

Empirical Asset Pricing

Assignment 01

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Question 1

(a) $f(v_t, \theta) =$

$$\begin{bmatrix} R_{t1} - \mu_1 \\ R_{t2} - \mu_2 \\ (R_{t1} - \mu_1)^2 - \sigma_1^2 \\ (R_{t2} - \mu_2)^2 - \sigma_2^2 \end{bmatrix}$$

$\theta =$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \sigma_1^2 \\ \sigma_2^2 \end{bmatrix}$$

$\frac{\partial f}{\partial \theta'}$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -2(R_{t1} - \mu_1) & 0 & -1 & 0 \\ 0 & -2(R_{t2} - \mu_2) & 0 & -1 \end{bmatrix}$$

$\hat{D}_T = \frac{1}{T} \sum \frac{\partial f}{\partial \theta'} =$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$A_T = I$

$A_T g_T(\theta) = 0$

$g_T(\theta) = 0 =$

$$\begin{bmatrix} \frac{\sum R_{t1}}{T} - \mu_1 \\ \frac{\sum R_{t2}}{T} - \mu_2 \\ \frac{\sum (R_{t1} - \mu_1)^2}{T} - \sigma_1^2 \\ \frac{\sum (R_{t2} - \mu_2)^2}{T} - \sigma_2^2 \end{bmatrix}$$

therefore,

$$\hat{\mu}_1 = \frac{\sum R_{t1}}{T} \quad \hat{\mu}_2 = \frac{\sum R_{t2}}{T} \quad \hat{\sigma}_1^2 = \frac{\sum (R_{t1} - \hat{\mu}_1)^2}{T} \quad \hat{\sigma}_2^2 = \frac{\sum (R_{t2} - \hat{\mu}_2)^2}{T}$$

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(b) $f(v, \theta)f(v, \theta)' =$

$$\begin{bmatrix} R_{t1} - \mu_1 \\ R_{t2} - \mu_2 \\ (R_{t1} - \mu_1)^2 - \sigma_1^2 \\ (R_{t2} - \mu_2)^2 - \sigma_2^2 \end{bmatrix} \begin{bmatrix} R_{t1} - \mu_1 & R_{t2} - \mu_2 & (R_{t1} - \mu_1)^2 - \sigma_1^2 & (R_{t2} - \mu_2)^2 - \sigma_2^2 \end{bmatrix}$$

given $R1 \sim N(\mu_1, \sigma_1^2)$ and $R2 \sim N(\mu_2, \sigma_2^2)$

$\hat{S}_T =$

$$\begin{bmatrix} \hat{\sigma}_1^2 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_2^2 & 0 & 0 \\ 0 & 0 & 2\hat{\sigma}_1^4 & 0 \\ 0 & 0 & 0 & 2\hat{\sigma}_2^4 \end{bmatrix}$$

(c) $f(v_t, \theta)f(v_{t-1}, \theta)' =$

$$\begin{bmatrix} R_{t1} - \mu_1 \\ R_{t2} - \mu_2 \\ (R_{t1} - \mu_1)^2 - \sigma_1^2 \\ (R_{t2} - \mu_2)^2 - \sigma_2^2 \end{bmatrix} \begin{bmatrix} R_{t1-1} - \mu_1 & R_{t2-1} - \mu_2 & (R_{t1-1} - \mu_1)^2 - \sigma_1^2 & (R_{t2-1} - \mu_2)^2 - \sigma_2^2 \end{bmatrix}$$

$\hat{\Gamma}_1 =$

$$\begin{bmatrix} \frac{\sum_2^T (R_{t1} - \mu_1)(R_{t1-1} - \mu_1)}{T-1} & 0 & \frac{\sum_2^T (R_{t1} - \mu_1)(R_{t1-1} - \mu_1)^2}{T-1} & 0 \\ 0 & \frac{\sum_2^T (R_{t2} - \mu_2)(R_{t2-1} - \mu_2)}{T-1} & 0 & \frac{\sum_2^T (R_{t2} - \mu_2)(R_{t2-1} - \mu_2)^2}{T-1} \\ \frac{\sum_2^T (R_{t1-1} - \mu_1)(R_{t1-1} - \mu_1)^2}{T-1} & 0 & \frac{\sum_2^T (R_{t1-1} - \mu_1)^2 (R_{t1-1} - \mu_1)^2}{T-1} + \sigma_1^4 & 0 \\ 0 & \frac{\sum_2^T (R_{t2-1} - \mu_2)(R_{t2-1} - \mu_2)^2}{T-1} & 0 & \frac{\sum_2^T (R_{t2-1} - \mu_2)^2 (R_{t2-1} - \mu_2)^2}{T-1} \end{bmatrix}$$

$\hat{S}_T =$

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & 2\sigma_1^4 & 0 \\ 0 & 0 & 0 & 2\sigma_2^4 \end{bmatrix} + \frac{1}{2}(\hat{\Gamma}_1 + \hat{\Gamma}_1')$$

(d) $H_0 : \mu_1 * \sigma_2 - \mu_2 * \sigma_1 = 0$

$R(\theta) = \mu_1 * \sigma_2 - \mu_2 * \sigma_1$

$\theta =$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$$

$\frac{\partial R(\theta)}{\partial \theta'} =$

$$\begin{bmatrix} \sigma_2 & -\sigma_1 & -\mu_2 & \mu_1 \end{bmatrix}$$

$T(R(\hat{\theta}_T)'[\frac{\partial R(\theta)}{\partial \theta'} \hat{V} \frac{\partial R(\theta)}{\partial \theta'}]^{-1}(R(\hat{\theta})) \sim \chi^2$

Sharpe Ratio Stock 1: 0.1113

Sharpe Ratio Stock 2: 0.0352

T-test Statistic: 1.4645

p-value: 0.1433 Fail to reject the null hypothesis: No significant difference in Sharpe ratios.

(e) After introducing the correlation matrix:

Sharpe Ratio Stock 1: 0.1113

Sharpe Ratio Stock 2: 0.0352

Sample Correlation: 0.4187
T-test Statistic: 12.5329
Degrees of Freedom: 1198
p-value: 0.0000
Reject the null hypothesis: Stocks have different Sharpe ratios.

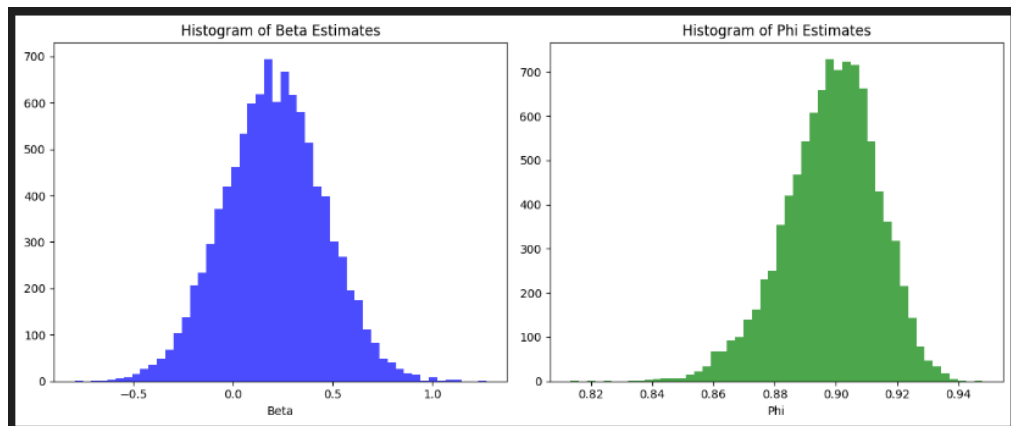
- (f) Observed Sharpe Ratio Difference: 0.0761
95% Confidence Interval: [-0.0325, 0.1561]
The sharp ratio difference is not significant at 5% level.

Question 2

- (a) Optional

Question 3

- (a) N=840



- (b) N=240
Looks more skewed to the right side for $\hat{\phi}$

