# Lecture 7: Time Series Forecasting with Machine Learning Models

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## Lecture 7: Overview

1. Time Series Forecasting with Machine Learning.
2. Applications.

# 1. Time Series

## Time Series

- Empirical economics focuses on causal inference and hypothesis testing.
- Machine learning emphasizes prediction.
- Overlap in objectives when forecasting in economics and finance.

## Time Series and Machine Learning

- Coulombe et al. (2019) discusses machine learning for time series econometrics.
- Potentially valuable tools: nonlinear models, regularization, cross validation, and alternative loss functions.
- Will focus on TensorFlow and deep learning models for time series.

**Sequential Models in Machine Learning**

- Specialized layers for neural networks to handle sequential data.
- Originally developed for natural language processing (NLP).
- Also applicable to time series contexts.
- Early example in Nakamura (2005) used neural networks for forecasting inflation.

**Dense Neural Networks**

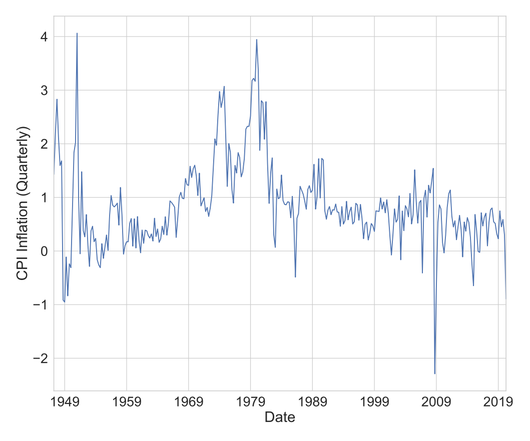
- Discussed earlier.
- Not previously adapted for sequential data.
- Forecasting exercise: Predicting quarterly inflation similar to Nakamura (2005).

## Forecasting Inflation

- Data on U.S. quarterly inflation from 1947:Q2 to 2020:Q2.
- Following Nakamura (2005), consider univariate models with lags of inflation.

# Time Series

**CPI Inflation: 1947:Q2 - 2020_Q2 (U.S. BLS)**




Source: Hull (2021).

## Pre-Processing Sequential Data

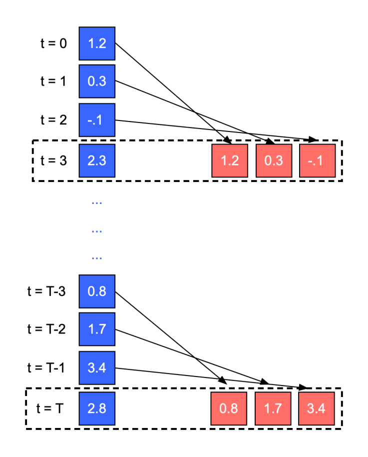
- Similar to text and image data, need to pre-process sequential data.
- Transform time series into fixed-length sequences.
- Decide on sequence length, e.g., number of lags for inputs.
- For a sequence length of three, use realizations in periods t, t-1, and t-3.

## Illustrating Pre-Processing

- Split single time series into overlapping sequences of consecutive observations.
- Use sequences to predict inflation for a single quarter ahead.

# Time Series

## Partition of Time Series




Source: Hull (2021).

## Sequence Generator for Inflation

```python
# Import packages.
import numpy as np
import tensorflow as tf
from tensorflow.keras.preprocessing.sequence import TimeseriesGenerator

# Load CSV data as pandas dataframe.
inflation = pd.read_csv('inflation.csv')

# Convert inflation column into numpy array.
inflation = np.array(inflation['Inflation'])

# Instantiate sequence generator.
generator = TimeseriesGenerator(inflation, inflation, length=4, batch_size=12)
```

## Model Training

- Using the generator object for batches of data.
- Use Keras Sequential() model.
- Sequential API for layer stacking.
- Define input, hidden, and output layers.
- Compile model with mean squared error and adam optimizer.
- Use fit_generator() for training.

## Train Neural Network with Sequences

```
# Define sequential model.
model = tf.keras.models.Sequential()

# Add input layer.
model.add(tf.keras.Input(shape=(4,)))

# Define dense layer.
model.add(tf.keras.layers.Dense(2, activation="relu"))
```

## Train Neural Network with Sequences

```
# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))

# Compile the model.
model.compile(loss="mse", optimizer="adam")

# Train the model.
model.fit_generator(generator, epochs=100)
```

## Train Neural Network with Sequences

*#Train for 25 steps*
Epoch 1/100
25/25 [==============================] - loss: 4.3247

Epoch 100/100
25/25 [==============================] - loss: 0.3816

**Model Performance**

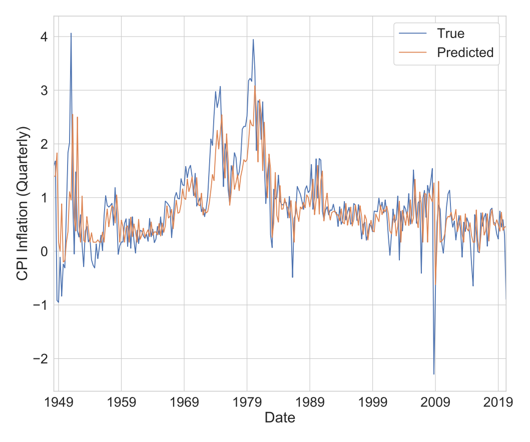
- Between epochs 1-100: Mean squared error drops from 4.32 to 0.38.
- No regularization or test sample split.
- Possible overfitting, but model has only 13 trainable parameters.

## Model Architecture Summary

```
print(model.summary())

_________________________________________________________________
Layer (type)          Output Shape          Param #
=================================================================
dense_1 (Dense)          (None, 2)            10
_________________________________________________________________
dense_1 (Dense)          (None, 1)             3
=================================================================
Total params: 13
Trainable params: 13
Non-trainable params: 0
_________________________________________________________________
```

## True Values of Inflation vs. Model's Prediction

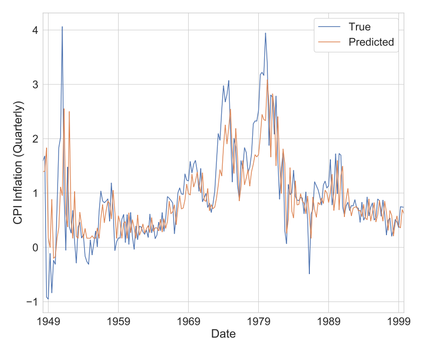



Source: Hull (2021).

## Model Evaluation

- Use model.predict_generator(generator) for predictions.
- Examine overfitting.
- Use pre-2000 values for training and post-2000 for testing.
- Construct a separate generator for predictions.

## Examining Potential Overfitting with Post-2000 Data




Source: Hull (2021).

**Recurrent Neural Networks (RNN)**

- RNN processes sequences using dense layers and recurrent layers.
- Inputs can be word vectors, musical notes, inflation measurements.
- Treatment follows Goodfellow et al. (2017).

## Recurrent Layer Structure

- Recurrent layer consists of cells.
- Each cell takes input value $x(t)$ and state $h(t-1)$.
- Produces an output value $o(t)$.

## Multiplication in RNN Cell

$$a(t) = b + Wh(t-1) + Ux(t)$$

- Take the state of the series, h(t-1).
- Multiply by weights, W.
- Take the input value, x(t), and multiply by a separate weights, U.
- Sum both terms together, along with a bias term, b.

## Activation Function in RNN Cell

$$h(t) = \mathsf{tanh}(a(t))$$

- Take output of multiplication step.
- Pass output to hyperbolic tangent activation function.
- Output is the updated state of the system, h(t).

## Output from RNN Cell

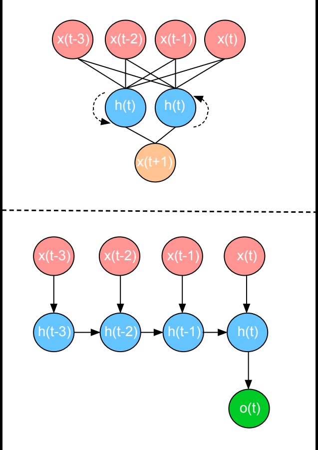
$$o(t) = c + Vh(t)$$

- Multiply updated state by a separate set of weights, V.
- Add bias term, c.

## Model Details

- In our example: inflation is the only feature.
- $x(t)$, $W$, $U$, and $V$ are scalars.
- Weights are shared across time periods.
- Only five parameters needed for one RNN cell layer.

## RNN (top) and unrolled RNN cell (bottom)




Source: Hull (2021).

**RNN Illustration**

- Pink nodes: input values (lags of inflation).
- Orange node: inflation for the next quarter (target).
- Blue nodes: individual RNN cells in an RNN layer.

**Unrolled RNN Cell**

- State combined with input to get next state.
- Last step gives output $o(t)$.
- Output is input to a final dense layer for next quarter's inflation prediction.

## RNN Characteristics

- Retains a state for sequential data.
- Updates state at each step.
- Reduces parameters through weight-sharing.
- No time-specific weights needed.
- Can handle sequences of varying length.

## Model Definition

- Define RNN model.
- Model similar in complexity to previous dense network.
- Uses ‘SimpleRNN‘ layer with 2 RNN cells.

## Sequence Generator for Inflation

```
import numpy as np
import tensorflow as tf
from tensorflow.keras.preprocessing.sequence import TimeseriesGenerator

# Load data.
inflation = pd.read_csv(data_path+'inflation.csv')

# Convert to numpy array.
inflation = np.array(inflation['Inflation'])
```

## Sequence Generator for Inflation

```
# Add dimension.
inflation = np.expand_dims(inflation, 1)

# Instantiate time series generator.
train_generator = TimeseriesGenerator(
inflation[:211], inflation[:211],
length = 4, batch_size = 12)
```

## Define RNN Model in Keras

```
# Define sequential model.
model = tf.keras.models.Sequential()

# Define recurrent layer.
model.add(tf.keras.layers.SimpleRNN(2, input_shape=(4, 1)))

# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))
```

## Define RNN Model in Keras

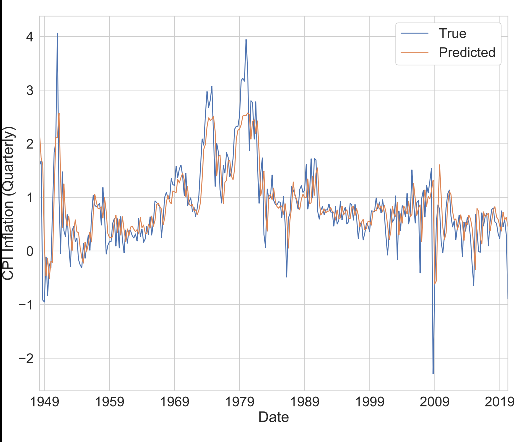
```
# Compile the model.
model.compile(loss="mse", optimizer="adam")

# Fit model to data using generator.
model.fit_generator(train_generator, epochs=100)
```

## Model Compilation and Training

- Mean squared error (MSE) is lower compared to dense network.
- Test sample performance post-2000 remains stable.

## Test Sample Performance Post-2000




Source: Hull (2021).

**Evaluate RNN Model in Keras**

```
Epoch 1/100
18/18 [==============================] - 1s 31ms/step - loss: 0.9206

Epoch 100/100
18/18 [==============================] - 0s 2ms/step - loss: 0.2594
```

## RNN vs. Dense Networks

- RNN has fewer parameters than dense networks.
- Single RNN cell layer requires 5 parameters.
- Examine architecture using ‘model.summary()‘.
- RNN model in this example: 11 parameters.
- Dense network used previously had more parameters.

## RNN Architecture in Keras

```
# Print model summary.
print(model.summary())
```

```
________________________________________________________________
Layer (type)                 Output Shape        Param #
================================================================
simple_rnn_1 (SimpleRNN)     (None, 2)           8
________________________________________________________________
dense_1 (Dense)              (None, 1)           3
================================================================
Total params: 11
Trainable params: 11
Non-trainable params: 0
________________________________________________________________
```

## RNN in Practice

- Challenges with basic RNN:
  - Vanishing gradient problem.
  - Original RNN struggles with long data sequences.
  - Doesn't account for distant temporal relationships.

## Long short-term memory (LSTM)

- RNNs suffer from vanishing gradient with long sequences.
- Solution: Use gated RNN cells.
- Two common gated RNN cells:
  - Long short-term memory (LSTM)
  - Gated recurrent units (GRUs)

**Long short-term memory (LSTM)**

- Introduced by Hochreiter and Schmidhuber (1997).
- Uses operators to limit flow of information in long sequences.

## Operations in LSTM

- Following Goodfellow et al. (2017) for LSTM operations.
- Three gates in LSTM:
  - Forget gate.
  - External input gate.
  - Output gate.
- Gates control flow of information in LSTM cell.

## Equations for LSTM Gates

- Forget gates:

$$f(t) = \sigma(b^f + W^f h(t-1) + U^f x(t))$$

- External input gates:

$$g(t) = \sigma(b^g + W^g h(t-1) + U^g x(t))$$

- Output gates:

$$q(t) = \sigma(b^q + W^q h(t-1) + U^q x(t))$$

## Internal States of LSTM

- Each gate has unique weights and biases.
- Gating procedure can be learned.
- Internal states are updated using definitions provided.
- Incorporates forget gate, external input gate, input sequence, and state.

## Equations for LSTM Gates

- Internal state:

$$s(t) = f^t s(t-1) + g(t)\sigma(b + Wh(t-1) + Ux(t))$$

- Hidden state:

$$h(t) = tanh(s(t))q(t)$$

## Use of Gates in LSTM

- Gates increase the number of model parameters.
- Significant improvement in handling long sequences.
- LSTM is typically the baseline ML model in time series analysis.

## LSTM Model Definition and Training

- Use tf.keras.layers.LSTM() instead of tf.keras.layers.SimpleRNN().
- Train for 100 epochs.
- Mean squared error is higher for LSTM than RNN after 100 epochs.
  - More weights in LSTM require more training epochs.
- Benefit from LSTM grows in sequence length.

## Train an LSTM Model in Keras

```
# Define sequential model.
model = tf.keras.models.Sequential()

# Define recurrent layer.
model.add(tf.keras.layers.LSTM(2, input_shape=(4, 1)))

# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))
```

## Train an LSTM Model in Keras

```
# Compile the model.
model.compile(loss="mse", optimizer="adam")

# Train the model.
model.fit_generator(train_generator, epochs=100)
```

## Train an LSTM Model in Keras

Epoch 1/100
18/18 [==============================] - 1s 62ms/step - loss: 3.1697

Epoch 100/100
18/18 [==============================] - 0s 3ms/step - loss: 0.5873

## LSTM Model's Architecture

- LSTM cells introduce additional operations:
  - Forget gate
  - External input gate
  - Output gate
- Each gate requires its own set of parameters.
- LSTM layer uses 32 parameters, which is 4 times as many as the RNN.

## Summarize LSTM Architecture in a Keras Model

```
# Print model architecture.
print(model.summary())
```

```
_________________________________________________________________
Layer (type)          Output Shape         Param #
=================================================================
lstm_1 (LSTM)           (None, 2)           32
_________________________________________________________________
dense_1 (Dense)         (None, 1)            3
=================================================================
Total params: 35
Trainable params: 35
Non-trainable params: 0
_________________________________________________________________
```

## Intermediate Hidden States

- By convention, LSTM only uses the final value of the hidden state.
- For instance:
  - Model uses $h(t)$.
  - Does not use $h(t-1), h(t-2), h(t-3)$.

## Use of Intermediate Hidden States

- Recent research shows benefits from using intermediate hidden states.
- Especially effective for modeling long-term dependencies in NLP.
- Typically done within attention models.

## Incorporating Hidden States in LSTM

- Modify LSTM cells to return hidden states.
- Set return_sequences to True.

## Code Implementation

```
# Modify LSTM to return hidden states
LSTM(return_sequences=True)
```

- After modification, we can review model's architecture using the summary() method.

## Code Implementation

```
# Define sequential model.
model = tf.keras.models.Sequential()

# Define recurrent layer to return hidden states.
model.add(tf.keras.layers.LSTM(2, return_sequences=True, input_shape=(4, 1)))

# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))

# Summarize model architecture.
model.summary()
```

## Code Implementation

```
Layer (type)          Output Shape          Param #
=================================================================
lstm_1 (LSTM)         (None, 4, 2)          32

dense_1 (Dense)       (None, 4, 1)          3
=================================================================
Total params: 35
Trainable params: 35
Non-trainable params: 0
```

## **Unusual Model Architecture**

- Model outputs a 4x1 vector, not a scalar prediction for each observation.
- This is due to the LSTM layer:
  - Outputs 4x1 vectors from its two LSTM cells.

## Making Use of LSTM Output

- Several methods to incorporate the LSTM output.
- Example: **Stacked LSTM** (Graves et al., 2013).
  - Pass full sequence hidden states to a second LSTM layer.
  - Adds depth to network for multiple levels of representation.

## Stacked LSTM Implementation

- Define a model with two LSTM layers.
- First LSTM layer:
  - Three LSTM cells.
  - Input shape of (4,1).
  - Set return_sequences to True.
  - Outputs 4x1 sequence of hidden states.

## Second LSTM Layer

- Accepts 3-tensor (4x1x3) from the first LSTM layer.
- Contains two cells.
- Only returns the final hidden states.
- Does not return intermediate state values.

## Define a Stacked LSTM Model

```python
# Define sequential model.
model = tf.keras.models.Sequential()

# Define recurrent layer to return hidden states.
model.add(tf.keras.layers.LSTM(3, return_sequences=True, input_shape=(4, 1)))

# Define second recurrent layer.
model.add(tf.keras.layers.LSTM(2))

# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))
```

## Summarize Stacked LSTM Architecture

```
# Summarize model architecture.
model.summary()
```

```
_________________________________________________________________
Layer (type)           Output Shape            Param #
=================================================================
lstm_1 (LSTM)          (None, 4, 3)            60
_________________________________________________________________
lstm_2 (LSTM)          (None, 2)               48
_________________________________________________________________
dense_1 (Dense)        (None, 1)               3
=================================================================
Total params: 111
Trainable params: 111
Non-trainable params: 0
_________________________________________________________________
```

**Focus and Context**

- Apply forecasting methods.
- Based examples on univariate inflation forecasting (Nakamura, 2005).
- All methods applicable to multivariate settings.

## Multivariate Forecasting

- Use both LSTM model and Gradient Boosted Trees.
- Forecast inflation monthly.
- Use five features instead of one.

## Data Preview and Model Implementation

- Prepare data and define model.
- Perform multivariate forecast using:
  - LSTM
  - Gradient boosted trees

## Features Details

- New features added:
  - Unemployment (measured in first differences).
  - Hours worked in manufacturing.
  - Hourly earnings in manufacturing.
  - Money supply measure (M1).
- Level variables transformed using percentage changes from previous period.

## Load and Preview Inflation Forecast Data

```
import pandas as pd

# Load data.
macroData = pd.read_csv(data_path+'macrodata.csv', index_col = 'Date')

# Preview data.
print(macroData.round(1).tail())
```

| Date | Inflation | Unemp | Hours | Earnings | M1 |
|---|---|---|---|---|---|
| 12/1/19 | -0.1 | 0.1 | 0.5 | 0.2 | 0.7 |
| 01/1/20 | 0.4 | 0.6 | -1.7 | -0.1 | 0.0 |
| 02/1/20 | 0.3 | -0.2 | 0.0 | 0.4 | 0.8 |
| 03/1/20 | -0.2 | 0.8 | -0.2 | 0.4 | 6.4 |
| 04/1/20 | -0.7 | 9.8 | -6.8 | 0.5 | 12.9 |

## LSTM Overview

- Recall data preparation for LSTM:
  - Instantiate a generator.
  - Convert target and features to np.array() objects.

## Data Generators

- Two data generators:
  - Training data
  - Test data

## Sequence Lengths

- Previous setup:
  - Quarterly data
  - 4-quarter sequence lengths
- Current setup:
  - Monthly data
  - 12-month sequence lengths

## Prepare Data for Use in LSTM Model

```
import numpy as np
import tensorflow as tf
from tensorflow.keras.preprocessing.sequence import TimeseriesGenerator

# Define target and features.
target = np.array(macroData['Inflation'])
features = np.array(macroData)

# Define train generator.
train_generator = TimeseriesGenerator(features[:393],
            target[:393], length = 12, batch_size = 6)
```

## Prepare Data for Use in LSTM Model

```
# Define test generator.
test_generator = TimeseriesGenerator(features[393:],
                 target[393:], length = 12, batch_size = 6)
```

## Generators Defined

- Train model.
- Model Specifications:
  - Two LSTM cells.
  - Input shape adjusted:
    - 12 elements in each sequence.
    - Five features.

## Training Outcome

- Concerns in macroeconomic forecasting:
  - Longer sequences and more variables.
  - Typically, concerns about number of parameters.

## Model Parameters

- Despite longer sequence length:
  - Doesn't increase the number of parameters.
- Model's total parameters:
  - Only 67 parameters.

## Load and Preview Inflation Forecast Data

```
# Define sequential model.
model = tf.keras.models.Sequential()

# Define LSTM model with two cells.
model.add(tf.keras.layers.LSTM(2, input_shape=(12, 5)))

# Define output layer.
model.add(tf.keras.layers.Dense(1, activation="linear"))

# Compile the model.
model.compile(loss="mse", optimizer="adam")
```

## Load and Preview Inflation Forecast Data

```
# Train the model.
model.fit_generator(train_generator, epochs=100)

Epoch 1/20
64/64 [==============================] - 2s 26ms/step - loss: 0.3065

Epoch 20/20
64/64 [==============================] - 0s 6ms/step - loss: 0.0663
```

## Use MSE to Evaluate Train and Test Sets

```
# Evaluate training set using MSE.
model.evaluate_generator(train_generator)

0.06527029448989197

# Evaluate test set using MSE.
model.evaluate_generator(test_generator)

0.15478561431742632
```

**Model Evaluation**

- Compare training sample results to test sample results.
- Training set performance generally better than test set.
- Not uncommon, but indicative of overfitting.

## Addressing Performance Gap

- If gap between training and test set performance is large:
  - Consider regularization.
  - Consider terminating training earlier.
  - Reduce number of epochs.

## Gradient Boosted Trees

- Compare GBT to deep learning.

## Data Preparation for Gradient Boosting

- Similar to LSTMs, data splitting into sequences is required.
- In TensorFlow, GBT trained using Estimator API.
- Must define feature columns for each of the five features.

## Data Generation Functions

- Define functions to generate data for both training and testing.
- Evaluate overfitting, analogous to the LSTM example.
- Sample split:
  - Train set: years before 2000.
  - Test set: years post-2000.

## Use MSE to Evaluate Train and Test Sets

```
# Define lagged inflation feature column.
inflation = tf.feature_column.numeric_column("inflation")

# Define unemployment feature column.
unemployment = tf.feature_column.numeric_column("unemployment")

# Define hours feature column.
hours = tf.feature_column.numeric_column("hours")
```

## Use MSE to Evaluate Train and Test Sets

```
# Define earnings feature column.
earnings = tf.feature_column.numeric_column("earnings")

# Define M1 feature column.
m1 = tf.feature_column.numeric_column("m1")

# Define feature list.
feature_list = [inflation, unemployment, hours, earnings, m1]
```

## Define the Data Generation Functions

```
# Define input function for training data.
def train_data():
    train = macroData.iloc[:392]
    features = {"inflation": train["Inflation"],
    "unemployment": train["Unemployment"],
    "hours": train["Hours"],
    "earnings": train["Earnings"],
    "m1": train["M1"]}
    labels = macroData["Inflation"].iloc[1:393]
    return features, labels
```

## Define the Data Generation Functions

```
# Define input function for test data.
def test_data():
    test = macroData.iloc[393:-1]
    features = {"inflation": test["Inflation"],
    "unemployment": test["Unemployment"],
    "hours": test["Hours"],
    "earnings": test["Earnings"],
    "m1": test["M1"]}
    labels = macroData["Inflation"].iloc[394:]
    return features, labels
```

## Train and Evaluate Model

```
# Instantiate boosted trees regressor.
model = tf.estimator.BoostedTreesRegressor(feature_columns =
    feature_list, n_batches_per_layer = 1)

# Train model.
model.train(train_data, steps=100)

# Evaluate train and test set.
train_eval = model.evaluate(train_data, steps = 1)
test_eval = model.evaluate(test_data, steps = 1)

# Print results.
print(pd.Series(train_eval))
print(pd.Series(test_eval))
```

## Train and Evaluate Model

```
average_loss         0.010534
label/mean           0.416240
loss                 0.010534
prediction/mean      0.416263
global_step        100.000000
dtype: float64

average_loss         0.145123
label/mean           0.172864
loss                 0.145123
prediction/mean      0.286285
global_step        100.000000
dtype: float64
```

**ML in Economics and Finance**

- ML focuses on prediction.
- Economics and finance often aim for causal inference and hypothesis testing.
- Overlap in fields: forecasting.

## Time Series Forecasting with ML

- Emphasis on deep learning models.
- Gradient boosted trees in TensorFlow.
- Early use of neural network in economics for forecasting: Nakamura (2005).
- Modern models: RNNs, LSTMs, stacked LSTMs.
- Applications extend to areas like NLP.

## Further Reading

- Macroeconomic time series forecasting: Cook and Hall (2017).
- Stock return & bond premium forecasting: Heaton et al. (2016), Messmer (2017), Rossi (2018), and Chen et al. (2019).
- High-dimensional time series regression and nowcasting: Babii et al. (2019, 2020).

# 2. Applications

# Applications

**Colab Tutorial**

- Time Series Prediction

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