Resource Reallocation with Carbon Emission Policies

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Motivation

- Climate crisis intensifies: rising temperatures, extreme weather.

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- Government interventions steer markets towards sustainability.
- Key policies: carbon pricing, renewable subsidies to curb emissions.
- Economic impacts:
  - Limitation in fossil fuel usage.
  - Adoption of renewable technologies.

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  - Limitation in fossil fuel usage.
  - Adoption of renewable technologies.
  - **Reallocation of resources to greener firms/industries.**

Research Question

- **What is the Economic Outcomes of environmental policies due to resources reallocation?**
    - Industry output
    - Firm-level productivity
    - Sector size
    - Emission intensity
    - Total Emission

Literature and Contribution

- Effectiveness of Carbon policies
  - Contribution: **Quantify substitution between green and brown capital** (Martinsson et al., 2024; Shapiro and Walker, 2018; Ahmadi, Yamazaki, and Kabore, 2022; Andersson, 2019)

Literature and Contribution

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- Misallocation
  - Contribution: **Misallocation (Reallocation) in the context of environmental policies** (Whited and Zhao, 2021; Hsieh and Klenow, 2009; Ai, Li, and Yang, 2020; Asker, Collard-Wexler, and De Loecker, 2014)

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- Climate Policy Design
  - Contribution: **Assess alternative instruments in Emission Intensity / resource reallocation**
    Acemoglu, Gancia, and Zilibotti (2012); Acemoglu et al. (2016); Oehmke and Opp (2023)

Road map

1. Develop Economic model with Emission
2. Characterize the allocation of resources
3. Estimate the model by Swedish data
4. Compare the optimal Policy with resource reallocation
5. Discuss the cost of the environmental policies

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Standard Framework

Hsieh and Klenow (2009)

- Heterogeneous monopolistic competitive firms
- Partial equilibrium
- Cobb-Douglas Production function
- CES aggregator for output
- Normal aggregation of emissions

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}$$

- $\hat{A}_{si}$: total factor of productivity

Firm's profit

Production functions

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} \quad , \qquad \hat{K} = (\alpha_s G_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha_s) B_{si}^{\frac{\gamma_s - 1}{\gamma_s}})^{\frac{\gamma_s}{\gamma_s - 1}}$$

$$E_{si} = \tilde{A}_{si} B_{si}$$

Emission General Model

- $\hat{A}_{si}$: total factor of productivity
- α_s: importance of Green capital in the production
- γ_s: elasticity of substitution between Green and Brown capital
- $\tilde{A}_{si}$: emission inefficiency
- Firms maximize over G, B, and L

Firm's profit

Estimation / Calibration

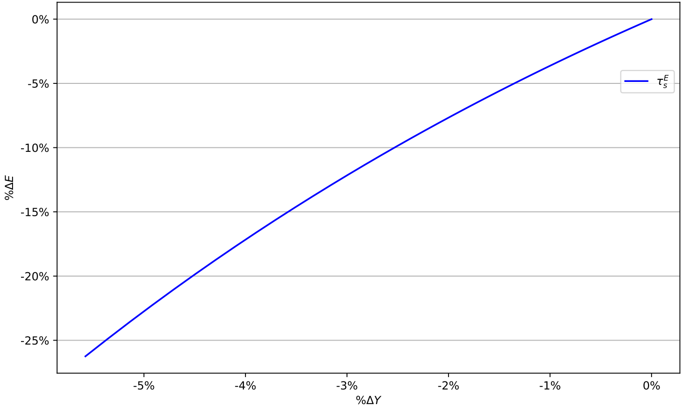
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	**Value**	**Source/Moment**
	Panel A: Estimated Value	
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
$\tilde{A}$	0.018	E/PY
	Panel B: Inputs	
σ	∞	Fully competitive
r	5%	-
w	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
	Panel C: Calibrated Value	
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B, Wiedemann (2023)

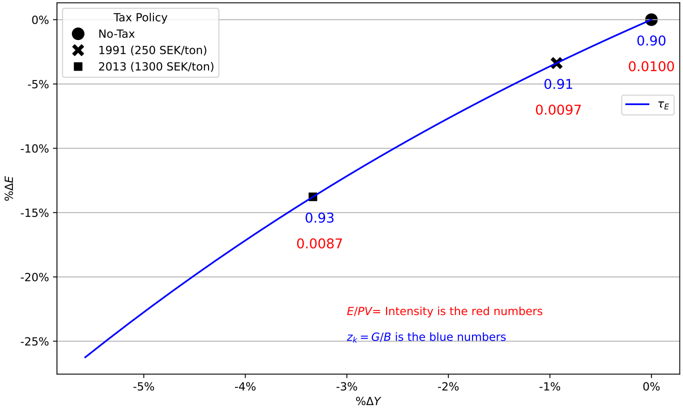
Sensitivity of α

Emission and Production

Results



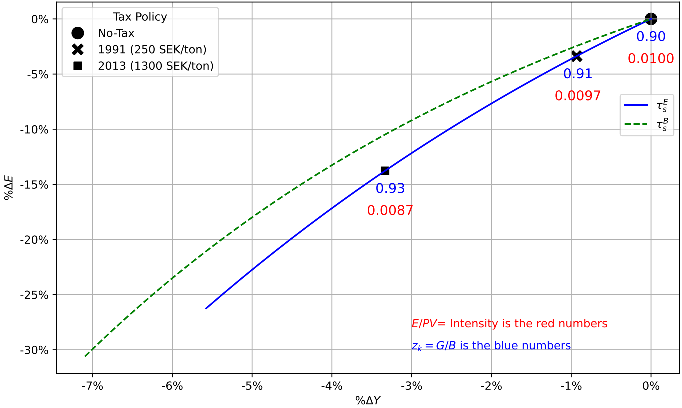
Carbon Intensity and Tax



Tax Policy
No-Tax
1991 (250 SEK/ton)
2013 (1300 SEK/ton)
τ_E
0.90
0.0100
0.91
0.0097
0.93
0.0087
E/PV= Intensity is the red numbers
$z_k = G/B$ is the blue numbers
0%
-5%
-10%
-15%
-20%
-25%
$\%\Delta E$
-5%
-4%
-3%
-2%
-1%
0%
$\%\Delta Y$

Emission and Production

Results



Future Steps

1. Develop Economic model with Emission
    - Firms could R&D
    - Add Household and Government
    - Firms could enter and exit the market
2. Characterize the allocation of resources
3. Provide a definition of Green and Brown capital
4. Estimate the model by Swedish data
5. Compare the optimal Policy with resource reallocation
6. Discuss the cost of the environmental policies

Thank you!

References I

Acemoglu, Daron, Gino Gancia, and Fabrizio Zilibotti. 2012. "Competing engines of growth: Innovation and standardization." *Journal of Economic Theory*, 147(2): 570–601.

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2016. "Transition to clean technology." *Journal of political economy*, 124(1): 52–104.

Ahmadi, Younes, Akio Yamazaki, and Philippe Kabore. 2022. "How do carbon taxes affect emissions? Plant-level evidence from manufacturing." *Environmental and Resource Economics*, 82(2): 285–325.

Ai, Hengjie, Kai Li, and Fang Yang. 2020. "Financial intermediation and capital reallocation." *Journal of Financial Economics*, 138(3): 663–686.

Andersson, Julius J. 2019. "Carbon taxes and CO2 emissions: Sweden as a case study." *American Economic Journal: Economic Policy*, 11(4): 1–30.

Asker, John, Allan Collard-Wexler, and Jan De Loecker. 2014. "Dynamic inputs and resource (mis) allocation." *Journal of Political Economy*, 122(5): 1013–1063.

Hsieh, Chang-Tai, and Peter J Klenow. 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly journal of economics*, 124(4): 1403–1448.

Krueger, Philipp, Daniel Metzger, and Jiaxin Wu. 2023. "The sustainability wage gap." *Swedish House of Finance Research Paper*, , (20-14): 21–17.

References II

Martinsson, Gustav, László Sajtos, Per Strömberg, and Christian Thomann. 2024. "The Effect of Carbon Pricing on Firm Emissions: Evidence from the Swedish CO2 Tax." *The Review of Financial Studies*, hhad097.

Oehmke, Martin, and Marcus M Opp. 2023. "A theory of socially responsible investment." *Swedish House of Finance Research Paper*, , (20-2).

Shapiro, Joseph S, and Reed Walker. 2018. "Why is pollution from US manufacturing declining? The roles of environmental regulation, productivity, and trade." *American Economic Review*, 108(12): 3814–3854.

Whited, Toni M, and Jake Zhao. 2021. "The misallocation of finance." *The Journal of Finance*, 76(5): 2359–2407.

Wiedemann, Moritz. 2023. "Green stewards: Responsible institutional investors foster green capex." *Available at SSRN 4618793*.

Emission General Model

- The firm's emission is:

$$E_{si} = \tilde{A}_{si} \tilde{K}_{si}^{\theta_s} L_{si}^{1-\theta_s} \quad , \qquad \tilde{K} = (\mu_s G_{si}^{\frac{\eta_s - 1}{\eta_s}} + (1-\mu_s) B_{si}^{\frac{\eta_s - 1}{\eta_s}})^{\frac{\eta_s}{\eta_s - 1}}$$

- The nominal profit for firms:

$$\pi_{si} = (1+\tau_{si}^{p}) P_{si} Y_{si} - \left(\left[(1+\tau_{G_{si}}) r_{si}^{G} G_{si} + (1+\tau_{B_{si}}) r_{si}^{B} B_{si} + (1+\tau_{l_{si}}) w_{si} l_{si} \right] + \tau_E E_{si} \right)$$

Back

Firm's profit

- The nominal profit for firms:

$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si}$$

- where
  - τ_s^p is the tax / Demand preference for the firm

Back

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- where
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  - τ_s^G is the Green capital subsidy / ESG preference of Financier

Back

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  - τ_s^p is the tax / Demand preference for the firm
  - τ_s^G is the Green capital subsidy / ESG preference of Financier
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  - τ_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)

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  - τ_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)
- The firm chooses the optimal capital and labor to minimize the cost of production and then chooses the price level to maximize the profit

Back

Firm Decision

$$\max_{G_{si}, B_{si}, L_{si}} \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

General Model Solution

Firm Decision

$$\max_{G_{si}, B_{si}, L_{si}} \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = z_{si}^{k}$$

General Model Solution

Firm Decision

$$\max_{G_{si}, B_{si}, L_{si}} \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = z_{si}^{k} = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B) r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G) r_s} \right)^{\frac{1}{\gamma_s}}$$

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$$\frac{L_{si}}{\hat{K}_{si}} = z_{si}^{l}$$

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$$\frac{L_{si}}{\hat{K}_{si}} = z_{si}^{l} = \frac{1-\beta}{\beta}\frac{1}{\alpha_s}\left(\alpha_s + (1-\alpha_s)z_{si}^{k\,1-\gamma_s}\right)^{\frac{1}{1-\gamma_s}}\frac{(1+\tau_s^G)r_s}{(1+\tau_s^W)w_{si}}$$

General Model Solution

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$$\frac{L_{si}}{\hat{K}_{si}} = z_{si}^{l} = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s) {z_{si}^{k}}^{1-\gamma_s} \right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^G) r_s}{(1+\tau_s^W) w_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s {z_{si}^{k}}^{\gamma_s - 1} + (1-\alpha_s) \right)^{\frac{\gamma_s}{1-\gamma_s}} {z_{si}^{l}}^{1-\beta} \bar{Y}_{si}$$

General Model Solution

Firm Decision

$$\max_{G_{si}, B_{si}, L_{si}} \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

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$$\frac{L_{si}}{\hat{K}_{si}} = z_{si}^l = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s) {z_{si}^k}^{1-\gamma_s} \right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^G) r_s}{(1+\tau_s^W) w_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s {z_{si}^k}^{\gamma_s - 1} + (1-\alpha_s) \right)^{\frac{\gamma_s}{1-\gamma_s}} {z_{si}^l}^{1-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}$$

General Model Solution

Firm Decision

$$\max_{G_{si}, B_{si}, L_{si}} \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

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$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s {z_{si}^{k}}^{\gamma_s - 1} + (1-\alpha_s) \right)^{\frac{\gamma_s}{1-\gamma_s}} {z_{si}^{l}}^{1-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}$$

- Firm will then charge markup over the marginal cost

General Model Solution

Model

Optimal Allocation

$$\max \quad -Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$z_{si}^{k} \equiv \frac{G_{si}}{B_{si}} = \left[\frac{\alpha_s}{1-\alpha_s}\frac{\frac{\partial}{\partial B}Cost_{si}}{\frac{\partial}{\partial G}Cost_{si}}\right]^{\gamma_s}$$

$$z_{si}^{l} \equiv \frac{L_{si}}{\hat{K}_{si}} = \frac{1-\beta_s}{\beta_s}\frac{1}{1-\alpha_s}(\alpha_s z_{si}^{k\,(\gamma_s-1)} + (1-\alpha_s))^{\frac{1}{1-\gamma_s}}\frac{\frac{\partial}{\partial B}Cost_{si}}{\frac{\partial}{\partial L}Cost_{si}}$$

$$= \frac{1-\beta_s}{\beta_s}\frac{1}{\alpha_s}(\alpha_s + (1-\alpha_s)z_{si}^{k\,(1-\gamma_s)})^{\frac{1}{1-\gamma_s}}\frac{\frac{\partial}{\partial G}Cost_{si}}{\frac{\partial}{\partial L}Cost_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}}\left(\frac{\phi_{si}}{z_{si}^{l}}\right)^{\theta_s} z_{si}^{l\;\beta_s}\bar{Y}_{si} = \psi_{si}\bar{Y}_{si}, \quad \text{where} \quad \phi_{si} = \frac{(\mu_s + (1-\mu_s)z_{si}^{k\,(1-\eta_s)})^{\frac{\eta_s}{\eta_s-1}}}{(\alpha_s + (1-\alpha_s)z_{si}^{k\,(1-\gamma_s)})^{\frac{\gamma_s}{\gamma_s-1}}}$$

Back

Model

Optimal firm level price

- Now Firm need to choose the price level to maximize the profit:

$$\max_{P_{si}} \quad \pi_{si} = P_{si}F_{si} - C_{si}F_{si}$$

- Firm-level real output is a function of the sector price, firm price, and sector real output (i.e. $F_{si} = (\frac{P_s}{P_{si}})^{\sigma_s} F_s$)
- Therefore, because the optimal ratio does not depend on the price, the ratio can be maximized out of the problem of the optimal determination of the price, leaving the firm's real output as just a function of price

$$P_{si} = \frac{1}{1+\tau^p_{si}} \frac{\sigma_s}{\sigma_s - 1} C_{si}$$

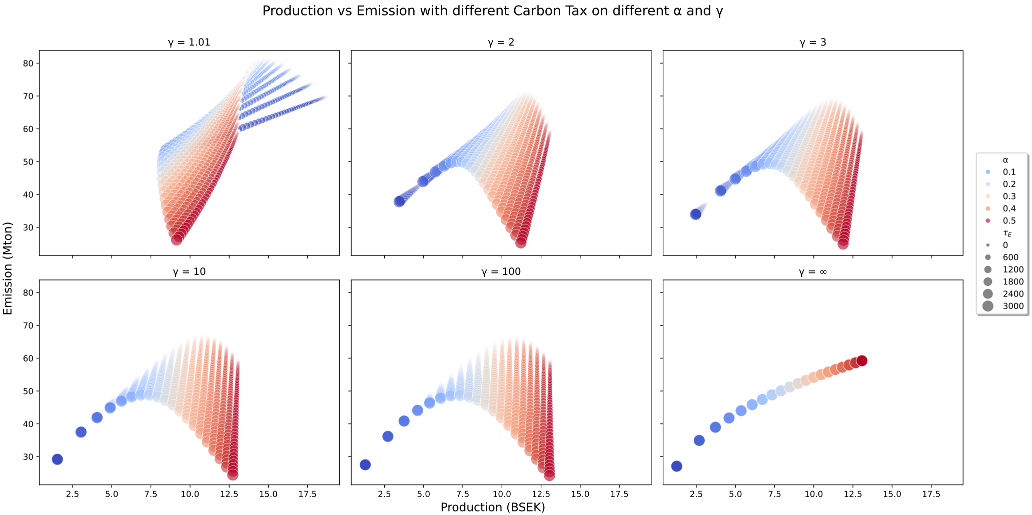
Estimation / Calibration

- My goal is to estimate the parameters sector by sector for Sweden
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	**Value**	**Source/Moment**
Panel A: Inputs		
σ	∞	Fully competitive
r	5%	-
w	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
Panel B: Calibrated Value		
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B, Wiedemann (2023)
Panel C: Estimated Value		
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
$\tilde{A}$	0.018	E/PY

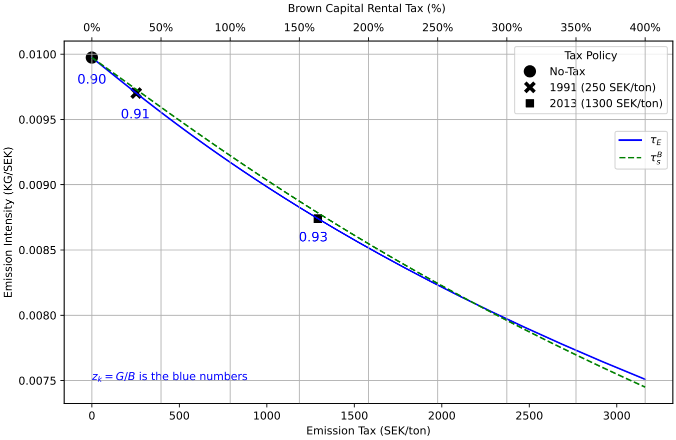
Sensitivity of α Back

Sensitivity analysis



Carbon Intensity and Tax

Counterfactual



τ_E	τ_s^B
100	14%
250	36%
500	66%
1300	171%
3000	360%

Reallocation

Resources allocation

- Now, we need to find the optimal allocation of resources in the economy under two scenarios:

Back

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$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} L_s$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{z_s^k}{1+z_s^k} K_s$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{1}{1+z_s^k} K_s$$

Back

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$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{1}{1+z_s^k} K_s$$

$$\tilde{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} L_s$$

Back

Reallocation

Resources allocation

- Now, we need to find the optimal allocation of resources in the economy under two scenarios:

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} L_s \qquad \tilde{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} L_s$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{z_s^k}{1+z_s^k} K_s \qquad \hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} \frac{z_s^k}{1+z_s^k} K_s$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{1}{1+z_s^k} K_s \qquad \hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} \frac{1}{1+z_s^k} K_s$$

Back