## Resource Reallocation with Carbon Emission Policies

Seyyed Morteza Aghajanzadeh

Stockholm School of Economics

June, 2024

#### Motivation

• Climate crisis intensifies: rising temperatures, extreme weather.

#### Motivation

- Climate crisis intensifies: rising temperatures, extreme weather.
- Government interventions steer markets towards sustainability.
- Key policies: carbon pricing, renewable subsidies to curb emissions.
- Economic impacts:
  - Limitation in fossil fuel usage.
  - Adoption of renewable technologies.

#### Motivation

- Climate crisis intensifies: rising temperatures, extreme weather.
- Government interventions steer markets towards sustainability.
- Key policies: carbon pricing, renewable subsidies to curb emissions.
- Economic impacts:
  - Limitation in fossil fuel usage.
  - Adoption of renewable technologies.
  - Reallocation of resources to greener firms/industries.

## Research Question

- What is the Economic Outcomes of environmental policies due to resources reallocation?
  - Industry output
  - Firm-level productivity
  - Sector size
  - Emission intensity
  - Total Emission

#### Literature and Contribution

- Effectiveness of Carbon policies:
  - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)

Contribution: Quantify substitution between green and brown capital

#### Literature and Contribution

- Effectiveness of Carbon policies:
  - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)
     Contribution: Quantify substitution between green and brown capital
- Misallocation:
  - Whited and Zhao (2021); Hsieh and Klenow (2009); Ai, Li, and Yang (2020); Asker, Collard-Wexler, and De Loecker (2014)
     Contribution: Misallocation (Reallocation) in the context of environmental policies

#### Literature and Contribution

- Effectiveness of Carbon policies:
  - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)
     Contribution: Quantify substitution between green and brown capital
- Misallocation:
  - Whited and Zhao (2021); Hsieh and Klenow (2009); Ai, Li, and Yang (2020); Asker, Collard-Wexler, and De Loecker (2014)
     Contribution: Misallocation (Reallocation) in the context of environmental policies
- Climate Policy Design:
  - Acemoglu, Gancia, and Zilibotti (2012); Acemoglu et al. (2016); Oehmke and Opp (2023)
     Contribution: Assess alternative instruments in Emission Intensity / resource
     reallocation trade off

## Road map

- Develop Economic model with Emission
- Characterize the allocation of resources
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Oiscuss the cost of the environmental policies

## Road map

- Develop Economic model with Emission ✓
- Characterize the allocation of resources
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Oiscuss the cost of the environmental policies

## Road map

- Develop Economic model with Emission ✓
- Characterize the allocation of resources
- 🧿 Estimate the model by Swedish data 📤
- 🧿 Compare the optimal Policy with resource reallocation 🛦
- Oiscuss the cost of the environmental policies A

#### Standard Framework

Hsieh and Klenow (2009)

- Heterogeneous monopolistic competitive firms
- Partial equilibrium
- Cobb-Douglas Production function
- CES aggregator for output
- Normal aggregation of emissions

## Extension

#### Production functions

• Industry s, firm i:

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}$$

•  $\hat{A}_{si}$ : total factor of productivity

Firm's profit



#### Extension

#### Production functions

• Industry s, firm i:

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} \quad , \qquad \hat{K} = \left(\alpha_s G_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1-\alpha_s) B_{si}^{\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{\gamma_s}{\gamma_s-1}}$$

$$E_{si} = \tilde{A}_{si}B_{si}$$

Emission General Model

- $\hat{A}_{si}$ : total factor of productivity
- ullet  $\alpha_s$ : importance of Green capital in the production
- ullet  $\gamma_s$ : elasticity of substitution between Green and Brown capital
- $\tilde{A}_{si}$ : emission inefficiency
- Firms maximize over G, B, and L

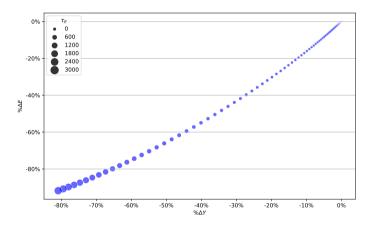
Firm's profit



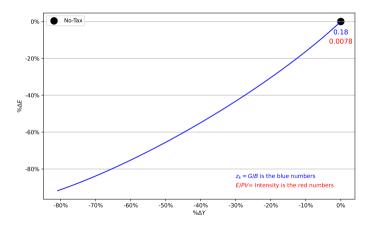
## Estimation / Calibration

• I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

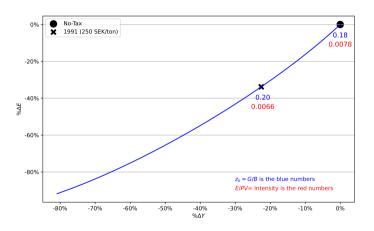
Parameter	Source/Moment
Panel A: Estimation	
$\alpha_s$	$\beta_G/\beta_B$
$\beta_s$	$\beta_G/\beta_L$
$\sigma_{s}$	WL/PY
$\gamma_s$	$\beta_{GB}/\beta_{B}\beta_{G}$
$Mean(\log(\hat{A}_{si}))$	$Mean(L_{si})$
$Sd(\log(\hat{A}))$	$Sd(L_{si})$
$Mean(\log(\tilde{A}_{si}))$	Mean(E/PY)
$Sd(\log(\tilde{A}))$	Sd(E/PY)
$Corr(\log(\hat{A}), \log(\tilde{A}))$	Corr(PY, E/PY)
Panel B: Additional Mome	nts
$\left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_s^B)r_{ti}+\tau_s^E\tilde{\Lambda}}{(1+\tau_s^G)r_{ti}}\right)^{\gamma_s}$	$z_k = G/B$
$\frac{1-\beta}{\beta}\frac{1}{\alpha}\left(\alpha_s+(1-\alpha_s)z_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{\gamma_s-1}}\frac{(1+r_s^C)r_{si}}{(1+r_s^C)w_{si}}$	$z_l = L/K$
$\frac{1}{\gamma}\frac{\ddot{A}}{r^B+\tau_E\ddot{A}}Z_k$	$\partial z_k/\partial \tau_E$
$\frac{\vec{A}/\gamma}{r^B + \tau_E \vec{A}} \frac{1}{\frac{\alpha}{1 - \alpha} z_k^{\gamma - 1} + 1} Z_I$	$\partial z_I/\partial \tau_E$
	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
Panel C: Calibration	
r	5%
w	500 TSEK



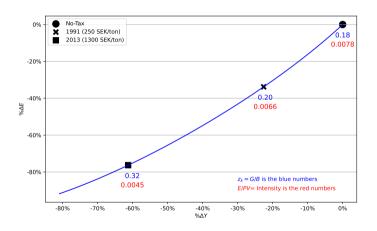


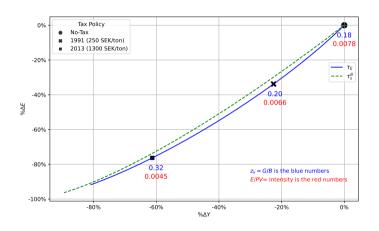














# Future Steps

- Develop Economic model with Emission
  - Firms could R&D
  - Add Household and Government
  - Firms could enter and exit the market
- 2 Characterize the allocation of resources
- Provide a definition of Green and Brown capital
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Objective the cost of the environmental policies

# Thank you!

## References I

- **Acemoglu, Daron, Gino Gancia, and Fabrizio Zilibotti.** 2012. "Competing engines of growth: Innovation and standardization." *Journal of Economic Theory*, 147(2): 570–601.
- Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2016. "Transition to clean technology." Journal of political economy, 124(1): 52–104.
- **Ahmadi, Younes, Akio Yamazaki, and Philippe Kabore.** 2022. "How do carbon taxes affect emissions? Plant-level evidence from manufacturing." *Environmental and Resource Economics*, 82(2): 285–325.
- Ai, Hengjie, Kai Li, and Fang Yang. 2020. "Financial intermediation and capital reallocation." *Journal of Financial Economics*, 138(3): 663–686.
- Andersson, Julius J. 2019. "Carbon taxes and CO2 emissions: Sweden as a case study." *American Economic Journal: Economic Policy*, 11(4): 1–30.
- **Asker, John, Allan Collard-Wexler, and Jan De Loecker.** 2014. "Dynamic inputs and resource (mis) allocation." *Journal of Political Economy*, 122(5): 1013–1063.
- **Hsieh, Chang-Tai, and Peter J Klenow.** 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly journal of economics*, 124(4): 1403–1448.
- Krueger, Philipp, Daniel Metzger, and Jiaxin Wu. 2023. "The sustainability wage gap." Swedish House of Finance Research Paper, , (20-14): 21–17.

#### References II

- Martinsson, Gustav, László Sajtos, Per Strömberg, and Christian Thomann. 2024. "The Effect of Carbon Pricing on Firm Emissions: Evidence from the Swedish CO2 Tax." *The Review of Financial Studies*, hhad097.
- **Oehmke, Martin, and Marcus M Opp.** 2023. "A theory of socially responsible investment." *Swedish House of Finance Research Paper*, , (20-2).
- **Shapiro**, **Joseph S**, and **Reed Walker**. 2018. "Why is pollution from US manufacturing declining? The roles of environmental regulation, productivity, and trade." *American Economic Review*, 108(12): 3814–3854.
- Whited, Toni M, and Jake Zhao. 2021. "The misallocation of finance." *The Journal of Finance*, 76(5): 2359–2407.
- Wiedemann, Moritz. 2023. "Green stewards: Responsible institutional investors foster green capex." Available at SSRN 4618793.

## **Emission General Model**

• The firm's emission is:

$$E_{si} = \tilde{A}_{si} \tilde{K}_{si}^{\theta_s} L_{si}^{1-\theta_s} \quad , \qquad \tilde{K} = (\mu_s G_{si}^{\frac{\eta_s-1}{\eta_s}} + (1-\mu_s) B_{si}^{\frac{\eta_s-1}{\eta_s}})^{\frac{\eta_s}{\eta_s-1}}$$

$$\pi_{si} = (1 + \tau_{si}^p) P_{si} Y_{si} - ([(1 + \tau_{G_s}) r_{si} G_{si} + (1 + \tau_{B_s}) r_{si} B_{si} + (1 + \tau_{I_s}) w_{si} I_{si}] + \tau_E E_{si})$$





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si}$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left( (1 + \tau_s^G) r_s G_{si} \right)$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm
  - ullet  $au_s^G$  is the Green capital subsidy / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s})P_{si}Y_{si} - \left((1 + \frac{\tau_s^G}{s})r_sG_{si} + (1 + \frac{\tau_s^B}{s})r_sB_{si}\right)$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm
  - $\tau_s^G$  is the Green capital subsidy / ESG preference of Financier
  - $\tau_s^B$  is the Brown capital tax / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si} - \left( (1 + \frac{\tau_s^G}{s}) r_s G_{si} + (1 + \frac{\tau_s^B}{s}) r_s B_{si} + (1 + \frac{\tau_s^W}{s}) w_{si} I_{si} \right)$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm
  - $\tau_s^G$  is the Green capital subsidy / ESG preference of Financier
  - $\tau_s^B$  is the Brown capital tax / ESG preference of Financier
  - $\bullet$   $au_s^W$  is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left( (1 + \tau_s^G) r_s G_{si} + (1 + \tau_s^B) r_s B_{si} + (1 + \tau_s^W) w_{si} I_{si} \right) - \tau_s^E E_{si}$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm
  - $\tau_s^G$  is the Green capital subsidy / ESG preference of Financier
  - $\tau_s^B$  is the Brown capital tax / ESG preference of Financier
  - $\bullet$   $au_s^W$  is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left( (1 + \tau_s^G) r_s G_{si} + (1 + \tau_s^B) r_s B_{si} + (1 + \tau_s^W) w_{si} I_{si} \right) - \tau_s^E E_{si}$$

- where
  - $\tau_s^p$  is the tax / Demand preference for the firm
  - $\tau_s^G$  is the Green capital subsidy / ESG preference of Financier
  - $\tau_s^B$  is the Brown capital tax / ESG preference of Financier
  - $\tau_s^W$  is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)
- The firm chooses the optimal capital and labor to minimize the cost of production and then chooses the price level to maximize the profit





$$\max_{G_{si},B_{si},L_{si}} \quad -Cost \quad ext{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{eta_s}L_{si}^{1-eta_s} = ar{Y}_{si}$$





$$\max_{\textit{G}_{\textit{si}},\textit{B}_{\textit{si}},\textit{L}_{\textit{si}}} \quad -\textit{Cost} \quad \text{s.t.} \qquad \hat{A}_{\textit{si}} \hat{K}_{\textit{si}}^{\beta_{\textit{s}}} L_{\textit{si}}^{1-\beta_{\textit{s}}} = \bar{Y}_{\textit{si}}$$

$$\frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^{k}$$





$$\begin{aligned} &\max_{G_{si},B_{si},L_{si}} &-Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ &\frac{G_{si}}{B_{si}} = \mathsf{z}_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_s^B)r_s + \tau_s^E\tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$





$$\begin{aligned} & \max_{G_{si},B_{si},L_{si}} & -Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ & \frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$

$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{l}$$





$$\begin{aligned} & \max_{G_{si},B_{si},L_{si}} & -Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ & \frac{G_{si}}{B_{si}} = Z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$

$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{\prime} = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)\mathbf{z}_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^{\mathbf{G}})r_s}{(1+\tau_s^{\mathbf{W}})w_{si}}$$

General Model Solution



# Firm Decision

$$\begin{aligned} &\max_{G_{si},B_{si},L_{si}} &-Cost \quad \text{s.t.} \quad &\hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ &\frac{G_{si}}{B_{si}} = z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$

$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{l} = \frac{1 - \beta}{\beta} \frac{1}{\alpha_{s}} \left( \alpha_{s} + (1 - \alpha_{s}) \mathbf{z}_{si}^{k - \frac{\gamma_{s} - 1}{\gamma_{s}}} \right)^{\frac{1}{1 - \gamma_{s}}} \frac{(1 + \tau_{s}^{G}) r_{s}}{(1 + \tau_{s}^{W}) w_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left( \alpha_{s} \mathbf{z}_{si}^{k \gamma_{s} - 1} + (1 - \alpha_{s}) \right)^{\frac{\gamma_{s}}{1 - \gamma_{s}}} \mathbf{z}_{si}^{l 1 - \beta} \bar{Y}_{si}$$

General Model Solution



# Firm Decision

$$\begin{aligned} &\max_{G_{si},B_{si},L_{si}} &- \textit{Cost} \quad \text{s.t.} \quad &\hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ &\frac{G_{si}}{B_{si}} = z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \end{aligned}$$

$$\frac{L_{si}}{\hat{K}_{si}} = z_{si}^{l} = \frac{1 - \beta}{\beta} \frac{1}{\alpha_{s}} \left( \alpha_{s} + (1 - \alpha_{s}) z_{si}^{k - \frac{\gamma_{s} - 1}{\gamma_{s}}} \right)^{\frac{1}{1 - \gamma_{s}}} \frac{(1 + \tau_{s}^{G}) r_{s}}{(1 + \tau_{s}^{W}) w_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left( \alpha_{s} z_{si}^{k \gamma_{s} - 1} + (1 - \alpha_{s}) \right)^{\frac{\gamma_{s}}{1 - \gamma_{s}}} z_{si}^{l 1 - \beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}$$

General Model Solution



## Firm Decision

$$\begin{aligned} \max_{G_{si},B_{si},L_{si}} &-Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ \frac{G_{si}}{B_{si}} &= z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\gamma_s} \\ \frac{L_{si}}{\hat{K}_{si}} &= z_{si}^l = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)z_{si}^{k-\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^G)r_s}{(1+\tau_s^W)w_{si}} \\ E_{si} &= \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k\gamma_s-1} + (1-\alpha_s)\right)^{\frac{\gamma_s}{1-\gamma_s}} z_{si}^{l-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si} \end{aligned}$$

• Firm will then charge markup over the marginal cost



### **Optimal Allocation**

$$\begin{aligned} \text{max} \quad & -\textit{Cost} \quad \text{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ z_{si}^k & \equiv \frac{G_{si}}{B_{si}} = \left[ \frac{\alpha_s}{1-\alpha_s} \frac{\frac{\partial}{\partial B} \textit{Cost}_{si}}{\frac{\partial}{\partial G} \textit{Cost}_{si}} \right]^{\gamma_s} \\ z_{si}^l & \equiv \frac{L_{si}}{\hat{K}_{si}} = \frac{1-\beta_s}{\beta_s} \frac{1}{1-\alpha_s} (\alpha_s z_{si}^{k(\gamma_s-1)} + (1-\alpha_s))^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial B} \textit{Cost}_{si}}{\frac{\partial}{\partial L} \textit{Cost}_{si}} \\ & = \frac{1-\beta_s}{\beta_s} \frac{1}{\alpha_s} (\alpha_s + (1-\alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial G} \textit{Cost}_{si}}{\frac{\partial}{\partial L} \textit{Cost}_{si}} \\ E_{si} & = \frac{\tilde{A}_{si}}{\hat{A}_{si}} (\frac{\phi_{si}}{z_{si}^l})^{\theta_s} z_{si}^{l} {}^{\beta_s} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}, \quad \text{where} \quad \phi_{si} & = \frac{(\mu_s + (1-\mu_s) z_{si}^{k(1-\eta_s)})^{\frac{\eta_s}{\eta_s-1}}}{(\alpha_s + (1-\alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{\gamma_s}{\gamma_s-1}}} \end{aligned}$$





# Model

### Optimal firm level price

• Now Firm need to choose the price level to maximize the profit:

$$\max_{P_{si}} \quad \pi_{si} = P_{si}F_{si} - C_{si}F_{si}$$

- Firm-level real output is a function of the sector price, firm price, and sector real output (i.e.  $F_{si} = (\frac{P_s}{P_{ci}})^{\sigma_s} F_s$ )
- Therefore, because the optimal ratio does not depend on the price, the ratio can be maximized out of the problem of the optimal determination of the price, leaving the firm's real output as just a function of price

$$P_{si} = \frac{1}{1 + \tau_{si}^p} \frac{\sigma_s}{\sigma_s - 1} C_{si}$$



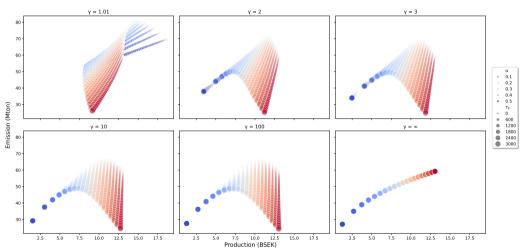
# Estimation / Calibration

- My goal is to estimate the parameters sector by sector for Sweden
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment
	Panel A: I	Inputs
$\sigma$	$\infty$	Fully competitive
r	5%	-
W	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
	Panel B: Calibr	rated Value
$\beta_s$	0.6	Martinsson et al. (2024)
$\alpha_s$	0.25	G/B, Wiedemann (2023)
Panel C: Estimated Value		
$\gamma$	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
$ ilde{ ilde{A}}$	0.018	E/PY

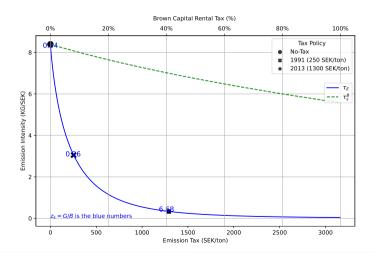
# Sensitivity analysis

Production vs Emission with different Carbon Tax on different  $\alpha$  and  $\gamma$ 



# Carbon Intensity and Tax

## Counterfactual



$ au_{ extsf{E}}$	$ au_s^B$
100	14%
250	36%
500	66%
1300	171%
3000	360%

### Resources allocation





#### Resources allocation

$$\hat{L}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$
 $\hat{G}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$ 
 $\hat{B}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{1}{1 + z_{s}^{k}} K_{s}$ 





#### Resources allocation

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} L_{s}$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} \frac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} \frac{1}{1 + z_{s}^{k}} K_{s}$$

$$\tilde{L}_{si} = \frac{A_{si}^{\sigma-1}/A_{si}^{\sigma}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s}$$





#### Resources allocation

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} \frac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} \frac{1}{1 + z_{s}^{k}} K_{s}$$

$$\begin{split} \tilde{L}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s} \\ \hat{G}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} \frac{z_{s}^{k}}{1+z_{s}^{k}} K_{s} \\ \hat{B}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{si}^{\sigma}} \frac{1}{1+z_{s}^{k}} K_{s} \end{split}$$



