# Empirical Asset Pricing
# Assignment 01

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## Question 1

(a) Let's define the variables that we need to use in the estimation.

$$f(v_t,\theta) = \begin{bmatrix} R_{t1}-\mu_1 \\ R_{t2}-\mu_2 \\ (R_{t1}-\mu_1)^2-\sigma_1^2 \\ (R_{t2}-\mu_2)^2-\sigma_2^2 \end{bmatrix} \quad , \quad \theta = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \sigma_1^2 \\ \sigma_2^2 \end{bmatrix}$$

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} f(v_t,\theta)$$

We know from the lecture that we need to calculate the $\frac{\partial f}{\partial \theta'}$ to get the $\hat{D_T}$:

$$\frac{\partial f(v_t,\theta)}{\partial \theta'} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -2(R_{t1}-\mu_1) & 0 & -1 & 0 \\ 0 & -2(R_{t2}-\mu_2) & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \hat{D_T} = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial f(v_t,\theta)}{\partial \theta'} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = -I$$

We also know that $A_T = I$ and $A_T g_T(\theta) = 0$. Therefore, we can calculate the $\hat{\theta}$:

$$A_T g_T(\theta) = 0 \Rightarrow \quad g_T(\theta) = 0$$

$$g_T(\theta) = \begin{bmatrix} \frac{\sum R_{t1}}{T} - \mu_1 \\ \frac{\sum R_{t2}}{T} - \mu_2 \\ \frac{\sum (R_{t1}-\mu_1)^2}{T} - \sigma_1^2 \\ \frac{\sum (R_{t2}-\mu_2)^2}{T} - \sigma_2^2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} \frac{\sum_{t=1}^{T} R_{t1}}{T} \\ \frac{\sum_{t=1}^{T} R_{t2}}{T} \\ \frac{\sum_{t=1}^{T} (R_{t1}-\hat{\mu_1})^2}{T} \\ \frac{\sum_{t=1}^{T} (R_{t2}-\hat{\mu_2})^2}{T} \end{bmatrix} = \begin{bmatrix} \hat{\mu_1} \\ \hat{\mu_2} \\ \hat{\sigma_1^2} \\ \hat{\sigma_2^2} \end{bmatrix} = \hat{\theta}$$

Our calculated $\hat{\theta}$ based on the given data is:

$$\hat{\theta} = \begin{bmatrix} 0.0162 \\ 0.0045 \\ 0.0212 \\ 0.0167 \end{bmatrix}$$

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mu_1 = sum(df['Stock1'])/len(df['Stock1'])
mu_2 = sum(df['Stock2'])/len(df['Stock2'])
sigma_1 = sum((df.Stock1 - mu_1)**2)/(len(df.Stock1))
sigma_2 = sum((df.Stock2 - mu_2)**2)/(len(df.Stock2))
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Listing 1: Python code for calculating $\hat{\theta}$

(b) Still we assume that there is no serial correlation in the moments. Therefore, we can calculate the $\hat{S_T}$ as follows:

$$\hat{S_T} = \frac{1}{T}\sum_{t=1}^{T} f(v_t,\theta)f(v_t,\theta)'$$

$$= \frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix} (R_{t1}-\mu_1)^2 & (R_{t1}-\mu_1)(R_{t2}-\mu_2) & (R_{t1}-\mu_1)^3-\sigma_1^2(R_{t1}-\mu_1) & (R_{t1}-\mu_1)(R_{t2}-\mu_2)^2-\sigma_2^2(R_{t1}-\mu_1) \\ (R_{t1}-\mu_1)(R_{t2}-\mu_2) & (R_{t2}-\mu_2)^2 & (R_{t1}-\mu_1)^2(R_{t2}-\mu_2)-\sigma_1^2(R_{t2}-\mu_2) & (R_{t2}-\mu_2)^3-\sigma_2^2(R_{t2}-\mu_2) \\ (R_{t1}-\mu_1)^3-\sigma_1^2(R_{t1}-\mu_1) & (R_{t1}-\mu_1)^2(R_{t2}-\mu_2)-\sigma_1^2(R_{t2}-\mu_2) & (R_{t1}-\mu_1)^4-2\sigma_1^2(R_{t1}-\mu_1)^2+\sigma_1^4 & (R_{t1}-\mu_1)^2(R_{t2}-\mu_2)^2-\sigma_1^2(R_{t1}-\mu_1)\sigma_2^2(R_{t2}-\mu_2) \\ (R_{t1}-\mu_1)(R_{t2}-\mu_2)^2-\sigma_2^2(R_{t1}-\mu_1) & (R_{t2}-\mu_2)^3-\sigma_2^2(R_{t2}-\mu_2) & (R_{t1}-\mu_1)^2(R_{t2}-\mu_2)^2-\sigma_1^2(R_{t1}-\mu_1)\sigma_2^2(R_{t2}-\mu_2) & (R_{t2}-\mu_2)^4-2\sigma_2^2(R_{t2}-\mu_2)^2+\sigma_2^4 \end{bmatrix}$$

(a) $f(v_t,\theta) =$

(b) $f(v,\theta)f(v,\theta)' =$

$$\begin{bmatrix} R_{t1}-\mu_1 \\ R_{t2}-\mu_2 \\ (R_{t1}-\mu_1)^2-\sigma_1^2 \\ (R_{t2}-\mu_2)^2-\sigma_2^2 \end{bmatrix}\begin{bmatrix} R_{t1}-\mu_1 & R_{t2}-\mu_2 & (R_{t1}-\mu_1)^2-\sigma_1^2 & (R_{t2}-\mu_2)^2-\sigma_2^2 \end{bmatrix}$$

given $R1 \sim N(\mu_1,\sigma_1^2)$ and $R2 \sim N(\mu_2,\sigma_2^2)$

$\hat{S_T} =$

$$\begin{bmatrix} \hat{\sigma_1}^2 & 0 & 0 & 0 \\ 0 & \hat{\sigma_2}^2 & 0 & 0 \\ 0 & 0 & 2\hat{\sigma_1}^4 & 0 \\ 0 & 0 & 0 & 2\hat{\sigma_2}^4 \end{bmatrix}$$

(c) $f(v_t,\theta)f(v_{t-1},\theta)' =$

$$\begin{bmatrix} R_{t1}-\mu_1 \\ R_{t2}-\mu_2 \\ (R_{t1}-\mu_1)^2-\sigma_1^2 \\ (R_{t2}-\mu_2)^2-\sigma_2^2 \end{bmatrix}\begin{bmatrix} R_{t1-1}-\mu_1 & R_{t2-1}-\mu_2 & (R_{t1-1}-\mu_1)^2-\sigma_1^2 & (R_{t2-1}-\mu_2)^2-\sigma_2^2 \end{bmatrix}$$

$\hat{\Gamma_1} =$

$$\begin{bmatrix} \frac{\sum_2^T(R_{t1}-\mu_1)(R_{t1-1}-\mu_1)}{T-1} & 0 & \frac{\sum_2^T(R_{t1}-\mu_1)(R_{t1-1}-\mu_1)^2}{T-1} & 0 \\ 0 & \frac{\sum_2^T(R_{t2}-\mu_2)(R_{t2-1}-\mu_2)}{T-1} & 0 & \frac{\sum_2^T(R_{t2}-\mu_2)(R_{t2-1}-\mu}{T-1} \\ \frac{\sum_2^T(R_{t1-1}-\mu_1)(R_{t1-1}-\mu_1)^2}{T-1} & 0 & \frac{\sum_2^T(R_{t1-1}-\mu_1)^2(R_{t1-1}-\mu_1)^2}{T-1}+\sigma_1^4 & 0 \\ 0 & \frac{\sum_2^T(R_{t2-1}-\mu_2)(R_{t2-1}-\mu_2)^2}{T-1} & 0 & \frac{\sum_2^T(R_{t2-1}-\mu_2)^2(R_{t2-1}-\mu_2)}{T-1} \end{bmatrix}$$

$\hat{S_T} =$

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & 2\sigma_1^4 & 0 \\ 0 & 0 & 0 & 2\sigma_2^4 \end{bmatrix} + \frac{1}{2}(\hat{\Gamma_1}+\hat{\Gamma_1}')$$

(d) $H_0 : \mu_1 * \sigma_2 - \mu_2 * \sigma_1 = 0$
$R(\theta) = \mu_1 * \sigma_2 - \mu_2 * \sigma_1$
$\theta =$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$$

$\frac{\partial R(\theta)}{\theta'} =$

$$\begin{bmatrix} \sigma_2 & -\sigma_1 & -\mu_2 & \mu_1 \end{bmatrix}$$

$T(R(\hat{\theta_T})'[\frac{\partial R(\theta)}{\theta'}\hat{V}\frac{\partial R(\theta)}{\theta'}']^{-1}(R(\hat{\theta}) \sim \chi^2$
Sharpe Ratio Stock 1: 0.1113
Sharpe Ratio Stock 2: 0.0352
T-test Statistic: 1.4645
p-value: 0.1433 Fail to reject the null hypothesis: No significant difference in Sharpe ratios.

(e) After introducing the correlation matrix:
Sharpe Ratio Stock 1: 0.1113
Sharpe Ratio Stock 2: 0.0352
Sample Correlation: 0.4187
T-test Statistic: 12.5329
Degrees of Freedom: 1198
p-value: 0.0000
Reject the null hypothesis: Stocks have different Sharpe ratios.

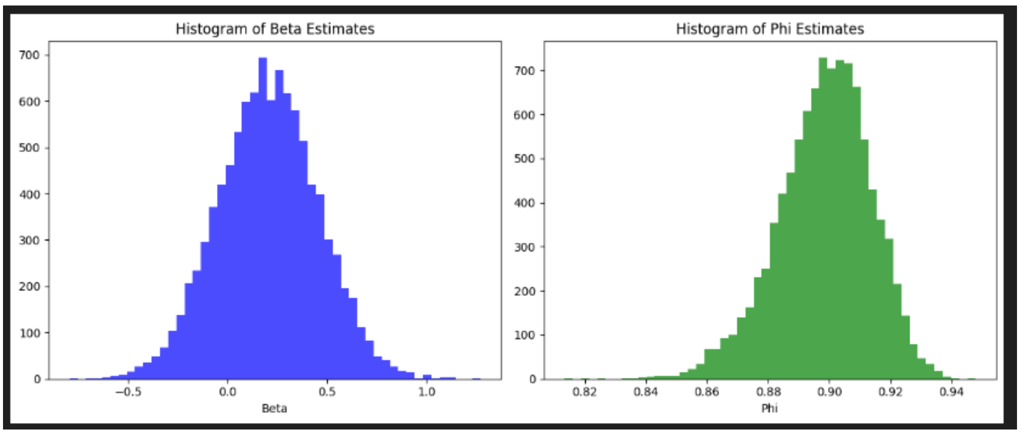
(f) Observed Sharpe Ratio Difference: 0.0761
95% Confidence Interval: [-0.0325, 0.1561]
The sharp ratio difference is not significant at 5% level.

## Question 2

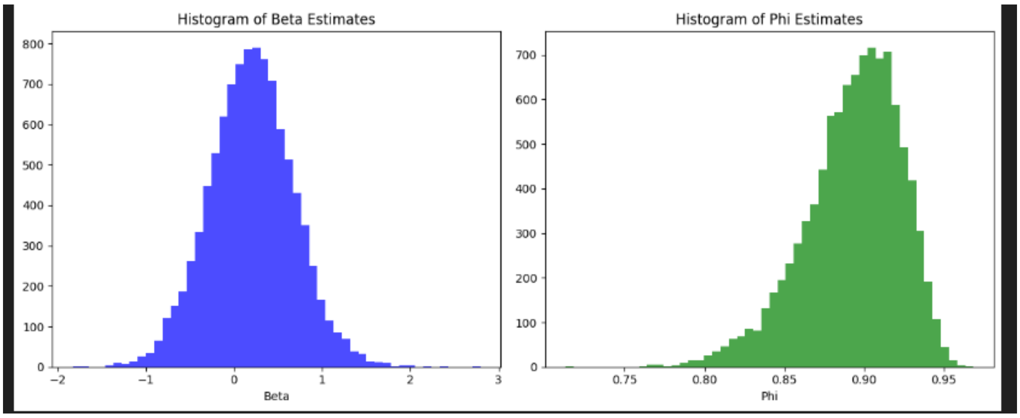
(a) Optional

## Question 3

(a) N=840




(b) N=240
Looks more skewed to the right side for $\hat{\phi}$


Histogram of Beta Estimates
800
700
600
500
400
300
200
100
0
−2
−1
0
1
2
3
Beta
Histogram of Phi Estimates
700
600
500
400
300
200
100
0
0.75
0.80
0.85
0.90
0.95
Phi