Household Finance Masterclass

Savings

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Outline

1. Consumption vs Savings
   - Consumption Smoothing
   - Precautionary saving
2. Financial Risk taking
   - Determinants of financial risk taking
   - Portfolio rebalancing
3. Diversification and Hedging
   - Diversification losses
   - Value and growth investors
4. Wealth Inequality
   - Income and wealth inequality
   - Returns to wealth and inequality

Wealth Management

Financial markets allow households to transfer resources across time and states of the world

- **By borrowing**, households are able to consume future income
- **By saving**, households transform current income into future consumption
  - By investing in different assets, households can hedge their labor income risk
  - By buying insurance products, households limit losses in the face of adversities such as accidents and death

Complexity of Financial Products

Financial markets are complex and do not necessarily evolve in the interest of households

- Households have limited financial literacy
  - Households lack basic precepts of financial theory:
    - Diversification not stock picking
    - Do not time the market
    - Importance of fund fees
  - Consequences of financial mistakes materialize only over long periods of time, if ever
    - No learning by doing
- Financial providers might obfuscate products and services
  - 25,000 equity funds investing in 50,000 stocks
  - Brokerage houses promote stock picking
  - Shrouded equilibrium (Gabaix and Leibson, 2006)

Shrouded Equilibrium

- Two types of consumers
  - Sophisticated/ Rational, can discern the true quality of a product/service
  - **Unsophisticated**, cannot evaluate the quality of a product/service
- Two products/services
  - High-quality
  - Low-quality (cheaper to produce) **shrouded** into being comparable to the high-quality

Equilibrium

1. All consumers are rational => only high-quality product offered
   - If offered, the low product will be competed out
2. With unsophisticated consumers => low-quality product offered as well
   - Firms can subsidize a lower price on the high-quality product than in rational equilibrium, by offering the low-quality product
   - No firm can compete away the low-quality product, since they would have to offer the high-quality product at a higher price and fail to attract even the sophisticated consumers

Financial Literacy

Concepts at the base of many financial decisions:

1. numeracy and capacity to do calculations related to interest rates, such as compound interest
2. understanding of inflation
3. understanding of risk diversification

> 1. Suppose you had $100 in a savings account and the interest rate was 2 percent per year. After 5 years, how much do you think you would have in the account if you left the money to grow: [more than $102; exactly $102; less than $102; do not know; refuse to answer.]
> 2. Imagine that the interest rate on your savings account was 1 percent per year and inflation was 2 percent per year. After 1 year, would you be able to buy: [more than, exactly the same as, or less than today with the money in this account; do not know; refuse to answer.]
> 3. Do you think that the following statement is true or false? "Buying a single company stock usually provides a safer return than a stock mutual fund." [true; false; do not know; refuse to answer.]

International results

TABLE 2
Comparative Statistics on Responses to Financial Literacy Questions around the World

Authors	Country	Year of data	Interest rate Correct	Interest rate DK	Inflation Correct	Inflation DK	Risk Diversification Correct	Risk Diversification DK	All 3 correct	At least 1 don't know	Number of Observations
Lusardi and Mitchell (2011d)	USA	2009	64.9%	13.5%	64.3%	14.2%	51.8%	33.7%	30.2%	42.4%	1,488
Alessie, VanRooij, and Lusardi (2011)	Netherlands	2010	84.8%	8.9%	76.9%	13.5%	51.9%	33.2%	44.8%	37.6%	1,665
Bucher-Koenen and Lusardi (2011)	Germany	2009	82.4%	11.0%	78.4%	17.0%	61.8%	32.3%	53.2%	37.0%	1,059
Sekita (2011)	Japan	2010	70.5%	12.5%	58.8%	28.6%	39.5%	56.1%	27.0%	61.5%	5,268
Agnew, Bateman, and Thorp (2013)	Australia	2012	83.1%	6.4%	69.3%	13.0%	54.7%	37.6%	42.7%	41.3%	1,024
Crossan, Feslier, and Hurnard (2011)	N. Zealand	2009	86.0%	4.0%	81.0%	5.0%	27.0%	2.0%*	24.0%*	7.0%	850
Brown and Graf (2013)	Switzerland	2011	79.3%	2.8%*	78.4%	4.2%*	73.5%*	13.0%*	50.1%*	16.9%*	1,500
Fornero and Monticone (2011)	Italy	2007	40.0%*	28.2%*	59.3%*	30.7%*	52.2%*	33.7%*	24.9%*	44.9%*	3,992
Almenberg and Säve-Söderbergh (2011)	Sweden	2010	35.2%*	15.6%*	59.5%	16.5%	68.4%	18.4%	21.4%*	34.7%*	1,302
Arrondel, Debbich, and Savignac (2013)	France	2011	48.0%*	11.5%*	61.2%	21.3%	66.8%*	14.6*	30.9%*	33.4%*	3,616
Klapper and Panos (2011)	Russia	2009	36.3%*	32.9%*	50.8%*	26.1%*	12.8%*	35.4%*	3.7%*	53.7%*	1,366
Beckmann (2013)	Romania	2011	41.3%	34.4%	31.8%*	40.4%*	14.7%	63.5%	3.8%*	75.5%*	1,030

Note: * indicates questions that have slightly different wording than the baseline financial literacy questions enumerated in the text.

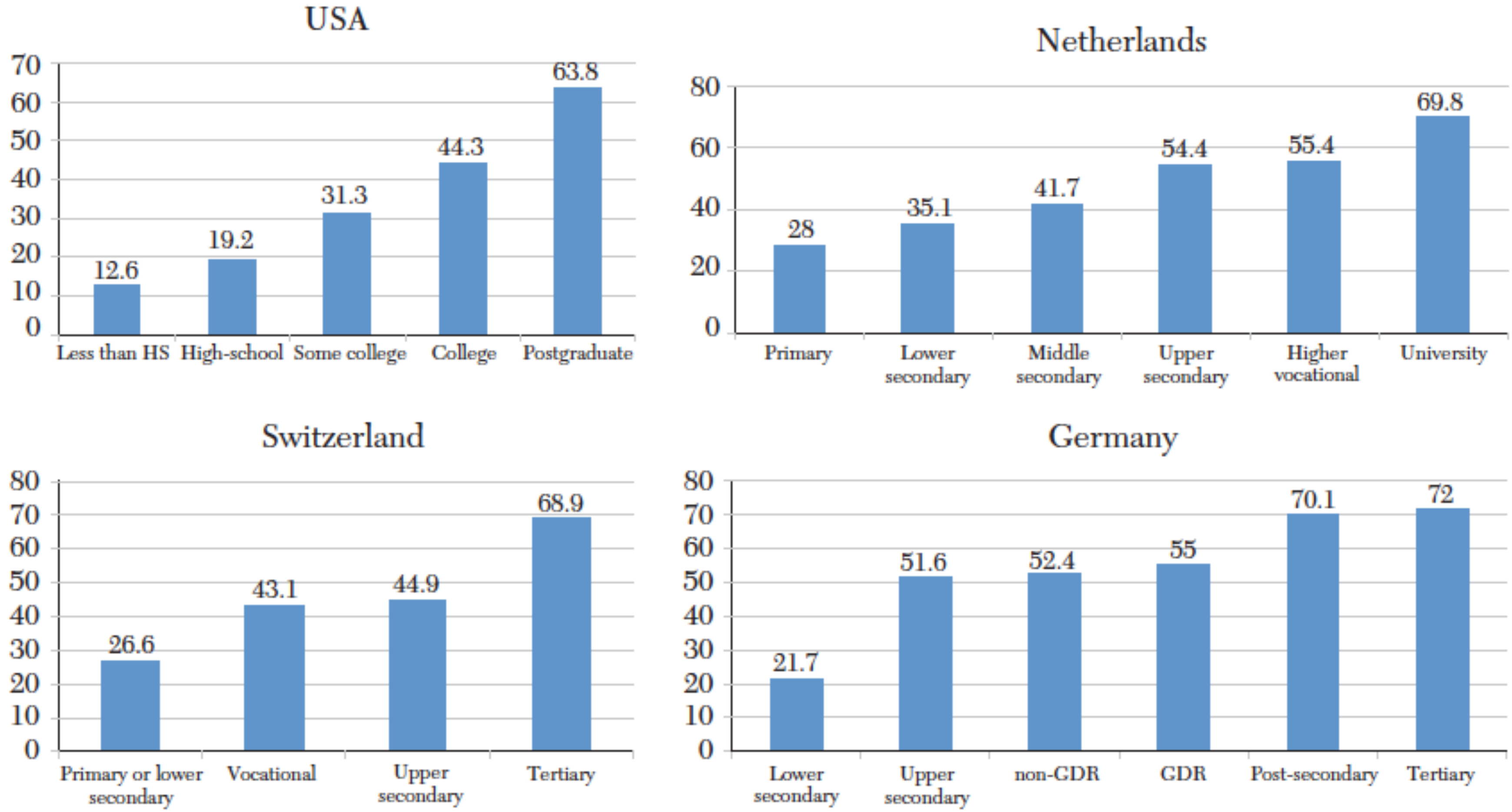
Source: Lusardi and Mitchell JEL (2014)

Best and worse performers change across questions

In Russia, it is rare that individual get all questions right

Sweden scores particularly low on the interest rate question

By Education



Financial literacy is strongly correlated with education across all countries

Financial Literacy

- In most countries a minority of households is able to answer correctly the three financial literacy questions
- Countries are not necessarily able to consistently score high in all questions
- Financial literacy is highly correlated with general education in all countries

Household Balance Sheets

Household Balance Sheet

GROSS WEALTH

Assets

Financial

- Cash/Bank
- Bonds
- Stocks
- Mutual funds

Liquid wealth

Real

- Real Estate
  - residential
  - commercial
- Private Equity

Illiquid wealth

Human Capital

Liabilities

Personal loans

- Student loans
- Credit card

Consumer loans

- Durables
- Cars/Boats

Mortgages

Net Wealth / Equity =
Gross wealth - Liabilities

Budget constraint

STOCKS: GROSS vs NET WEALTH

Net wealth is the difference between the value of total assets (gross wealth) *A* and the value of total debt *D* in the household balance sheet

$$W_t = A_t - D_t$$

- *W* is positive when the value of household's assets are larger than the liabilities
  - This is typically the case when *A* includes real estate and *D* mortgages. But even in this case households can be underwater if house prices collapse
- *W* can be negative when *D* is mostly consumer debt or uncollateralized
  - A prime example are student loans at the beginning of working life

FLOWS

Households

- earn **labor income** *L*
  - Educational choices and age effects
- earn **capital income**, i.e. dividends and net capital gains on each asset *a* that they own, $r_a A_a$, where A_a is the value of asset *a*, and

$$r_a = R_a - 1 = \frac{P_{a,1} - P_{a,0}}{P_{a,0}} + y_{a,1},$$

  where $P_{a,t}$ is the price and $y_{a,1}$ the yield (dividend or interest earn per unit of price) of asset *a*
- pay interests on debt, $i_d D_d$, where D_d is the value of debt *d* and i_d is the interest rate on *d*
- decide how much to **consume** *C*

Budget Constraint

DYNAMIC/INTERTEMPORAL Budget Constraint

defines stock of *net* wealth W_{t+1} in the end of each period given the net wealth on the previous period W_t

$$W_{t+1} = R_{t+1} W_t + L_{t+1} - C_{t+1}$$ **Dynamic/ Intertemporal BC**

The household decides

1. how much to work L_{t+1}
   – exogenous in most household finance models
2. how much to consume C_{t+1}
   – and how to consume in some models
3. level and composition of debt and gross wealth, which results in the return R_{t+1} on net wealth W_t

RETURN ON GROSS WEALTH

Each **asset class** provides **nominal gross return:**

$$R_{a,t+1} = 1 + r_{a,t+1}$$

and real return $R^r_{a,t+1} = 1 + r_{a,t+1} - \pi_{t+1}$, where π is the inflation rate

The **total nominal return** on **gross wealth** is the weighted average of returns on household's assets portfolio (i.e., total return on its gross wealth)

$$R_{A,t+1} = \sum_{a \in A} \omega_{a,t} * R_{a,t+1}$$

where $\boldsymbol{\omega_{a,t}}$ are weights of each asset *a* out of gross wealth.

Similarly, the real return is $R^r_{A,t+1} = \sum_{a \in A} \omega_{a,t} * R^r_{a,t+1}$

INTEREST ON LIABILITIES

The interest paid on each type of **debt** or **loan** *d* is:

$$I_{d,t+1} = 1 + i_{d,t+1}$$

The **total interest on debt** is the weighted average of interests paid on liabilities:

$$I_{D,t+1} = \sum_{d \in D} \omega_{d,t} * I_{d,t+1}$$

where $\boldsymbol{\omega_{d,t+1}}$ are weights of each type of debt out of total liabilities

- $I_{D,t+1}$ can also be expressed in real terms by subtracting inflation

$I^r_{d,t+1} = 1 + i_{d,t+1} - \pi_{t+1}$ and $I^r_{D,t+1} = \sum_{d \in D} \omega_{d,t} * I^r_{d,t+1}$

RETURN ON NET WEALTH

The **return on net wealth**, or **return on equity**, is given by

$$R_{t+1} = R_{A,t+1}\frac{A_t}{W_t} - I_{D,t+1}\frac{D_t}{W_t}$$

- Note that if $R_A > I_D$ and W>0, the return on net wealth can reach infinity as the household borrows more and more against its assets
  - Households can be their own hedge funds!
- Note that the budget constrain / wealth accumulation equation holds even if net wealth is negative
- Unless noted, net wealth and returns are calculated in real terms (since consumption is real)

$$W_{t+1} = R_{t+1}W_t + L_{t+1} - C_{t+1}$$

BORROWING CONSTRAINT

Due to moral hazard, risk management and financial stability issues, household leverage is typically capped

- Mortgages are collateralized by real estate values

$$M \leq LTV \cdot RE$$

where RE is the real estate market value, M is mortgage debt, and LTV is the maximum loan to value ratio allowed

- Debt, especially when it is mostly composed by uncollateralized loans (e.g. student and credit card debt), is also capped by income, as it must be serviced over time

$$D \leq LTI \cdot L$$

where LTI is the maximum loan to income ratio allowed

Consumption – Saving Decision

Consumption Saving Decision

To focus on the consumption / saving decision, we assume

1. Exogenous labor income
   - Deterministic in the first part of the lecture
2. No borrowing constraints
   - Although I will highlight when they can play a role
3. Only one asset: deterministic payoff
   - an asset that pays a constant real return *R* forever
   - we often consider a real perpetuity, i.e. an asset that pays a constant real return
     - in practice a portfolio of t-bills and inflation linked bonds are the closest asset class
4. Same borrowing and saving rate

Multi-period Consolidated Budget Constraint

From the intertemporal budget constraint

$$W_{t+1} = R\,W_t + L_{t+1} - C_{t+1}$$

integrate forward to get

$$W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t} = \frac{W_T}{R^T} + \sum_{t=1}^{T} \frac{C_t}{R^t}$$

and since $W_T = 0$

$$W_0 + \underbrace{\boxed{\sum_{t=1}^{T} \frac{L_t}{R^t}}}_{HC_0} = \sum_{t=1}^{T} \frac{C_t}{R^t}$$

Consolidated BC

Lifecycle Model

$$\max_{\{C_t\}_{t=1,\dots T}} u(C_1, C_2, \dots, C_T)$$

$$s.t.\ W_{t+1} = R\, W_t + L_{t+1} - C_{t+1}, \text{for all } t$$

- Time additive utility: $u(C_1, C_2, \dots, C_T) = \sum_{t=1}^{T} u_t(C_t)$
- $u_t(C_t)$ satisfies the usual Inada conditions:

$$u_t'(C) > 0, u_t'(0) = +\infty, u_t''(C) < 0$$

- Time preferences rate (TPR = Δ) and state dependence:

$$u_t(C_t) = \frac{1}{\Delta^t} u(C_t, z_t)$$

  preferences depend on the realization of a state variable z_t, e.g. family composition

- We will initially consider $u_t(C_t) = \frac{1}{\Delta^t} u(C_t)$

Multi-period Model

- The derivation is similar for the general case

$$u'(C_t^*) = \lambda \left(\frac{\Delta}{R}\right)^t \qquad \text{(FO)}$$

where λ is the Lagrange multiplier of the consolidated budget constraint and thus does not depend on *t*

Consumption *evolution* over the lifecycle depends only on interest rates and preferences – not on how income is distributed over the lifecycle

- In order to satisfy (FO), the agent borrows in periods when income is too low and saves when it is too high

Euler Equation

(FO) $\Rightarrow$

$$\lambda = u'(C_t^*)\left(\frac{R}{\Delta}\right)^t$$

$$\lambda = u'(C_{t+1}^*)\left(\frac{R}{\Delta}\right)^{t+1}$$

$$u_t'(C_t^*) = \frac{R}{\Delta}u'(C_{t+1}^*) \quad \text{(EE)}$$

$$\frac{u'(C_{t+1}^*)}{u'(C_t^*)} = \frac{\Delta}{R}$$

$$MRS_{t+1} = \frac{u'(C_{t+1}^*)/\Delta^{t+1}}{u'(C_t^*)/\Delta^t} = \frac{1}{R} = MRT_{t+1}$$

- If $u'(C_t^*) > \frac{R}{\Delta}u'(C_{t+1}^*)$, the household is better off by increasing C_t^* marginally (say of one unit) by saving less or borrowing, which has value $u'(C_t^*)$, and thus decreasing next period consumption by R, which has value in *t* of $\frac{R}{\Delta}u'(C_{t+1}^*)$
- The household will continue to do so until the marginal rate of substitution MRS is equal to the marginal rate of transformation MRT

Consumption Growth

- Take derivative in (FO) wrt to time

$$\frac{\partial C_t^* \backslash \partial t}{C_t^*} = -\frac{u'(C_t^*)}{C_t^* u''(C_t^*)}(r - \delta) \qquad \text{(CG)}$$

where $1 + r = R$ and $1 + \delta = \Delta$

- $r - \delta$ the extent to which the rate of return exceed the rate of time preference, represents the incentive to save and thus increase consumption growth
- The agent reaction to $r - \delta$ is regulated by the **elasticity of intertemporal substitution**

$$EIS = -\frac{u'(C_t^*)}{C_t^* u''(C_t^*)}$$

Euler Equation: derivation

Take derivative of optimal consumption path $u'(C_t^*) = \lambda \left(\frac{\Delta}{R}\right)^t$ w.r.t t:

$$\frac{\partial u'(C_t^*)}{\partial t} = \frac{\partial \left[\lambda \left(\frac{\Delta}{R}\right)^t\right]}{\partial t}$$

- Note that $\left(\frac{\Delta}{R}\right)^t = exp\left[\ln\left(\frac{\Delta}{R}\right)^t\right] = exp\left[t\ln\left(\frac{\Delta}{R}\right)\right]$
- Take the derivative of the RHS:

$$\frac{\partial\left[\lambda\left(\frac{\Delta}{R}\right)^t\right]}{\partial t} = \lambda\, \frac{\partial\left[\exp\left[t\ln\left(\frac{\Delta}{R}\right)\right]\right]}{\partial t} = \lambda\, \ln\left(\frac{\Delta}{R}\right)\exp\left[t\ln\left(\frac{\Delta}{R}\right)\right] = \lambda\,(\ln\Delta - \ln R)\left(\frac{\Delta}{R}\right)^t$$

- Taking the derivative of the LHS $\Rightarrow$ $u''(C_t^*)\frac{\partial C_t^*}{\partial t} = \lambda\,(\ln\Delta - \ln R)\left(\frac{\Delta}{R}\right)^t$

Use the fact that $\lambda = u'(C_t^*)\left(\frac{\Delta}{R}\right)^{-t}$ from $u'(C_t^*) = \lambda\left(\frac{\Delta}{R}\right)^t$:

$$u''(C_t^*)\frac{\partial C_t^*}{\partial t} = u'(C_t^*)\left(\frac{\Delta}{R}\right)^{-t}(\ln\Delta - \ln R)\left(\frac{\Delta}{R}\right)^t = u'(C_t^*)\,(\ln\Delta - \ln R)$$

Since $R = 1 + r$ and $\Delta = 1 + \delta$, and $\ln(1+x) \approx x$ for small x

$$\frac{\partial C_t^*}{\partial t} = \frac{u'(C_t^*)}{u''(C_t^*)}(\delta - r) \Rightarrow \boldsymbol{\frac{\partial C_t^* \backslash \partial t}{C_t^*} = -\frac{u'(C_t^*)}{C_t^*\,u''(C_t^*)}(r - \delta)}$$

Consumption Smoothing

We consider several cases in order of complexity that highlight the implications of the consumption saving model

I. $\boldsymbol{R = \Delta}$

- $R = \Delta = 1, L_t = L$
  - Friedman (1952)
  - Brumberg and Modigliani (1954)
- $R = \Delta > 1, L_{t+1} = GL_t$
  - borrowing

Constant Consumption

II. $\boldsymbol{R \neq \Delta}, L_t = L \ or \ L_{t+1} = GL_t$ (CRRA utility)

Constant Consumption Growth

Example: R = Δ

- the consolidated BC is:

$$W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t} = \sum_{t=1}^{T} \frac{C_t}{R^t}$$

- $R = \Delta \Rightarrow$ (FO) becomes: $u'(C_t^*) = \lambda \left(\frac{\Delta}{R}\right)^t = \lambda \Rightarrow C_t^* \equiv \bar{C}$
- Since $C_t^* \equiv \bar{C} \Rightarrow$ the consolidated BC is: $W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t} = \bar{C} \sum_{t=1}^{T} \frac{1}{R^t}$
- Hence consumption is constant: $\bar{C} = \frac{W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t}}{\sum_{t=1}^{T} \frac{1}{R^t}}$ (1)
- Yearly realizations of income do not affect consumption, which is fully smoothed over the lifecycle

Example: $R = \Delta = 1$, $L_t = L$, $W_0 = 0$

1. $W_0 = 0$ implies that the consolidated BC becomes:

$$\sum_{t=1}^{T} \frac{L_t}{R^t} = \sum_{t=1}^{T} \frac{C_t}{R^t}$$

2. $R = 1$ and $L_t = L$ further implies that:

$$L\,T = \sum_{t=1}^{T} C_t$$

3. $R = \Delta$ implies $u'(C_t^*) = \lambda$ so consumption is constant over time

$$C_t^* \equiv \bar{C}$$

Hence the BC implies $T\bar{C} = TL$ so that:

$$C_t^* = L$$

and savings out of labor income are zero: $S_t = L - C_t^* = 0$

Friedman (1952): PIH

Permanent increase in income ΔL from time τ onwards $\Rightarrow$

for all $t \geq \tau$, BC: $(T-\tau)\ (L+\Delta L) = (T-\tau)\bar{C}$, hence:

$$C_t^* = L + \Delta L, \qquad S_t = L + \Delta L - C_t^* = 0, \qquad t \geq \tau$$

- consumption fully absorbs the permanent increase in income

Temporary increase in income ΔL at time $\tau \Rightarrow$

for $t \geq \tau$, BC: $(T-\tau)L + \Delta L = (T-\tau)\bar{C}$, hence:

$$C_t^* = L + \frac{\Delta L}{T-\tau}, t \geq \tau$$

$$S_\tau = L + \Delta L - C_t^* = L + \Delta L - \left(L + \frac{\Delta L}{T-\tau}\right) = \Delta L\,\frac{T-\tau-1}{T-\tau} \approx \Delta L$$

$$S_t = L - C_t^* = L - \left(L + \frac{\Delta L}{T-\tau}\right) = -\frac{\Delta L}{T-\tau}, \qquad t > \tau$$

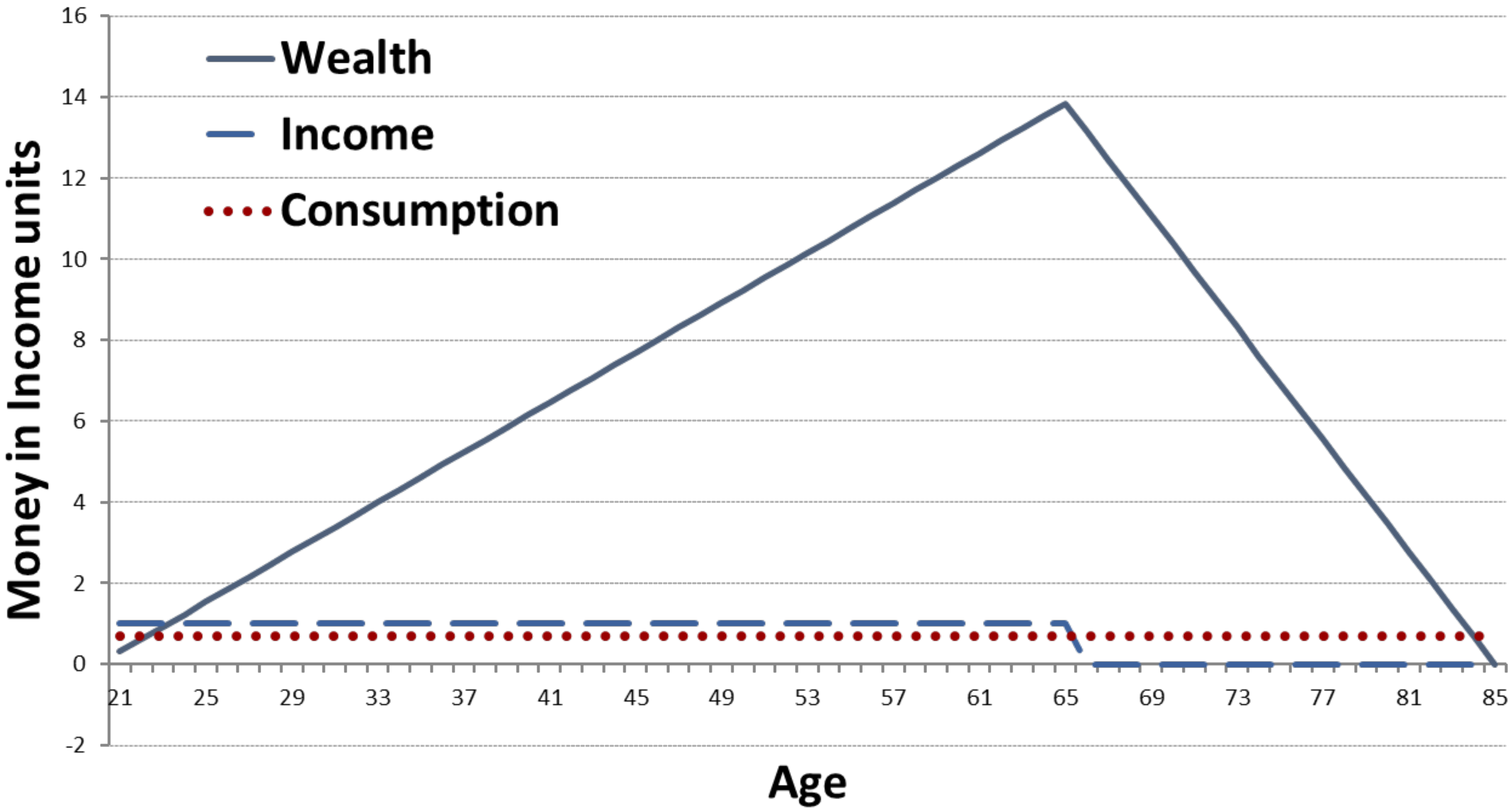
- temporary increase in income is saved and consumption increases only slightly

Correlation between income and savings is due to consumption smoothing: save temporary income shocks

Modigliani Brumberg (1954)
$L_t = L$ while working, $L_t = 0$ after

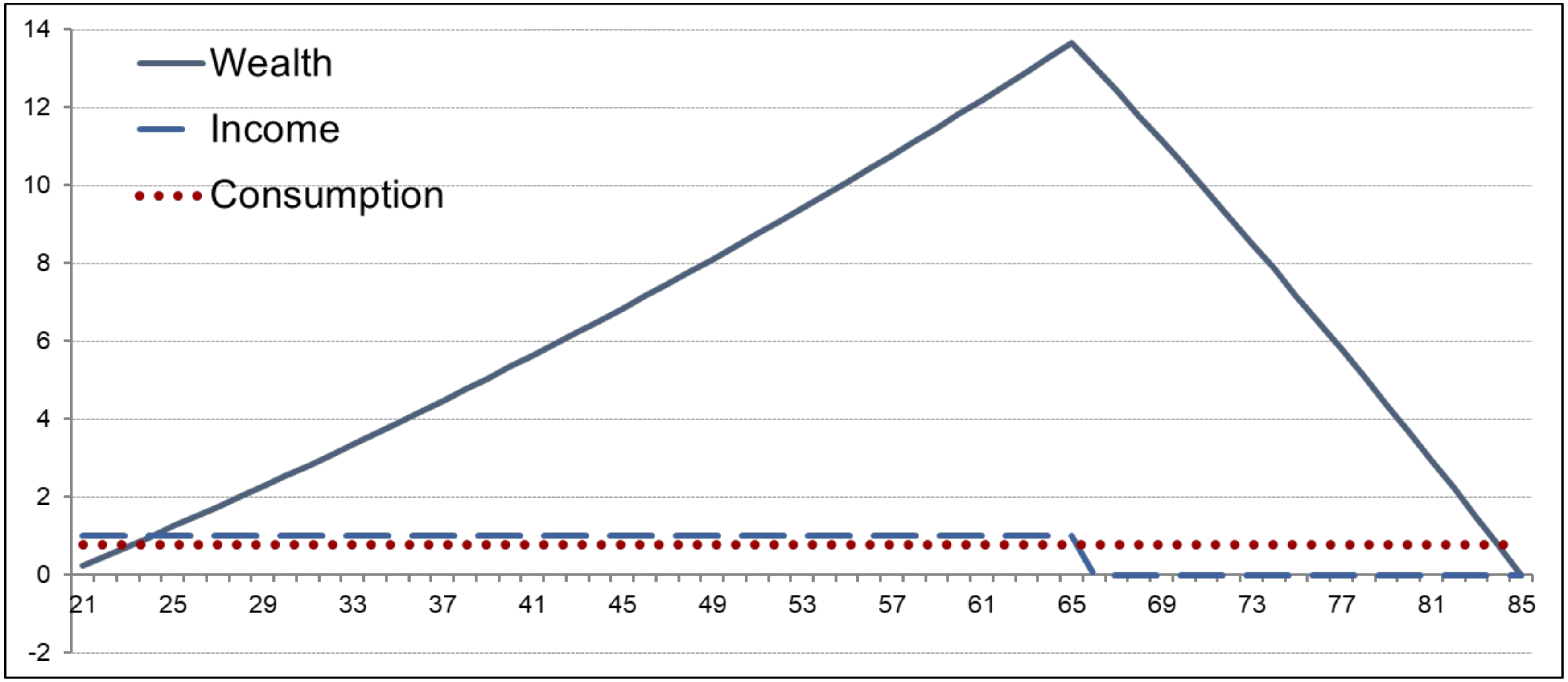
- Labor income earned for T_L **= number of working years**
- Consume for $T = T_L + T_R$, where T_R **= number of retired years**
- Since $W_0 = 0 \Rightarrow$ the **consolidated BC** is: $\sum_{t=1}^{T_L} \frac{L_t}{R^t} = \sum_{t=1}^{T} \frac{C_t}{R^t}$
  - $R = \Delta \Rightarrow$ **consumption is constant** over time: $C_t^* \equiv \bar{C}$
  - R**=1**, $L_t = L \Rightarrow$ the **consolidated BC** becomes: $T_L L = (T_L + T_R)C$
- Hence:
  - **consumption** is $C=LT_L/T$,
  - **savings** out of labor income are $S_L=L-C=LT_R/T$ during working life and while retired $S_R=-C$ per year.
  - **Wealth** grows to a maximum at retirement ($t=T_L$): $T_L L T_R/T$

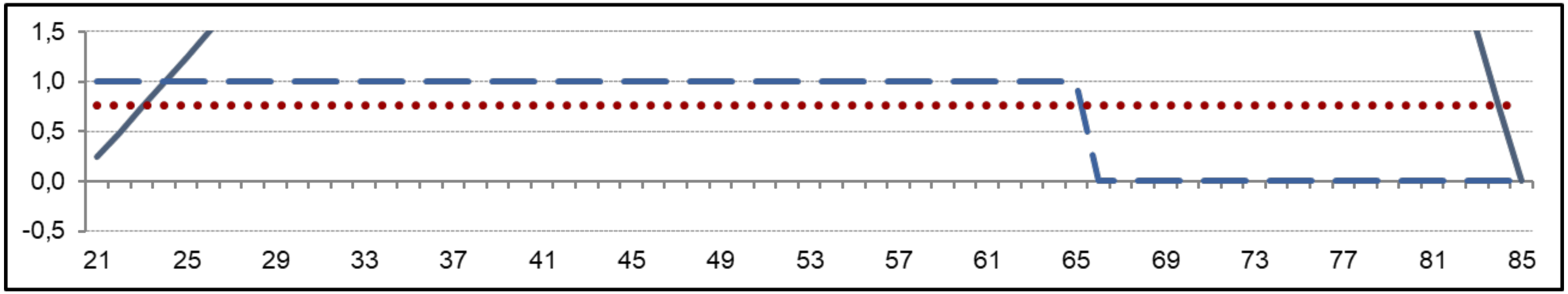
Modigliani Brumberg (1954)



Consumption, income and assets in the stripped-down life-cycle

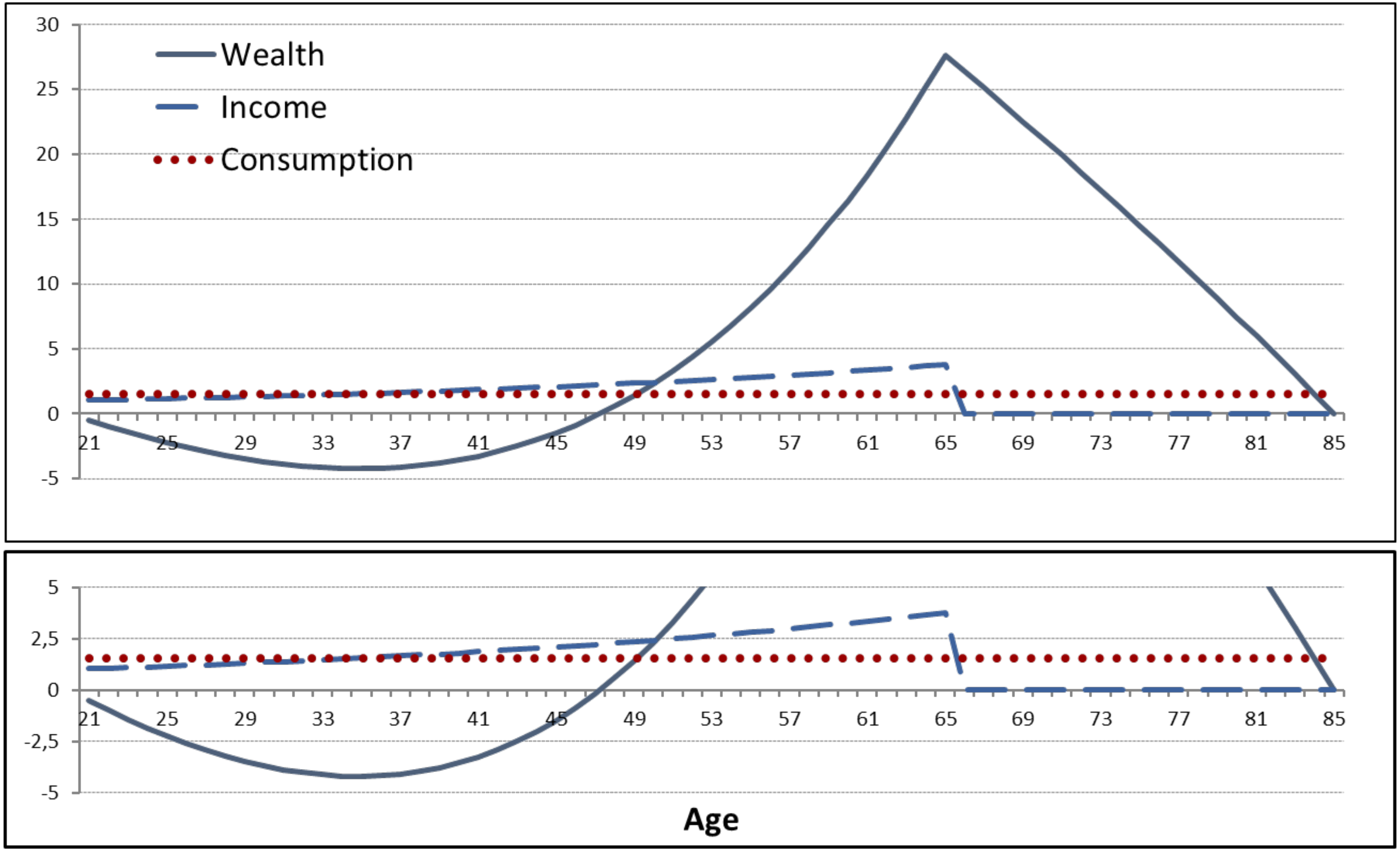
Lifecycle Profile: R = Δ > 1





Consumption perfectly smoothed over the lifecycle

Lifecycle Profile: Income Growth, $L_{t+1} = G\ L_t$



The investor borrow in the first years of working life

Income Growth and Net Wealth

- Since income will be higher later in working life, households smooth consumption by borrowing
  - Consumption for the young is higher than income
  - net wealth becomes more and more negative as long as household borrow (C>L)
  - net wealth recovers when the households saves by paying off its debt (C<L)
- Borrowing constraint: if young individuals cannot borrow, then their consumption can only track their income, and they will not be able to smooth consumption

$R \neq \Delta$: CRRA

CRRA preferences take the form

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \Rightarrow \quad EIS = -\frac{u'(C)}{Cu''(C)} = -\frac{(C)^{-\gamma}}{C\left(-\gamma (C)^{-\gamma-1}\right)} = \frac{1}{\gamma}$$

- (CG) becomes $\dfrac{dC_t^* / dt}{C_t^*} \approx \dfrac{1}{\gamma}(r-\delta)$
- consumption growth equals $(r-\delta)/\gamma$ and it is constant over time: consumption peaks at the end ($r > \delta$) or at the beginning ($r < \delta$) of the lifecycle
- If $r = \delta$, consumption is constant and its level is given by lifetime resources

CRRA: Consumption Level

(FO) becomes: $c_t^* = \left[\frac{1}{\lambda}\left(\frac{R}{\Delta}\right)^t\right]^{1/\gamma} = \frac{1}{\lambda^{1/\gamma}}\left(\frac{R}{\Delta}\right)^{t/\gamma}$

Plug into the consolidated budget constraint

$$W_0 + \sum_{t=1}^{T}\frac{L_t}{R^t} = \frac{1}{\lambda^{1/\gamma}}\sum_{t=1}^{T}\frac{\left(\frac{R}{\Delta}\right)^{t/\gamma}}{R^t}$$

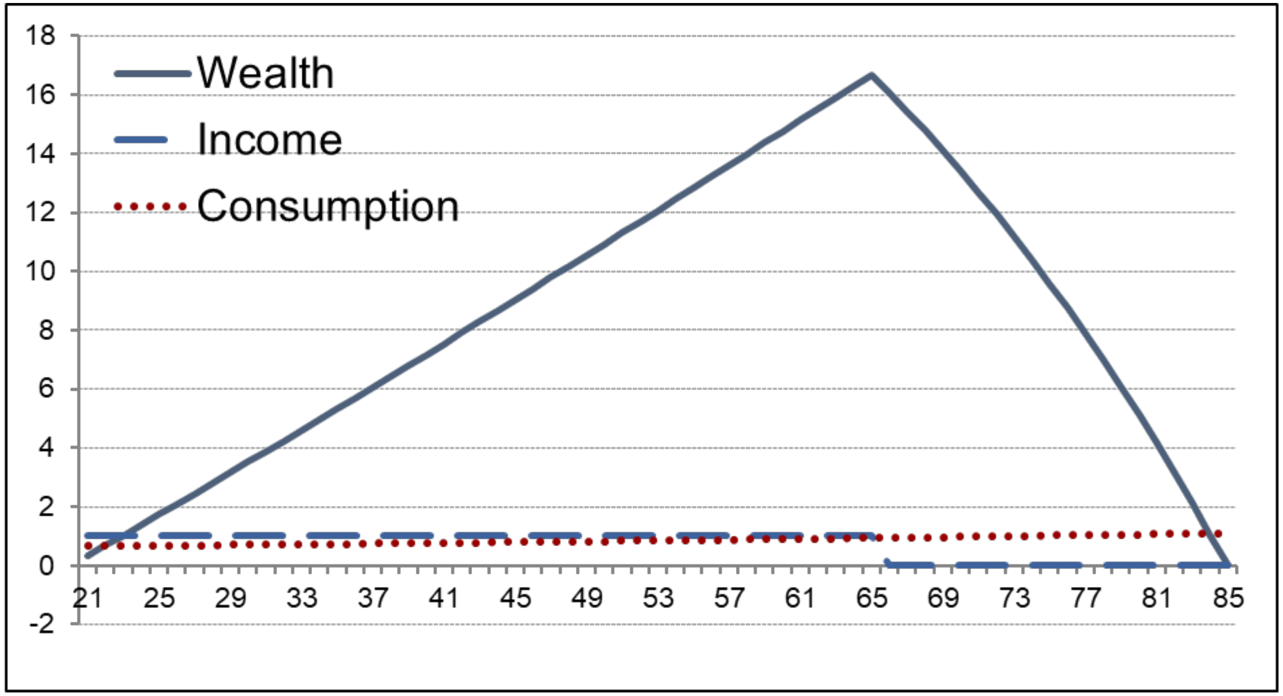
So that:

$$\frac{1}{\lambda^{1/\gamma}} = \left(W_0 + \sum_{t=1}^{T}\frac{L_t}{R^t}\right) \Big/ \sum_{t=1}^{T}\left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{t}{\gamma}}$$ and: $$c_t^* = \left(\frac{R}{\Delta}\right)^{\frac{t}{\gamma}} \boxed{\frac{W_0 + \sum_{\tau=1}^{T}\frac{L_\tau}{R^\tau}}{\sum_{\tau=1}^{T}\left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{\tau}{\gamma}}}}$$

Constant over time

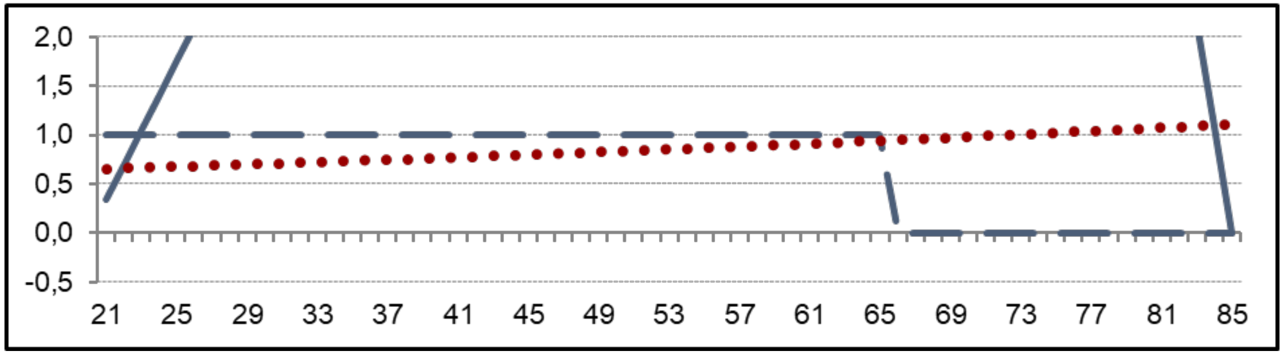
Consumption is proportional to lifetime resources and grows over time if $R > \Delta$

CRRA: $R > \Delta > 1$, $L_t = L$



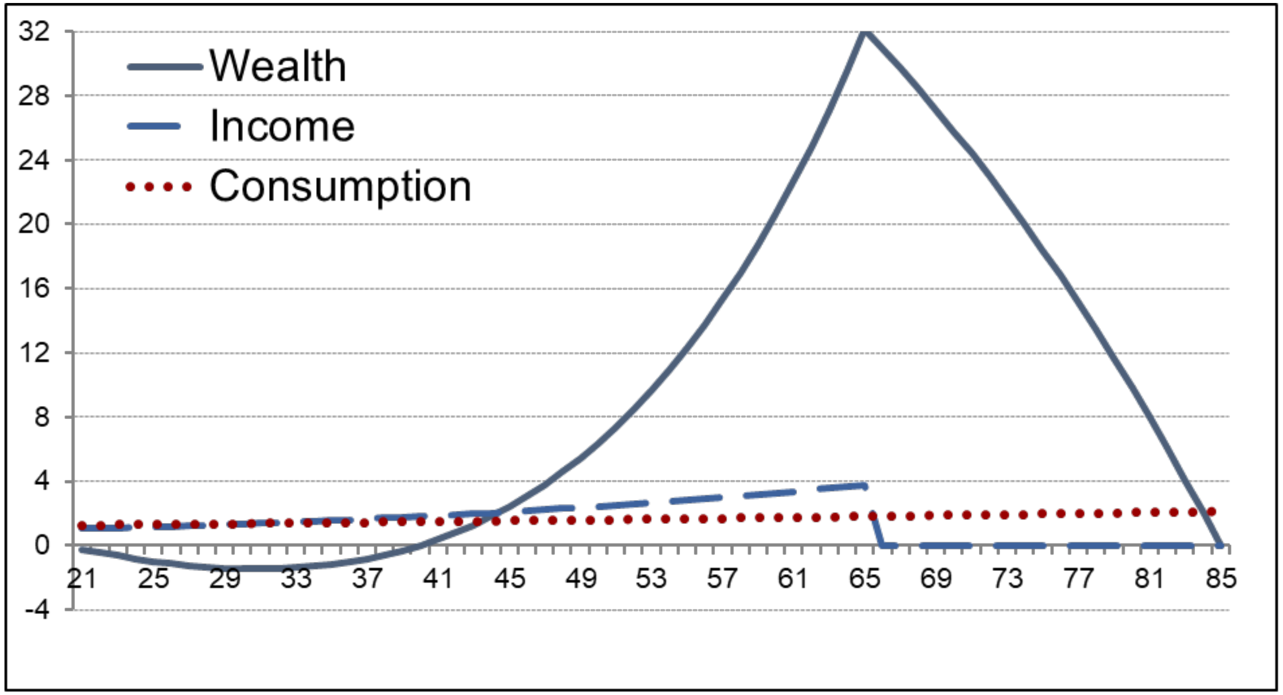
Real rate	2%
TPR	1%
Income growth	0%
Working age	21 yo
Retirement age	65 yo
Life expectancy	85 yo
Gamma	1.2

C/L at 21 yo	66%
C/L at 65 yo	94%



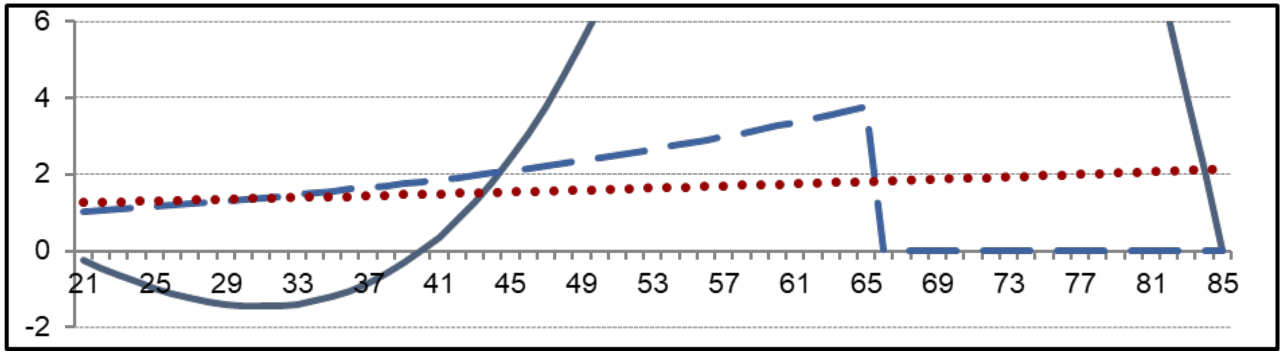
Consumption increases over the whole lifecycle

CRRA: $R > \Delta > 1$, $L_{t+1} = G\ L_t$



Real rate	2%
TPR	1%
Income growth	3%
Working age	21 yo
Retirement age	65 yo
Life expectancy	85 yo
Gamma	1.2

C/L at 21 yo	122%
C/L at 65 yo	48%



Borrow when young if income growth is large enough

If g=2%, initial C/L =98%

Summary

- The lifecycle model can generate several insights on optimal consumption and savings over the lifecycle
  - Consumption smoothing
    - Permanent Income Hypothesis
  - Accumulation and decumulation of wealth
    - **Borrow** early in life if income is expected to grow substantially, otherwise save for retirement
  - Consumption smoothing might be hampered by borrowing constraints: early in life cannot consume more than income
  - Saving behavior and consumption growth depend on how the rate of time preferences TPR relates to the market interest rate R
    - TPR>R (TPR<R), the consumer is more impatient (patient) than the market and consumption declines (grows) over time
- Constant marginal utility of consumption produces either ever constant or ever-declining or ever-growing consumption paths

Income and Consumption

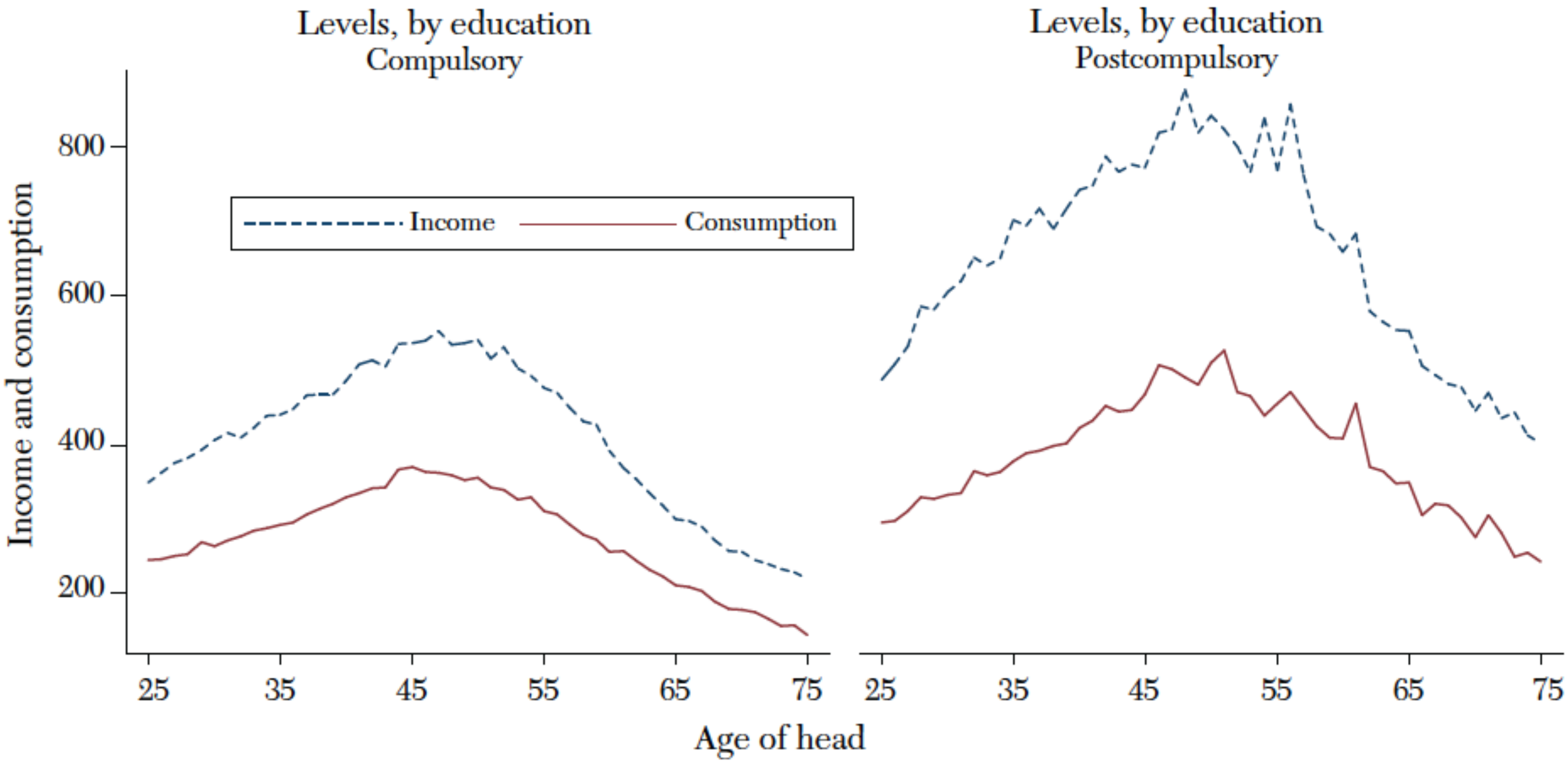


Figure 1. Average Income and (Nondurable) Consumption by Education

Source: U.K. Family Expenditure Survey, 1978–2007.

Attanasio and Weber, 2010, Consumption and Savings

Reconciling Facts and Theory

Consumption should be mostly smooth over time, instead it tracks income, why?

- Individuals don't manage their finances optimally (role for advice/policy)
- Lifecycle model abstracts from institutional environment
  - borrowing constraints early in working life: young households cannot borrow to smooth consumption
- Consumption or income measured incorrectly
  - after retirement, home production: households produce at home part of the goods and services they used to buy
  - The conometrician cannot distinguish permanent vs transitory imcome shocks
- Marginal utility of consumption might not be constant over time and might peak at middle age
  - habit model

State Dependence

- Variation over time can depend on state variables z_t (career effects, family composition …), so that $u_t(C_t\ ) = u(C_t, z_t)$, and the Euler Equation (EE) becomes

$$u'(C_t^*, z_t) = \frac{R}{\Delta} u'(C_{t+1}^*, z_{t+1}), \text{ or } \frac{u'(C_{t+1}^*, z_{t+1})}{u'(C_t^*, z_t)} = \frac{\Delta}{R}$$

- This might explain why in the data consumption varies over the lifecycle and peaks at middle age

Habit

$$u\left(C_t, z_t\right) = u\left(C_t - X_t\right)$$

X_t is a level of consumption below which the agent does not want to fall, typically called a habit level. It can represent

- minimum consumption subsistence
- consumption commitments
- standards of living

vary over lifecycle

Internal habit represents the agent standard of living (might be endogenous)

External habit represents the standard of living of the agent's peers (in terms of age, geographical location, social status, etc.)

In two periods the problem becomes (same budget constraint!!):

$$\max_{C_1, C_2} \frac{1}{\Delta} u(C_1 - X_1) + \frac{1}{\Delta^2} u(C_2 - X_2), \;\; s.t. \; W_0 + \frac{L_1}{R} + \frac{L_2}{R^2} = \frac{C_1}{R} + \frac{C_2}{R^2}$$

Example: CRRA and Habit

- CRRA preferences with habit: $u(C,X)=\dfrac{(C-X)^{1-\gamma}}{1-\gamma}$

$$EIS=-\frac{u'(C-X)}{Cu''(C-X)}=-\frac{(C-X)^{-\gamma}}{C\left(-\gamma(C-X)^{-\gamma-1}\right)}=\frac{1}{\gamma}\left(1-\frac{X}{C}\right)$$

- EIS is state dependent
- Note that $C_t^* > X_t$ since $u'(C-X)=+\infty$ for $C \approx X$
- If the interest rate is larger than the time preference rate ($r > \delta$), consumption growth is lower (and so are savings) in periods when habit is high and close to current consumption

Habit: Consumption Level

(FO) give $C_t^* = X_t + \left[\frac{1}{\lambda}\left(\frac{R}{\Delta}\right)^t\right]^{1/\gamma} = X_t + \frac{1}{\lambda^{1/\gamma}}\left(\frac{R}{\Delta}\right)^{t/\gamma}$

Plug into consolidated BC as in CRRA case:

$$W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t} = \sum_{t=1}^{T} \frac{1}{R^t}\left[\mathrm{X_t} + \frac{1}{\lambda^{1/\gamma}}\left(\frac{\mathrm{R}}{\Delta}\right)^{t/\gamma}\right]$$

Hence: $$\frac{1}{\lambda^{1/\gamma}} = \left(W_0 + \sum_{t=1}^{T} \frac{L_t}{R^t} - \sum_{t=1}^{T} \frac{X_t}{R^t}\right) \Big/ \sum_{t=1}^{T} \left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{t}{\gamma}}$$

And: $$C_t^* = X_t + \left(\frac{R}{\Delta}\right)^{\frac{t}{\gamma}} \boxed{\frac{W_0 + \sum_{\tau=1}^{T} \frac{L_\tau}{R^\tau} - \sum_{\boldsymbol{\tau}=\mathbf{1}}^{T} \frac{X_{\boldsymbol{\tau}}}{R^{\boldsymbol{\tau}}}}{\sum_{\tau=1}^{T} \left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{\tau}{\gamma}}}} \qquad \text{(H)}$$

Constant over time

Consumption covers habit and is proportional to lifetime resources minus the cost of maintaining the habit over the lifecycle

CRRA vs Habit

$$C_t^* = \left(\frac{R}{\Delta}\right)^{\frac{t}{\gamma}} \frac{W_0 + \sum_{\tau=1}^{T} \frac{L_\tau}{R^\tau}}{\sum_{\tau=1}^{T} \left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{\tau}{\gamma}}}$$

$$C_t^* = \boxed{X_t} + \left(\frac{R}{\Delta}\right)^{\frac{t}{\gamma}} \frac{W_0 + \sum_{\tau=1}^{T} \frac{L_\tau}{R^\tau} - \boxed{\sum_{\tau=1}^{T} \frac{X_\tau}{R^\tau}}}{\sum_{\tau=1}^{T} \left(\frac{R^{1-\gamma}}{\Delta}\right)^{\frac{\tau}{\gamma}}}$$

- Even if $R = \Delta$, consumption is not necessarily constant over time, agents consume more (and save less) during periods of higher habit
- Consumption depends on lifecycle resources *net* of the present value of maintaining the habit over time, its fluctuations over time depend on the level of the habit
- The hump shaped pattern of consumption over age can be explaining by hump shaped habit, as many expenditures related to standard of living (housing, children related, etc.) peak at 45-50yo

Summary: State Dependent Utility

Lifecycle consumption/savings model in the data

- Constant marginal utility of consumption implies full consumption smoothing: inconsistent with the data
- Solutions:
  - Borrowing constraints
  - time varying marginal utility (habit model)

State dependent utility and habit

- Save more or borrow for periods in which marginal utility of consumption is higher
- Higher marginal utlity might depends on career stages, family situations, past standards of living, etc.

Uncertainty: Value Function

By defining the value function V_t as the sum of current and future expected utility at the optimum consumption policy

$$V_t(W_t) = E_t\left[\sum_{\tau=0}^{T-t} u_t(C_{t+\tau}^*)\right]$$

with the budget constraint: $W_{t+1} = (W_t - C_t)R_{t+1} + L_{t+1}$

We can rewrite the maximization problem as a recursive problem

$$V_t(W_t) = \max_{C_t}\left\{u_t(C_t) + E_t\left[V_{t+1}\big((W_t - C_t)R_{t+1} + L_{t+1}\big)\right]\right\} \qquad \text{(VF)}$$

which has FOC:

$$u_t'(C_t) = E_t[R_{t+1}V_{t+1}'(W_{t+1})]$$

The utility of consuming marginally more (saving marginally less) today should equal the marginal value of wealth that it can generate tomorrow through the interest rate R_{t+1}

Uncertainty: Characterization

Taking the derivative of (VF) wrt wealth at the optimum, and using the envelope theorem, we have

$$V_t'(W_t) = E_t[R_{t+1}V_{t+1}'(W_{t+1})]$$

hence, using the FOC, we have: $u_t'(C_t) = V_t'(W_t)$

so that we obtain the famous **Euler Equation**:

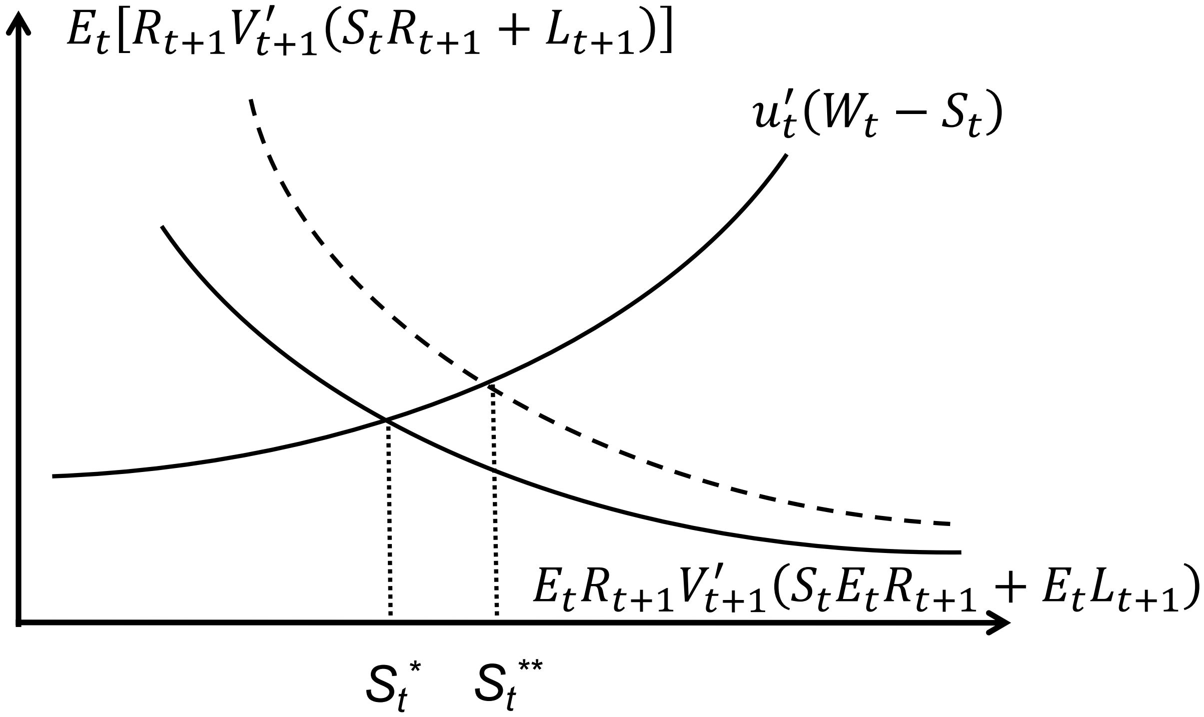
$$u_t'(C_t) = E_t[R_{t+1}u_{t+1}'(C_{t+1})] \qquad \text{(EE)}$$

Without uncertainty (EE) says that marginal rate of substitution of consumption today wrt consumption tomorrow $u_t'(C_t)/u_{t+1}'(C_{t+1})$ should be equal to the real discount factor $1/R_{t+1}$, i.e. the value of one unit of consumption tomorrow for the agent should be equal to its discounted value using the real interest rate rate

Precautionary Motive

In terms of savings $S_t = W_t - C_t$ the FOC are:

$$u'_t(W_t - S_t) = E_t[R_{t+1}V'_{t+1}(S_tR_{t+1} + L_{t+1})]$$



If *V'* is convex, its expectation (dotted line) is higher with uncertainty, and so are savings

The precautionary motive depends on the third derivative of *V*

Note: No precautionary savings with quadratic utility

Summary: Lifecycle Hypothesis

- The lifecycle consumption/savings model
  - Consolidated budget constraint and consumption smoothing
  - Rate of time preferences and market interest rates
  - Growth rate of consumption and the elasticity of intertemporal substitution
  - Borrowing constraints, time varying marginal utility and habit
  - Precautionary savings