Lecture 2
Second-year Ph.D. course in Household Finance

Roine Vestman[1]

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[1]Stockholm University and Centre for Economic Policy Research

Outline today

- Extensions to the consumer's problem:
    - Epstein-Zin preferences
    - Portfolio choice and optimal glide paths
    - Pension systems
    - Housing and mortgages
- The curse of dimensionality
- EGM and discrete control variables
- Guide to Uppmax
- Assistance with the problem set

Epstein-Zin (1989,1991) preferences

At age t, each individual maximizes the following:

$$U_t = \left(C_t^{1-\rho} + \beta E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{1}$$

$$U_T = C_T, \tag{2}$$

where β is the discount factor, $\psi = 1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion. For $t = 25, 26, ..., T$ with $T = 100$.
For notational convenience adopt the notation of Lars Hansen and others and define the operator $\mathcal{R}_t(U_{t+1}) \equiv E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$.

The rest of the model is the same as before.

Income

Let $y_{it} = \ln(Y_{it})$. Then for $t \leq 45$:

$$
\begin{aligned}
y_{it} &= g_t + z_{it} + \omega_{it}, && (3)\\
z_{it} &= \rho z_{it-1} + \eta_{it}, && (4)\\
y_{it} &\geq \ln(\underline{Y}). && (5)
\end{aligned}
$$

where g_t is a deterministic life-cycle trend, $\rho \in (0,1]$, $\underline{Y} \geq 0$. Retirement:

$$
\begin{aligned}
Y_{i,t} &= \lambda Y^{p}_{i,45} \\
&= \lambda \exp(g_{45} + z_{i45}), \quad t > 45. && (6)
\end{aligned}
$$

Stochastics:

$$
\begin{aligned}
z_{i1} &\sim N\left(-\sigma_z^2/2, \sigma_z^2\right). && (7)\\
\eta_{it} &\sim N\left(-\sigma_\eta^2/2, \sigma_\eta^2\right). && (8)\\
\omega_{it} &\sim N\left(-\sigma_\omega^2/2, \sigma_\omega^2\right). && (9)
\end{aligned}
$$

Budget constraints and laws of motions

Let $\mathcal{S}_t = \{z_t, X_t\}$ denote the state variables for $t = 1, ..., T$. Budget constraints and laws of motions for state variables:

$$C_t + A_t = X_t, \tag{10}$$

$$A_t \geq 0, \tag{11}$$

where X_t denotes cash on hand. Its law of motion is:

$$X_{25} = \hat{A}_{25} + Y_{25}, \tag{12}$$

where $\hat{A}_{25}$ is initial financial wealth.

$$\begin{aligned} X_{t+1} &= A_t R + Y_{t+1} && (13) \\ &= A_t R + e^{g_{t+1} + \rho z_t + \eta_{t+1} + \omega_{t+1}} \quad t = 25, ..., 63 && (14) \end{aligned}$$

$$\begin{aligned} X_{t+1} &= A_t R + Y_{t+1} \\ &= A_t R + \lambda e^{g_{64} + z_{t+1}} = A_t R + \lambda e^{g_{64} + z_{64}} \quad t = 64, ..., 98 && (15) \end{aligned}$$

For $t < 64$, the law of motion for z_t is given by (4). For $t \geq 64$, $z_{t+1} = z_t$.

Applying the endogenous grid point method to the problem

The optimization problem for working life is given by:

$$V_t(\mathcal{S}_t) = \max_{A_t} \left\{ \underbrace{\left((X_t - A_t)^{1-\rho} + \beta \mathcal{R}_t(V_{t+1})^{1-\rho} \right)^{\frac{1}{1-\rho}}}_{\equiv \underline{V}_t} \right\},$$

subject to (3)–(5), (11)–(14).

- Take FOC w.r.t A_t
- Envelope condition

FOC w.r.t A_t

$$\left(\frac{1}{1-\rho}\right)\underline{V}_t^{\frac{1}{1-\rho}-1}\Big[-(1-\rho)\left(X_t-A_t\right)^{-\rho}+\beta(1-\rho)\mathcal{R}_t(V_{t+1})^{-\rho}\frac{\partial}{\partial A_t}\left(\mathcal{R}_t(V_{t+1})\right)\Big]=0 \qquad (16)$$

$\frac{\partial}{\partial A_t}\left(\mathcal{R}_t(V_{t+1})\right)$ =?

$$
\begin{aligned}
\frac{\partial}{\partial A_t}\left(\mathcal{R}_t(V_{t+1})\right) &= \frac{\partial}{\partial A_t}\left(\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}\right) \\
&= \frac{1}{1-\gamma}\left(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\right)^{\frac{1}{1-\gamma}-1}\cdot\frac{\partial}{\partial A_t}\left(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\right) \\
&= \frac{1}{1-\gamma}\left(\mathbb{E}_t\, V_{t+1}^{1-\gamma})\right)^{\frac{1}{1-\gamma}-1}\cdot\mathbb{E}_t\left[\frac{\partial}{\partial A_t}\left(V_{t+1}^{1-\gamma}\right)\right] \\
&= \mathbb{E}_t[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}-1}\cdot\mathbb{E}_t\left[V_{t+1}^{-\gamma}V'_{t+1}R\right] \\
&= \mathbb{E}_t[V_{t+1}^{1-\gamma}]^{\frac{\gamma}{1-\gamma}}\cdot\mathbb{E}_t\left[V_{t+1}^{-\gamma}V'_{t+1}R\right] \\
&= \mathcal{R}_t(V_{t+1})^{\gamma}\,\mathbb{E}_t\left[V_{t+1}^{-\gamma}V'_{t+1}R\right] \qquad (17)
\end{aligned}
$$

where going from the second to the third line invokes Leibniz rule:
`https://en.wikipedia.org/wiki/Leibniz_integral_rule`

Substituting

Substitute (17) into (16) and simplifying:

$$(X_t - A_t)^{-\rho} = \beta \mathcal{R}_t(V_{t+1})^{-\rho} \mathcal{R}_t(V_{t+1})^{\gamma}\, \mathbb{E}_t \left[V_{t+1}^{-\gamma} V'_{t+1} R \right] \tag{18}$$

$V'_{t+1} = ?$

Invoke the Envelope condition:

$$\begin{aligned} V'_t \quad = \tfrac{\partial V_t}{\partial X_t} \quad &= \left(\frac{1}{1-\rho}\right) \underline{V}_t^{\frac{1}{1-\rho}-1} \cdot (1-\rho)\,(X_t - A_t)^{-\rho} = \\ &= \quad (X_t - A_t)^{-\rho} \underline{V}_t^{\frac{\rho}{1-\rho}} = \\ &= \quad (X_t - A_t)^{-\rho} V_t^{\rho} \end{aligned} \tag{19}$$

$V'_{t+1} =?$

Lead (19) forward and substitute into (18):

$$(X_t - A_t)^{-\rho} = \beta\, \mathbb{E}_t \left[\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\rho-\gamma} (X_{t+1} - A_{t+1})^{-\rho} R \right] \tag{20}$$

Or:

$$1 = \mathbb{E}_t \left[\underbrace{\beta \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}}_{=M_{t+1}} R \right] \tag{21}$$

Compare to the stochastic discount factor for CRRA preferences:
$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \Leftrightarrow \rho = \gamma$

Can show that return on "total wealth", R^W_{t+1}, equals:

$$R^W_{t+1} = \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{1-\rho} \tag{22}$$

See e.g. John Campbell's textbook, equation (6.42).

Why Epstein-Zin?

- The risk-free rate puzzle of Weil (1989): why is the risk-free rate so low if $\rho > \gamma$?
- Allows to impose a reasonable EIS while increasing RRA
- Bansal and Yaron (JF, 2004): long-run risk in C_t and $\rho < 1$ (EIS>1) requires representative investor to earn high equity premium
- Sargent: $\rho < \gamma$: preference for early resolution of uncertainty
- Also useful for life-cycle portfolio choice models (Gomes and Michaelides, JF 2005; Dahlquist, Setty, and Vestman JF 2018, Vestman RFS 2019)
- Increasingly common in macroeconomics

The endogenous grid point method and Epstein-Zin preferences

- Notice that (20) allows for the same kind of inversion as in the CRRA case to obtain $X_t(a^i)$:

$$X_t(a^i) = a^i + \left(\beta\,\mathbb{E}_t\left[\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\rho-\gamma}(X_{t+1} - A_{t+1})^{-\rho} R\right]\right)^{-1/\rho} \tag{23}$$

- Note difference from CRRA utility: We always need to the value function so need to approximate V_t on a dense grid also in the contrained region (i.e., $X_t < X(a^1)$) to capture its curvature.

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Life-cycle portfolio choice models

Choice between saving in a risk-free bond and in the stock market:

- Merton (1969,71): abstracting from risky income
- Viceira (JF, 2001)
- Cocco, Gomes and Maenhout (RFS, 2005): Carroll (1997) + choice of equity share = **Part 2 of the problem set**
- Gomes and Michaelides (JF, 2005): addition of extensive margin decision (stock market participation)

2 financial assets

- B_t with return R_f (safe)
- S_t with return R_{t+1} (stochastic)

Most common assumptions are:

$$S_t \geq 0$$
$$B_t \geq 0 \tag{24}$$

In this case:

$$A_t = B_t + S_t \tag{25}$$
$$A_t \geq 0 \tag{26}$$
$$\alpha_t = \frac{S_t}{A_t} \in [0, 1] \tag{27}$$
$$R_{t+1} = \exp(\ln(R_f) + \mu + \nu_{t+1}) \tag{28}$$

where $\nu_{t+1} \sim N(-\sigma_\nu^2/2, \sigma_\nu^2)$ and where the income process is modified from (4) to

$$z_{it} = \rho z_{it-1} + \eta_{it} + \theta \nu_{t+1}, \tag{29}$$

where $\theta > 0$ but $\theta << 1$. $corr(\Delta y_{it}, R_t) \approx 0.1 - 0.2$

Return on portfolio

$$R_{it+1} = \alpha_{it} \exp(\ln(R_f) + \mu + \nu_{t+1}) + (1 - \alpha_{it}) \exp(\ln(R_f)) \tag{30}$$
$$= R_f + \alpha_{it}(R_{t+1} - R_f)$$
$$\mathbb{E}_t[R_{it+1}] = R_f + \alpha_{it}(\mathbb{E}_t[R_{t+1}] - R_f)) \tag{31}$$

3 modifications to Carroll (1997):

- New control variable: α_{it} with constraint (27).
- Law of motion for X_{it+1} is a function of $(A_{it}, \alpha_{it}, z_{it})$.
- X_{it} is stochastic also in retirement.

Optimality conditions: 2 Euler equations

$$1 = \mathbb{E}_t[M_{it+1}R_f] \tag{32}$$
$$1 = \mathbb{E}_t[M_{it+1}R_{t+1}] \tag{33}$$

Or:

$$1 = \mathbb{E}_t[M_{it+1}R_f] \tag{34}$$
$$0 = \mathbb{E}_t[M_{it+1}(R_{t+1} - R_f)] \tag{35}$$

For CRRA preferences, (34)-(35) are equivalent to:

$$(X_t - A_t)^{-\gamma} = \mathbb{E}_t[\beta(X_{t+1} - A_{t+1})^{-\gamma}R_f] \tag{36}$$
$$0 = \mathbb{E}_t[\beta(X_{t+1} - A_{t+1})^{-\gamma}(R_{t+1} - R_f)] \tag{37}$$

Extension to the endogenous grid point method

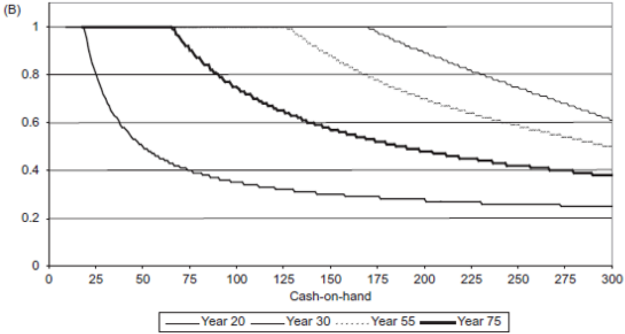
- Pick $a^i \in \mathcal{A}$.
- Optimize on $\alpha \in [0,1]$:
  - Guess on α
  - Given (a^i, α, z) evaluate X_{t+1} and then the RHS of (37).
  - If $RHS > 0$ then $\alpha^* > \alpha$, elseif $RHS < 0$ then $\alpha^* < \alpha$.
- Given α^*, use (36) to back out $X_t(a^i, \alpha^*, z_t)$.

Since α is bounded between 0 and 1 a suitable optimization algorithm is Golden-section search – see `https://en.wikipedia.org/wiki/Golden-section_search` or the bracketing algorithm, p. 94-95 in Judd (1998).

The algorithm keeps track of $\alpha^{low} \leq \alpha^* \leq \alpha^{high}$ and stops when $\alpha^{high} - \alpha^{low} \leq tolerance$.

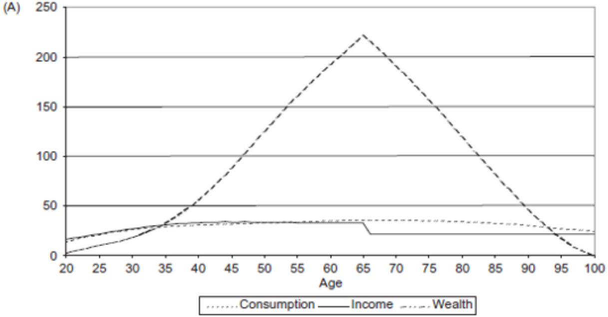
What do we gain from using EGM in this setting?

Policy function $\alpha_t(X_t, z_t)$ for the equity share (Cocco et al. (RFS 2005))

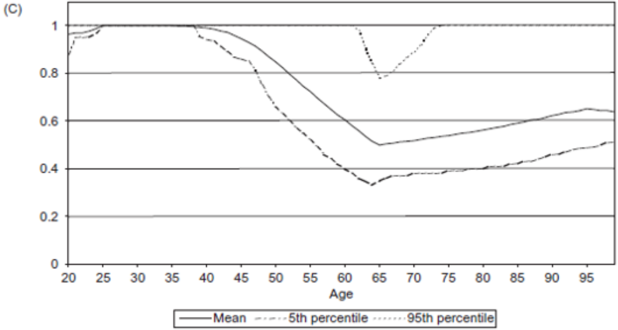


$PDV(Y_{it})$ is bond-like in terms of risk. Furthermore, it is much larger than A_{it}. Investable wealth is a small share of total wealth, and substantial risk can be tolerated.

Life-cycle profiles for C, Y, A



Life-cycle profiles (glide paths) for the equity share (α)



- Stock market returns are i.i.d and yet down-ward sloping glide paths. So not a consequence of mean reversion/predictability of the stock market.
- Rather, consequency of bond-like "human wealth" (non-investable wealth).

How to think about A?

- Replacement rate (λ) between 0.68 and 0.93.
- $\Rightarrow A$ is voluntary savings? ("Third pillar"). $\beta = 0.96$ and $\mathbb{E}[R] = 1.08$ implies too much A?
- Subsequent literature by Kaplan, Violante, and Weidner on hand-to-mouth and wealthy hand-to-mouth households: 35% of households live paycheck to paycheck. Andersson and Vestman (2021): 35% of Swedish households have less than 4 months of disposable income in liquid wealth.
- The state space is scaled: $\hat{X}_t = X_t / Y^p$, $\hat{A}_t = A_t / Y^p$. No role for progressivity, and no role for heterogeneity in financial literacy being correlated with Y^p. Any circumstance that implies A_t / Y^p non-constant for different Y^p implies model is wrong.

My view of the life-cycle portfolio choice literature

- Very mature, pure technical additions do not make it into the top journals
- Two strategies that have potential: (i) make models more similar to the actual institutional setting; (ii) map model closer to detailed data
- Both strategies require model extensions, but guidance on how comes from real life, not from having programming skills

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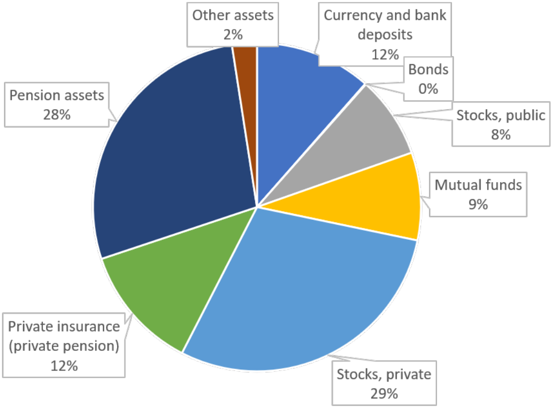
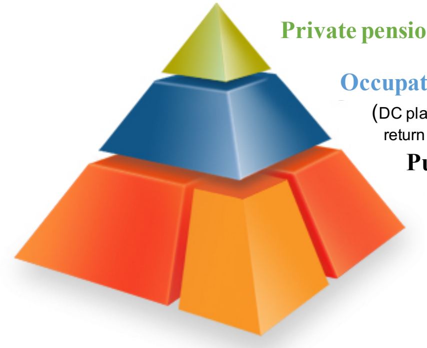
Aggregate financial wealth statistics for Swedish households (2021)



Two observations:

- Liquid financial wealth is only 29% of total wealth
- Even in the aggregate 40% of total wealth is allocated to pension funds and pension plans (for many individuals the pension assets' share will be much greater) $\Rightarrow$ optimal policy design on pension assets associated with large welfare gains



Adapted from the Swedish Pensions Agency

Private pension scheme

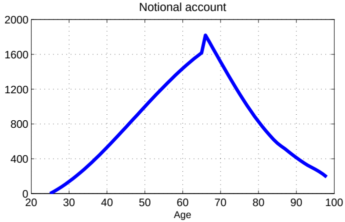
Occupational pension:

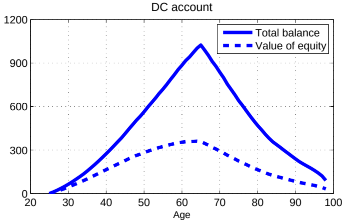
(DC plan; 4.5% contribution; return depends on choice)

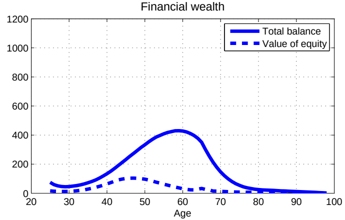
Public pension system:

- **Income pension** (notional DC plan; 16% contribution; return like wage growth)
- **Premium pension** (DC plan; 2.5% contribution; return depends on choice)
- **Guaranteed pension**

The DC accounts matter a lot for pension income







Pension accounts become state variables

$$X_{it+1} = A_{it}(R_f + \alpha_{it} R_{t+1}) + Y_{it+1} \tag{38}$$
$$A^{DC}_{it+1} = A^{DC}_{it}(R_f + \alpha^{DC}_{it} R_{t+1}) + \lambda^{DC} Y_{it} \tag{39}$$
$$N_{it+1} = R_f N_{it} + \lambda^{N} Y_{it} \tag{40}$$

- Why are A^{DC}_{it+1} and N_{it+1} state variables?
- Dahlquist, Setty and Vestman (JF 2018): optimal α^{DC}_{it} in a default pension fund
- Schlafmann, Setty and Vestman (2022): optimal λ^{DC}_{it}

The Strategic Asset Allocation of Default Pension Fund (Dahlquist, Setty, and Vestman 2018)

- Pension investor characteristics in mandatory DC pension plans vary a lot:
  - Age
  - Income
  - Financial wealth (net worth)
  - Equity exposure in liquid financial wealth (αA_{it})
  - Some investors are not at all exposed to equity in their liquid financial wealth ($I_{it} = 0$)
- **Question:** What is the welfare gain of customizing α_{it}^{DC} to individual characteristics? Relates to ongoing financial innovation and algorithmic financial advising (roboadvising).
- The default fund in a pension plan is the natural policy tool (Sweden's premium pension plan: AP7).

Panel data set on individual investors

- Detailed data from 2000 to 2007 on:
  - **Fund holdings in the government-mandated premium (DC) pension plan and number of fund changes**
  - Holdings outside the pension system (as in Calvet, Campbell, Sodini 2007, 2009)
  - Individuals' socio-demographics
- We define two investor types based on activity in the pension plan:
  1. Passive (60.5%): 31.3% default investors + 29.2% one-time initially active
  2. Active (39.5%)
  - Definition based on Dahlquist, Martinez, and Söderlind (2007)

Averages of variables

	All	Passive	Active
Investors			
Number of investors	301,632	182,487	119,145
Fraction of investors	1.000	0.605	0.395
State variables			
Age	46.8	46.6	47.0
Labor income	248,420	224,526	285,017
Financial wealth	248,039	217,846	294,284
Stock market exposure			
Participation dummy	0.520	0.455	0.619
Equity share (unconditional)	0.234	0.196	0.290
Equity share (conditional)	0.449	0.432	0.469
Educational dummies			
Elementary school	0.157	0.184	0.116
High school	0.544	0.539	0.551
College	0.288	0.267	0.320
PhD	0.011	0.010	0.013

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Regression analysis: non-participation outside and passivity inside pension system are positively correlated conditional on observables.

Model

- Individuals live from age 25 up to at most age 100 (retirement at 65).
- Epstein-Zin preferences over a single consumption good.
- Uninsurable risky labor income during working age, **annuity payments** from pension accounts upon retirement.
- Save outside the pension system:
  - A risk-free bond and a stock market index:
    choose consumption/savings, stock market entry (costly), equity share
  - **A one-time participation cost:** κ_i, cross-sectionally distributed
- Save inside the pension system in 2 accounts:
  1. (Notional pension account: income-based, return of the risk-free bond)
  2. **DC account (premium pension plus occupational pension plan)**
  - Fixed contribution rates
  - Annuities are actuarially fair and insure against longevity risk
  - **A one-time activity (opt out) cost:** $\kappa_i^{\mathbf{DC}}$, cross-sectionally distributed

Opt-out decision and asset allocation in the DC account

Active investors

- Opt out at a cost κ^{DC}
- Choose the equity share in the DC account, α_t^{DC}, fully rationally

Default investors

- Stay in the default fund and do not pay cost κ^{DC}
- Default designs for α_t^{DC}:
  1. "100-minus-age"
  2. The average optimal age-based equity share: a glide path that conditions only on age
  3. The rule of thumb: conditions on a sub-set of state variables
  4. The optimal equity share: conditions on all of the state variables (including κ_i, κ_i^{DC})

Calibration

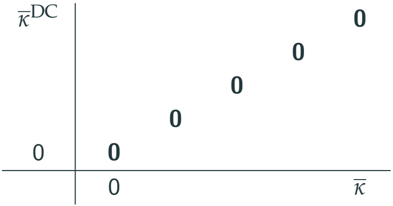
“Exogenously” / Standard:

- EIS, risk-free rate, equity premium, equity volatility
- Life-cycle profile for labor income, labor income shocks
- Contribution rates (16%+7%)
- Floor on annuity from notional account
- Age-based DC equity share: “100-minus-age”

Matched to fit the data

1. Discount factor (match financial wealth / labor income 25-64).
2. Risk aversion coefficient (match weighted conditional equity share 25-69).
3. The joint distribution of $(\kappa, \kappa^{\text{DC}})$

The joint distribution of $(\kappa, \kappa^{\mathrm{DC}})$



- Square matrix $\Rightarrow$ the two marginal distributions have same shape and are symmetric
- Solve and simulate the model to determine:
  1. $\overline{\kappa}$: SEK 15,600 (USD 2,000)
  2. $\overline{\kappa}^{\mathrm{DC}}$: SEK 3,600 (USD 460)
  3. Layers off diagonal: 3

$$
\begin{array}{c|ccccc}
\overline{\kappa}^{\text{DC}} & & & & \mathbf{1} & \mathbf{0} \\
 & & & \mathbf{1} & \mathbf{0} & \mathbf{1} \\
 & & \mathbf{1} & \mathbf{0} & \mathbf{1} & \\
 & \mathbf{1} & \mathbf{0} & \mathbf{1} & & \\
0 & \mathbf{0} & \mathbf{1} & & & \\
\hline
 & 0 & & & & \overline{\kappa}
\end{array}
$$

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The joint distribution of $(\kappa, \kappa^{\mathsf{DC}})$

$$
\begin{array}{c|ccccc}
\overline{\kappa}^{\mathrm{DC}} & & & \mathbf{2} & \mathbf{1} & \mathbf{0} \\
 & & \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\
 & \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} \\
 & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & \\
0 & \mathbf{0} & \mathbf{1} & \mathbf{2} & & \\
\hline
 & 0 & & & & \overline{\kappa}
\end{array}
$$

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The joint distribution of $(\kappa, \kappa^{\mathsf{DC}})$

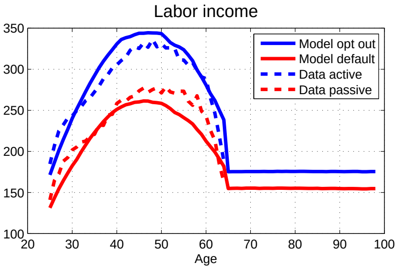
$\overline{\kappa}^{\text{DC}}$		**3**	**2**	**1**	**0**
	3	**2**	**1**	**0**	**1**
	2	**1**	**0**	**1**	**2**
	1	**0**	**1**	**2**	**3**
0	**0**	**1**	**2**	**3**	
	0				$\overline{\kappa}$

- Square matrix $\Rightarrow$ the two marginal distributions have same shape and are symmetric
- Solve and simulate the model to determine:
    1. $\overline{\kappa}$: SEK 15,600 (USD 2,000)
    2. $\overline{\kappa}^{\text{DC}}$: SEK 3,600 (USD 460)
    3. Layers off diagonal: 3
- Equal weight on 23 types implies a correlation between κ and κ^{DC} of 0.2
- Low average costs: SEK 7,800 (USD 1,000) for participation and SEK 1,800 (USD 230) for opt-out

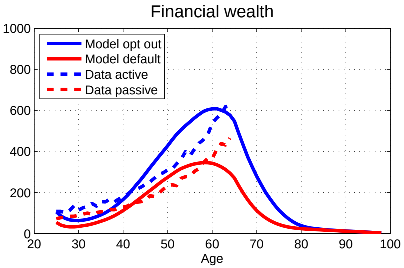
Matched moments

	Data	Model
Active (opting out) / non-participation	0.151	0.158
Active (opting out) / participation	0.244	0.255
Passive (default) / non-participation	0.330	0.316
Passive (default) / participation	0.275	0.271

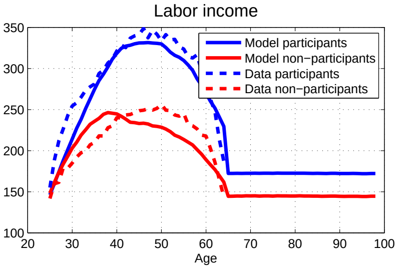
Model fit



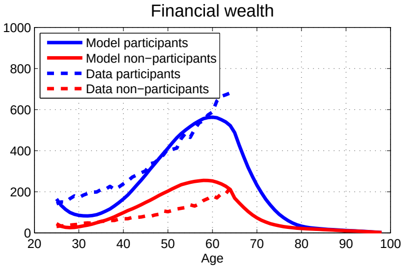
Labor income
Model opt out
Model default
Data active
Data passive
350
300
250
200
150
100
20
30
40
50
60
70
80
90
100
Age



Financial wealth
Model opt out
Model default
Data active
Data passive
1000
800
600
400
200
0
20
30
40
50
60
70
80
90
100
Age



Labor income
Model participants
Model non−participants
Data participants
Data non−participants
350
300
250
200
150
100
20
30
40
50
60
70
80
90
100
Age

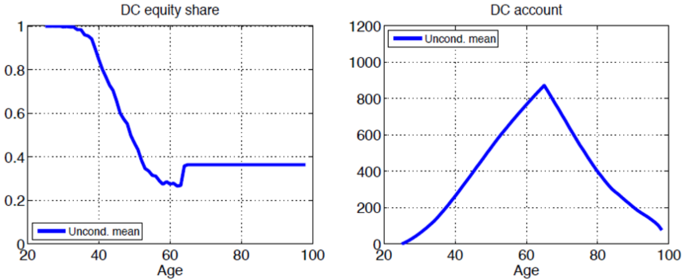


Financial wealth
Model participants
Model non−participants
Data participants
Data non−participants
1000
800
600
400
200
0
20
30
40
50
60
70
80
90
100
Age

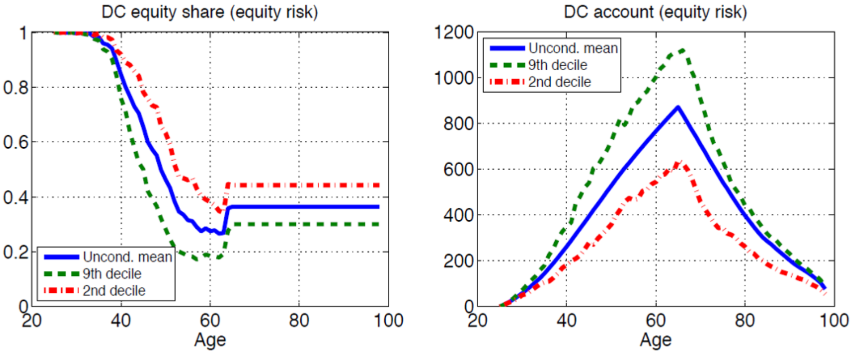
Simulations to characterize the optimal DC equity share

- Simulation method similar to Campbell and Cocco (JF, 2015)
- Two sources of risk:
    1. Aggregate – shocks to stock market (equity risk)
    2. Idiosyncratic – uninsurable labor income shocks (inequality)
- An economy: life-cycle path for one birth cohort exposed to common equity returns
- Simulate many economies with different returns & common income shocks
- 3 ways to characterize the optimal asset allocation and other outcomes:
    1. Unconditional mean (Average optimal)
    2. Equity risk
    3. Inequality

DC equity share: unconditional mean

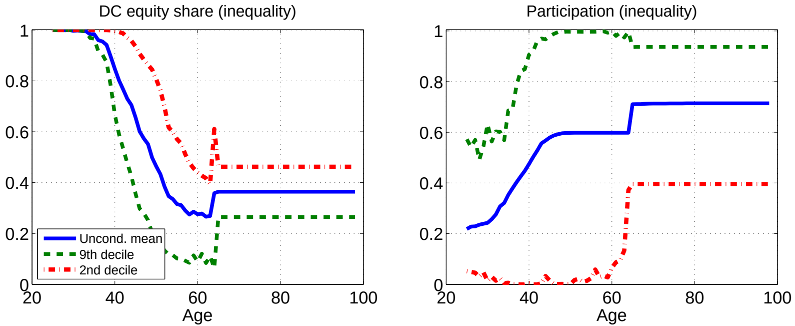


DC equity share: Equity risk



- High realized returns increase the DC account
- Optimal asset allocation reduces equity risk in pension income
- Cohort effects

DC equity share: Inequality



- Participation rates correspond to the equity share deciles
- Optimal asset allocation compensates for non-participation outside

Regressions on simulated data

	I	II	III	IV	V	VI	VII
Constant	1.746***	1.873***	1.585***	1.738***	1.313***	1.347***	1.266***
	(0.016)	(0.015)	(0.018)	(0.016)	(0.013)	(0.011)	(0.012)
Age	–0.024***	–0.023***	–0.018***	–0.022***	–0.009***	–0.008***	–0.007***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Labor income		–0.760***					0.262***
		(0.039)					(0.025)
Fin. wealth			–0.565***				–0.096***
			(0.041)				(0.032)
Participation				–0.233***		–0.196***	–0.198***
				(0.006)		(0.003)	(0.004)
DC account					–0.666***	–0.603***	–0.618***
					(0.026)	(0.022)	(0.017)
R-squared	0.630	0.687	0.740	0.730	0.786	0.855	0.859

Our proposal for rule of thumb in red

Welfare analysis: Does customization matter?

- Compare welfare of gradual customization for default investors
- Certainty equivalent consumption based on expected utility at 25
- Welfare measure is ex ante – captures both risk and return
- In addition, we study changes in opt-out rates and pension income

Welfare analysis

	100-minus-age	Average optimal	Rule of thumb	Optimal
Cumulative welfare gain	—			1.5%
Share of default investors	0.587			1.000
<u>Regressions</u>				
Constant				
Age				
Participation dummy				
DC account balance				
R-squared				
<u>Pension income</u>				
Mean				
Equity risk				
Inequality				

Welfare analysis

	100-minus-age	Average optimal	Rule of thumb	Optimal
Cumulative welfare gain	—	0.3%	0.9%	1.5%
Share of default investors	0.587	0.679	0.753	1.000
Regressions				
Constant				
Age				
Participation dummy				
DC account balance				
R-squared				
Pension income				
Mean				
Equity risk				
Inequality				

Welfare analysis

	100-minus-age	Average optimal	Rule of thumb	Optimal
Cumulative welfare gain	—	0.3%	0.9%	1.5%
Share of default investors	0.587	0.679	0.753	1.000
Regressions				
Constant	1.347	1.363	1.384	1.411
Age	–0.008	–0.009	–0.009	–0.010
Participation dummy	–0.196	–0.199	–0.198	–0.195
DC account balance	–0.603	–0.564	–0.533	–0.505
R-squared	0.855	0.855	0.853	0.850
Pension income				
Mean				
Equity risk				
Inequality				

Welfare analysis

	100-minus-age	Average optimal	Rule of thumb	Optimal
Cumulative welfare gain	—	0.3%	0.9%	1.5%
Share of default investors	0.587	0.679	0.753	1.000
Regressions				
Constant	1.347	1.363	1.384	1.411
Age	–0.008	–0.009	–0.009	–0.010
Participation dummy	–0.196	–0.199	–0.198	–0.195
DC account balance	–0.603	–0.564	–0.533	–0.505
R-squared	0.855	0.855	0.853	0.850
Pension income				
Mean	154,880	155,461	158,952	152,281
Equity risk	0.121	0.122	0.127	0.087
Inequality	0.234	0.233	0.194	0.196

Results are robust to:

1. Left-skewed equity returns and a low equity premium
2. Implementing a rule of thumb from a misspecified model
3. Simple forms of investment mistakes ("Down or Out") outside the DC account
4. A higher correlation between labor income and equity returns (combined with left-skewness)
5. Accounting for wealth tied in real estate

Outline today

- The books
- Extensions to the consumer's problem
  - Epstein-Zin preferences
  - Portfolio choice and optimal glide paths
  - Pension systems
  - Housing and mortgages
- The curse of dimensionality
- EGM and discrete control variables
- Guide to Uppmax for those interested
- Assistance with the problem set

Housing and mortgages

- Modelling housing and mortgages was for a long time considered prohibively computationally expensive, or at least cost-benefit trade-off not so attractive
- Housing: both consumption good and investment asset
- Mortgage: long-term contract (not 1-period): to be realistic, cannot fold into A_t and X_{t+1}
- Transaction costs
- Discrete choice whether to adjust the house or the mortgage
- In the early literature (pre Global Financial Crisis) a lot of short-cuts were used
- The GFC made the computational costs worthwhile

The housing good and asset

Utility:

$$C_t = c_t^{1-\theta} {h_t}^{\theta} \tag{41}$$

$$u(C_t) = \frac{(c_t^{1-\theta} {h_t}^{\theta})^{1-\gamma}}{1-\gamma} \tag{42}$$

Simplest constraints for a homeowner:

$$c_t + (1+\phi) P_t^h h_t + A_t = X_t \tag{43}$$

$$A_t \geq -(1-\delta) P_t^h h_t \tag{44}$$

The mortgage is folded into A_t and is rolled over every period (1-period debt).

Define X_t to be cash-in-hand if the household sells its house.

3 choices and laws of motions and budget constraint

If the household sells and rents in $t+1$:

$$X_{t+1} = Y_{t+1} + A_t R + R^h_{t+1} P^h_t h_t (1-\psi) \tag{45}$$

$$c_{t+1} + \tau P^h_{t+1} h_{t+1} + A_{t+1} = X_{t+1} \tag{46}$$

$$A_t \geq 0 \tag{47}$$

If the household sells and buys (rebalances):

$$X_{t+1} = Y_{t+1} + A_t R + R^h_{t+1} P^h_t h_t (1-\psi) \tag{48}$$

$$c_{t+1} + (1+\phi) P^h_{t+1} h_{t+1} + A_{t+1} = X_{t+1} \tag{49}$$

$$A_{t+1} \geq -(1-\delta) P^h_{t+1} h_{t+1} \tag{50}$$

If the household stays in $t+1$:

$$X_{t+1} = Y_{t+1} + A_t R + R^h_{t+1} P^h_t h_t (1-\psi) \tag{51}$$

$$c_{t+1} + (\phi-\psi) P^h_{t+1} h_t + A_{t+1} = X_{t+1} \tag{52}$$

$$A_{t+1} \geq -(1-\delta) P^h_{t+1} h_{t+1} \tag{53}$$

State variables

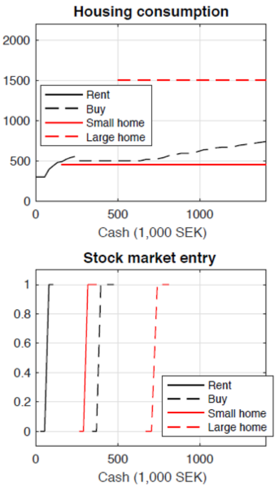
- Renter: z_t, X_t.
- Stayer: z_t, X_t, h_t.
- Buyer: z_t, X_t.
- P_t^h?
  - Renter: with Cobb-Douglas expenditure shares and no frictions in the rental market the problem can be reduced to choosing A_t
  - To be able to omit P_t^h also from stayer and buyer: relies on defining X_{t+1} to be cash in hand after the household has sold.
- Drawbacks of not having long-term mortgage as a separate state variable:
  - If $R_{t+1}^h < 1$ then budget constraint tightens
  - X_{t+1} can become negative: under water and cash-flow deficit. Mortgage default with U.S. non-recourse.

Housing and portfolio choice

- Grossman and Laroque, Flavin and Yamashita.
- Homeownership crowds out financial wealth but makes optimal conditional equity share higher (Cocco, RFS 2005; Yao and Zhang, RFS 2005).
- Preference heterogeneity required to generate gap in stock market partcipation rates (Vestman, RFS 2019).

	United States		**Sweden**	
	Renters	Homeowners	Renters	Homeowners
Fraction of households	24.8%	75.2%	35.8%	64.2%
Total financial wealth	24.9	276.5	21.8	73.3
Net worth	19.0	496.0	3.0	162.5
Wage income	27.5	61.0	20.2	52.1
Housing wealth	0	320.4	0	195.2
Home equity	0	175.6	0	89.2
Stock market participation	25.7%	61.9%	37.6%	77.6%

Policy functions for type 1 (high savings motive)

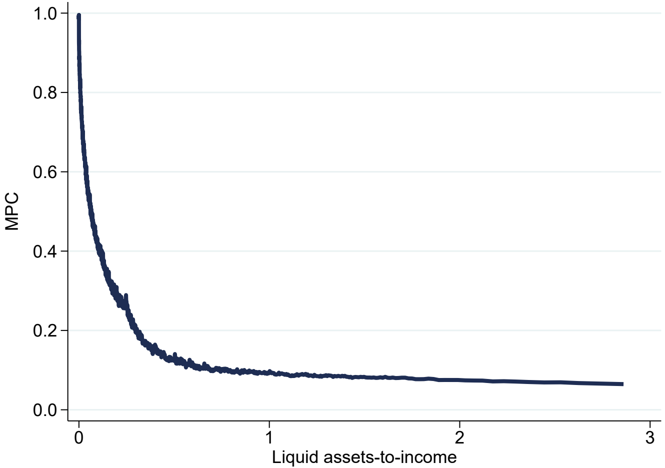


Policy functions for type 1 (high sav. motive) and type 2 (low sav. motive)

Housing and response to shocks

- Response to shocks: generates large MPCs: $\partial C_t / \partial Y_i$ (temporary shock ω_{it})
- Partial equilibrium house price shock $\partial C_t / \partial P^h$ because of wealthy HtM households
  - Macroeconomic literature on how much general equilibrium (GE) scales up the PE response (Guren, McKay, Nakamura and Steinsson)
- Mortgage defaults and fire sales generates general equilibrium / feedback effects

HtM households and temporary shocks to income, MPC^Y



Source: Kilström, Sigurdsson, and Vestman (2022). See also Kaplan and Violante (JEP, 2018): "Microeconomic Heterogeneity and Macroeconomi Shocks"

- Response to a partial equilibrium house price shock (i.e., ignoring GE effects or the cause of the shock)

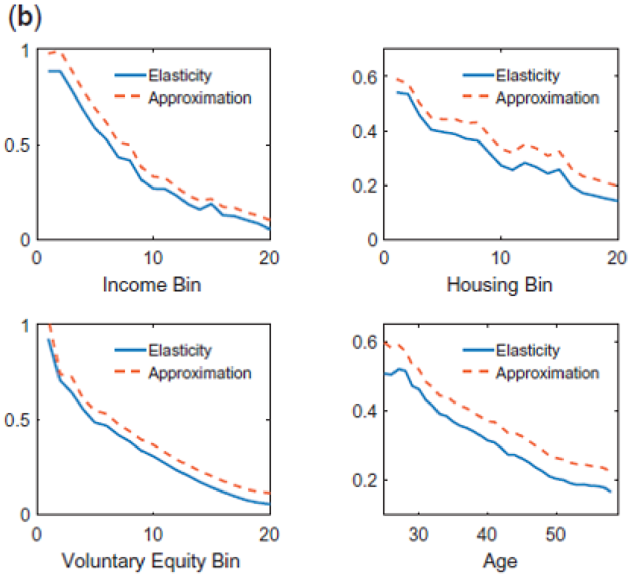
$$\Delta C_{it} = MPC_{it}^{Y} \times \frac{(1-\delta)P_{t-1}H_{it-1}}{(1-\delta)P_t H_{it} + (1+r)A_{it}} \times \Delta W_{it}. \tag{54}$$

where MPC_{it}^{Y} is the MPC to a temporary shock to income.

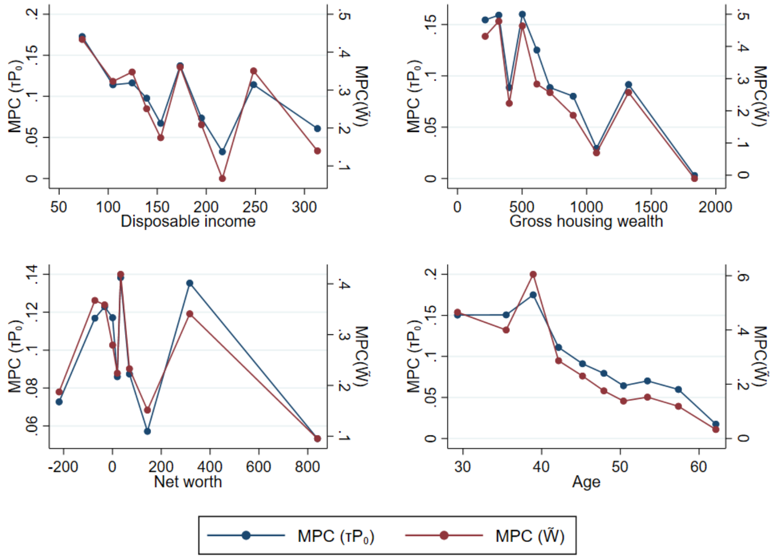
Compare to the simple model in Sodini, Van Nieuwerburgh, Von Liliendfeld, Vestman (2022):

$$c^o - c^r = \left(\frac{r}{1+r}\right)\left(1 - \frac{1}{(1+r)^{T+1}}\right)^{-1} \tau P_0 \left(\frac{R_h}{R}\right)^{T+1} \tag{55}$$

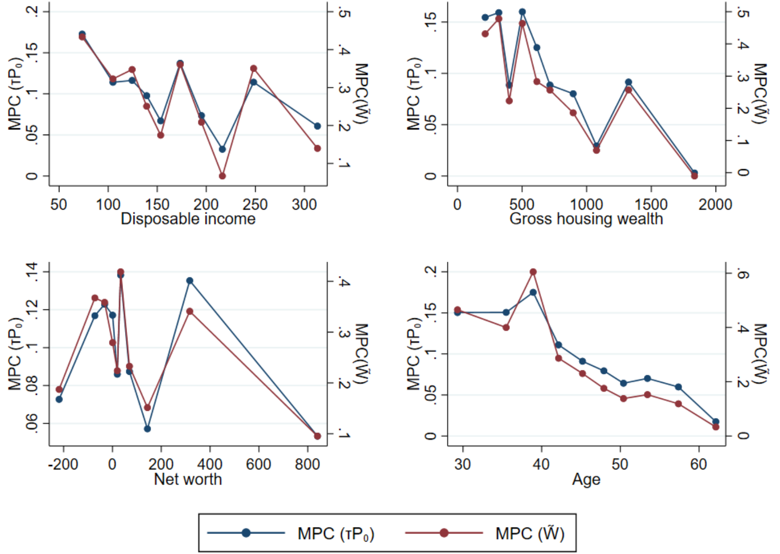
Cross-sectional variation – Berger et al. (ReStud 2018)



Cross-sectional variation – Sodini et al. (2022)



Cross-sectional variation – Sodini et al. (2022)



Related empirical evidence on housing wealth shocks

- Mian, Rao, and Sufi (QJE 2013)
- Aladangady (AER 201x)
- Kaplan, Mitman, and Violante (JPubE 2020)
- Böjeryd, Vestman, Tyrefors, and Kessel (2022): "The Housing Wealth Effect: Quasi-Experimental Evidence" – see references in their for complete overview

Mortgage design

With proper modelling of the mortgage as a separate state variable new questions can be answered:

- Mitman (AER, 2016): "Macroeconomic Effects of Bankruptcy and Foreclosure Policies"
- Kaplan, Mitman, and Violante (JPE, 2020): The Housing Boom and Bust: Model Meets Evidence
- Mortgage design:
  - Campbell and Cocco (JF, 2015): "A Model of Mortgage Default"
  - Campbell, Clara, Cocco (JF, 2021): "Structuring Mortgages for Macroeconomic Stability"
  - Guren, Krishnamurthy and McQuade (JF, 2021): "Mortgage Design in an Equilbrium Model of the Housing Market"
  - Greenwald, Landvoigt, and Van Nieuwerburgh (JF, 2021): "Financial Fragility with SAM?"
  - Karlman, Kinnerud, Kragh-Sorensen (2022): "Down-payment requirements and consumption responses to income shocks"
- Monetary policy transmission: Arlene Wong (Princeton), Karin Kinnerud (BI Oslo)

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Curse of dimensionality

- For every additional state variable the state space grows exponentially
- Tricks:
  - Exogenous state variables: Maliar and Maliar + Jakob Almerud and Anders Österling
  - Higher-order splines and sparse grids (collocation points)
  - ...

EGM and discrete choices

- $\hat{V}_t = max\{V_t^{rent}, V_t^{buy}\}$
- $V_t^{rent} = max\{u(c_t) + \beta \hat{V}_{t+1}\}$
- Non-convex set for constraints. FOC necessary but not sufficient.
- See Jeppe Druedahl (Computational Economics 2020) for pseudo-algorithm.

Uppmax

- `https://www.uppmax.uu.se/`
- Need to apply for user account + small project with computational budget (core hours / month)
- Requires ssh (e.g., Putty) and scp clients (e.g., WinSCP)
- Need to know how to run (and compile) your source code
- Need to know the scheduler (SLURM)