Household Finance
PhD Course

Risk Aversion

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Outline

- Lifecycle model
  - Participation decision
- Risk aversion
  - Relative and absolute risk aversion
- Elicitation of preferences
  - Surveys: qualitative vs quantitative
  - Revealed preferences

Lifecycle Model

$$\max_{\{C_t, w_t\}_{t=1,\dots T}} E_0\left[\sum_{t=1}^{T} \frac{1}{\Delta^t} u(C_t)\right]$$

$$\text{s.t. } W_{t+1} = R_{t+1} W_t + L_{t+1} - C_{t+1}$$

$$R_{t+1} = 1 + r_f + w_t r_{t+1}^e$$

- Households choose not only how much to consume in each period (i.e. how much to borrow or save), but also how much risk to take in their portfolio, i.e. the the share of net wealth invested in the risky asset $\boldsymbol{w_t}$, or risky share
- The utility function $u(C_t)$ will not only regulate impatience, EIS, and precautionary saving, but also the household's attitude towards risk, its risk preferences
- Wealth is only liquid wealth, no real estate for the moment
- Borrowing at the risk free rate

Participation Decision

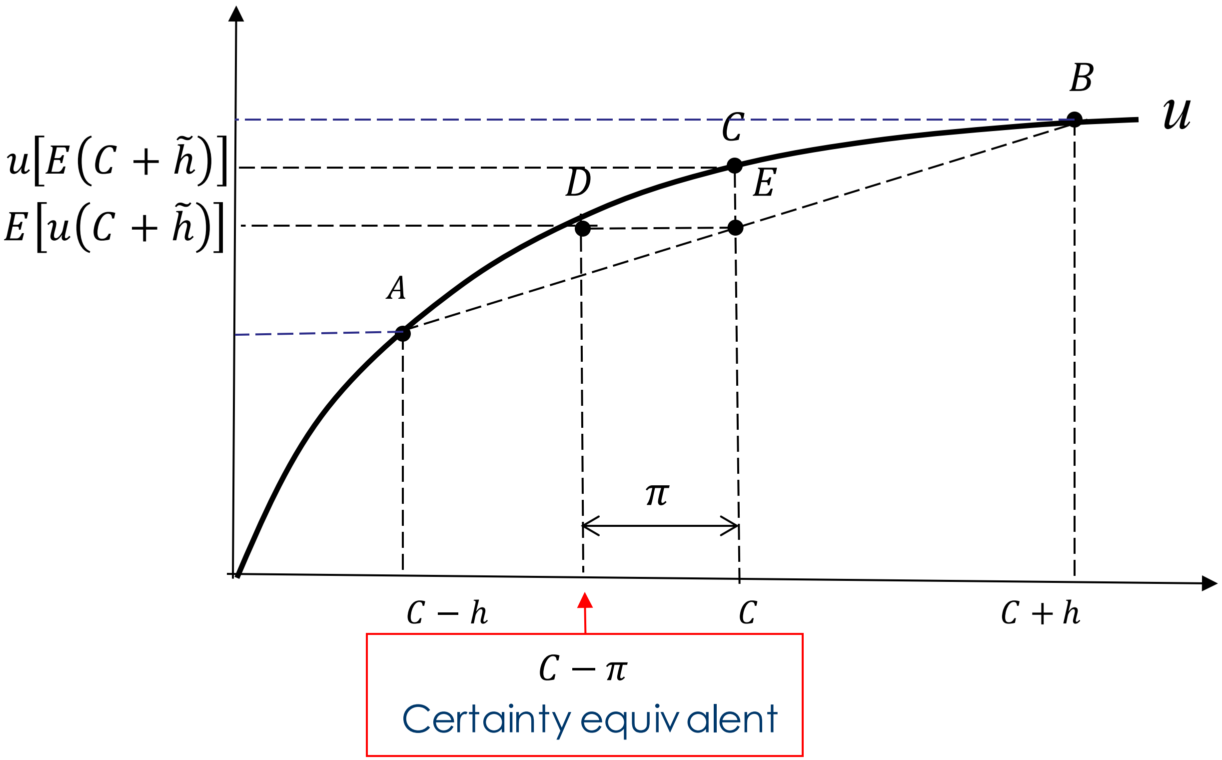
$$\max_{\{C_t, w_t\}_{t=1,\ldots T}} E_0\left[\sum_{t=1}^{T} \frac{1}{\Delta^t} u(C_t)\right]$$

$$\text{s.t. } W_{t+1} = R_{t+1} W_t + L_{t+1} - C_{t+1} - K\, \mathbb{I}(w_t \neq 0)$$
$$R_{t+1} = 1 + r_f + w_t r_{t+1}^e$$

- *K* is a participation cost that the agent has to pay if she decides invests in the risky asset
  - $\mathbb{I}(\ldots)$ is an indicator function that is one if the condition in the parenthesis is true and otherwise it is zero
  - Different modelling: Catherine (forthcoming), Galvez (2019)
- The participation cost will reduce consumption so it usually interpreted broadly:
  - Transaction/trading costs/fees or costs related to getting financial advice … but also
  - The opportunity cost of the time spent learning about financial markets, which can be larger for less educated households

Risk preferences

Coefficient of Risk Aversion



Fair Gamble

$$E(\tilde{h}) = 0$$

$$\Downarrow$$

$$E(C+\tilde{h}) = C$$

Coefficient of relative risk aversion $RRA = -\frac{C\,u''(C)}{u'(C)}$

Coefficient of absolute risk aversion $ARA = -\frac{u''(C)}{u'(C)}$

Interpretation

The risk premium π is how much the investor requires to be paid to take the fair gamble $\tilde{h}$

- RRA: proportional risk premium and gamble

$$E\left[u\left(C(1+\tilde{h})\right)\right] = u\left(C(1-\pi)\right) \Rightarrow \pi \cong \frac{1}{2}\underbrace{\left[-\frac{Cu''(C)}{u'(C)}\right]}_{\text{RRA}} Var(\tilde{h})$$

- ARA: additive risk premium and gamble

$$E\left[u(C+\tilde{h})\right] = u(C-\pi) \Rightarrow \pi \cong \frac{1}{2}\underbrace{\left[-\frac{u''(C)}{u'(C)}\right]}_{\text{ARA}} Var(\tilde{h})$$

RRA: derivation

Consider a fair gamble $\tilde{h}$, i.e. $E(\tilde{h}) = 0$, **proportional to wealth**

The risk premium $\tilde{\pi}$ is how much the investor requires to be paid to take the gamble:

$$\boldsymbol{E[u(W(1+\tilde{h}))] = u(W(1-\tilde{\pi}))}$$

For small gambles and small risk premia, we can use the Taylor expansion: $f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$

1. Approximate around *W (a=W)*

Second-order Taylor expansion of the LHS:

$$\boldsymbol{E}\left[\boldsymbol{u}\left(\underbrace{\boldsymbol{W(1+\tilde{h})}}_{\boldsymbol{W+W\tilde{h}=x}}\right)\right] \approx E\left[u(W) + u'(W)\left(\underbrace{W + W\tilde{h}}_{\boldsymbol{x}} - W\right) + \frac{u''(W)}{2}\left(\underbrace{W + W\tilde{h}}_{\boldsymbol{x}} - W\right)^2\right] \Rightarrow$$

$$\boldsymbol{E[u(W(1+\tilde{h}))]} \approx u(W) + u'(W)W\underbrace{E(\tilde{h})}_{\boldsymbol{0}} + \frac{1}{2}u''(W)W^2\underbrace{E(\tilde{h}^2)}_{Var(\tilde{h})}$$

First-order Taylor expansion of the RHS:

$$\boldsymbol{u}\left(\underbrace{\boldsymbol{W(1-\tilde{\pi})}}_{\boldsymbol{W-W\tilde{\pi}=x}}\right) \approx u(W) + u'(W)\left(\underbrace{W - W\tilde{\pi}}_{\boldsymbol{x}} - W\right) \Rightarrow \boldsymbol{u(W(1-\tilde{\pi}))} \approx u(W) - u'(W)W\tilde{\pi}$$

2. Solve for $\tilde{\pi}$: $\tilde{\pi} = \frac{1}{2}\underbrace{\left[-\frac{\boldsymbol{Wu''(W)}}{\boldsymbol{u'(W)}}\right]}_{\boldsymbol{RRA}} Var(\tilde{h})$

If U is concave $\Rightarrow$
$u''(W) < 0 \Rightarrow RRA > 0$

ARA: derivation

Consider a fair gamble h, i.e. $E(h) = 0$, **additive**

Additive risk premium π how much investor requires to be paid to take the gamble:

$$\boldsymbol{E[u(W+h)] = u(W-\pi)}$$

For small gambles and small risk premia, we can use the Taylor expansion: $f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$

1. Approximate around W ($a=W$)

<u>Second-order Taylor expansion of the LHS:</u>

$$\boldsymbol{E}\left[\boldsymbol{u}\underbrace{(\boldsymbol{W}+\boldsymbol{h})}_{\boldsymbol{W}+\boldsymbol{h}=\boldsymbol{x}}\right] \approx E\left[u(W) + u'(W)\left(\underbrace{W+h}_{\boldsymbol{x}} - W\right) + \frac{u''(W)}{2}\left(\underbrace{W+h}_{\boldsymbol{x}} - W\right)^2\right] \Rightarrow$$

$$\boldsymbol{E[u(W+h)]} \approx u(W) + u'(W)\underbrace{E(\tilde{h})}_{\boldsymbol{0}} + \frac{1}{2}u''(W)\underbrace{E(\tilde{h}^2)}_{Var(\tilde{h})}$$

<u>First-order Taylor expansion of the RHS:</u>

$$\boldsymbol{u}\left(\underbrace{\boldsymbol{W}-\boldsymbol{\pi}}_{\boldsymbol{W}-\boldsymbol{\pi}=\boldsymbol{x}}\right) \approx u(W) + u'(W)\left(\underbrace{W-\pi}_{\boldsymbol{x}} - W\right) \Rightarrow \boldsymbol{u(W-\pi)} \approx u(W) - u'(W)\pi$$

2. Solve for π: $\pi = \frac{1}{2}\underbrace{\left[-\frac{\boldsymbol{u''(W)}}{\boldsymbol{u'(W)}}\right]}_{\boldsymbol{ARA}}Var(\tilde{h})$

If U is concave $\Rightarrow$ $u''(W) < 0 \Rightarrow ARA > 0$

Estimating Risk Preferences

Why?

- Essential part of financial advice
  - Comply with EU regulations (European Investment Service Directive – MiFID)
- Assessment of financial mistakes

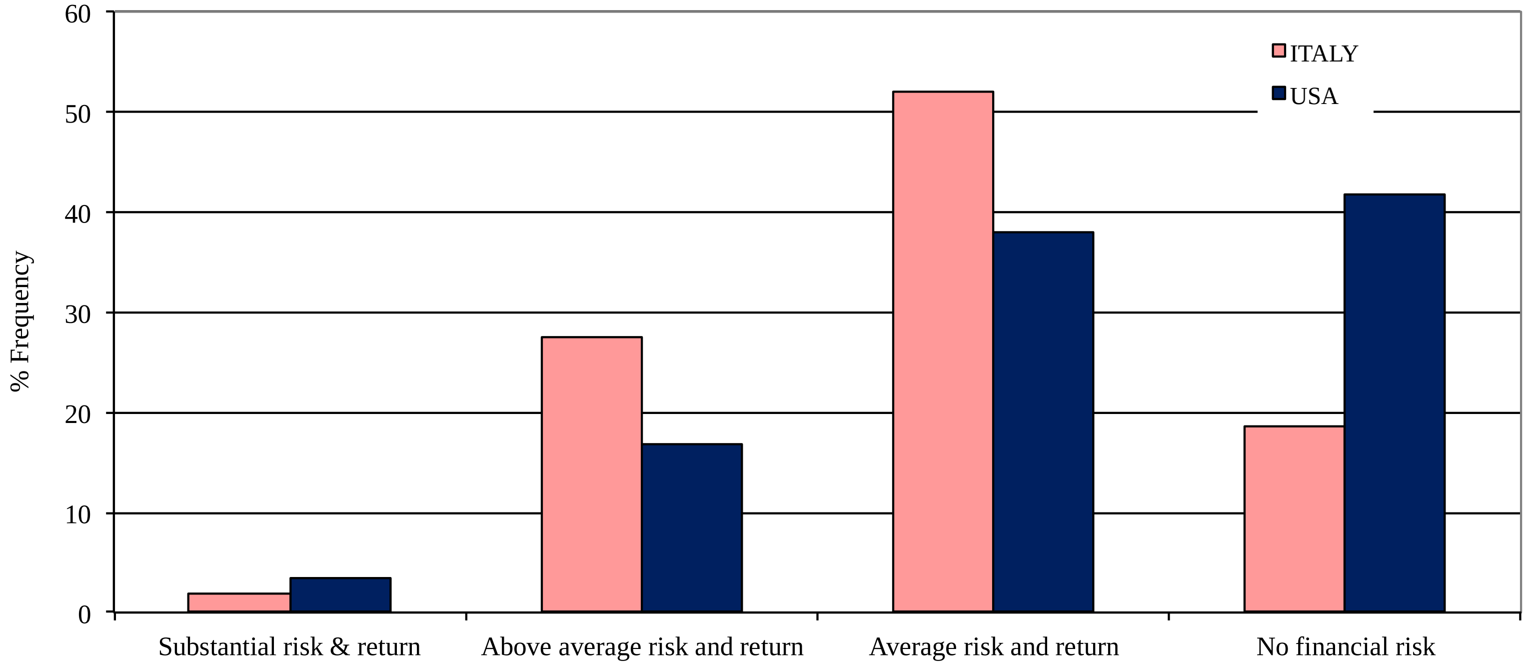
How?

- **Elicitation**: survey data (Barsky et al., 1997)
- **Revealed preference**: data on observed decisions (Friend and Blume, 1975)
  - Useful to infer risk aversion from choices in one domain to study choices in another domain

Qualitative Elicitation Measures

Which of the following statements comes closest to the amount of financial risk that you are willing to take when you make your financial investment?

1. Take substantial financial risks expecting to earn substantial returns;
2. Take above average financial risks expecting to earn above average returns;
3. Take average financial risks expecting to earn average returns;
4. Not willing to take any financial risks



Survey of Consumer Finances (US) and Unicredit Survey (Italy)

Quantitative Elicitation Measures

The US SCF or the Italian SHIW ask questions like:

SHIW (Italy): Willingness to pay for 5000EUR or 0 with probability ½

However the gamble is not proportional to wealth or consumption, i.e. it helps to measure absolute risk aversion ARA but not relative risk aversion RRA

To measure relative risk aversion, Barsky et al. (97) ask the following question to PSID respondents and find an average RRA coefficient of about 4:

Situation: As the only income earner in the family you have a job that gives you income yearly for life.
Opportunity: 50% chance double income, 50% chance cut income by 1/3

Accept

Opportunity: 50% chance double income
50% chance cut income by 1/2

Accept / Decline

Decline

Opportunity: 50% chance double income
50% chance cut income by 1/5

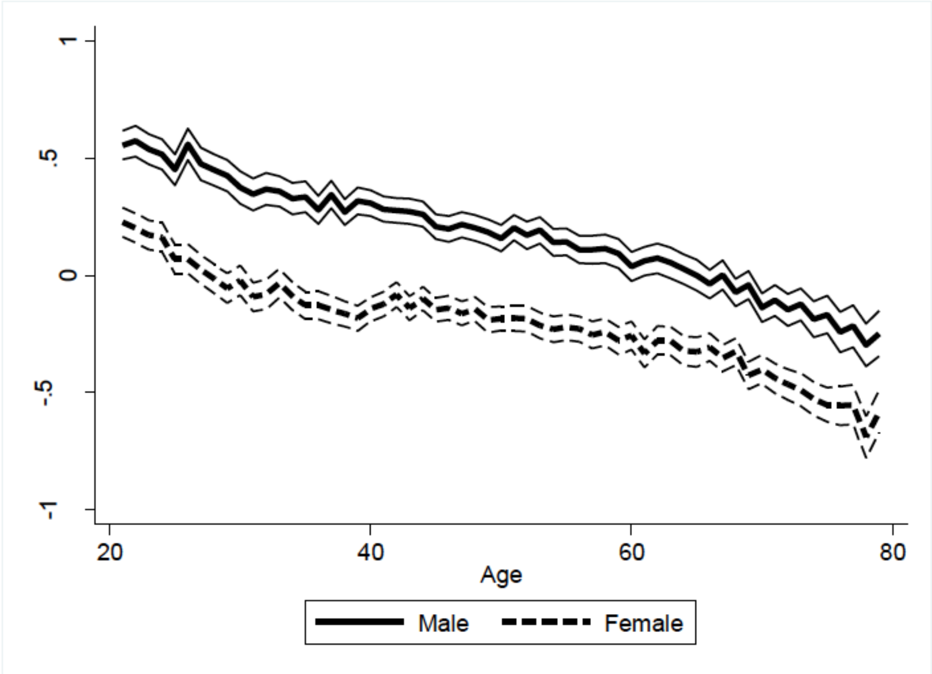
Accept / Decline

External Validity

Risk aversion measures correlates well risk taking in other domains, e.g. car driving, health, career, sport and leisure (Barsky et al., 97)

They also correlate as expected with individual characteristics:

Risk Attitudes across Age



Source: SOEP

Dohmen et al. 2015 - Risk attitudes across the life course

General Willingness to Take Risks	
Female	-0.614***
	(0.056)
Age (in years)	-0.020***
	(0.003)
School Degree	0.505***
	(0.173)
Retired (Pension)	-0.335***
	(0.137)
Log(household income 2002)	0.107***
	(0.007)
Log(household income 2003)	0.066***
	(0.012)
Log(household income 2002)	0.043***
	(0.013)
Life Satisfaction	0.102***
	(0.014)
Constant	1.171
	(0.815)
Observations	14,773

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

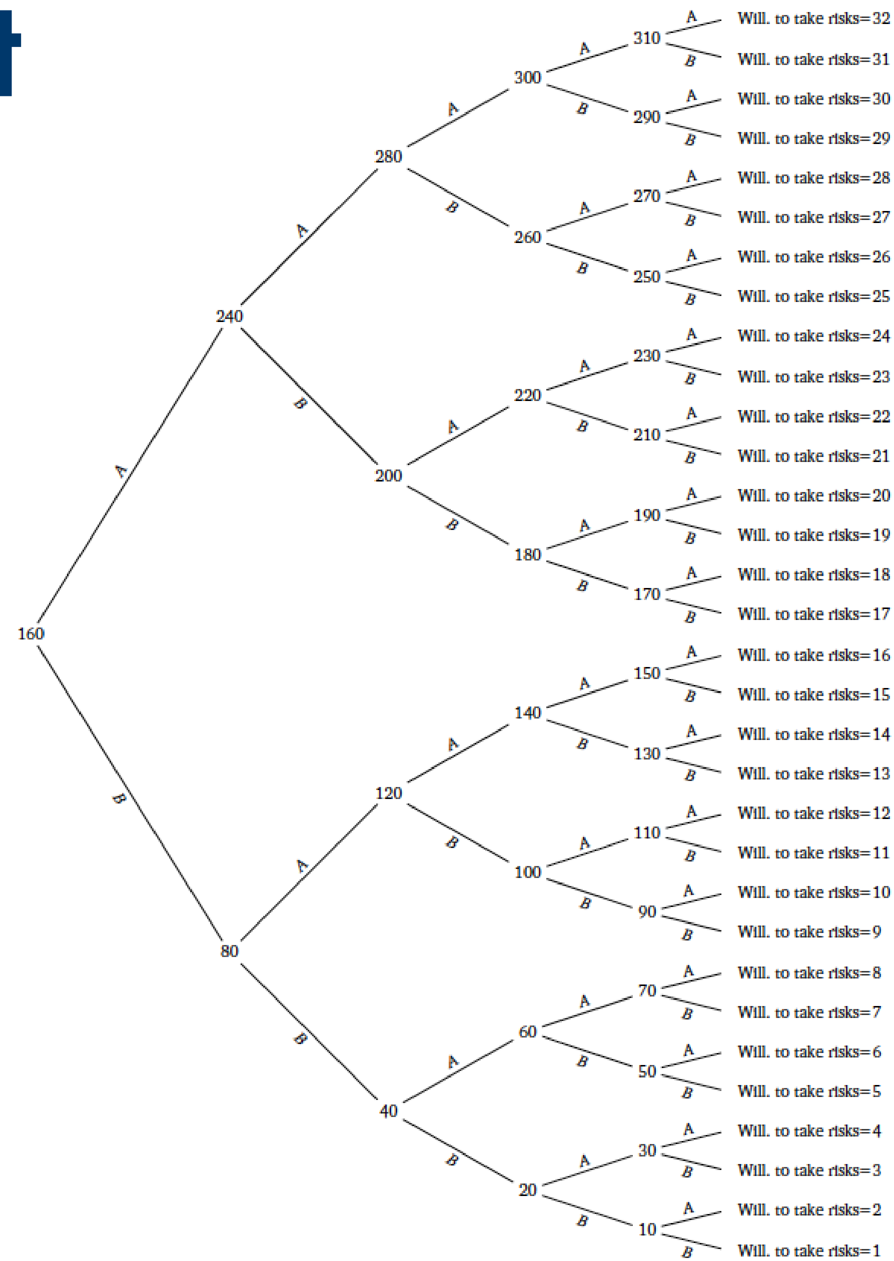
Dohmen et al. 2011 – Individual risk attitudes

State of the art

Falk et al. (QJE 2018)

Please imagine the following situation.

- You can choose between a sure payment of a particular amount of money, or a draw, where you would have an equal chance of getting amount x or getting nothing.
- We will present to you five different situations. What would you prefer: a draw with a 50 percent chance of receiving amount 300Euro, and the same 50 percent chance of receiving nothing, or the amount of y as a sure payment (160Euro at the beginning then A if the gamble is accepted and B if the gamble is turned down)?



Elicitation of TPR

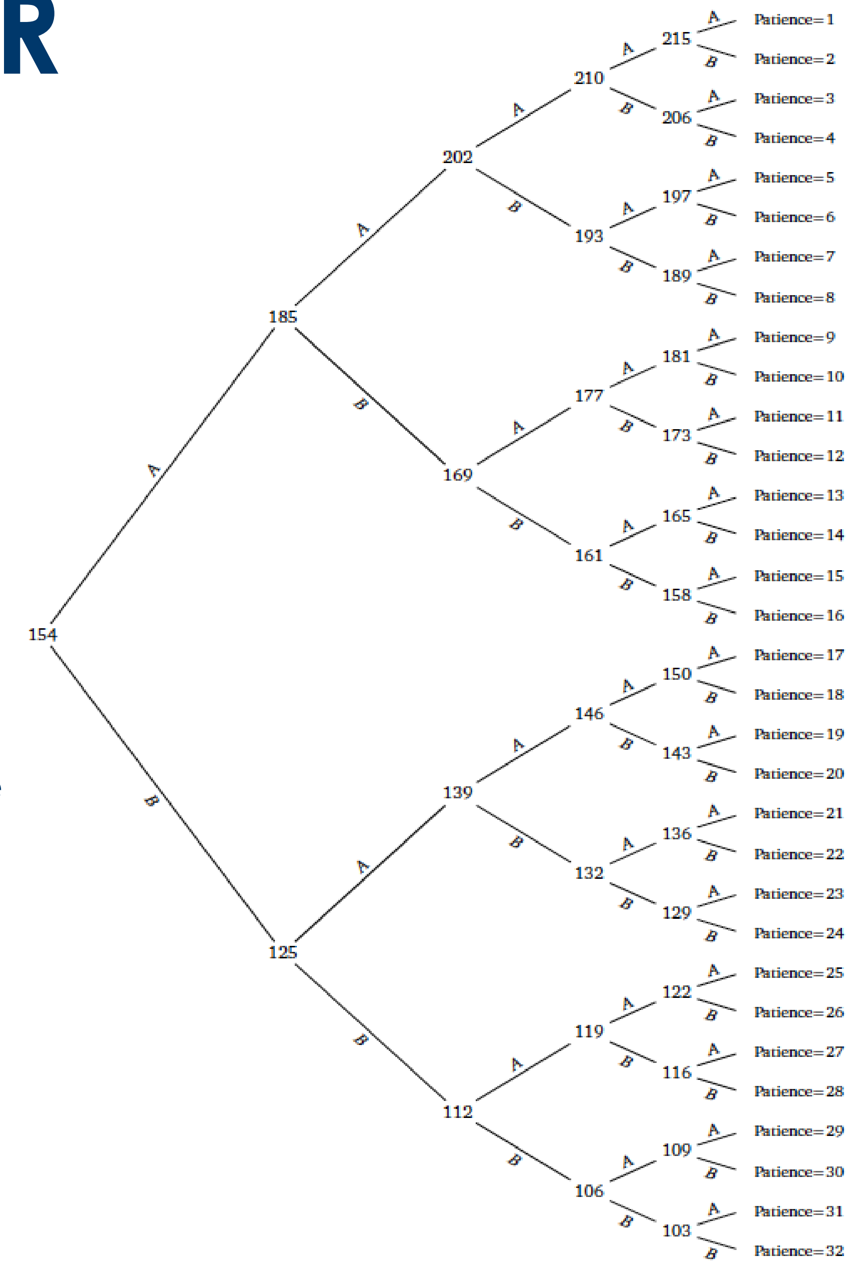
Falk et al. (QJE 2018)

Suppose you were given the choice between receiving a payment today or a payment in 12 months.

- We will now present to you five situations. The payment today is the same in each of these situations. The payment in 12 months is different in every situation. For each of these situations we would like to know which you would choose. Please assume there is no inflation, i.e, future prices are the same as today's prices.
- Would you rather receive 100 Euro today or x Euro in 12 months?

(A: 100 today, B: x in 12 months)

Note: huge TPR!!



Qualitative vs Quantitative Measures

Qualitative measures

- are simple to ask and have high response rates
- do not distinguish between aversion to risk and risk perception / beliefs

Quantitative measure

- elicit precise measures of ARA, RRA or TPR
- have lower response ratio, are more complex
- respondents tend to under-report willingness to pay
- answers change depending on how questions are asked

Hypothetical elicitation might not reflect risk preferences in real life

Merton Model

Merton Model

- Simplest and fundamental framework for financial risk taking

$$\max_{\{C_t, w_t\}_{t=1,\ldots T}} E_0\left[\sum_{t=1}^{T} \frac{1}{\Delta^t} u(C_t)\right]$$

$$\text{s.t. } W_{t+1} = \left(1 + r_f + w_t r_{t+1}^e\right) W_t - C_{t+1}$$

- Δ is the rate of time preferences
- Preferences are constant over time otherwise (no z_t)
- No labor income
  - Consumption out of capital income or wealth itself
  - Rich household
- Borrowing and lending at the risk free rate r_f
- Excess return on the risky asset is i.i.d.
- Finite horizon T

CRRA Utility

If the investor has CRRA utility $u(C_t) = \frac{C^{1-\gamma}}{1-\gamma}$ the optimal risky share is (Merton, 1969)

$$w_t^M = \frac{E_t(r_{t+1}^e)}{\gamma Var_t(r_{t+1}^e)} \tag{1}$$

The share of wealth optimally invested in risky assets

- does not depend on household wealth
- does not depend on horizon
- varies over time if beliefs and/or risk aversion changes
- One to one mapping between γ and w_t^M

γ is the coefficient of relative risk aversion and regulates how much risk the agent is willing to take

CRRA Utility: Calibration

- 2% risk free rate
- 4% equity premium
- 16% volatility

- Risk aversion $\gamma = 3 \Longrightarrow w_t^* = \frac{E_t(r_{t+1}^e)}{\gamma Var_t(r_{t+1}^e)} = \frac{4\%}{3 \times 0.16^2} = 52\%$
- Risk aversion $\gamma = 8 \Longrightarrow w_t^* = \frac{E_t(r_{t+1}^e)}{\gamma Var_t(r_{t+1}^e)} = \frac{4\%}{8 \times 0.16^2} = 19.5\%$
- Risk aversion $\gamma = 2 \Longrightarrow w_t^* = \frac{E_t(r_{t+1}^e)}{\gamma Var_t(r_{t+1}^e)} = \frac{4\%}{2 \times 0.16^2} = 78.1\%$

Elicitation by Revealed Preferences

Solve for γ in (1): $\gamma = \dfrac{E_t(r_{t+1}^e)}{w_t^M Var_t(r_{t+1}^e)}$

Assume an expected excess return of 6.2% and a volatility of 20% in the first two columns

SCF: Survey of Consumer Finances – US

Swedish wealth registry contains individual assets information => estimate $E(r^e)$ and $Var(r^e)$

Source: Guiso and Sodini, 2013

	Relative Risk Aversion Coefficient	
	Imputed Risky Portfolios (full diversification)	
Percentiles	US SCF	Swedish Wealth Registry
1	1.6	1.6
p5	1.6	1.7
10	1.8	1.9
25	2.2	2.4
50	3.5	3.8
75	7.1	8.6
90	16.4	24.9
95	30.8	50.1
99	136.4	189.6

Administrative vs Survey Data

- Administrative data is very reliable but lacks information on preferences and beliefs
- Preferences can be elicited in experiments and surveys
  - Very challenging (e.g. Epper et al. 2020 for impatience)
  - Yet crucial to test models
- Surveys can shed light on alternative theories
  - Choi and Robertson 2020

Some Data Sources

- Europe: HFCS
  - available at the ECB: https://www.ecb.europa.eu/stats/ecb_surveys/hfcs/html/index.en.html
- US: SCF
  - available at the FED: https://www.federalreserve.gov/econres/scfindex.htm
- China: CHFS
  - administered by the Southwestern University of Finance and Economics: https://e.swufe.edu.cn
- See also:
  - http://www.household-finance.net/datasets?field_datacountry_tid=All&field_availability_tid=All&field_observation_level_tid=All&field_data_type_tid=All&field_research_field_tid=All&field_publication_status_tid=All&sort_by=changed&sort_order=DESC”
  - https://www.eui.eu/Research/Library/ResearchGuides/Economics/Statistics/MicroDataSet

Optimal Participation

With a participation cost K. i.e. with a BC:

$$W_{t+1} = \left(1 + r_f + w_t r_{t+1}^e\right) W_t - C_{t+1} - K\, \mathbb{I}(w_t \neq 0)$$

The optimal solution is to participate (i.e. choose $w_t \neq 0$) if the optimal Merton solution w_t^M is above a certain threshold $\bar{w}$

In other words, the optimal solution becomes:

$$w_t^{pM} = w_t^M = \frac{E_t(r_{t+1}^e)}{\gamma Var_t(r_{t+1}^e)}, \qquad if w_t^M \geq \bar{w}$$

$$w_t^{pM} = 0, \qquad if w_t^* < \bar{w}$$

$\bar{w}$ depends on the participation cost K

Again the optimal risky share does not change over time unless believes change and in particular it does not depend on wealth unless the participation cost itself does