PhD 426: Household Finance

Problem Set 1

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Computational details

I solve the model using MATLAB. The `exercise_1.m` file is the main script, and contains parameter definitions, code to solve and simulate the model, and code to create graphs. The `functions` sub-folder contains a few user-defined functions that are called from the main script. In particular, the `solveLifeCycleHH` function solves the model for a given set of parameters, and returns cash-on-hand grids, policy and value functions for each age.

Given that the numéraire is SEK 10,000, I use a log-linear grid over asset holdings ranging from $\underline{a} = 0$ to $\bar{a} = 1000$. I use 500 grid points; results remain virtually unchanged if I use 1,000 or 1,500 points instead. I discretize the distribution of transitory income shocks using a Gauss-Hermite quadrature with 5 nodes (see function `ghqnorm`). For the permanent income process, I use the Tauchen-Hussey (1991) discretization scheme with a fixed grid and transition matrix (see function `tauchenHussey`). I use 15 points on the grid. Ideally, one would use a widening set of grids over the life cycle, since the process is non-stationary (i.e. $\rho = 1$). However, the fixed-grid scheme I use appears to work well, and the results are unchanged when using more grid points.

I use the discretized income processes when simulating the economy. In particular, I generate the initial permanent income using normally-distributed random numbers, and then interpolate them on the nearest point of the discrete income grid obtained through Tauchen-Hussey (1991). Hence the income process is never off-grid, so that the policy functions can be used exactly. Another option would be to let the income process be fully continuous when simulating, while interpolating policy choices over income states. I expect this to yield similar results.

Finally, I note that the assumed floor on disposable income is too low to matter with the specifications detailed above. Indeed, a household aged 25 has a lowest possible income of

$$Y_{i,25}^{\min} = \exp\left(g_{25} + \underline{z} + \underline{\omega}\right) \approx 11.57 > \underline{Y} = 4.8$$

where $\underline{z}, \underline{\omega}$ are the lower endpoints on the log-grids for permanent and transitory income, respectively. Of course, using wider grids would make it more likely for $\underline{Y}$ to be reached.

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Results

Baseline calibration

I first present results for the baseline specification (i.e. as specified in the problem set instructions). Figure 1 plots the consumption policy function against cash-on-hand, for different ages. The left panel assumes the lowest possible permanent income state, while the right panel plots the highest state. I omit later retirement ages (> 65). The dotted black line represents the 45° line; as expected constrained households consume the entirety of their cash-on-hand.

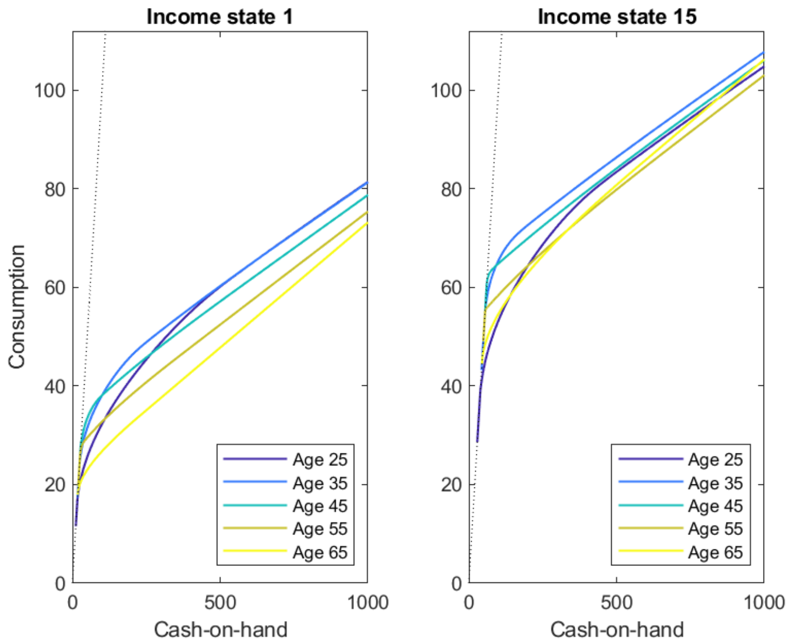

Figure 1: Optimal consumption as a function of cash-on-hand, at different ages.

Figure 2 plots the average life-cycle profiles for consumption, income and assets. The lower panel plots the average savings rate (i.e. $s_{i,t} = 1 - c_{i,t}/x_{i,t}$). As expected, average consumption follows the same humped-shaped pattern as income during the working life. During retirement average consumption decreases steadily as assets deplete, until it reaches the replacement income level around age 80. This pattern is due to the relative impatience of households ($\beta = 0.945$), which makes it optimal to front-load consumption through retirement by using savings.

Average asset holdings follow the expected pattern. They decrease in earlier periods due to rich households prioritizing consumption. They start rising again around age 40, due to all households saving for retirement, and peak at the retirement age (denoted by the vertical dotted line). Remarkably, asset holdings are quite low over the whole life cycle, in that they are of similar magnitude to consumption. This is due to the relative impatience of households ($\beta = 0.945$) and the high pension replacement income, which together provide little incentives to save.

Figure 3 plots the life-cycle of two selected individuals, namely those at the 5^{th} and 95^{th} percentiles of the initial cash-on-hand distribution. Consistent with previous observations, consumption appears as a smoothed version of income, and asset holdings are strongly correlated with (lagged) income shocks. While the initially-poorer household (top panel) has low asset holdings

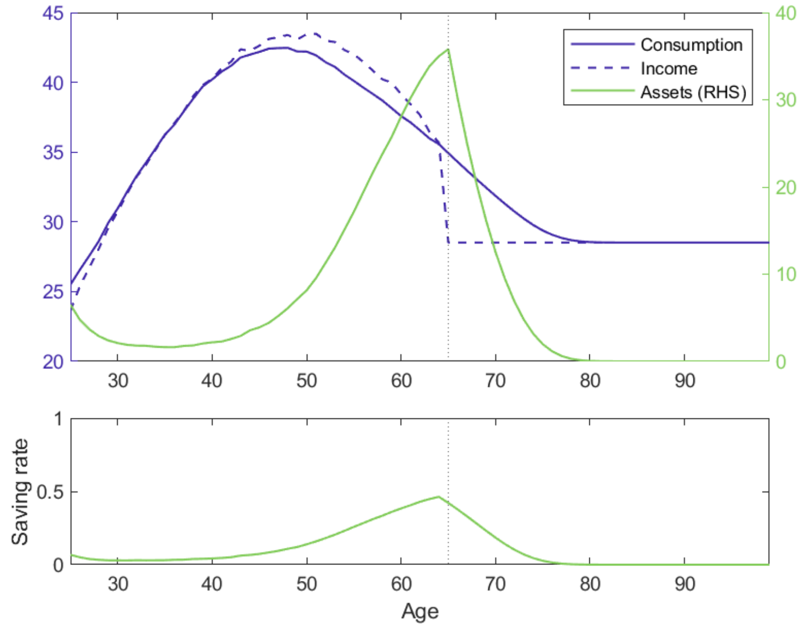

Figure 2: Average life-cycle profile.

over its life cycle, the initially-richer household (bottom panel) does not hold much more assets. As mentioned above, this is due to a lack of incentives to save overall. Wealth inequality would thus be quite low in this particular calibration of the model.

Alternative calibrations

I now study two variations of the baseline calibration.

First, I set $\beta = 0.99$, i.e. households are now more patient. The resulting optimal consumption functions are flatter, which translates into higher savings at all ages. Figure 4 plots the resulting life-cycle profile. As expected, average consumption is less front-loaded, and increases over the entire working life. In particular, households start saving earlier, around age 30, and accumulate large asset holdings thanks to the high income levels in ages 40–50. During retirement, consumption is roughly linear, and assets deplete steadily until the end of life. In stark contrast to the baseline calibration, average assets are much higher, as are saving rates.

Figure 5 reports the life-cycle profiles of the two individuals studied in Figure 3. The difference is striking; in particular it is clear that savings are now primarily driven by a replacement-income motive rather than a precautionary savings motive. At the same time, consumption is now less volatile due to the larger buffer in assets.

Next, I set $\lambda = 0$, i.e. there is no replacement income during retirement (and $\beta = 0.945$). Figure 6 plots the resulting consumption policy functions, which are almost linear in cash-on-hand. In particular, consumption appears to decrease with age. Figure 7 plots the corresponding average life-cycle profile. Although average consumption still exhibits a hump around age 35, it is smoothly decreasing thereafter, including during retirement. Households tend to save from age 35 onward, as in the $\beta = 0.99$ calibration. Saving rates are extremely high, and peak at 95% around retirement, as expected given the lack of a replacement income.

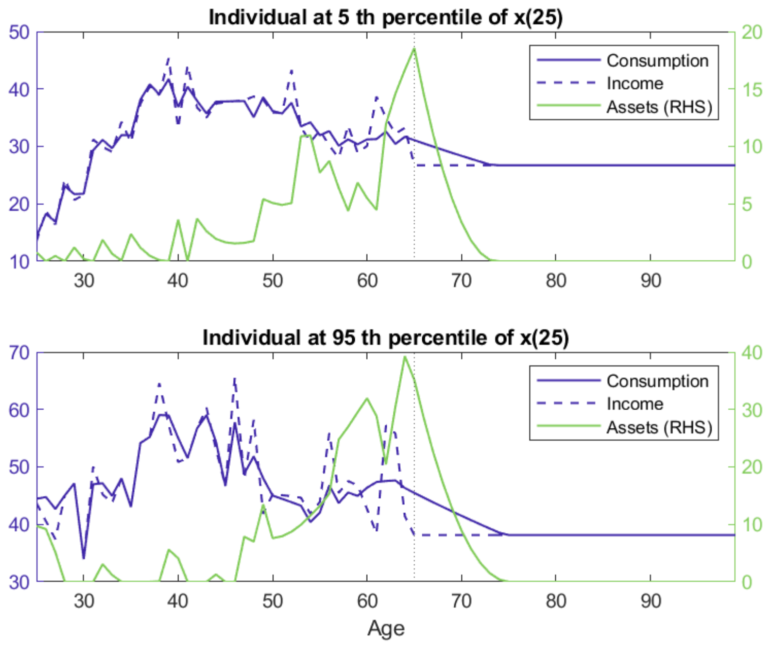

Figure 3: Life-cycle profiles for two selected individuals.

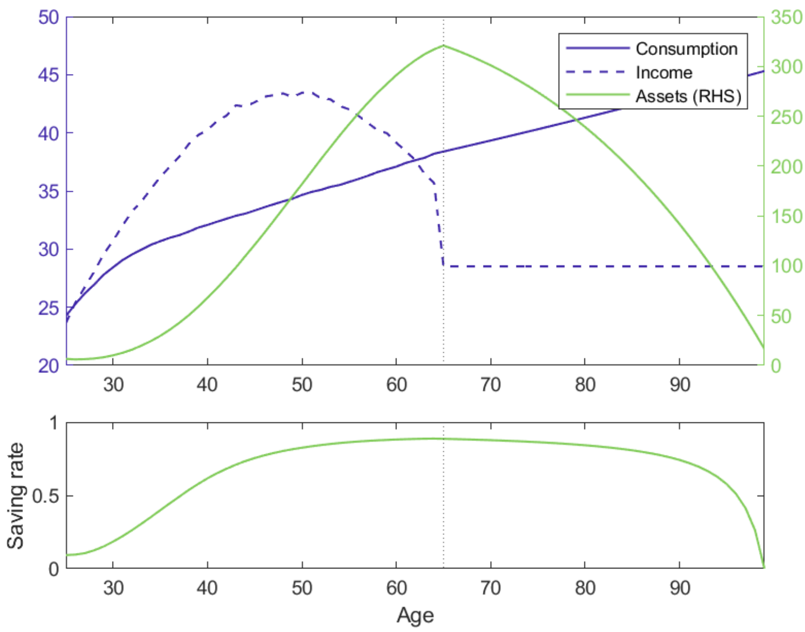

Figure 4: Average life-cycle profile with $\beta = 0.99$.

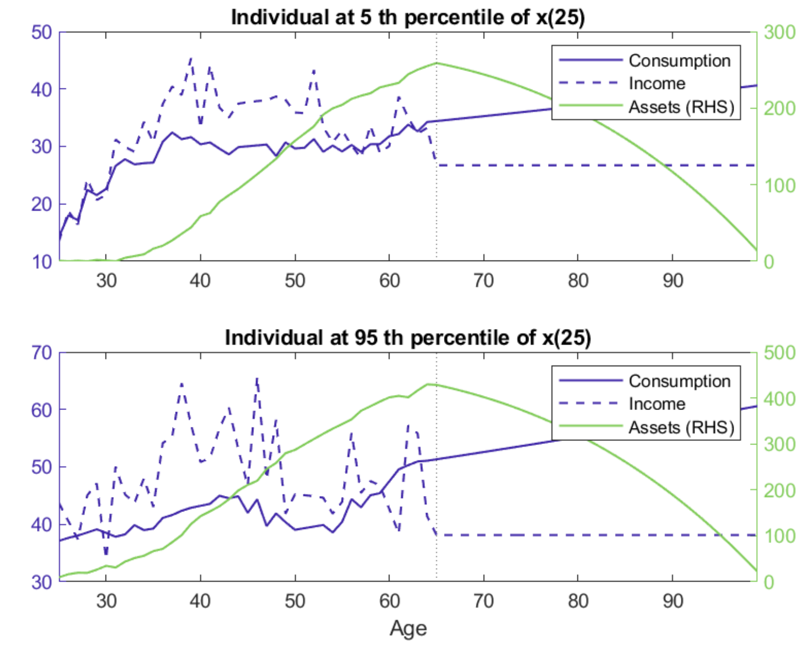

Figure 5: Life-cycle profiles for the two individuals in Fig. 3, with $\beta = 0.99$.

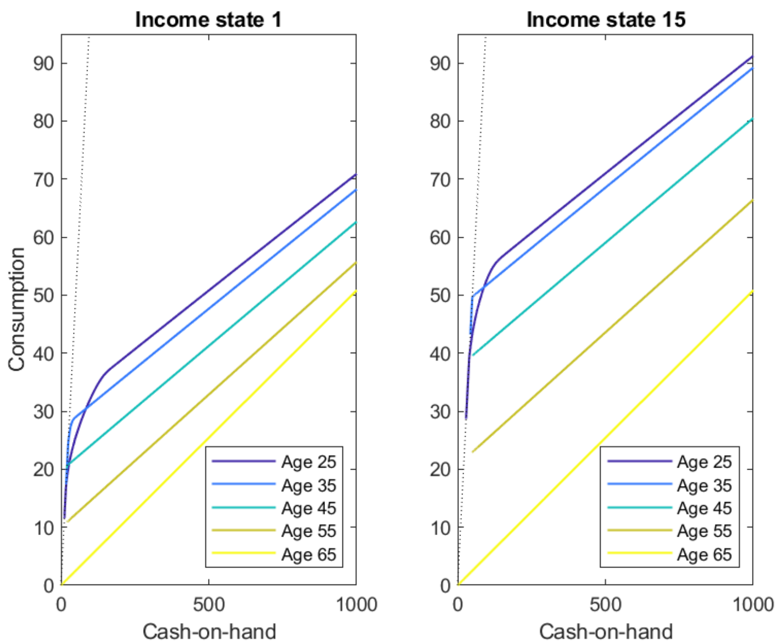

Figure 6: Optimal consumption as a function of cash-on-hand, at different ages, with $\lambda = 0$.

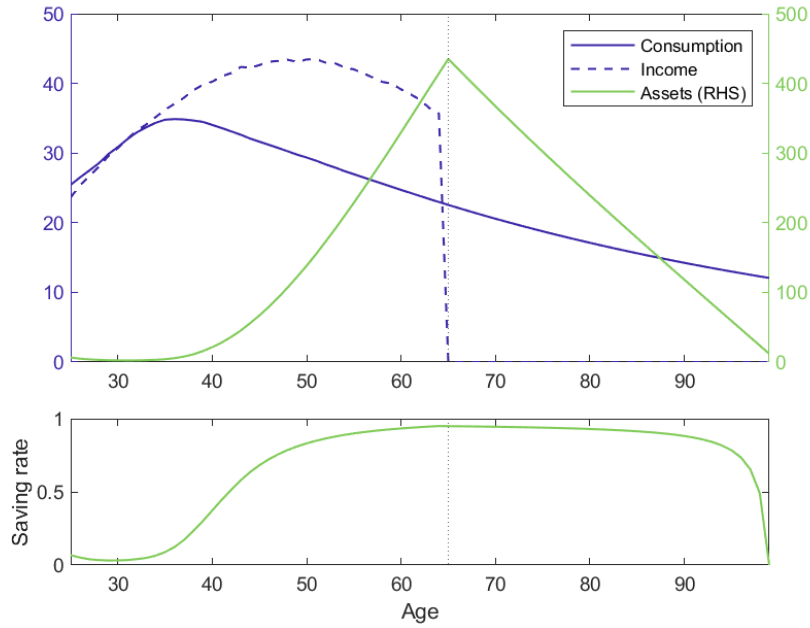

Figure 7: Average life-cycle profile with $\lambda = 0$.