Life cycle models in macroeconomics

Quantitative Macroeconomic Methods I

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Outline

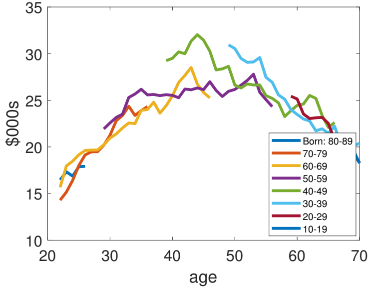
Introduction to life cycle models

Why study the life cycle?

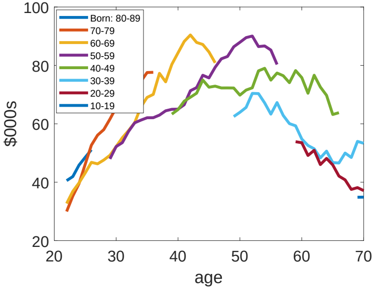
- So far we have studied the economy with the infinite horizon model
- Implication of this are:
  - Household that lives forever or cares about their children / successors in a way that is analogous
  - Apart from shocks life stays the same
- In reality, there are strong trends along the life cycle
  - Income tends to grow particularly at the beginning of life
  - We retire (and require savings to replace labor income)
  - Households tend to start by renting before purchasing a house
  - In later life we might face health shocks and increased medical expenses
  - Consumption is **hump shaped** over LC, even accounting for family composition
- To understand these phenomena it is useful to have a model that captures life cycle dynamics
- Life cycle approach can be more **computationally feasible**
  - Solving a finite number of periods rather than value function convergence
    - *richer dynamics in other dimensions*
  - Not necessarily true in transition where age is a state variable

Life cycle profiles

Cohort average life cycle profiles: consumption has hump shaped pattern, flatter than income



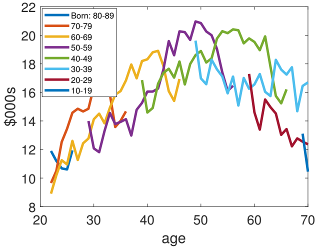
(a) non-durable consumption



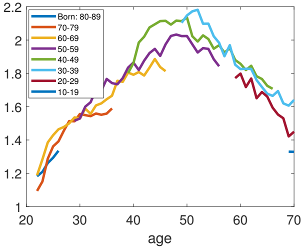
(b) income

Data from Consumer Expenditure Survey (US)

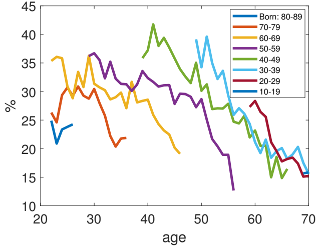
Life cycle profiles: cars



(a) car stock

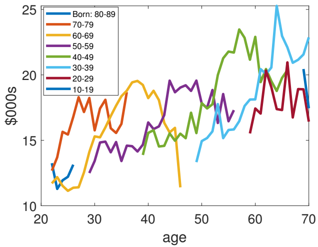


(b) number of cars



,

(c) % hh purchasing



(d) value of purchase

Life cycle profiles: UK data

Similar patterns in UK data, Attanasio and Weber (2010)

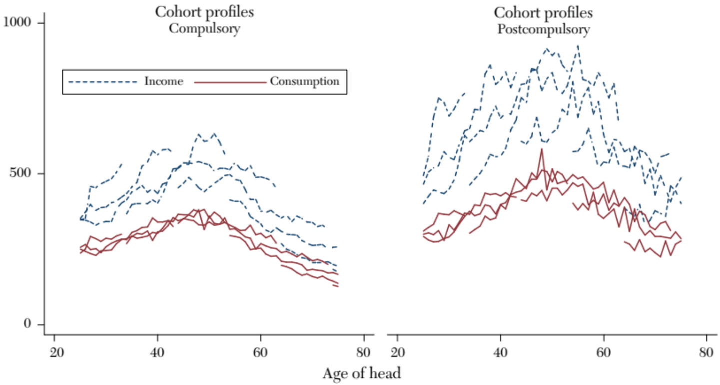


Figure: consumption and income in the UK

Data from UK Family Expenditure Survey

Life cycle profiles: wealth and housing

Durables over the life cycle, Fernandez-Villaverde and Kruger (2011)

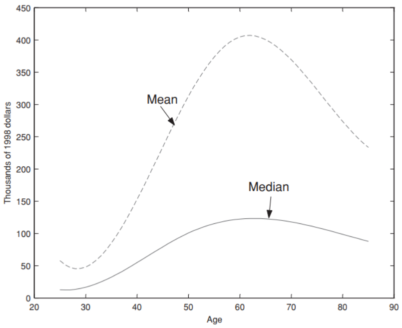


FIGURE 7. Household net worth, 1995 SCF.

(a) net worth

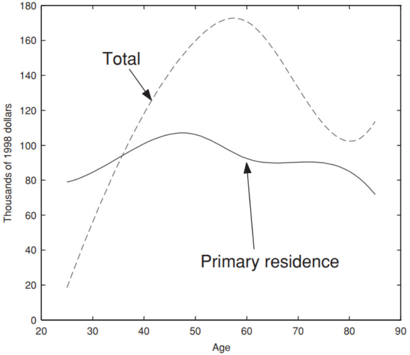


FIGURE 9. Median household assets, 1995 SCF.

(b) assets

Data from Survey of Consumer Finance (US)

Life cycle profiles: wealth and housing

Median housing and assets, Diaz and Luengo-Prado (2008)

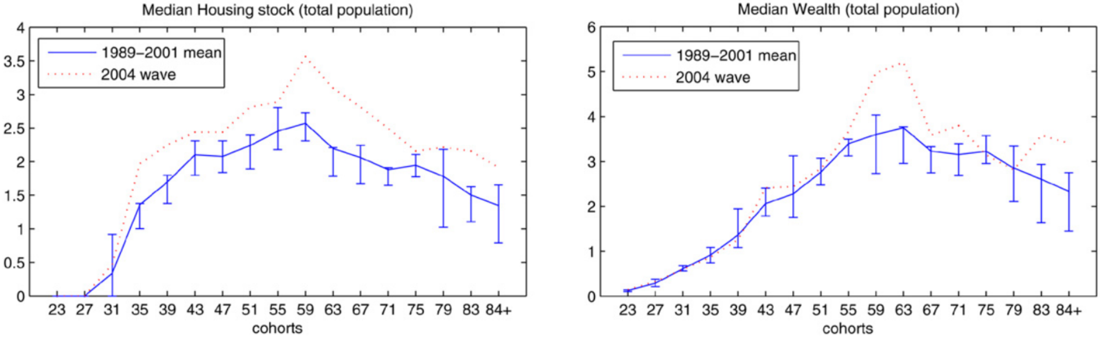


Figure: Median housing and assets

Data from Survey of Consumer Finance (US)

Life cycle profiles: wealth and housing

Homeownership, Diaz and Luengo-Prado (2008)

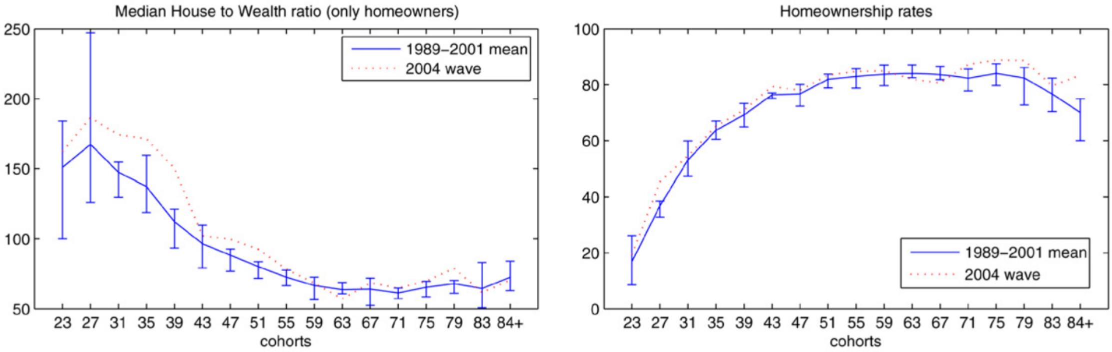


Figure: Share of wealth and homeownership

Data from Survey of Consumer Finance (US)

Life cycle profiles: cross sectional variance

Increasing variance of income and consumption along life cycle, Guvenen (2011)

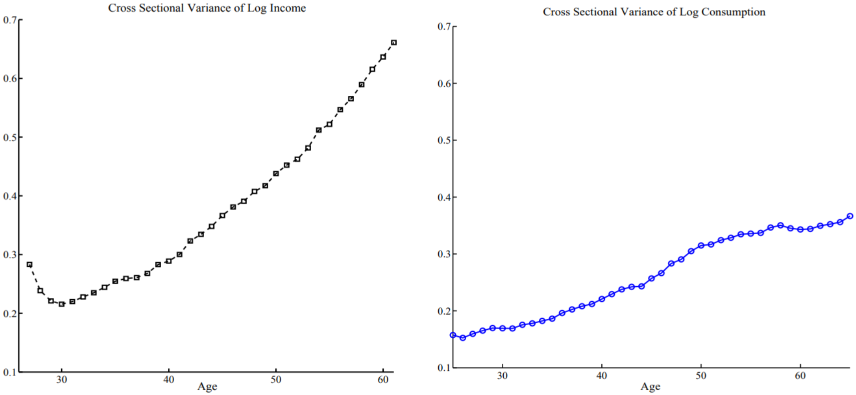


Figure: Cross section variance income and consumption

Data from Panel Study of Income Dynamics (US)

Life cycle profiles: wealth inequality declines

Share of wealth held by top 1% falls (Halvorsen Hubmer Ozkan Salgado, 2021)

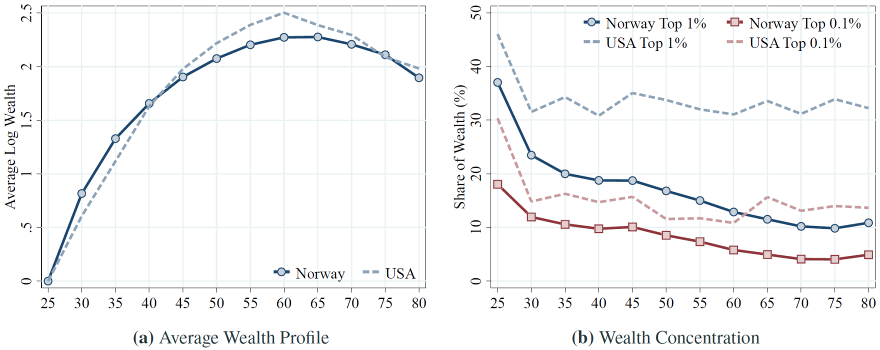


Figure: Wealth inequality in Norway and US during life cycle

Data from Statistics Norway and SCF

Life cycle profiles: wealth dynamics

No convergence at the top: decline in lifetime inequality comes from the lower-half catching up

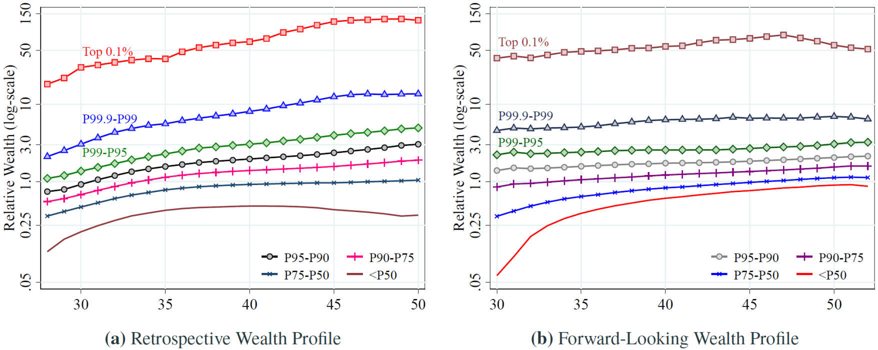
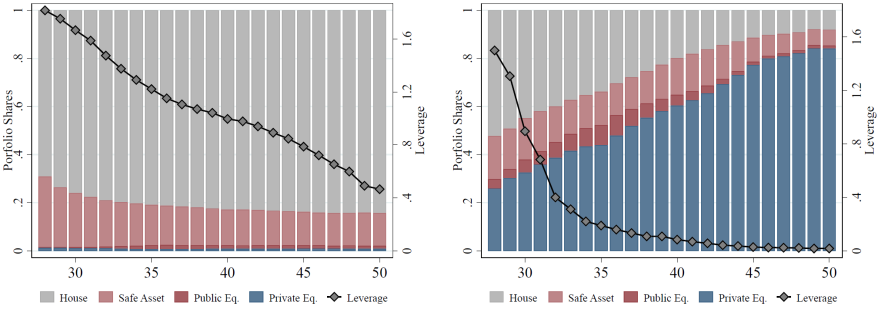


Figure: Dynamics average wealth profiles

Data from Statistics Norway

Life cycle profiles: portfolio share

Rich accumulate large share in Private Equity at age 25/29; Low-mid wealth have mostly housing



(a) Households at bottom 50%

(b) Households at top 0.1%

Figure: Retrospective Portfolio Shares for 50/54 years-old in 2015

Data from Statistics Norway

Life cycle profiles: wealth dynamics

HHs that reach the top experience higher average lifetime returns, mostly from equity

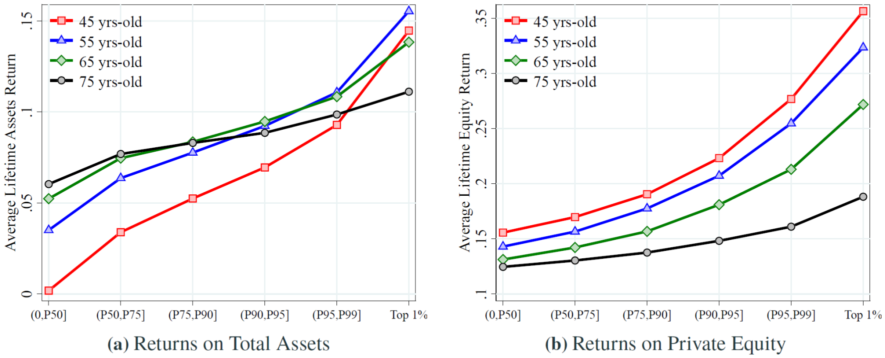


Figure: Lifetime Returns on Assets

Data from Statistics Norway

Life cycle facts summary

Are they consistent with a permanent income life cycle model?

- Life cycle profiles of consumption and income track each other and are hump shaped
  - PIH: perfectly smooth consumption
  - Changing family composition
- Evidence of hump shaped patterns in durables
- Homeownership rates increase quickly at beginning of life
  - Indicative of liquidity constraints
- Household wealth peaks around retirement, but elderly have significant wealth
- Cohort variance of income and consumption rises over life cycle
  - Against perfect risk sharing. Differing slopes evidence of transitory component / insurance
- Declining cohort wealth inequality during lifecycle. Importance of heterogeneous returns

Important facts not shown

- Large predictable fall in income and consumption at retirement (Banks, Blundell & Tanner 98)
- Evidence of “excess smoothness” of consumption to predictable income changes (Campbell & Deaton 89, Attanasio & Pavoni 07)

Classic life cycle consumption-savings problem

Study model similar (not exactly same) to Gourinchas and Parker 2002

- Period of model is annual. Discount factor $\beta \in (0, 1)$
- Let j be household age $j = 1, ..., N$
  - $j = 1$ is the normalization of the first age e.g. 25
- Households work for the first N^w periods. Retired for N^r periods
- Each period the household face an age dependent probability of death

$$\varrho_{j+1} = Pr(alive_{j+1} | alive_j)$$

- with $\varrho_1 = 1$
- Die with probability 1 in period $N + 1$, $\varrho_{N+1} = 0$

Classic life cycle consumption-savings problem

Asset market

- Households hold wealth W_j
- Interest rate on saving is r
- Restrict households borrowing
  - can relax, in GP (02) actually natural borrowing constraint

$$W_{j+1} \geq 0$$

- In general need terminal asset condition: $W_{N+1} \geq 0$
- For the moment lets not worry about *general equilibrium*

Permanent-transitory income

- Income is the permanent-transitory decomposition in logs
- Let income each period be Y_j
- Income has a permanent P_j and transitory U_j component.
- When working let income be:

$$Y_j = P_j U_j$$
$$P_j = G_j P_{j-1} N_j$$

- Transitory income U_j is i.i.d.
  - log normal $\log U_j \sim \mathcal{N}(0, \sigma_u^2)$
  - in GP (02) with probability p it takes value $0 \rightarrow$ n.b.c $W_{j+1} > 0$
- G_j is a deterministic component capturing the income profile
- N_j is a i.i.d permanent income shock $\log N_j \sim \mathcal{N}(0, \sigma_n^2)$
- When retired household receives a pension based on end of working life wage: ϕP_{N^w}

Preferences

- Household preferences are given by:

$$\mathbf{E}_1 \sum_{j=1}^{N} \beta^{j-1} \Big(\prod_{s=1}^{j} \varrho_s \Big) \nu(Z_j) u(C_j)$$

- C_j is the consumption of non-durables
- Z_j age specific taste shifter e.g. household size
- The felicity utility function is CRRA:

$$u(C_j) = \frac{C_j^{1-\gamma}}{(1-\gamma)}$$

- Where $\gamma > 1$ is the inverse of the *Intertemporal Elasticity of Substitution*
- The budget constraint is:

$$W_{j+1} = (1+r)(W_j + Y_j - C_j)$$

Redefine in terms of cash on hand

- Currently three dimensional state vector (W_j, Y_j, P_j)
- Redefine problem in terms of cash on hand X_j:

$$
\begin{aligned}
X_{j+1} &= (1+r)(X_j - C_j) + Y_{j+1} \\
&= W_{j+1} + Y_{j+1}
\end{aligned}
$$

- We have saved one state variable!

Household maximization: Bellman equation

Working age households

- The household maximization problem can be written in the recursive form:
- for periods $j = 1, ..., N^w$

$$V^j(X_j, P_j) = \max_{C_j} \ \nu(Z_j)u(C_j) + \beta \varrho_{j+1} \mathbf{E_j} V^{j+1}(X_{j+1}, P_{j+1})$$

$$X_{j+1} = (1+r)(X_j - C_j) + Y_{j+1}$$

$$C_j \leq X_j$$

- Plus the income process:

$$Y_j = P_j U_j$$

$$P_j = G_j P_{j-1} N_j$$

Household maximization: Bellman equation

Retired households

- for periods $j = N^w + 1, .., N - 1$

$$\begin{aligned} V^j(X_j, P_{N^w}) &= \max_{C_j} \; \nu(Z_j)u(C_j) + \beta \varrho_{j+1} V^{j+1}(X_{j+1}, P_{N^w}) \\ X_{j+1} &= (1 + r)(X_j - C_j) + \phi P_{N^w} \\ C_j &\leq X_j \end{aligned}$$

- and period $j = N$

$$\begin{aligned} V^N(X_N, P_{N^w}) &= \max_{C_N} \; \nu(Z_N)u(C_N) \\ C_N &\leq X_N \end{aligned}$$

Household solution

- The solution to the household problem is a sequence of
  - household value function $\left\{ V_j(X_j, P_j) \right\}_{j=1}^{N}$
  - consumption policy functions $\left\{ C_j(X_j, P_j) \right\}_{j=1}^{N}$
  - wealth accumulation defined as above
- Notice the policy functions are now non-stationary and age specific
- In general the propensity to consume out of “permanent”’ shocks changes as the time horizon shortens
  - permanent shocks become more temporary

Permanent income normalization

Divide through by permanent income

- Before characterizing the solution notice we can reduce state space further
- Because of homothetic utility function problem is homogenous of degree $1-\gamma$
- Normalize all variables such that $x_j \equiv X_j / P_j$ and $c_j \equiv C_j / P_j$
- Asset accumulation equation:

$$\frac{X_{j+1}}{P_{j+1}} = \frac{P_j}{P_{j+1}} \frac{(1+r)(X_j - C_j)}{P_j} + \frac{Y_{j+1}}{P_{j+1}}$$

$$x_{j+1} = (1+r)\frac{(x_j - c_j)}{G_{j+1} N_{j+1}} + U_{j+1}$$

Permanent income normalization

Normalizing the value function

- Divide value function through by $P_j^{1-\gamma}$

$$\frac{1}{P_j^{1-\gamma}} V^j(X_j, P_j) = \max_{C_j} \ \nu(Z_j) \frac{C_j^{1-\gamma}}{1-\gamma} \frac{1}{P_j^{1-\gamma}} +$$

$$\beta \varrho_{j+1} \mathbf{E_j} \left(\frac{P_{j+1}}{P_j} \right)^{1-\gamma} \frac{1}{P_{j+1}^{1-\gamma}} V^{j+1}(X_{j+1}, P_{j+1})$$

$$\tilde{V}^j(x_j) = \max_{c_j} \ \nu(Z_j) \frac{c_j^{1-\gamma}}{1-\gamma} +$$

$$\beta \varrho_{j+1} \mathbf{E_j} (G_{j+1} N_{j+1})^{1-\gamma} \tilde{V}^{j+1}(x_{j+1})$$

- Where $\tilde{V}^j(\cdot) = \frac{1}{P_j^{1-\gamma}} V^j(\cdot)$
- Now the only state variable to keep track of is cash on hand

Permanent income normalization

Divide through by permanent income

- We can characterize the solution with the Euler equation
- for the working age household:

$$c_j^{-\gamma} \geq \beta \varrho_{j+1}(1+r)\frac{\nu(Z_{j+1})}{\nu(Z_j)}\mathbf{E_j}\left(G_{j+1}N_{j+1}\right)^{1-\gamma}c_{j+1}^{-\gamma}$$

  - Where $\mathbf{E_j}$ is over values of N_{j+1} and U_{j+1}
- for the retired household:

$$c_j^{-\gamma} \geq \beta \varrho_{j+1}(1+r)\frac{\nu(Z_{j+1})}{\nu(Z_j)}c_{j+1}^{-\gamma}$$

- Solve back from terminal value of consumption c_N recursively with endogenous grid method

Retirement problem simplification

- Notice in this simple setting the retirement problem is entirely deterministic (cake eating problem)
- Replace income in retirement with a one off lump sum hP_{N^w} in period $N^w + 1$ (as in GP 02)
- We could actually solve back to find a closed form solution for the value of wealth at retirement
  - Solving forward with the Euler equation:

$$c_{N^w+j} = \left[(\beta(1+r))^{j-1} \left(\prod_{s=N^w+1}^{N^w+j} \varrho_s \right) \varrho_{N^w+1}^{-1} \frac{\nu(Z_{N^w+j})}{\nu(Z_{N^w+1})} \right]^{1/\gamma} c_{N^w+1}$$

$$c_{N^w+j} = \left[(\beta(1+r))^{j-1} \Omega_j \right]^{1/\gamma} c_{N^w+1}$$

Retirement problem simplification

- Using the present value budget constraint:

$$
\begin{aligned}
w_{N^w+1} &= \sum_{j=1}^{N-N^w} \frac{1}{(1+r)^{j-1}} c_{N^w+j} \\
&= \sum_{j=1}^{N-N^w} (1+r)^{(j-1)(1-\gamma)/\gamma} \left[\beta^{j-1}\Omega_j\right]^{1/\gamma} c_{N^w+1} \\
&\equiv \Lambda^{-1} c_{N^w+1}
\end{aligned}
$$

Retirement problem simplification

- Substitute into value function:

$$\tilde{V}^{N^w+1}(w_{N^w+1}) = \sum_{j=1}^{N-N^w} \beta^{j-1} \Big(\prod_{s=N^w+1}^{N^w+j} \varrho_s \Big) \varrho_{N^w+1}^{-1} \nu(Z_{N^w+j}) \times$$

$$\frac{1}{1-\gamma} \Bigg(\underbrace{\Big[(\beta(1+r))^{j-1} \Omega_{N^w+j} \Big]^{1/\gamma} \Lambda w_{N^w+1}}_{c_{N^w+j}} \Bigg)^{1-\gamma}$$

$$\tilde{V}^{N^w+1}(w_{N^w+1}) = \Gamma \frac{1}{1-\gamma} w_{N^w+1}^{1-\gamma}$$

Retirement problem simplification

- Consumption is a linear function of wealth at retirement:

$$C_{N^w+1} = \varphi_1 (X_{N^w+1} + H_{N^w+1})$$

- Equivalently, can be written as a linear function of cash on hand with $\varphi_0 = \varphi_1 h$:

$$c_{N^w+1} = \varphi_0 + \varphi_1 x_{N^w+1}$$

  - Setting $Z_j = \overline{Z}$ constant and probability of death to zero $\varrho_j = 1$ we can show:

$$\varphi_1 = \frac{1 - \beta^{1/\gamma}(1+r)^{1/\gamma - 1}}{1 - (\beta^{1/\gamma}(1+r)^{1/\gamma-1})^{N-N^w}}$$

- In general we can't do this if we add post-retirement uncertainty

Inspecting consumption growth

How does the model compare with complete markets benchmark?

Complete markets baseline

$$c_j^{-\gamma} = \beta(1+r)c_{j+1}^{-\gamma}$$

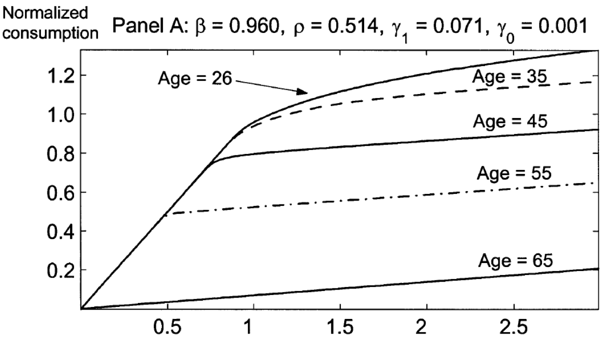
Canonical life cycle

$$c_j^{-\gamma} \geq \beta \varrho_{j+1}(1+r)\frac{\nu(Z_{j+1})}{\nu(Z_j)}\mathbf{E_j}\left(G_{j+1}N_{j+1}\right)^{1-\gamma}c_{j+1}^{-\gamma}$$

- Complete markets model predicts perfect consumption smoothing
- In life cycle model with uncertainty a number of additional terms
  - Early in life: liquidity constraints prevent household equalizing marginal utility
  - Uncertainty causes households to want to insure, reducing consumption today relative to tomorrow
  - Taste shifters (household size) alter timing of consumption peak
  - In later life: increased death probability means households are more impatient

Consumption functions in Gourinchas & Parker (02)

- Consumption concave in cash in hand
- Early in life *buffer stock* behaviour
  - Low cash on hand consume most but not all income $\rightarrow$ low liquid assets at $t+1$
  - At high wealth precautionary motive small, run down wealth



Model estimation: Method of Simulated Moments

- Use income parameters from PSID by education, occupation type:
  - And probability of unemployment (zero income shock)
  - Also interest rate
  - Distribution of wealth at age 26 (CEX)
- Simulate the the model and use the MSM to estimate remaining parameters to match cohort CEX profiles
- Minimize difference between consumption at each age and model:
  - Parameters: θ, state $\hat{\chi}$

$$g(\theta; \hat{\chi}) = \underbrace{\frac{1}{I}\sum_{i=1}^{I_t} \log C_{i,t}}_{data} - \underbrace{\log(\hat{C}_t(\theta, \hat{\chi})}_{model}$$

- Minimize: $g(\theta; \hat{\chi})' W g(\theta; \hat{\chi})$

Model estimation: Method of Simulated Moments

- In cross sectional data (CEX) estimate:

$$\log \tilde{C}_i = f_i\pi_1 + a_i\pi_2 + b_i\pi_3 + \mathcal{U}_i\pi_4 + Ret_i\pi_5 + \epsilon_i$$

  - Family size=f_i, age=a_i, cohort=b_i, state unemployment rate=$\mathcal{U}_i$, retirement=Ret_i

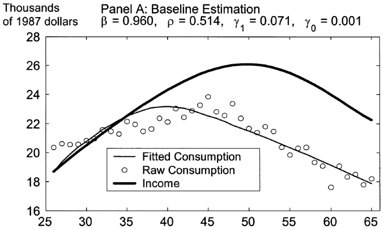
- Model equivalent of average household aged a:

$$\log C_a = \bar{f}\hat{\pi}_1 + a\hat{\pi}_2 + \bar{\mathcal{U}}\hat{\pi}_4$$

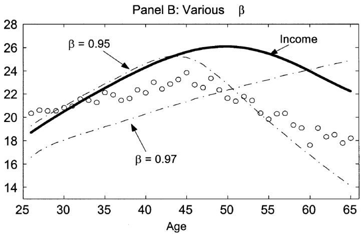
- Smooth data series with fifth order polynomial
- Parameters to estimate: β, risk aversion=ρ, retirement rule=(γ_0, γ_1)
- Baseline estimate for average household
  - Then estimate by education/occupation subgroup

Example life cycle profile in Gourinchas & Parker (02)

Fairly convincing fit to consumption profile, family size effects held constant



(a) baseline

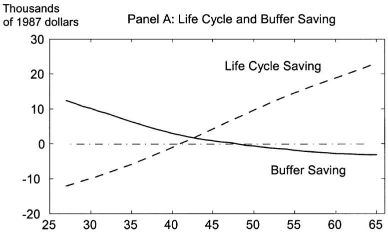


(b) Alternative β

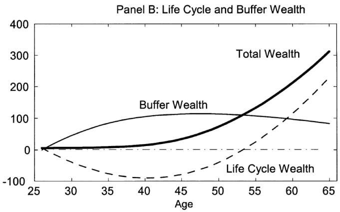
- Better fit than Permanent Income Complete Markets model (linear rule)
- Misses borrowing in early part of life
  - In data, consumption is slightly flatter and peaks later
- Panel B shows more sensitivity to β calibration
- Fairly low estimate for risk aversion parameter $\rho = 0.514$

Determinants of wealth accumulation

- Compare behavior to a certainty equivalent life cycle alternative
  - Decompose *precautionary* motive and *life cycle* motive
- In model young households face rising income, want to consumer more today
  - Constrained by liquidity constraint
  - Hold precautionary buffer savings against uninsurable income risk
- After age 40 build up assets for retirement (life cycle) purposes
  - Provides source of insurance $\rightarrow$ dominates requirement for buffer stock



(a) savings



(b) wealth

Life cycle model insights

- Sufficiently rich life cycle model can capture many features of data
- Emphasize precautionary motive and liquidity constraints when young
- Life cycle savings for retirement when older
- Methods such as GP (02) show how micro data can be used to estimate parameters of interest
    - Provide a rich set of targets to calibrate to and over identifying moments to check models against
- Retirement in these models is a bit of an uninteresting state

What do we know about the income process

Income process and insurance

- Income process faced by households is important for predictions of life cycle model
- Many papers have tried to study how much income risk households face and its features/moments
- Benchmark permanent income model would suggest consumption:
  - fully adjusts to permanent shocks
  - (almost) no adjustment to transitory shocks
- In permanent income model, over life cycle increase in variance of log consumption tells about size of permanent shocks

$$p_t^i = p_{t-1}^i + \zeta_t^i$$

$$var_i(c_t^i) \approx var_i(c_{t-1}^i) + var(\zeta_t)$$

- Linearly increasing consumption variance over life cycle would seem to support this

Income and consumption variance increased in 80s

Blundell, Pistaferri and Preston (08)

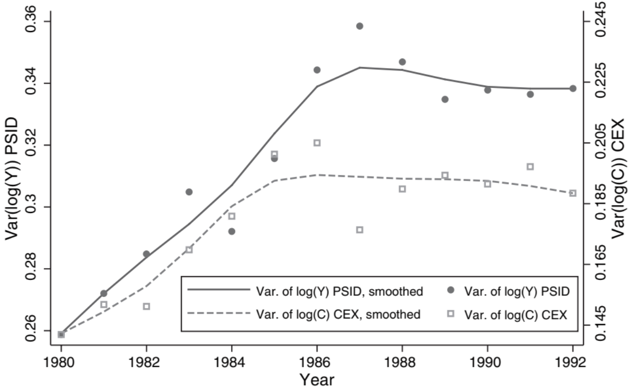


FIGURE 1. OVERALL PATTERN OF INEQUALITY

Figure: Cross sectional variance income and consumption

Data from PSID and CEX (US)

Consumption inequality increasing linearly with age

Higher inequality for younger cohorts

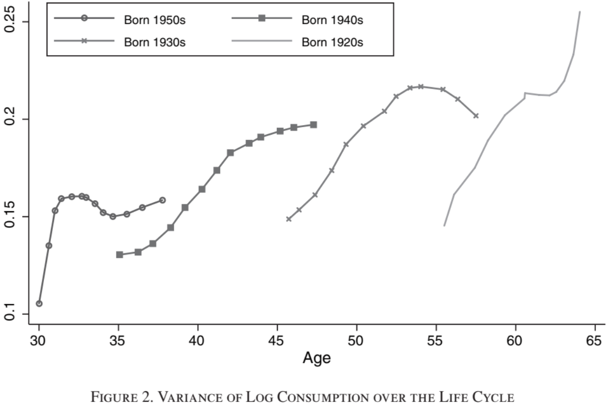


Figure: Cohort growth in consumption variance

Data from CEX (US)

A model of partial insurance

Combining CEX and PSID to study consumption insurance

- BPP (08) combined income data from the PSID and consumption data from the CEX to study change in income shocks
- Assume additional unmodeled sources of insurance
  - in logs: consumption c_t^i, permanent income shocks, ζ_t^i and transitory income shocks ϵ_t^i
  - Insurance parameters ϕ_t and ψ_t

$$\Delta c_t^i = \phi_t \zeta_t^i + \psi_t \epsilon_t^i + \xi_t^i$$

- Income growth: $\Delta y_t^i = \zeta_t^i + \Delta \nu_t^i$
- With MA(q) representation of transitory shocks

$$\nu_t^i = \sum_{j=0}^{q} \theta_j \epsilon_{t-j}^i$$

- Relationship is exact for quadratic preferences and an approximation for CRRA

A model of partial insurance

Combining CEX and PSID to study consumption insurance

- Model provides a variety of covariance restrictions on the data
- For income:

$$cov(\Delta y_t, \Delta y_{t+s}) = \begin{cases} var(\zeta_t) + var(\Delta \nu_t) & \text{for } s = 0 \\ cov(\Delta \nu_t, \Delta \nu_{t+s}) & s \neq 0 \end{cases}$$

- Can identify order of MA(q) process
  - If MA(0): $var(\zeta_t) = cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})$ and $var(\epsilon_t) = -cov(\Delta y_t, \Delta y_{t+1})$
- For insurance coefficients

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\Delta \epsilon_t) & \text{for } s = 0 \\ \psi_t cov(\Delta \epsilon_t, \Delta \nu_{t+s}) & s \neq 0 \end{cases}$$

- From consumption growth:

$$cov(\Delta c_t, \Delta c_{t+s}) = \begin{cases} \phi_t^2 var(\zeta_t) + \psi_t^2 var(\Delta \epsilon_t) + var(\xi_t) & \text{for } s = 0 \\ 0 & s \neq 0 \end{cases}$$

- Estimate moments with minimum distance estimator

Data

Combining CEX and PSID to study consumption insurance

- A problem for the analysis is that there isn't a panel data set with *both* income and consumption data
  - PSID provide panel data on income and narrow food measure of consumption
  - CEX has rich cross sectional information on consumption
- **Idea:** use data on consumption from the CEX to impute a measure of non-durable consumption in the PSID
  - Comparable food consumption in both datasets
- Estimate demand for food in CEX, given observables and non-durable consumption
- Invert demand and use parameter estimates to impute non-durable consumption at the household level in the PSID
- Growth in variance of log consumption in both similar - *imputation hopefully ok*

BPP estimates

Find partial insurance to perm shocks around 2/3

- **Data:** until mid 80s $var(\Delta y_t)\uparrow$, $cov(\Delta y_t, \Delta y_{t+1})\uparrow$, and $var(\Delta c_t)\uparrow$
- In early 80s $cov(\Delta c_t, \Delta y_t)\uparrow$. Flat or declining after
- $cov(\Delta c_t, \Delta y_{t+1})$ almost zero $\rightarrow$ *transitory shocks* with insurance

	Whole sample	No college	College	Born 1940s	Born 1930s
ϕ	0.6423	0.9439	0.4194	0.7928	0.6889
(Partial insurance perm. shock)	(0.0945)	(0.1783)	(0.0924)	(0.1848)	(0.2393)
ψ	0.0533	0.0768	0.0273	0.0675	−0.0381
(Partial insurance trans. shock)	(0.0435)	(0.0602)	(0.0550)	(0.0705)	(0.0737)
p-value test of equal ϕ	23%	99%	8%	81%	18%
p-value test of equal ψ	75%	33%	29%	76%	4%

Figure: estimated insurance parameters

- Partial insurance against permanent shocks
- Almost full insurance against transitory shocks
- Reject change in insurance pre and post 1985

Interpreting the data

- Consumption growth equation can be written

$$\Delta var(\Delta c_t) \approx \phi^2 \Delta var(\zeta_t) + \psi^2 var(\Delta \epsilon_t)$$

- Early 1980s we saw increase in variance of permanent shocks and consumption variance
- Partial insurance meant not all increase from income to consumption
- Later on mainly increase in transitory shocks but these are well insured. No impact on consumption inequality
- Note: under CRRA approximation self insurance means we would still expect $\phi < 1$
  - depends on households assets as a share of total financial wealth and future labor income
- Kaplan & Violante (2010) show in a life cycle model with AR(1) process for persistent income can get coefficient close to 2/3

Heterogeneous income profiles

Alternative reading of the data is that households face heterogeneous income profiles

- So far we've considered models where the persistence component of income follows a unit root or ρ is high > 0.9.
- Model in logs:

$$
\begin{aligned}
y^i_{t,age} &= \alpha^i_t + p^i_t + g^i_{age} + \epsilon_t \\
p^i_t &= \rho p^i_{t-1} + \zeta^i_t
\end{aligned}
$$

- This follows estimates from a large literature e.g. MaCurdy (82), Abowd & Card (89), Floden & Linde (01) and Storesletten (04)
- These papers imposed a common g_{age} following a statistical test
- Guvenen (09) suggests the test has *low power* to identify heterogeneous g^i_{age}
- Particularly problem given size of datasets

Ignoring HIP biases ρ upwards

Attributes fanning out of income to shocks. (Guvenen 09)

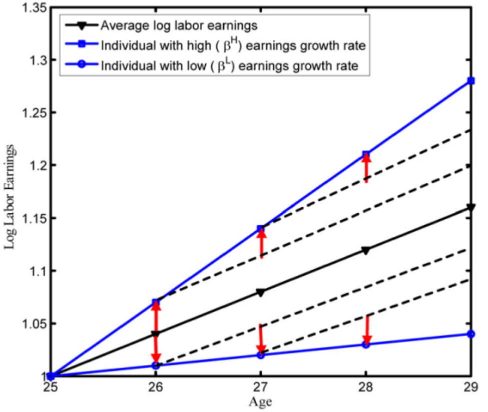


Fig. 1. Ignoring profile heterogeneity results in an upward bias in estimated persistence.

Figure: Bias of restricted income profiles

Ignoring HIP biases ρ upwards

Guvenen 09

- When estimating process with HIP finds lower persistence: $\sim$0.8
- Heterogeneity in profiles accounts for 50-75 % of income inequality at age 55
- Evidence of dispersion within education groups and higher dispersion for college educated individuals
- Still not necessarily evidence about a lack of income uncertainty
  - Depends about individuals knowledge of profile
- **Further paper** (Guvenen & Smith 14): Using joint data on choices and income
- Households have substantial knowledge about slope profile and that income shocks not that persistent $\rho = 0.7 - 0.8$
  - Less uninsurable risk than typically assumed

Administrative data

More data, more moments

- Most research based on PSID
  - Core sample: 3,000 households before sample selection
- Access to administrative tax records has allowed for much richer analysis
- *Non-parametric* approaches
- Drawbacks: limited demographics, individuals not households, no hours, miss unemployment
- Lots of evidence of non-normality in income process
- Higher order moments vary significantly by age and earnings level

US administrative data: key facts

Guvenen, Karahan, Ozkan & Song (21)

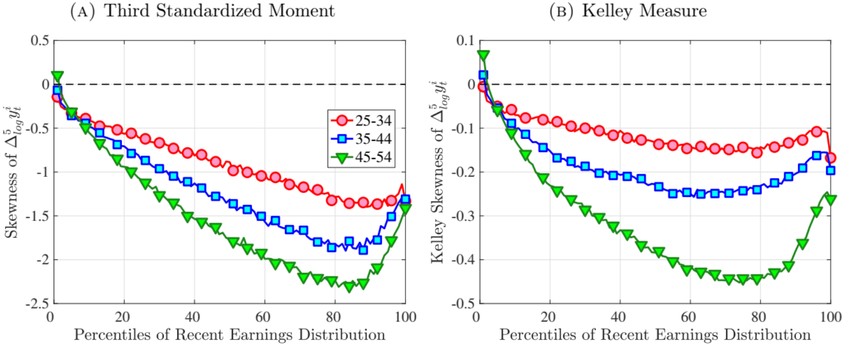
Summarizing new income facts:

1. Risk is left (negatively) skewed and increases with age and earnings
2. Earnings growth displays high kurtosis
3. Positive earnings changes are transitory for high income earners. Negative changes are persistent
    - Opposite pattern for low earners
4. Huge magnitude of variation of lifetime wage growth
5. Evidence of extremely persistence long-term nonemployment
6. Previous: left skewness in idiosyncratic risk rises in recessions
    - reject countercyclical variance

Fact 1. Left skewness increases over lifecycle

Left skewness also increasing in past earnings

FIGURE 4 – Skewness of Five-Year Log Earnings Growth

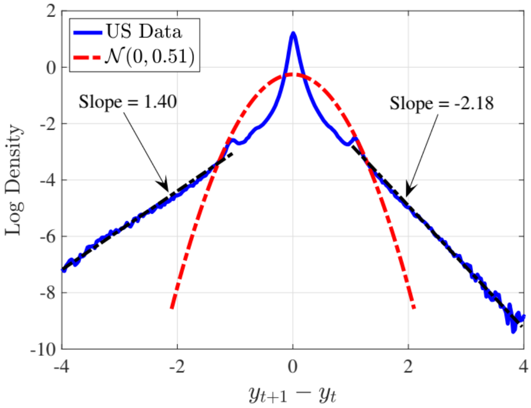


- Decline likelihood of large gains with age (largest factor)
- Increasing likelihood of fall after 45 for above median earners

Fact 2. Earnings growth displays high Kurtosis

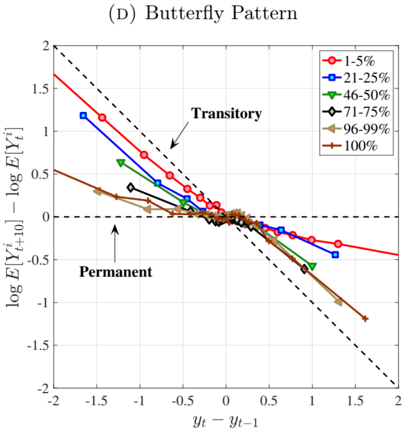
31% of annual changes less than 5%

FIGURE 6 – Double-Pareto Tails of the U.S. Annual Earnings Growth Distribution



Fact 3. High earners: +ve shock transitory, -ve perm

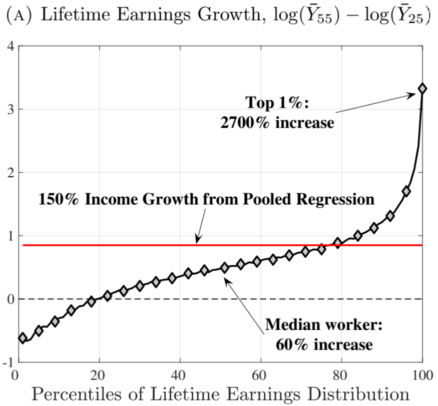
For low earners: -ve earnings growth transitory, while +ve earning growth permanent



- **Impulse Response Function:** y-axis shows 10-year growth predicted by change between $t-1$ and t $\rightarrow$ 45-degrees indicates mean reversion

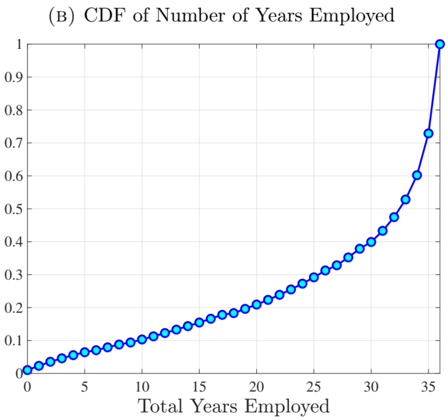
Fact 4. Large lifetime earnings growth of top earners

Top 1 % see 28 fold increase in earning between 25 and 55



Fact 5. Tail of HHs with weak labor market attachment

18% of men spend 18 years non-employed (one year)



Fact 6. Negative skewness is increasing in Recessions

Once account for skewness no evidence of increasing variance

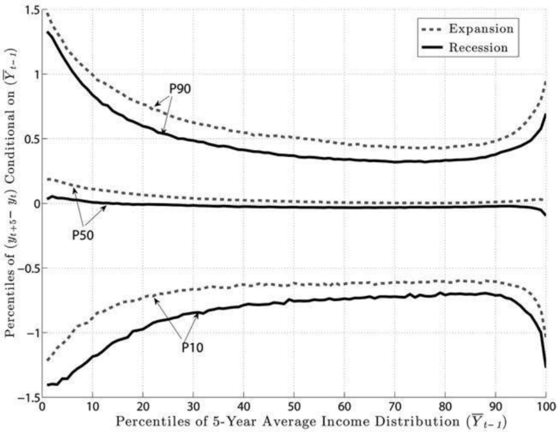


Figure: Skewness in permanent earnings growth

Non-linear approach to partial insurance

Arellano, Blundell & Bonhomme (17)

- Significant heterogeneity in income shocks / risk faced across distribution
- ABB (17) use a quantile based method to estimate persistence of income in response to shocks
- Evidence of non-linear income persistence in PSID and Norwegian admin data at different parts of distribution
  - Initial income
  - Size of shock
- Use to estimate non-linear statistical “model” of consumption
  - PSID now has consumption data! But biannual
- Generalize previous BPP (08) measure of partial insurance

Evidence of nonlinearity in persistence of earnings

Positive (negative) shocks for poor (high) earning households wipe out history

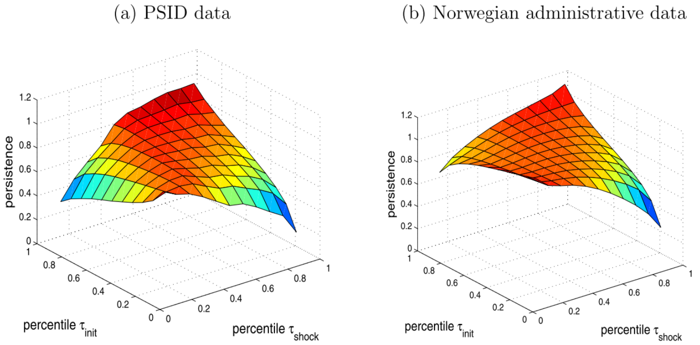


Figure: Quantile regression of log-earnings

- "Persistence" of earnings history $\rightarrow$ derivative of quantile function wrt y_{t-1}
- $\rho(\eta_{i,t-1}, \tau)$ (not shown) in estimated earnings mode is:
  - often around 1
  - 0.6-0.8 when bad shocks hit high earner / good shocks hit low earners

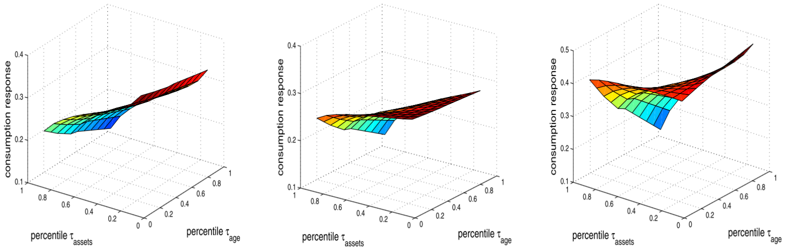
Non-linear consumption response

Older households with higher assets display smaller consumption response

(a) Response to earnings PSID data

(b) Response to earnings Nonlinear model

(c) Response to η_{it} Nonlinear model



- Insurance coefficient between 0.6 and 0.7 (panel C)
- Simulations suggest asset holdings attenuate consumption response to negative shocks
  - Particularly later in life

Male lifetime incomes have declined by 10-19% since 1967

Female lifetime incomes increased from low base (Guvenen Kaplan Song Weidner, 2019)

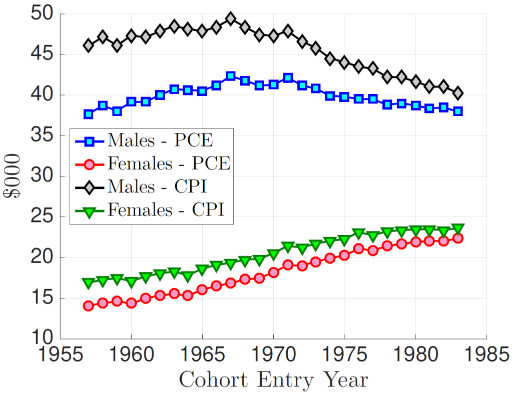


Figure: Median Lifetime Income by Cohort and Gender

Decline for men due to fall at young ages. Women's entry income ↑

Steady decline in male median incomes at ages 25 and 35

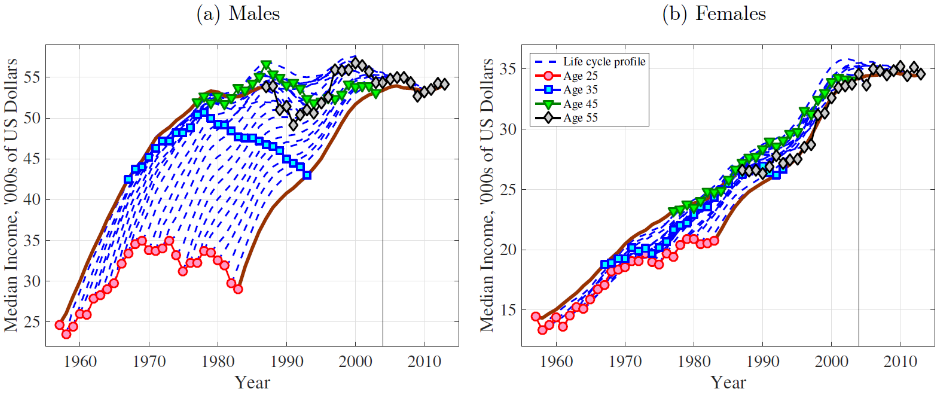


Figure: Age Profiles of Median Income by Cohort

Rising inequality within gender, falling inequality between genders

Female to male lifetime income ratio rose from 40 to 60 percent

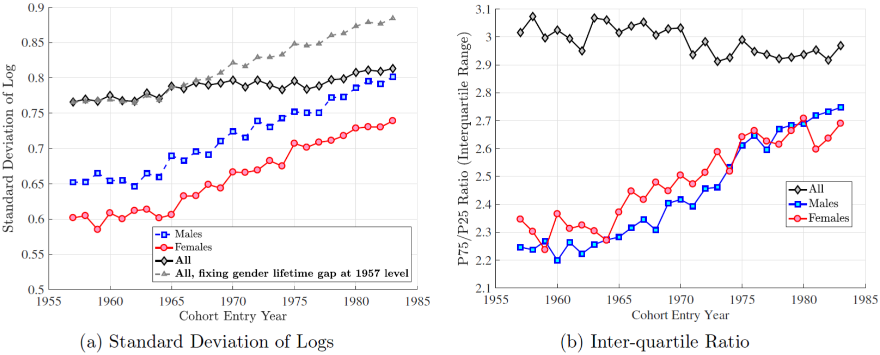


Figure: Cohort Lifetime Inequality, Overall and by Gender

- Reconciled by falling gender wage gap (e.g. dotted gray line panel (a))

Rise in male inequality due to inequality when entering labor market

More complex pattern for female lifetime inequality. Difference driven by <P50 (see paper)

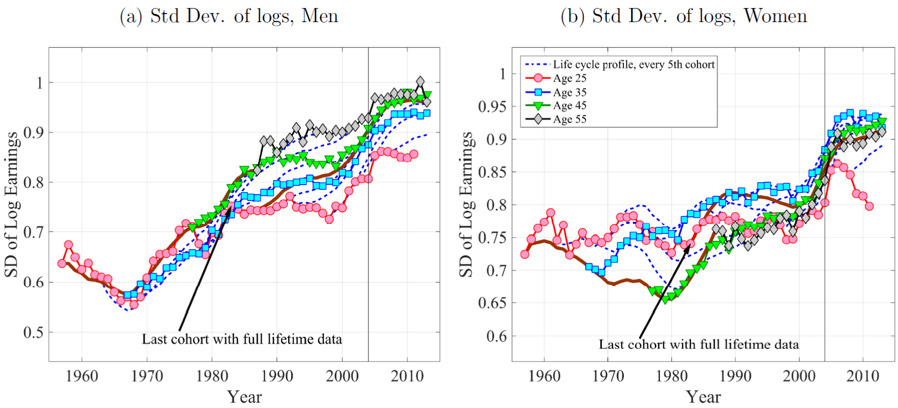


Figure: Age Profiles of Cross-Sectional Inequality, by Cohort

Income process summary

- Empirical literature originally thought income was well approximated by permanent/transitory decomposition
- If not random walk, permanent component very persistent
  - AR(1) income process still widely used in quantitative applications
- New empirical evidence reveals lots of heterogeneity. Higher order moments far from Gaussian
- Suggestive of jobs ladder model
- *Question?* The importance for consumption macro models & which moments of income important to get right
  - Doesn't resolve wealth distribution concentration (De Nardi, Fella & Paz Pardo (20))
  - Countercyclical income risk / left skewness seems important
- *Lifetime income*: across cohorts, males have seen falling driven by lower entry wages. Opposite pattern for females.
  - For males, much of rise in inequality looks a bit like a fixed effect

Bequest motives

What happens when households die?

- The stark assumption of the baseline model was that households place zero value on the world after their death
- Prediction is households consume and run down their assets during retirement and are unconcerned with long-run adjustments
- In the data we see households die with significant wealth
- The assumptions made are extremely important for the resulting wealth distribution
- The correct level of estate taxation is an important policy question
- Consider two approaches
  1. Infinite horizon dynastic households (Casteneda, Diaz-Gimenez Rios-Rull 03) seen already
  2. Bequest motive (De Nardi 04)

Bequest motives

Alternative to dynastic models is to assume households value the wealth at end of life

- Previous approach: life cycle added to infinite horizon model
- De Nardi (04): *add bequests to life cycle model*
  - similar motivation of trying to fit wealth distribution
- Additional observation that wealth is concentrated even in old age
  - Data estimates suggest private transfers across generations is 60% of current wealth (including accrued interest)
  - Cite estimate of 40% lower bound for Sweden

Modeling strategy

- Overlapping generations model. Young and old households linked by accidental and voluntary bequests
- Large estates accumulated over multiple generations, with voluntary bequests
  - accidental bequests are insufficient
- Key innovation is that bequests are **luxury good**
  - modelling somewhat ad hoc. What would be a reasonable test?

Wealth and earnings in US and Sweden

Similar level of concentration. Less inequality in Sweden, lower savings at bottom

TABLE 1

Wealth

Capital or wealth output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top					
			1%	5%	20%	40%	60%	80%
U.S. data								
3·0	0·60	0·78	29	53	80	93	98	100
Swedish data								
1·7	>0·51	0·73	17	37	75	99	100	100

TABLE 2

Gross earnings

Gini coeff.	Percentage earnings in the top				
	1%	5%	20%	40%	80%
U.S. data					
0·46	6	19	48	72	98
Swedish data					
0·40	4	15	42	68	98

Figure: Wealth and earnings distributions US and Sweden

Environment

- Model period is 5 years
- Three stages: young, middle age (with child), retired
  - $j = 1$ (20 years old), $j = 2$ (have children), $j = 10$ (65, retire), $N = 14$ (die before 90)
  - Child starts working (enters model) at j=6 (45yr). No death before (j=9) (60yr)
- Parents derive utility from *warm glow* bequest $\phi(b_t)$
- Worker have exogenous life cycle age efficiency profile ϵ_t
- Face stochastic shocks y_j with transition function Q_y
- Productivity of parent age 40 is transmitted to child with Q_{yh}
- Children do not know parents assets (form expectations) but know productivity aged 40
- Tax on labor τ_l, assets τ_a and estates τ_b with exemption ex_b
- State space: (j, a, y, yp)
  - yp parents productivity aged 40. Set to $yp = 0$ after inheritance
- Household inherits bequest once in lifetime at random period of parents death

Household decision problem

Decision problem when young

- For age 20-30 ($j = 1 - 3$). No receipt of bequest

$$V^j(a_j, y_j, yp_j) = \max_{c_j, a_{j+1}} \; u(c_j) + \beta \mathbf{E_j} V^{j+1}(a_{j+1}, y_{j+1}, yp_j)$$

$$a_{j+1} = (1 + r(1 - \tau_a))a_j - c_j + \epsilon_j y_j(1 - \tau_l)$$

$$a_{j+1} \geq 0$$

Household decision problem

Decision problem when young

- For age 35-55 ($j = 4 - 8$). No death, but expectation of bequest if not yet received $yp = 0$. Survival probability: s_j

$$V^j(a_j, y_j, yp_j) = \max_{c_j, a_{j+1}} \; u(c_j) + \beta \mathbf{E_j} V^{j+1}(a_{j+1}, y_{j+1}, yp_{j+1})$$

$$a_{j+1} = (1 + r(1 - \tau_a))a_j - c_j + \epsilon_j y_j(1 - \tau_l) +$$

$$b_{j+1} \cdot \mathbf{1}[yp_j > 0]\mathbf{1}[yp_{j+1} = 0]$$

$$a_{j+1} \geq 0$$

$$yp_{j+1} = \begin{cases} yp_j & \textit{with}\ \mathrm{Pr.}\ s_{j+5} \\ 0 & \textit{with}\ \mathrm{Pr.}\ 1 - s_{j+5} \end{cases}$$

Household decision problem

Decision problem when face probability of dying

- After tax bequest is $b(a_{j+1}) = a_{j+1} - \tau_b \cdot \max\{0, a_{j+1} - ex_b\}$
- For age 60 ($j_r - 1$)

$$V^j(a_j, y_j, yp_j) = \max_{c_j, a_{j+1}} \; u(c_j) + s_j \beta W^{j+1}(a_{j+1}) + (1 - s_j)\phi(b(a_{j+1}))$$

$$a_{j+1} = (1 + r(1 - \tau_a))a_j - c_j + \epsilon_j y_j (1 - \tau_l) + \\ b_{j+1} \cdot \mathbf{1}[yp_j > 0]\mathbf{1}[yp_{j+1} = 0]$$

$$a_{j+1} \geq 0$$

$$yp_{j+1} = \begin{cases} yp_j & \textit{with } \Pr.\ s_{j+5} \\ 0 & \textit{with } \Pr.\ 1 - s_{j+5} \end{cases}$$

Household decision problem

Decision problem when face probability of dying

- For age 65-85. Bequest already received

$$W^j(a_j) = \max_{c_j, a_{j+1}} \; u(c_j) + s_j \beta \; W^{j+1}(a_{j+1}) + (1 - s_j)\phi(b(a_{j+1}))$$

$$a_{j+1} = (1 + r(1 - \tau_a))a_j - c_j + p$$

$$a_{j+1} \geq 0$$

Warm glow bequest

How to value wealth passed to children?

- With warm glow bequest motive specification:

$$\phi(b_j) = \phi_1 \left(1 + \frac{b_j}{\phi_2}\right)^{1-\gamma}$$

- Valuation of children's welfare: ϕ_1, luxury good: ϕ_2, EIS: $1/\gamma$
  - Larger value of ϕ_2 more of a luxury
- Notice for the younger household b_j is a random object with distribution $\mu_b^j(x, \cdot)$
  - Needs to be consistent with parental behavior

Stationary equilibrium definition

- Collecting states in vector x
- A stationary equilibrium is
  - prices (r, w)
  - policies $c^j(x), a^j_+(x)$
  - government tax and transfers
  - a distribution over bequest $\mu^j_b(x, \cdot)$
  - a constant distribution of people over states $m^j(x)$
- Such that:
  - Given prices, gov. policies, and μ_b, the policies $c^j(x), a^j_+(x)$ solve the household problem
  - The tax rate is such that the gov. budget holds every period
  - The invariant distribution m^* is consistent with household policies and transition probabilities
  - Asset and labor markets clear and prices equal the marginal products
  - The expected bequest distribution $\mu^j_b(x, \cdot)$ is consistent with bequests left by parents

Calibration

- Calibrate to US and Swedish targets (Sweden as small open economy)
- Many fairly standard targets
- ϕ_1 targets transfer share of wealth: 60%
- ϕ_2 targets ratio of average bequest at median ($32k) and 30 percentile ($2k)
  - $\phi_1 = -9.5$ and $\phi_2 = 11.6$
- In US estate tax parameters target:
  - estate tax revenues:GDP and share of households paying tax 1.5%
    - $\tau_b = 0.1$ and $ex_b = 40y$
- In Sweden estate tax parameters target:
  - revenue from bequest and gift taxes
    - $\tau_b = 0.15$ and $ex_b = 10y$
  - Swedish inheritance tax abolished in 2004

Results

Adding warm glow bequest increases wealth concentration, particularly top 1%

TABLE 5

Results for the U.S. calibration

Capital–output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with negative or zero wealth
U.S. data								
3·0	0·60	0·78	29	53	80	93	98	5·8–15·0
No intergenerational links, equal bequests to all								
3·0	0·67	0·67	7	27	69	90	98	17
No intergenerational links, unequal bequests to children								
3·0	0·38	0·68	7	27	69	91	99	17
One link: productivity inheritance								
3·0	0·38	0·69	8	29	70	92	99	17
One link: parent's bequest motive								
3·0	0·55	0·74	14	37	76	95	100	19
Both links: parent's bequest motive and productivity inheritance								
3·0	0·60	0·76	18	42	79	95	100	19

Figure: US results under various specifications

- Less extreme change for Sweden

Results

Model simulation. Below median save for retirement, above save for bequest

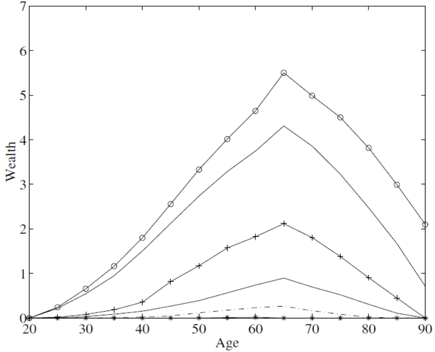


FIGURE 2

U.S. wealth 0·1, 0·3, 0·5, 0·7, 0·9, 0·95 quantiles, by age. Experiment with bequest motive only

Figure: US life cycle simulation

- Consistent with Dynan, Parker and Zeldes (04)

Key mechanisms

- Precautionary saving motive and retirement savings are insufficient to generate wealth accumulation as seen in the data
- Intergenerational transmission of productivity alone does not affect results much
- With luxury bequests households that have high income or receive high bequest have stronger savings motive
  - Large estates are accumulated from one generation to the next which concentrates wealth
- Calibration implies good match of bequest size up to 70 percentile (50:30 ratio target)
- Above 70 percentile predicts larger bequests than in data
  - top coding?

Summary

Comparison with dynastic life cycle model of Casteneda Diaz-Gimenez Rios-Rull (03)

- Two papers incorporate life cycle dimension and inheritance to try and match US wealth distribution
  - Come to different conclusions about source of wealth concentration
- Also encapsulate different ideas about how household's value the future
  - With bequest motive only the total wealth left matters
  - In dynastic model environment e.g. price, taxes etc also relevant
- In life cycle setting bequests seem important to explain wealth of households in old age
- Warm glow bequest motives have become widely used, but micro foundations still controversial

Reading list

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