Resource Reallocation with Carbon Emission Policies

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Motivation

• Climate crisis intensifies: rising temperatures, extreme weather.

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- Government interventions steer markets towards sustainability.
- Key policies: carbon pricing, renewable subsidies to curb emissions.
- Economic impacts:
 - Limitation in fossil fuel usage.
 - Adoption of renewable technologies.

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- Economic impacts:
 - Limitation in fossil fuel usage.
 - Adoption of renewable technologies.
 - Reallocation of resources to greener firms/industries.

Research Question

- What is the Economic Outcomes of environmental policies due to resources reallocation?
 - Industry output
 - Firm-level productivity
 - Sector size
 - Emission intensity
 - Total Emission

Literature and Contribution

- Effectiveness of Carbon policies:
 - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)

Contribution: Quantify substitution between green and brown capital

Literature and Contribution

- Effectiveness of Carbon policies:
 - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)
 Contribution: Quantify substitution between green and brown capital
- Misallocation:
 - Whited and Zhao (2021); Hsieh and Klenow (2009); Ai, Li, and Yang (2020); Asker, Collard-Wexler, and De Loecker (2014)
 Contribution: Misallocation (Reallocation) in the context of environmental policies

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 Contribution: Misallocation (Reallocation) in the context of environmental policies
- Climate Policy Design:
 - Acemoglu, Gancia, and Zilibotti (2012); Acemoglu et al. (2016); Oehmke and Opp (2023)
 Contribution: Assess alternative instruments in Emission Intensity / resource
 reallocation trade off

Road map

- Develop Economic model with Emission
- Characterize the allocation of resources
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Oiscuss the cost of the environmental policies

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- 🧿 Compare the optimal Policy with resource reallocation 🛦
- Oiscuss the cost of the environmental policies A

Standard Framework

Hsieh and Klenow (2009)

- Heterogeneous monopolistic competitive firms
- Partial equilibrium
- Cobb-Douglas Production function
- CES aggregator for output
- Normal aggregation of emissions

Extension

Production functions

• Industry s, firm i:

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}$$

• \hat{A}_{si} : total factor of productivity

Firm's profit



Extension

Production functions

• Industry s, firm i:

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} \quad , \qquad \hat{K} = \left(\alpha_s G_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1-\alpha_s) B_{si}^{\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{\gamma_s}{\gamma_s-1}}$$

$$E_{si} = \tilde{A}_{si}B_{si}$$

Emission General Model

- \hat{A}_{si} : total factor of productivity
- ullet α_s : importance of Green capital in the production
- ullet γ_s : elasticity of substitution between Green and Brown capital
- \tilde{A}_{si} : emission inefficiency
- Firms maximize over G, B, and L

Firm's profit



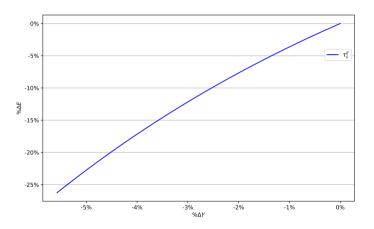
Estimation / Calibration

• I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Source/Moment	
Panel A: Estimation		
α_s	β_G/β_B	
β_s	β_G/β_L	
σ_{s}	WL/PY	
γ_s	$\beta_{GB}/\beta_{B}\beta_{G}$	
$Mean(\log(\hat{A}_{si}))$	$Mean(L_{si})$	
$Sd(\log(\hat{A}))$	$Sd(L_{si})$	
$Mean(\log(\tilde{A}_{si}))$	Mean(E/PY)	
$Sd(\log(\tilde{A}))$	Sd(E/PY)	
$Corr(\log(\hat{A}), \log(\tilde{A}))$	Corr(PY, E/PY)	
Panel B: Additional Mome	nts	
$\left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_s^B)r_{ti}+\tau_s^E\tilde{\Lambda}}{(1+\tau_s^G)r_{ti}}\right)^{\gamma_s}$	$z_k = G/B$	
$\frac{1-\beta}{\beta}\frac{1}{\alpha}\left(\alpha_s+(1-\alpha_s)z_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{\gamma_s-1}}\frac{(1+r_s^C)r_{si}}{(1+r_s^C)w_{si}}$	$z_l = L/K$	
$\frac{1}{\gamma}\frac{\ddot{A}}{r^B+\tau_E\ddot{A}}Z_k$	$\partial z_k/\partial \tau_E$	
$\frac{\vec{A}/\gamma}{r^B + \tau_E \vec{A}} \frac{1}{\frac{\alpha}{1 - \alpha} z_k^{\gamma - 1} + 1} Z_I$	$\partial z_I/\partial \tau_E$	
	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$	
Panel C: Calibration		
r	5%	
w	500 TSEK	

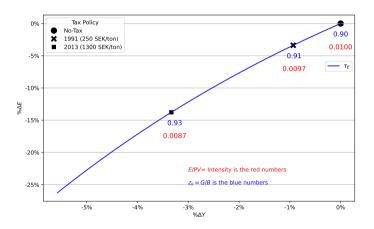
Emission and Production

Results



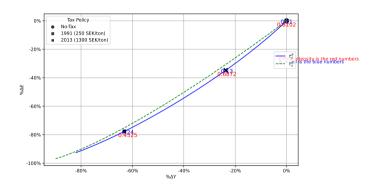
Emission and Production

Results



Emission and Production

Results



Carbon Intensity and Tax



Future Steps

- Develop Economic model with Emission
 - Firms could R&D
 - Add Household and Government
 - Firms could enter and exit the market
- 2 Characterize the allocation of resources
- Provide a definition of Green and Brown capital
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Objective the cost of the environmental policies

Thank you!

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Emission General Model

• The firm's emission is:

$$E_{si} = \tilde{A}_{si} \tilde{K}_{si}^{\theta_s} L_{si}^{1-\theta_s} \quad , \qquad \tilde{K} = (\mu_s G_{si}^{\frac{\eta_s - 1}{\eta_s}} + (1 - \mu_s) B_{si}^{\frac{\eta_s - 1}{\eta_s}})^{\frac{\eta_s}{\eta_s - 1}}$$

$$\pi_{si} = (1 + \tau_{si}^p) P_{si} Y_{si} - ([(1 + \tau_{G_s}) r_{si} G_{si} + (1 + \tau_{B_s}) r_{si} B_{si} + (1 + \tau_{I_s}) w_{si} I_{si}] + \tau_E E_{si})$$





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si}$$

- where
 - τ_s^p is the tax / Demand preference for the firm





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left((1 + \tau_s^G) r_s G_{si} \right)$$

- where
 - τ_s^p is the tax / Demand preference for the firm
 - ullet au_s^G is the Green capital subsidy / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s})P_{si}Y_{si} - \left((1 + \frac{\tau_s^G}{s})r_sG_{si} + (1 + \frac{\tau_s^B}{s})r_sB_{si}\right)$$

- where
 - τ_s^p is the tax / Demand preference for the firm
 - τ_s^G is the Green capital subsidy / ESG preference of Financier
 - τ_s^B is the Brown capital tax / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si} - \left((1 + \frac{\tau_s^G}{s}) r_s G_{si} + (1 + \frac{\tau_s^B}{s}) r_s B_{si} + (1 + \frac{\tau_s^W}{s}) w_{si} I_{si} \right)$$

- where
 - τ_s^p is the tax / Demand preference for the firm
 - τ_s^G is the Green capital subsidy / ESG preference of Financier
 - τ_s^B is the Brown capital tax / ESG preference of Financier
 - \bullet au_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left((1 + \tau_s^G) r_s G_{si} + (1 + \tau_s^B) r_s B_{si} + (1 + \tau_s^W) w_{si} I_{si} \right) - \tau_s^E E_{si}$$

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$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left((1 + \tau_s^G) r_s G_{si} + (1 + \tau_s^B) r_s B_{si} + (1 + \tau_s^W) w_{si} I_{si} \right) - \tau_s^E E_{si}$$

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 - τ_s^p is the tax / Demand preference for the firm
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 - τ_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)
- The firm chooses the optimal capital and labor to minimize the cost of production and then chooses the price level to maximize the profit





$$\max_{G_{si},B_{si},L_{si}} \quad -Cost \quad ext{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{eta_s}L_{si}^{1-eta_s} = ar{Y}_{si}$$





$$egin{array}{ll} \max_{G_{si},B_{si},L_{si}} & -Cost \quad ext{s.t.} & \hat{A}_{si}\hat{K}_{si}^{eta_s}L_{si}^{1-eta_s} = ar{Y}_{si} \ & rac{G_{si}}{B_{si}} = z_{si}^k \end{array}$$

$$\begin{aligned} &\max_{G_{si},B_{si},L_{si}} &-Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ &\frac{G_{si}}{B_{si}} = \mathsf{z}_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_s^B)r_s + \tau_s^E\tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$





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$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{l}$$





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$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{\prime} = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)\mathbf{z}_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^{\mathbf{G}})r_s}{(1+\tau_s^{\mathbf{W}})w_{si}}$$



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$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{l} = \frac{1 - \beta}{\beta} \frac{1}{\alpha_{s}} \left(\alpha_{s} + (1 - \alpha_{s}) \mathbf{z}_{si}^{k - \frac{\gamma_{s} - 1}{\gamma_{s}}} \right)^{\frac{1}{1 - \gamma_{s}}} \frac{(1 + \tau_{s}^{G}) r_{s}}{(1 + \tau_{s}^{W}) w_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_{s} \mathbf{z}_{si}^{k \gamma_{s} - 1} + (1 - \alpha_{s}) \right)^{\frac{\gamma_{s}}{1 - \gamma_{s}}} \mathbf{z}_{si}^{l 1 - \beta} \bar{Y}_{si}$$



$$\begin{aligned} &\max_{G_{si},B_{si},L_{si}} &- \textit{Cost} \quad \text{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ &\frac{G_{si}}{B_{si}} = z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$

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$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_{s} z_{si}^{k \gamma_{s} - 1} + (1 - \alpha_{s}) \right)^{\frac{\gamma_{s}}{1 - \gamma_{s}}} z_{si}^{l 1 - \beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}$$



$$\begin{aligned} \max_{G_{si},B_{si},L_{si}} &-Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ \frac{G_{si}}{B_{si}} &= z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \\ \frac{L_{si}}{\hat{K}_{si}} &= z_{si}^l = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)z_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^G)r_s}{(1+\tau_s^W)w_{si}} \\ E_{si} &= \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k\gamma_s-1} + (1-\alpha_s)\right)^{\frac{\gamma_s}{1-\gamma_s}} z_{si}^{l-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si} \end{aligned}$$

• Firm will then charge markup over the marginal cost



Optimal Allocation

$$\max \quad -Cost \quad \text{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$z_{si}^k \equiv \frac{G_{si}}{B_{si}} = \left[\frac{\alpha_s}{1-\alpha_s} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial G} Cost_{si}} \right]^{\gamma_s}$$

$$z_{si}^l \equiv \frac{L_{si}}{\hat{K}_{si}} = \frac{1-\beta_s}{\beta_s} \frac{1}{1-\alpha_s} (\alpha_s z_{si}^{k(\gamma_s-1)} + (1-\alpha_s))^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}}$$

$$= \frac{1-\beta_s}{\beta_s} \frac{1}{\alpha_s} (\alpha_s + (1-\alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial G} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\frac{\phi_{si}}{z_{si}^{l}}\right)^{\theta_{s}} z_{si}^{l} \overset{\beta_{s}}{\bar{\gamma}} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}, \quad \text{where} \quad \phi_{si} = \frac{\left(\mu_{s} + (1 - \mu_{s}) z_{si}^{k(1 - \eta_{s})}\right)^{\frac{\eta_{s}}{\eta_{s} - 1}}}{\left(\alpha_{s} + (1 - \alpha_{s}) z_{si}^{k(1 - \gamma_{s})}\right)^{\frac{\gamma_{s}}{\gamma_{s} - 1}}}$$



Sevved Morteza Aghaianzadeh (SSE)



Model

Optimal firm level price

• Now Firm need to choose the price level to maximize the profit:

$$\max_{P_{si}} \quad \pi_{si} = P_{si}F_{si} - C_{si}F_{si}$$

- Firm-level real output is a function of the sector price, firm price, and sector real output (i.e. $F_{si} = (\frac{P_s}{P_{ci}})^{\sigma_s} F_s$)
- Therefore, because the optimal ratio does not depend on the price, the ratio can be maximized out of the problem of the optimal determination of the price, leaving the firm's real output as just a function of price

$$P_{si} = \frac{1}{1 + \tau_{si}^p} \frac{\sigma_s}{\sigma_s - 1} C_{si}$$



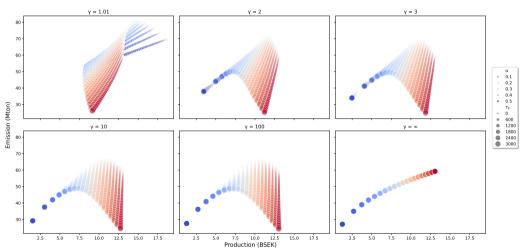
Estimation / Calibration

- My goal is to estimate the parameters sector by sector for Sweden
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment	
	Panel A: I	Inputs	
σ	∞	Fully competitive	
r	5%	-	
W	500 TSEK	-	
L	250 (sd = 900)	Martinsson et al. (2024)	
Panel B: Calibrated Value			
β_s	0.6	Martinsson et al. (2024)	
α_s	0.25	G/B, Wiedemann (2023)	
Panel C: Estimated Value			
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$	
$ ilde{ ilde{A}}$	0.018	E/PY	

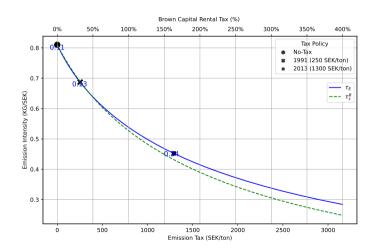
Sensitivity analysis

Production vs Emission with different Carbon Tax on different α and γ



Carbon Intensity and Tax

Counterfactual



$ au_{ extsf{E}}$	$ au_s^B$
100	14%
250	36%
500	66%
1300	171%
3000	360%

Resources allocation





Resources allocation

$$\hat{L}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$
 $\hat{G}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$
 $\hat{B}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{1}{1 + z_{s}^{k}} K_{s}$





Resources allocation

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} L_{s}$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} \frac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} \frac{1}{1 + z_{s}^{k}} K_{s}$$

$$\tilde{L}_{si} = \frac{A_{si}^{\sigma-1}/A_{si}^{\sigma}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s}$$





Resources allocation

$$\hat{L}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$
 $\hat{G}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$
 $\hat{B}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{1}{1 + z_{s}^{k}} K_{s}$

$$\begin{split} \tilde{L}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s} \\ \hat{G}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} \frac{z_{s}^{k}}{1+z_{s}^{k}} K_{s} \\ \hat{B}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} \frac{1}{1+z_{s}^{k}} K_{s} \end{split}$$



