1 Problem set 1 – Ph.D. course in Household Finance

Your task is to solve and simulate the buffer-stock savings model of Carroll (1997). The deadline is November 7 at 23:59 and should be submitted individually on the course website in Canvas.

1.1 Model setup

Individuals start their economic life at the age of $t = 25$ and we follow them until the end of their lives at $T = 100$. One time period is a year. The life cycle consists of two phases: a working phase and a retirement phase with exogenous retirement at age 65.

Stochastic process for disposable income We consider the income process of Carroll and Samwick (1997). A working-age individual i, at age $t = 25, 26, ..., 64$, receives disposable income Y_{it}. It has a deterministic hump-shaped life-cycle trend, g_t, a permanent income component z_{it}, and a transitory idiosyncratic income shock, ω_{it}. Disposable income cannot be less than $\underline{Y}$ which is a parsimonious way to account for welfare and transfers. Let $y_{it} = \ln(Y_{it})$. Then for $t \leq 64$:

$$
\begin{aligned}
y_{it} &= g_t + z_{it} + \omega_{it}, && (1)\\
z_{it} &= \rho z_{it-1} + \eta_{it}, && (2)\\
y_{it} &\geq \ln(\underline{Y}). && (3)
\end{aligned}
$$

The random variable η_{it} is an idiosyncratic shock to permanent income, which is distributed $\mathrm{N}\left(-\sigma_\eta^2/2, \sigma_\eta^2\right)$. There is income heterogeneity already at age 25 through the initial persistent shock, z_{i25}, which is distributed $\mathrm{N}\left(-\sigma_z^2/2, \sigma_z^2\right)$. The random variable ω_{it} is a transitory income shock. It is distributed

$$\omega_{it} \sim N\left(-\sigma_\omega^2/2, \sigma_\omega^2\right). \qquad (4)$$

Upon retirement, which happens at $t = 65$, individuals have a safe pension income. It is modeled as a deterministic replacement rate, λ, relative to permanent labor income at 64:

$$
\begin{aligned}
Y_{i,t} &= \lambda Y_{i,64}^p \\
&= \lambda \exp(g_{64} + z_{i64}), \quad t \geq 65. && (5)
\end{aligned}
$$

Preferences Individuals have CRRA preferences over consumption:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \qquad (6)$$

where γ is the coefficient of risk aversion.

Budget constraints and laws of motions Individuals choose consumption (C_t) and savings (A_t) in every period subject to a no-borrowing constraint:

$$C_t + A_t = X_t, \tag{7}$$

$$A_t \geq 0, \tag{8}$$

where X_t denotes cash on hand. Its law of motion is:

$$X_{25} = \hat{A}_{25} + Y_{25}, \tag{9}$$

$$X_{t+1} = A_t R + Y_{t+1}, \quad t = 25, ..., 99 \tag{10}$$

where initial wealth is distributed

$$\ln(\hat{A}_{25}) \sim N(\mu_a - \sigma_A^2/2, \sigma_A^2). \tag{11}$$

The optimization problem Let $\mathcal{S}_t = \{Z_t, X_t\}$ denote the state variables for $t = 1, ..., T$. The optimization problem is given by:

$$V_t(\mathcal{S}_t) = \max_{C_t, A_t} u(C_t) + \beta E_t [V_{t+1}], \tag{12}$$

subject to (1)-(4) and (5)-(10), and where $V_{T+1} = 0$. The parameter β is the discount factor.

1.2 Parameter values

Let the numeraire of the model be SEK 10,000. The parameter values are:

- A life-cycle is from 25 to 100 years ($T = 75$)
- 40 years of working life ($T_w = 40$)
- 35 years of retirement ($T_r = 35$)
- The life-cycle profile for e^{g_t} is provided in the Excel file Income_profile.xlsx on Canvas. The unit is SEK 10,000.
- $\underline{Y} = 4.8$
- $\lambda = 0.80$
- $R = 1.02$
- $\beta = 0.945$
- $\gamma = 2$
- $\rho = 1$

- $\sigma_\omega = 0.10$
- $\sigma_\nu = 0.0713$
- $\sigma_z = 0.589$
- $\mu_A = 1.916$
- $\sigma_A = 2.129$

1.3 Assignment

- Solve the model above in a programming language of your choice, using dynamic programming. In particular, use the endogenous grid point method of Carroll (2006). Report policy functions in a graph, akin to the lecture notes.
- Simulate the model with 1,000 individuals. Display the average life-cycle profiles for Y_t, A_t and C_t. In another graph, report simulated paths for a particular consumer.

1.4 Useful sources

- For the Tauchen or Roewenherst algorithms:
  - Giulio Fella: `https://giuliofella.net/`
  - Martin Flodén: A Note on the Accuracy of Markov-Chain Approximations to Highly Persistent AR(1)-Processes, Economics Letters, June 2008, 99, 516-520. Matlab code: `https://martinfloden.net/files/ar1_processes_matlab_code.zip`
- Lecture 2 of Ben Moll's first-year course: `https://benjaminmoll.com/wp-content/uploads/2021/04/Lecture2_EC442_Moll.pdf`
- Ben Moll and Greg Kaplan have a lot of code: `http://benjaminmoll.com/ha_codes/`
  - Have a look at the readme.txt file and egp_IID_lifecycle.m if you want
- Lot's of other code, e.g. Chris Caroll: `http://econ.jhu.edu/People/CCarroll/EndogenousArchi` `zip`

1.5 Some concrete tips

- Separate model parameters from technical parameters – you want flexilibity also on the technical ones
- Construct the objects necessary for integration just once, before the solution algorithm starts

- Define subroutines that represent the laws of motions for X and z: $X_{t+1}(a, z_{j,t})$ and $z_{t+1}(z_{j,t})$. Notice that they return vectors $(N_z \cdot N_q) \times 1$ and each outcome is associated with probability $w_i \cdot \pi(z_{k,t+1}|z_{j,t})$.
- The simulation:
  - Use the same set of shocks every time to be able to debug efficiently. Draw them once, then save them and then load them every time.
  - Re-use as much of the code from the solution as possible, for instance the law of motions for the state variables.
  - Make the problem minimalistic first: no need to start with large T, or with dense grids, or with income uncertainty

References

Carroll, C. (2006, September). The method of the endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters 91*(3), 312–320.

Carroll, C. D. (1997, 02). Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis*. *The Quarterly Journal of Economics 112*(1), 1–55.

Carroll, C. D. and A. A. Samwick (1997). The nature of precautionary wealth. *Journal of Monetary Economics 40*, 41–71.