates her information on the value of asset i and whether she has suffered a liquidity shock. Sometimes we will define the type as referring only to v_i, in which case it refers to both $(v_i,0)$ and (v_i,L).

(ii) If $\underline{v}(1-\tau) < L/n < \underline{v}$, then there exists an equilibrium in which

$$x^*_{co}(v_i,\theta) = \begin{cases} 0 & \text{if } v_i = \overline{v} \text{ and } \theta = 0 \\ \overline{x}_{co} = \frac{\underline{v}+\frac{L/n-\underline{v}}{\tau}}{\overline{p}_{co}} < 1 & \text{if } v_i = \underline{v} \text{ and } \theta = 0, \text{ or } v_i = \overline{v} \text{ and } \theta = L \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases} \tag{7}$$

and prices are:

$$p^*_i(x_i) = \begin{cases} \overline{v} & \text{if } x_i = 0 \\ \overline{p}_{co} = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)} & \text{if } x_i \in (0, \overline{x}_{co}], \\ \underline{v} & \text{if } x_i > \overline{x}_{co}. \end{cases} \tag{8}$$

(iii) If $L/n \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, then there exists an equilibrium as described by Lemma 1, except $\overline{x}_{so}$ is replaced by $\overline{x}_{so}/n$.

(iv) No other equilibrium exists.

The intuition is as follows. Under common ownership, the investor decides not only how much of her portfolio to sell, but also which firms. If $L/n \leq \underline{v}(1-\tau)$, the liquidity shock is sufficiently small that it can be satisfied by selling only bad firms. She thus retains all good firms, regardless of whether she suffers a shock. If there is no shock, which occurs with probability (w.p.) $1-\beta$, the investor no longer has strict incentives to sell bad firms because doing so is fully revealing. Thus, while there is an equilibrium in which the investor at least partially sells all bad firms ($\underline{x}(0) > 0$), there is also an equilibrium in which she fully retains them ($\underline{x}(0) = 0$). Overall, bad firms are retained w.p. $(1-\beta)\cdot\mathbf{1}_{\underline{x}(0)=0}$; as a result, a retained firm is not fully revealed as good and only priced at $\underline{v} + \Delta\frac{\tau}{\tau+(1-\tau)(1-\beta)\cdot\mathbf{1}_{\underline{x}(0)=0}}$ rather than $\overline{v}$. Any firm that is at least partially sold is fully revealed as bad and priced at $\underline{v}$.

For $\underline{v}(1-\tau) < L/n < \underline{v}$, the shock is sufficiently large that the investor cannot satisfy it by only fully selling bad firms, but sufficiently small that she can still sell good firms less. She sells $\overline{x}_{co}$ from each good firm. Thus, upon no shock, she no longer retains bad firms but sells $\overline{x}_{co}$ to disguise her sale as that of good firms driven by a shock. As a result, $(\overline{v}, L)$ is pooled with $(\underline{v}, 0)$—but not $(\underline{v}, L)$, and so the price is higher than under separate ownership. Retained firms are fully revealed as good and priced at $\overline{v}$.

Finally, for $L/n \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, the shock is sufficiently large that it forces the investor to sell good firms as much as bad firms. Thus, $(\overline{v}, L)$ is pooled with not only $(\underline{v}, 0)$ (as in the moderate-shock case) but also $(\underline{v}, L)$, further reducing its price below $\overline{v}$ and increasing the price of $(\underline{v}, L)$ above $\underline{v}$. Since the investor's trading strategy is the same as under separate ownership ($(\overline{v}, L)$, $(\underline{v}, 0)$, and $(\underline{v}, L)$ are all pooled), prices are exactly the same.[8]

We denote the expected equilibrium price of firm i, given value v_i, under common and separate ownership by $P_{co}(v_i)$ and $P_{so}(v_i)$, respectively. Price informativeness measures the closeness of the expected equilibrium price to fundamental value, which is $\overline{v}$ for a good firm and $\underline{v}$ for a bad firm. Thus, price informativeness is higher under common ownership if $P_{co}(\overline{v})$ is higher than $P_{so}(\overline{v})$ and thus closer to $\overline{v}$, and also $P_{co}(\underline{v})$ is lower than $P_{so}(\underline{v})$ and thus closer to $\underline{v}$. Proposition 1 gives conditions under which this is the case.

Proposition 1 (Price informativeness):

(i) If $L/n > \underline{v}(1-\tau)$ or $\beta \geq \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$, then in any equilibrium

$$P_{co}(\overline{v}) \geq P_{so}(\overline{v}) \ \ \textit{and}\ P_{co}(\underline{v}) \leq P_{so}(\underline{v}), \tag{9}$$

that is, price informativeness is weakly higher under common ownership, with strict inequalities if $L/n \leq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$.

(ii) If $L/n \leq \underline{v}(1-\tau)$ and $\beta < \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$, then there is an equilibrium with $\underline{x}(0) = 0$ and

$$P_{co}(\overline{v}) < P_{so}(\overline{v}) \ \ \textit{and}\ P_{co}(\underline{v}) > P_{so}(\underline{v}), \tag{10}$$

that is, price informativeness is strictly lower under common ownership. In all other equilibria (i.e., if $\underline{x}(0) > 0$), we have Equation (9).

[8] Note that, for $\underline{v}\frac{1-\tau}{\beta\tau+1-\tau} \leq L/n < \underline{v}$, both the equilibria in parts (ii) and (iii) can be sustained. While the seller has the option to satisfy a shock by selling bad firms more, she may also sell good firms to the same degree as bad firms. Doing so increases her trading losses on good firms but reduces them on bad firms, since bad firms are now pooled with good firms upon a shock.

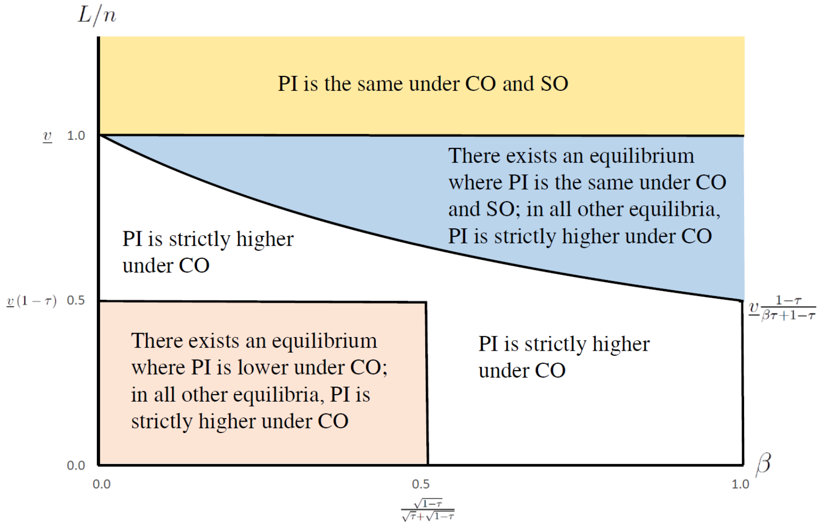

Figure 1

Comparison of price informativeness when $\tau = 0.5$ and $\underline{v} = 1$

Proposition 1 shows that whether price informativeness is higher, the same, or lower under common ownership depends on the values of L/n and β. This dependence is illustrated in Figure 1, and the intuition is as follows. Under common ownership, the investor has a diversified portfolio of good and bad firms. This allows her to choose which firms to sell upon a shock—in particular, she sells bad firms first. In the moderate-shock equilibrium of Lemma 2, part (ii), a shock causes her to fully sell bad firms and partially retain good firms. Thus, bad firms are fully revealed upon a shock and priced at $\underline{v}$, in contrast to separate ownership, where they are always pooled with $(\overline{v}, L)$ and $(\underline{v}, 0)$. As a result, bad firms receive a lower expected price under common ownership.[9] Turning to good firms, they are retained and thus fully revealed upon no shock. Upon a shock, a good firm is sold, but only partially. The market maker knows that if the firm were bad and the investor had suffered a shock, it would have been sold fully. Thus, it is priced at $\underline{v} + \Delta\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$ (i.e., pooled with only $(\underline{v}, 0)$) rather than $\underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ (i.e., pooled with $(\underline{v}, 0)$ and $(\underline{v}, L)$) under separate ownership.

A similar intuition applies to the small-shock equilibrium of Lemma 2, part (i), where the investor fully retains good firms. As a result, the sale of firm i cannot be attributed to a shock

[9] In He (2009), the price impact of a sale is stronger if the asset is more correlated with other assets in the seller's portfolio. Retaining an asset is even more costly when it is positively correlated with the rest of the portfolio, and particularly so when the asset is low quality. Thus, retention is a stronger signal of asset quality, leading to a steeper pricing function. His model features risk aversion rather than liquidity shocks.

because if firm i were good and the investor had needed liquidity, she would have sold other firms instead. Thus, a sold firm is fully revealed as being bad and priced at $\underline{v}$. On the other hand, since sold firms are fully revealed as bad, the investor no longer has strict incentives to sell bad firms upon no shock. If she fully retains bad firms ($\underline{x}(0) = 0$) upon no shock (which occurs w.p. $1 - \beta$), then being retained is no longer fully revealing. If β is sufficiently high ($\beta \geq \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$), this case is infrequent and so price informativeness is higher under common ownership in any equilibrium. If β is low, the equilibrium with $\underline{x}(0) = 0$ is less informative than the separate ownership benchmark. However, there will always also exist an equilibrium with $\underline{x}(0) > 0$ (i.e., the investor sells bad firms upon no shock). This equilibrium is more informative than under separate ownership and will be selected under the efficiency criterion (Section 2 shows that, when firm value is endogenous, the most informative equilibrium is the most efficient.) If $L/n > \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, we are in the large-shock equilibrium of Lemma 2, part (iii). The investor's trading behavior is exactly the same as under separate ownership, and so price informativeness is the same.

We end this section with two observations to clarify the intuition and mechanics behind our price informativeness result. First, note that the benchmark is identical if the investor holds one unit in a continuum of perfectly correlated firms of mass n. Then, moving from the benchmark to common ownership holds constant the number of firms and only reduces their correlation. This comparison shows that price informativeness is increasing in the diversification of an investor's portfolio. It also highlights that the effect on price informativeness stems from diversification, rather than simply giving the investor additional firms. Similarly, diversification alone is insufficient; the investor must have flexibility over which firms to sell. An index fund is diversified, but constrained to selling all firms equally; we analyze index funds in Section 3.2.

Second, the results show that diversifying by adding more firms to the investor's portfolio is different from adding financial slack, that is, liquid assets (such as Treasury bills or cash) on which the investor has no private information, or risk-free borrowing capacity. Consider the effect of adding $A < L$ dollars of liquid assets to the separate ownership model.[10] The proofs of Appendix A show that, if the investor holds A in liquid assets, the results of this section remain the same except that the liquidity shock is now reduced to $L - A$. Since price informativeness

[10]If $A \geq L$, then the addition effectively insulates the seller from a liquidity shock—the net liquidity shock, $L-A$, is now negative. For nearly all investors, there will always be non-informational reasons for selling assets, such as the emergence of a new investment opportunity, so we consider the case of $A < L$.

under separate ownership is independent of the size of the shock, it is unaffected by the liquid assets. Liquid assets reduce the amount that the owner of a good firm must sell upon a shock; the owner of a bad firm reduces the amount that she sells by the same margin, and so $(\overline{v}, L)$, $(\underline{v}, 0)$, and $(\underline{v}, L)$ remain pooled.

Now consider $A < L$ dollars instead being added as an investment in a new firm j. We consider the case in which firms i and j are negatively correlated, but the result only requires less than perfect correlation so that there is a non-zero probability that firms i and j may have different values. Upon a shock, if the initial firm i is good (and new firm j is bad), the investor will sell j first and only partially sell i—the same as if the investor had instead added liquid assets. However, if firm i is bad (and new firm j is good), the investor will not sell j first because doing so would entail a loss, unlike selling liquid assets, which are always fairly priced. She instead fully sells firm i. Put differently, liquid assets provide uncontingent liquidity (they are always sold first, so i never has to be fully sold) but firm j provides only contingent liquidity (it is not sold first if it is good, so i may still have to be fully sold). Contingent liquidity depends on a firm's value, and so the sale of a firm is more likely to be driven by its value and is thus more informative. In sum, adding liquid assets reduces the net liquidity shock but keeps us within the separate ownership model and so price informativeness is unchanged. Adding an illiquid asset moves us to the common ownership model and so price informativeness rises.

2 Governance

The core model has shown that common ownership can improve price informativeness. We now endogenize firm value as depending on a real action to demonstrate the implications of common ownership for corporate governance.

2.1 Governance through exit

This section extends governance-through-exit models to the case of multiple firms, and shows how the strength of exit depends on the investor's holdings of potentially unrelated firms. Each firm is now run by a separate manager, who takes action $a_i \in \{0, 1\}$ at $t = 1$. When $a_i = 1$ (0), the value of each security is $v_i = \overline{v}$ ($\underline{v}$). Examples of $a_i = 0$ include shirking, cash flow diversion, perk consumption, and empire building. We refer to $a_i = 0$ as "shirking" and $a_i = 1$

as "working." A good (bad) firm is one in which the manager has worked (shirked). Action $a_i = 1$ imposes a cost $\widetilde{c}_i \in [0, \infty)$ on manager i, which he privately observes prior to deciding his action.[11] The probability density function of $\widetilde{c}_i$ is given by f and its cumulative distribution function by F. Both are continuous and have full support. We assume $\widetilde{c}_i$ are i.i.d. across firms, and that $E[\widetilde{c}_i] \leq \Delta$ so that $a_i = 1$ is ex ante efficient. Firm value is $\overline{R}$ if $v = \overline{v}$ and $\underline{R}$ if $v = \underline{v}$.[12]

Manager i's objective function is given by:

$$u_{M,i}(a_i, p_i, \widetilde{c}_i, \omega) = (1 - \omega)\, v\,(a_i) + \omega p_i - \widetilde{c}_i a_i. \tag{11}$$

The manager cares about both the security's fundamental value and its $t = 2$ price; these price concerns are captured by $\omega \in (0, 1)$. If the security is equity, ω refers to stock price concerns, which are standard in exit models and can stem from a number of sources introduced in prior work—takeover threat (Stein 1988), reputational concerns (Narayanan 1985), or the manager expecting to sell his own shares at $t = 2$ (Stein 1989). Many of these sources are over and above any stock price incentives provided by the manager's contract, and so exist regardless of what contract he is given. Empirically, Coughlan and Schmidt (1985), Warner, Watts, and Wruck (1988), and Weisbach (1998) find that a fall in stock performance significantly increases CEO turnover, and Edmans, Goldstein, and Jiang (2012) find a similar effect on takeover probability. If the security is debt, the manager may care about the short-term debt price, or the firm's reputation in debt markets, as it will affect the ease with which he can raise additional debt (Diamond 1989). The application to debtholders shows how they can exert governance even if they do not have control (e.g., through a default or covenant violation), as assumed by prior research. Note that we do not require the manager's utility to be linear in the stock price p_i, only to be increasing in it; we assume linearity for simplicity.

As in the core model, the investor privately observes v_i under separate ownership and $\mathbf{v} \equiv [v_i]_{i=0}^{n}$ under common ownership, but neither she nor the market makers observe $\widetilde{\mathbf{c}} \equiv [\widetilde{c}_i]_{i=0}^{n}$. We will abuse language slightly by using the phrase "the manager will be sold" to refer to the

[11] Given that the investor observes v_i, the analysis would not change if the investor also observes $\widetilde{c}_i$. The key assumption is that the market maker does not observe $\widetilde{c}_i$ or v_i.

[12] For the case in which there are multiple classes of securities (in addition to the class owned by the investor), we assume $m\Delta \leq \overline{R} - \underline{R}$. This assumption means that the aggregate gain across the m securities from $R_i = \overline{R}$ cannot exceed the overall gain in firm value, otherwise the value of any other classes of securities would be decreasing in R and so their owners would have incentives to reduce firm value (cf. Innes 1990). This assumption guarantees that stronger governance increases firm value.

securities of the firm run by the manager being sold. We continue to focus on symmetric equilibria, in which the managers follow the same strategy and each market maker uses a symmetric pricing function. The equilibrium concept is as in Section 1 with the following additions: (vii) a decision rule by each manager i that maximizes his expected utility $u_{M,i}$ given his information on $\widetilde{c}_i$, other managers' strategies, the market maker's pricing rule, and the investor's trading strategy, and (viii) each market maker forms expectations about τ that are consistent with (vii), instead of taking it as given. Indeed, since firm value is now endogenous (it depends on the manager's action), the proportion of good firms τ is now also endogenous.

The analysis below shows that, in any equilibrium and under any ownership structure, the manager follows a threshold strategy—he works if and only if $\widetilde{c}_i \leq c^*$. Therefore, the probability of a good firm is given by $F(c^*)$, which replaces τ in the expressions in the trade-only model of Section 1. Lemma 3 characterizes the equilibrium under separate ownership.

Lemma 3 (Separate ownership, exit): An equilibrium under separate ownership always exists. There is a unique $c^*_{so,exit} > 0$ such that, in any equilibrium, manager i chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^*_{so,exit}$. The working threshold $c^*_{so,exit}$ is given by the solution of $c^* = \phi_{exit}(F(c^*))$ where

$$\phi_{exit}(\tau) = \Delta - \frac{\omega\Delta}{\tau + \frac{1-\tau}{\beta}}. \tag{12}$$

Prices and trading strategies are characterized by Lemma 1, where τ is given by $\tau^*_{so,exit} \equiv F\left(c^*_{so,exit}\right)$.

Lemma 4 characterizes the most efficient equilibrium under common ownership—the equilibrium that maximizes total surplus (aggregate value of all firms minus the aggregate cost of all managers' efforts). The proof of Lemma 7 in Appendix B formally shows that a higher c^* increases total surplus. Thus, given the ownership structure, the most efficient equilibrium involves the largest working threshold.

Lemma 4 (Common ownership, exit): An equilibrium under common ownership always exists. In any equilibrium there is $c^* > 0$ such that manager i chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^*$. The working threshold in the most efficient equilibrium is given by

$$c^{**}_{co,exit} = \begin{cases} \Delta & \textit{if } L/n \leq \underline{v}(1 - F(\Delta)) \\ c^{**}_{exit} & \textit{else,} \end{cases} \tag{13}$$

where $c^{**}_{exit} < \Delta$.[13] Prices and trading strategies are characterized by Lemma 2, where τ is given by $\tau^{**}_{co,exit} \equiv F\left(c^{**}_{co,exit}\right)$.

The next result compares governance through exit under the two ownership structures.

Proposition 2 (Comparison of most efficient equilibria, exit): The working threshold and total surplus are strictly higher in the most efficient equilibrium under common ownership than under separate ownership if $L/n \leq \underline{v}\,(1 - F\,(\Delta))$.[14]

The investor exerts governance by selling the firm if the manager shirks. Doing so reduces the security price and punishes the manager ex post; the threat of exit increases his incentives to work ex ante. However, the punishment for shirking depends not on the action to sell but the price impact of the sale—and hence whether the investor is concentrated or diversified. Under separate ownership, the manager's incentives to work are low for two reasons. First, the reward for working is low. If he works, the investor may suffer a liquidity shock and be forced to sell, and so the security price is lower than its fundamental value of $\overline{v}$. Second, the punishment for shirking is low. If he shirks and is sold, the sale is consistent with a working manager being sold due to a liquidity shock, and so the security price is not too low. Proposition 2 states that, if $L/n \leq \underline{v}\,(1 - F\,(\Delta))$, governance is strictly superior in the most efficient equilibrium under common ownership than under separate ownership, because common ownership alleviates the above two concerns. First, common ownership increases the reward for working, because a working manager is no longer automatically sold upon a shock—he is retained upon a small shock and only partially sold upon a moderate shock. Second, common ownership increases the punishment for shirking. Under a small shock, being sold to any degree is fully revealing of shirking and leads to the lowest possible price of $\underline{v}$. Under a moderate shock, the investor fully sells bad firms and only partially sells good firms, so being fully sold reveals a firm as bad.

[13] We characterize the manager's working threshold when $L/n \leq \underline{v}\,(1 - F\,(\Delta))$, which is the only case necessary for the main result of this section. The proof of Lemma 4 fully characterizes the threshold when $L/n > \underline{v}\,(1 - F\,(\Delta))$.

[14] $L/n \leq v(1 - F(\Delta))$ is a sufficient condition, but is not necessary. Even if $L/n > v(1 - F(\Delta))$, a type (i) equilibrium is possible. It would not be with a working threshold of Δ, as there would too few bad firms for the investor to satisfy the shock by selling only them. However, by lowering the working threshold to $F^{-1}(1 - \frac{L/n}{\underline{v}})$ (through an unshocked investor retaining bad firms with positive probability), the equilibrium can "create" enough bad firms to satisfy the shock. In addition, if $L/n > v(1 - F(\Delta))$, a type (ii) equilibrium is possible. In both type (i) and type (ii) equilibria, the investor sells good firms less than bad firms upon a shock, increasing price informativeness. Thus, the working threshold may still be higher than under separate ownership (even though it is less than Δ, as shown by Lemma 4.)

In both cases, common ownership is a commitment device by the investor to sell bad firms more than good firms. Intuitively, it creates a tournament between the n managers, who know that the investor will sell the worst performers regardless of whether she is hit by a liquidity shock. Since the market also knows that the worst performers are sold, it gives a low price to a sold firm. Moreover, this tournament means that, even if there is no explicit relative performance evaluation in the managers' contracts, the investor will engage in relative performance evaluation. With a large shock, common ownership does not provide flexibility and so this effect is absent.[15]

Proposition 2 gives conditions under which the working threshold is larger under common ownership, that is, $c^{**}_{co,exit} > c^{**}_{so,exit}$. This result implies that common ownership is more efficient than separate ownership since, for each firm, governance is stronger and the increased firm value exceeds the increased cost of managerial effort. However, note that common ownership can be more efficient even if $c^{**}_{co,exit} < c^{**}_{so,exit}$, because the investor exerts governance over $n > 1$ firms. Thus, weaker governance over several firms can improve total surplus more than stronger governance over one firm.[16]

We close this section with two remarks. First, the analysis has considered portfolio structure as exogenous and compared the investor's trading strategy under both separate and common ownership. When portfolio structure is endogenous, she will choose the structure that maximizes governance as long as the purchase of her initial position is not fully observed (as in, e.g., Kyle and Vila 1991).[17] This is because she can acquire securities at less than their fundamental value, and thus shares in the value created by improved governance. If her trade is fully observed, she has to acquire her initial position at their full value, including any gover-

[15] While Proposition 2 compares the benchmark to the most efficient equilibrium under common ownership, which will be chosen under the efficiency criterion, Online Appendix E.1 considers all equilibria and derives a condition under which any equilibrium under common ownership is strictly more efficient than under separate ownership.

[16] As stated in Section 1.1, we assume that any securities not held by the investor are held by households, who do not engage in governance. This is to hold constant the number of securities held by governing investors across both models. If, when moving from common to separate ownership, the stakes in the other $n-1$ firms are now held by other large investors, then this force disappears—separate ownership no longer reduces the number of firms that are being governed.

[17] Recall that the structure that maximizes the working threshold c^* need not maximize total surplus—even if the threshold is lower under common ownership, total surplus is higher since the investor governs n rather than 1 firms. This issue does not arise when maximizing the value of the investor's portfolio, rather than total surplus, since she owns n units under both ownership structures—thus, the structure that maximizes c^* also maximizes the value of her portfolio.

nance benefits, and so would be indifferent. In addition to sharing in the gains from improved governance under diversification (if her trade is not fully observed), the investor may choose to be diversified for reasons outside the model, for instance, risk reduction concerns, “prudent man” rules, or downward-sloping demand curves for a single security. In this case, our results suggest that diversification for private risk reduction or price impact reasons can have a social benefit by improving governance.

Second, in the end of Section 1.3, we discussed how a concentrated investor adding risky securities to her portfolio (i.e., moving to common ownership) leads to critically different results from adding $A < L$ of liquid assets—that is, the effect of substituting liquid assets for risky securities. Here, we discuss the effect of adding liquid assets while holding risky securities constant—that is, if a diversified investor were able to add A of liquid assets before receiving her private information. As discussed at the end of Section 1.3, a liquidity buffer of A effectively reduces the liquidity shock to $L - A$. The investor will thus choose A such that $\frac{L-A}{n} \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$, so that we are in the small-shock equilibrium of Lemma 2, part (i), where trading flexibility, and thus price informativeness and governance, are maximized. Indeed, the condition $\frac{L-A}{n} \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$ in Proposition 2 (extended to incorporate a liquidity buffer of A) is now automatically satisfied. Governance is always strictly higher under common than separate ownership when the investor can endogenously choose her liquidity buffer, strengthening the results of our core model. Finally, we consider the case in which the investor can increase the number of firms n that she owns. The effect is the same as increasing A, as it reduces the per-firm liquidity shock L/n. Thus, she would similarly choose n such that $\frac{L-A}{n} \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$.

2.2 Governance through voice

Rather than being taken by the manager, the action a_i could instead be taken by the investor, who also bears its (privately observed) cost.[18] Examples include advising the firm on strategy,

[18] The cost of monitoring will depend on firm-specific factors that are, in part, privately known to the investor (as in Landier, Sraer, and Thesmar 2009). For example, she may have private information on the business ties that she may lose if she engages in perk prevention, on her ability to use her business connections to benefit the firm, or on the extent to which she can extract private benefits. The results are robust to assuming a publicly known monitoring cost and probabilistic monitoring—that is, paying this cost leads to $v_i = \overline{v}$ w.p. $\iota < 1$, not w.p. 1. We only require the investor to have private information on firm value, which can arise from either $\widetilde{c}_i$ being privately observed or monitoring being probabilistic. This analysis is available upon request.

In Faure-Grimaud and Gromb (2004), the investor only trades (as in our core model) and the action is undertaken by a separate “monitor” who is also concerned about the $t = 2$ security price. This model is using her business connections to benefit the firm, preventing the firm's manager from extracting perks or empire-building, or choosing not to take private benefits for herself. We refer to $a_i = 1$ as "monitoring" and $a_i = 0$ as "not monitoring." A good (bad) firm is now one that has been monitored (not monitored). Through her private knowledge of $\mathbf{a} \equiv [a_i]_{i=0}^n$, the investor continues to have private information on $\mathbf{v}$. In addition to demonstrating the applicability of the model to governance through voice, a quite separate contribution of this section is to show how our unifying model can be applied to both voice and exit. Thus far, these literatures have developed largely independently and been modeled with quite different frameworks.

The investor's utility conditional on $\mathbf{x}$ and the realization $\mathbf{c} \equiv [c_i]_{i=0}^n$ of $\widetilde{\mathbf{c}}$ is now given by:

$$u_{I,voice}(\mathbf{x}, \mathbf{a}, \mathbf{p}, \mathbf{c}, \theta) = u_I(\mathbf{x}, \mathbf{a}, \mathbf{p}, \theta) - \int_0^n c_i a_i di. \tag{14}$$

under common ownership, and analogously under separate ownership.[19] The equilibrium definition is similar to Section 1, with the following additions: (vii) the investor's monitoring rule in each firm i maximizes her expected utility given $\widetilde{\mathbf{c}}$, her expected trading strategy, and each market maker's pricing rule, and (viii) each market maker forms expectations about τ that are consistent with (vii).

Similar to the exit model, in any equilibrium and under any ownership structure, the investor follows a threshold strategy—she monitors firm i if and only if $\widetilde{c}_i \leq c^*$. Lemma 5 characterizes the equilibrium under separate ownership.

Lemma 5 (Separate ownership, voice): An equilibrium under separate ownership always exists.In any equilibrium, there is $c^*_{so,voice} > 0$ such that the investor chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^*_{so,voice}$. The monitoring threshold $c^*_{so,voice}$ is given by the solution of $c^*/n = \phi_{voice}(F(c^*))$, where

$$\phi_{voice}(\tau) \equiv \Delta \left[1 - \beta \min\left\{\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau}, \frac{1}{\beta\tau + 1 - \tau}\right\}\right]. \tag{15}$$

Prices and trading strategies are characterized by Lemma 1, where τ is given by $\tau^*_{so,voice} \equiv F\left(c^*_{so,voice}\right)$.

Lemma 6 characterizes the equilibrium under common ownership.[20]

identical to the model of Section 2.1, with the monitor replacing the manager.

[19] We implicitly assume $\int_0^n c_i di < \infty$.

[20] Similar to the exit model, $c^{**}_{co,voice}$ is fully characterized only when $L/n \leq \underline{v}(1 - F(\Delta))$. For completeness,

Lemma 6 (Common ownership, voice): An equilibrium under common ownership always exists. In any equilibrium there is $c^* > 0$ such that the investor chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^*$. The monitoring threshold in equilibrium is given by

$$c^{**}_{co,voice} = \begin{cases} \Delta & \text{if } L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right) \\ c^{**}_{voice} & \text{otherwise} \end{cases}, \tag{16}$$

where $c^{**}_{voice} < \Delta$. Prices and trading strategies are characterized by Lemma 2, where τ is given by $\tau^{**}_{co,voice} \equiv F\left(c^{**}_{co,voice}\right)$.

As in the exit model, we define efficiency as total surplus. Proposition 3 compares the efficiency of governance under the two ownership structures.

Proposition 3 (Comparison of equilibria, voice): For any $L \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$ there exists $\underline{n}\left(L\right) > 1$ such that if $1 < n < \underline{n}\left(L\right)$, the monitoring threshold and total surplus in any equilibrium under common ownership are strictly larger than any equilibrium under separate ownership.[21]

The intuition is as follows, which mirrors that of Section 2.1. Under separate ownership, the investor's incentives to monitor are low for two reasons. First, if she monitors and increases the firm value to $\overline{v}$, she may suffer a liquidity shock and be forced to sell for a price below $\overline{v}$. This reduces the payoff to monitoring. Second, if she does not monitor, she can sell some firms ("cut and run") and pretend that the sale is of a good firm but motivated by a liquidity shock, as in Kahn and Winton (1998) and Maug (1998). This increases the payoff to not monitoring. The same two reasons mean that the flexibility stemming from common ownership increases the investor's payoff from monitoring. With a small shock, the investor never needs to sell a monitored firm. With a moderate shock, the investor is forced to sell a monitored firm but only partially, and so receives a higher price than under separate ownership. In addition, the payoff to cutting and running is now lower since prices are more informative. A sale is more indicative that the investor has not monitored, since if she had monitored and suffered a

we fully characterize the monitoring threshold $c^{**}_{co,voice}$ when $L/n > \underline{v}\left(1 - F\left(\Delta\right)\right)$ in Online Appendix E.2. Note that the analysis of these cases is not necessary for our main result.

[21] As in Proposition 2, the condition $L \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$ is sufficient but not necessary, for similar reasons as in note 14.

liquidity shock, she would have sold other firms instead. In both cases, common ownership is a commitment device by the investor to monitor rather than cut and run. With a large shock, common ownership does not provide flexibility and so this benefit is absent.

The positive effect of flexibility must be weighed against the fact that the investor now only has 1 rather than n units in each firm, which reduces her incentive to monitor. Thus, Proposition 3 shows that governance is superior under common ownership if the number of firms is sufficiently low, so that the decline in the number of units from n to 1 and thus the effect of being spread too thinly is small, and the liquidity shock is not large, so that common ownership provides flexibility.[22] Analogous to the exit model, common ownership can be more efficient even if $c^{**}_{co,voice} < c^{**}_{so,voice}$, because the investor exerts governance over $n > 1$ firms. Thus, weaker governance over several firms can improve total surplus more than stronger governance over one firm. While diversification does spread an investor more thinly with respect to one firm, the flip side is that it spreads an investor across more firms. This second effect is typically ignored by the literature, which focuses on the governance of a single firm, but must be taken into account in any analysis of total surplus.

Also similar to the exit model, if the investor could endogenously choose ownership structure, she would select the one that maximizes her expected portfolio value. The only difference is that in the voice model, this value is net of her expected monitoring costs (in the exit model, action costs are borne by the managers). Thus, if she could acquire her initial position anonymously, she would internalize the effect of her monitoring on her z units rather than the entire firm, and the entirety of the monitoring costs. Proposition 4 below gives a condition under which the investor will prefer common ownership. Intuitively, if the number of firms under common ownership is sufficiently small, her monitoring is concentrated over fewer firms and so she bears lower aggregate monitoring costs.

Proposition 4 (Investor's choice of equilibrium, voice): For any $L \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$, there exists $1 < \underline{\underline{n}}\left(L\right) \leq \underline{n}\left(L\right)$ such that, if $1 < n < \underline{\underline{n}}\left(L\right)$, the investor's expected payoff net of monitoring costs in any equilibrium under common ownership is strictly higher than in any equilibrium under separate ownership.

[22]Note that, unlike Proposition 2 for the exit model, which studies the most efficient equilibrium under common ownership, Proposition 3 for the voice model holds under all equilibria, for the reasons discussed in Online Appendix E.2.

As with the exit model, we discuss the effect of adding liquid assets while holding risky securities constant. Similar to that model, the investor will choose A such that $\frac{L-A}{n} \leq \underline{v}(1 - F(\Delta))$, so that we are in the small-shock equilibrium of Lemma 2, part (i), where trading flexibility, and thus price informativeness and governance, are maximized. The increase in the value of her portfolio outweighs the greater cost of monitoring. Finally, we consider the case in which the investor now owns n/κ units in κ distinct firms. In the exit model, only one effect was relevant: a rise in the number of firms reduces the per-firm liquidity shock. Here, there is a counteracting effect: a rise in the number of firms reduces her stake in each individual firm and thus her incentives to monitor. Thus, the investor will choose κ sufficiently small such that governance overall improves. While her choice of the number of firms will be different from the exit model (she will choose few rather than many firms), the same conclusion holds—both Proposition 2 and Proposition 4 are strengthened if the investor can choose either a liquidity buffer (while holding risky securities constant) or the number of firms she holds under common ownership.

3 Extensions

3.1 Imperfect information under common ownership

Thus far, we have assumed that the investor is equally endowed with private information under both separate and common ownership. This applies to cases in which owning and operating the asset automatically gives the seller information. This is termed "learning by holding" by Plantin (2009); see Acharya and Johnson (2007), Bushee, Jung, and Miller (2017), Solomon and Soltes (2015), and Waters and Mirkin (2016) for evidence. For example, large shareholders have greater access to firm management, and lenders are able to request information from borrowers at little cost. However, if information acquisition is endogenous, the investor may be less informed under common ownership for two reasons. First, she has a smaller stake in each individual firm, and Edmans (2009) shows that an investor's information acquisition is increasing in her initial stake size, up to a point; alternatively, her access to information may be increasing in stake size (rather than the stake only needing to exceed a threshold to grant her access). Second, if the investor has the option to acquire information before forming her portfolio, she may choose to hold only stocks about which she is informed—that is, have a concentrated portfolio, as shown by Van Nieuwerburgh and Veldkamp (2009, 2010) and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014, 2016). Thus, an investor who has chosen common ownership is one who has acquired less information to begin with.

This section analyzes whether our results continue to hold when the investor is less informed under common ownership. We continue to assume that, under separate ownership, the investor perfectly observes v_i, while under common ownership she observes signal $s_i \in \{\underline{v}, \overline{v}\}$ on v_i where

$$\Pr[y_i = v_i | v_i] = \rho \in (\frac{1}{2}, 1]. \tag{17}$$

Parameter ρ captures the precision of the diversified investor's private information, and $\rho < 1$ implies that she has less information on each asset than a concentrated investor.

Proposition 5 compares price informativeness under the two ownership structures.

Proposition 5 (Price informativeness, imperfect information under common ownership): There are $0 < \underline{y} \leq \overline{y}$, independent of ρ and β, such that:

(i) If $L/n < \underline{y}$, then there is $\rho^* \in [\frac{1}{2}, 1)$ such that, if $\rho > \rho^*$, there exists an equilibrium in which price informativeness is strictly higher under common ownership. If in addition $\beta \geq \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$, then price informativeness is strictly higher in any equilibrium under common ownership when $\rho > \rho^*$. If $\beta = 1$, then $\rho^* = \frac{1}{2}$.

(ii) If $L/n \geq \overline{y}$ and $\rho < 1$, then there exists an equilibrium in which price informativeness is strictly lower under common ownership.

(iii) If $\beta < 1$, there is $\underline{\rho} \in (\frac{1}{2}, 1)$ such that, if $\rho < \underline{\rho}$, then in any equilibrium, price informativeness is strictly lower under common ownership.

Part (i) of Proposition 5 shows that, if the shock L/n is sufficiently small and the diversified investor's information precision ρ is sufficiently high, there exists an equilibrium in which price informativeness is strictly higher under common ownership. Intuitively, price informativeness depends on two factors: how much information the investor has, and how revealing her trades are of this information (the key contribution of this paper). Proposition 1 showed that, when the investor is perfectly informed under both ownership structures (i.e., the first factor is the same in both), price informativeness is higher under common ownership and a sufficiently small shock due to the second factor—her trading flexibility means that her trades are informative. Proposition 5 shows that this result continues to hold when the common owner has imprecise information, as long as the imprecision is not sufficiently large to outweigh the effect of flexibility.[23] If $\beta = 1$, then prices are fully uninformative under separate ownership as the investor always sells her securities, regardless of her information—thus, prices are more informative under common ownership as long as precision is strictly positive.

If ρ is sufficiently low, then imprecision outweighs flexibility. The first effect dominates and price informativeness is lower, as stated in part (iii). Proposition 1 also showed that, if the liquidity shock is sufficiently large, price informativeness is the same under both ownership structures because the common owner has no flexibility to disproportionately sell bad assets. Thus, if she has imprecise information, price informativeness will be lower—as confirmed by part (ii). As in Section 2, the different levels of price informativeness under each ownership structure then imply different levels of governance.

3.2 Index funds

Common ownership increases price informativeness through allowing the investor to sell bad assets first. This section highlights the importance of trading flexibility for this result by analyzing index funds, which are unable to trade flexibly. Index funds have become increasingly important in recent years. Appel, Gormley, and Keim (2018) show that their ownership share of the U.S. stock market quadrupled between 1998 and 2014, and they now account for over a third of all mutual fund assets. Appel, Gormley, and Keim (2016) show empirically that index funds engage in monitoring, and such activity will likely increase over time. For example, BlackRock CEO Larry Fink's 2018 open letter to CEOs announced that it would double the size of its stewardship department over the next three years.[24] The European Union Shareholder Rights Directive (adopted in 2017) and the ongoing (2018) revision of the UK Stewardship Code places greater monitoring obligations on all investors, both active and passive. Since index funds have an increasingly important role in monitoring, it is interesting to study whether they have greater monitoring incentives than otherwise-identical active funds.

An index fund is diversified, but unlike the diversified investor in Lemma 6, its trading is

[23] Also as in Proposition 1, part (i) shows that price informativeness is higher in *any* equilibrium if β is sufficiently high.

[24] https://www.blackrock.com/corporate/investor-relations/larry-fink-ceo-letter.

constrained in two ways. First, it must sell all assets to the same degree. Second, it only sells if there is a liquidity shock: it must raise exactly θ in revenue. We include the second constraint as it is a feature of index funds in real life, but it is not the driver of our results—only the requirement to sell all assets to the same degree is.

Proposition 6 (Index fund, voice): An equilibrium for ownership by an index fund always exists. In any equilibrium, the monitoring threshold of an index fund, $c^{**}_{co,voice,index}$, is given by the solution of $c^* = \xi\left(F(c^*)\right)$, where

$$\xi\left(\tau\right) \equiv \Delta\left[1 - \beta \min\left\{1, \frac{L/n}{\underline{v} + \tau\Delta}\right\}\right] \tag{18}$$

and $c^{**}_{co,voice,index} < \Delta$. If $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$, then $c^{**}_{co,voice,index} < c^{**}_{co,voice}$, the monitoring threshold for an unconstrained investor under common ownership.

Some commentators (e.g., Bhide 1993) argue that the ability to cut and run reduces monitoring incentives. One may think that index funds may therefore have greater monitoring incentives than the active funds considered in the core model—since index funds cannot disproportionately sell bad firms, they are locked in to monitor. Our model shows that this need not be the case: if $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$, then $c^{**}_{co,voice} = \Delta > c^{**}_{co,voice,index}$, and so active funds monitor more than index funds. The intuition is twofold and echoes the two reasons for why active funds monitor more than concentrated owners discussed in Section 2.2. First, the flip side of index funds' inability to cut and run—to disproportionately sell bad firms—is that they are also unable to disproportionately retain good firms if they suffer a shock. A shock forces them to sell good firms to the same extent as bad firms. This reduces the payoff to monitoring. Second, because index funds sell good firms to the same extent as bad firms, they receive a relatively high price upon selling. This increases the payoff to not monitoring. In contrast, if an active fund does not monitor a firm and subsequently sells it, it receives the lowers possible price of $\underline{v}$ if $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$. Intuitively, since an index fund does not have the flexibility to sell bad firms more than good firms, its trades (and thus prices) are uninformative.

3.3 Robustness

3.3.1. Two firms. The model uses a continuum of firms to invoke the law of large numbers, in turn leading to significant tractability—since we know that the investor will have a proportion

τ of good firms, this is the only case that we need to consider. Online Appendix F.1 shows that the results continue to hold with two firms, albeit with more cases to consider. The intuition is as follows. With two firms, we must also consider cases in which the firms are either both good or both bad, but these cases are uninteresting because the investor has no trading flexibility. She has trading flexibility—due to owning one good and one bad firm—$2\tau(1-\tau)$ of the time, rather than all of the time under a continuum. Flexibility still improves compared to the case of separate ownership, where she never has trading flexibility because all units are necessarily perfectly correlated with each other. Diversification—whether to a finite number or a continuum of firms — provides trading flexibility since individual shares need not be perfectly correlated. The increment to flexibility is increasing in the number of firms. Specifically, with N firms, the probability that firms are either all good or all bad (and thus there is zero trading flexibility) is $\tau^N + (1-\tau)^N$, which decreases with N. Our core model captures this force by studying the two polar cases, of separate ownership (no flexibility) and owning a continuum of firms (full flexibility), to highlight the benefits of common ownership most clearly.

3.3.2. Single market maker. The model assumes that there is a separate market maker for each firm i. This is for two reasons. The first is realism: in reality, there are several market makers for traded securities, and different market makers specialize in different securities. Away from a securities application, a conglomerate selling multiple subsidiaries, or a private equity firm selling multiple businesses, will likely be selling them to different buyers, and so the buyer of one subsidiary does not know the details of the sale of the other subsidiary in real time. The second is to highlight the economic forces behind our result—that common ownership gives the investor trading flexibility. Our result does not arise because the market maker can observe the investor's trades in other firms and compare her trade in firm i to that in firm j. Merely knowing that she has other firms in her portfolio, that she could have sold upon a shock, is sufficient for the market maker to give a low price to a sold firm.

An alternative assumption is to have a single market maker who observes the trades in all firms, such one who makes markets for many securities. This assumption also likely holds if securities are traded on a blockchain (Yermack 2017). Online Appendix F.2 shows that price informativeness can be even higher under common ownership than with separate market makers (i.e., the results become stronger), since the single market maker is able to engage in "relative performance evaluation." For example, consider the moderate-shock equilibrium of part (ii) of Lemma 2, where $(\overline{v}, L)$ is pooled with $(\underline{v}, 0)$ under separate market makers because both are partially sold. With a single market maker, prices depend not on the absolute trade in a given firm, but the trade relative to that in other firms. If other firms are sold more (less), the market maker infers a shock (no shock) and thus that the partially sold firm is good (bad). Thus, $(\overline{v}, L)$ and $(\underline{v}, 0)$ can now be fully distinguished. By comparing the trade in firm i to that in firm j, the market maker can better discern whether a sale was due to a liquidity shock or low firm value, leading to perfect price informativeness.[25]

Online Appendix F.2 also shows that there also exists an equilibrium under common ownership where prices are fully uninformative. The intuition is as follows. The market maker knows with certainty the value of the investor's portfolio, which is $\underline{v} + \Delta\tau$ by the law of large numbers. As a result, if the investor sells a tranche of her entire portfolio (engages in "balanced exit"), the market maker pays $\underline{v} + \Delta\tau$ for a portfolio worth $\underline{v} + \Delta\tau$.[26] Intuitively, by selling all firms to the same degree, the investor loses on the good firms but gains on the bad firms since the market maker cannot distinguish the two. If the investor sells bad firms more than good firms (engages in "imbalanced exit"), the market maker knows that firms sold more are bad and so pays $\underline{v}$ for firms worth $\underline{v}$. The investor thus makes zero profit under both balanced and imbalanced exit (regardless of whether she has suffered a shock) and is thus indifferent between the two trading strategies. As a result, there is also an equilibrium in which she retains all firms when $\theta = 0$ and sells all firms when $\theta = L$. Since the investor's trade is independent of firm value, prices are fully uninformative. Section 2 showed that, when firm values are endogenous, higher price informativeness leads to higher real efficiency. Thus, under the efficiency criterion, the equilibrium with greater price informativeness will be selected. Moreover, Online Appendix F.1.2 analyzes the single market maker model with two firms, where the law of large numbers does not apply and so the market maker does not know the value of the investor's

[25] Gervais, Lynch, and Musto (2005) show that mutual fund families can add value by monitoring multiple managers, since firing one manager increases sellers' perceived skill of retained managers. Inderst, Mueller, and Münnich (2007) show that when a seller finances several entrepreneurs, an individual entrepreneur may exert greater effort. To obtain refinancing, he needs to deliver not only good absolute performance, but also good performance relative to his peers. In Fulghieri and Sevilir (2009), multiple entrepreneurs compete for the limited human capital of a single venture capitalist. These effects are similar to the relative performance evaluation channel under a single buyer, but will not arise in the case of our core model in which trades in other assets are unobservable, and so are fundamentally different from the effect of flexibility in this paper.

[26] This contrasts with both the case of concentration and the case of diversification with separate buyers, since the buyer for an individual asset does not know whether it is worth $\overline{v}$ or $\underline{v}$.

portfolio. It derives conditions under which price informativeness with a single market maker is higher under any equilibrium under common ownership than any equilibrium under separate ownership. Thus, our results are robust to the assumption of a single market maker.

3.3.3. Unobserved initial portfolio and information endowment. In the core model, whether the investor is concentrated or diversified is common knowledge. Indeed, large investors have to file their quarterly positions under Schedule 13F, and all investors have to file stakes that exceed 5% via Schedule 13D or 13G. In the application to debt, loan sales are often over the counter and decentralized, so the seller's identity is observable. Note also that a diversified investor has incentives to disclose her position, as it strengthens governance and thus the value of her portfolio.

Online Appendix F.3 considers the case in which the investor's initial portfolio is her private information. Now, a diversified investor may attempt to mimic the concentrated investor's trades, to reduce the price impact of her sale. We show that price informativeness is strictly increasing in the probability that the investor is diversified. Our main model analyzes the limit case in which the probability is either zero (separate ownership) or one (common ownership). This extension shows that when probabilities can be interior, a greater probability of common ownership increases price informativeness, echoing the main model's result.

In addition to allowing her initial portfolio to be her private information, Online Appendix F.3 also allows the investor to be either informed or uninformed, and for her information endowment to be her private information (as well as independent of her portfolio structure). Thus, the investor is sometimes both uninformed and forced to trade due to a liquidity shock, similar to a noise trader in the Kyle (1985) model. The results continue to hold.

3.3.4. Noise traders. In general, informed investors can make profits on their information for two reasons. First, their trade may be unobservable, because it is pooled with that of noise traders, as modeled by Kyle (1985) in a securities application. Second, their trade may be observable, but the market maker does not know whether it is due to information or a liquidity shock, as modeled by Glosten and Milgrom (1985) in a securities application. Our model uses the second framework, since it is the fact that liquidity shocks are at the portfolio level that leads to connection between unrelated firms. This applies most closely to securities that are traded over the counter, in a secondary distribution, or on a blockchain securities market (Yermack 2017).

The extension in Online Appendix F.3 described earlier allows the liquidity trader to sometimes be a noise trader. We conjecture that the results will be robust to adding noise traders to the model in addition to the (always-informed) investor, as with securities traded anonymously on an exchange.[27] The market maker is now only able to partially infer the probability that a sale comes from the informed investor (rather than uninformed noise traders), rather than observing it directly. However, given a probability that the informed investor has sold firm i, the likelihood that this sale was due to negative information is higher under common ownership due to the investor's flexibility over which firms to sell to satisfy a shock. The model only requires a strictly positive probability of a liquidity shock for common ownership to improve price informativeness (it allows for any $\beta \in (0, 1]$, including $\beta = 1$, that is, no private information about the liquidity shock), because it is the portfolio-wide liquidity shock that creates the link between trading in the individual firms, and it can accommodate any volume of noise trader demand.

Note that the comparison with the classic microstructure models of Glosten and Milgrom (1985) and Kyle (1985) leads to an interesting intuition. Even though there is no separate noise trader in our model, the liquidity shock can be thought of as effectively creating a noise trader—the investor in the liquidity-shock state, with whom the investor is camouflaging. Common ownership reduces this camouflage, since a shocked investor now has flexibility over what to sell. She is an informed trader, not a noise trader, and so common ownership effectively removes the noise trader from the model. As a result, an unshocked investor cannot pretend that her trades are not driven by information.

3.3.5. Heterogeneous firms. Online Appendix F.4 considers the case in which firms have different Δ, which parameterizes uncertainty or information asymmetry. As a result, the price impact of selling—and thus the security's liquidity—differs across firms. It remains the case that price informativeness is strictly higher under common ownership when the shock is small. Regardless of Δ and thus price impact or liquidity, the investor always receives (weakly) more than $\underline{v}$ by selling a bad firm and less than $\overline{v}$ by selling a good firm, and so is always better off by selling firms that she knows to be bad and retaining firms she knows to be good. Thus, it

[27] As in Kyle (1985), this extension assumes that the volume of noise traders is independent of price informativeness and thus their trading losses, since they are forced to trade due to a liquidity shock (similar to the seller's liquidity shock in our model). The results will likely go through even if noise traders adjust part of their trading volume based on their expected losses, as long as at least part of the trading volume is nondiscretionary.

remains the case that common ownership allows the investor to fully retain good firms upon a small shock, and so a sale fully reveals that a firm is bad. Note that the analysis of holding cash (see the discussion at end of Section 1.3) also shows that the model is robust to heterogeneous firms, since cash has information asymmetry of $\Delta = 0$.

3.3.6. Discontinuing relationships. In our model, the investor is concerned with the price impact of her sale, as she receives the sale price. Online Appendix F.5 extends the model to the case in which the investor is not concerned with price impact, as in the case of a headquarters closing a division rather than selling it. In this case, there is no sale price, and the headquarters' payoff from shutdown is its alternative use of capital, which is independent of the market's perception of the shutdown business—but, in an exit model, the business manager will still care about his reputation implied by the shutdown. We show that, even with a fixed reservation payoff, price informativeness is always weakly higher under common ownership, because it remains the case that the investor has a choice of what firm to sell when she suffers a shock, and so her sale decisions convey information. This model can also apply to other discontinuation decisions, such as a bank ceasing to lend or a venture capitalist not investing in a future financing round,[28] as well as to stakeholders other than investors, such as a supplier's or customer's decision to terminate its relationship with a firm. The threat of being the only business, out of many, with which the stakeholder terminates the relationship improves the manager's effort incentives.

3.3.7. Distribution of liquidity shocks. While our model assumes a binary liquidity shock $\theta \in \{0, L\}$, we conjecture that our core mechanism, that common ownership provides flexibility, applies regardless of the distribution of θ. Even with a more general distribution of liquidity shocks, as long as there is a strictly positive probability that the shock is not large, a diversified investor can sometimes sell good firms less than bad ones upon a shock, whereas a concentrated investor is always forced to sell them to the same degree.

[28] In this model, the discontinuation decision has no direct effect on firm value, for example, because there are other banks or venture capitalists who can provide financing. This highlights the channel through which the trading / discontinuation decision affects firm value—indirectly through affecting incentives.

4 Applications and Empirical Predictions

This section discusses the applications and empirical predictions of the model. Some are consistent with existing findings; others are new and potentially fruitful avenues for future research. We start with the trade-only model of Section 1. Lemma 2 shows that, under common ownership, an investor sells her worst assets first. Huang, Ringgenberg, and Zhang (2016) find that mutual funds do so upon a liquidity shock, Maksimovic and Phillips (2001) show that conglomerates tend to sell their least efficient plants, and Berndt and Gupta (2009) find that borrowers whose loans are sold in the secondary market underperform their peers. Moreover, our model highlights that even an investor subject to a liquidity shock can trade on information, if she is diversified. This has important implications for empirical studies that investigate the effect of non-informationally motivated trades, for example, as an exogenous shock to prices. It is insufficient to show that an investor has been subject to a liquidity shock to claim that her trades are not motivated by information. Some papers study the discretionary trades of mutual funds that have suffered outflows, but such trades could still be information-driven as the mutual funds can choose which stocks to sell to fund the outflows. Edmans, Goldstein, and Jiang (2012) thus study the hypothetical sales that mutual funds would have made if they scaled back their existing portfolio in proportion, rather than their actual sales.

A second prediction concerns the price decline upon a sale of debt or equity. Comparing Lemmas 1 and 2 shows that a price decline upon a sale of securities should be greater when the investor is diversified. Scholes (1972), Mikkelson and Partch (1985), Holthausen, Leftwich, and Mayers (1990), and Sias, Starks, and Titman (2006) show that equity sales by large shareholders reduce the stock price due to conveying negative information, and Dahiya, Puri, and Saunders (2003) find similar results for loan sales. However, the effect of the rest of the investor's portfolio on the price decline is a new prediction that, to our knowledge, has not previously been tested. Relatedly, the price decline should be smaller the larger the investor's liquidity shock, as a larger shock means that sales are less likely to be driven by private information. Darmouni (2017) finds that when a lender suffers a liquidity shock, borrowers tend to have their loans terminated. The greater the shock, the greater the likelihood that the borrower finds a new loan, suggesting that the reputational damage from termination is smaller.

A third prediction, stemming from Proposition 1, is that price informativeness may be higher under diversification. Existing research (e.g., Van Nieuwerburgh and Veldkamp 2009, 2010; Kacperczyk, Van Nieuwerburgh, and Veldkamp 2014, 2016) suggests that price informativeness should be lower since a diversified investor is likely less informed. Our model points out that, even if a diversified investor is indeed less informed, price informativeness may still be higher because her trades are more revealing. Since price informativeness depends on both the amount of information an investor has and the extent to which it is revealed in prices, examining the link between investor diversification and price informativeness will help identify the settings in which each of the two forces is likely stronger.[29]

More broadly, while existing research on common ownership has focused on its corporate finance implications (e.g., Azar, Schmalz, and Tecu 2018; Kang, Luo, and Na 2018), the implications discussed earlier relate common ownership to asset pricing. Moreover, the insights from the trade-only model can be applied outside a trading context to corporate finance settings, such as a director's decision to quit a board, a firm's decision to exit or scale back a line of business, or an employer's decision to fire a worker. In all of these cases, the negative inference resulting from termination is stronger if the decision-maker had many other relationships that she could have terminated instead. Even in a trading context, the "price" can refer to market perceptions of quality rather than a literal price. For example, if a bank sells a loan or stops lending to a borrower, that borrower's perceived creditworthiness falls more if the bank had other borrowers it could have stopped lending to instead. Beyond security trading, a director's decision to quit a firm is a more negative signal if he serves on other boards; a conglomerate's decision to exit a business line is a more negative signal of industry prospects than if a focused firm scaled back its operations.

Moving to the governance model, both the exit and voice applications suggest that diversification can strengthen governance; for voice, this arises if the greater price informativeness outweighs any potential loss from the investor being spread too thinly—that is, as long as the investor does not diversify to too many firms. This application has the potential to justify why shareholders own blocks in multiple firms, despite the free-rider problem. Existing justifications are typically based on diversification of risk. While conventional wisdom might suggest that the diversification induced by risk concerns necessarily weakens governance, our model high-

[29] Cohen and Frazzini (2008) and Hoberg and Phillips (forthcoming) show that return predictability is lower when two (related) stocks are held by a common owner. This is consistent with our model's prediction that common ownership leads to higher price informativeness—prices being closer to fundamentals. However, our model does not make predictions on how past price changes in firm i relate to future price changes in firm j, which is what they specifically test.

lights an opposing force. Indeed, Kang, Luo, and Na (2018) find that institutional investors are more effective at governance the more blocks they have in other companies, controlling for portfolio size. This superior governance may arise due to greater price informativeness as in our paper, or other channels such as additional blocks leading to learning-by-doing. Similarly, our model shows that common ownership may improve governance, and offers an alternative explanation for its association with higher product prices documented by Azar, Schmalz, and Tecu (2018)—these higher prices may stem from superior governance, which leads to better product quality or more efficient pricing. Moreover, the results suggest that mergers between investors—which generate the benefits of diversification without the costs of being spread too thinly—should improve governance, even if the investors do not have common holdings and so the merger does not increase their stake in a given firm.

There are a number of potential ways to distinguish between the channel through which common ownership has real effects in this paper, and those documented by prior research. To the best of our knowledge, they have not yet been tested and may be promising questions for future research. First, channels based on learning-by-doing or anti-competitive behavior are stronger when the firms under common ownership are in related industries; they predict no or negligible effect in unrelated industries. This paper shows that common ownership may have real effects even if the firms are unrelated, and in fact the effects may be stronger because the investor is more diversified. Second, the channel of greater price informativeness can be tested directly; other channels have no implications for price informativeness. Gallagher, Gardner, and Swan (2013) provide evidence for governance through exit by showing that blockholders lead to an improvement in not only firm performance but also price informativeness; a similar analysis could be undertaken for blockholders who own multiple stakes. Third, the exit model suggests that the effects of common ownership should be stronger if the manager's wealth is more sensitive to the stock price.

Relatedly, while existing studies typically use the size of the largest blockholder or the number of blockholders as a measure of governance, both the exit and voice applications theoretically motivate a new measure—the number of other large stakes owned by its main shareholder or creditor, as studied by Kang, Luo, and Na (2018). Faccio, Marchica, and Mura (2011) empirically study a related measure, the concentration of a firm in an investor's portfolio. They argue that diversification can be desirable because a concentrated investor will turn down risky, positive-NPV projects, unlike our channel.

The investor in governance models is typically a large shareholder in a public firm, but our model is general and applies to any investor who has a large amount of an asset—not necessarily traded equity. This includes a bank that trades loans, a debtholder who trades bonds, or a company headquarters that sells divisions. Thus, the model can be applied to a corporate headquarters' or private equity general partner's decision to diversify into multiple uncorrelated business lines rather than focus on a single business or have multiple correlated businesses. Our results suggest that diversified firms face a more severe adverse selection problem when divesting than concentrated firms, since it is harder to justify a divestment as resulting from a liquidity shock.[30] Stein (1997) shows that an advantage of conglomeration is "winner-picking"–the headquarters can invest surplus funds into the division that has the best investment opportunities at the time. One may think that a related advantage is "loser-picking"–if it suffers a liquidity shock, it can choose to sell the most poorly-performing business. However, potential market makers know this, and so the headquarters face a greater, not smaller, adverse selection problem when selling. On the other hand, this greater adverse selection may strengthen incentives. Under the exit application, it is the divisional manager whose actions affect firm value, and his reputation may be affected by the market's perceived value of his division. Under the voice application, it is the headquarters whose actions affect firm value, and its incentives to monitor are greater under common ownership.

5 Conclusion

This paper has shown that common ownership can improve governance through both voice and exit. The common channel is that common ownership gives the investor a diversified portfolio of good and bad firms. As a result, she has greater flexibility over which firms to sell, and will sell bad firms first. This intensifies the adverse selection problem—the sale of a firm is a stronger signal that it is bad, since if it were good and the investor had suffered a liquidity shock, she would have sold bad firms first. In addition to reducing the expected price for a bad

[30] Where the asset is a business, it may also be sold for strategic reasons such as dissynergies. However, evidence suggests that liquidity needs are an important motive for asset sales (e.g., Borisova, John, and Salotti 2013; Campello, Graham, and Harvey 2010). Our analysis studies how diversification affects the likelihood that a sale is driven by a liquidity shock rather than private information, effectively holding synergy motives constant when comparing diversification to concentration. Edmans and Mann (forthcoming) consider both synergy and overvaluation motives for asset sales. Their model features exogenous asset values, a publicly known liquidity shock, and only consider the case of diversified and not concentrated ownership.

firm, common ownership also increases the expected price for a good firm, since the investor may not have to sell it, or may only have to sell it partially, if she suffers a shock. Moreover, common ownership has a different effect from reducing the size of the liquidity shock or adding financial slack to the investor's portfolio.

This greater price informativeness in turn improves governance through both exit and voice. If the manager works or the investor monitors, the firm is more likely to be retained since the investor has other firms that she can sell upon a shock. If the manager shirks or the investor cuts and runs, the stock price is lower than under separate ownership. This result suggests that concentrating ownership of many firms within a small number of investors may strengthen governance. Thus, mergers of investors and demergers of firms may improve governance; demergers of investors and mergers of firms reduce it. Similarly, diversification by a corporate headquarters, private equity general partner, venture capitalist, or bank can improve managers' incentives to work and investors' incentives to monitor. In addition, our paper identifies a novel channel through which common ownership has a positive real effect on firm value, even if the firms are in unrelated industries. This contrasts conventional wisdom that diversification necessarily weakens governance by spreading an investor too thinly, or that common ownership necessarily has negative real effects by leading to anti-competitive behavior.

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Appendix A. Proofs of Trade-Only Model

We prove Lemmas 1 and 2 for the case in which the investor also holds $A \in [0, L)$ liquid assets, to accommodate the extensions at the end of Sections 1.3 and 2.1. We sometimes refer to the firm as the illiquid asset. Since the price of the liquid asset always equals its true value, the investor always makes zero profit by selling the liquid asset. Therefore, her payoff depends only on her profits from selling the illiquid asset and losses from not satisfying her liquidity need. The proofs of the two lemmas below show that the effect of $A > 0$ on the equilibrium strategies can be summarized by the substitution of L with $L - A$. Thus, Proposition 1 continues to hold with the exception that L is replaced by $L - A$.

Proof of Lemma 1. Since $L - A > 0$, the investor must sell some illiquid assets to satisfy her liquidity need. Let $x^*(v, \theta)$ and $q^*(v, \theta)$ be an equilibrium strategy for type (v, θ) of selling the illiquid asset and liquid asset, respectively. We start by proving that there is a unique $\overline{x} > 0$ such that ("s.t.") $x^*(\underline{v}, L) = x^*(\underline{v}, 0) = x^*(\overline{v}, L) = \overline{x} > x^*(\overline{v}, 0)$. We argue six points:

1. In any equilibrium, $x^*(\underline{v}, L) > 0$, $x^*(\underline{v}, 0) > 0$, and $x^*(\overline{v}, L) > 0$. Proof: Suppose on the contrary that $x^*(\overline{v}, L) = 0$. The investor can raise strictly more revenue by deviating to selling n units of the illiquid asset. Since $\beta > 0$ and $L - A > 0$, this deviation will strictly increase her payoff by reducing the penalty from not meeting her liquidity need, a contradiction. Note that $x^*(\overline{v}, L) > 0$ implies $p\left(x^*(\overline{v}, L)\right) > \underline{v}$. Next, suppose on the contrary $x^*(\underline{v}, \theta) = 0$. The investor's payoff is $\underline{v}$. However, since $x^*(\overline{v}, L) > 0$ and $p\left(x^*(\overline{v}, L)\right) > \underline{v}$, by deviating to $x^*(\overline{v}, L)$ the investor obtains more revenue and a payoff strictly higher than $\underline{v}$.

2. In any equilibrium, $x^*(\overline{v}, 0) \neq x^*(\underline{v}, 0)$ and $x^*(\overline{v}, 0) \neq x^*(\underline{v}, L)$. Proof: If on the contrary $x^*(\overline{v}, 0) = x^*(\underline{v}, \theta)$ for some $\theta \in \{0, L\}$, then $p\left(x^*(\overline{v}, 0)\right) < \overline{v}$. Based on point 1, $x^*(\underline{v}, \theta) > 0$. Therefore, the payoff of type $(\overline{v}, 0)$ is strictly smaller than $\overline{v}$ and so she will deviate to fully retaining the firm.

3. In any equilibrium, $x^*(\overline{v}, 0) \neq x^*(\overline{v}, L)$. Proof: Suppose not. Based on point 1, $x^*(\overline{v}, L) > 0$. Based on point 2, $p\left(x^*(\overline{v}, 0)\right) = \overline{v}$. Therefore, $x^*(\overline{v}, 0) = x^*(\overline{v}, L)$ implies $p\left(x^*(\overline{v}, L)\right) = \overline{v}$, $p\left(x^*(\overline{v}, L)\right) = \overline{v}$ implies $x^*(\underline{v}, 0) \neq x^*(\overline{v}, \theta)$, and $x^*(\underline{v}, 0) \neq x^*(\overline{v}, \theta)$ implies $p\left(x^*(\underline{v}, 0)\right)) = \underline{v}$. However, type $(\underline{v}, 0)$ can obtain a strictly higher payoff by deviating to $x^*(\overline{v}, L) > 0$.

4. In any equilibrium, there is $\overline{x} > 0$ such that $x^*(\underline{v}, L) = x^*(\underline{v}, 0) = x^*(\overline{v}, L) = \overline{x}$. Proof: Let $x^*(\overline{v}, L) = \overline{x}$, and note that based on point 1, $\overline{x} > 0$. Moreover, the optimality of $\overline{x}$ implies that either the investor can satisfy her liquidity need by selling $\overline{x}$, or she obtains the highest revenue possible by following strategy $\overline{x}$. Suppose on the contrary, $x^*(\underline{v}, \theta) \neq \overline{x}$ for some $\theta \in \{0, L\}$. Then, based on point 2, it must be $p\left(x^*(\underline{v}, \theta)\right) = \underline{v}$, and the payoff of type $(\underline{v}, \theta)$ is $\underline{v}$. However, type $(\underline{v}, \theta)$ can deviate to $\overline{x}$ and generate a payoff strictly higher than $\underline{v}$ and meet her liquidity need (as best as she can).

5. In any equilibrium, $x^*(\overline{v}, 0) < \overline{x}$. Proof: From points 2 and 3, $x^*(\overline{v}, 0) \neq \overline{x}$, and so $p\left(x^*(\overline{v}, 0)\right) = \overline{v}$. Suppose on the contrary $x^*(\overline{v}, 0) > \overline{x}$. Then, type $(\underline{v}, 0)$ has a strictly profitable deviation from $\overline{x}$ to $x^*(\overline{v}, 0)$: she can sell strictly more units from a bad firm at a strictly higher price.

6. In any equilibrium, $q^*(\overline{v}, L) = A$. Proof: Suppose on the contrary that $q^*(\overline{v}, L) < A$. Based on point 1, $x^*(\overline{v}, L) > 0$. Based on point 4, $p\left(x^*(\overline{v}, L)\right) < \overline{v}$. Thus, type $(\overline{v}, L)$ has a profitable deviation to selling $x^*(\overline{v}, L) - \varepsilon$ from the firm and A from the liquid asset, where $\varepsilon \in (0, x^*(\overline{v}, L))$. Since $p(\cdot)$ is non-increasing, $p\left(x^*(\overline{v}, L) - \varepsilon\right) \geq p\left(x^*(\overline{v}, L)\right)$. Therefore, this deviation increases the payoff of type $(\overline{v}, L)$ by at least $\varepsilon\left(\overline{v} - p\left(x^*(\overline{v}, L)\right)\right) > 0$. Moreover, it increases the revenue by at least $A - q^*(\overline{v}, L) - \varepsilon p\left(x^*(\overline{v}, L)\right)$, which is always positive for $\varepsilon > 0$ sufficiently small. Therefore, it must be $q^*(\overline{v}, L) = A$.

Given the claims above, Bayes' rule implies $p_i(\overline{x}) = \overline{p}_{so}$, as given by the lemma. We prove that in any equilibrium $\overline{x} \leq \overline{x}_{so} \equiv n \min\{\frac{(L-A)/n}{\overline{p}_{so}}, 1\}$. Suppose on the contrary that $\overline{x} > \overline{x}_{so}$. Then it has to be $\overline{x}_{so} = \frac{L-A}{\overline{p}_{so}} < n$, and so $\overline{x}\overline{p}_{so} > L - A$. Since the pricing function is non-increasing, there is $\varepsilon > 0$ such that $(\overline{x} - \varepsilon)\, p\,(\overline{x} - \varepsilon) \geq \frac{L-A}{n}$. This implies that type $(\overline{v}, L)$ will strictly prefer deviating to $\overline{x} - \varepsilon$, a contradiction. We conclude $\overline{x} \leq \overline{x}_{so}$. Next, we prove that if $\frac{L-A}{n} \leq \underline{v}$ then $\overline{x} = \overline{x}_{so}$ in any equilibrium. Suppose on the contrary that $\overline{x} < \overline{x}_{so}$. Then type $(\overline{v}, L)$ does not raise L in equilibrium by selling $\overline{x}$ units of the firm. Consider a deviation to selling all units. Since $p(n) \geq \underline{v}$, the revenue raised would be at least $n\underline{v} \geq L - A$, and so the deviation is optimal, a contradiction. Suppose $\frac{L-A}{n} > \underline{v}$. In this case, any $\frac{n\underline{v}}{\overline{p}_{so}} \leq \overline{x} \leq \overline{x}_{so}$ can be an equilibrium. Indeed, we can show that $\overline{x} < \frac{n\underline{v}}{\overline{p}_{so}}$ cannot occur in an equilibrium with $\frac{L-A}{n} > \underline{v}$ by noting that there is an optimal deviation to selling n units. Among all $\frac{n\underline{v}}{\overline{p}_{so}} \leq \overline{x} \leq \overline{x}_{so}$, we select the equilibrium in which the investor's liquidity need is satisfied, which is $\overline{x}_{so}$.

Next, we argue $q^*(\underline{v}, L) = A$. Suppose not. Recall $q^*(\overline{v}, L) = A$ and $x^*(\underline{v}, L) = x^*(\overline{v}, L) = \overline{x}_{so}$. Note that type $(\overline{v}, L)$ never raises revenue strictly higher than L. Therefore, $\overline{x}_{so}\overline{p}_{so} + A \leq L$.

Since $q^*(\underline{v}, L) < A$, type $(\underline{v}, L)$ raises revenue strictly smaller than L as well. Therefore, he has an optimal deviation to $q^*(\underline{v}, L) = A$, which does not affect her profit on selling the firm, but strictly increases her revenue.

Next, since $x^*(\overline{v}, 0) = 0$, the pricing function given by Equation (4) is consistent with Equation (3) and is non-increasing. Note that Equation (3) is incentive compatible given Equation (4). First, the equilibrium payoff of type $(\overline{v}, 0)$ is $\overline{v}$, the highest possible. Second, since $\overline{p}_{so}\overline{x}_{so} \leq L - A$ and $p^*(x)$ is flat on $(0, \overline{x}_{so}]$, deviating to $(0, \overline{x}_{so}]$ generates revenue strictly lower than $L - A$, and so is suboptimal if $\theta = L$. Moreover, since $x > \overline{x}_{so} \Rightarrow p^*(x) = \underline{v}$, the investor has no optimal deviation to $x > \overline{x}_{so}$, regardless of the firm's value. Last, it is easy to see that $x = \overline{x}_{so}$ is optimal for type $(\underline{v}, 0)$.

Finally, the arguments above imply that the effect of $A > 0$ on the equilibrium strategies (quantities x^* and prices p^*) can be summarized by the substitution of L with $L - A$. ■

Proof of Lemma 2. We let $q(\theta) \in [0, A]$ be the amount of liquid assets the investor sells given θ. Suppose $\frac{L-A}{n} \leq \underline{v}(1-\tau)$. The investor can raise at least L by selling only bad firms and liquid assets, even if she receives the lowest possible price of $\underline{v}$ for bad firms. Since the investor is never forced to sell a good firm, she sells a positive amount $x_i' > 0$ from a good firm only if $p(x_i') = \overline{v}$—that is, she does not sell x_i' from a bad firm. We first argue that, in any equilibrium, $x_i > 0 \Rightarrow p(x_i) < \overline{v}$. Suppose on the contrary there is $x_i' > 0$ s.t. $p(x_i') = \overline{v}$, and let x_i' be the highest quantity with this property. The investor chooses not to sell x_i' from a bad firm only if there is x_i'' that she chooses with strictly positive probability, where

$$x_i'' p_i(x_i'') + (1 - x_i'')\underline{v} \geq x_i' p_i(x_i') + (1 - x_i')\underline{v}. \tag{19}$$

The above inequality requires $p_i(x_i'') > \underline{v}$. Since she sells x_i'' from a bad firm with positive probability, we have $p_i(x_i'') < \overline{v}$. Given this price, she will never sell x_i'' from a good firm, contradicting $p_i(x_i'') > \underline{v}$. Therefore, she sells x_i' from a bad firm with strictly positive probability, contradicting $p(x_i') = \overline{v}$. We conclude that in any equilibrium $x_i > 0 \Rightarrow p(x_i) < \overline{v}$, and so $v_i = \overline{v} \Rightarrow x_i = 0$. Note that the condition on $\underline{x}(\theta) \in [\frac{\max\{\theta - q(\theta), 0\}/n}{\underline{v}(1-\tau)}, 1]$ simply requires that the investor sells enough of the bad firms (and the liquid assets) to meet her liquidity need, given by the realization of θ. Last, $p^*(0)$ follows from Bayes' rule and the observation that

$v_i = \overline{v} \Rightarrow x_i = 0$. This completes part (i).

Next, suppose $\frac{L-A}{n} > \underline{v}\,(1-\tau)$. We proceed by proving the following claims.

1. In any equilibrium there is a unique $\overline{x} > 0$ s.t. $x_i^*(\overline{v}, L) = x_i^*(\underline{v}, 0) = \overline{x}$. To prove this, let $\overline{x} = x_i^*(\overline{v}, L)$. Since $\frac{L-A}{n} > \underline{v}\,(1-\tau)$, the investor must sell some good firms to meet her liquidity need, and so $\overline{x} > 0$. We denote $p_i\,(\overline{x}) = \overline{p}$. Since the investor sells $\overline{x}$ of a good firm, $\overline{p} > \underline{v}$. We argue that, in any equilibrium, if $\theta = 0$, then she sells $\overline{x}$ of every bad firm. Suppose not. Recall that $p_i\,(x_i^*(\overline{v}, 0)) = \overline{v}$ implies that she does not sell $x_i^*(\overline{v}, 0)$ of a bad firm in equilibrium (indeed, if $\theta = L$, the investor will never sell a good firm for a price lower than $\overline{v}$). Since $x_i^*(\underline{v}, 0) \neq \overline{x}$ and $x_i^*(\underline{v}, 0) \neq x_i^*(\overline{v}, 0)$, we must have $p_i\,(x_i^*(\underline{v}, 0)) = \underline{v}$, which yields a payoff of $\underline{v}$. This creates a contradiction since the investor has strict incentives to deviate and sell $\overline{x} > 0$ of a bad firm, thereby obtaining a payoff strictly above $\underline{v}$. Note that this implies that $\overline{p} < \overline{v}$.

2. In any equilibrium the investor sells all liquid assets if $\theta = L$, that is, $q\,(L) = A$. Suppose not. Since $\overline{x} > 0$ and $\overline{p} < \overline{v}$, the investor has a profitable deviation to selling $\varepsilon > 0$ less of the good firms and instead selling $\varepsilon \overline{xp}$ more of the liquid asset. This deviation yields exactly the same revenue to the investor, but increases her profit by $\varepsilon \overline{x}\,(\overline{v} - \overline{p}) > 0$. Since the investor did not sell all liquid assets in the first place, there exists $\varepsilon > 0$ sufficiently small as required.

3. In any equilibrium, either $x_i^*(\underline{v}, L) = \overline{x}$ or $x_i^*(\underline{v}, L) = 1$, where $\overline{x}$ is defined as in claim 1. To prove this, note that if $x_i^*(\overline{v}, 0) > 0$ then the investor cannot sell $x_i^*(\overline{v}, 0)$ of a bad firm in equilibrium. Therefore, if $x_i^*(\underline{v}, L) \neq \overline{x}$, then $p_i\,(x_i^*(\underline{v}, L)) = \underline{v}$. Suppose $x_i^*(\underline{v}, L) \neq \overline{x}$ and $x_i^*(\underline{v}, L) < 1$. Then, she can always deviate to fully selling a bad firm, and not selling some good firms, keeping revenue constant. Her payoff from selling a bad firm is no lower (since she previously received $\underline{v}$ for each bad firm), but by not selling some good firms, for which she previously received $\overline{xp} + (1 - \overline{x})\,\overline{v} < \overline{v}$, she increases her payoff. Therefore, $x_i^*(\underline{v}, L) \in \{\overline{x}, 1\}$, as required.

4. If in equilibrium $x_i^*(\underline{v}, L) = 1$ and $\overline{x} < 1$, then $\frac{L-A}{n} < \underline{v}$ and $\overline{x} = \overline{x}_{co} = \frac{\underline{v} + \frac{(L-A)/n - \underline{v}}{\tau}}{\overline{p}_{co}}$, as given by Equation (7). To prove this, since $x_i^*(\underline{v}, L) = 1$ and $v_i = \overline{v} \Rightarrow x_i^* < 1$, $p_i\,(1) = \underline{v}$.

Moreover, given claims 1 and 3, and by Bayes' rule, $\overline{p}$ is given by $\overline{p}_{co}$, as given by Equation (8). Suppose $\theta = L$. Since $\overline{p}_{co} > \underline{v}$, the investor chooses $x_i^*(\underline{v}, L) = 1$ only if the revenue from selling $\overline{x}$ from all firms is strictly smaller than $L - A$ (so she cannot raise enough revenue this way) and also the revenue from selling $\overline{x}$ of all good firms and 1 from all bad firms, that is,

$$\overline{x}\overline{p}_{co} < \min\left\{(1-\tau)\underline{v} + \tau\overline{x}\overline{p}_{co}, \frac{L-A}{n}\right\} \Leftrightarrow \overline{x}\overline{p}_{co} < \min\{\underline{v}, \frac{L-A}{n}\}.$$

Intuitively, we require $\overline{x}\overline{p}_{co} < \underline{v}$, since the investor receives $\overline{x}\overline{p}_{co}$ by partially selling $\overline{x}$ of a good firm for $\overline{p}_{co}$, and $\underline{v}$ by fully selling a bad firm for $\underline{v}$. In equilibrium, she would only fully sell a bad firm if doing so raises more revenue.

We now prove that

$$(1-\tau)\underline{v} + \tau\overline{x}\overline{p}_{co} = \frac{L-A}{n}, \tag{20}$$

that is, fully selling bad firms and selling $\overline{x}$ of good firms raises exactly $L - A$. We do so in two steps. We first argue that this strategy cannot raise more than $L - A$, that is,

$$(1-\tau)\underline{v} + \tau\overline{x}\overline{p}_{co} \leq \frac{L-A}{n}. \tag{21}$$

Suppose not. Then, the investor has slack: she can deviate by selling only $\overline{x} - \varepsilon$ instead of $\overline{x}$ from each good firm, while still meeting her liquidity need. Since prices are non-increasing, $p_i(\overline{x} - \varepsilon) \geq \overline{p}_{co}$, and so for small $\varepsilon > 0$, she still raises at least $L - A$. Her payoff is strictly higher since she sells less from the good firms. We next argue that this strategy cannot raise less than $L - A$, that is,

$$(1-\tau)\underline{v} + \tau\overline{x}\overline{p}_{co} \geq \frac{L-A}{n}. \tag{22}$$

Suppose not. If the strategy did not raise $L-A$, then it must be that $\underline{v} \leq (1-\tau)\underline{v} + \tau\overline{x}\overline{p}_{co}$, that is, the alternative strategy of fully selling her entire portfolio raises even less revenue (otherwise, the investor would have deviated to that strategy). Therefore, $\underline{v} \leq \overline{x}\overline{p}_{co}$, which contradicts $\overline{x}\overline{p}_{co} < \underline{v}$ that we proved above. Intuitively, if fully selling a firm for $\underline{v}$ raises less revenue than selling $\overline{x}$ of a firm for $\overline{p}_{co}$, then the investor would not fully sell bad firms. Combining Conditions (21) and (22) yields Equation (20) as required, implying $\overline{x} = \overline{x}_{co}$, and $\overline{x}_{co}\overline{p}_{co} < \underline{v}$ implies $\frac{L-A}{n} < \underline{v}$ as required.

5. If in equilibrium $x_i^*(\underline{v}, L) = \overline{x}$ then $\frac{L-A}{n} \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, $\overline{p} = \overline{p}_{so}$ and $\overline{x} = \overline{x}_{so}/n$. To prove this, since prices are non-increasing, we must have $\overline{xp} \leq \frac{L-A}{n}$. Otherwise, if $\theta = L$, the investor deviates by selling $\overline{x} - \varepsilon$ instead of $\overline{x}$ from a good firm. For small $\varepsilon > 0$, she can raise enough revenue but sell less from the good firms. Note that $x_i^*(\underline{v}, L) = \overline{x} \Rightarrow \overline{p} = \overline{p}_{so}$. Suppose on the contrary that $\frac{L-A}{n} < \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$.We argue that there is an optimal deviation to fully selling all bad firms, and selling x' from good firms, for some $x' \in (0, \overline{x}]$. Since $\frac{L-A}{n\underline{v}} < \frac{1-\tau}{\beta\tau+1-\tau} < 1$, the investor can always raise at least $L - A$ by selling all firms. Therefore, it must be that if the investor sells $\overline{x}$ from each firm, she meets her liquidity need, that is, $\overline{xp}_{so} = \frac{L-A}{n}$. Moreover, $\overline{p}_{so} > \underline{v} \Rightarrow \overline{x} < 1$. Since $\overline{x}$ is an equilibrium, $xp\,(x) < \frac{L-A}{n}$ for any $x < \overline{x}$. Let

$$x' = \frac{\frac{L-A}{n} - (1-\tau)\,\underline{v}}{\tau\overline{p}_{so}}$$

Note that $\frac{L-A}{n} - (1-\tau)\,\underline{v} > 0$ implies $x' > 0$ and $\overline{xp}_{so} = \frac{L-A}{n} < \underline{v}$ implies $x' < \overline{x}$. By deviating to fully selling all bad firms and selling only $x' \leq \overline{x}$ from all good firms, the revenue raised is at least $L - A$. This deviation generates a higher payoff if and only if

$$x'\tau\overline{p}_{so} + (1-x')\,\tau\overline{v} + (1-\tau)\,\underline{v} > \overline{xp}_{so} + (1-\overline{x})\left(\tau\overline{v} + (1-\tau)\,\underline{v}\right).$$

Using $\overline{xp}_{so} = \frac{L-A}{n}$, $x' = \frac{(L-A)/n-(1-\tau)\underline{v}}{\tau\overline{p}_{so}}$, and $\overline{p}_{so} = \underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}$, we obtain $\frac{L-A}{n} < \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, which implies that this deviation is optimal, a contradiction. We conclude that $\frac{L-A}{n} \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$ as required. Intuitively, if the shock were smaller, the investor would retain more of good firms. For the same reasons as in the benchmark, $\overline{x} = \overline{x}_{so}/n$.

Consider part (ii). We show that, if $\underline{v}\,(1-\tau) < \frac{L-A}{n} < \underline{v}$, the specified equilibrium exists. First, note that $\frac{L-A}{n} < \underline{v} \Rightarrow \overline{x}_{co} < 1$. Second, note that the prices in Equation (8) are consistent with the trading strategy given by Equation (7). Moreover, the pricing function in Equation (8) is non-increasing. Third, we show that given the pricing function in Equation (8), the investor's trading strategy in Equation (7) is indeed optimal. Suppose $\theta = 0$. Given Equation (8), the investor's optimal response is $v_i = \overline{v} \Rightarrow x_i = 0$ and $v_i = \underline{v} \Rightarrow x_i = \overline{x}_{co}$, as prescribed by Equation (7). Suppose $\theta = L$. Given Equation (8), the investor's most profitable deviation involves selling $\overline{x}_{co}$ from each bad firm, selling all liquid assets, and selling the least amount of a good firm, such that she raises at least L in total. However, recall that by the construction of $\overline{x}_{co}$, $(1-\tau)\underline{v} + \tau\overline{x}_{co}\overline{p}_{co} = \frac{L-A}{n}$. Also note that $\frac{L-A}{n} < \underline{v} \Rightarrow \overline{x}_{co}\overline{p}_{co} < \frac{L-A}{n}$. Therefore, the most profitable deviation generates revenue strictly lower than $L-A$, and hence is suboptimal. This concludes part (ii).

Consider part (iii). We show that, if $\frac{L-A}{n} \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, the specified equilibrium exists. The proof is as described by Lemma 1, where $\overline{x}_{so}$ is replaced by $\overline{x}_{so}/n$. The only addition is that we note that as per the proof of claim 5, $\frac{L-A}{n} \geq \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$ guarantees that, if $\theta = L$, the investor has no profitable deviation. The proof that the investor has no profitable deviation when $\theta = 0$ is the same as in the proof of part (ii) above.

Part (iv) follows from claims 1–5.

Finally, the arguments above imply that the effect of $A > 0$ on the equilibrium strategies (quantities x^* and prices p^*) can be summarized by the substitution of L with $L - A$. ■

Proof of Proposition 1. From Lemma 1

$$P_{so}(v_i) = \begin{cases} \overline{p}_{so} & \text{if } v_i = \underline{v} \\ \beta\overline{p}_{so} + (1-\beta)\overline{v} & \text{if } v_i = \overline{v}. \end{cases} \tag{23}$$

Let $\gamma = (1-\beta)\cdot \mathbf{1}_{\underline{x}(0)=0}$. Then from Lemma 2:

$$P_{co}(\underline{v}) = \begin{cases} \gamma[\underline{v} + \Delta\frac{\tau}{\tau+\gamma(1-\tau)}] + (1-\gamma)\,\underline{v} & \textit{if } \ L/n \leq \underline{v}\,(1-\tau) \\ \{\beta\underline{v} + (1-\beta)\,\overline{p}_{co}, P_{so}(\underline{v})\} & \textit{if } \ \underline{v}\,(1-\tau) < L/n < \underline{v} \\ P_{so}(\underline{v}) & \textit{if } \ L/n \geq \underline{v} \end{cases}$$

$$P_{co}(\overline{v}) = \begin{cases} p_i^*(0) & \textit{if } \ L/n \leq \underline{v}\,(1-\tau) \\ \{\beta\overline{p}_{co} + (1-\beta)\,\overline{v}, P_{so}(\overline{v})\} & \textit{if } \ \underline{v}\,(1-\tau) < L/n < \underline{v} \\ P_{so}(\overline{v}) & \textit{if } \ L/n \geq \underline{v}, \end{cases}$$

and the curly brackets encompass the two possible equilibria (types (ii) and (iii)) that can exist when $\underline{v}\,(1-\tau) < L/n < \underline{v}$.

To prove Condition (9), first suppose $L/n > \underline{v}\,(1-\tau)$. It is sufficient to note that

$$\beta\underline{v} + (1-\beta)\,\overline{p}_{co} < \overline{p}_{so}$$

and

$$\beta\overline{p}_{co} + (1-\beta)\,\overline{v} > \beta\overline{p}_{so} + (1-\beta)\,\overline{v}.$$

The latter holds given that $\overline{p}_{co} > \overline{p}_{so}$, and the former holds given that

$$\begin{gathered} \beta\underline{v} + (1-\beta)\,\overline{p}_{co} < \overline{p}_{so} \Leftrightarrow \\ \frac{1-\beta}{\beta\tau + (1-\beta)(1-\tau)} < \frac{1}{\beta\tau + 1 - \tau} \Leftrightarrow \\ (1-\beta)\beta\tau < \beta\tau, \end{gathered}$$

which holds.

Next, suppose $L/n \leq \underline{v}\,(1-\tau)$. Note that

$$\begin{aligned} \overline{p}_{so} &\geq \gamma\left[\underline{v} + \Delta\frac{\tau}{\tau+\gamma\,(1-\tau)}\right] + (1-\gamma)\,\underline{v} \Leftrightarrow \gamma \leq \frac{\beta\tau}{\beta\tau + (1-\beta)\,(1-\tau)} \\ \beta\overline{p}_{so} + (1-\beta)\,\overline{v} &< \underline{v} + \Delta\frac{\tau}{\tau+\gamma\,(1-\tau)} \Leftrightarrow \gamma < \frac{\beta\tau}{\beta\tau + (1-\beta)\,(1-\tau)}. \end{aligned}$$

Therefore, this condition holds if $\underline{x}(0) > 0$ or, $\underline{x}(0) = 0$ and $1 - \beta < \frac{\beta\tau}{\beta\tau + (1-\beta)(1-\tau)} \Leftrightarrow \beta > \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$, as required. ■

Appendix B. Proofs of Governance Through Exit

Here, we state and prove an auxiliary lemma used in the proofs of Lemma 3, Lemma 4, and Proposition 2.

Lemma 7 (Threshold, exit): In any equilibrium and under any ownership structure, there is a $c^* > 0$ such that manager i chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^*$. A higher c^* increases total surplus.

Proof. Suppose in equilibrium, under ownership structure $\chi \in \{so, co\}$, the market maker believes that the manager works w.p. τ^*_χ. From Equation (11), if the manager works, his expected utility is $(1-\omega)\overline{v} + \omega P_\chi\left(\overline{v}, \tau^*_\chi\right) - \widetilde{c}_i$, and if he shirks it is $(1-\omega)\underline{v} + \omega P_\chi\left(\underline{v}, \tau^*_\chi\right)$. Therefore, he works if and only if $\widetilde{c}_i \leq c^* \equiv (1-\omega)\Delta + \omega\left[P_\chi\left(\overline{v}, \tau^*_\chi\right) - P_\chi\left(\underline{v}, \tau^*_\chi\right)\right]$. (We now refer to prices and trading quantities as functions of τ^*_χ to make explicit their dependence on the anticipated working probability.)

To show that a higher c^* increases total surplus, note that manager i works only if his weight on the value gain $(1-\omega)\Delta$ plus ω times the expected price rise exceeds his cost. The maximum price rise is Δ, which arises if the price is fully informative. Thus, in any equilibrium, $c^* \leq (1-\omega)\Delta + \omega\Delta = \Delta$. Ex ante total surplus (firm value minus the cost of effort) in equilibrium is increasing in c^* if and only if $c^* \leq \Delta$. Since the manager always chooses $c^* \leq \Delta$, a higher c^* always increases total surplus. ■

Proof of Lemma 3. In equilibrium, the market maker and the investor believe the manager follows threshold c^*. Given Equations (3) and (4), the manager expects the price to be $P_{so}(v_i, F(c^*))$ if he chooses v_i. Therefore, he works if and only if

$$(1-\omega)\overline{v} + \omega P_{so}(\overline{v}, F(c^*)) - \widetilde{c}_i \geq (1-\omega)\underline{v} + \omega P_{so}(\underline{v}, F(c^*)),$$

where $P_{so}(v_i)$ is given in Equation (23). In equilibrium, c^* must be such that the above incentive constraint binds at $\widetilde{c}_i = c^*$, that is, $c^* = \phi_{exit}(F(c^*))$. Note that $\phi_{exit}(\tau)$ is decreasing in τ and is bounded from above and below. Therefore, a solution always exists and is unique, as required. The equilibrium is characterized by Lemma 1, where $\tau = \tau^*_{so,exit}$. ■

Proof of Lemma 4. We use "type (i) equilibrium," "type (ii) equilibrium," and "type (iii) equilibrium" to refer to the equilibria given in parts (i), (ii), and (iii) of Lemma 2, respectively. We prove that the working threshold in the most efficient equilibrium is given by

$$c^{**}_{co,exit} = \begin{cases} \Delta & \textit{if } L/n \leq \underline{v}(1 - F(\Delta)) \\ c^{**}_{i,exit} \equiv \text{the largest solution of } c^* = \psi_{exit}(F(c^*)) & \textit{if } \underline{v}(1 - F(\Delta)) < L/n \leq \underline{v}\left(1 - \tau^{**}_{ii,exit}\right) \\ c^{**}_{ii,exit} \equiv \text{the largest solution of } c^* = \zeta_{exit}(F(c^*)) & \textit{if } \underline{v}\left(1 - \tau^{**}_{ii,exit}\right) < L/n < \underline{v} \\ c^*_{so,exit} & \textit{if } L/n \geq \underline{v}, \end{cases} \tag{24}$$

where $\tau^{**}_{ii,exit} \equiv F\left(c^{**}_{ii,exit}\right)$,

$$\zeta_{exit}(\tau) = \Delta - \frac{\omega\Delta}{\frac{\tau}{1-\beta} + \frac{1-\tau}{\beta}}. \tag{25}$$

and

$$\psi_{exit}(\tau) = \Delta - \frac{\omega\Delta}{\frac{\tau}{1-\beta} + 1 - \tau}. \tag{26}$$

If true, $\underline{v}(1 - F(\Delta)) < L/n$ implies $c^{**}_{co,exit} < \Delta$ as required. The value c^{**}_{exit} in Lemma 4 refers to $c^{**}_{i,exit}$, $c^{**}_{ii,exit}$, or $c^*_{so,exit}$ depending on the level of L/n.

First, suppose $L/n \geq \underline{v}$. Based on Lemma 2, any equilibrium is type (iii). Therefore, $c^{**}_{co,exit} = c^*_{so,exit}$. Similar to the proofs of Lemma 2 part (iii) and Lemma 3, such an equilibrium exists.

Second, suppose $\underline{v}\left(1 - \tau^{**}_{ii,exit}\right) \leq L/n < \underline{v}$. Consider a type (ii) equilibrium. The manager works if and only if

$$(1-\omega)\overline{v} + \omega\left[\beta\overline{p}_{co}(\tau^*) + (1-\beta)\overline{v}\right] - \widetilde{c}_i \geq (1-\omega)\underline{v} + \omega\left[\beta\underline{v} + (1-\beta)\overline{p}_{co}(\tau^*)\right].$$

Using $\overline{p}_{co}(\tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$, we obtain $v_i = \overline{v} \Leftrightarrow \widetilde{c}_i \leq \zeta_{exit}(\tau^*)$. Therefore, c^* must solve $c^* = \zeta_{exit}(F(c^*))$. Similar to the proof of Lemma 2, part (ii), if $\tau = \tau^{**}_{ii,exit}$ an equilibrium with these properties exists. By definition of $c^{**}_{ii,exit}$, such an equilibrium is more efficient than any other type (ii) equilibrium. Moreover, simple algebra shows that $\zeta_{exit}(\tau) > \phi_{exit}(\tau)$, and so $c^{**}_{ii,exit} > c^*_{so,exit}$. That is, an equilibrium with $\tau = \tau^{**}_{ii,exit}$ is more efficient than any type (iii) equilibrium. Finally, we show that an equilibrium with $\tau = \tau^{**}_{ii,exit}$ is more efficient than any type (i) equilibrium. Based on part (i) of Lemma 2, the threshold of the alternative equilibrium must satisfy $L/n \leq \underline{v}(1-\tau^*)$. However, since by assumption $L/n > \underline{v}\left(1-\tau^{**}_{ii,exit}\right)$, it follows that $\tau^* < \tau^{**}_{ii,exit}$, that is, the alternative equilibrium is less efficient.

Third, suppose $L/n < \underline{v}\left(1-\tau^{**}_{ii,exit}\right)$. Note that type (ii) or type (iii) equilibria do not exist. Indeed, by the definition of $\tau^{**}_{ii,exit}$, any other type (ii) equilibrium satisfies $\tau^* \leq \tau^{**}_{ii,exit}$, and since $L/n < \underline{v}\left(1-\tau^{**}_{ii,exit}\right)$, it also satisfies $L/n < \underline{v}(1-\tau^*)$, which contradicts it being a type (ii) equilibrium. Any type (iii) equilibrium must have $\tau^* = F\left(c^*_{so,exit}\right)$. Since $c^*_{so,exit} < c^{**}_{ii,exit}$ and $L/n < \underline{v}\left(1-\tau^{**}_{ii,exit}\right)$, it must be $L/n < \underline{v}(1-\tau^*)$, which contradicts it being a type (iii) equilibrium. Next, we argue that a type (i) equilibrium exists. Recall that there are only two kinds of type (i) equilibrium: those in which the investor sells bad firms if $\theta = 0$ and those in which she retains them. We argue that the following strategies are an equilibrium: the manager's working threshold is

$$c^{**} = \begin{cases} \Delta & \text{if } L/n \leq \underline{v}(1-F(\Delta)) \\ c^{**}_{i,exit} \equiv \text{the largest solution of } c^* = \psi_{exit}(F(c^*)) & \text{if } \underline{v}(1-F(\Delta)) < L/n \leq \underline{v}\left(1-\tau^{**}_{ii,exit}\right), \end{cases}$$

the investor's trading strategy is

$$x^*(v_i,\theta) = \begin{cases} 0 & \text{if } v_i = \overline{v} \\ \eta^* & \text{if } v_i = \underline{v} \text{ and } \theta = 0 \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases}$$

where

$$\eta^* = \begin{cases} 1 & \text{if } L/n \leq \underline{v}\,(1 - F(\Delta)) \\ 0 & \text{if } \underline{v}\,(1 - F(\Delta)) < L/n < \underline{v}\left(1 - \tau^{**}_{ii,exit}\right) \end{cases},$$

and prices are

$$p^*_i(x_i) = \begin{cases} \underline{v} + \Delta \frac{F(c^{**})}{F(c^{**}) + (1-\beta)(1-\eta^*)(1-F(c^{**}))} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases}$$

To see that the above equilibrium exists, first note that the prices in this equilibrium follow from the investor's trading strategy and Bayes' rule. Second, given these prices, the investor's trading strategy is optimal. Indeed, note that $L/n \leq \underline{v}\,(1 - F(c^{**}))$, and so the investor can satisfy her liquidity need by selling only bad firms. Since $x_i > 0 \Rightarrow p^*_i = \underline{v}$, she has strict incentives to fully retain good firms, and weak incentives to sell bad firms. The manager works if and only if

$$\begin{aligned} (1-\omega)\,\overline{v} + \omega p^*_i(0) - \widetilde{c}_i &\geq (1-\omega)\,\underline{v} + \omega\left[\beta\underline{v} + (1-\beta)\left((1-\eta^*)p^*_i(0) + \eta^*\underline{v}\right)\right] \Leftrightarrow \\ (1-\omega)\,\Delta + \omega\,(1 - (1-\beta)(1-\eta^*))\,(p^*_i(0) - \underline{v}) &\geq \widetilde{c}_i. \end{aligned}$$

Using the explicit form of $p^*_i(0)$ as given above, this yields

$$\begin{aligned} (1-\omega)\,\Delta + \omega\,(1 - (1-\beta)(1-\eta^*))\,\Delta \frac{F(c^{**})}{F(c^{**}) + (1-\beta)(1-\eta^*)(1-F(c^{**}))} &\geq \widetilde{c}_i \Leftrightarrow \\ H(\eta^*, c^{**}) &\geq \widetilde{c} \end{aligned}$$

where

$$H(\eta, c) = \Delta - \frac{\omega\Delta}{\frac{F(c)}{1-\beta}\frac{1}{1-\eta} + 1 - F(c)}.$$

Existence follows if there is $\eta^* \in \{0, 1\}$ such that $H(\eta^*, c^{**}) = c^{**}$ and $L/n \leq \underline{v}\,(1 - F(c^{**}))$. Note that $\eta^* = 1$ requires $c^{**} = \Delta$. Therefore, a type (i) equilibrium with $\eta^* = 1$ exists if and only if $L/n \leq \underline{v}\,(1 - F(\Delta))$. Since this equilibrium attains the first best, it is the most efficient one. Suppose $\underline{v}\,(1 - F(\Delta)) < L/n \leq \underline{v}\left(1 - \tau^{**}_{ii,exit}\right)$. The only candidate equilibrium is a type (i) equilibrium with $\eta^* = 0$, and so it must be the most efficient equilibrium. The working threshold must solve $H(0, c^{**}) = c^{**}$; a solution always exists. Also note that $H(0, c^{**}) =$

$\psi_{exit}\left(F\left(c^{**}\right)\right) < \zeta_{exit}\left(\tau\right)$ for any $\tau \in (0,1)$, and so $c^{**}_{i,exit} < c^{**}_{ii,exit}$. Since $L/n < \underline{v}\left(1 - \tau^{**}_{ii,exit}\right)$, it must be $L/n < \underline{v}\left(1 - \tau^{**}_{i,exit}\right)$, which shows that a type (i) equilibrium exists. ■

Proof of Proposition 2. According to Lemma 3, the working threshold under separate ownership, $c^{*}_{so,exit}$, is given by the unique solution of $c^{*} = \phi_{exit}\left(F\left(c^{*}\right)\right)$, where $\phi_{exit}\left(\tau\right) = \Delta - \frac{\omega\Delta}{\tau + \frac{1-\tau}{\beta}}$. Therefore, $c^{*}_{so,exit} < \Delta$. In addition, according to Lemma 4, under common ownership, the working threshold in the most efficient equilibrium, $c^{**}_{co,exit}$, is given by Δ if $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$. Note that total surplus under separate and common ownership, respectively, is

$$\begin{aligned} \Omega_{so,exit}\left(c\right) &= \underline{R} + F\left(c\right)\left(\overline{R} - \underline{R}\right) - F\left(c\right)E\left[c_i | c_i < c\right] \text{ and} \\ \Omega_{co,exit}\left(n, c\right) &= n\Omega_{so,exit}\left(c\right) \end{aligned}$$

Therefore, if $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$ then

$$\begin{aligned} \Omega_{co,exit}\left(n, \Delta\right) &= n\Omega_{so,exit}\left(\Delta\right) \\ &> n\Omega_{so,exit}\left(c^{*}_{so,exit}\right) \\ &> \Omega_{so,exit}\left(c^{*}_{so,exit}\right). \end{aligned}$$

The proof trivially follows from these two results. ■

Appendix C. Proofs of Governance Through Voice

Here, we state and prove an auxiliary lemma used in the proofs of Lemma 5, Lemma 6, Proposition 3, and Proposition 4.

Lemma 8 (Threshold, voice): In any equilibrium and under any ownership structure, there is a $c^{*} > 0$ such that the investor chooses $a_i = 1$ if and only if $\widetilde{c}_i \leq c^{*}$. A higher c^{*} increases total surplus.

Proof. Consider separate ownership and suppose the market maker anticipates monitoring probability τ^{*}. Regardless of her monitoring decision, the investor still faces prices as given by

Equation (4), evaluated at $\tau = \tau^*$. Therefore, as in the proof of Lemma 1, the investor follows the trading strategy prescribed by Equation (3). She thus monitors if and only if:

$$n\underline{v} + \overline{x}_{so}(\tau^*)(\overline{p}_{so}(\tau^*) - \underline{v}) \leq n\overline{v} - \beta\overline{x}_{so}(\tau^*)(\overline{v} - \overline{p}_{so}(\tau^*)) - \widetilde{c}_i.$$

This inequality can be rearranged as

$$\widetilde{c}_i/n \leq \overline{v} - \beta\frac{\overline{x}_{so}(\tau^*)}{n}(\overline{v} - \overline{p}_{so}(\tau^*)) - \left(\underline{v} + \frac{\overline{x}_{so}(\tau^*)}{n}(\overline{p}_{so}(\tau^*) - \underline{v})\right).$$

which implies a threshold strategy.

Consider common ownership, and suppose the investor decides to monitor a mass of $n\tau$ firms. Since all firms are ex ante identical, the investor will monitor the mass of $n\tau$ firms with the lowest monitoring costs. That is, the investor will monitor firm i if and only if $\widetilde{c}_i \leq F^{-1}(\tau)$—a threshold strategy.

To show that a higher c^* increases total surplus, first note that ex ante total surplus (firm value minus the cost of monitoring) in equilibrium under separate and common ownership, respectively, is

$$\begin{aligned}
\Omega_{so,voice}(c) &= \underline{R} + F(c)\left(\overline{R} - \underline{R}\right) - F(c)E[c_i|c_i < c] \text{ and} \\
\Omega_{co,voice}(n,c) &= n\Omega_{so,voice}(c).
\end{aligned}$$

The only difference is that, under common ownership, governance is exerted on n firms. It is immediate to see that both terms increase in c^* if and only if $c^* \leq \overline{R} - \underline{R}$. Second, given a threshold c and number of firms n, the investor's net payoffs under separate and common ownership, respectively, are

$$\begin{aligned}
\Pi_{so,voice}(n,c) &= n(\underline{v} + F(c)\Delta) - F(c)E[c_i|c_i < c] \text{ and} \\
\Pi_{co,voice}(n,c) &= n(\underline{v} + F(c)\Delta) - nF(c)E[c_i|c_i < c].
\end{aligned}$$

Note that $\Pi_{co,voice}(n,c)$ and $\Pi_{so,voice}(n,c)$ have a unique maximum at Δ and $n\Delta$ respectively, and that in any equilibrium $c^*_{co,voice} \leq \Delta$ and $c^*_{so,voice} < n\Delta$. Since $1 < n \leq m$ and $m\Delta \leq \overline{R} - \underline{R}$, given the ownership structure, a higher c^* in equilibrium necessarily implies a higher surplus, even net of the investor's monitoring costs. ■

Proof of Lemma 5. The proof of Lemma 8 shows that, under separate ownership, the investor monitors if and only if

$$\widetilde{c}_i/n \leq \overline{v} - \beta\frac{\overline{x}_{so}(\tau^*)}{n}\left(\overline{v} - \overline{p}_{so}(\tau^*)\right) - \underline{v} - \frac{\overline{x}_{so}(\tau^*)}{n}\left(\overline{p}_{so}(\tau^*) - \underline{v}\right). \tag{27}$$

This holds if and only if

$$\widetilde{c}_i/n \leq \Delta\left[1 - \beta\frac{\overline{x}_{so}(\tau^*)}{n}\frac{1}{\beta\tau^* + 1 - \tau^*}\right] \Leftrightarrow \widetilde{c}_i/n \leq \phi_{voice}(\tau^*).$$

Thus, the cutoff in any equilibrium must satisfy $c^*/n = \phi_{voice}(\tau^*)$. In equilibrium, $\tau^* = F(c^*)$, and so $c^*_{so,voice}$ must solve $c^*/n = \phi_{voice}(F(c^*))$, as required. Note that, as a function of c^*, $\phi_{voice}(F(c^*))$ is strictly positive and bounded from above. Therefore, a strictly positive solution always exists. Since $V_{so}(v_i, \tau)$ is derived from Lemma 1, the equilibrium is characterized by Lemma 1, where τ is given by $\tau^*_{so,voice}$. ■

Proof of Lemma 6. Suppose $L/n \leq \underline{v}(1 - F(\Delta))$. Recall that in any equilibrium under common ownership, $c^* \leq \Delta$. Therefore, $L/n \leq \underline{v}(1 - F(\Delta))$ implies $L/n \leq \underline{v}(1 - \tau^*)$. From Lemma 2, this implies that if an equilibrium exists, it must be type (i). For any cutoff c (which could differ from the cutoff c^* anticipated by the market makers), she expects to sell $x^*(c) \in [0, 1]$ of a good firm upon a shock, where

$$x^*(c) = \min\left\{1, \max\left\{0, \frac{L/n - \underline{v}(1 - F(c))}{\underline{v}F(c)}\right\}\right\} \tag{28}$$

is increasing in c. Indeed, she will sell good firms only if she cannot satisfy her liquidity need by selling all $(1 - F(c))$ bad firms. Since $L/n \leq \underline{v}(1 - \tau^*) < \underline{v}$, we can rewrite

$$x^*(c) = \max\left\{0, 1 - \frac{1 - \frac{L/n}{\underline{v}}}{F(c)}\right\}.$$

Let $\Pi(c^*, c)$ be the investor's expected payoff, including the possibility of trade, given that the market makers set prices under the belief there are $F(c^*)$ monitored firms, and the investor monitors a fraction $F(c)$ of all firms, where we allow for $c \neq c^*$. Therefore,

$$\Pi(c^*, c)/n = F(c)(\overline{v} - x^*(c)\beta\Delta) + (1 - F(c))\underline{v} - F(c)E[c_i|c_i < c] \tag{29}$$

$$= \underline{v} + \begin{cases} F(c)(\Delta - E[c_i|c_i < c]) & \textit{if } c \leq F^{-1}(1 - \frac{L/n}{\underline{v}}) \\ \beta\Delta(1 - \frac{L/n}{\underline{v}}) + F(c)(\Delta(1-\beta) - E[c_i|c_i < c]) & \textit{otherwise.} \end{cases} \tag{30}$$

Since $L/n \leq \underline{v}(1 - F(\Delta)) \Rightarrow \Delta \leq F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$, and it is always suboptimal to choose $c > \Delta$ regardless of market makers' beliefs, the investor's optimal threshold solves $c^* \in \arg\max_{c\in[0,\Delta]} F(c)(\Delta - E[c_i|c_i < c])$. The first derivative with respect to c is $f(c)(\Delta - c)$ and the second derivative is $f'(c)(\Delta - c) - f(c)$. Therefore, if an equilibrium exists, it must entail $c^* = \Delta$. Since $L/n \leq \underline{v}(1 - F(\Delta))$, by construction, there is a type (i) equilibrium with such a threshold. Lemma 9 in Online Appendix E.2 shows that an equilibrium always exists when $L/n > \underline{v}(1 - F(\Delta))$, and that in those cases the monitoring threshold is smaller than Δ. ■

Proof of Proposition 3. First note that, according to Lemma 5, the monitoring threshold under separate ownership must solve $c^* = n\phi_{voice}(F(c^*))$. Since $\phi_{voice}(F(c^*)) < \Delta$, there is $\underline{n}(L) > 1$ such that the largest solution of $c^* = n\phi_{voice}(F(c^*))$, denoted by $\overline{c}^*_{so}(n, L)$, is strictly smaller than Δ if $n < \underline{n}(L)$. Note that $\underline{n}(L)$ satisfies $\overline{c}^*_{so}(\underline{n}(L), L) = \Delta$. Second, Lemma 6 shows that if $L/n \leq \underline{v}(1 - F(\Delta))$, the monitoring threshold is Δ in any equilibrium under common ownership, and an equilibrium always exists. Therefore,

$$\begin{aligned} \Omega_{co,voice}(n, \Delta) &= n\Omega_{so,voice}(\Delta) \\ &> n\Omega_{so,voice}\left(c^*_{so,voice}\right) \\ &> \Omega_{so,voice}\left(c^*_{so,voice}\right). \end{aligned}$$

We conclude that, if $L \leq \underline{v}(1 - F(\Delta))$, there is $\underline{n}(L) > 1$ such that if $1 < n < \underline{n}(L)$, then any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership. ■

Proof of Proposition 4. According to Proposition 3, if $0 < L < \underline{v}(1 - F(\Delta))$ and $1 \leq n < \underline{n}(L)$, then in any equilibrium $c^*_{so,voice}(n) < \Delta = c^*_{co,voice}$. Since

$$\begin{aligned}\Pi_{so,voice}\left(1, c^*_{so,voice}(1)\right) &< \Pi_{so,voice}(1, \Delta) \\ &= \Pi_{co,voice}(1, \Delta),\end{aligned}$$

from continuity of $\Pi_{so,voice}\left(n, c^*_{so,voice}(n)\right)$ and $\Pi_{co,voice}(n, \Delta)$ in n, for all $0 < L < \underline{v}(1 - F(\Delta))$ there is $1 < \underline{\underline{n}}(L) \leq \underline{n}(L)$ such that if $1 < n < \underline{\underline{n}}(L)$, then $\Pi_{so,voice}\left(n, c^*_{so,voice}(n)\right) < \Pi_{co,voice}(n, \Delta)$, as required. ■

Appendix D. Proofs of Extensions

Proof of Proposition 5. We define $\hat{\tau}$ as the probability that the diversified investor receives signal $\overline{v}$, and $\overline{e}$ ($\underline{e}$) as the expected firm value upon signal $\overline{v}$ ($\underline{v}$), that is,

$$\begin{aligned}\hat{\tau} &\equiv \Pr[y_i = \overline{v}] = \tau\rho + (1-\tau)(1-\rho) \\ \overline{e} &\equiv E[v_i | y_i = \overline{v}] = \underline{v} + \Delta\tau\frac{\rho}{\tau\rho + (1-\tau)(1-\rho)} \\ \underline{e} &\equiv E[v_i | y_i = \underline{v}] = \underline{v} + \Delta\tau\frac{1-\rho}{\tau(1-\rho) + (1-\tau)\rho}.\end{aligned}$$

Define $\hat{\Delta} \equiv \overline{e} - \underline{e} > 0$ as the difference in expected firm value when the signal increases from $\underline{e}$ to $\overline{e}$. Lemma 2 continues to hold if $(\overline{v}, \underline{v}, \tau, \Delta)$ are replaced by $(\overline{e}, \underline{e}, \hat{\tau}, \hat{\Delta})$ everywhere. Therefore, the properties of the equilibrium remain generically the same when the investor has imperfect information under common ownership.

From Lemma 1,

$$P_{so}(v_i) = \begin{cases} \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau} & \text{if } v_i = \underline{v} \\ \beta(\underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}) + (1-\beta)\overline{v} & \text{if } v_i = \overline{v}. \end{cases}$$

Let $\gamma = (1-\beta)\cdot \mathbf{1}_{\underline{x}(0)=0}$. From Lemma 2, the expected price in equilibrium conditional on e_i is

$$P_{co}(\underline{e}) = \begin{cases} \gamma[\underline{e} + \hat{\Delta}\frac{\hat{\tau}}{\hat{\tau}+\gamma(1-\hat{\tau})}] + (1-\gamma)\,\underline{e} & \text{if } L/n \leq \underline{e}\,(1-\hat{\tau}) \\ \{\beta\underline{e} + (1-\beta)\,(\underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+(1-\beta)(1-\hat{\tau})}), \underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}\} & \text{if } \underline{e}\,(1-\hat{\tau}) < L/n < \underline{e} \\ \underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}} & \text{if } L/n \geq \underline{e} \end{cases}$$

$$P_{co}(\overline{e}) = \begin{cases} p_i^*(0) = \underline{e} + \hat{\Delta}\frac{\hat{\tau}}{\hat{\tau}+\gamma(1-\hat{\tau})} & \text{if } L/n \leq \underline{e}\,(1-\hat{\tau}) \\ \{\beta(\underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+(1-\beta)(1-\hat{\tau})}) + (1-\beta)\,\overline{e}, \beta(\underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}) + (1-\beta)\,\overline{e}\} & \text{if } \underline{e}\,(1-\hat{\tau}) < L/n < \underline{e} \\ \beta(\underline{e} + \hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}) + (1-\beta)\,\overline{e} & \text{if } L/n \geq \underline{e} \end{cases}$$

and the curly brackets encompass the two possible equilibria (types (ii) and (iii)) that can exist when $\underline{e}\,(1-\hat{\tau}) < L/n < \underline{e}$. Note that the expected price in equilibrium conditional on v_i is

$$\begin{cases} (1-\rho)\,P_{co}(\overline{e}) + \rho P_{co}(\underline{e}) & \text{if } v_i = \underline{v} \\ \rho P_{co}(\overline{e}) + (1-\rho)\,P_{co}(\underline{e}) & \text{if } v_i = \overline{v}. \end{cases}$$

Price informativeness is higher under common ownership if and only if

$$\rho P_{co}(\overline{e}) + (1-\rho)\,P_{co}(\underline{e}) > P_{so}(\overline{v}) \text{ and } (1-\rho)\,P_{co}(\overline{e}) + \rho P_{co}(\underline{e}) < P_{so}(\underline{v}).$$

However, since

$$\begin{aligned} \tau P_{so}(\overline{v}) + (1-\tau)\,P_{so}(\underline{v}) &= \underline{v} + \tau\Delta \\ &= \hat{\tau} P_{co}(\overline{e}) + (1-\hat{\tau})\,P_{co}(\underline{e}) \end{aligned}$$

and

$$\hat{\tau} P_{co}(\overline{e}) + (1-\hat{\tau})\,P_{co}(\underline{e}) = \tau\left[\rho P_{co}(\overline{e}) + (1-\rho)\,P_{co}(\underline{e})\right] + (1-\tau)\left[(1-\rho)\,P_{co}(\overline{e}) + \rho P_{co}(\underline{e})\right],$$

we have

$$\begin{aligned}(1-\rho)\,P_{co}\left(\overline{e}\right)+\rho P_{co}\left(\underline{e}\right) &< P_{so}\left(\underline{v}\right)\Leftrightarrow\\ \rho P_{co}\left(\overline{e}\right)+(1-\rho)\,P_{co}\left(\underline{e}\right) &> P_{so}\left(\overline{v}\right).\end{aligned}$$

Therefore, it is sufficient to focus on the condition

$$(1-\rho)\,P_{co}\left(\overline{e}\right)+\rho P_{co}\left(\underline{e}\right)<P_{so}\left(\underline{v}\right). \tag{31}$$

Consider first part (iii). Note that given β, as $\rho\rightarrow\frac{1}{2}$ we have $\hat{\tau}\rightarrow\frac{1}{2}$, $\underline{e}\rightarrow\underline{v}+\tau\Delta$, $\overline{e}\rightarrow\underline{v}+\tau\Delta$, and $\hat{\Delta}\rightarrow 0$. Therefore,

$$(1-\rho)\,P_{co}\left(\overline{e}\right)+\rho P_{co}\left(\underline{e}\right)\rightarrow\underline{v}+\tau\Delta.$$

However, note that $\tau P_{so}\left(\overline{v}\right)+(1-\tau)\,P_{so}\left(\underline{v}\right)=\underline{v}+\tau\Delta$ and $P_{so}\left(\overline{v}\right)>P_{so}\left(\underline{v}\right)$ imply $P_{so}\left(\underline{v}\right)<\underline{v}+\tau\Delta$. Since $\beta<1$ implies $P_{so}\left(\underline{v}\right)<\underline{v}+\tau\Delta$, by continuity, there is $\underline{\rho}\in(\frac{1}{2},1)$ as required.

Next, consider part (ii). If $L/n\geq\underline{v}+\Delta\tau$ then $L/n\geq\underline{e}$ for all $\rho\in[\frac{1}{2},1)$. Therefore, a type (iii) equilibrium always exists. In this case, Condition (31) is violated if and only if

$$\begin{aligned}(1-\rho)\left[\beta\left(\underline{e}+\hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}\right)+(1-\beta)\,\overline{e}\right]+\rho\left[\underline{e}+\hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}\right] &\geq \underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}\Leftrightarrow\\ \underline{e}+\hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}+(1-\rho)\,\hat{\Delta}\frac{(1-\hat{\tau})\,(1-\beta)}{\beta\hat{\tau}+1-\hat{\tau}} &\geq \underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}.\end{aligned}$$

Note, however, that

$$\underline{e}+\hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}=\underline{v}+\Delta\tau\frac{1-(1-\beta)\,\rho}{1-(1-\beta)\left[\tau\rho+(1-\tau)\,(1-\rho)\right]},$$

which is a decreasing function of ρ. Therefore, $\underline{e}+\hat{\Delta}\frac{\beta\hat{\tau}}{\beta\hat{\tau}+1-\hat{\tau}}>\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ for any $\rho<1$, which implies that Condition (31) is violated as required. That is, if a type (iii) equilibrium exists, price informativeness is lower under common ownership. Letting $\overline{y}=\underline{v}+\Delta\tau$ completes this proof.

Next, consider part (i). Note that

$$\begin{aligned}
\underline{e}\left(1-\hat{\tau}\right) &= \left(\underline{v}+\Delta\tau\frac{1-\rho}{\tau\left(1-\rho\right)+\left(1-\tau\right)\rho}\right)\left[\tau\left(1-\rho\right)+\left(1-\tau\right)\rho\right] \\
&= \underline{v}\left[\tau\left(1-\rho\right)+\left(1-\tau\right)\rho\right]+\Delta\tau\left(1-\rho\right) \\
&= \underline{v}\left(1-\tau\right)+\left(1-\rho\right)\left(\tau\left(2\underline{v}+\Delta\right)-\underline{v}\right)
\end{aligned}$$

Thus, if $L/n<\underline{y}\equiv\min\{\frac{\underline{v}+\tau\Delta}{2},\underline{v}\left(1-\tau\right)\}$ then $L/n<\underline{e}\left(1-\hat{\tau}\right)$ for all $\rho\in(\frac{1}{2},1]$. Under this condition, the equilibrium must be of type (i). Therefore, Condition (31) holds if and only if

$$\begin{aligned}
\left(1-\rho\right)\left[\underline{e}+\hat{\Delta}\frac{\hat{\tau}}{\hat{\tau}+\gamma\left(1-\hat{\tau}\right)}\right]+\rho\left[\gamma[\underline{e}+\hat{\Delta}\frac{\hat{\tau}}{\hat{\tau}+\gamma\left(1-\hat{\tau}\right)}]+\left(1-\gamma\right)\underline{e}\right] &< \underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}\Leftrightarrow \\
\underline{e}+\left(1-\rho+\rho\gamma\right)\frac{\hat{\Delta}\hat{\tau}}{\hat{\tau}+\gamma\left(1-\hat{\tau}\right)} &< \underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}\Leftrightarrow \\
\gamma\left(\rho\hat{\Delta}\hat{\tau}-\left(1-\hat{\tau}\right)\left[\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}-\underline{e}\right]\right) &< \\
\hat{\tau}\left[\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}-\underline{e}\right]-\left(1-\rho\right)\hat{\Delta}\hat{\tau} &\Leftrightarrow \\
\frac{\hat{\tau}\left[\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}-\underline{e}\right]-\left(1-\rho\right)\hat{\Delta}\hat{\tau}}{\hat{\Delta}\hat{\tau}-\left[\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}-\underline{e}\right]+\hat{\tau}\left[\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}-\underline{e}\right]-\left(1-\rho\right)\hat{\Delta}\hat{\tau}} &> \gamma
\end{aligned}$$

Note that the left-hand side converges to $\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$ as $\rho\to1$. Also recall $\gamma=(1-\beta)\cdot\mathbf{1}_{\underline{x}(0)=0}$ and note that

$$1-\beta<\frac{\beta\tau}{\beta\tau+\left(1-\beta\right)\left(1-\tau\right)}\Leftrightarrow\beta>\frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}.$$

Therefore, for any β there exists $\rho^{*}\in(\frac{1}{2},1)$ such that if $\rho\in(\rho^{*},1]$ then there is a type (i) equilibrium in which $\gamma=0$ (i.e., $\underline{x}\left(0\right)=0$) and Condition (31) holds. If $\beta>\frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$ then the same argument holds for any equilibrium. If $\beta=1$ then $\gamma=0$ even if $\underline{x}\left(0\right)=0$, and Condition

(31) holds if and only if

$$
\begin{aligned}
(1-\rho)\left[\underline{e}+\hat{\Delta}\right]+\rho\underline{e} &< \underline{v}+\Delta\tau \Leftrightarrow \\
\frac{(1-\rho)\,\rho}{\tau\rho+(1-\tau)\,(1-\rho)}+\frac{(1-\rho)\,\rho}{\tau\,(1-\rho)+(1-\tau)\,\rho} &< 1 \Leftrightarrow \\
1-4\rho+4\rho^2 &> 0 \Leftrightarrow \\
(2\rho-1)^2 &> 0,
\end{aligned}
$$

which always holds. ■

Proof of Proposition 6. The index fund raises exactly θ and must sell the same proportion from all firms. If the market maker expects the investor to monitor a fraction τ^* of all firms, then the price of any on-equilibrium trade must be $\overline{p}=\underline{v}+\tau^*\Delta$. Since the investor only raises θ, she sells $\overline{x}\,(\tau^*)=\min\{1,\frac{L/n}{\underline{v}+\tau^*\Delta}\}$ from each firm. The investor's expected payoff is given by

$$
\begin{aligned}
\Pi\,(c,c^*) &= n\left(\beta\left[\overline{x}\,(\tau^*)\left(\underline{v}+\tau^*\Delta\right)+\left(1-\overline{x}\,(\tau^*)\left(\underline{v}+F\,(c)\,\Delta\right)\right)\right]+(1-\beta)\left(\underline{v}+F\,(c)\,\Delta\right)\right) \\
&\quad -nF\,(c)\,E\left[c_i|c_i<c\right] \\
&= n\left[\beta\overline{x}\,(\tau^*)\left(\underline{v}+\tau^*\Delta\right)+\left(1-\beta\overline{x}\,(\tau^*)\right)\left(\underline{v}+F\,(c)\,\Delta\right)-F\,(c)\,E\left[c_i|c_i<c\right]\right]
\end{aligned}
$$

The first-order condition with respect to c implies $c^*=(1-\beta\overline{x}\,(\tau^*))\,\Delta$. Therefore, the equilibrium monitoring threshold must be a solution of $c^*=\Delta\left[1-\beta\overline{x}\,(F\,(c^*))\right]$, and note that the RHS is $\xi\,(F\,(c^*))$, as required. Finally, note that $c^{**}_{co,voice,index}<\Delta$, the threshold for an unconstrained investor under common ownership if $L/n\leq\underline{v}\,(1-F\,(\Delta))$. ■

Online Appendix for "Governance Under Common Ownership"

Appendix E. Governance: Additional Equilibria

This section considers additional equilibria to the most efficient equilibria focused on in Section 2.

E.1 Governance through exit: additional equilibria

While Section 2.1 compared the most efficient equilibrium under common ownership with the benchmark, Proposition 7 considers all equilibria under common ownership.

Proposition 7 (Comparison of equilibria, exit): Suppose $L/n \leq \underline{v}(1-F(\Delta))$. There is $\beta^* \in [0,1)$ s.t., if $\beta \geq \beta^*$, any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership.

Proof of Proposition 7. Suppose $L/n \leq \underline{v}(1-F(\Delta))$. From Lemma 3, the unique working threshold in any equilibrium under separate ownership is strictly smaller than Δ— that is, $c^*_{so,exit} < \Delta$. Moreover, since $c^*_{so,exit}$ solves $\phi_{exit}(F(c^*)) = c^*$, $\lim_{\beta\to 1} c^*_{so,exit} = \Delta(1-\omega)$. From Lemma 4, a type (i) equilibrium under common ownership exists in which the working threshold is Δ. From the proof of Lemma 4, the only other possible equilibrium is a type (i) equilibrium in which the investor retains bad firms if $\theta = 0$. If such an equilibrium exists, the working threshold must satisfy $\psi_{exit}(F(c^*)) = c^*$. Note that as $\beta \to 1$ any solution of $\psi_{exit}(F(c^*)) = c^*$ converges to Δ. Therefore, if β is sufficiently close to 1, any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership. ■

Proposition 7 shows that, if β is sufficiently high, *any* equilibrium under common ownership is strictly more efficient than the separate ownership benchmark. If β is sufficiently low, there exist less efficient equilibria — these are the small-shock equilibria where the investor retains some bad firms. In such equilibria, the incentives to work are decreasing in the frequency with which a bad firm is retained. This frequency is greater if β is low, since a bad firm is always retained upon no shock, and if L is low, since a smaller shock allows the investor to retain more bad firms upon a shock. However, under the efficiency criterion, the most efficient equilibrium will be chosen and so governance is always weakly stronger under common ownership.

E.2 Governance through voice: full analysis

Recall that Proposition 2, for the exit model, studied the most efficient equilibrium under common ownership and that Proposition 7 in Online Appendix E.1 required an extra condition on β to guarantee that governance is stronger under all equilibria under common ownership. In contrast, Proposition 3, for the voice model, holds for all equilibria under common ownership, without the need for an extra condition on β. This is because, while multiple equilibria exist, they differ only in terms of the investor's trading strategy, and not her monitoring strategy. For $L/n \leq \underline{v}(1-F(\Delta))$, even if price informativeness is lower under common ownership, per-security monitoring incentives remain higher. This is because the only way in which price informativeness can be lower is if the investor retains bad firms upon a shock, and so being retained is not fully revealing that a firm has been monitored. However, this does not affect the investor's incentives to monitor, since her payoff of a monitored and retained firm is its fundamental value of $\overline{v}$, regardless of the stock price. Thus, the threshold is Δ in *any* equilibrium for which $L/n \leq \underline{v}(1-F(\Delta))$.

While unnecessary for the main result in Section 2.2 (which holds for $L/n \leq \underline{v}(1-F(\Delta))$), for completeness, Lemma 9 gives the most efficient equilibrium under common ownership when $L/n > \underline{v}(1-F(\Delta))$.

Lemma 9 Suppose $L/n > \underline{v}(1-F(\Delta))$. There are $\underline{v}(1-F(\Delta)) < \underline{y} \leq \overline{y} \leq \underline{v}$ such that the monitoring threshold in the most efficient equilibrium is given by

$$c^{**}_{co,voice} = \begin{cases} c^{**}_{ii} \equiv \textit{the largest solution of } c^* = \zeta_{voice}\left(F\left(c^*\right)\right) & \textit{if } \underline{v}\left(1-F\left(\Delta\right)\right) < L/n < \underline{y} \\ \max\{c^{**}_{ii}, c^{**}_{iii}\} & \textit{if } \underline{y} \leq L/n < \overline{y} \\ c^{**}_{iii} \equiv \textit{the largest solution of } c^* = \phi_{voice}\left(F(c^*)\right) & \textit{if } L/n \geq \overline{y}, \end{cases} \tag{32}$$

where

$$\zeta_{voice}\left(\tau\right) \equiv \Delta\left[1 - \frac{L/n - \underline{v}\left(1-\tau\right)}{\underline{v}(\frac{1-\beta}{\beta}\left(1-\tau\right)+\tau) + \Delta\tau}\frac{1-\beta+\beta\tau}{\tau}\right]. \tag{33}$$

Prices and trading strategies are characterized by Lemma 2.

Below we state and prove an auxiliary lemma used in the proof of Lemma 9.

Lemma 10 Suppose $L/n > \underline{v}(1 - F(\Delta))$. Consider an equilibrium under common ownership in which each firm is good w.p. $\tau^* = F(c^*)$ where $L/n > \underline{v}(1 - \tau^*)$. Then:

(i) If the equilibrium is type (ii), then

$$\zeta_{voice}(F(c^*)) - \beta(1 - \overline{x}_{co}(F(c^*)))\Delta \leq c^* \leq \zeta_{voice}(F(c^*)), \tag{34}$$

where $\zeta_{voice}(\cdot)$ is given by Equation (33)

(ii) If the equilibrium is type (iii), then

$$c^* = \phi_{voice}(F(c^*)), \tag{35}$$

where $\phi_{voice}(\cdot)$ is given by Equation (15).

Proof. Let $\Pi(c^*, c)$ be as defined in the proof of Lemma 6. Then, $c^* = F^{-1}(\tau^*)$ is an equilibrium only if $c^* \in \arg\max_{c \geq 0} \Pi(c^*, c)$. Also, let $\overline{x}^* = \overline{x}_{co}(\tau^*)$ and $\overline{p}^* = \overline{p}_{co}(\tau^*)$ if the equilibrium is type (ii) and let $\overline{x}^* = \overline{x}_{so}(\tau^*)/n$ and $\overline{p}^* = \overline{p}_{so}(\tau^*)$ if the equilibrium is type (iii).

If $\theta = 0$, the investor obtains a payoff of $\overline{v}$ from a good firm. She can obtain a payoff of $\overline{x}^*\overline{p}^* + (1 - \overline{x}^*)\underline{v}$ from a bad firm by selling $\overline{x}^*$ of each bad firm, which generates the highest payoff. Recall that, from Lemma 2, the investor has no incentives to sell less than $\overline{x}^*$ of a bad firm when $\theta = 0$. Thus, for any $x_1' < \overline{x}^*$,

$$\begin{aligned} \overline{x}^*\overline{p}^* + (1 - \overline{x}^*)\underline{v} &> x_i' p(x_i') + (1 - x_i')\underline{v} \\ &\Rightarrow \overline{x}^*\overline{p}^* - x_i' p(x_i') > (\overline{x}^* - x_i')\underline{v} > 0. \end{aligned}$$

Therefore, in any equilibrium, the pricing function in the range $[0, \overline{x}^*)$ must satisfy this condition. This condition also implies that the investor cannot raise more revenue from a single deviation to selling $x' < \overline{x}^*$ from one particular bad firm. Without loss of generality and to simplify the exposition, hereafter we assume $x_i^*(\overline{v},0)=0$ and $x'\in(0,\overline{x}^*)\Rightarrow p(x_i')=\overline{p}^*$. These off-equilibrium prices preserve monotonicity, and $\overline{x}^*\overline{p}^*-x_i'p(x_i')>(\overline{x}^*-x_i')\underline{v}$. Moreover, note that if the investor found it optimal to monitor τ^* firms under the general pricing function (i.e., before specializing to $p(x_i')=\overline{p}^*$), deviating to sell $x'<\overline{x}^*$ from a given firm cannot be sufficiently beneficial to induce deviation from monitoring τ^* firms under the pricing rule $x'\in(0,\overline{x}^*)\Rightarrow p(x_i')=\overline{p}^*$. Indeed, since the pricing rule $x'\in(0,\overline{x}^*)\Rightarrow p(x_i')=\overline{p}^*$ is the lowest that satisfies monotonicity, a deviation from monitoring τ^* firms is less beneficial than under the general pricing function, and hence suboptimal as well. Intuitively, if she sold less than $\overline{x}^*$ from bad firms, she would receive the same price as if she sold $\overline{x}^*$, and so her incentives to monitor are no different.

Suppose $\theta=L$, and consider a type (iii) equilibrium. We argue that the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm. We consider two cases.

1. Suppose $L/n\geq\underline{v}$. We first argue $\overline{x}^*\overline{p}^*\geq\underline{v}$. To see why, recall that in this case that $\overline{x}^*=\min\{\frac{L/n}{\overline{p}^*},1\}$. Therefore, either $\overline{x}^*\overline{p}^*=L/n\geq\underline{v}$ or $\overline{x}^*=1$. Since $\overline{p}^*\geq\underline{v}$, $\overline{x}^*=1\Rightarrow\overline{x}^*\overline{p}^*\geq\underline{v}$. Second, since $\overline{x}^*\overline{p}^*\geq\underline{v}$, the investor raises more funds when she chooses $x_i=\overline{x}^*$ rather than $x_i=1$ (when $\overline{x}^*<1$). Therefore, she will sell $\overline{x}^*$ from each bad firm and $\frac{L/n-\overline{x}^*\overline{p}^*(1-\tau)}{\overline{p}^*\tau}$ from each good firm. If $\overline{x}^*\overline{p}^*=L/n$ then $\frac{L/n-\overline{x}^*\overline{p}^*(1-\tau)}{\overline{p}^*\tau}=\overline{x}^*$, and if $\overline{x}=1$ then $\overline{p}^*\leq L/n$, which implies that she has to sell the entire portfolio to raise L. Either way, she will sell $\overline{x}^*$ from each firm in her portfolio.

2. Suppose $L/n<\underline{v}$. Note that $\overline{x}^*=\min\{\frac{L/n}{\overline{p}^*},1\}$ and $\overline{p}^*>\underline{v}>L/n$ imply $\overline{x}^*\overline{p}^*=L/n$, and so $\overline{x}^*\overline{p}^*<\underline{v}$. If the investor deviates from selling $\overline{x}^*$ from each firm, she would deviate to fully selling $\min\{\frac{L/n}{\underline{v}},1-\tau\}$ bad firms, selling a fraction $\overline{x}^*$ of $(1-\tau)-\min\{\frac{L/n}{\underline{v}},1-\tau\}$ bad firms, and selling a fraction $\hat{x}=\max\{0,\frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}^*}\}$ of all good firms. Deviation is not strictly preferred if and only if

$$\begin{aligned}
& \tau\left[\hat{x}\overline{p}^*+(1-\hat{x})\overline{v}\right]+\left[(1-\tau)-\min\left\{\frac{L/n}{\underline{v}},(1-\tau)\right\}\right]\left[\overline{x}^*\overline{p}^*+(1-\overline{x}^*)\underline{v}\right]+ \\
& \min\left\{\frac{L/n}{\underline{v}},(1-\tau)\right\}\underline{v} \\
\leq\; & \tau\left[\overline{x}^*\overline{p}^*+(1-\overline{x}^*)\overline{v}\right]+(1-\tau)\left[\overline{x}^*\overline{p}^*+(1-\overline{x}^*)\underline{v}\right]\Leftrightarrow
\end{aligned}$$

$$\max\left\{0, \frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}^*}\right\} \geq \overline{x}^*\left[1-\min\left\{\frac{L/n}{\underline{v}}, 1-\tau\right\}\frac{(\overline{p}^*-\underline{v})}{\tau(\overline{v}-\overline{p}^*)}\right] \tag{36}$$

If $\tau \leq 1-\frac{L/n}{\underline{v}}$, then Condition (36) holds if and only if $\tau \leq \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*}$, where we used $\overline{p}^* = \overline{p}_{so}(\tau^*)$. Note that

$$1-\frac{L/n}{\underline{v}} < \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*} \Leftrightarrow \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v} < L/n$$

which must hold if the equilibrium is type (iii). If $\tau > 1-\frac{L/n}{\underline{v}}$ then Condition (36) holds if and only if $L/n \geq \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v}$, which must hold if the equilibrium is type (iii). Therefore, Condition (36) holds, and deviation is suboptimal.

Combining cases 1 and 2 above, the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm, and so her payoff is given by

$$\begin{aligned}\Pi(c^*,c) &= F(c)\left[\overline{v}-\beta\overline{x}^*(\overline{v}-\overline{p}^*)\right]+(1-F(c))\left[\underline{v}+\overline{x}^*(\overline{p}^*-\underline{v})\right]-F(c)E\left[c_i|c_i<c\right] \\ &= \underline{v}+\Delta\left[F(c)+\overline{x}^*(\tau^*-F(c))\frac{\beta}{\beta\tau^*+1-\tau^*}\right]-F(c)E\left[c_i|c_i<c\right],\end{aligned} \tag{37}$$

and so

$$\frac{\partial\Pi(c^*,c)}{\partial c}\frac{1}{f(c)} = \Delta\left[1-\overline{x}^*\frac{\beta}{\beta\tau^*+1-\tau^*}\right]-c.$$

Substituting $\overline{x}^* = \overline{x}_{so}(\tau^*)$ into the first-order condition $\frac{\partial\Pi(c^*,c)}{\partial c} = 0$, yields $c^* = \phi_{voice}(F(c^*))$, as required. This completes part (iii).

Suppose $\theta = L$, and consider the type (ii) equilibrium. Recall that we must have $L/n < \underline{v}$. Also recall that, by construction,

$$\tau^*\overline{x}^*\overline{p}^*+(1-\tau^*)\underline{v} = L/n.$$

Therefore, the investor can raise L by fully selling $(1-\tau^*)n$ bad firms and selling a fraction $\overline{x}^*$ of τ^*n good firms. Also, since $\overline{x}^*\overline{p}^* < \underline{v}$, selling $\overline{x}^*$ from every firm will not raise enough revenue to satisfy the shock. Note that, regardless of firm value, the investor has strict incentives to sell $\overline{x}^*$ rather 1. Indeed, in the latter case the payoff is $\underline{v}$, the lowest possible. Therefore, regardless of the proportion of good firms (i.e., even if $\tau \neq \tau^*$), she will fully sell exactly $(1-\tau^*)n$ firms and a fraction $\overline{x}^*$ of all other firms. The investor will prefer fully selling a bad firm if there are sufficient numbers. Therefore, her expected payoff from choosing cutoff c is:

$$\begin{aligned}\Pi\left(c^*,c\right) &= \left(1-F\left(c\right)\right)\left[\begin{array}{c}\left(1-\beta\right)\left[\overline{x}^*\overline{p}^*+\left(1-\overline{x}^*\right)\underline{v}\right]\\ +\beta\left(\begin{array}{c}\underline{v}\min\left\{1,\frac{1-\tau^*}{1-F(c)}\right\}\\ +\left[\overline{x}^*\overline{p}^*+\left(1-\overline{x}^*\right)\underline{v}\right]\left(1-\min\left\{1,\frac{1-\tau^*}{1-F(c)}\right\}\right)\end{array}\right)\end{array}\right]\\ &\quad +F\left(c\right)\left[\left(1-\beta\right)\overline{v}+\beta\left(\begin{array}{c}\underline{v}\max\left\{0,\frac{F(c)-\tau^*}{F(c)}\right\}\\ +\left[\overline{x}^*\overline{p}^*+\left(1-\overline{x}^*\right)\overline{v}\right]\left(1-\max\left\{0,\frac{F(c)-\tau^*}{F(c)}\right\}\right)\end{array}\right)\right]\\ &\quad -F\left(c\right)E\left[c_i|c_i<c\right].\end{aligned}$$

Using $\overline{x}^*=\overline{x}_{co}\left(\tau^*\right)$ and $\overline{p}^*=\overline{p}_{co}\left(\tau^*\right)$, we obtain:

$$\begin{aligned}\Pi\left(c^*,c\right) &= \underline{v}+F\left(c\right)\Delta+\overline{x}^*\left(\tau^*-F\left(c\right)\right)\beta\Delta\frac{1-\beta+\beta\tau^*}{\beta\tau^*+\left(1-\beta\right)\left(1-\tau^*\right)}-\beta\left(1-\overline{x}^*\right)\Delta\max\left\{F\left(c\right)-\tau^*,0\right\}\\ &\quad -F\left(c\right)E\left[c_i|c_i<c\right].\end{aligned}$$

Note that

$$\frac{\partial\Pi\left(c^*,c\right)}{\partial c}\frac{1}{f\left(c\right)}=\zeta_{voice}\left(\tau^*\right)-c-\begin{cases}0 & if\;\;F\left(c\right)<\tau^*\\ \beta\left(1-\overline{x}^*\right)\Delta & if\;\;F\left(c\right)>\tau^*,\end{cases}$$

which is a strictly decreasing function of c, and $\zeta_{voice}\left(\cdot\right)$ is given by Equation (33). Since f is strictly positive, $f\left(0\right)>0$ and $\frac{\partial\Pi(c^*,c)}{\partial c}\frac{1}{f(c)}|_{c=0}>0$. Therefore, $\Pi\left(c^*,c\right)$ is maximized at

$$\arg\max_{c\geq 0}\Pi\left(\tau^*,c\right)=\begin{cases}\zeta_{voice}\left(\tau^*\right) & if\;\;F\left(\zeta_{voice}\left(\tau^*\right)\right)<\tau^*\\ F^{-1}\left(\tau^*\right) & if\;\;F\left(\zeta_{voice}\left(\tau^*\right)-\beta\left(1-\overline{x}\right)\Delta\right)\leq\tau^*\leq F\left(\zeta_{voice}\left(\tau^*\right)\right)\\ \zeta_{voice}\left(\tau^*\right)-\beta\left(1-\overline{x}\right)\Delta & if\;\;\tau^*<F\left(\zeta_{voice}\left(\tau^*\right)-\beta\left(1-\overline{x}\right)\Delta\right).\end{cases}$$

In equilibrium, we require $c^*\left(\tau^*\right)=F^{-1}\left(\tau^*\right)$. Therefore, τ^* must satisfy

$$\zeta_{voice}\left(\tau^*\right)-\beta\left(1-\overline{x}\left(\tau^*\right)\right)\Delta\leq F^{-1}\left(\tau^*\right)\leq\zeta_{voice}\left(\tau^*\right),$$

as required. ■

The full proof of Lemma 9 now follows.

Proof of Lemma 9. We prove the result in three steps. First, suppose $L/n \geq \underline{v}$. Based on Lemma 2, the equilibrium must be type (iii). Based on part (ii) of Lemma 10, the monitoring threshold must solve $c^* = \phi_{voice}(F(c^*))$. Note that $\phi_{voice}(F(c))$ is continuous, $\phi_{voice}(F(0)) = \Delta(1-\beta)$ and $\phi_{voice}(1) = \Delta(1 - \min\{\frac{L/n}{\underline{v}+\Delta}, 1\})$, and hence, by the intermediate value theorem, a solution always exists. Given a threshold that satisfies $c^* = \phi_{voice}(F(c^*))$, by construction there is a type (iii) equilibrium with this threshold.

Second, we prove that if $\underline{v}(1 - F(\Delta)) < L/n < \underline{v}$, there always exists a type (ii) equilibrium where the monitoring threshold is given by part (i) of Lemma 10— that is, the largest solution of $c^* = \zeta_{voice}(F(c^*))$. In particular, it is sufficient to show that $c^* = \zeta_{voice}(F(c^*))$ has a solution such that $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) < c^*$ (which is equivalent to $\underline{v}(1-\tau^*) < L/n$). Indeed, when $c^* = F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$ then $\zeta_{voice}(F(c^*)) = \Delta$. Since $\underline{v}(1 - F(\Delta)) < L/n$, then $c^* = F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) \Rightarrow \zeta_{voice}(F(c^*)) > F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$. Furthermore, when $F(c^*) = 1$ then $\zeta_{voice}(F(c^*)) = \Delta\left[1 - \frac{L/n}{\underline{v}+\Delta\tau}\right] < \infty$, since $F(c^*) = \tau^* = 1$. Since $\zeta_{voice}(F(c^*))$ is continuous in c^*, by the intermediate value theorem, a solution strictly greater than $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$ always exists. By construction, there is a type (ii) equilibrium with such a threshold.

Third, suppose $\underline{v}(1 - F(\Delta)) < L/n < \underline{v}$. We compare the efficiency of the sustainable equilibria. First note that any type (i) equilibrium is less efficient than a type (ii) equilibrium. Indeed, in the former case the equilibrium threshold c_i^* must satisfy $L/n \leq \underline{v}(1 - F(c_i^*))$, and in the latter case the equilibrium threshold c_{ii}^* must satisfy $L/n > \underline{v}(1 - F(c_{ii}^*))$. Therefore, $c_{ii}^* > c_i^*$, as required. Next, consider type (iii) equilibria. When $L/n < \underline{v}$, such equilibria exhibit $\overline{x}^*\overline{p}^* = L/n$, where $\overline{p}^* = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$. Therefore, whenever these equilibria exist,

$$\phi_{voice}(\tau) \equiv \Delta\left[1 - \beta\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau}\right].$$

Note that $\zeta_{voice}(\tau) > \phi_{voice}(\tau)$ if and only if

$$\Delta\left[1-\frac{L/n-\underline{v}(1-\tau)}{\underline{v}\left(\frac{1-\beta}{\beta}(1-\tau)+\tau\right)+\Delta\tau}\frac{1-\beta+\beta\tau}{\tau}\right] > \Delta\left[1-\beta\frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right] \Leftrightarrow$$
$$\left(\frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right)L/n < \underline{v}. \quad (38)$$

Also note that

$$1 \geq \frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} \Leftrightarrow \beta \geq \frac{\underline{v}}{\underline{v}+\Delta}.$$

Therefore, if $\beta \geq \frac{\underline{v}}{\underline{v}+\Delta}$, then Condition (38) always holds, which implies that the most efficient equilibrium is type (ii). In this case, $\underline{y}=\overline{y}=\underline{v}$. In other words, whenever a type (ii) equilibrium exists (i.e., $\underline{v}(1-F(\Delta)) < L/n < \underline{v}$), it is the most efficient equilibrium.

Suppose $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$. Note that Condition (38) is equivalent to $\Gamma(\tau) < 0$, where

$$\Gamma(\tau) = \tau^2 - \tau\left[\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right]\frac{\underline{v}-L/n}{\underline{v}} - \frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}-L/n}{\underline{v}}.$$

Note that $\min\Gamma(\tau) < 0$. Also, recall $\underline{v}(1-\tau_{ii}^{**}) < L/n$. Therefore, it is sufficient to focus on $\underline{v}(1-\tau) < L/n \Leftrightarrow \frac{\underline{v}-L/n}{\underline{v}} < \tau$. It can be verified that $\Gamma\left(\frac{\underline{v}-L/n}{\underline{v}}\right) < 0$. Therefore, there is $\hat{\tau} > \frac{\underline{v}-L/n}{\underline{v}}$ such that $\Gamma(\tau) \geq 0 \Leftrightarrow \tau \geq \hat{\tau}$ where $\hat{\tau}$ is the largest root of $\Gamma(\tau)$, given by

$$\hat{\tau} = \frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right)+\frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\sqrt{\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right)^2+4\frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}}{\underline{v}-L/n}}. \quad (39)$$

Note that a type (iii) equilibrium requires

$$\underline{v}\frac{1-\tau}{\beta\tau+1-\tau} < L/n \Leftrightarrow \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} < \tau$$

where $\frac{\underline{v}-L/n}{\underline{v}} < \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. Also note that $\tau^* < F(\Delta)$ in both a type (ii) and type (iii) equilibrium. Therefore, the relevant range is $\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} \leq \tau \leq F(\Delta)$. This interval is non-empty if and only if

$$\frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta} < L/n \Leftrightarrow \frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta.$$

Note that $\underline{v}(1-F(\Delta)) < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta}$ for all β. Since $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$ if $L/n < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\frac{\underline{v}}{\underline{v}+\Delta}}$, the most efficient equilibrium is type (ii). This establishes the existence of $\underline{y}$, the threshold below which a type (ii) equilibrium is most efficient.

Suppose

$$\frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta < \frac{\underline{v}}{\underline{v}+\Delta}. \tag{40}$$

If $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$, then $\phi_{voice}(\tau)$ is a decreasing function, and so τ^{**}_{iii}, given by the solution of $\tau = F(\phi_{voice}(\tau))$, is unique. Therefore, the equilibrium with τ^{**}_{iii} is most efficient if and only if

$$\max\left\{\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}, \hat{\tau}\right\} < \tau^{**}_{iii}.$$

We now prove that $\hat{\tau} \geq \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. To do so, we first prove that

$$\hat{\tau} < x \Leftrightarrow \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} < x\frac{\frac{1-\beta}{\beta}+x\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}. \tag{41}$$

To see this, note that

$$\hat{\tau} \geq x \Leftrightarrow \sqrt{\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right)^2+4\frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}}{\underline{v}-L/n}} \geq 2x\frac{\underline{v}}{\underline{v}-L/n}-\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow$$

$$\begin{aligned}
2x\frac{\underline{v}}{\underline{v}-L/n}-\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &< 0 \text{ or} \\
2x\frac{\underline{v}}{\underline{v}-L/n}-\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &\geq 0 \text{ and} \\
4\frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}}{\underline{v}-L/n} &\geq \left[2x\frac{\underline{v}}{\underline{v}-L/n}\right]^2-2\left[2x\frac{\underline{v}}{\underline{v}-L/n}\right]\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow
\end{aligned}$$

$$2x\frac{\underline{v}}{\underline{v}-L/n}+\frac{1-\beta}{\beta} < \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} \text{ or}$$

$$2x\frac{\underline{v}}{\underline{v}-L/n}+\frac{1-\beta}{\beta} \geq \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} \text{ and } \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} \geq \frac{x\frac{1-\beta}{\beta}+x^2\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}$$

Note

$$2x\frac{\underline{v}}{\underline{v}-L/n}+\frac{1-\beta}{\beta} > \frac{x\frac{1-\beta}{\beta}+x^2\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x} \Leftrightarrow 2x\frac{\underline{v}}{\underline{v}-L/n}\frac{1-\beta}{\beta}+x^2\frac{\underline{v}}{\underline{v}-L/n}+\left[\frac{1-\beta}{\beta}\right]^2 > 0,$$

which proves Condition (41). Using Condition (41), we have

$$\hat{\tau} > \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} \Leftrightarrow \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} > \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}}.$$

This eventually yields

$$\frac{\beta}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\left(1+\frac{L/n}{\underline{v}-L/n}\beta\right) > -\frac{L/n}{\underline{v}-L/n}\frac{\beta\frac{L/n}{\underline{v}-L/n}(1-\beta)}{\frac{1}{\beta}+\frac{L/n}{\underline{v}-L/n}(1-\beta)},$$

which always holds.

Since $\hat{\tau} \geq \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$, τ^{**}_{iii} is most efficient only if $\hat{\tau} < \tau^{**}_{iii}$ and $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$, that is,

$$\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} < \tau^{**}_{iii}\frac{\frac{1-\beta}{\beta}+\tau^{**}_{iii}\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+\tau^{**}_{iii}}.$$

Note that $\lim_{L/n\to\underline{v}}\tau^{**}_{iii} > 0 = \lim_{L/n\to\underline{v}}\hat{\tau}$. By continuity, there is $\overline{y} \in [\underline{y}, \underline{v})$ such that, if $L/n > \underline{y}$, the most efficient equilibrium is type (iii). ■

Appendix F. Robustness: Full Analysis

F.1 Two firms

We define $\mathbf{x} \equiv (x_i, x_j)$, $\mathbf{v} \equiv (v_i, v_j)$, $\mathbf{x}^T \equiv (x_j, x_i)$, $\mathbf{v}^T \equiv (v_j, v_i)$, $\overline{\mathbf{v}} \equiv (\overline{v}, \overline{v})$, and $\underline{\mathbf{v}} \equiv (\underline{v}, \underline{v})$. We denote by $\mathbf{e}(\theta, \mathbf{v}) \equiv (e_i(\theta, \mathbf{v}), e_j(\theta, \mathbf{v}))$ the equilibrium strategy of type $(\theta, \mathbf{v})$. By symmetry, $p_i(\mathbf{x}) = p_j(\mathbf{x}^T)$ for all x_i and x_j, and $e_j(\theta, \mathbf{v}) = e_i(\theta, \mathbf{v}^T)$. We therefore omit the subscript whenever there is no risk of confusion. Let $\Pi^*(\theta, \mathbf{v}) \equiv \Pi(\mathbf{e}(\theta, \mathbf{v}), \mathbf{v})$ denote the equilibrium payoff of type $(\theta, \mathbf{v})$, where

$$\Pi(\mathbf{x}, \mathbf{v}) = x_i p_i(\mathbf{x}) + (n/2 - x_i) v_i + x_j p_i(\mathbf{x}^T) + (n/2 - x_j) v_j.$$

We start by analyzing the case of separate market makers and then a single market maker. We focus on the case of small liquidity shocks ($L/n \leq \underline{v}/2$, so that a shock can be met by fully selling one bad firm) since this is where our results are strongest.

F.1.1 Separate market makers

Lemma 11 (Common ownership, two firms, separate market makers): Suppose $L/n \leq \underline{v}/2$. An equilibrium under common ownership always exists and is unique.[31]

(i) If $\tau \geq \frac{1}{1+\beta}$, then

$$x^*_{co,i}(v_i, v_j, \theta) = \begin{cases} 0 & \text{if } v_i = \overline{v} \text{ and } \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & \text{else} \end{cases} \tag{42}$$

and prices of firm i are

$$p^*_i(x_i) = \begin{cases} \overline{v} & \text{if } x_i = 0 \\ \overline{p}_{co}(\tau) = \underline{v} + \frac{\beta\tau}{\beta\tau + 1 - \tau}\Delta & \text{if } x_i \in (0, \overline{x}_{co}(\tau)], \\ \underline{v} & \text{if } x_i > \overline{x}_{co}(\tau). \end{cases}$$

[31] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\sqrt{\beta}}$ then the equilibria in part (i) and part (ii) coexist.

(ii) If $\tau < \frac{1}{1+\beta}$, then

$$x_{co,i}^{*}(\mathbf{v},\theta) = \begin{cases} 0 & \textit{if } \mathbf{v} = (\overline{v}, \underline{v}), \textit{ or } \mathbf{v} = \overline{\mathbf{v}} \textit{ and } \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & \textit{if } \mathbf{v} = (\underline{v}, \overline{v}) \textit{ and } \theta = 0,\ \mathbf{v} = \underline{\mathbf{v}}\ , \textit{ or } \mathbf{v} = \overline{\mathbf{v}} \textit{ and } \theta = L \\ n/2 & \textit{if } \mathbf{v} = (\underline{v}, \overline{v}) \textit{ and } \theta = L, \end{cases} \tag{43}$$

and prices of firm i are

$$p_i^{*}(x_i) = \begin{cases} \overline{v} & \textit{if } x_i = 0 \\ \overline{p}_{co}(\tau) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)(1-\tau)+(\tau^2+(1-\tau)^2)\beta}\Delta & \textit{if } x_i \in (0, \overline{x}_{co}(\tau)], \\ \underline{v} & \textit{if } x_i > \overline{x}_{co}(\tau). \end{cases} \tag{44}$$

The intuition is as follows. If $\tau \geq \frac{1}{1+\beta}$, then the investor retains a firm only if it is good and she suffers no shock. If the firm is bad, or if she suffers a shock, she sells the firm to the same degree ($\overline{x}_{co}(\tau)$ in this case), just as under separate ownership. In particular, even if firm i is good and firm j is bad, she still sells firm i upon a shock. Even though she could meet her liquidity need by selling only firm j, doing so would lead to the lowest possible price of $\underline{v}$. When $\tau \geq \frac{1}{1+\beta}$, the probability of a good firm (τ) and the probability of a liquidity shock (β) are both high, and so the price of a partially sold firm $\overline{p}_{co}(\tau)$ is also high as there is a high probability that this is a good firm sold due to a shock. Thus, the investor prefers to meet the shock by selling only $\overline{x}_{co}(\tau)$ of the bad firm, even though doing so also requires her to sell $\overline{x}_{co}(\tau)$ of the good firm to meet the shock. Put differently, even though common ownership gives the investor the flexibility to meet the shock by selling only bad firms, she chooses not to take advantage of this flexibility. This issue does not arise with a continuum of firms since it is never the case that all firms are good. Thus, a sold firm cannot be a good firm sold due to a shock, and so it receives the lowest possible price of $\underline{v}$.

If $\tau < \frac{1}{1+\beta}$, then the price of a partially-sold firm is sufficiently low that an investor with exactly one bad firm does take advantage of her flexibility to meet a liquidity shock by selling only the bad firm and retaining the good one.

F.1.2 Single market maker

Here, the market maker can condition p_i on x_j. We focus on equilibria with non-increasing prices in the following sense: If $0 \leq x < x + \varepsilon \leq n/2$ then $p_i(\mathbf{x}) \geq p_i(\mathbf{x}+\varepsilon)$ and $p_j(\mathbf{x}) \geq p_j(\mathbf{x}+\varepsilon)$. In other words, among balanced exit strategies, if an investor sells less of all firms, the prices of all firms must (weakly) increase.

Lemma 12 (Common ownership, two firms, single market maker): Suppose $L/n \leq \underline{v}/2$. An equilibrium under common ownership always exists.[32]

(i) If $\tau > \frac{1}{1+\sqrt{\beta}}$, then the equilibrium is unique where:

$$x^*_{co,i}(v_i, v_j, \theta) = \begin{cases} 0 & \textit{if } \mathbf{v} = (\overline{v}, \overline{v}) \textit{ and } \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & \textit{else} \end{cases} \tag{45}$$

and prices of firm i are

$$p^*_i(x_i, x_j) = \begin{cases} \overline{v} & \textit{if } x_i = 0 \\ \overline{p}_{co}(\tau) = \underline{v} + \tau \frac{\beta + (1-\beta)(1-\tau)}{\beta + (1-\beta)(1-\tau^2)} \Delta & \textit{if } 0 < x_i \leq \min\{\overline{x}_{co}(\tau), x_j\}, \\ \underline{v} & \textit{if } \min\{\overline{x}_{co}(\tau), x_j\} < x_i \end{cases}$$

(ii) If $\tau < \frac{1}{1+\sqrt{\beta}}$, then in any equilibrium

$$x^*_{co,i}(\mathbf{v}, \theta) = \begin{cases} 0 & \textit{if } \mathbf{v} = \overline{\mathbf{v}} \textit{ and } \theta = 0, \textit{ or } \mathbf{v} = (\overline{v}, \underline{v}) \textit{ and } \theta = L \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & \textit{if } \mathbf{v} = \overline{\mathbf{v}} \textit{ and } \theta = L, \textit{ or } \mathbf{v} = \underline{\mathbf{v}} \\ n/2 & \textit{if } \mathbf{v} = (\underline{v}, \overline{v}) \textit{ and } \theta = L \end{cases} \tag{46}$$

and

$$\left(x^*_{co,i}((\underline{v}, \overline{v}), 0), x^*_{co,j}((\underline{v}, \overline{v}), 0)\right) \neq \left(\overline{x}_{co}(\tau), \overline{x}_{co}(\tau)\right).$$

[32] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\sqrt{\beta}}$ then the equilibria in part (i) and part (ii) coexist.

Moreover,

$$\overline{p}_{co}(\tau) = \underline{v} + \frac{\beta\tau^2}{\beta\left((1-\tau)^2+\tau^2\right) + (1-\beta)(1-\tau)^2}\Delta.$$

The equilibrium with the lowest price informativeness satisfies

$$\left(x^*_{co,i}\left((\underline{v},\overline{v}),0\right), x^*_{co,j}\left((\underline{v},\overline{v}),0\right)\right) = (0,0)$$

and

$$p^*_i(x_i,x_j) = \begin{cases} p(0) = \underline{v} + \Delta\frac{\tau(1-\tau)+\tau^2}{\tau(1-\tau)+(1-\tau)\tau+\tau^2} & \text{if } x_i = 0 \\ \overline{p}_{co}(\tau) & \text{if } 0 < x_i \le \min\{\overline{x}_{co}(\tau), x_j\} \\ \underline{v} & \text{if } \min\{\overline{x}_{co}(\tau), x_j\} < x_i \end{cases}.$$

The equilibria are similar to the separate market maker case, except for when the investor has exactly one bad firm and does not suffer a shock. With separate market makers, the investor voluntarily sells $\overline{x}_{co}(\tau)$ of the bad firm, to disguise the sale as being of a good firm and due to a shock, and retains the good firm. With a single market maker, such disguise is no longer possible, since the market maker would see that the good firm is fully retained, infer that there has been no shock, and price the partially-sold firm at $\underline{v}$. She may thus choose to sell both the bad and good firm to the same degree, to disguise their sale as being motivated by a shock. The lowest level of price informativeness arises when the investor retains both firms upon no shock, because now being fully retained no longer fully reveals that a firm is good.

F.1.3 Price informativeness

Proposition 8 (Price informativeness, two firms): Suppose $L/n \le \underline{v}/2$.

(i) Separate market makers: If $\tau < \frac{1}{1+\beta}$, then in any equilibrium under separate and common ownership:

$$P_{co,Separate}(\overline{v},\tau) > P_{so}(\overline{v}) \text{ and } P_{co,Separate}(\underline{v},\tau) < P_{so}(\underline{v}).$$

(ii) Single market maker: There is $\overline{\beta} \in (0,1)$ such that, if $\tau < \frac{1}{2}$ and $\beta \in (\overline{\beta}, 1]$, then in any equilibrium under separate and common ownership:

$$P_{co,Single}(\overline{v}, \tau) > P_{so}(\overline{v}) \text{ and } P_{co,Single}(\underline{v}, \tau) < P_{so}(\underline{v}).$$

The intuition is as follows; we discuss the required parameters for the separate market makers case but the intuition is similar for the single market maker case. Price informativeness can be higher under common ownership only if $\tau < \frac{1}{1+\sqrt{\beta}}$, because only in this case does an investor with one bad firm take advantage of her flexibility to meet a liquidity shock by selling only the bad firm. For price informativeness to be higher under common ownership in *any* equilibrium, the price of a bad firm must be lower than under separate ownership. Under separate ownership, a bad firm's price is increasing in β, since the higher the probability of a shock, the higher the probability that a partial sale is of a good firm due to a shock. Thus, a high β (to increase the price of a bad firm under separate ownership) and a low τ (so that $\tau < \frac{1}{1+\sqrt{\beta}}$) ensure that price informativeness is higher under common ownership in any equilibrium.

F.1.4 Proofs

Proof of Lemma 11. We start by proving several claims.

1. In any equilibrium, there is a unique $\overline{x}_G \in (0, n/2)$ such that $\mathbf{e}(L, \overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$. Proof: A symmetric equilibrium requires the investor to sell the same quantities from both firms if they have the same value. A pure strategy equilibrium requires $\overline{x}_G$ to be unique. Since $\beta > 0$, such $\overline{x}_G$ exists. Since $L > 0$, if $\overline{x}_G = 0$ then the investor has a profitable deviation to selling $n/2$ units from each firm and raising more revenue. If $\overline{x}_G = n/2$, since $p(\overline{x}_G) \geq \underline{v}$, $\overline{x}_G p(\overline{x}_G) \geq n/2\underline{v} \geq L > L/2$, which contradicts $\overline{x}_G p(\overline{x}_G) = L/2$ that we prove in claim 2 below.

2. In any equilibrium, (i) $p_i(\overline{x}_G) \in (\underline{v}, \overline{v})$; (ii) $\overline{x}_G p(\overline{x}_G) = L/2$. Proof of part (i): Since $\beta > 0$, $p(\overline{x}_G) > \underline{v}$. Suppose on the contrary $p(\overline{x}_G) = \overline{v}$. Since prices are non-increasing, there is $\bar{x} \geq \overline{x}_G$ such that $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$ and either $\bar{x} = n/2$ or $x > \bar{x} \Rightarrow p(x) < \overline{v}$.

Let $\overline{x}_B = e_i(L, \underline{\mathbf{v}}) = e_j(L, \underline{\mathbf{v}})$. Then, $p(\overline{x}_B) < \overline{v}$. Moreover, since $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$, it must be $\overline{x}_B > \bar{x}$. If $p(\overline{x}_B) = \underline{v}$ then type $\underline{\mathbf{v}}$ has a profitable deviation to $(\bar{x}, \bar{x})$. Therefore, $p(\overline{x}_B) > \underline{v}$, which requires either $\mathbf{e}(L, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_B$ or $\mathbf{e}(0, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_B$. Type $(\overline{v}, \underline{v})$ prefers $\overline{\mathbf{x}}_B$ over $\overline{\mathbf{x}}_G$ only if

$$\begin{aligned} 2\overline{x}_B p(\overline{x}_B) + (n/2 - \overline{x}_B)(\underline{v} + \overline{v}) &\geq 2\overline{x}_G \overline{v} + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) \Leftrightarrow \\ p(\overline{x}_B) &\geq \frac{\overline{x}_G}{\overline{x}_B} \frac{\overline{v} - \underline{v}}{2} + \frac{\overline{v} + \underline{v}}{2} \end{aligned}$$

which is strictly greater than $\frac{\overline{v}+\underline{v}}{2}$. However, note that

$$p(\overline{x}_B) \leq \underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)}$$

that is, the highest possible value of $p(\overline{x}_B)$ arises when type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_B$ regardless of her liquidity need and type $\underline{\mathbf{v}}$ chooses $\overline{\mathbf{x}}_B$ only when there is a shock. In addition,

$$\underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)} < \frac{\overline{v} + \underline{v}}{2} \Leftrightarrow 0 < \beta\left(1 - \tau^2\right)$$

which always holds. Therefore, type $(\overline{v}, \underline{v})$ never chooses $\overline{\mathbf{x}}_B$, a contradiction.

Proof of part (ii): Suppose on the contrary $\overline{x}_G p(\overline{x}_G) > L/2$. Since prices are non-increasing, there is $\varepsilon > 0$ such that $(\overline{x}_G - \varepsilon) p(\overline{x}_G - \varepsilon) \geq L/2$. Since $p(\overline{x}_G) < \overline{v}$, if $\theta = L$ then type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to $(\overline{x}_G - \varepsilon, \overline{x}_G - \varepsilon)$. Suppose on the contrary $\overline{x}_G p(\overline{x}_G) < L/2$. Since $L/n \leq \underline{v}/2$, the investor can sell $n/2$ from both firms, get a price no lower than $\underline{v}$, and thus raise enough liquidity. This creates a profitable deviation.

3. In any equilibrium, $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ and $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$. Proof: Since the investor can fully retain both firms, $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$. Suppose on the contrary $e_i(0, \overline{\mathbf{v}}) \geq \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$ and $p(\overline{x}_G) < \overline{v}$ (from claim 2), it cannot be $e_i(0, \overline{\mathbf{v}}) = \overline{x}_G$. Suppose $e_i(0, \overline{\mathbf{v}}) > \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v} > p(\overline{x}_G)$, if $\theta = L$ then type $\overline{\mathbf{v}}$ has a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{e}(0, \overline{\mathbf{v}})$.

4. In any equilibrium, $e_i(0, (\overline{v}, \underline{v})) < \overline{x}_G$ and $p(e_i(0, (\overline{v}, \underline{v}))) = \overline{v}$. Proof: the same as claim

3.

5. In any equilibrium, if the investor sells x_B from a bad firm, either $x_B = \overline{x}_G$ or $p(x_B) = \underline{v}$. Proof: Suppose on the contrary the investor sells $x_B \neq \overline{x}_G$ from the bad firm w.p. $\gamma > 0$ in equilibrium, and $p(x_B) > \underline{v}$. Therefore, $p(x_B) < \overline{v}$. Based on claims 1-4, $p(x_B) > \underline{v}$ requires the investor to sell x_B from the good firm when $\theta = L$ and the other firm is bad. Let $\hat{x}$ be the quantity sold from the bad firm in this case. That is, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, \hat{x})$. Note that $\hat{x} \geq x_B$. Otherwise, type $(\overline{v}, \underline{v})$ has a strictly optimal deviation from $(x_B, \hat{x})$ to $(\hat{x}, x_B)$, that is, selling more from the bad firm and still meeting her liquidity need. Moreover, note that $\hat{x} = x_B$. To show this, if $\hat{x} \neq x_B$ then based on claims 1-4, we must have $x_B < \hat{x} = \overline{x}_G$ or $p(\hat{x}) = \underline{v}$.

   (a) Suppose $x_B < \hat{x} = \overline{x}_G$. If type $(\overline{v}, \underline{v})$ prefers $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$, then type $\overline{\mathbf{v}}$ must also prefer $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$. However, since $\mathbf{e}(L, \overline{\mathbf{v}}) = (\overline{x}_G, \overline{x}_G)$, type $\overline{\mathbf{v}}$ must be indifferent between the two. Both strategies must generate the same payoff, but also raise exactly L, otherwise the investor can always sell less from the good firm and still meet her liquidity need. However, this implies $x_B = \overline{x}_G$, a contradiction.

   (b) Suppose $p(\hat{x}) = \underline{v}$. Then, type $(\overline{v}, \underline{v})$ has a strictly profitable deviation from $(x_B, \hat{x})$ to $(0, n/2)$. Under both strategies, she raises enough revenue (recall $\underline{v}n/2 \geq L$). However, under the latter strategy her payoff is $n/2(\underline{v} + \overline{v})$, and under the former strategy her payoff is strictly smaller: she is getting a payoff $\underline{v}$ for the bad firm, but since $p(x_B) < \overline{v}$, her payoff for the good firm is strictly smaller than $\overline{v}$.

Therefore, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, x_B)$. By revealed preference,

$$\Pi((x_B, x_B), (\overline{v}, \underline{v})) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(x_B) \geq \frac{\underline{v} + \overline{v}}{2}.$$

Suppose type $(\overline{v}, \underline{v})$ chooses (x_B, x_B) w.p. $\sigma > 0$, then

$$p(x_B) = \underline{v} + \Delta \frac{\sigma\tau(1-\tau)}{\sigma[\tau(1-\tau) + (1-\tau)\tau] + \gamma(1-\tau)} < \frac{\underline{v} + \overline{v}}{2},$$

a contradiction.

6. In any equilibrium, if $\theta = 0$, then the investor sells $\overline{x}_G$ from a bad firm. Proof: Based on claim 5, if the investor sells x_B from a bad firm, either $x_B = \overline{x}_G$ or $p(x_B) = \underline{v}$. Since $p(\overline{x}_G) > \underline{v}$, if the investor does not need liquidity, she will maximize her payoff by choosing $\overline{x}_G$.

7. In any equilibrium, $\mathbf{e}(L, \underline{\mathbf{v}}) = (\overline{x}_G, \overline{x}_G)$. Proof: By symmetry, the investor must sell in equilibrium the same quantity from both firms. Based on claim 5, if $\mathbf{e}(L, \underline{\mathbf{v}}) \neq (\overline{x}_G, \overline{x}_G)$, the investor must receive $\underline{v}$ from the firms, and so her payoff is $\underline{v}$. Since $p(\overline{x}_G) > \underline{v}$ and $\overline{x}_G p(\overline{x}_G) = L/2$, the investor maximizes her payoff by choosing $(\overline{x}_G, \overline{x}_G)$.

8. In any equilibrium, if $p(\overline{x}_G) > \frac{\underline{v}+\overline{v}}{2}$, then $\mathbf{e}(L, (\overline{v}, \underline{v})) = (\overline{x}_G, \overline{x}_G)$, and if $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$, then $e_i(L, (\overline{v}, \underline{v})) < \overline{x}_G < e_j(L, (\overline{v}, \underline{v}))$. Proof: Recall $L/n \leq \underline{v}/2$ implies that selling $(0, n/2)$ yields type $(\overline{v}, \underline{v})$ a payoff of $n/2(\underline{v} + \overline{v})$ and enough revenue to cover her liquidity need. Therefore, she prefers $(\overline{x}_G, \overline{x}_G)$ if and only if

$$\Pi((\overline{x}_G, \overline{x}_G), (\overline{v}, \underline{v})) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{x}_G) \geq \frac{\underline{v} + \overline{v}}{2}.$$

Suppose $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$. If the investor chooses balanced exit $(x, x) \notin \{(\overline{x}_G, \overline{x}_G), (0, 0)\}$, then by symmetry, $p(x) = \frac{\underline{v}+\overline{v}}{2}$. The revenue raised must be exactly L, else she can sell slightly less from the good firm and make a strictly higher profit. Therefore, it cannot be $x < \overline{x}_G$, else type $\overline{\mathbf{v}}$ will deviate from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Also, it cannot be $x > \overline{x}_G$. Indeed, if $x > \overline{x}_G$ then $\Pi((x, x), (\overline{v}, \underline{v})) \geq \Pi((\overline{x}_G, \overline{x}_G), (\overline{v}, \underline{v}))$ implies $\Pi((\overline{x}_G, \overline{x}_G), (\underline{v}, \underline{v})) > \Pi((x, x), (\underline{v}, \underline{v}))$, which means that type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Therefore, the investor cannot choose balanced exit when $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$.

   Next, suppose type $(\overline{v}, \underline{v})$ chooses imbalanced exit (x_G, x_B). Note that the investor always sells more from the bad firm—that is, $x_G < x_B$. Also, since the investor can always meet her liquidity need choosing $(0, n/2)$, she must raise enough liquidity by choosing (x_G, x_B). We wish to prove $e_i(L, (\overline{v}, \underline{v})) < \overline{x}_G < e_j(L, (\overline{v}, \underline{v}))$, which requires us to prove $x_G < \overline{x}_G < x_B$. Suppose on the contrary $x_B \leq \overline{x}_G$. Since prices are non-increasing,

$p(x_B) \geq p(x_G) \geq p(\overline{x}_G)$, and type $\overline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_B) when $\theta = L$, a contradiction. Therefore, $\overline{x}_G < x_B$. Suppose $\overline{x}_G \leq x_G$. If $x_G > \overline{x}_G$ then the based on all the claims above, no other type is choosing x_G, and so $p(x_G) = \overline{v}$. However, since $x_G > \overline{x}_G$ type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_G). Suppose $x_G = \overline{x}_G$. Note that according to claim 5, $x_B > \overline{x}_G$ implies $p(x_B) = \underline{v}$. Therefore, the investor's payoff from the bad firm is $\underline{v}$. Since $p(\overline{x}_G) < \overline{v}$, the investor's payoff from the good firm is strictly lower than $\overline{v}$. Therefore, the investor's overall payoff is strictly smaller than $n/2(\underline{v} + \overline{v})$, which means that she has a profitable deviation to $(0, n/2)$. We conclude that $x_G < \overline{x}_G < x_B$, as required.

Based on claims 3 and 4, we can assume without loss of generality that $e_i(0, \overline{\mathbf{v}}) = e_i(0, (\overline{v}, \underline{v})) = 0$. In addition, the above claims imply there are both balanced exit and imbalanced exit equilibria. Consider first the balanced exit equilibrium of part (i). In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. Note that, based on claim 8, $p(\overline{x}_G) \geq \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{co}(\tau)$, given the expression of $\overline{p}_{co}(\tau)$ in part (i), it must be $\tau \geq \frac{1}{1+\beta}$.

We now show profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$, the investor cannot receive more than $\overline{v}$ from a good firm and $\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\underline{v}$ from a bad firm, which she does by following the equilibrium strategies.

2. Suppose $\theta = L$ and both firms are bad. Then, the equilibrium strategy yields the highest possible payoff given prices.

3. Suppose $\theta = L$ and both firms are good. The only possible profitable deviation is selling $\varepsilon \in (0, n/2 - \overline{x}_{co}(\tau)]$ more from one firm and $\delta \in (0, \overline{x}_{co}(\tau)]$ less from the other, while still raising enough revenue. This deviation generates at least L in revenue if and only if

$$(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} + (\overline{x}_{co}(\tau) - \delta)\overline{p}_{co}(\tau) \geq L \Leftrightarrow \delta \leq \frac{(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} - L/2}{\overline{p}_{co}(\tau)}. \tag{47}$$

This deviation increases the investor's payoff if and only if

$$\begin{aligned}
&\left(\overline{x}_{co}(\tau)+\varepsilon\right)\underline{v}+\left(n/2-\left(\overline{x}_{co}(\tau)+\varepsilon\right)\right)\overline{v}+\left(\overline{x}_{co}(\tau)-\delta\right)\overline{p}_{co}(\tau)+\left(n/2-\left(\overline{x}_{co}(\tau)-\delta\right)\right)\overline{v}\\
&>2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau)+2\left(n/2-\overline{x}_{co}(\tau)\right)\overline{v}\Leftrightarrow\\
\delta &> \frac{\varepsilon\Delta+\overline{x}_{co}(\tau)\left(\overline{p}_{co}(\tau)-\underline{v}\right)}{\overline{v}-\overline{p}_{co}(\tau)}.
\end{aligned}$$

However, note that

$$\frac{\varepsilon\Delta+\overline{x}_{co}(\tau)\left(\overline{p}_{co}(\tau)-\underline{v}\right)}{\overline{v}-\overline{p}_{co}(\tau)}>\frac{\left(\overline{x}_{co}(\tau)+\varepsilon\right)\underline{v}-L/2}{\overline{p}_{co}(\tau)}\Leftrightarrow\overline{p}_{co}(\tau)>\underline{v},$$

and so a profitable deviation does not exist.

4. Suppose $\theta = L$ and exactly one firm is bad. The only possible profitable deviation is selling $\varepsilon \in (0, n/2-\overline{x}_{co}(\tau)]$ more from the bad firm and $\delta \in (0, \overline{x}_{co}(\tau)]$ less from the good firm. As in case 3, this deviation generates at least L in revenue if and only if $\delta \leq \frac{(\overline{x}_{co}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{co}(\tau)}$. This deviation is profitable if and only if

$$\begin{aligned}
&\left(\overline{x}_{co}(\tau)+\varepsilon\right)\underline{v}+\left(n/2-\left(\overline{x}_{co}(\tau)+\varepsilon\right)\right)\underline{v}+\left(\overline{x}_{co}(\tau)-\delta\right)\overline{p}_{co}(\tau)+\left(n/2-\left(\overline{x}_{co}(\tau)-\delta\right)\right)\overline{v}\\
&>2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau)+\left(n/2-\overline{x}_{co}(\tau)\right)\left(\underline{v}+\overline{v}\right)\Leftrightarrow\\
\delta &> \overline{x}_{co}(\tau)\,\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)}.
\end{aligned}$$

Overall, the deviation is feasible and profitable if and only if

$$\overline{x}_{co}(\tau)\,\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)}<\delta\leq\min\left\{\overline{x}_{co}(\tau),\frac{\left(\overline{x}_{co}(\tau)+\varepsilon\right)\underline{v}-L/2}{\overline{p}_{co}(\tau)}\right\}.$$

Note that a larger δ implies a more profitable deviation, and that the profitability of the deviation is invariant to ε as long as $\varepsilon > 0$. Therefore, it is sufficient to focus on

$\varepsilon = n/2 - \overline{x}_{co}(\tau)$. Therefore, a profitable deviation exists if and only if

$$\begin{aligned}
\overline{x}_{co}(\tau)\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< \min\left\{\overline{x}_{co}(\tau), \frac{(\overline{x}_{co}(\tau)+n/2-\overline{x}_G)\underline{v}-L/2}{\overline{p}_{co}(\tau)}\right\} \Leftrightarrow \\
\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< \min\left\{1, \frac{n\underline{v}-L}{L}\right\} \Leftrightarrow \\
\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< 1 \Leftrightarrow \\
\tau &< \frac{1}{1+\beta}
\end{aligned}$$

which contradicts $\tau \geq \frac{1}{1+\beta}$.

Next, consider the imbalanced exit equilibrium of part (ii). Given claim 8, we assume without loss of generality that $\mathbf{e}(L, (\overline{v}, \underline{v})) = (0, n/2)$. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the proposition. Note that, based on claim 8, $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{co}(\tau)$, given the expression of $\overline{p}_{co}(\tau)$ in part (ii), it must be $\tau < \frac{1}{1+\beta}$.

We now show that profitable deviations do not exist. As in the balanced exit equilibrium, it is straightforward to see that there is no profitable deviation when $\theta = 0$, when $\theta = L$ and both firms are bad, or when $\theta = L$ and both firms are good. Suppose $\theta = L$ and exactly one firm is bad. First note that selling more of the good firm and less from the bad firm (but still more than $\overline{x}_{co}(\tau)$ units) is suboptimal: the investor's payoff from the bad firm does not change, but since $x > 0 \Rightarrow p(x) < \overline{v}$, her payoff from the good firm decreases. Therefore, a profitable deviation requires selling $\overline{x}_{co}(\tau)$ from the bad firm. However, since by construction $\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) = L/2$, this deviation generates revenue of at least L if and only if the investor also sells $\overline{x}_{co}(\tau)$ from the good firm. Such a deviation is profitable if and only if

$$\begin{aligned}
&\underline{v}n/2 + \overline{v}n/2 < \overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\underline{v} + \overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\overline{v} \Leftrightarrow \\
&\qquad \frac{\underline{v}+\overline{v}}{2} < \overline{p}_{co}(\tau) \Leftrightarrow \frac{1}{1+\beta} < \tau,
\end{aligned}$$

which contradicts $\tau > \frac{1}{1+\beta}$. ■

Proof of Lemma 12. We start by proving several claims.

1. In any equilibrium, if $\mathbf{x}$ such that $x_i < x_j$ is on the path, then $p_i(\mathbf{x}) = \overline{v}$ and $p_j(\mathbf{x}) = \underline{v}$. Proof: by symmetry, $\mathbf{v} \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$. The investor's payoff is $\Pi(\mathbf{x}, \mathbf{v})$. Note that for $\mathbf{x}$ satisfying $x_i < x_j$
$$\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j.$$
Since $\mathbf{x}$ is on the equilibrium path (by symmetry, so is x^T), $p_i(\mathbf{x}) \geq p_j(\mathbf{x})$. Since $x_i < x_j$ implies $v \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$, it must be $\mathbf{v} = (\overline{v}, \underline{v})$, and so $p_i(\mathbf{x}) = \overline{v}$ and $p_j(\mathbf{x}) = \underline{v}$, as required.

2. In any equilibrium, if $v_i > v_j$, then $x_i \leq x_j$. Proof: suppose on the contrary $v_i > v_j$ and $x_i < x_j$. Since $\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j$, the investor has a profitable deviation to sell x_i units from firm v_j and x_j units from v_i.

3. In any equilibrium, there is a unique $\overline{x}_G \in (0, n/2)$ such that $\mathbf{e}(L, \overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$. Proof: the same as claim 1 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

4. In any equilibrium, (i) $p_i(\overline{\mathbf{x}}_G) \in (\underline{v}, \overline{v})$; (ii) $\overline{x}_G p(\overline{\mathbf{x}}_G) = L/2$. Proof: the same as claim 2 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

5. In any equilibrium, $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ and $p(\mathbf{e}(0, \overline{\mathbf{v}})) = \overline{v}$. Proof: the same as claim 3 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

6. In any equilibrium, $e_i(0, \underline{\mathbf{v}}) = e_i(L, \underline{\mathbf{v}}) = \overline{x}_G$. Proof: By symmetry, if $v_i = v_j = \underline{v}$ then $e_i(\theta, \underline{\mathbf{v}}) = e_j(\theta, \underline{\mathbf{v}})$. Let $\mathbf{x}_B(\theta) \equiv \mathbf{e}(\theta, \underline{\mathbf{v}})$. Note that if $x < \overline{x}_G$ then $xp(x, x) < L/2$. Otherwise, since $p(\overline{\mathbf{x}}_G) < \overline{v}$, claim 1 implies that type $(\overline{\mathbf{v}}, L)$ has strict incentives to deviate and sell less than $\overline{x}_G$ from each firm. Therefore, $x_B(L) \geq \overline{x}_G$. Suppose that, instead of $x_B(L) = \overline{x}_G$, we have $x_B(L) > \overline{x}_G$. So far we have shown it cannot be $x_B(L) < \overline{x}_G$. Now, we rule out $x_B(L) > \overline{x}_G$. Since $p(\overline{\mathbf{x}}_G) > \underline{v}$, type $\underline{v}$ chooses $\mathbf{x}_B(L)$ only if $p(\mathbf{x}_B(L)) > \underline{v}$. This is possible only if $\mathbf{e}(L, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$ or $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$. Note that type $(\overline{v}, \underline{v})$ can always obtain a payoff of $n/2(\underline{v} + \overline{v})$ and revenue of $n/2\underline{v} \geq L$

by choosing $(0, n/2)$. Therefore, she would choose $\mathbf{x}_B(L)$ only if

$$2x_B(L)\,p(\mathbf{x}_B(L)) + (n/2 - x_B(L))(\underline{v} + \overline{v}) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\mathbf{x}_B(L)) \geq \frac{\underline{v} + \overline{v}}{2}$$

Let $\gamma > 0$ be the probability type $\underline{v}$ chooses $\mathbf{x}_B(L)$, then the highest possible value of $p(\mathbf{x}_B(L)))$ arises when type $(\overline{v}, \underline{v})$ chooses $\mathbf{x}_B(L)$ w.p. one. In this case,

$$p(\mathbf{x}_B(L)) = \underline{v} + \Delta \frac{\tau(1-\tau)}{[\tau(1-\tau) + (1-\tau)\tau] + \gamma(1-\tau)^2} < \frac{\underline{v} + \overline{v}}{2},$$

a contradiction.

7. In any equilibrium, if $p(\overline{\mathbf{x}}_G) > \frac{\underline{v}+\overline{v}}{2}$, then $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(L, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$, and if $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$ then $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ and $\mathbf{e}(L, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$. Proof: Note that if type $(\overline{v}, \underline{v})$ chooses $(0, n/2)$ then she obtains a payoff of $n/2(\underline{v} + \overline{v})$, and since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity need. Therefore, regardless of her liquidity need, type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_G$ if and only if

$$2\overline{x}_G p(\overline{\mathbf{x}}_G) + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) > n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{\mathbf{x}}_G) > \frac{\underline{v} + \overline{v}}{2}, \tag{48}$$

as required.

Based on the claims above, there are two types of equilibria: those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$ (balanced exit), and those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ (which can potentially be imbalanced exit). Consider first the balanced exit equilibrium. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. In particular $\overline{x}_G = \overline{x}_{co}(\tau)$. Note that Condition (48) and $p(\overline{\mathbf{x}}_G) = \overline{p}_{co}(\tau)$ imply that $\tau \geq \frac{1}{1+\sqrt{\beta}}$ is necessary. We now show profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$ and $\mathbf{v} = \overline{\mathbf{v}}$, the investor cannot receive more than $\overline{v}$ on each of her good firms, and so fully retains both.

2. Suppose $\mathbf{v} = \underline{\mathbf{v}}$. The equilibrium payoff is

$$\Pi^*(\underline{\mathbf{v}}) = 2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + 2(n/2 - \overline{x}_{co}(\tau))\,\underline{v} > \underline{v}.$$

Any deviation to sell $x > \overline{x}_{co}(\tau)$ from one of the firms generates a strictly lower payoff: the price of the sold firm falls to $\underline{v}$, and the price of the other firm does not increase unless it is fully retained, in which case it generates a payoff of $\underline{v}$ since the firm is bad. Any deviation to sell $x < \overline{x}_{co}(\tau)$ from one of the firms also generates a strictly lower payoff: the price of either firm does not increase unless it is fully retained in which case its payoff is its fundamental value of $\underline{v}$. Therefore, there is no profitable deviation.

3. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. Note that by construction $\overline{\mathbf{x}}_G p(\overline{\mathbf{x}}_G) = L/2$, and so the investor can meet her liquidity need by choosing $\overline{\mathbf{x}}_G$. Since Condition (48) holds, by choosing $\overline{\mathbf{x}}_G$ the investor obtains a payoff higher than $n/2(\underline{v} + \overline{v})$. Based on the pricing function, any deviation that involves selling strictly more than $\overline{x}_{co}$ from at least one firm is dominated by fully selling the bad firm and fully retaining the good firm, a strategy that generates a payoff of $n/2(\underline{v} + \overline{v})$. Consider a deviation that involves selling strictly $x < \overline{x}_{co}$ from at least one firm. If it is a balanced exit strategy, it generates a payoff of $2x\overline{p}_{co}(\tau) + 2(n/2 - x)\frac{\overline{v}+\underline{v}}{2}$. However,

$$
\begin{aligned}
x\overline{p}_{co}(\tau) + (n/2 - x)\frac{\overline{v}+\underline{v}}{2} &< \overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\frac{\overline{v}+\underline{v}}{2} \Leftrightarrow \\
x[\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2}] &< \overline{x}_{co}(\tau)[\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2}].
\end{aligned}
$$

Since $x < \overline{x}_{co}$ and $\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2} \geq 0$, choosing $x < \overline{x}_{co}$ is suboptimal. If it is an imbalanced exit strategy, the investor can profit from a deviation only if she sells less from the good firm. However, any such deviation lowers the price of the bad firm to $\underline{v}$. Therefore, an optimal deviation must involve the investor fully retaining the good firm. However, this deviation creates a payoff of $n/2(\underline{v} + \overline{v})$, which is lower than the equilibrium payoff.

4. Suppose $\theta = L$ and $\mathbf{v} = \overline{\mathbf{v}}$. The equilibrium payoff is

$$
\Pi^*(\overline{\mathbf{v}}) = 2\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + 2(n/2 - \overline{x}_{co}(\tau))\overline{v},
$$

Based on the pricing function, any deviation to selling more than $\overline{x}_{co}(\tau)$ from both firms requires the investor to sell more of the good firm but receive a price strictly lower than $\overline{p}_{co}(\tau)$ and $\overline{v}$, and hence generates a strictly lower payoff. Moreover, since $\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) = L/2$, any deviation to selling less than $\overline{x}_{co}(\tau)$ from both firms does not generate enough revenue to meet the liquidity need, and is thus suboptimal. Consider a deviation to imbalanced exit (x_i, x_j) such that $x_i < \overline{x}_{co}(\tau) < x_j$. The payoff and revenue are, respectively,

$$\hat{\Pi} = x_i \overline{p}_{co}(\tau) + x_j \underline{v} + (n - (x_i + x_j))\,\overline{v},$$

and

$$\hat{R} = x_i \overline{p}_{co}(\tau) + x_j \underline{v}.$$

This deviation is optimal only if $\hat{\Pi} \geq \Pi^*(\overline{\mathbf{v}})$ and $\hat{R} \geq L$. If there is such a deviation, then there is another optimal deviation in which $\hat{R} = L$ (selling less from either firm does not decrease the price). Since $2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) = L$, $\hat{\Pi} \geq \Pi^*(\overline{\mathbf{v}})$ requires

$$(n - (x_i + x_j))\,\overline{v} \geq 2\,(n/2 - \overline{x}_{co}(\tau))\,\overline{v} \Leftrightarrow (x_i + x_j) \leq 2\overline{x}_{co}(\tau)$$

Using $x_i \overline{p}_{co}(\tau) + x_j \underline{v} = L$ and $\overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)}$, the condition becomes $x_i \geq 2\overline{x}_{co}(\tau)$, which contradicts $x_i < \overline{x}_{co}(\tau)$. Therefore, there is no profitable deviation.

Next, consider the equilibrium with $\mathbf{e}(\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$. This implies that $\overline{x}_G = \overline{x}_{co}(\tau)$, and from Bayes' rule, $p(\overline{\mathbf{x}}_G) = \overline{p}_{co}(\tau)$ is given as in part (ii) of the proposition. Since we require $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, the expression for $\overline{p}_{co}(\tau)$ in part (ii) implies that $\tau < \frac{1}{1+\sqrt{\beta}}$.

We argue that if $\theta = L$ then type $(\overline{v}, \underline{v})$ must follow imbalanced exit, that is $e_i(L, (\overline{v}, \underline{v})) < e_j(L, (\overline{v}, \underline{v}))$. Suppose on the contrary $\mathbf{e}(L, (\overline{v}, \underline{v})) = \mathbf{x} = (x, x)$ where $x \neq \overline{x}_G$. Based on claims 3, 5, and 6, no other type is choosing $\mathbf{x}$. Therefore, $p(\mathbf{x}) = \frac{\underline{v}+\overline{v}}{2}$. Since $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, if $x > \overline{x}_G$ then type $\underline{v}$ has a strictly profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$: she can sell more units from each firm for a price higher than $p(\overline{\mathbf{x}}_G)$, and thereby get a strictly larger payoff. Consider $x < \overline{x}_G$. Note that $xp(\mathbf{x}) < L/2$, otherwise, type $\overline{\mathbf{v}}$ would have a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$ when $\theta = L$. Therefore, it cannot be $x < \overline{x}_G$. Thus, $e_i(L, (\overline{v}, \underline{v})) \neq e_j(L, (\overline{v}, \underline{v}))$, and from claim 2 we have $e_i(L, (\overline{v}, \underline{v})) < e_j(L, (\overline{v}, \underline{v}))$. Given claims 1 and 2, we can assume without loss of generality (and to ease the exposition) that the imbalanced strategy involves $\mathbf{e}(L, (\overline{v}, \underline{v})) = (0, n/2)$. Since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity need.

The equilibrium therefore hinges on type $(\overline{v}, \underline{v})$'s strategy when $\theta = 0$. The lowest price informativeness arises when type $(\overline{v}, \underline{v})$ follows balanced exit. There are two cases to consider:

1. If $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \mathbf{e}(0, \overline{\mathbf{v}})$, then

$$\begin{aligned} P'_{co,single}(\underline{v}) &= \tau\left(\beta\underline{v} + (1-\beta)\frac{\overline{v}+\underline{v}}{2}\right) + (1-\tau)\overline{p}_{co}(\tau) \\ P'_{co,single}(\overline{v}) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta)\tau\overline{v} + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)\frac{\overline{v}+\underline{v}}{2}. \end{aligned}$$

2. If $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(0, \overline{\mathbf{v}})$, then

$$\begin{aligned} P''_{co,single}(\underline{v}) &= \tau(\beta\underline{v} + (1-\beta)p(0)) + (1-\tau)\overline{p}_{co}(\tau) \\ P''_{co,single}(\overline{v}) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta)\tau p(0) + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)p(0). \end{aligned}$$

Note that

$$p(0) \in \left(\frac{\overline{v}+\underline{v}}{2}, \tau\overline{v} + (1-\tau)\frac{\overline{v}+\underline{v}}{2}\right)$$

and

$$\begin{aligned} P''_{co,single}(\underline{v}) &> P'_{co,single}(\underline{v}) \Leftrightarrow p(0) > \frac{\overline{v}+\underline{v}}{2}, \text{ and} \\ P''_{co,single}(\overline{v}) &< P'_{co,single}(\overline{v}) \Leftrightarrow p(0) < \tau\overline{v} + (1-\tau)\frac{\overline{v}+\underline{v}}{2}. \end{aligned}$$

Therefore, the equilibrium with the lowest price informativeness is obtained when type $(\overline{v}, \underline{v})$ chooses $(0, 0)$ when $\theta = 0$.

We now show that an equilibrium with $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(0, \overline{\mathbf{v}})$ exists. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the proposition. We now show that profitable deviations do not exist. As in the balanced exit equilibrium, it is straightforward to see that there is no profitable deviation when $\mathbf{v} = \overline{\mathbf{v}}$ or $\mathbf{v} = \underline{\mathbf{v}}$ (cases 1, 2, and 4 above). Indeed, the equilibrium strategies of these type do not change. Moreover, the proof only depend on the property $\overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)}$, which holds. One difference from the balanced exit is that price $p(0,0) < \overline{v}$, but this does not affect the investor's incentives since her payoff does not depend on the price when she fully retains the firms.

There is one remaining case to consider. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. The equilibrium payoff is $n/2(\underline{v} + \overline{v})$ and the revenue is when $\theta = L$ is $\underline{v}n/2 \geq L$ (the investor can meet her liquidity need by choosing $(0, n/2)$). Note that any deviation to imbalanced exit will continue to generate a payoff of $\underline{v}$ from the bad firm and at most $\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\,\overline{v} < \overline{v}$ from the good firm. Therefore, such a deviation is suboptimal. Any deviation to balanced exit (x, x), where $x > \overline{x}_{co}(\tau)$, will generate a payoff of $\underline{v}$ on the bad firm and a payoff of at most $\overline{x}_{co}(\tau)\,\underline{v} + (n/2 - \overline{x}_{co}(\tau))\,\overline{v} < \overline{v}$, on the good firm, which is suboptimal. Consider a deviation to balanced exit (x, x) where $x \leq \overline{x}_{co}(\tau)$. The investor's payoff is

$$2x\overline{p}_{co}(\tau) + (n/2 - x)(\overline{v} + \underline{v}),$$

which is smaller than $n/2(\underline{v} + \overline{v})$ if and only if $\overline{p}_{co}(\tau) \leq \frac{\overline{v}+\underline{v}}{2}$, which always if $\tau < \frac{1}{1+\sqrt{\beta}}$. Therefore, the investor has no profitable deviation, as required. Note that that monotonicity is preserved: $p(0) > \overline{p}_{co}(\tau) \Leftrightarrow \frac{(1-\tau)}{\tau^2} > \beta$, which always holds if $\tau < \frac{1}{1+\sqrt{\beta}}$. ■

Proof of Proposition 8. Recall that according to Lemma 1, $P_{so}(\underline{v}) = \overline{p}_{so}(\tau)$ and $P_{so}(\overline{v}) = \beta\overline{p}_{so}(\tau) + (1-\beta)\,\overline{v}$, where

$$\overline{p}_{so}(\tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}.$$

Consider part (i). Based on part (i) of Lemma 11, if $\tau \geq \frac{1}{1+\beta}$ then the equilibrium is the same as under the benchmark, and so price informativeness is the same. If $\tau < \frac{1}{1+\beta}$ then

$$\begin{aligned} P_{co,Separate}(\underline{v}, \tau) &= \beta\tau\underline{v} + (1-\beta\tau)\,\overline{p}_{co}(\tau) \\ P_{co,Separate}(\overline{v}, \tau) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta\tau)\,\overline{v} \end{aligned}$$

Using

$$\overline{p}_{co}(\tau) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)(1-\tau) + (\tau^2 + (1-\tau)^2)\beta}\Delta.$$

Using simple algebra, it can be verified that

$$P_{co,Separate}(\overline{v},\tau) > P_{so}(\overline{v}) \text{ and } P_{co,Separate}(\underline{v},\tau) < P_{so}(\underline{v}). \tag{49}$$

Consider part (ii). Based on Lemma 12, price informativeness is higher with two firms only if $\tau < \frac{1}{1+\sqrt{\beta}}$. Part (ii) of Lemma 12 also gives the fully uninformative equilibrium. In this equilibrium

$$\begin{aligned} P_{co,Single}(\underline{v},\tau) &= \tau\left(\beta\underline{v} + (1-\beta)\,p(0)\right) + (1-\tau)\,\overline{p}_{co}(\tau) \\ P_{co,Single}(\overline{v},\tau) &= \beta\tau\overline{p}_{co}(\tau) + \beta(1-\tau)\,\overline{v} + \left((1-\beta)\tau + (1-\beta)(1-\tau)\right)p(0) \end{aligned}$$

where

$$\begin{aligned} p(0) &= \underline{v} + \Delta\frac{\tau(1-\tau)+\tau^2}{\tau(1-\tau)+(1-\tau)\tau+\tau^2} = \underline{v} + \Delta\frac{1}{2-\tau} \\ \overline{p}_{co}(\tau) &= \underline{v} + \frac{\beta\tau^2}{\beta\left((1-\tau)^2+\tau^2\right)+(1-\beta)(1-\tau)^2}\Delta = \underline{v} + \frac{\beta\tau^2}{\beta\tau^2+(1-\tau)^2}\Delta \end{aligned}$$

Using simple algebra, it can be verified that

$$\begin{aligned} P_{co,Single}(\underline{v},\tau) &< P_{so}(\underline{v}) \Leftrightarrow \\ \frac{1-\beta}{2-\tau} + \frac{(1-\tau)\beta\tau}{\beta\tau^2+(1-\tau)^2} &< \frac{\beta}{\beta\tau+1-\tau} \end{aligned}$$

and

$$\begin{aligned} P_{co,Single}(\overline{v},\tau) &> P_{so}(\overline{v}) \Leftrightarrow \\ \frac{\beta^2\tau^3}{\beta\tau^2+(1-\tau)^2} + (1-\tau)\frac{3\beta-1-\tau\beta}{2-\tau} &> \frac{\beta^2\tau}{\beta\tau+1-\tau} \end{aligned}$$

Note that as $\beta \to 1$, the two conditions hold for any τ. Also note that if $\tau < \frac{1}{2}$ then $\tau < \frac{1}{1+\sqrt{\beta}}$ for all $\beta \in [0,1]$. Combined, it concludes the proof. ■

F.2 Single market maker

This section considers the case of a single market maker, who observes the order flows of all firms when setting prices. Since liquidity shocks are i.i.d. across investors under separate ownership, x_j contains no information relevant for the pricing of firm $i \neq j$. Therefore, the equilibria under separate ownership do not change. Below we derive the equilibria under common ownership. We focus on equilibria with monotonic prices, defined as in the two firm, single market maker case.

Proposition 9 (Single market maker): For all L, there exists an equilibrium under common ownership where price informativeness is strictly higher than under separate ownership. There also exists an equilibrium under common ownership where price informativeness is strictly lower than under separate ownership.

Proof. First, suppose $L/n \leq \underline{v}(1-\tau)$. Since the market maker observes all trades, and due to the law of large numbers, in equilibrium he knows (ex post) the exact measure of bad firms sold across all firms he buys. Thus, the investor will always receive the expected value of her portfolio ($\underline{v}+\tau\Delta$) in all states.[33] We show that there is an equilibrium in which, under common ownership, $x^*(\overline{v},\theta) = 0$ and $x^*(\underline{v},\theta) = \overline{x}\ \forall\ \theta \in \{0, L\}$, with $\overline{x} = \frac{L/n}{\underline{v}(1-\tau)} \leq 1$. For this candidate equilibrium, prices can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i = 0$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the investor generates revenue of $\overline{x}(1-\tau)\underline{v}n = L$, which is sufficient to cover her shock. Finally, prices are clearly monotonic. Thus, this is an equilibrium.

Next, suppose $L/n \in (\underline{v}(1-\tau), \underline{v}+\Delta\tau)$. Then, the investor must sell a positive amount of good firms upon a shock. Consider the candidate equilibrium where $x^*(\overline{v}, L) = \overline{x}$ and $x^*(\underline{v}, L) = 1$, with $\overline{x} = \frac{L/n - \underline{v}(1-\tau)}{\overline{v}\tau} < 1$, and $x^*(\overline{v}, 0) = 0$ and $x^*(\underline{v}, 0) = 1$. Prices in this

[33] With separate market makers, a maker maker does not observe the entire vector of trades, and thus may not know the exact measure of bad firms among the purchased assets.

equilibrium can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i \leq \overline{x} \text{ and } \int_{j:x_j>\overline{x}} dj = (1-\tau)\,n$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the investor by construction generates revenue of L, which is sufficient to cover her shock. Finally, prices are monotonic. Thus, this is an equilibrium.

Given these first two cases, if $L/n < \underline{v}+\Delta\tau$, then there exists an equilibrium under common ownership in which $P_{co}(\overline{v},\tau) = \overline{v}$ and $P_{co}(\underline{v},\tau) = \underline{v}$—that is, prices are fully informative and thus more informative than under separate ownership.

There is also an equilibrium under common ownership with fully uninformative prices when $L/n < \underline{v} + \Delta\tau$. Consider a candidate equilibrium in which $x^*(v,\theta) = \overline{x} \equiv \frac{L/n}{\underline{v}+\Delta\tau} < 1$ for all v and θ. Let equilibrium prices be

$$p_i(\mathbf{x}) = \underline{v} + \Delta\tau \text{ if } x_i = x_j : \forall i, j$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Under these strategies and prices, the investor raises $L = n\overline{xp}$ and receives a payoff of $\underline{v} + \Delta\tau$. Furthermore, there are no profitable deviations: deviating to another balanced exit strategy will still yield $\underline{v} + \Delta\tau$, while deviating to imbalanced exit will yield at most $\underline{v} + \Delta\tau$. Finally, since the prices for balanced exit are always $\underline{v} + \Delta\tau$, they satisfy monotonicity. Since prices are $\underline{v} + \Delta\tau$ for all firms, they are fully uninformative.

Finally, consider the case when $L/n \geq \underline{v} + \Delta\tau$. Then, in any equilibrium under common ownership, the investor must fully sell all firms when $\theta = L$. This implies that $p_i(\mathbf{x}) = \underline{v} + \Delta\tau$ when $x_i = 1$ for all i. The investor has two potential strategies when $\theta = 0$. The first is balanced exit, where trades and thus prices are fully uninformative. The second is imbalanced exit, where she sells $\underline{x}$ when $v_i = \overline{v}$, and $\overline{x} > \underline{x}$ when $v_i = \underline{v}$. Expected prices are:

$$P_{co}(\overline{v}, \tau) = (1-\beta)\overline{v} + \beta(\underline{v} + \Delta\tau)$$
$$P_{co}(\underline{v}, \tau) = (1-\beta)\underline{v} + \beta(\underline{v} + \Delta\tau) = \underline{v} + \Delta\tau\beta.$$

(Prices off-equilibrium can be described in a similar way to $L/n \in (\underline{v}(1-\tau), \underline{v} + \Delta\tau)$.) Under separate ownership, expected prices are:

$$P_{so}(\overline{v}, \tau) = (1-\beta)\overline{v} + \beta\left(\underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}\right)$$
$$P_{so}(\underline{v}, \tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}.$$

Since $P_{so}(\underline{v}, \tau) > P_{co}(\underline{v}, \tau)$ and $P_{so}(\overline{v}, \tau) < P_{co}(\overline{v}, \tau)$, prices are strictly more informative under common ownership. ■

F.3 Unobserved initial portfolio and information endowment

This section analyzes the trade-only model when the market makers do not observe whether the investor is concentrated or diversified. It also allows for the investor to be uninformed; her information endowment is also her private information. The market maker of firm i believes the investor is concentrated w.p. $\mu \in (0,1)$ and diversified w.p. $1-\mu$. Moreover, the market maker believes that she is informed with probability $\lambda \in (0,1)$ and uninformed with probability $1-\lambda$. The concentration of the investor's portfolio and her information endowment are independent of each other. For simplicity, we assume $L/n \leq \underline{v}(1-\tau)$ so that we are in the small-shock equilibrium of Lemma 2 where our results are strongest. We also assume that $L \leq \underline{v}$, so that the investor can satisfy her liquidity needs by selling one unit of an asset. This assumption guarantees that the concentrated investor does not have incentives to separate herself from the diversified investor by simply selling more than one unit.[34]

Proposition 10 presents the equilibrium. We slightly abuse notation by using $v_i = \underline{v} + \tau\Delta$

[34] If such separation is feasible, the anonymity of the investor plays no significant role as the investor's trade can reveal her portfolio. Note that since the investor's information endowment does not affect the set of feasible trades, this assumption does not change the investor's ability or incentives to reveal her information endowment through her trades.

to denote the case in which the investor is uninformed.

Proposition 10 (Anonymous Trade): Suppose the investor is concentrated w.p. μ and informed w.p. λ. If $L \leq \underline{v}$ and $1 \leq n(1-\tau)$, then in any equilibrium the concentrated investor's trading strategy is[35]

$$x_{so}^{*}(v_i,\theta) = \begin{cases} 0 & \textit{if } v_i \in \{\overline{v}, \underline{v}+\tau\Delta\} \;\; \textit{and } \theta = 0 \\ \overline{x}^{*} \equiv L/\overline{p} & \textit{otherwise,} \end{cases} \tag{50}$$

the diversified investor's trading strategy is

$$x_{co}^{*}(v_i,\theta) = \begin{cases} 0 & \textit{if } v_i = \overline{v} \;\; \textit{or, } v_i = \underline{v}+\tau\Delta \;\; \textit{and } \theta = 0 \\ \overline{x}^{*} & \textit{if } v_i = \underline{v} \;\; \textit{or, } v_i = \underline{v}+\tau\Delta \;\; \textit{and } \theta = L \end{cases}, \tag{51}$$

and prices are

$$p_i^{*}(x_i) = \begin{cases} \underline{v} + \Delta\frac{\tau\lambda(1-\beta\mu)+\tau(1-\lambda)(1-\beta)}{\tau\lambda(1-\beta\mu)+(1-\lambda)(1-\beta)} & \textit{if } x_i = 0 \\ \overline{p} \equiv \underline{v} + \Delta\frac{\lambda\mu\beta\tau+(1-\lambda)\beta\tau}{\lambda(\mu\beta\tau+1-\tau)+(1-\lambda)\beta} & \textit{if } x_i \in (0,\overline{x}] \\ \underline{v} & \textit{if } x_i > \overline{x}. \end{cases} \tag{52}$$

Moreover, price informativeness in equilibrium decreases with μ.

Intuitively, since $L/n < \underline{v}(1-\tau)$, if the market maker knows that a sale is from a diversified and informed investor, she knows that the asset must be bad for the same reasons as in the core model. Thus, a diversified investor will pool with a concentrated investor who suffers a liquidity shock. Thus, the equilibrium is similar to that under separate ownership except that the market maker takes into account the fact that a sale may be of a bad asset by a diversified investor, or by an uninformed investor who needs liquidity. This yields the expression for $\overline{p}$, the price upon a sale order of $\overline{x}^{*}$. Note that $\overline{p}$ increases in μ and that $p_i^{*}(0)$, which reflects the possibility that the uninformed investor fully retains her assets when unshocked, is decreasing in μ. Overall, price

[35] There is another equilibrium which is identical to the one described in the Proposition except that, if $v_i = \underline{v} + \tau\Delta$ and $\theta = 0$, then the investor sells $x_i > 0$ units, where $p(x_i) = \underline{v} + \tau\Delta$ and x_i is sufficiently small such that no other type mimics this strategy. In the proof we show that, in this equilibrium, price informativeness also decreases with μ.

informativeness decreases with the probability that the investor is concentrated. This extends the result of Proposition 1 to the case in which the investor's portfolio is private information, and so the market maker attaches an interior probability to her being concentrated.

Proof of Proposition 10. We first note that $L \leq \underline{v}$ and $1 \leq n(1-\tau)$ imply $L/n < \underline{v}(1-\tau)$. Therefore, the diversified (and informed) investor sells a positive amount of her good assets only if the price is $\overline{v}$. Without loss of generality, we assume that the diversified investor fully retains good assets—that is, $x^*_{co}(\overline{v}, \theta) = 0$. Similarly, the concentrated (and informed) investor who does not suffer a liquidity shock sells a positive amount from her good asset only if the price is $\overline{v}$, so without loss of generality $x^*_{so}(\overline{v}, 0) = 0$ as well.

We first show a set of results regarding the possible equilibrium selling strategies, similar to the proof of Lemma 1.

1. We prove that in any equilibrium, if $x_i > 1$ is on the path, then $p_i(x_i) = \underline{v}$. Let $\hat{x}_i$ be the highest quantity sold in equilibrium such that $p_i(\hat{x}_i) > \underline{v}$. Suppose on the contrary that $\hat{x}_i > 1$. Since $p_i(\hat{x}_i) > \underline{v}$, it must be that the investor sells $\hat{x}_i$ if $v_i \in \{\overline{v}, \underline{v} + \tau\Delta\}$. Since $L \leq \underline{v}$ and $1 \leq n(1-\tau)$, it must be $\hat{x}_i p_i(\hat{x}_i) > L$. Since prices are non-increasing, the investor can raise L in revenue by selling strictly less than $\hat{x}_i$. Therefore, selling $\hat{x}_i$ is weakly optimal only if $p_i(\hat{x}_i) = \overline{v}$ when $v_i = \overline{v}$ and $p_i(\hat{x}_i) \geq \underline{v} + \tau\Delta$ when $v_i = \underline{v} + \tau\Delta$. Suppose $p_i(\hat{x}_i) = \overline{v}$. Then, the investor has strict incentives to sell $\hat{x}_i$ units of firm i when $v_i = \underline{v}$. Indeed, since $\hat{x}_i$ is the highest quantity sold in equilibrium such that $p_i(\hat{x}_i) > \underline{v}$ and $p_i(\hat{x}_i) = \overline{v}$ is the highest price possible, by selling $\hat{x}_i$ the investor maximizes her profit. Therefore, it cannot be $p_i(\hat{x}_i) = \overline{v}$. Suppose $p_i(\hat{x}_i) \in [\underline{v} + \tau\Delta, \overline{v})$. Since $p_i(\hat{x}_i) < \overline{v}$, the investor never chooses $\hat{x}_i$ when $(v_i, \theta) = (\overline{v}, 0)$. Moreover, since $\hat{x}_i p_i(\hat{x}_i) > L$ and prices are non-increasing, there exists $x_i < \hat{x}_i$ such that $x_i p_i(x_i) \geq L$. Since $p_i(\hat{x}_i) < \overline{v}$, the investor has strict incentives to choose $x_i < \hat{x}_i$ when $(v_i, \theta) = (\overline{v}, L)$. Therefore, it must be $p_i(\hat{x}_i) = \underline{v} + \tau\Delta$, which implies that the investor sells $\hat{x}_i$ units when $v_i = \underline{v} + \tau\Delta$, but she never sells $\hat{x}_i$ units when $v_i = \underline{v}$. If the investor never sells $\hat{x}_i$ units when $v_i = \underline{v}$, then there must be $x'_i < \hat{x}_i$ such that $p_i(x'_i) > \underline{v} + \tau\Delta$ (or else, the investor has strict preferences to sell $\hat{x}_i$ units when $v_i = \underline{v}$, for the same reasons as in the case where $p_i(\hat{x}_i) = \overline{v}$). Since $p_i(x'_i) > \underline{v} + \tau\Delta$, the investor must be selling x'_i units when $v_i = \overline{v}$. Since the investor sells x'_i units when $v_i = \underline{v}$, $p_i(x'_i) < \overline{v}$. Since $p_i(x'_i) < \overline{v}$, the investor

sells x_i' units when $v_i = \overline{v}$ only if she needs liquidity. Therefore, it must be $x_i' p_i(x_i') \geq L$, or else, the investor has incentives to sells strictly more than x_i' when $(v_i, \theta) = (\overline{v}, L)$ in order to meet her liquidly needs. However, note that the investor strictly prefers selling x_i' units over selling $\hat{x}_i$ unit when $v_i = \underline{v} + \tau\Delta$, irrespective of her needs for liquidity. Indeed, by selling $\hat{x}_i$ units the investor's payoff is exactly $\underline{v} + \tau\Delta$. However, by selling x_i' units the investor raises enough revenue to meet her liquidity needs and she is getting $p_i(x_i') > \underline{v} + \tau\Delta$ for the x_i' units she is selling, which increases her profit. This implies that $\hat{x}_i$ must be off-equilibrium, a contradiction.

2. $x_\chi(\underline{v}, \theta) > 0$ and $x_\chi(\underline{v} + \tau\Delta, L) > 0$. To see this, note that if $\theta = L$, the investor will sell a positive amount. If $\theta = 0$ and $v_i = \underline{v}$, suppose that $x_i = 0$. Her payoff in this case is $\underline{v}$. Since $x_\chi(\underline{v} + \tau\Delta, L) > 0$, $p_i(x_\chi(\underline{v} + \tau\Delta, L)) > \underline{v}$, and so type $\underline{v}$ has a profitable deviation to $x_\chi(\underline{v} + \tau\Delta, L)$.

3. $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$. Suppose not. Then, it cannot be $x_\chi(\underline{v}, \theta) \notin \{x_{so}(\overline{v}, 0), x_{co}(\overline{v}, \theta), x_{so}(\underline{v} + \Delta\tau, 0), x_{co}(\underline{v} + \Delta\tau, 0)\}$, since then $p(x_\chi(\underline{v}, \theta)) = \underline{v}$ and the investor's payoff is $\underline{v}$. Indeed, since $x_{co}(\underline{v}+\tau\Delta, L) > 0$ and $p_i(x_{co}(\underline{v} + \tau\Delta, L)) > \underline{v}$, the investor has a profitable deviation to $x_{co}(\underline{v} + \tau\Delta, L) > 0$. However, since $x_{so}(\overline{v}, 0) = x_{co}(\overline{v}, \theta) = 0$, it must be $x_\chi(\underline{v}, \theta) \in \{x_{so}(\underline{v} + \Delta\tau, 0), x_{co}(\underline{v} + \Delta\tau, 0)\}$. Moreover, if $x_\chi(\underline{v} + \Delta\tau, 0) > 0$, then it must be $x_\chi(\underline{v} + \Delta\tau, 0) \notin \{x_{so}(\overline{v}, 0), x_{co}(\overline{v}, \theta)\}$. Therefore, if $x_\chi(\underline{v}, \theta) = x_{\chi'}(\underline{v} + \Delta\tau, 0) > 0$, then it must be $p_i(x_{\chi'}(\underline{v} + \Delta\tau, 0)) < \underline{v} + \Delta\tau$. However, if $p_i(x_{\chi'}(\underline{v} + \Delta\tau, 0)) < \underline{v} + \Delta\tau$, type $(\underline{v} + \Delta\tau, 0)$ has a profitable deviation to $x = 0$, which implies it must be $x_{\chi'}(\underline{v} + \Delta\tau, 0) = 0$, and therefore, $x_\chi(\underline{v}, \theta) = 0$. However, this contradicts point 2. Therefore, $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$ as required.

4. The investor sells strictly less than one unit from firm i. Suppose not. Based on point 1, if $x_i \geq 1$, then the price is $\underline{v}$, which means that the investor never sells more than one unit when $v_i \in \{\overline{v}, \underline{v} + \Delta\tau\}$. Based on point 3, it must be $x_\chi(\underline{v}, \theta) < 1$ as well.

5. $x_{so}(\underline{v}, 0) = x_{so}(\underline{v}, L) = x_{co}(\underline{v}, 0) = x_{co}(\underline{v}, L) = \overline{x} > 0$. Suppose on the contrary this is not the case. Based on point 4, $x_\chi(\underline{v}, \theta) < 1$. Since $n > 1$ and $\underline{v} \geq L$, the investor always meets her liquidity needs in equilibrium. Therefore, the liquidity constraint does

not bind, and the strategies of the investor when $v_i = \underline{v}$ must generate the same payoff. Suppose there are $(\chi', \theta') \neq (\chi'', \theta'')$ such that $x_{\chi'}(\underline{v}, \theta') = x' \neq x'' = x_{\chi''}(\underline{v}, \theta'')$. Payoffs are the same when $v_i = \underline{v}$ if

$$x'p_i(x') + (1 - x')\underline{v} = x''p_i(x'') + (1 - x'')\underline{v} \Leftrightarrow$$
$$x'p_i(x') - x''p_i(x'') = (x' - x'')\underline{v}.$$

Recall $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$. There are two cases. First $x' = x_{so}(\overline{v}, L)$ and $x'' = x_\chi(\underline{v} + \Delta\tau, L)$. Note that $x' \neq x''$ implies $x_{so}(\overline{v}, L) \neq x_\chi(\underline{v} + \Delta\tau, L)$, and by revealed preference:

$$\begin{aligned} x'p_i(x') - x''p_i(x'') &\geq (x' - x'')\left(\underline{v} + \Delta\tau\right) \text{ and,} \\ x'p_i(x') - x_L p_i(x'') &\leq (x' - x'')\overline{v} \end{aligned}$$

which holds if and only if

$$(x' - x'')\overline{v} \geq (x' - x'')\underline{v} \geq (x' - x'')\left(\underline{v} + \Delta\tau\right),$$

which can never hold. Second, $x' = x_{co}(\underline{v} + \Delta\tau, L)$ and $x'' = x_{so}(\underline{v} + \Delta\tau, L)$. Note that $x' \neq x''$ implies $x_{co}(\underline{v} + \Delta\tau, L) \neq x_{so}(\underline{v} + \Delta\tau, L)$, and by revealed preferences

$$\begin{aligned} x'p_i(x') - x''p_i(x'') &\geq (x' - x'')\left(\underline{v} + \Delta\tau\right) \text{ and,} \\ x'p_i(x') - x_L p_i(x'') &\leq (x' - x'')\left(\underline{v} + \Delta\tau\right) \end{aligned}$$

which holds if and only if

$$(x' - x'')\left(\underline{v} + \Delta\tau\right) \geq (x' - x'')\underline{v} \geq (x' - x'')\left(\underline{v} + \Delta\tau\right),$$

which holds if and only if $x' = x''$, a contradiction.

6. $x_\chi(\underline{v} + \Delta\tau, L) = \overline{x}$. Suppose instead $x_\chi(\underline{v} + \Delta\tau, L) = x'_\chi \neq \overline{x}$. Since $x'_\chi \neq \overline{x}$, point 5 yields $p_i(x'_\chi) \geq \underline{v} + \Delta\tau$. Moreover, since $n > 1$ and $\underline{v} \geq L$, the investor meets her liquidity

needs by selling x'_χ units. There are two cases:

(a) Suppose $p_i(\overline{x}) < p_i(x'_\chi)$. If $x'_\chi > \overline{x}$, then type $(\underline{v}, 0)$ has a profitable deviation to x'_χ, since she can sell more shares for a higher price. If $x'_\chi < \overline{x}$, then if $x_{so}(\overline{v}, L) = \overline{x}$, then type $(\overline{v}, L)$ has a profitable deviation to x'_χ (which satisfies her liquidity need). Therefore, it must be $x_{so}(\overline{v}, L) \neq \overline{x}$ and $p_i(\overline{x}) < \underline{v} + \Delta\tau$. This implies that $x_{\chi'}(\underline{v} + \Delta\tau, L) \neq \overline{x}$ as well: indeed, if $x_i = \overline{x}$ and $v_i = \underline{v} + \Delta\tau$, then the investor's payoff is strictly smaller than $\underline{v} + \Delta\tau$, but if $x_i = x'_\chi$ and $v_i = \underline{v} + \Delta\tau$, then the investor's payoff is strictly larger as she sells fewer units at a higher price. Therefore, it must be that $p_i(\overline{x}) = \underline{v}$. However, in this case and type $(\underline{v}, 0)$ has a profitable deviation to x'_i, a contradiction.

(b) Suppose $p_i(\overline{x}) \geq p_i(x'_i)$. Since $p_i(x'_\chi) \geq \underline{v} + \Delta\tau$, type $(\overline{v}, L)$ must play $\overline{x}$ with positive probability. By revealed preference, this means that

$$\overline{x} p_i(\overline{x}) - x_{\chi'} p_i(x_{\chi'}) \geq (\overline{x} - x_{\chi'})\overline{v}.$$

Since type $\underline{v}$ also weakly prefers $\overline{x}$ over $x_{\chi'}$,

$$\overline{x} p_i(\overline{x}) - x_{\chi'} p_i(x_{\chi'}) \geq (\overline{x} - x_{\chi'})\underline{v}.$$

However, type $(\underline{v} + \Delta\tau)$ weakly prefer $x_{\chi'}$ over $\overline{x}$,

$$\overline{x} p_i(\overline{x}) - x_{\chi'} p_i(x_{\chi'}) \leq (\overline{x} - x_{\chi'})\left(\underline{v} + \Delta\tau\right).$$

The combination of the three conditions implies $\overline{x} - x_{\chi'} = 0$, a contradiction.

7. $p_i(\overline{x}) < \underline{v} + \tau\Delta$. Based on points 1–6 and the application of Bayes' rule,

$$p_i(\overline{x}) \leq \max_{\gamma \in [0,1]} \left\{ \underline{v} + \Delta \frac{\lambda\mu\beta\tau + (1-\lambda)\left(\beta + (1-\beta)\gamma\right)\tau}{\lambda\left[\mu\left(\beta\tau + 1 - \tau\right) + (1-\mu)(1-\tau)\right] + (1-\lambda)\left(\beta + (1-\beta)\gamma\right)} \right\}.$$

Indeed, the highest possible value of $p_i(\overline{x})$ arises if $x_{so}(\overline{v}, L) = \overline{x}$ w.p. 1, and $x_\chi(\underline{v} + \Delta\tau, 0) = \overline{x}$ w.p. γ. Note that since $\tau \in (0,1)$ and $\lambda \in (0,1)$, for every $\gamma \in [0,1]$ the RHS is strictly smaller than $\underline{v} + \Delta\tau$.

8. $x_{so}(\overline{v}, L) = \overline{x}$. Suppose instead $x_{so}(\overline{v}, L) = x_i' \neq \overline{x}$. Based on points 6 and 7, $p_i(x_i') > \underline{v} + \Delta\tau > p_i(\overline{x})$. Therefore, type $(\underline{v} + \Delta\tau, L)$ has strict incentives to deviate to x_i' since it leads to a trading profit and also satisfies her liquidity need.

9. $x_\chi(\underline{v} + \Delta\tau, 0) \neq \overline{x}$. Suppose instead that $x_\chi(\underline{v} + \Delta\tau, 0) = \overline{x}$. Based on point 7, $p_i(\overline{x}) < \underline{v} + \Delta\tau$. Therefore, type $(\underline{v} + \Delta\tau, 0)$ has strict incentives to deviate from $x_i = \overline{x}$ to $x_i = 0$.

Given the claims above, Bayes' rule implies $p_i(\overline{x}) = \overline{p}$. Note that $L \leq \underline{v}$ implies $\overline{x}^* = L/\overline{p} \leq 1$. We prove that, in any equilibrium, $\overline{x} = \overline{x}^*$. Suppose on the contrary that $\overline{x} > \overline{x}^*$. Then, it has to be $\overline{xp} > \overline{x}^*\overline{p} = L$. Since the pricing function is non-increasing, there is $\varepsilon > 0$ such that $(\overline{x} - \varepsilon)\, p\,(\overline{x} - \varepsilon) \geq L$. This implies that type $(\overline{v}, L)$ of the concentrated investor will strictly prefer deviating to $\overline{x} - \varepsilon$, a contradiction. We conclude that $\overline{x} \leq \overline{x}^*$. Next, suppose on the contrary $\overline{x} < \overline{x}^*$. Then, it has to be $\overline{xp} < \overline{x}^*\overline{p} = L$. Type $(\overline{v}, L)$ of the concentrated investor does not raise revenue of L in equilibrium by selling $\overline{x}$ units of the firm. Consider a deviation to selling one unit. Since $p\,(1) \geq \underline{v}$, the revenue raised would be at least $\underline{v} \geq L$, and so the deviation is optimal, a contradiction.

Note that $x_\chi(\underline{v} + \Delta\tau, 0) \neq 0$ can be sustained in equilibrium, if it is small enough to prevent mimicking by types who choose $\overline{x}^*$. At the same time, $x_\chi(\underline{v} + \Delta\tau, 0) = 0$ can also be in equilibrium. We consider these two cases separately below:

1. Suppose $x_\chi(\underline{v} + \Delta\tau, 0) = 0$. The pricing function in Proposition 10 is consistent with the trading strategy and it is non-increasing. Note that the trading strategy is incentive-compatible given prices. Consider the concentrated investor. First, the equilibrium payoff of type $(\overline{v}, 0)$ is $\overline{v}$, the highest possible. Second, since $\overline{x}^*\overline{p} = L$ and $p^*\,(x)$ is flat on $(0, \overline{x}^*]$, deviating to $(0, \overline{x}^*]$ generates revenue strictly lower than L, and so is suboptimal if $\theta = L$. Moreover, since $x > \overline{x}^* \Rightarrow p^*\,(x) = \underline{v}$, the investor has no optimal deviation to $x > \overline{x}^*$, regardless of firm value. Last, it is easy to see that $x = \overline{x}^*$ is optimal for type $(\underline{v}, 0)$. Consider the diversified investor. Since $\overline{p} \in (\underline{v}, \underline{v} + \Delta\tau)$, $p^*\,(x)$ is flat on $(0, \overline{x}^*]$, and $x > \overline{x}^* \Rightarrow p^*\,(x) = \underline{v}$, she has incentives to sell $\overline{x}^*$ from all bad assets. Moreover, since

$\overline{x}^* \overline{p} = L$, $L \leq \underline{v}$ and $1 \leq n(1-\tau)$ imply that by selling $\overline{x}^*$ the diversified investor raises enough revenue to meet her liquidity need.

Note that the expected price in equilibrium conditional on $\underline{v}$ and $\overline{v}$, respectively is,

$$\begin{aligned} P(\underline{v}) &= \lambda \overline{p} + (1-\lambda)\left[\beta \overline{p} + (1-\beta) p^*(0)\right] \\ P(\overline{v}) &= \lambda\left(\beta \mu \overline{p} + (1-\beta\mu) p^*(0)\right) + (1-\lambda)\left[\beta \overline{p} + (1-\beta) p^*(0)\right]. \end{aligned}$$

It can be verified that $P(\underline{v})$ is increasing in μ. Since $\tau P(\overline{v}) + (1-\tau) P(\underline{v}) = \underline{v} + \Delta\tau$, it follows that higher μ implies higher $P(\underline{v})$ and a lower $P(\overline{v})$, and therefore, price informativeness decreases in μ.

2. Suppose $x_\chi(\underline{v} + \Delta\tau, 0) \neq 0$. The only difference from the previous case is that $p^*(0) = \overline{v}$ and $p^*(x_\chi(\underline{v} + \Delta\tau, 0)) = \underline{v} + \Delta\tau$. The proof that the trading strategy is incentive-compatible given prices is very similar to the case 1 above, and for brevity it is omitted. The expected price in equilibrium conditional on $\underline{v}$ and $\overline{v}$, respectively is,

$$\begin{aligned} P(\underline{v}) &= \lambda \overline{p} + (1-\lambda)\left[\beta \overline{p} + (1-\beta)(\underline{v} + \Delta\tau)\right] \\ P(\overline{v}) &= \lambda\left(\beta \mu \overline{p} + (1-\beta\mu)\overline{v}\right) + (1-\lambda)\left[\beta \overline{p} + (1-\beta)(\underline{v} + \Delta\tau)\right]. \end{aligned}$$

It can be verified that $\overline{p}$ is increasing in μ. Therefore, $P(\underline{v})$ is also increasing in μ, which implies that price informativeness decreases in μ.

■

F.4 Heterogeneous firms

In the core model, all firms have the same valuation distribution Δ. We now analyze the case in which firms have different valuation distributions, and thus differ in their information asymmetry and the price impact of selling. For brevity, we consider the small shock case, since this is where our results are strongest.

Let there be $J \geq 1$ classes of firms. The valuation of firm i in class $j \in \{1, ..., J\}$ is $v_{i,j} \in \{\underline{v}_j, \overline{v}_j\}$, where $\Delta_j \equiv \overline{v}_j - \underline{v}_j > 0$. We assume $\underline{v}_{j'} < \overline{v}_{j''}$ for all $j' \in J$ and $j'' \in J$—that is, the worst good firm is more valuable than the best bad firm. We also index by j the exogenous parameters τ, $\overline{v}$, and $\underline{v}$. As in the core model, each firm has its own market maker, and the class to which firm i belongs is common knowledge. All random variables are independent across firms and classes.

The analysis of separate ownership remain unchanged, with the addition of a subscript j to denote that quantities apply to a firm of class j. Under common ownership, we assume the investor owns a mass of $n_j \geq 0$ firms from class j.

Proposition 11 (Heterogeneous firms): Suppose $L \leq \sum_{j=1}^{J} n_j \underline{v}_j (1-\tau_j)$, and let γ_j denote the probability that $x_{j,i} = 0$ when $v_{j,i} = \underline{v}_j$. For firm class j, if $\gamma_j \leq \frac{\beta\tau_j}{\beta\tau_j + (1-\beta)(1-\tau_j)}$, price informativeness is weakly higher under common ownership than separate ownership. Conversely, if $\gamma_j > \frac{\beta\tau_j}{\beta\tau_j + (1-\beta)(1-\tau_j)}$, price informativeness is strictly higher under separate ownership.

Proof. Since $L \leq \sum_{j=1}^{J} n_j \underline{v}_j (1-\tau_j)$, the investor can meet her liquidity need by selling bad firms alone. Then, in any equilibrium, as in the core model it must be that for firm i in class j, $x_i > 0 \to p_{j,i}(x_i) = \underline{v}_j$. Then, if $v_{j,i} = \overline{v}_j$, $x^*_{j,div}(v_{j,i}, \theta) = 0$. If $v_{j,i} = \underline{v}_j$, similar to the core model, $x^*_{j,div}(v_{j,i}, \theta) = \underline{x}_j(\theta) \in \{0, 1\}$, with $\underline{x}_j$ such that $\sum_{j=1}^{J} \underline{x}_j(\theta) n_j \underline{v}_j (1-\tau_j) \in \left[\theta, \sum_{j=1}^{J} n_j \underline{v}_j (1-\tau_j)\right]$. Letting $\gamma_j \in \{0, \beta, 1-\beta, 1\}$ denote the probability that $x_{j,i} = 0$ when $v_{j,i} = \underline{v}_j$, prices on the equilibrium path are then:

$$
\begin{aligned}
p_{j,i}(x_{j,i}) &= \underline{v}_j \;\; \textit{if}\, x_{j,i} > 0 \\
p_{j,i}(x_{j,i}) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j} \right) \;\; \textit{if}\, x_{j,i} = 0.
\end{aligned}
$$

Then, expected prices under common ownership are:

$$
\begin{aligned}
P_{j,div}(\overline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j} \right) \\
P_{j,div}(\underline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \gamma_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j} \right).
\end{aligned}
$$

under separate ownership, expected prices are:

$$P_{j,con}(\overline{v}_j,\tau_j) = \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j + (1-\beta)(1-\tau_j)}{\beta\tau_j + (1-\tau_j)} \right)$$
$$P_{j,con}(\underline{v}_j,\tau_j) = \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j}{\beta\tau_j + (1-\tau_j)} \right).$$

Then, we have

$$\begin{aligned}
P_{j,div}(\underline{v}_j,\tau_j) \leq P_{j,con}(\underline{v}_j,\tau_j) &\iff \\
\frac{\gamma_j\tau_j}{\tau_j + (1-\tau_j)\gamma_j} \leq \frac{\beta\tau_j}{\beta\tau_j + (1-\tau_j)} &\iff \\
\beta\tau_j\gamma_j + (1-\tau_j)\gamma_j \leq \beta\tau_j + \beta(1-\tau_j)\gamma_j &\iff \\
\frac{\beta\tau_j}{\beta\tau_j + (1-\beta)(1-\tau_j)} \geq \gamma_j. &
\end{aligned}$$

Price informativeness is higher under common ownership if this inequality holds, which is the same as in the core model (with subscript j added). ■

F.5 Discontinuing relationships

In this section, we apply our model to situations in which the investor decides whether to (partly) discontinue a relationship with the firm. We thus distinguish between two concepts—the price $p_i(x_i)$ reflects the impact of (dis)continuation on firm i's reputation, and the payoff upon selling is what the investor receives. In the core model, these were the same, but now the latter is the investor's outside option, $r < \overline{v}$.[36] Importantly, unlike in the core model, this reservation payoff is fixed and independent of the impact of sale on the firm's reputation. However, we nevertheless show that common ownership can still improve price informativeness.

We consider two cases based on the magnitude of r. In the first case, $r < \underline{v}$, and so the investor discontinues only if she needs liquidity. In the second case, $r \in (\underline{v}, \overline{v})$, and so the investor discontinues her relationship with bad firms, but retains it with good firms.

[36] If $r \geq \overline{v}$, the analysis is trivial since the investor is weakly better off discontinuing all assets, regardless of their value and her liquidity needs. This behavior will result in identical price informativeness under separate and common ownership and, in particular, expected prices will be the same for both good and bad firms.

Proposition 12 (Discontinuing relationships, low reservation payoff): Suppose $r < \underline{v}$. Price informativeness under common ownership is always weakly higher than under separate ownership, and strictly higher when $L/n < r$.

Proof. First, consider separate ownership. The optimal strategies are $x_i^*(v_i, L) = \overline{x} \equiv \min\{\frac{L/n}{r}, 1\}$ for all v_i, and $x_I^*(v_i, 0) = 0$ for all v_i. Then, prices are fully uninformative.

Next, consider common ownership. Note first that, for all L/n, it is optimal not to discontinue when $\theta = 0$. Now, suppose $L/n \leq r(1-\tau)$. Then, when $\theta = L$, the optimal strategies are $x_i^*(\overline{v}, L) = 0$ and $x_i^*(\underline{v}, L) = \overline{x} \equiv \frac{L/n}{(1-\tau)r}$. In that case $p(x_i) = \underline{v}$ for $x_i = \overline{x}$, and $p(x_i) = \underline{v} + \Delta\frac{\tau}{\tau+(1-\beta)(1-\tau)}$ for $x_i = 0$. If, instead, $L/n \in (r(1-\tau), r)$, optimal strategies when $\theta = L$ are $x_i^*(\underline{v}, L) = 1$ and $x_i^*(\overline{v}, L) = \frac{L/n - r(1-\tau)}{r\tau}$. Thus, again prices are partially informative. Finally, if $L/n \geq r$, we have $x_i^*(v_i, L) = 1$ for all v_i, and so prices are fully uninformative.

Therefore, prices are strictly more informative under common ownership when $L/n < r$, and equally (un)informative when $L/n \geq r$. ■

Proposition 13 (Discontinuing relationships, high reservation payoff): Suppose $r \in (\underline{v}, \overline{v})$. Price informativeness under common ownership is always identical to that under separate ownership.

Proof. Note that when $r \in (\underline{v}, \overline{v})$, the investor always wants to discontinue bad firms fully. First, consider separate ownership. Then, $x_i^*(\underline{v}, \theta) = 1$ for all θ,$x_i^*(\overline{v}, 0) = 0$, and $x_i^*(\overline{v}, L) = \overline{x} \equiv \min\{\frac{L/n}{r}, 1\}$. Then, if $L/n < r$, prices are $p_i(x_i) = \underline{v}$ if $x_i = 1$, and $p_i(x_i) = \overline{v}$ is $x_i \in \{0, \overline{x}\}$.[37] Otherwise, they are $p_i(x_i) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ if $x_i = 1$ and $p_i(x_i) = \overline{v}$ if $x_i = 0$.

Under common ownership, $x_i^*(\underline{v}, \theta) = 1$ for all θ, and $x_i^*(\overline{v}, 0) = 0$. If $\frac{L}{n} < r$, then $x_i^*(\overline{v}, L) = \overline{x} \equiv \max\{0, \frac{L/n - r(1-\tau)}{\tau r}\}$. In this case, prices are $p_i(x_i) = \overline{v}$ if $x_i \in \{0, \overline{x}\}$, and $p_i(x_i) = \underline{v}$ if $x_i = 1$.

Alternatively, if $\frac{L}{n} \geq r$, then $x_i^*(\overline{v}, L) = 1$. Prices are $p_i(x_i) = \overline{v}$ if $x_i = 0$, and $p_i(x_i) = \underline{v} + \Delta\tau$ if $x_i = 1$.

Thus, if $L/n < r$,

$$P_{so}(\overline{v}, \tau) = \overline{v} \ = P_{co}(\overline{v}, \tau),$$

[37] Since off-equilibrium prices are irrelevant for the seller's discontinuation decision, we do not specify them to ease the exposition.

and

$$P_{so}(\underline{v}, \tau) = \underline{v} = P_{co}(\underline{v}, \tau).$$

If instead $L/n \geq r$,

$$\begin{aligned} P_{so}(\overline{v}, \tau) &= \underline{v} + \Delta \frac{\beta\tau + (1-\beta)(1-\tau)}{\beta\tau + 1 - \tau} = P_{co}(\overline{v}, \tau) \\ P_{so}(\underline{v}, \tau) &= \underline{v} + \Delta \frac{\beta\tau}{\beta\tau + 1 - \tau} = P_{co}(\underline{v}, \tau) \end{aligned}$$

Thus, price informativeness under separate ownership is identical to that under common ownership. ■

The intuition is as follows. Since the investor is unconcerned with price impact, she sells a bad firm fully and thus receives the lowest possible price of $\underline{v}$ under any ownership structure. Thus, if the shock is not large enough to force the sale of her entire portfolio ($\frac{L}{n} < r$), prices are already fully informative under separate ownership and so no higher under common ownership.