Problem set 4: Life cycle savings

Kieran Larkin

September 26, 2022

1 Solve the life cycle model with endogenous grid method

1.1 Life cycle model without bequests

In this problem set we are going to solve a basic life cycle model with the *endogenous grid method* and simulate a panel of households.

Consider a life cycle model with no bequests:

$$\mathbf{E_1} \sum_{j=1}^{N} \beta^{j-1} \Big(\prod_{s=1}^{j} \varrho_s \Big) \frac{C_j^{1-\gamma}}{(1-\gamma)}$$

subject to:

$$\begin{aligned} C_j + A_{j+1} &= A_j(1+r) + Y_j \\ A_{j+1} &\geq 0 \end{aligned}$$

Income during working life is:

$$\begin{aligned} Y_j &= P_j U_j G_j \\ p_j &= \log(P_j) \\ p_j &= \rho p_{j-1} + n_j \end{aligned}$$

- Transitory income U_j is i.i.d.
- G_j is a deterministic component capturing the income profile
- n_j is a i.i.d permanent income shock $n_j \sim \mathcal{N}(0, \sigma_n^2)$

When retired household receives a pension based on average end of working life wage: ψG_{N^w}

1.1.1 Calibration:

- Annual model. Discount factor $\beta = 0.94$. Interest rate $r = 0.04$
- Set household to work for $N^w = 40$ periods and retire for $N^r = 20$ periods before dying with probability 1.
- Set the survival probability $\varrho = 1$ during working period of life
- For $j > N^w$, set the survival probability as a linearly decreasing trend starting at 1 for $j = N^w + 1$ and reaching 0.92 in period N
- Coefficient of relative risk aversion $\gamma = 1.5$
- For permanent income process:
  - $\rho = 0.97$ and $\sigma_\epsilon^2 = 0.015$
  - Use the Floden weighting for the Markov approximation of the income process
- For temporary income process:

$$U_j = \begin{cases} 0.4 & \textit{with probability } p_u = 0.01 \\ 1 & \textit{otherwise} \end{cases}$$

- For G_j download the file `lc_profile.csv` from the website. This can be loaded into matlab with `csvread` command. Age 25 is $j = 1$ in model.
- Set the pension replacement rate to $\psi = 0.6$
- Use 5 grid points in the permanent income dimension
- Use 2 grid points in the transitory income dimension
- Use 1,000 uniformly spaced grid points in tomorrow's assets dimension
- Set min asset choice to $\underline{a} = 0$ and max assets $\overline{a} = 80$

1.1.2 Endogenous grid method

- Solve the model backwards with the endogenous grid method using the Euler equation
- In the last period enforce $A_{N+1} = 0$ such that household consumes all cash in hand
  - Try: $C_N = X_N = \mathcal{G}_{A'} + 0.01$[1]
- Remember to store the endogenous grid over cash in hand $\mathcal{G}_{X_j}$ each period

[1] This is a bit unnecessary as no household will hold this much assets in final period

- You will need to make use of the `interp1` function in Matlab and include the option for extrapolation

- Given policies defined over cash on hand, each period you'll need to construct $X_{j+1} = A_{j+1}(1+r)+Y_{j+1}$. Cash on hand tomorrow given asset choice $A_{j+1} \in \mathcal{G}_{A'}$ today
- Add a point $C_j = X_j = 0$ to your consumption policy function to capture behaviour of constrained households

1.2 Simulate economy

Once you have solved the policies of the household, simulate a lifecycle panel of 5,000 households.

1.2.1 Simulation settings:

- Set initial income to 1 ($Y_1 = 1,\ U_1 = 1$)
- Draw initial assets from the following log normal[2] distribution: $A_1 \sim \log \mathcal{N}(\mu_{a0}, \sigma_{a0}^2)$
- With parameters $\mu_{a0} = -2.5$ and $\sigma_{a0}^2 = 4$
  - limit the maximum initial assets to $\exp(\mu_{a0} + 3 \times \sigma_{a0})$
- Store the permanant income shocks $\{n_{ij}\}\ \forall i, j$ for later
- When you simulate the income process it's fine (for this problem set) to use the policy for income on the nearest grid point, while letting the actual continuous income variable enter cash in hand.
  - i.e. you don't need to undertake two-dimensional interpolation of the policy function $c_j(X, Y)$

1.3 Life cycle questions

1. Plot the average life cycle profiles for consumption, income, assets and share of households still alive
2. Report the growth in average log consumption and average log income from j=1 to the maximum lifecycle value.
   (a) How does income growth compare to consumption growth?
3. Plot the evolution of the cross-sectional variance of log consumption and log income as households age

[2] I.e. take the exponential of the normal distibution as with income

1.4 Insurance questions

We want to understand how much insurance there is to permanent income shocks in this life cycle model. One measure *in the spirit of* Blundell, Pistaferri and Preston (2008) is to calculate:

$$\phi = 1 - \frac{cov(\Delta c_{it}, n_{it})}{var(n_{it})}$$

Where Δc_{it} is the growth log consumption from time $t-1$ to time t. n_{it} is the permanent income shock.

1. Calculate this ϕ for the economy using working age households

2. Kaplan & Violante (2010) report that insurance is decreasing in the persisence of the income process. Evaluate $\phi(\rho)$ for 10 uniform grid points between 0.7 and 0.995.

   (a) Plot the result with ρ on the x-axis and 1- $\phi(\rho)$ on the y-axis.

   - Tip: when comparing simulations/calibrating its a good idea to store the shocks before hand, so you are using the same set of shocks for every iteration