

# Household Finance PhD Course

## Human Capital

Paolo Sodini

2023

Copyright © Paolo Sodini. All rights reserved.

# Outline

- Asset Valuation
  - Stochastic discount factor
  - Human capital
  - Tangible and intangible components
- Background risk
  - Risk taking vs precautionary savings
- Lifecycle model
  - Bond-like human capital
  - Stock-like human capital
- Tradable component of human capital
  - The beta of human capital
- Risk taking over the lifecycle and wealth
  - Habit
  - Labor and stock market

# Lifecycle Model – Labor Income

$$\max_{\{C_t, w_t\}_{t=1, \dots, T}} E_0 \left[ \sum_{t=1}^T \frac{1}{\Delta^t} u(C_t) \right]$$

$$\text{s.t. } W_{t+1} = (1 + r_f + w_t r_{t+1}^e) W_t + L_{t+1} - C_{t+1}$$

- Decision between a riskless and a risky asset
  - Risk free asset with return
    - $R_f = 1 + r_f$
  - Excess return on the risky asset
    - $r_t^e = r_t - r_f = R_t - R_f$
- Labor income  $L_t$

# CRRA Utility

If the investor has CRRA utility  $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$  the optimal risky share is (Merton, 1969)

$$w_t^M = \frac{E_t(r_{t+1}^e)}{\gamma \text{Var}_t(r_{t+1}^e)} \quad (1)$$

The share of wealth optimally invested in risky assets

- does not depend on household wealth
- does not depend on horizon
- varies over time if beliefs and/or risk aversion changes

$\gamma$  is the coefficient of relative risk aversion and regulates how much risk the agent is willing to take

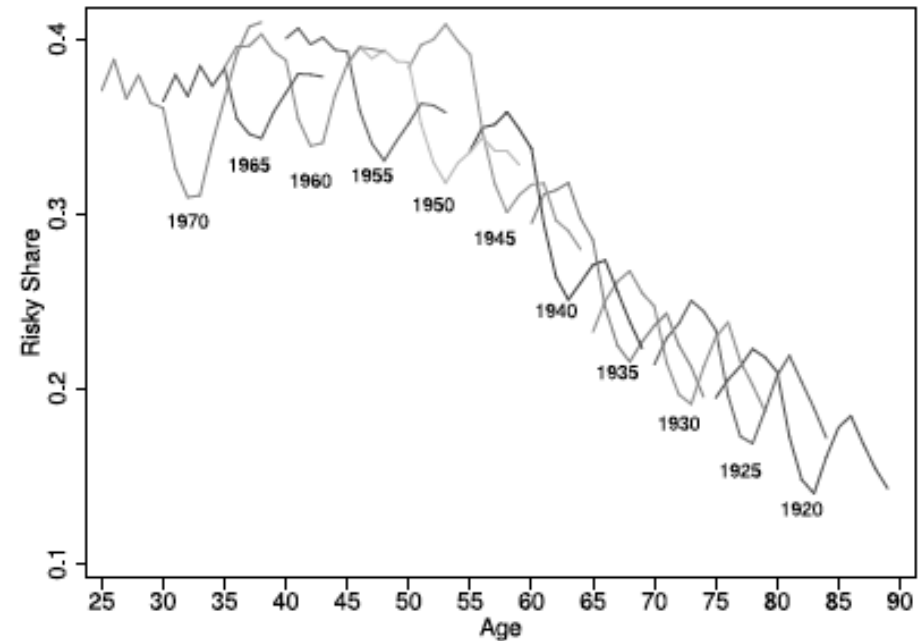
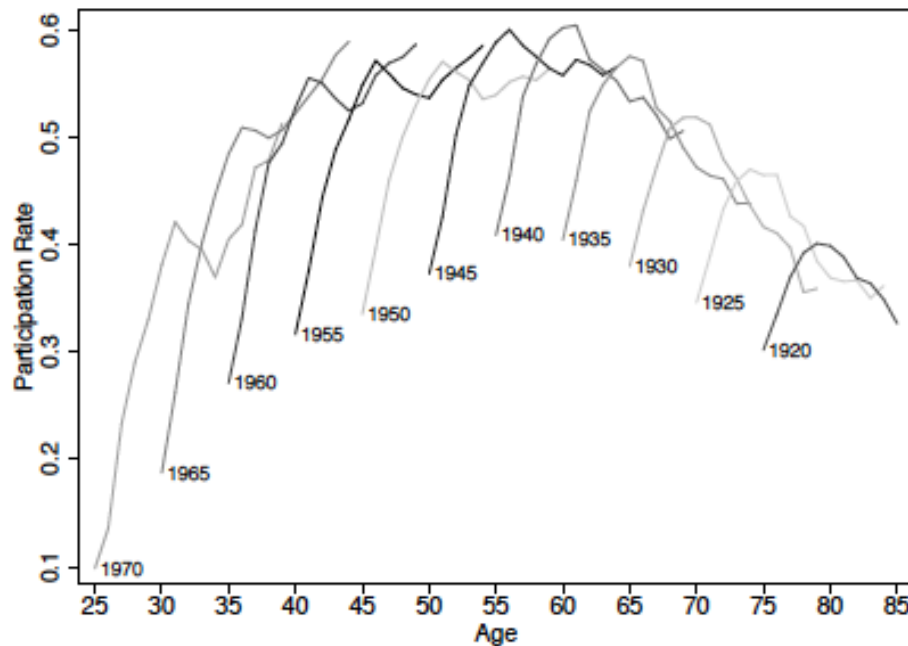
The dollar invested in risky financial assets is a constant fraction of financial wealth, i.e.

$$RFW_t = w_t^M \times W_t$$

# Risk Taking over the Lifecycle

Do households keep their risky share constant over the life-cycle?

- Evidence from Norway



Source: Fagereng et al. 2017

# Labor Income Process

# Estimate Labor Income Process

Campbell, Cocco, Gomes and Maenhout 01 (refined)

$$\ln(L_{h,t}) = \alpha_c + \beta' x_{h,t} + v_{h,t} + \eta_{h,t}$$

$\alpha_c$  is a cohort fixed effect and  $x_{h,t}$  contain household characteristics including age dummies

- cannot distinguish age, time and cohort fixed effects (Ameriks and Zeldes, 2004)
- see Gomes and Smirnova (2021) for a solution using household fixed effects

$v_{h,t}$  is a **permanent** shock with systematic and idiosyncratic components:

$$v_{h,t} = \xi_t + z_{ht}$$

Systematic

 $\xi_t = \xi_{t-1} + \chi_t$

Idiosyncratic

 $z_{h,t} = z_{h,t-1} + \theta_{h,t}$

**All shocks  
are normal**

$\eta_{h,t}$  is an **idiosyncratic transitory** shock

# Labor Income Risk Estimation

Systematic component of permanent income risk  $\sigma_\chi$ :

- Average out the unpredictable component of labor income

$$\tilde{\xi}_t = \frac{1}{H} \sum_h v_{h,t} + \varepsilon_{h,t} = \frac{1}{H} \sum_h \ln(L_{h,t}) - \alpha_c - \beta' x_{h,t}$$

- Estimate the sample std dev of the innovations  $\tilde{\xi}_t - \tilde{\xi}_{t-1} = \chi_t$

Idiosyncratic risk:

- Consider only innovations in idiosyncratic shocks

$$v_{h,t} + \eta_{h,t} - \tilde{\xi}_t$$

- Use Carroll and Samwick ('97) to estimate permanent  $\sigma_\theta$  and transitory  $\sigma_\eta$  components of idiosyncratic income risk



# Carroll and Samwick 97

Variance of cumulative income growth innovations  $V_{h,d}$

$$V_{h,d} = \text{Var}(u_{h,t-d+1} + \dots + u_{h,t}) = d\sigma_{\theta,h}^2 + 2\sigma_{\eta,h}^2$$

where  $u_{h,t} = \ln(L_{h,t}/L_{h,t-1}) - \beta'(x_{h,t} - x_{h,t-1}) - (\tilde{\xi}_t - \tilde{\xi}_{t-1})$

- Stack the  $V_{h,d}$  of (groups/individual) households for a sufficient number of  $d \geq 2$
- Run OLS of the resulting vector  $\begin{pmatrix} V_{h,2} \\ \vdots \\ V_{h,d} \end{pmatrix}$  on  $\begin{pmatrix} 2 & 2 \\ \vdots & \vdots \\ d & 2 \end{pmatrix}$ 
  - One regression for each individual household (noisy), or for sub-samples of households, such as same education or business sector
  - $d$  can be different across households in the same group
- First coefficient estimates  $\sigma_{\theta,h}^2$ , second estimates  $\sigma_{\eta,h}^2$

# Human Capital

# Asset Valuation

- Consider an asset with returns  $R_t$  that pays until time  $T$  (note that  $T$  could be infinity). From the Euler Equation

$$u'_t(C_t^*) = E_t \left[ R_{t+1} \frac{1}{\Delta} u'_{t+1}(C_{t+1}^*) \right] \quad \text{or} \quad 1 = E_t \left[ \frac{u'_{t+1}(C_{t+1}^*)}{\Delta u'_t(C_t^*)} R_{t+1} \right]$$

- and since  $R_{t+1} = \frac{d_{t+1} + P_{t+1}}{P_t}$

$$P_t = E_t \left[ \frac{u'_{t+1}(C_{t+1}^*)}{\Delta u'_t(C_t^*)} (d_{t+1} + P_{t+1}) \right]$$

- Hence, rolling over one period

$$P_t = E_t \left[ \frac{u'_{t+1}(C_{t+1}^*)}{\Delta u'_t(C_t^*)} d_{t+1} + \frac{u'_{t+2}(C_{t+2}^*)}{\Delta^2 u'_t(C_t^*)} (d_{t+2} + P_{t+2}) \right]$$

- in which we can keep substituting  $P_{t+\tau}$  to obtain

# Stochastic Discount Factor

$$P_t = E_t \left[ \sum_{\tau=1}^{T-t} \frac{u'_{t+\tau}(C_{t+\tau}^*)}{\Delta^\tau u'_t(C_t^*)} d_{t+\tau} \right] = E_t \left[ \sum_{\tau=1}^{T-t} m_{t,t+\tau} d_{t+\tau} \right] \quad (2)$$

- where  $m_{t,t+\tau} = u'_{t+\tau}(C_{t+\tau}^*) / \Delta^\tau u'_t(C_t^*)$  is called the **stochastic discount factor** (SDF) or **marginal rate of substitution** (MRS)
- $P_t$  is the price the agent is willing to pay for asset with return  $R_t$
- $P_t$  can differ across agents as they have different optimal consumption paths  $C_t^*$  and utility functions  $u_t$
- For traded assets, market clearing insures that valuations are equalized across agents (law of one price)
- In **complete markets**, all times and states are spanned by existing assets, and the SDFs themselves are fully equalized:  $m_{t,\tau}^h = m_{t,\tau}^i$  for all individuals  $h$  and  $i$

# The Value of Human Capital

- Human capital is an asset that pays a dividend stream equal to labor income in each period  $L_t$ , hence its value is defined as

$$HC_t = E_t \left[ \sum_{\tau=1}^{T-t} m_{t,t+\tau} L_{t+\tau} \right]$$

- Since human capital is not traded its value can differ across individuals
- Part of human capital can be spanned by traded securities, i.e. we can form a portfolio of traded risky assets  $j$  so that

$$L_t = \sum_j v_j d_{j,t} + \varepsilon_t = v' d_t + \varepsilon_t$$

where  $v_j$  is the weight on asset  $j$  and  $d_{j,t}$  is the dividend of asset  $j$  at time  $t$ .  $v$  and  $d_t$  are the corresponding vectors. Note:  $E_t(\varepsilon_t) = 0$

- $v$  is estimated from an OLS regression, hence it spans optimally  $HC$ , i.e. captures as much of the variation in the labor income stream as possible (highest  $R^2$ ), and is uncorrelated with  $\varepsilon_t$

# Tangibility of Human Capital

- We can decompose the value of HC into its tradable and untradable/intangible components

$$HC_t = E_t \left[ \underbrace{\sum_{\tau=1}^{T-t} m_{t,t+\tau} v' d_{t+\tau}}_{\text{Tangible, tradable}} \right] + E_t \left[ \underbrace{\sum_{\tau=1}^{T-t} m_{t,t+\tau} \varepsilon_{t+\tau}}_{\text{Intangible, untradable}} \right]$$

**Tangible, tradable**, has value equal to the market value of the portfolio  $v$

**Intangible, untradable**, its value can differ across individuals

- The intangible component  $\varepsilon_t$  of human capital, even though has zero expectation, is valued insofar it covaries with the SDF

$$\underbrace{E_t \left[ \sum_{\tau=1}^{T-t} m_{t,t+\tau} \varepsilon_{t+\tau} \right]}_{\text{Second moment}}$$

# Estimate Mimicking Portfolio

In the CGM specification, only the systematic component  $\xi_t$  can be potentially correlated with financial assets

$\varepsilon_t$  will then contain the untradable component of systematic risk and all idiosyncratic risk, both permanent  $z_{h,t}$  and transitory  $\eta_{h,t}$

Hugget Kaplan (2011 and 2016)

- How much the stock market spans human capital?

$$\ln(L_{h,t}) = v \ln(R_t) + \varepsilon_t$$

where  $R_t$  is the real return on the S&P500 and  $L_{h,t}$  is per-capita real labor income. But the return to HC is

$$\ln(R_t^{HC}) \approx \frac{HC_t - HC_{t-1} + L_t}{HC_{t-1}}$$

- However, if  $m_{t,\tau} = m_{t,s}$  for all  $\tau, s$  and the systematic labor income shocks are stationary, then  $\ln(R_t^{HC}) = \ln(L_t/HC)$

# Background Risk

$$Var(L_t) = Var(v'd_t) + \underbrace{Var(\varepsilon_t)}_{\text{Background risk}}$$

The part of labour income risk that is not tradable cannot be hedged by trading financial assets, households are stuck with it, so it crowds out risk taking (**background risk effect**)

- If we are “endowed” with a risk that we can’t eliminate then we are less willing to take on other risks.
- Note that we do not allow the variance to vary over time

The presence of labour income risk makes investors:

- less willing to hold stocks than otherwise
- increase precautionary savings

How large should be the effect of background risk on savings and financial risk taking?

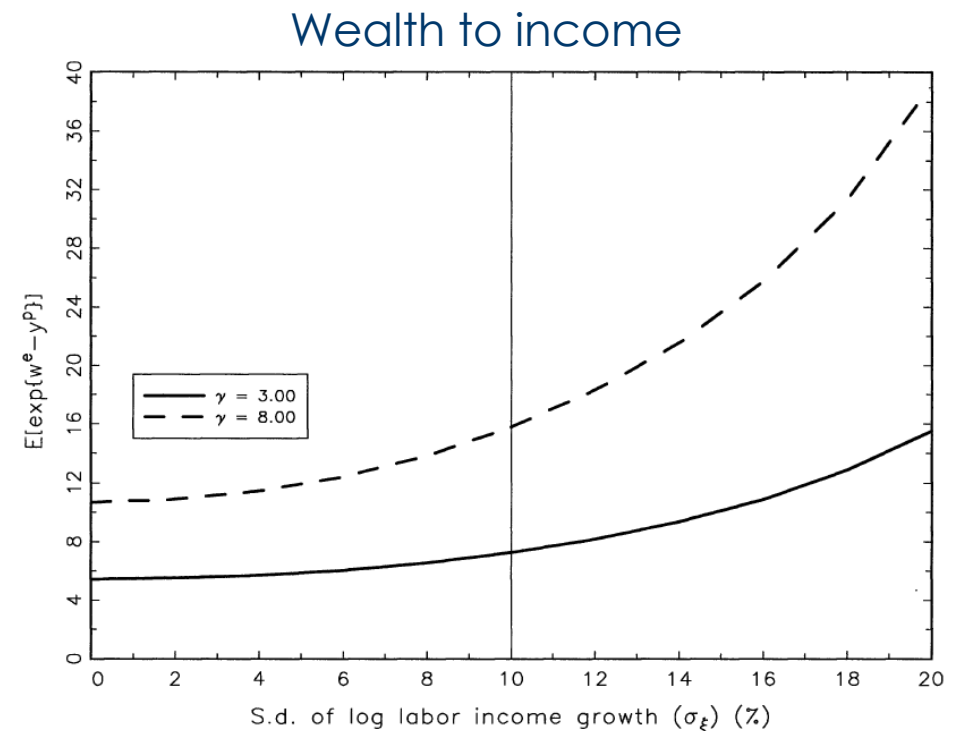
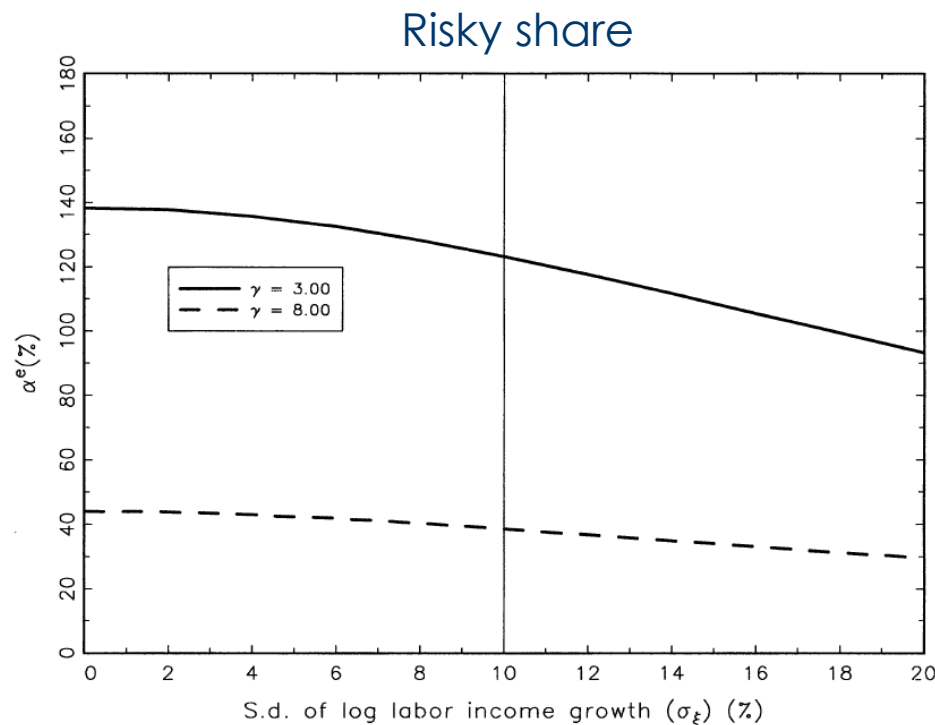


# Effect of Background Risk

Background risk has limited effect on the risky share

- Strong effect through precautionary savings

Limited effect on risk taking of increase in mean-preserving (fair) income risk with CRRA utility



Source: Viceira (2001): "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income"

# Implications

- The impact on the risky share of background risk is limited
  - Note that in the Merton model the risky share were 52% and 19.5% for  $\gamma = 3$  and 8 respectively, hence the presence of non-tradeable HC increases dramatically the risk share independently on its riskiness.
  - It remains above 1 for most of the range for  $\gamma = 3$ , and around 40% for  $\gamma = 8$
- Background risk has a strong effect on precautionary saving
  - Note that the effect of precautionary savings is stronger than the effect of EIS on wealth accumulation (since lower risk aversion implies higher EIS)

# Permanent vs Transitory Shocks

$$\varepsilon_t = \underbrace{\tilde{v}_t}_{\substack{\text{Untradable} \\ \text{permanent} \\ \text{component}}} + \underbrace{\eta_t}_{\substack{\text{Transitory} \\ \text{shock}}}$$

Only permanent shocks  $\tilde{v}_t$ , have an effect on human capital and thus should have an effect on the risky share

- Guiso, Jappelli, and Terilizzesse (1996) show that background risk reduces the risky share: one standard deviation increase in income variance reduces the risky share by 2.4 percentage points
- Angerer and Lam (2009) show that the impact of permanent income risk is larger than the impact of transitory income risk
- Fagereng Guiso and Pistaferri (2018) show that permanent income risk causally affects the portfolio risky share

# Background Risk: Current Research

- Large administrative dataset on individual income now available in the US
  - representative 10% panel sample of the U.S. population from the Master Earnings File (MEF) of the U.S. Social Security Administration
  - From 1978 to now
- Advances in understanding background risk
  - Arellano, Blundell, Bonhomme (2017)
- ... and its implications for portfolio choice
  - Galvez and Paz-Pardo (2021)
  - Azzalini (2023)

# Beta of Human Capital and Portfolio Choice

# Campbell and Viceira (2001)

$$RFW_t = \underbrace{w_t^M \times W_t}_{\text{Original Merton term}} + \underbrace{w_t^M \times HC_t}_{\text{Investment in HC}} - \underbrace{\beta^{HC} \times HC_t}_{\text{Reduction in investment in risky assets}}$$

Original Merton term, with

$$w_t^M = \frac{1}{\gamma} \frac{E_t(r_{t+1}^e)}{Var_t(r_{t+1}^e)}$$

HC increases the household balance sheet and correspondingly the investment in risky assets as for W

$\beta^{HC}$  is the beta of human capital

$$\beta^{HC} = \frac{Cov(r_t^{HC}, r_t^e)}{Var(r_t^e)}$$

where  $r_t^{HC} = (HC_t - HC_{t-1} + L_t)/HC_{t-1}$

The impact of HC on the investment in risky assets is reduced by the extent it commoves with risky asset returns

- $\beta^{HC} = 0$  : HC affects risky holdings as W

Temporary income shocks  $\varepsilon_t$  have little impact on HC and thus on its return. HC returns are instead highly affected by permanent income shocks.

Idiosyncratic income shocks, permanent or temporary, do not commove with the return on the market  $r_t^e$  and thus do not affect  $\beta^{HC}$

	Idiosyncratic	Systematic
Permanent	✗	✓
Transitory	✗	✗

# Measuring the Beta of $HC$

Define  $H_{t-1,ht}$  as the certainty equivalent of  $H_{h,t}$  in period  $t - 1$

- agent  $h$  is indifferent between receiving either the deterministic value  $H_{t-1,ht}$  or the uncertain value of his human capital  $H_{h,t}$ .

For an agent with CRRA preferences and risk aversion  $\gamma$ :

$$H_{t-1,ht} \approx \bar{H}_{h,t} - \frac{\gamma}{2} \frac{Var_{t-1}(H_{h,t})}{W_{h,t} + \bar{H}_{h,t}} + \frac{\gamma(\gamma + 1)}{6} \frac{Skew_{t-1}(H_{h,t})}{(W_{h,t} + \bar{H}_{h,t})^2}$$

where  $\bar{H}_{h,t} = E_{t-1}(H_{h,t})$ . Then (dropping subscripts):

$$\beta^{HC} = \frac{Cov\left(\frac{\Delta \bar{H}}{\bar{H}}, r^e\right)}{\sigma_r^2} - \frac{\gamma}{2} \omega_H \frac{Cov(\Delta Var(\tilde{\varepsilon}), r^e)}{\sigma_r^2} + \frac{\gamma(\gamma + 1)}{6} \omega_H^2 \frac{Cov(\Delta Skew(\tilde{\varepsilon}), r^e)}{\sigma_r^2}$$

where  $\omega_H = \frac{\bar{H}_{h,t}}{W_{h,t} + \bar{H}_{h,t}}$  and  $\tilde{\varepsilon} = \frac{H_{h,t}}{\bar{H}_{h,t}}$

Note: proofs in Catherine, Sodini and Zhang (forthcoming)

# Measuring the Beta of $HC$

Since  $HC$  is difficult to estimate, academics have proxied the return on  $HC$  with the labor income innovations growth rate

$$g_t^L = \Delta u_t / u_{t-1}$$

First moment: covariance  $\beta^{HC} = \frac{Cov(g_t^L, r_t^e)}{Var(r_t^e)}$

- In the data  $Cov(g_t^L, r_t^e) \approx 0$ , hence  $HC$  has a huge effect on financial risk taking for young households

**Participation Puzzle:** portfolio choice models cannot reconcile why young households participate so little (Cocco, Gomes and Menhout, 2005)





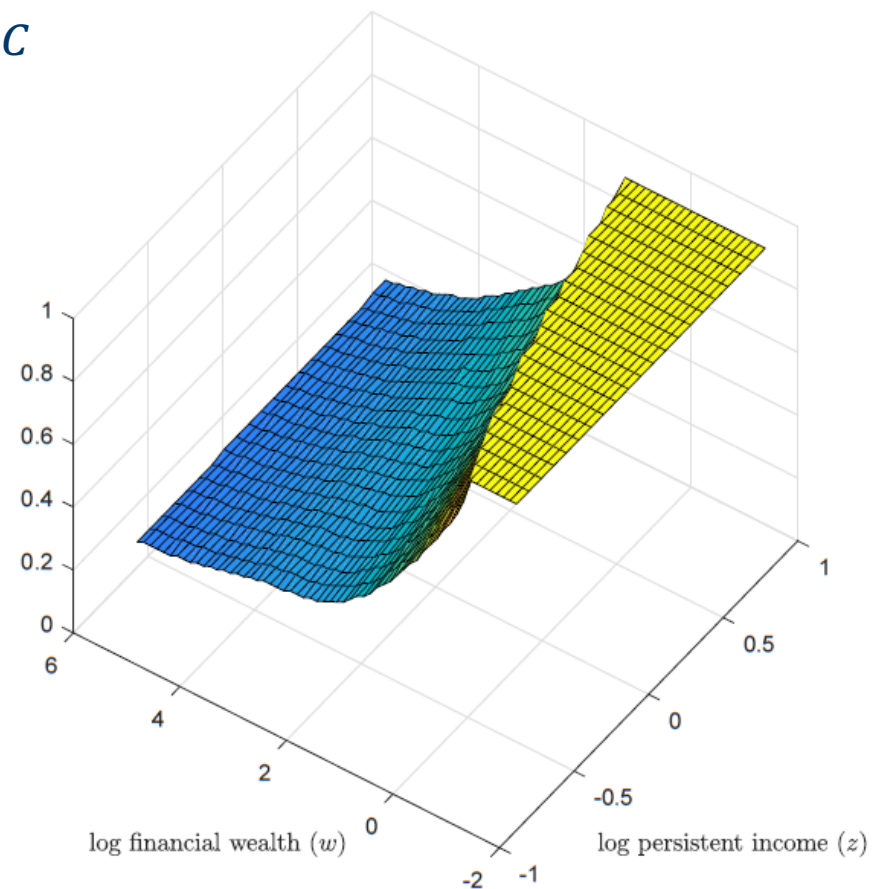
# Risk Taking and Financial Wealth

$$w_t = \left(1 + \frac{HC_t}{W_t}\right) w_t^M - \frac{HC_t}{W_t} \beta_{HC}$$

Financial wealth reduces the optimal equity share for reasonable calibrations

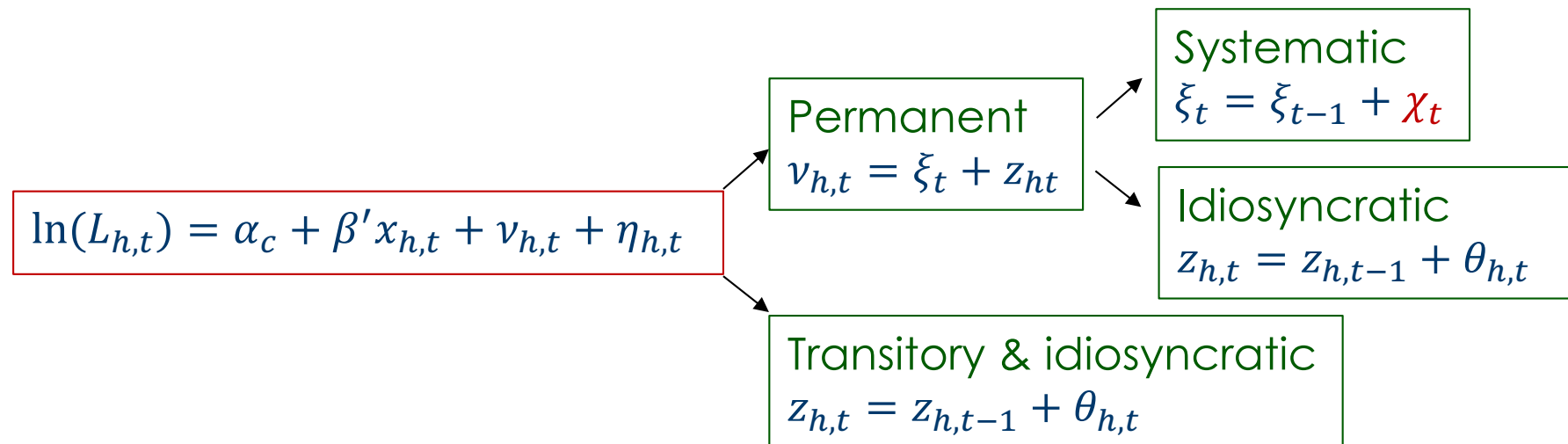
Source: Catherine (2018)  
“Countercyclical labor income risk and portfolio choices of the life-cycle”.

Optimal equity share at 40



# Estimation of $\beta_{HC}$

- Benzoni et al. (2007) shows that the cointegration between stock market and per capital labor income is strong
- Bagliano et al. (2021) proposes a new methodology to estimate  $\beta_{HC}$  and find it is substantial

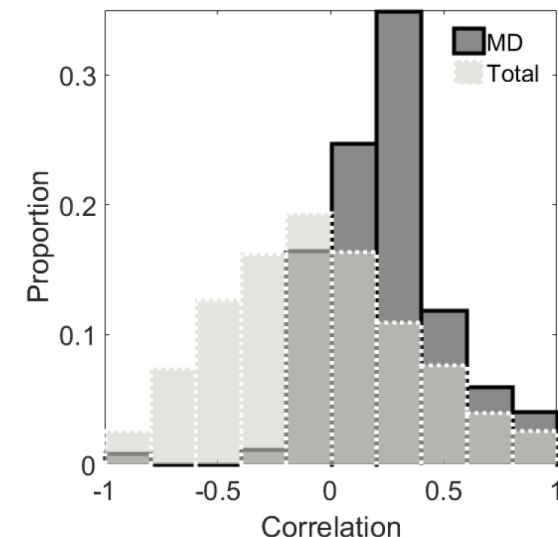


- Define  $e_{h,t} = v_{h,t} + \eta_{h,t}$
- Assume  $\chi_t = \beta_{HC} R_{Mt}^e \Rightarrow cov(\Delta e_{h,t}, \Delta e_{k,t}) = \sigma_{R_M}^2 \beta_{HC}$ , for all  $h, k$
- Additional  $N \times N$  moment conditions ( $N = \text{num of hh}$ )

# Evidence on $\beta_{HC}$ and hh Behavior

## Empirical evidence on **household behavior**

- Massa and Simonov (2006) find that households with higher **correlations** of their labor income with the stocks in their portfolio invest more in the stock market
- Calvet and Sodini (2014) find that  $\beta_{HC}$  does not have an effect on financial risk taking
  - Correlations overweight households with low income risk, who are less likely to care about hedging human capital
- Bonaparte et al. (2014) finds that households with higher  $\beta_{HC}$  participate less and have a lower risky share
- Bagliano et al. (2021) find that the distribution of the correlations of permanent labor income and the stock market is not centered around 0 and that household hedge permanent income risk.
- They cluster households into 80 groups by education, gender, urban/rural area, and high/low: risk aversion and financial literacy
- MD: estimate from their model, Total: sample correlation

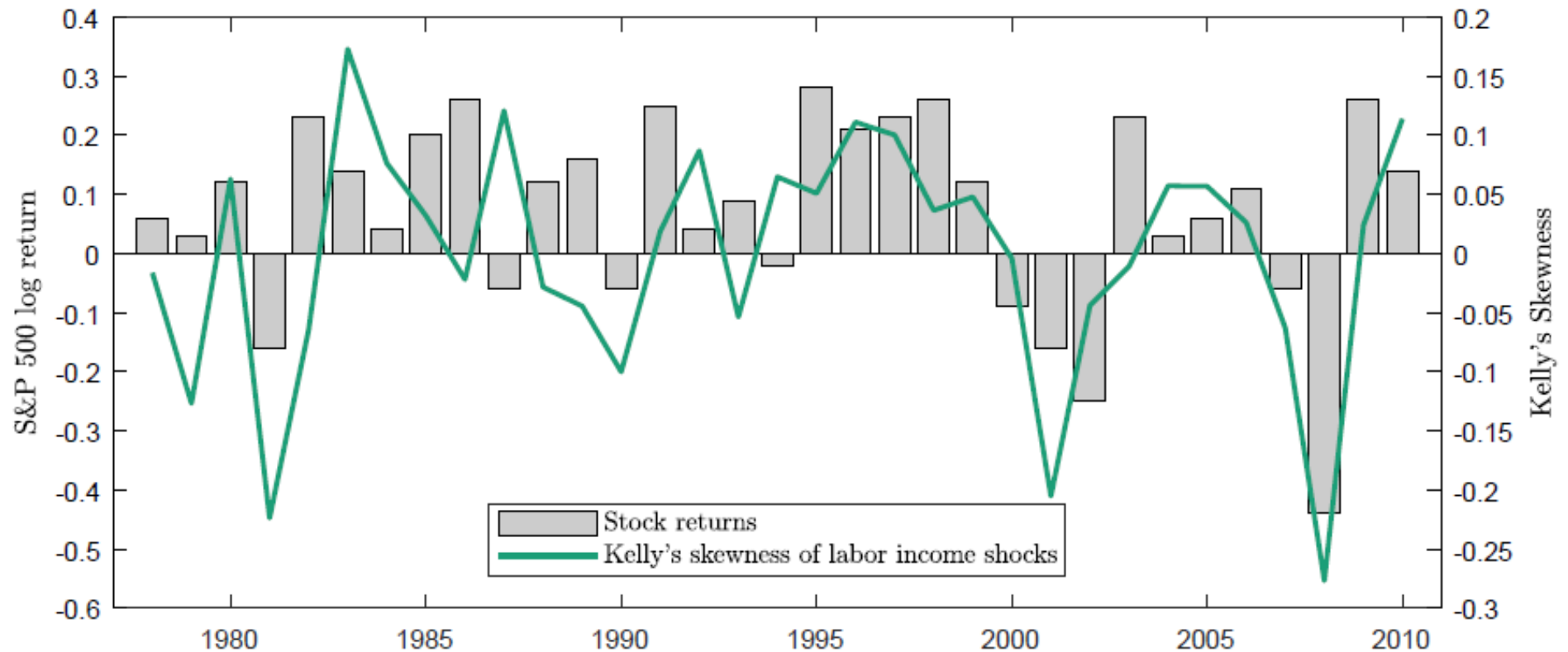


# Measuring the Beta of HC

- Second moment: **counter-cyclical variance**  $\beta^{HC} = \frac{Cov(Var(g_t^L), r_t^e)}{Var(r_t^e)}$ 
  - The dispersion of labor income outcomes  $Var(g_t^L)$  increases significantly in bad times when measured with the PSID survey (Lynch and Tan, 2011)
  - However Guvenen et al. (2014) finds this evidence weak once administrative data is used
- Third moment: **cyclical Skewness**  $\beta^{HC} = \frac{Cov(Skew(g_t^L), r_t^e)}{Var(r_t^e)}$ 
  - Tail risk (skewness) is highly correlated with stock market outcomes (Guvenen et al. 2014)
    - During bad times layoffs are more common
    - During good times bonuses and wage raises are more likely to happen
  - The  $\beta^{HC}$  is large and can rationalize why young households stay out or invest little in financial markets (Catherine, 2022) solving the participation puzzle

# HC Skewness and Stocks

## Skewness of income shocks and stock returns in the US



Skewness:  $\frac{(d_9 - d_5) - (d_5 - d_1)}{d_9 - d_1}$ , where  $d_i$  is  $i^{\text{th}}$  decile of log change in wage.

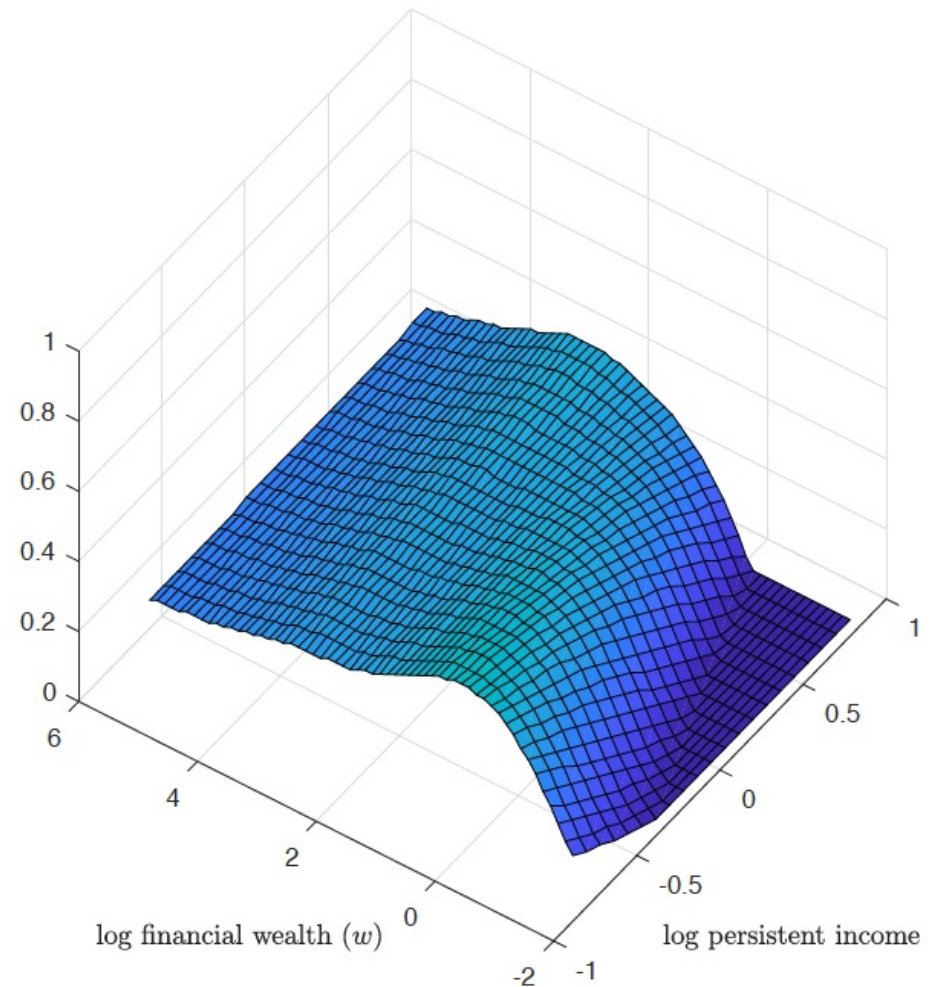
Source: Catherine (2020) "Countercyclical labor income risk and portfolio choices of the life-cycle".

# Risk Taking and Financial Wealth

With cyclical skewness, financial wealth increases the optimal equity share at least for low to moderate levels of financial wealth

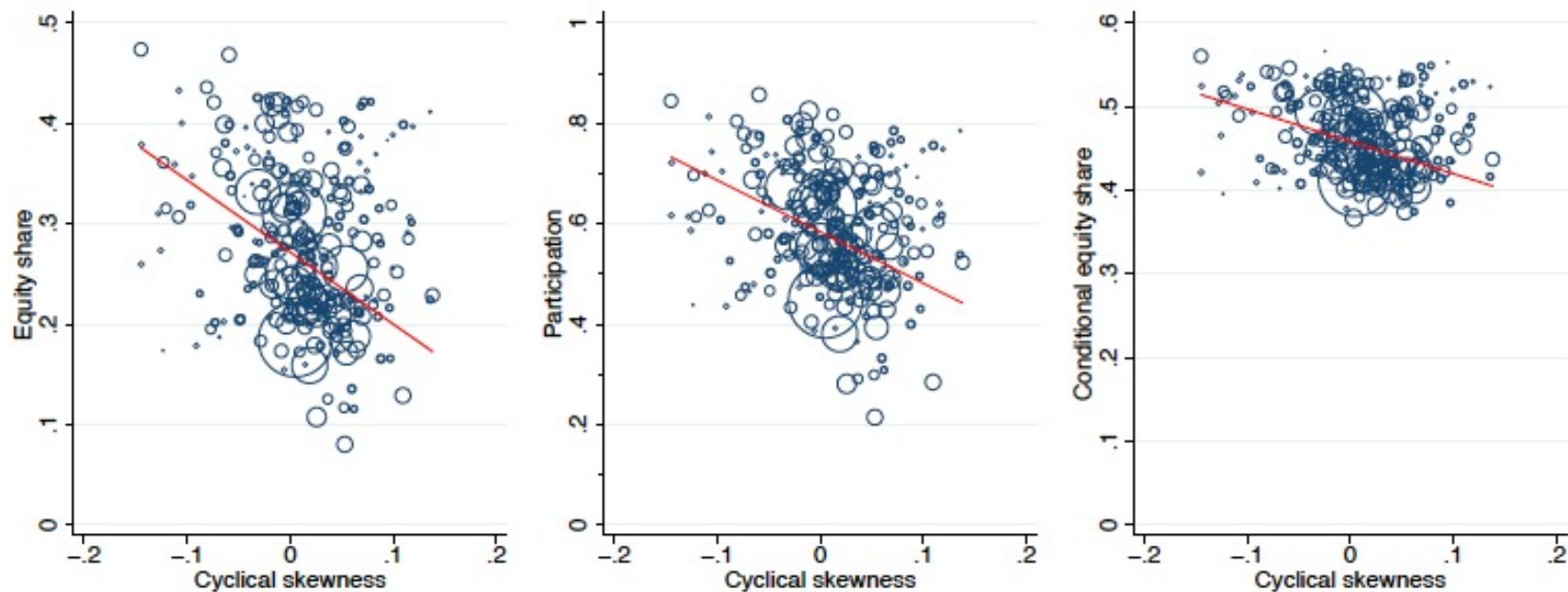
Source: Catherine (2022)  
“Countercyclical labor income risk and portfolio choices of the life-cycle”.

Optimal equity share at 40



# Cyclical Skewness: Evidence

Catherine, Sodini and Zhang (forthcoming) show that individuals with education and working in sectors with higher cyclical skewness participate less in risky financial markets and have somewhat lower conditional risky share: the effect is large!



Other measures of the beta of  $HC$  do not affect financial risk taking  
The effect of cyclical skewness declined with the ratio of  $HC/FW$  as predicted by the extended Merton formula

# Non-Standard Preferences



# Habit

With (external) habit utility  $u(C) = \frac{(C - X)^{1-\gamma}}{1-\gamma}$  the optimal risky share is

$$w_t^h = w_t^M \left( 1 - \frac{PV(X)}{W_t} \right) \quad \text{elasticity of } w_t^h \text{ wrt } W_t: \eta_t^h = \frac{d \ln(w_t^h)}{d \ln(W_t)} = \frac{y_t^h}{1 - y_t^h}$$


$PV(X)$  is the present value of the habit: the **habit liability**

- $w_t^h$  increases with liquid wealth and decreases with habit
  - wealthy households with modest habits should behave like CRRA: risk taking purely driven by risk attitude, insensitive to wealth (low elasticity)
  - poor households wrt their standards of living, take less risk than their risk preferences would dictate, they are very sensitive to changes in wealth (high elasticity)

Campbell and Cochrane (99), Chetty and Szeidl (07 and 15), Constantinides (90)

# Habit and Human Capital

If human capital is riskless, and the household resources can maintain the habit over an infinite horizon:

$$W_t + HC_t > PV(X)$$

the optimal risky share with (external) habit and (riskless) labor income is

$$w_t^{hHC} = w_t^M \left( 1 - \frac{PV(X) - HC_t}{W_t} \right) \quad \eta_t^{hHC} = \frac{d \ln(w_t^{hHC})}{d \ln(W_t)} = \frac{y_t^{hHC}}{1 - y_t^{hHC}}$$

Effect of liquid wealth on financial risk taking depends whether the habit liability is larger than human capital

- $PV(X) > HC_t$  risk taking increases with liquid wealth, as the household becomes less preoccupied with maintaining the habit, which is not covered by HC
- $PV(X) < HC_t$  risk taking decreases with liquid wealth, as the habit liability is covered by human capital, and the household does not tolerate too much risk in its liquid wealth as it grows compared to human capital.

# DRRA Literature

## Habit preferences

- Gomes Michaelides (2003)
- Polkovnichenko (2006)

## Not much mileage quantitatively

- habit cannot be that tight with labor income risk
- introduction of unemployment insurance might vanish the impact of habit

## DRRA

- Wachter and Yogo (2011)
- Meeuwis (2022)

# Luxury Good: Wachter and Yogo (2011)

Two consumption goods:  $C$  and  $L$  (luxury)

$$u(C, L) = \frac{v(C, L)^{1-\gamma}}{1-\gamma}, \text{ where } v(C, L) = \left( C^{1-\lambda} \frac{\alpha(1-\lambda)}{1-\phi} L^{1-\phi} \right)^{\frac{1}{1-\lambda}}$$

$\lambda \geq \phi \geq 1$ : luxury consumption is not necessary, basic consumption has higher marginal utility

Optimally, both  $C$  and  $L$  increase in total expenditure  $E = C + qL$  but the share of  $L$  also increases ( $q$  is the price of  $L$  per unit of  $C$ ).

$$RRA = \left( \frac{\gamma}{\lambda} C + \frac{\xi(1-\lambda)}{\phi(1-\phi)} qL \right) / \left( \frac{1}{\lambda} \frac{C}{E} + \frac{1}{\phi} \frac{qL}{E} \right) \left( C + \frac{1-\lambda}{1-\phi} qL \right), \text{ and } \xi = \frac{\gamma(1-\phi) + \phi - \lambda}{1-\lambda} < \gamma$$

- Poorer households consume mostly  $C \Rightarrow RRA$  is close to  $\gamma$ .
- Wealthier households consume mostly  $L \Rightarrow RRA$  is close to  $\xi$ :

**Wealthier households are less risk averse!**

# Summary

- Merton Solution
  - Model for the rich: no HC
  - No predictability and CRRA implies constant risky share as long as beliefs and risk aversion do not change
  - Participate if your risky share is large enough
- Two types of income risk
  - Uncorrelated - background risk induces mostly precautionary saving and little effect on the risky share
  - Correlated with financial markets (including the risk free bonds) - tradable component: beta of HC
- Risky share depends on the value of HC and the beta of HC
  - HC bond like
    - risky share (much) higher than Merton solution since wealth in HC is like a bond
    - Increasing in HC/W
  - HC stock like
    - Risky share lower than Merton solution since wealth in HC is like a stock
    - Decreasing in HC/W
  - Empirically, HC bond like
- Labor and financial markets
  - Cyclical skewness should impact strongly financial risk taking (not cyclical variance)
  - Cyclical skewness varies strongly across sectors
- Non-standard preferences
  - Habit (consumption commitments, minimum standards of living)
  - Luxury good