Allocative Efficiency of Green Finance Instruments *

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March 15, 2024

Abstract

This paper investigates the allocative efficiency of green finance instruments through a general equilibrium model with heterogeneous firms and financial frictions. We emphasize the impact of the timing of financial mechanisms—'ex-post', such as carbon taxes, versus 'ex-ante', like green credit schemes—on the distribution of dirty capital and its environmental implications. Our study reveals that ex-post measures inadvertently direct dirty capital towards financially constrained firms, potentially exacerbating emission intensity. Such theoretical prediction explains empirical observations of Hartzmark and Shue (2023), indicating such strategies may be counterproductive. Conversely, ex-ante approaches yield beneficial redistributions. The study emphasizes the significance of incorporating the distributive effects of green finance tools into their design and advocates for a general equilibrium viewpoint to evaluate their effectiveness comprehensively, highlighting the pivotal role of instrument timing.

JEL Codes: E22, E23, G32, Q4, Q5
Keywords: Green Finance Instruments; Allocative Efficiency; Green Technology; Financial Friction; Climate Externality

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1 Introduction

Green finance has witnessed a remarkable upswing, fueled by hope that it could influence firms' decisions to adopt green technology and serve as a key driver in the shift towards a sustainable economy. However, recent debates have cast doubt on the effectiveness of current green finance instruments. Concerns have been raised about whether these instruments are capable of inducing significant improvements in firm behavior (Berk and Van Binsbergen, 2021). The aggregate impact of these tools could manifest in various ways: it could be positive (Green and Vallee, 2022), neutral (Arnold, 2023), or even counterproductive (Hartzmark and Shue, 2023).

Our paper aims to delve deeper into this issue by providing a unified framework to analyze the impact of green finance instruments on firms' adoption of green technology. In particular, we explore the differential effects of these green instruments on financially constrained versus unconstrained firms. We reveal that this distinction leads to notable distributional outcomes of dirty capital allocation, which in turn critically influence the overall effectiveness of such instruments. Our study highlights the nuanced interaction of green finance instruments and financial constraints, offering valuable insights for policymakers and stakeholders in sustainable development.

We present a tractable general equilibrium model that incorporates financial frictions and heterogeneous enterprises, similar to Lanteri and Rampini (2023), together with an imperfect elastic supply of capital and variable capital utilization. This model explores the decision-making process of enterprises in choosing between environmentally sustainable ('green') and pollutant ('dirty') technologies under different green finance instruments. Our analysis reveals that green finance instruments can lead to varied distributional outcomes, significantly influencing their overall effectiveness.

Specifically, green instruments that reward or penalize based on the retrospective environmental impact, which we refer to as *'ex-post instruments'* (like carbon taxes), tend to redirect dirty capital towards financially constrained firms. In contrast, *'ex-ante instruments,'* which offer upfront subsidies for green financing or penalties for dirty financing (such as green credit schemes), tend to shift dirty capital towards less financially constrained firms.

A critical consideration is the pollution intensity of constrained firms: if these firms emit more pollutants per unit of dirty capital (often due to higher capital utilization and lower maintenance), then the redistributional impact of these green instruments becomes crucial. ex-post instruments might inadvertently increase total emissions by channeling dirty capital to more polluting, constrained firms. Conversely, ex-ante instruments effectively encourage these firms to transition to green technology, potentially enhancing their environmental impact. Therefore, the overall effectiveness of green finance instruments in this model is significantly influenced by the distributional shift of dirty capital and the pollution intensity of financially constrained firms.

The underlying rationale for the distinct redistributive impacts of these green instruments lies in the varying financial conditions of firms. Financially constrained firms, lacking in internal capital, prioritize upfront cash requirements (or down payments) when making investment decisions; conversely, financially unconstrained firms, with ample cash reserves, focus more on the frictionless cost of capital usage (user cost of capital).

The influence of green instruments on the relative attractiveness of green versus dirty capital can be dissected into two effects: a direct effect on either the down payment or the user cost of capital, and an indirect effect stemming from equilibrium changes in capital price, which affects both down payment and user cost.

Ex-post instruments directly increase the user cost of dirty capital while leaving the down payment unchanged. However, given the limited elasticity of capital supply, a general equilibrium effect emerges: a decrease in the demand for dirty capital leads to a lower market price. This indirectly lowers both the user cost and down payment for dirty capital. In this context, the primary outcome is an increase in the user cost of capital due to direct effects, while the initial down payment predominantly decreases due to indirect market adjustments. Consequently, this dual effect causes a redistribution of dirty capital, shifting it from financially unconstrained firms to constrained firms, owning to the simultaneous increase in user cost and reduction in down payment for dirty capital.

In contrast, ex-ante instruments directly increase the down payment for dirty capital while maintaining the user cost unchanged. On the other hand, the indirect effect from capital price adjustments reduces both the user cost and down payment for dirty capital.

This combined influence of increased down payment and decreased user cost for dirty capital results in a redistribution of dirty capital from constrained to unconstrained firms.

The distributional effect of green finance instruments becomes particularly significant considering dirty capital generates more emissions in financially constrained firms than unconstrained firms. Our model captures this through a higher capital utilization intensity among financially constrained firms. Constrained firms often resort to over-utilizing the same amount of dirty capital to maximize output, leading to increased emissions. This is a direct consequence of their limited access to capital; they need to extract as much value as possible from their existing assets. In contrast, unconstrained firms, with better access to capital, are more likely to use their resources efficiently and responsibly, leading to lower emissions from the same amount of dirty capital. This variance in emission intensity, when coupled with the distributional effects of green finance instruments, plays a critical role in shaping both the aggregate environmental outcome and the effectiveness of these instruments.

The prevailing approach in sustainable investing typically involves channeling investments towards companies with a positive environmental impact and avoiding those with a negative impact. Although this strategy is somewhat linked to external financing, it aligns more closely with 'ex-post instruments' like carbon taxes rather than upfront green financing models like green credit. This is because it rewards environmental friendliness ex-post, offering financing benefits for demonstrated greenness rather than providing upfront funding for green initiatives.

For financially constrained firms, this strategy poses a challenge. These firms, lacking sufficient internal capital, are unlikely to increase their investment in green capital for a future financial benefit. This is due to their immediate capital limitations. Additionally, the general equilibrium effect of this investing strategy, which tends to lower the price of dirty capital, inadvertently channels more of dirty capital towards financially constrained firms. While this investment approach may effectively reduce the overall use of dirty capital at a social level, it could also be counterproductive. It might unintentionally increase total emissions due to the higher emission intensity of financially constrained firms.

Our model offers a potential bridge over the existing gaps in green finance literature.

For instance, Duchin et al. (2022) find that in response to ESG investing pressures, large firms divest polluting assets, selling them off to smaller, private firms – a phenomenon that exactly echoes our model's predictions. This suggests that the prevailing sustainable investment strategies, due to their ″ex-post″ nature, might be counterproductive (Hartzmark and Shue, 2023). Conversely, the coal lending bans, because of their ″ex-ante″ nature, have successfully compelled firms that rely on bank financing to retire their power plants, proving to be effective (Green and Vallee, 2022).

Our analysis of ″ex-ante″ versus ″ex-post″ green financial instruments also offers broader insights into the practical application of green policies and sustainable investment strategies. In Table 1, we organize current green financial tools into ″ex-ante″ and ″ex-post″ classifications. Additionally, Section 6.1 provides an in-depth examination of each instrument, explaining the basis for their categorization into these distinct groups.

[Place Table 1 about here]

The rest of the paper is structured as follows: Section 2 reviews relevant literature, setting the stage for our analysis. Section 3 introduces a simple two-period model to elucidate how different green finance instruments influence the investment decisions in green and dirty capital among financially constrained and unconstrained firms. Section 4 conducts a numerical exercise to demonstrate the equilibrium impacts of these green instruments on total emissions and allocative efficiency. In Section 5, we extend our model to a dynamic framework, allowing for more realistic calibration of firm financial constraints and a broader range of policy scenarios. Section 6 engages in an in-depth discussion on the practical application of green finance instruments, weighing the advantages and disadvantages of ″ex-ante″ versus ″ex-post″ approaches beyond our model. The paper concludes in Section 7, summarizing our findings and their implications for policies and future research.

2 Related Literature

In exploring the complex dynamics between green finance instruments and firm investment in green technologies, our study draws upon and contributes to several strands of academic literature.

First, we build on quantitative general equilibrium models of climate change, i.e. the dynamic integrated climate-economy (DICE) model of William Nordhaus, as in Nordhaus (2014). Recent contributions extending the DICE model include Acemoglu et al. (2012), Acemoglu et al. (2016), Golosov et al. (2014) and Hassler and Krusell (2012), which study optimal carbon taxation and directed technological change. These studies offer a crucial understanding of the broader economic impacts of climate policies but often neglect the detailed implications of financial constraints faced by firms. Our research seeks to fill this gap by integrating these constraints into the general equilibrium framework, thereby enhancing the model's applicability to real-world scenarios.

Another significant aspect of our study is examining how financial constraints influence corporate environmental responsibility. Empirical studies by Hong et al. (2012), Goetz (2018), and Xu and Kim (2022) have demonstrated that financially unconstrained firms tend to engage more in social and environmental responsibilities. Lanteri and Rampini (2023) further elucidates the theory behind firms' choices between dirty and clean technology under financial constraints. Our work builds on these foundations to explore the implications of green finance instruments on these choices, incorporating factors like variable utilization and inelastic capital supply which have significant implications for aggregate outcomes.

We also contribute to the literature evaluating the effects of sustainable investing strategies on firm investment decisions and overall economic outcomes. Notable works in this realm include those by Heinkel et al. (2001), Davies and Van Wesep (2018), Pástor et al. (2021), Berk and Van Binsbergen (2021), Broccardo et al. (2022), Edmans et al. (2022), and Pedersen (2023). Recent empirical research indicates that sustainable investing has succeeded in increasing the cost of capital for environmentally harmful ('brown') firms compared to their 'green' counterparts (see, e.g., Chava (2014),Van der Beck (2021), Kacperczyk and Peydró (2022), Pástor et al. (2022), Aron-Dine et al. (2023), Green and Vallee (2022), and Gormsen et al. (2023)). Nonetheless, research by Akey and Appel (2021), Bartram et al. (2022), and Hartzmark and Shue (2023) highlights the potential unintended consequences of green finance schemes, questioning their overall effectiveness. In particular, the findings of Hartzmark and Shue (2023) suggest that reducing financing costs for green firms may result in minimal environmental improvements compared to the effects on brown firms. Our paper contributes to this discourse by offering a unified framework to assess the effectiveness of various green instruments within the context of financial constraints, focusing on both 'ex-ante' and 'ex-post' instruments and their distributional effects.

Lastly, our study also extensively relies on the theories of constrained efficiency in macroeconomics, drawing on seminal works like those of Diamond (1967), Stiglitz (1982), and Davila et al. (2012). These theories are pivotal in understanding the interactions between climate externalities, financial frictions, and optimal policy making, as reviewed in Nuño and Moll (2018).

3 Two-period Model

In this section, we present a simple two-period model that integrates key aspects that is needed to convey the basic intuition, including financial frictions, heterogeneous enterprises, variable capital utilization, and a capital supply characterized by imperfect elasticity. Our goal is to shed light on the ways in which various green finance instruments influence investment choices in green and dirty capital. Special attention is given to the redistribution of dirty capital between financially constrained and unconstrained firms, highlighting the allocative impact of these green instruments on the total emission.

3.1 Model Setup

Time is discrete and there are two periods for the economy, called time 0 and 1. A representative household is risk-neutral, with a discount factor β for the utility from the second period. The economy is a small open economy, and private agents can have access to the world financial market with gross interest rate fixed at β^{-1} unless additional restrictions in financial transactions are specified.

3.1.1 Firms' Problem

There are heterogeneous firms with different initial net worth and different productivity. The number of firms are infinite with a measure of one. The representative household owns all the firms in the economy.[1]

Production Technology. There are two types of capital that can be used for firm production, called dirty and green (denoted k^d and k^g, respectively). Firms utilize a combination of dirty and green capital for their production processes. The composite capital goods are denoted by $k = g(k^d, k^g)$, in which g is the function combining dirty and green capital into composite capital goods.

During production, a firm can determine the intensity with which it utilizes its capital, represented by h. This intensity directly influences production, as the output is a function of the utilized capital, denoted as $h \times k$. Firms are subject to decreasing returns to scale, characterized by the function $f = (h \times k)^\alpha$, with $0 < \alpha < 1$. This implies that, even for firms with the highest level of productivity and sufficiently large net worth, the firm size and the amount of capital used are still bounded. Utilizing capital incurs costs, which differ based on the type of capital. The cost functions for dirty and green capital are denoted as $c^d(h)$ and $c^g(h)$, respectively. Moreover, the utilization of dirty capital results in emissions, an aspect that will be discussed in more detail later in the analysis.

Firms purchase capital goods k^d and k^g at $t = 0$ at market prices of q^d and q^g, respectively. Production occurs at $t = 1$, after which all capital is fully depreciated. We assume there are no uncertainty in productivity going to the next period for simplicity. But we do allow for different levels of productivity, so that firms will have different marginal product

[1] By assuming this, it implies that when we consider firms' optimizations over time, we should always use the representative household's discount factor as the firm's one. In addition, as we consider the social planner's problem in the next section , we then only need to consider the representative household's welfare. Alternatively, if we had assumed these heterogenous firms are owned by independent, separate firm owners in the economy (such as those in Stiglitz (1982), Lorenzoni (2008),Dávila and Korinek (2018) and so on), we must consider the Pareto weights of different agents in the economy when formulating the social planner's problem.

for the same level of capital inputs.

Financial Frictions. Firms are endowed with some initial net worth w. They can engage in borrowing within the credit, subject to Kiyotaki and Moore (1997) type collateral constraints. Specifically, firms can borrow up to a θ fraction of their purchased capital's value. This rule can be motivated by the limited enforcement scenario where, at the end of the first period, the manager has the opportunity to default and divert a $1-\theta$ fraction of the capital, but cannot divert any capital in the second period.

We consider an extreme form of financial friction in equity financing: in the first period, firms are unable to issue external equity. In general models we consider firms that could issue external equities for finance, but typically with some additional issuance costs.

Optimization Problem For a typical individual firm i with an initial net worth of w_i (where $w_i > 0$) and a productivity level z_i, its optimization problem in the absence of any policy intervention is formulated as follows (omitting subscript i for simplicity):

$$\max\ d_0 + \beta d_1 \tag{1}$$

subject to the constraints:

$$\begin{aligned} w + b - d_0 &\geq q^d k^d + q^g k^g, && (2) \\ zf(h \times k) - c^d(h)k^d - c^d(h)k^g &\geq d_1 + \beta^{-1} b, && (3) \\ \theta q^d k^d + \theta q^g k^g &\geq b, && (4) \\ d_0 &\geq 0 && (5) \end{aligned}$$

The firm aims to maximize its discounted dividends over two periods by choosing dividend payout d_0, capital investments k^d and k^g, external debt b and capital utilization intensity h. Emissions are not considered in this optimization. Constraint (2) is the firm's budget constraint at time 0, indicating the use of internal capital w and debt b for dividend payouts d_0 and capital purchases. Constraint (3) is the budget constraint at time 1, where the firm's revenue is allocated for debt repayment and dividend distribution. Constraint (4) is the firm's borrowing constraint, which captures that firms are only able to borrow up to a θ

fraction of their capital's value. Finally, Constraint (5) specifies that firms cannot issue external capital, as indicated by $d_0 > 0$.

3.1.2 Capital Goods Market

For the supply of capital, we assume the representative household is endowed with a technology to supply capital goods: it requires $\chi^j(k) = p^j k^j + \frac{\kappa^j}{2}(k^j)^2,$ for $j \in \{d, g\}$ units of final goods (also the consumption goods, serving as the numeraire) to produce k units of type j capital goods. p^j controls the relative expensiveness of dirty and green capital while κ^j controls the elasticity of capital supply curve. We assume dirty and green capital have the same supply elasticity, $\kappa^d = \kappa^g = \kappa$, for simplicity.

The capital market is competitive, and the representative household takes the capital prices, q^d and q^g, as given when supplying capital goods. Market equilibrium for the two types of capital goods is maintained through the following market-clearing conditions:

$$\int k_i^d \, di = K^d, \quad \int k_i^g \, di = K^g. \tag{6}$$

This indicates that the total demand of each type of capital good from all firms (k_i^d and k_i^g for each firm i) equals the aggregate supply of each capital good in the market (K^d and K^g).

3.1.3 Emissions and Climate Goods

The use of dirty capital in the economy results in climate damage. Specifically, emissions are directly proportional to the utilized dirty capital. For an individual firm i, its emissions are given by $e_i = h \times k_i^d$. To calculate the total emissions, we integrate individual firm emissions across all firms, represented as $\int e_i \, di$. This model setup links a firm's emissions explicitly to its utilized dirty capital. Consequently, two firms with the same amount of dirty capital may have different emission levels if their utilization intensities vary. This aspect is crucial for capturing the varying environmental impacts of dirty capital utilized in different types of firms and aligns with the broader objectives of our model to evaluate the allocative efficiency of green finance instruments.

Households, in our model, derive utility not only from consumption goods but also from climate goods. The utility from climate goods is inversely affected by the aggregate emission E, which results from using dirty capital in production. We model the climate goods' utility impact as $-d \times E$, where d represents the damage to household utility per unit of emission.

For simplicity, we assume that climate damage directly influences utility. An alternative approach, as explored in studies like Golosov et al. (2014) and Acemoglu et al. (2016), considers the overall productivity being negatively impacted by aggregate emissions. However, the fundamental insights of this paper are not contingent on the specific nature of this assumption. The critical aspect is that emissions exert a negative externality on the aggregate economy, and these externalities are not internalized by private firms.

Emissions may include a variety of pollutants, such as carbon dioxide, sulfur dioxide, nitrogen dioxide, as well as the release of chemical contaminants, toxic substances, particulate matter, among others. The principles discussed here are broadly applicable to any negative externality. Nonetheless, it's important to recognize that in practical scenarios, for certain pollutants and emissions, alternative policy approaches are more commonly adopted. In these cases, rather than leveraging green finance instruments, regulatory mechanisms like quantity limits, legal penalties, and other enforcement methods are frequently employed.

3.1.4 Green Finance Instruments

We categorize green instruments into two types: 'ex-post instruments' that reward or penalize based on the retrospective environmental impact, like carbon taxes; and 'ex-ante instruments,' that offer upfront subsidies for green financing or penalties for dirty financing, like green credit schemes.

The firm's optimization problem incorporating these two green instruments is formulated as:

$$\max\ d_0 + \beta d_1 \tag{7}$$

subject to

$$
\begin{aligned}
w + b - d_0 &\geq q^d k^d + q^g k^g, && (8) \\
zf(h \times k) - c^d(h)k^d - c^d(h)k^g &\geq d_1 + \beta^{-1}b + \tau^d hk^d + \tau^g hk^g, && (9) \\
\xi^d \theta q^d k^d + \xi^g \theta q^g k^g &\geq b, && (10) \\
d_0 &\geq 0. && (11)
\end{aligned}
$$

In the context of 'ex-post instruments', we examine carbon tax as an illustrative example. Carbon tax is a form of environmental tax levied on carbon emissions. It is intended to encourage the reduction of greenhouse gas emissions by penalizing the use of carbon-intensive, or 'dirty', capital while incentivizing cleaner alternatives. In this framework, the government can impose a tax $\tau^d > 0$ on carbon emissions to discourage dirty capital use, reflecting the external costs of environmental damage. Conversely, a subsidy $\tau^g < 0$ can be applied to the utilization of green capital, promoting investments in environmentally friendly technologies. The policymaker is constrained by the tax equation:

$$\tau^d \int h_i k_i^d + \tau^g \int h_i k_i^g \geq 0 \tag{12}$$

This equation ensures that the overall tax policy is revenue-neutral or revenue-positive.

Regarding 'ex-ante instruments', we consider green credit schemes as an example. These schemes are designed to support and incentivize environmentally friendly projects, primarily through preferential loan conditions for green projects. In our framework, the emphasis is on the differential collateralizability of green and dirty capital. Specifically, the government can adjust collateral requirements of debt financing, making them less favorable for dirty capital ($\xi^d < 1$) and more favorable for green capital ($\xi^g > 1$). This approach aims to redirect external debt financing away from environmentally harmful projects towards those that are environmentally friendly. The policymaker is constrained by the green credit equation:

$$\xi^d \theta q^d K^d + \xi^g \theta q^g K^g \leq \theta q^d K^d + \theta q^g K^g \tag{13}$$

This constraint ensures that the total amount of financing, once adjusted for the green credit instrument, remains within the standard financing limits.

Furthermore, we can explore the external equity financing of firms based on the environmental impact of their projects. We modify the last dividend equation as $d_0 \geq \varsigma^d q^d k^d + \varsigma^g q^g k^g$. In this case, the government can impose $\varsigma^d > 0$ to penalize external equity financing of firms using dirty capital and $\varsigma^g < 0$ to incentivize financing with green capital. In this simplified two-period model without risk and default, the instruments for external debt and equity financing are isomorphic. Therefore, for simplicity, we only focus on the green credit scheme.

3.2 Capital Choice without Green Instruments

In this subsection, we analyze the firm's optimal allocation between dirty and green capital in the absence of green instruments. Our focus is particularly on the differing choices made by financially constrained and unconstrained firms.

For exposition, the firm's utilization rate is set to $h = 1$ when analyzing capital choices and the allocation impact of green finance instruments.[2] In this case, the utilization costs for both dirty and green capital are set to be constant, with $c^d(h) = c_0^d$ for dirty capital and $c^g(h) = c_0^g$ for green capital, respectively. We also assume that utilizing dirty capital is more costly than green capital, i.e., $c_0^d > c_0^g$. We define the output function as $F = zf(k^d, k^g)$, where F' represents the marginal product of utilized capital. We introduce the multipliers η for the firm's dividend constraint, and λ for the collateral constraint, respectively. The firm's first-order conditions are derived as follows:

$$d_0 \quad : \quad \mu = 1 + \eta, \tag{14}$$

$$k^d \quad : \quad \mu q^d - \lambda \theta q^d = \beta F' g_1 - \beta c_0^d, \tag{15}$$

$$k^g \quad : \quad \mu q^g - \lambda \theta q^g = \beta F' g_2 - \beta c_0^g, \tag{16}$$

$$b \quad : \quad \mu = 1 + \lambda. \tag{17}$$

[2] The pattern of firm capital choices and the allocation effect of green instruments is independent of capital utilization. For simplicity, utilization choices are abstracted from this analysis. However, different levels of capital utilization will influence overall emissions, which is addressed in Section 3.3.3 concerning the aggregate impact of green instruments.

Adopting the framework of Jorgenson (1963), we use U to denote the frictionless user cost of capital and Φ for the down payment, which is the minimum internal funds required per unit of capital. The user cost and down payment for dirty (U^d and Φ^d) and green (U^g and Φ^g) capital are defined as:

$$\begin{aligned} U^d &= q^d + \beta c_0^d, && (18) \\ U^g &= q^g + \beta c_0^g, && (19) \\ \Phi^d &= q^d(1-\theta), && (20) \\ \Phi^g &= q^g(1-\theta). && (21) \end{aligned}$$

These costs reflect the immediate (*down payment*) and total frictionless (*user cost*) financial commitments required to invest in each type of capital.

We employ a general CES (Constant Elasticity of Substitution) composite capital goods function to model the combination of dirty and green capital:

$$g(k^d, k^g) = \left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{22}$$

In this function, σ is the elasticity of substitution between the two types of capital, while γ determines their respective shares in the composite capital.

Incorporating this composite capital goods function into the first order conditions, the optimal ratio of dirty to green capital in a firm's investment is given by:

$$\frac{k^d}{k^g} = \left(\frac{\gamma}{1-\gamma}\right)^{\sigma} \left(\frac{U^d + \Phi^d \lambda}{U^g + \Phi^g \lambda}\right)^{-\sigma}. \tag{23}$$

Here, $U^j + \Phi^j \lambda$ represents the firm's *total cost* for each type of capital, integrating not just the frictionless user cost but also the shadow cost related to the down payment. This shadow cost, emerging from financial constraints, varies by firm and hinges on each firm's financing condition. For firms with ample financial resources ($\lambda = 0$), this total cost simplifies to the basic user cost, indicating indifference towards down payments. Conversely, for financially constrained firms (characterized by a high λ), the significance of down payments increases, impacting the firm's cost considerations and investment decisions more profoundly.

Equation (23) reveals that the ratio of dirty to green capital depends on their relative total costs: $U^d + \Phi^d \lambda$ for dirty capital and $U^g + \Phi^g \lambda$ for green capital. Financially unconstrained firms prioritize the relative user cost of capital, U. In contrast, financially constrained firms place more emphasis on the relative down payment requirement, Φ. This distinction in focus drives the different capital allocation choices between the two types of firms.

Given our assumptions that the utilization cost of dirty capital is higher than that of green capital, $c_0^d > c_0^g$, an equilibrium condition emerges: $\frac{U^d}{U^g} > \frac{\Phi^d}{\Phi^g}$. This implies that in equilibrium, dirty capital is characterized by a lower initial down payment but incurs higher total costs during usage. Conversely, green capital requires a higher down payment but is cheaper to use over time.

This equilibrium dynamic has significant implications for investment choices between financially unconstrained and constrained firms. Financially unconstrained firms, which are less sensitive to initial down payments, tend to invest more in green capital due to its lower user costs. On the other hand, financially constrained firms, which are more affected by the initial down payment, tend to invest more in dirty capital, despite its higher user costs. This investment pattern aligns with the theoretical framework proposed by Lanteri and Rampini (2023) and is consistent with empirical findings documented in Hong et al. (2012) and Xu and Kim (2022), which observe similar behaviors in capital allocation decisions.

3.3 Impact of Green Finance Instruments

In this subsection, we explore the influence of green finance instruments on firms' decisions regarding dirty and green capital, and the resultant distributional effects.

3.3.1 Capital Choice with Green Instruments

Incorporating the effects of carbon tax and green credit schemes, the firm's first-order conditions are modified as follows:

$$
\begin{aligned}
d_0 &: \quad \mu = 1 + \eta, && (24)\\
k^d &: \quad \mu q^d - \lambda \xi^d \theta q^d = \beta F' g_1 - \beta \left(c_0^d + \tau^d \right), && (25)\\
k^g &: \quad \mu q^g - \lambda \xi^g \theta q^g = \beta F' g_2 - \beta \left(c_0^g + \tau^g \right), && (26)\\
b &: \quad \mu = 1 + \lambda. && (27)
\end{aligned}
$$

These conditions reflect the additional costs or benefits associated with the carbon tax (τ^d and τ^g) and the modified collateral requirements ($\xi^d \theta$ and $\xi^g \theta$) under green credit.

The user cost and down payment for dirty and green capital under these green instruments are recalculated as:

$$
\begin{aligned}
U^d &= q^d + \beta \left(c_0^d + \tau^d \right), && (28)\\
U^g &= q^g + \beta \left(c_0^g + \tau^g \right), && (29)\\
\Phi^d &= q^d (1 - \xi^d \theta), && (30)\\
\Phi^g &= q^g (1 - \xi^g \theta). && (31)
\end{aligned}
$$

which take into account the additional factors introduced by green finance instruments. We also define $u^j = \beta \left(c_0^j + \tau^j \right)$ and $\varphi^j = (1 - \xi^j \theta)$. As it becomes evident, u^j and φ^j are directly affected by green finance instruments. In combination with the equilibrium price q^j, these parameters the user cost U^j and down payment Φ^j.

The total cost of dirty and green capital can be represented as $(q^j + u^j) + q^j \varphi^j \lambda$ for $j \in \{d, g\}$. The optimal ratio of dirty to green capital, as derived in equation (23), still applies but with the updated user cost and down payment values. This equation continues to be a critical factor in determining how firms allocate their resources between dirty and green capital, particularly under the influence of green finance instruments.

3.3.2 Direct and Indirect impact of Green Instruments

The influence of green instruments on the relative costs of capital can be decomposed into two channels: a direct effect and an indirect effect resulting from equilibrium changes in capital prices. Green instruments typically exert direct effects on U^j or Φ^j via changes in u^j or φ^j, and induce indirect effects on both U^j and Φ^j through equilibrium changes in capital price q^j.

If the capital goods market is characterized by fixed prices (perfectly elastic supply), then green instruments primarily exert direct effects. For example, a carbon tax impacting u^j and green credit affecting φ^j will encourage both financially unconstrained and constrained firms to shift towards green capital. However, a carbon tax is more effective for unconstrained firms, while green credit is more beneficial for financially constrained firms due to variations in the tightness of constraints, represented by λ.

The dynamics become more nuanced when the supply of capital goods is not perfectly elastic. Under these conditions, the effects of green instruments diverge significantly between financially constrained and unconstrained firms.

3.3.3 Distributional Effects of Green Instruments

Ex-post Instruments such as Carbon Tax: The introduction of a carbon tax targets the component u^d, causing a direct increase, while leaving φ^d unchanged. This policy also triggers general equilibrium effects. A decrease in the overall demand for dirty capital reduces its market price q^d, indirectly affecting both the user cost (U^d) and the down payment (Φ^d) for dirty capital. The direct impact of the tax predominates in the case of user cost, whereas the indirect effect, through the change in q^d, is more significant for the down payment.

Consequently, a carbon tax raises the user cost of dirty capital (U^d), while reducing its down payment (Φ^d). This results in differing responses from firms: financially unconstrained firms, more sensitive to user cost, are likely to decrease their investment in dirty capital, favoring green capital instead. On the other hand, financially constrained firms, influenced more by down payment considerations, may increase their investment in dirty capital due to the lowered capital price. This leads to a shift in the allocation of dirty capital towards financially constrained firms.

Ex-ante Instruments such as Green Credit: Green credit schemes directly affect φ^d while keeping u^d unchanged.[3,4] The general equilibrium effect, however, decreases dirty capital price q^d, influencing both U^d and Φ^d. In this context, the indirect effect on the user cost of capital becomes more pronounced, whereas the direct effect significantly influences the down payment.

Consequently, green credit schemes tend to lower the user cost of dirty capital (U^d), while increasing its down payment (Φ^d). Financially unconstrained firms, focusing more on user cost, may shift their investment away from green capital towards dirty capital. In contrast, financially constrained firms, influenced more by down payment considerations, might find the financing advantages of green capital more appealing, leading to an increased investment in green capital. This dynamic causes a redistribution of dirty capital towards financially unconstrained firms.

Other Green Instruments and a General Principle: Real-world green instruments might be multifaceted and impact both the user cost of capital and the down payment. Under an imperfectly elastic capital supply, these instruments can generate varying distributional effects depending on the relative strengths of their direct and indirect influences on user cost and down payment. A key principle is that green instruments influencing the user cost tend to be more effective for unconstrained firms, potentially redistributing dirty capital to constrained firms. In contrast, green instruments impacting the down payment are more beneficial for constrained firms, possibly reallocating dirty capital to unconstrained firms. The timing of cash flows plays a crucial role in determining whether a green instrument exerts a stronger influence on the user cost or the down payment. For instance, prevailing green investment strategies, which provide financing advantages based on past environmental performance, are analogous to carbon tax in our two-period model. This

[3]For simplicity, when discussing the effects of green credit, we assume that these instruments also penalize dirty investment. This is a reasonable simplification as the choice between dirty and clean capital is based on their relative costs.

[4]Our analysis focuses on scenarios where the marginal cost of over-utilization is sufficiently high, ensuring that financially unconstrained firms always opt for the normal utilization rate $\underline{h} = 1$ and highly financially constrained firms choose the maximum utilization $\overline{h}$.

similarity arises because the financing benefits are deferred to a later period rather than provided upfront. Therefore, a financially constrained firm, with limited internal capital, is less likely to invest in green capital in anticipation of future financing benefits.

Table 2 summarizes the impact of these instruments on financially constrained and unconstrained firms.

[Place Table 2 about here]

3.4 From Distributional Effect to Aggregate Outcome

The distributional effect of green finance instruments becomes particularly significant when dirty capital generates higher levels of emissions in financially constrained than unconstrained firms. This difference can significantly influence the aggregate environmental impact. Specifically, ”ex-ante” instruments direct dirty capital towards firms that are unconstrained and less emission-intensive, thereby being allocatively efficient and effectively reducing total emissions. In contrast, ”ex-post” instruments divert dirty capital towards more constrained and emission-intensive firms, resulting in allocative inefficiency and can potentially be counter-productive in reducing emission.

3.4.1 Variable Capital Utilization and Aggregate Emission

One primary reason for this disparity is the differing utilization intensity between these firms. Constrained firms often resort to over-utilizing the same amount of dirty capital to maximize output, leading to increased emissions. This is a direct consequence of their limited access to capital; they need to extract as much value as possible from their existing assets. In contrast, unconstrained firms, with easier access to capital, can afford to use their resources more efficiently and responsibly, resulting in lower emissions for the same amount of capital.

To capture this feature, we introduce variable capital utilization into our model. Since emissions are directly linked to the utilization of dirty capital, there is a direct correlation between increased utilization intensity and higher emissions. More precisely, our model allows firms to choose their capital utilization intensity within a bounded range, $[\underline{h}, \overline{h}]$, indicating limited flexibility in utilization. Here, $\underline{h}$ represents normal utilization, while $\overline{h}$ indicates the threshold for over-utilization. Without loss of generality, we set the normal utilization level as $\underline{h} = 1$.The utilization cost functions, $c^d(h)$ and $c^g(h)$, are increasing and convex, that is, $c^{j\prime}(h) > 0$ and $c^{j\prime\prime}(h) \geq 0$ for $j \in \{d, g\}$. This means that the costs increase with higher utilization intensity and do so at an accelerating rate. We also assume that the cost for utilizing dirty capital, $c^d(h)$, consistently exceeds that for green capital, $c^g(h)$, highlighting a higher cost associated with dirty capital usage.

We now turn our attention to characterizing the capital utilization strategies of firms. Firms face a decision: to either expand their capital base for less intensive use or to utilize a smaller amount of capital more intensively. Detailed in Appendix A, we outline the first-order conditions of the firm's decision-making process regarding variable capital utilization and discuss their preferences in utilization. Typically, a firm's capital utilization rate increases in response to financial constraints; constrained firms tend to leverage their capital more heavily to maximize production.

It's important to note that when discussing increased utilization intensity, we are essentially referring to over-utilization, as opposed to normal utilization. Over-utilization often comes with constraints and leads to higher depreciation and maintenance costs. To prevent the trivial outcome where all firms maximize their capital utilization to the upper limit, we introduce an assumption: the marginal cost of over-utilizing capital is prohibitively high for financially unconstrained firms. This means that such firms will never find it optimal to over-utilize their capital. We summarize this assumption in the following lemma:

Lemma 1. *Let $U^j(1) = q^j + \beta c^j(1) + \tau^j$ represent the user cost of utilizing type j capital at normal utilization intensity (i.e., $\underline{h} = 1$). If $\min\left(c^{d\prime}(h), c^{g\prime}(h)\right) > \beta^{-1} \max\left(U^d(1), U^g(1)\right)$, then unconstrained firms ($\lambda = 0$) will choose normal utilization intensity $\underline{h} = 1$.*

Proof. See Appendix A. □

This lemma suggests that for financially unconstrained firms to avoid over-utilizing their capital, the marginal cost of increasing utilization intensity must surpass the user cost associated with employing additional capital in a less intensive manner. This parameter restriction will be maintained throughout our analysis.

In contrast, financially constrained firms, being more sensitive to the initial down payment, are more inclined to over-utilize their capital with a smaller base. This preference stems from the fact that over-utilization, unlike initial capital investment, does not require immediate cash outflows. Given the limited flexibility in capital utilization in our model, we propose the following lemma for these firms:

Lemma 2. *For sufficiently constrained firms (i.e., with a large λ), they will opt for the maximum possible utilization intensity $\overline{h}$.*

Proof. See Appendix A. □

The distinction in capital utilization between constrained and unconstrained firms is critical as it influences the aggregate outcomes. Specifically, a unit of dirty capital in a financially constrained firm will lead to greater emissions compared to the same unit in an unconstrained firm, due to the tendency of the former to over-utilize. This difference in emissions, combined with the distributional impact of green finance instruments, plays a decisive role in determining the efficacy of such instruments.

3.4.2 Allocative Efficiency beyond Our Model

There might be other factors contributing to higher emissions in financially constrained firms. For instance, financially constrained firms tend to implement cost-cutting strategies. These firms often reduce their expenditure on crucial areas such as pollution control, environmental management systems, and regular maintenance. Such reductions can inadvertently lead to increased emissions due to less efficient and poorly maintained equipment and processes.

The impact of these strategies is further intensified by enforcement challenges in smaller firms. The success of green instruments largely relies on effective enforcement. However, smaller and financially constrained firms often escape the attention of regulatory bodies, resulting in a gap in policy implementation. Consequently, perceiving a lower risk of regulatory action or penalties, these firms might not strictly adhere to environmental standards, leading to heightened emissions.

In light of these differences, it becomes evident that the allocation of dirty capital to financially constrained firms is less 'efficient' than to unconstrained firms. As a result, green finance instruments that inadvertently shift dirty capital towards constrained firms can lead to unintended environmental consequences. For instance, a carbon tax, while intended to discourage the use of dirty capital, might paradoxically increase total emissions if it leads to a higher concentration of dirty capital in constrained firms. On the other hand, green credit schemes, by reallocating some dirty capital to unconstrained firms, could effectively reduce overall emissions. These nuanced effects underscore the importance of considering firm-specific characteristics and behaviors in policy design and implementation.

In our subsequent numerical exercises, we will demonstrate the impact of these instruments. By analyzing how dirty capital is reallocated between financially constrained and unconstrained firms, we will illustrate the nuanced effects of various green instruments. Specifically, we will explore how ex-post instruments like carbon taxes might be counterproductive in certain scenarios by increasing total emissions, whereas ex-ante instruments like green credit schemes could be more effective in reducing overall emissions.

4 Numerical Analysis

In this section, we conduct a numerical analysis to assess the allocative effects of various green instruments, as discussed in our preceding theoretical analysis. Specifically, we demonstrate how ex-post instruments may redirect dirty capital towards constrained firms, whereas ex-ante instruments are likely to channel dirty capital towards unconstrained firms, leading to significantly different aggregate outcomes.

We will first examine the capital allocation decisions of both constrained and unconstrained firms in the absence of any green instruments. Subsequently, we utilize a carbon tax, representative of ex-post instruments, and green credits, as an example of ex-ante instruments, to investigate their redistributive impacts. This includes a focused comparison of their direct effects in partial equilibrium scenarios and their broader implications in general equilibrium contexts. Finally, we evaluate the overall impact of these instruments on total emissions, breaking down the aggregate change in emissions into a level effect and an allocative effect, and discuss the allocative efficiency of each instrument.

Table 3 outlines the parameters for our two-period model. We have selected a relatively high elasticity of substitution between dirty capital (k^d) and green capital (k^g), set at 40.0, to capture the significant potential for substitution between these two types of capital in production processes. Our analysis begins with a scenario of fixed capital utilization (with $h = 1$) to elucidate firms' capital choices and the redistributive impacts of various green instruments on capital allocation. We then extend our analysis to variable capital utilization scenarios to explore the aggregate impacts of green instruments on total emission. We consider a symmetric case for the two types of capital, with the notable exception that the utilization cost for dirty capital is set at 1.0, which is higher than the 0.8 utilization cost for green capital. Both types of capital are characterized by an imperfectly elastic supply function and share a common collateralizability parameter ($\theta = 0.4$). Firm productivity is assumed to follow a uniform distribution within the range of $[1, 50]$.

[Place Table 3 about here]

4.1 Capital Choices and the Distributive Effects of Green Instruments

Figure 1 and Figure 2 first present the results without any policy intervention, setting a baseline for our examination. Specifically, Figure 1 illustrates the comparison of the relative total cost, represented as $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$, between dirty and clean capital across different values of the firm multiplier λ. Given our parameters, the higher utilization cost of dirty capital leads to a market equilibrium where dirty capital is priced lower than clean capital, $q^d < q^g$. This results in dirty capital having a higher frictionless user cost but a lower initial down payment, i.e., $U^d > U^g$ and $\Phi^d < \Phi^g$. Consequently, it becomes evident that for lower values of λ, the relative total cost approaches $\frac{U^d}{U^g} > 1$, whereas for higher λ values, it approaches to $\frac{\Phi^d}{\Phi^g} < 1$. As derived in equation (23), the relative total cost plays a critical role in determining a firm's preference for dirty versus green capital.

[Place Figure 1 about here]

Figure 2 dives deeper into firms' optimal capital choice in relation to their productivity levels. The panels (a) through (d) detail the firm's multiplier λ, the relative importance of down payment, the relative total cost, and the dirty to green capital ratio as a function of productivity, respectively. In our model, all firms begin with identical net worth; variations arise from differences in productivity. Consequently, firms with higher productivity, despite having the same initial net worth, face greater financial constraints. Therefore, as shown in Figure 2 Panel (a), a firm's shadow price of constraints, λ, increases with firm productivity. When firms are less financially constrained, the user cost predominates their cost considerations; as constraints tighten, the down payment's significance grows. Due to the higher user cost but lower down payment requirement of dirty capital, financially unconstrained firms show a preference for green capital, whereas financially constrained firms prefer dirty capital. This preference is visually captured in Panel (d) by an upward-sloping curve of the dirty to green capital ratio.

[Place Figure 2 about here]

Next, we turn to the different distributive consequences of implementing ex-ante and ex-post green instruments. In particular, we compare their direct partial equilibrium (PE) effect without equilibrium price change and their general equilibrium effect (GE) with equilibrium price change. We employ a carbon tax (with $\tau^d = 0.5$) as an example of an ex-post instrument and a dirty capital lending ban (with $\xi^d = 0.8$) as an example of an ex-ante instrument. The results are illustrated in Figure 3 and Figure 4, with each figure's Panel (a) to (d) showing relative total cost, the dirty to green capital ratio, and the percentage change in dirty and green capital, respectively. The solid red line represents the GE effect, the dashed red line the PE effect, and the blue line serves as a benchmark, showing results in the absence of any green interventions for comparison.

The carbon tax's direct PE effect raises the user cost of dirty capital without altering its down payment. Figure 3 demonstrates that this increases the relative total cost of dirty capital for all firms, with a more pronounced effect on unconstrained firms, for whom the user cost is a more critical component of total cost. This PE effect prompts a shift away from dirty capital towards green capital for all firms, with unconstrained firms showing a stronger response. However, the introduction of the GE effect, considering equilibrium price adjustments, reveals nuanced outcomes. The decline in dirty capital's equilibrium price, caused by reduced demand, lowers both its user cost and down payment. The direct PE effect predominates for the user cost, thereby still increasing the relative total cost for unconstrained firms versus the benchmark, albeit to a lesser degree. Conversely, for down payments, the indirect GE effect prevails, reducing the relative total cost for constrained firms compared to the benchmark and thus inversely affecting their investment behaviors. This leads to a decrease in investments in dirty capital for unconstrained firms and an increase in investments in dirty capital for constrained firms. That is, the GE effect of the carbon tax reallocates dirty capital from unconstrained to constrained firms.

[Place Figure 3 about here]

In stark contrast, the ex-ante instrument—dirty capital lending ban—exhibits an inverse pattern to the ex-post instrument. Its direct PE effect increases the down payment required for dirty capital while leaving the user cost unchanged. As illustrated in Figure 4, this effect marginally raises the relative total cost of dirty capital for all firms, with a more significant impact on constrained firms, while leaving the relative total cost for unconstrained firms ($\lambda = 0$) unchanged. This effect also drives a universal reduction in dirty capital investment, more so among constrained firms. Incorporating the GE effect, which accounts for the decline in dirty capital price, the outcome reverses for unconstrained firms and becomes attenuated for constrained firms. Although the GE effect reduces the user cost, it does not fully counteract the direct PE effect of increasing the down payment. This leads to a reduced dirty capital investment by constrained firms but an increased investment by unconstrained firms. That is, the GE effect of the lending ban shifts dirty capital from constrained to unconstrained firms. Intriguingly, the substantial impact of the lending ban reverses the preference for dirty versus green capital for both types of firms. As a result, unconstrained firms exhibit a higher dirty to green capital ratio compared to constrained firms.

[Place Figure 4 about here]

4.2 The Impact Elasticity

Hartzmark and Shue (2023) demonstrate that reducing financing costs yields less environmental benefit for green firms compared to dirty firms. Our model echoes this finding, replicating their observed pattern in scenarios without any policy interventions or with ex-post instruments. However, this pattern dissipates when strong enough ex-ante policies are applied. Intuitively, reducing financing costs eases financial constraints more significantly for firms that are already financially constrained than for those that are not. In scenarios without green instruments or with the application of ex-post instruments, financially constrained firms are dirty firms. Thus, easing financial constraints under these conditions tends to shift firm preferences towards greener capital, resulting in a more pronounced improvement for dirty firms than for their greener counterparts. Conversely, with a sufficiently strong ex-ante policy in place, this preference flips, with financially unconstrained firms showing a preference for dirty capital. In such a context, relaxing financial constraints instead leads to increased investment in dirty capital.

In Figure 5, we leverage on our model to explore how firms respond to injection of net worth in different economies. We examine three scenarios: the benchmark equilibrium without any policy interventions, the equilibrium with carbon tax ($\tau^d = 0.5$), and the equilibrium with smaller dirty capital collateralizability ($\xi^d = 0.8$, as opposed to $\xi^d = 1.0$ in the benchmark case). For each setting, we maintain constant capital prices and introduce a 1% increase in firm net worth. The resulting changes in the ratio of dirty to green capital (k^d/k^g) are depicted in the right column of Figure 5.

[Place Figure 5 about here]

In economies without policy interventions and those employing ex-post instruments, financially constrained firms reduce their dirty capital holdings more than their unconstrained counterparts. The impact is more pronounced under ex-post instruments because these policies increase the disparity in down payments required for dirty versus green capital. This increases the difference in greenness between constrained and unconstrained firms, which in turn amplify the improvement given one unit of net worth injection. Interestingly, in economies with strong ex-ante green instruments, the preference between dirty and green capital reverses. Consequently, easing a firm's financial constraints in such contexts leads to an increase in its dirty capital, contradicting existing literature findings.

Our model predictions are largely consistent with Hartzmark and Shue (2023), noting that the green instruments currently in use are predominantly ex-post. However, careful interpretation of these results is necessary. The findings presented in Hartzmark and Shue (2023) can be partially attributed to the equilibrium effects stemming from the prevalent sustainable investment strategy. This strategy, by reducing the market price of dirty capital, consequently lowers its associated down payment. This effect in turn amplifies the financial disparity between investments in dirty and green capital. As a result, the potential for environmental improvement among financially constrained firms becomes increasingly reliant on their financing conditions. In the absence of this investment strategy, the influence of capital costs on environmental improvements for these constrained firms might not be as significant. This is due to a narrower gap in the down payments required for dirty versus green capital investments, leading to a less pronounced sensitivity to their financial constraints. Furthermore, in scenarios dominated by ex-ante instruments, these findings may no longer apply.

4.3 Aggregate Effects on Total Emission

The impact of "ex-ante" and "ex-post" green instruments on aggregate emissions can vary significantly. Notably, ex-post green instruments could be counterproductive by inadvertently increasing emissions. This unintended consequence arises as these instruments can redistribute dirty capital towards financially constrained firms. Such firms tend to use their capital more intensively, leading to higher emissions.

To study the aggregate effect on emissions, we introduce variable capital utilization rates for firms, allowing them to choose between a standard rate ($h = 1$) and an intensified rate ($h = 2$). We set the cost associated with over-utilization high enough that only the most financially constrained firms opt for this strategy. This model enables an examination of each green instrument's impact on total emissions, quantified as the sum of utilized dirty capital across all firms, $E = \int h_i k_i^d$. Furthermore, to assess the efficiency of capital allocation, we calculate the average utilization intensity of dirty capital, defined as the total emissions divided by the aggregate dirty capital, $H^d = \frac{E}{K^d}$. An increased average utilization intensity suggests higher emissions for the same amount of dirty capital. Through this framework, we can dissect the change in total emissions into two distinct components: the change due to shifts in dirty capital allocation among firms (allocative effect) and the change resulting from variations in the overall level of dirty capital (level effect), as illustrated below:

$$K^d H^d - K^d_{bench} H^d_{bench} = \underbrace{\left(K^d H^d - K^d H^d_{bench}\right)}_{\text{Allocative Effect}} + \underbrace{\left(K^d H^d_{bench} - K^d_{bench} H^d_{bench}\right)}_{\text{Level Effect}} \tag{32}$$

The outcomes of applying "ex-post" and "ex-ante" green instruments are presented in Figures 6 and 7, with Panels (a) through (d) showcasing total emissions, emission intensity, average dirty capital utilization intensity, and the breakdown of emission changes in response to policy adjustments, respectively.

In Figure 6, we increase the carbon tax parameter from 0 to 4.[5] As the carbon tax increases, both total emissions and emission intensity exhibit a hump-shaped pattern, where emissions first increase and then decrease. Such a pattern suggests that, under certain conditions, ex-post instruments may inadvertently increase emissions rather than mitigate them. This unintended consequence stems from the redistribution of dirty capital towards financially constrained firms, which in turn amplifies the average utilization rate of such capital. Panel (c) illustrates this dynamic, indicating a consistent increase in the average utilization rate as the carbon tax increases. Furthermore, Panel (d) decomposes the change in emissions into two distinct components. It becomes apparent that, although the imposition of a carbon tax reduces the aggregate level of dirty capital—potentially reducing total emissions—the deterioration in allocative efficiency, conversely, heightens emissions. When this adverse allocative effect dominates, ex-post instruments become counterproductive.

[Place Figure 6 about here]

[5]For an individual firm with $z = 2.0$, $\tau^d = 4$ results in a cost equivalent to approximately 10% of its output.

The allocative effect is opposite for ex-ante instruments. In Figure 7, we change the collateralizability parameter for dirty capital, ξ^d, reducing it from a benchmark value of 1 to 0.2. This reduction implies the implementation of an increasingly stringent lending restriction for dirty capital. Correspondingly, both total emissions and emission intensity demonstrate a consistent downward trend. The average dirty capital utilization also decreases as dirty capital are redistributed towards financially unconstrained firms, which tend to utilize capital less intensively. This effect is further elucidated in Panel (d), revealing that the economy gains from both a decrease in the overall level of dirty capital and enhanced allocative efficiency. Collectively, these factors contribute to a significant reduction in total emissions.

[Place Figure 7 about here]

Our analysis reveals that ex-post instruments may unintentionally increase emissions by reallocating dirty capital to financially constrained firms. In contrast, ex-ante instruments lead to a decrease in emissions by directing dirty capital away from these firms. In this sense, ex-post instruments are allocatively inefficient while ex-ante instruments are allocatively efficient.

5 Dynamic Model

Previously, we can obtain tractable analysis from the two-period model; however, in that model we assume firms' distributions of productivity and net worth are exogenous. Since firms may have incentives to accumulate net worth endogenously over time and potentially mitigate their financial constraints, it is important to study a dynamic model and evaluate our mechanisms quantitatively.

To do so, we extend the two-period model to infinity periods. Specifically, the firm's optimization problem can be formulated by the following Bellman equation:

$$V(z_t, w_t; S_t) = \max_{\{d_t, b_{t+1}, k_t^d, k_t^g, h_t\}} d_t + \beta E_t V(z_{t+1}, w_{t+1}; S_{t+1}) \tag{33}$$

subject to

$$
\begin{aligned}
d_t &= w_t + b_{t+1} - q_t^d k_t^d - q_t^g k_t^g, && (34) \\
w_{t+1} &= F(z_{t+1}, h_{t+1}, k_t) - c^d(h_{t+1})k_t^d - c^g(h_{t+1})k_t^g - Rb_{t+1} - \tau^d h_{t+1} k_t^d - \tau^g h_{t+1} k^g, && (35) \\
b_{t+1} &\leq \xi^d \theta q_{t+1}^d k_t^d + \xi^g \theta q_{t+1}^g k_t^g && (36) \\
d_t &\geq 0. && (37)
\end{aligned}
$$

That is, in the optimal problem, the firm's individual state variables include productivity z_t and net worth w_t at the beginning of time t. We use S_t and S_{t+1} to denote aggregate state variables. At time t, the firm needs to choose dividend d_t, borrowing b_{t+1}, capital investment k_t^d, k_t^g for the next period, and also capital utilization h_t for time t production. Going to the next period, after z_{t+1} is realized, the firm chooses next period's production and repayment. The firm is also subject to the same collateral constraint in the credit market as in the two-period model.

The representative household's preference is:

$$
\max \sum_{t \geq 0} \beta^t (C_t - \phi^d E_t), \tag{38}
$$

and as before, she chooses consumption, saving and capital supplies.

We plan to first calibrate the model, and then explore the dynamic model quantitatively when there different types of green finance instruments, i.e., ex-ante versus ex-post instruments.

6 Discussion on "Ex-ante" versus "Ex-post"

6.1 Green Instruments in Practice

Our research presents a novel framework for categorizing green finance instruments into two distinct classifications: "Ex-ante" versus "Ex-post." This distinction is crucial due to their different implications on dirty capital allocation, highlighting the need for a nuanced discussion on the classification of existing green finance instruments. The fundamental criterion for this classification is the timing of financial incentives or penalties. In ex-ante instruments, incentives or penalties are applied upfront, based on the commitment to future environmental sustainability, whereas ex-post instruments apply incentives or penalties retrospectively, based on past environmental performance.

A carbon tax is a financial mechanism designed to charge emitters for the amount of carbon dioxide they release into the atmosphere, functioning as a critical tool for encouraging the reduction of greenhouse gas emissions. It operates on the principle of making polluters pay for the environmental damage their emissions cause, thereby incentivizing companies and individuals to reduce their carbon footprint through cleaner practices and technologies. The implementation of carbon taxes across the globe has become an increasingly prevalent approach to address climate change. Direct carbon pricing instruments now cover almost a quarter of global greenhouse gas emissions. This demonstrates significant progress from a decade ago when only 7% of global emissions were covered by such policies (World Bank, 2023).

A carbon tax is considered an ex-post instrument because it's applied retroactively, based on the actual emissions firms have produced. This method allows for an accurate reflection of the pollution generated, with taxes typically due at set intervals (e.g., annually or quarterly), based on a firm's carbon emissions. The delayed payment system inherent in this approach reinforces its classification as ex-post, emphasizing the policy's retrospective nature in assessing and addressing environmental impacts.

The carbon credits trading system, part of the broader carbon pricing strategy alongside carbon taxes, offers a dynamic approach to managing emissions. It allows firms to buy or sell emission allowances, providing flexibility and incentivizing reductions in greenhouse gases. This system's design enables companies to strategically plan for emissions reductions, which can potentially serve as an ex-ante approach due to its forward-looking nature.

However, the system's flexibility regarding the timing of trades means it often operates effectively as an ex-post instrument in practice. This is because firms, especially those facing higher emissions or financial limitations, frequently opt to buy credits retrospectively.

This interplay between prospective planning and retrospective adjustment highlights the real-world complexity in achieving desired allocation outcomes.

Carbon offsets are mechanisms for compensating for emissions by funding equivalent carbon dioxide saving projects. While similar to carbon credits in contributing to emission reduction goals, offsets are usually voluntary and can support projects unrelated to the buyer's direct emissions. This flexibility allows entities to support environmental projects globally. However, the voluntary nature and lack of specificity in project relevance can lead to accusations of greenwashing, where companies claim environmental efforts that are not as impactful as presented. Like carbon credits, offsets could be considered ex-ante for their proactive funding of emission reduction projects, but their voluntary aspect and potential for misuse complicate their impact.

Historical performance-based sustainable investing. Recently, sustainable investing has surged in popularity. Investors committed to sustainability actively reallocate their capital from environmentally harmful firms to those with a positive environmental impact, effectively reducing the cost of capital for green firms and increasing it for dirty firms. The prevalent strategy in this movement involves classifying companies as "green" or "dirty" based on their historical environmental performance indicators, such as past emission intensity and ESG scores. This historical-based approach is often chosen due to the challenges in making credible green commitments in advance and concerns over greenwashing, where firms may overstate their environmental commitments. As such, relying on a firm's track record of environmental performance provides a more reliable measure of its commitment to sustainability. Consequently, given its reliance on historical environmental performance to guide investment decisions, sustainable investing inherently adopts an ex-post approach.

Sustainable investing with commitments. Part of the commitment issue stems from the fact that dominant sustainable investing occurs at the firm level, where it's challenging for investors to verify how their funds are being used—whether for financing environmentally friendly projects or those detrimental to the environment. This uncertainty impedes firms from making credible green commitments in advance. However, project financing within the debt market offers a viable solution to this issue. At the project level, the environmental impact assessment becomes more straightforward, enabling sustainable investors to preferentially support green initiatives with financing advantages from the start. Tools such as green debt, green bonds, and policies like coal lending bans exemplify how this strategy can be applied, proving to be effective in addressing climate change (Green and Vallee, 2022). Due to their forward-looking nature, these instruments are categorized as ex-ante instruments.

6.2 "Ex-ante" vs. "Ex-post" beyond Our Model

Understanding the distinctive distributional impacts of 'ex-ante' and 'ex-post' green finance instruments is essential for assessing their overall effectiveness in enhancing aggregate environmental sustainability. However, the real-world implementation of these instruments often faces challenges, and their impacts must be considered across both intensive and extensive margins.

Implementation Issue: While 'ex-ante' instruments are theoretically more allocatively efficient, their practical implementation faces significant challenges. The primary issue lies in the incomplete contracts and the difficulty for firms to credibly commit to the greenness of their projects. This lack of commitment can be attributed to the inherent uncertainty and complexity in predicting the environmental impact of a project in its nascent stages. The absence of standardized metrics and benchmarks for greenness complicates this process, making it harder to distinguish genuinely sustainable projects from those that are not.

In contrast, 'ex-post' instruments are generally easier to implement due to their reliance on past performance metrics. The retrospective nature of these instruments simplifies the validation process, as it is based on tangible, historical data. This ease of implementation contributes to the prevalence of 'ex-post' strategies in current green instruments and investment strategies. Nonetheless, project-specific financing, an 'ex-ante' approach, is gaining more popularity in the green debt market, as evidenced by Green and Vallee (2022).

Intensive Margin vs. Extensive Margin: Under our framework, firms are faced with the choice between existing green and dirty technologies, which is essentially an intensive margin decision. In this context, ’ex-ante’ instruments are more effective as they shift dirty capital from financially constrained firm towards unconstrained firms. By providing upfront incentives, these instruments make green technologies more accessible, especially for financially constrained firms.

However, the potential benefits of ’ex-post’ instruments at the extensive margin are not captured in our model. These green instruments predominantly impact financially unconstrained firms, potentially spurring green innovation. Innovation, typically undertaken by less financially constrained entities, is incentivized under ’ex-post’ instruments. These firms, motivated by the prospect of future benefits based on their environmental performance, are likely to invest in new, cleaner technologies. This innovation not only enhances the firm’s green portfolio but also has the potential for broader spillover effects, contributing to the overall greening of the economy.

In conclusion, while ’ex-ante’ instruments are more efficient in driving immediate technology adoption, especially for constrained firms, ’ex-post’ instruments play a critical role in fostering long-term green innovation. This underscores the importance of a balanced policy approach that addresses both the immediate adoption needs of existing technologies (intensive margin) and the longer-term innovation incentives (extensive margin) to achieve comprehensive environmental sustainability.

7 Conclusion

Our study reveals that a more thoughtful application of green finance instruments is crucial. ’Ex-ante’ instruments, which provide upfront subsidies for green technologies, are shown to be more allocatively efficient, effectively guiding financially constrained firms towards cleaner technologies. On the other hand, ’ex-post’ instruments like carbon taxes tend to be less efficient in this regard, often shifting dirty capital towards these constrained firms and potentially leading to higher overall emissions due to their higher emission intensity. This highlights the necessity of designing green finance instruments that are specifically tailored to the varied financial conditions of firms, ensuring a more effective and sustainable environmental impact.

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Tables and Figures

Table 1: Classification of Green Finance Instruments into Ex-ante and Ex-post Categories

Green Instruments	Ex-post Instruments	Ex-ante Instruments
Carbon Tax	✓	
Carbon Credits Trading System*		✓
Carbon Offsets*		✓
Historical Performance-Based Sustainable Investing	✓	
Sustainable Investing with Commitments		✓

This table classifies various green finance instruments into either ex-ante or ex-post categories based on the timing of their implementation and impact. Instruments marked with an asterisk (*) are theoretically ex-ante, due to their proactive approach, but can function effectively as ex-post in practice.

Table 2: The Impact of Green Instruments under Financial Constraints

Instruments	Ex-post Instruments		Ex-ante Instruments	
Firm Type	Constrained	Unconstrained	Constrained	Unconstrained
Direct Effect	$u^d \uparrow, \varphi^d \rightarrow, u^g \downarrow, \varphi^g \rightarrow$		$u^d \rightarrow, \varphi^d \uparrow, u^g \rightarrow, \varphi^g \downarrow$	
Indirect Effect	$q^d \downarrow, q^g \uparrow$		$q^d \downarrow, q^g \uparrow$	
User Cost	$U^d \uparrow, U^g \downarrow$		$U^d \downarrow, U^g \uparrow$	
Down Payment	$\Phi^d \downarrow, \Phi^g \uparrow$		$\Phi^d \uparrow, \Phi^g \downarrow$	
Relative Total Cost	$\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda} \downarrow$	$\frac{U^d}{U^g} \uparrow$	$\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda} \uparrow$	$\frac{U^d}{U^g} \downarrow$
Capital Investment	$k^d \uparrow, k^g \downarrow$	$k^d \downarrow, k^g \uparrow$	$k^d \downarrow, k^g \uparrow$	$k^d \uparrow, k^g \downarrow$

This table summarizes the impact of green instruments on highly constrained firms (large λ) and unconstrained firms ($\lambda = 0$). $\uparrow$ indicates an increase, $\downarrow$ indicates a decrease, $\rightarrow$ indicates unchanged.

Table 3: Parameters for the Two-period Model

Parameters	Meaning	Value
β	Firm discount factor	0.95
R	Gross interest rate	$1/\beta$
σ	Elasticity of substitution between k^d and k^g	40.0
γ	Share of k^d	0.5
w_0	Firm initial net worth	5.0
α	Decreasing return to scale	0.85
θ	Fraction of capital as collateral	0.4
$c^d(h)$	Utilization cost of k^d	$c^d(h) = 1.0 + 30(h-1)$
$c^g(h)$	Utilization cost of k^g	$c^g(h) = 0.8 + 30(h-1)$
$\chi^d(K^d)$	K^d Capital supply cost	$\chi^d(K^d) = 0.5 + 2.0K^d$
$\chi^g(K^g)$	K^g Capital supply cost	$\chi^g(K^g) = 0.5 + 2.0K^g$

This table summarizes the parameters for the two-period model.

Figure 1: **User Cost versus. Down Payment**

This figure illustrates the relative total costs of dirty versus green capital, represented by the equation $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$, as a function of varying levels of the firm's multiplier, denoted by λ. Detailed parameters underlying this model are presented in Table 3.

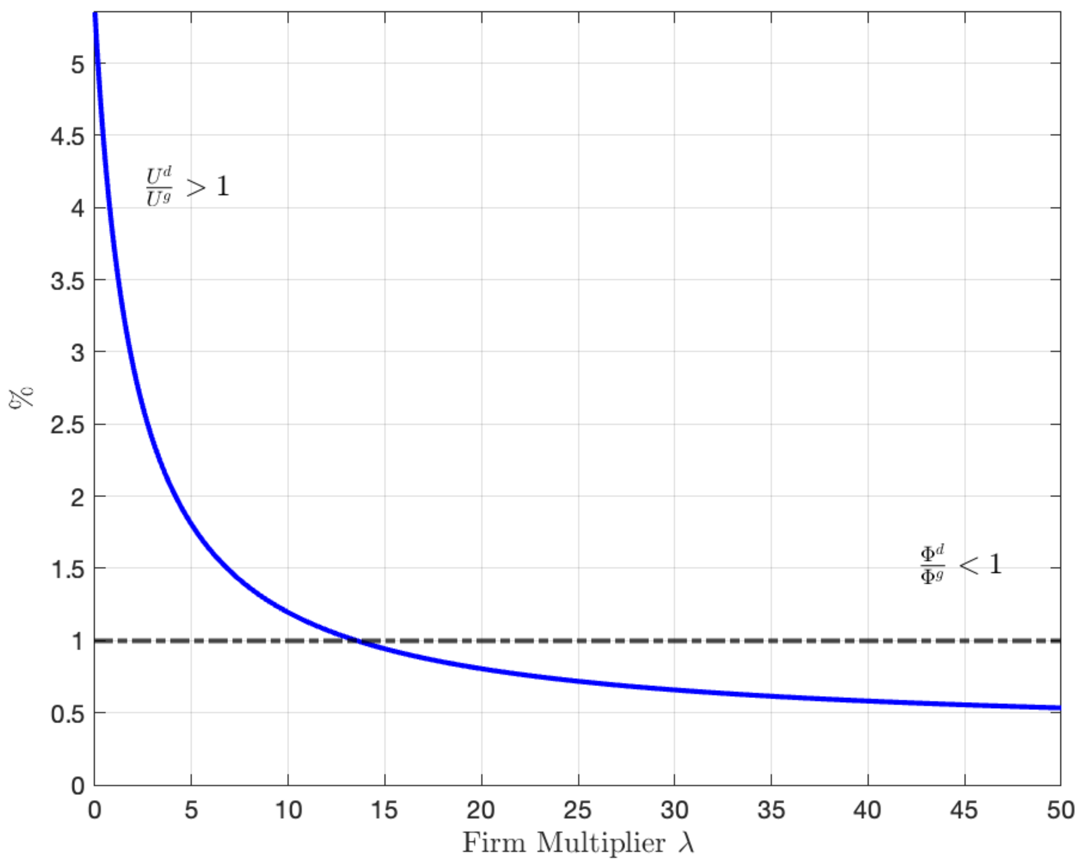


Figure 2: **Capital Choice without Green Instruments**

This figure presents firm capital choices in the benchmark case without green instruments. It is divided into four panels: Panel (a) shows the firm's multiplier, λ; Panel (b) presents the relative importance of down payment, $\frac{\Phi^d\lambda}{U^d+\Phi^d\lambda}$; Panel (c) depicts the relative total cost of dirty versus green, $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$; and Panel (d) illustrates the dirty to green capital ratio, $\frac{k^d}{k^g}$, all as functions of firm productivity. Detailed parameters for the underlying model are provided in Table 3.

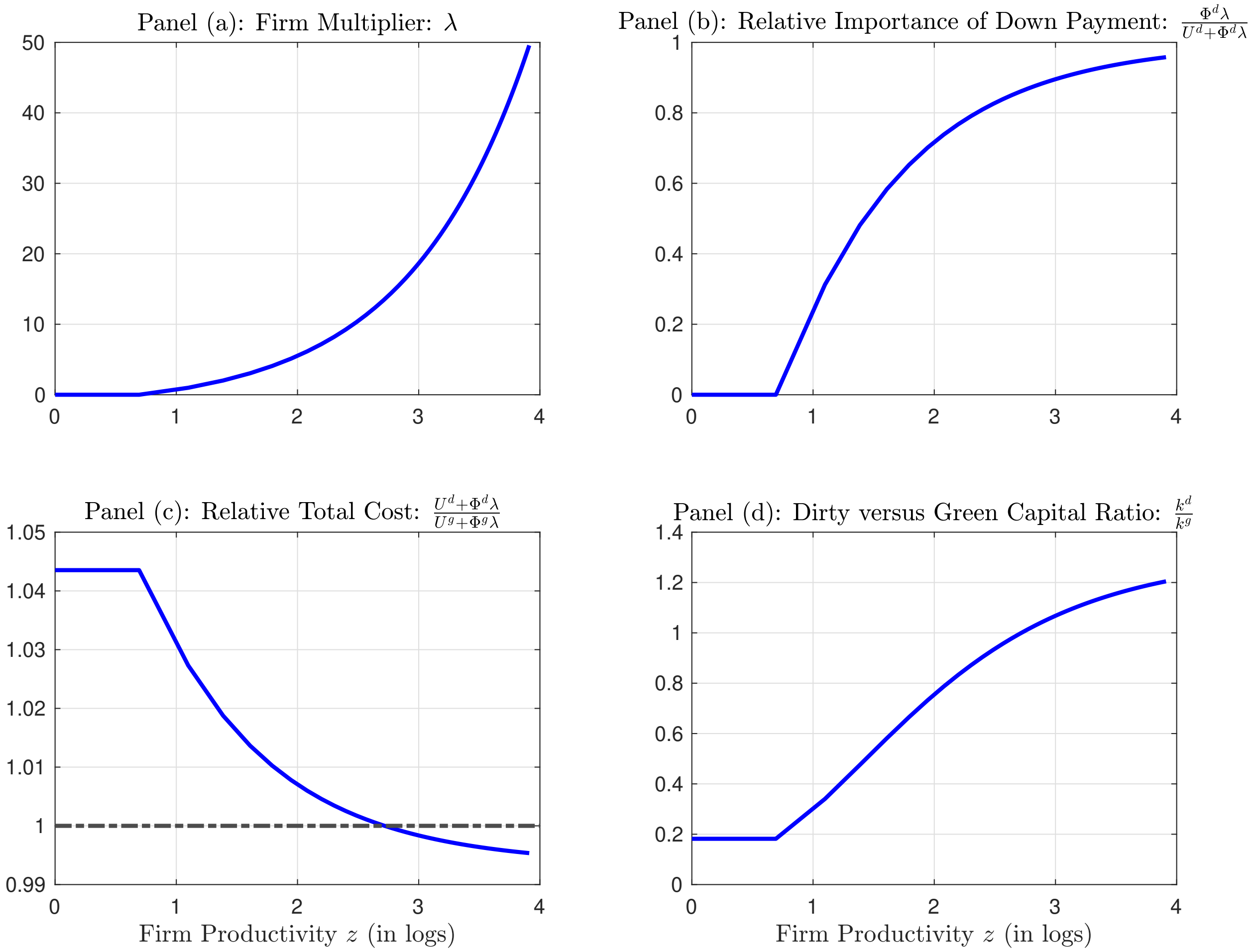


Figure 3: **Distributional Effect: Ex-post Instrument**

This figure illustrates how firm capital choices are influenced by a carbon tax ($\tau^d = 0.5$). It is divided into four panels: Panel (a) compares the relative total costs of dirty versus green capital, expressed as $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$; Panel (b) shows the ratio of dirty to green capital, $\frac{k^d}{k^g}$; Panel (c) depicts the percentage change in dirty capital investment relative to the benchmark; and Panel (d) illustrates the percentage change in green capital investment relative to the benchmark, all as functions of firm productivity. The benchmark case results are represented by a blue curve for comparison. The impact of the carbon tax is shown in two ways: the dashed red curve represents the partial equilibrium effect, assuming constant prices for dirty and green capital as in the benchmark case, while the solid red curve indicates the general equilibrium effect, including changes in capital prices due to the tax. Detailed parameters for the underlying model are provided in Table 3.

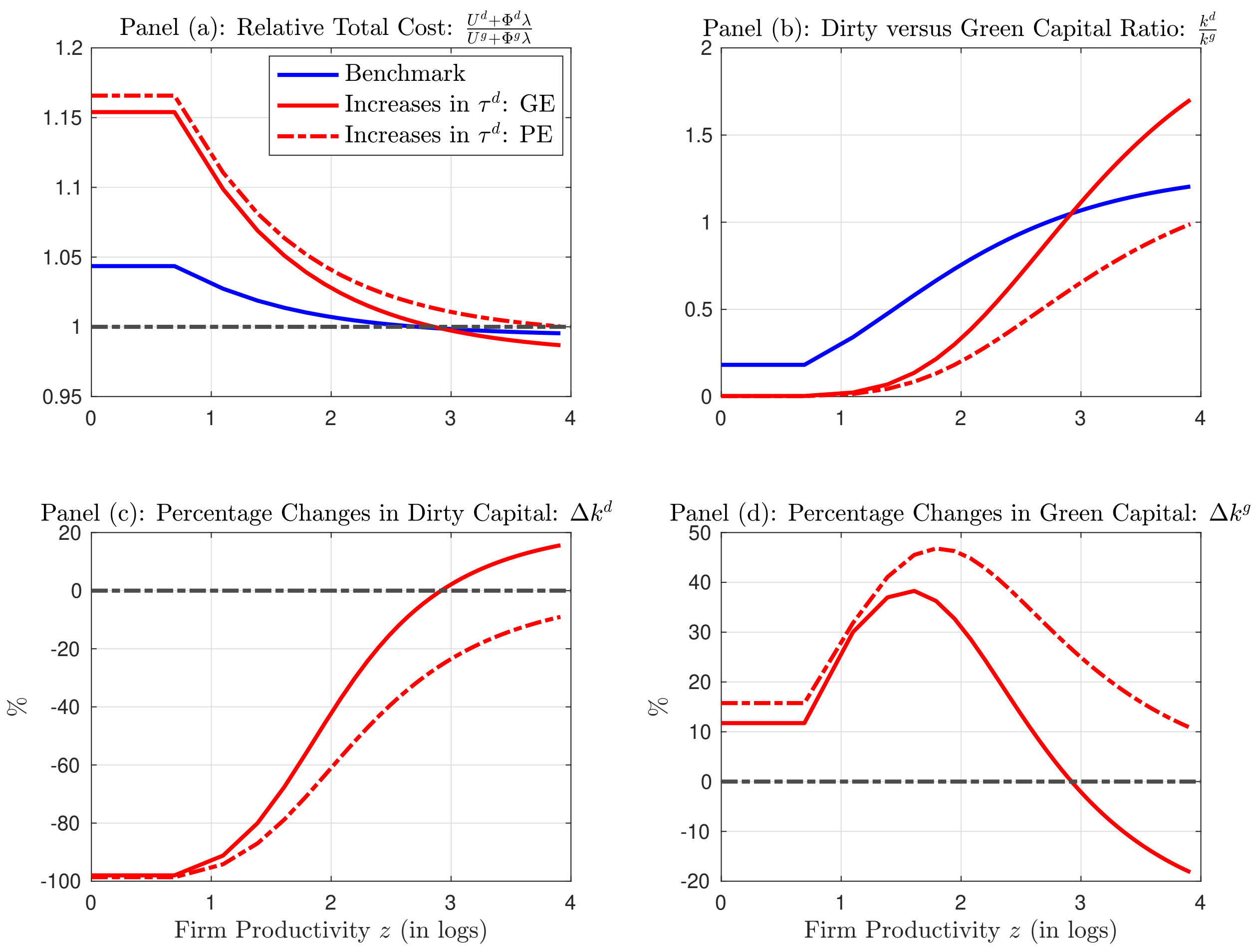


Figure 4: **Distributional Effect: Ex-ante Instrument**

This figure illustrates how firm capital choices are influenced by a dirty capital lending ban ($\xi^d = 0.8$). It is divided into four panels: Panel (a) compares the relative total costs of dirty versus green capital, expressed as $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$; Panel (b) shows the ratio of dirty to green capital, $\frac{k^d}{k^g}$; Panel (c) depicts the percentage change in dirty capital investment relative to the benchmark; and Panel (d) illustrates the percentage change in green capital investment relative to the benchmark, all as functions of firm productivity. The benchmark case results are represented by a blue curve for comparison. The impact of the lending ban is shown in two ways: the dashed red curve represents the partial equilibrium effect, assuming constant prices for dirty and green capital as in the benchmark case, while the solid red curve indicates the general equilibrium effect, including changes in capital prices due to the lending ban. Detailed parameters for the underlying model are provided in Table 3.

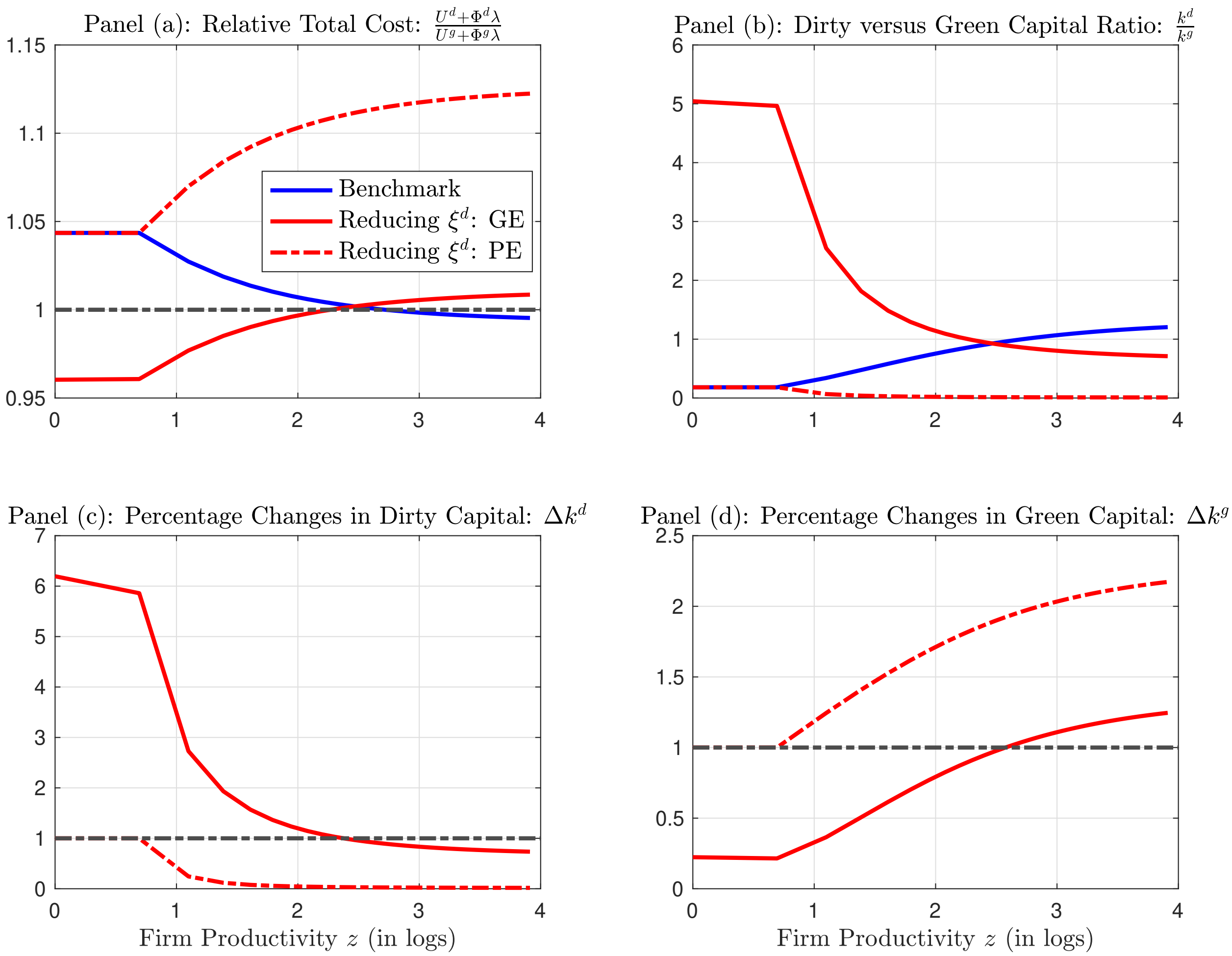


Figure 5: **Impact Elasticity**

This figure illustrates the impact elasticity, defined as the environmental improvement resulting from a one-unit injection of net worth, across various firms in three distinct economies. We explore three scenarios: the benchmark equilibrium without any policy interventions, the equilibrium with carbon tax ($\tau^d = 0.5$), and the equilibrium with dirty capital lending ban ($\xi^d = 0.8$). The left column shows the dirty to green capital ratio for firms within each economy. For each scenario, capital prices are held constant at their original equilibrium values, and a 1% increase in firm net worth is introduced. The resulting changes in the ratio of dirty to green capital (k^d/k^g) are depicted in the right column. Detailed parameters for the underlying model are provided in Table 3.

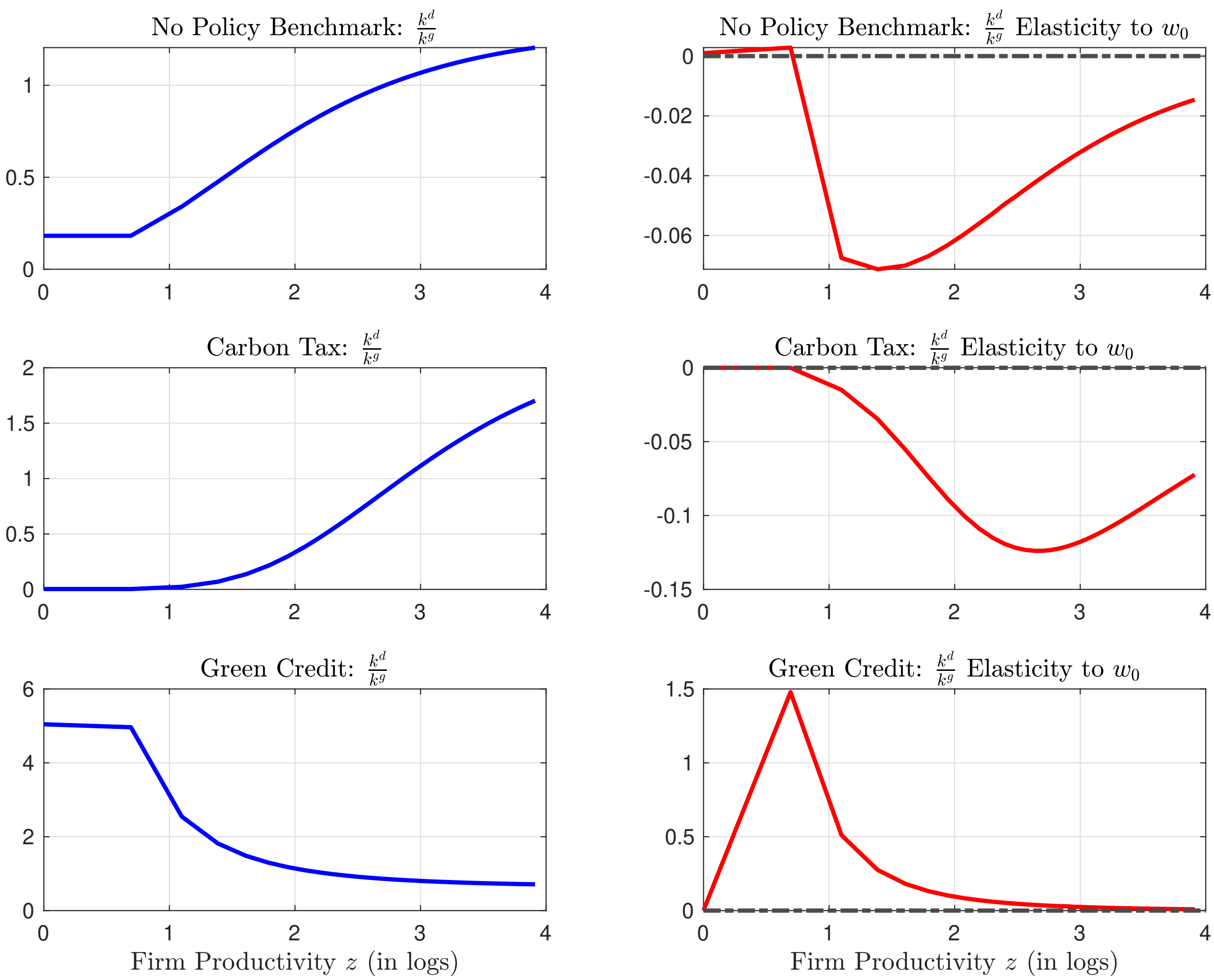


Figure 6: **Aggregate Effect, Ex-post Instrument**

This figure shows the aggregate impact of carbon tax. It is divided into four panels: Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E = \int h_i k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output; Panel (c) illustrates average dirty capital utilization intensity, $H^d = \frac{\int h_i k_i^d}{\int k_i^d}$; Panel (d) decompose the change in total emissions relative to the benchmark into two components: the allocative effect, calculated as $K^d H^d - K^d H^d_{bench}$, and the level effect, calculated as $K^d H^d_{bench} - K^d_{bench} H^d_{bench}$. Each variable is plotted against the carbon tax rate, τ^d. Detailed parameters for the underlying model are provided in Table 3.

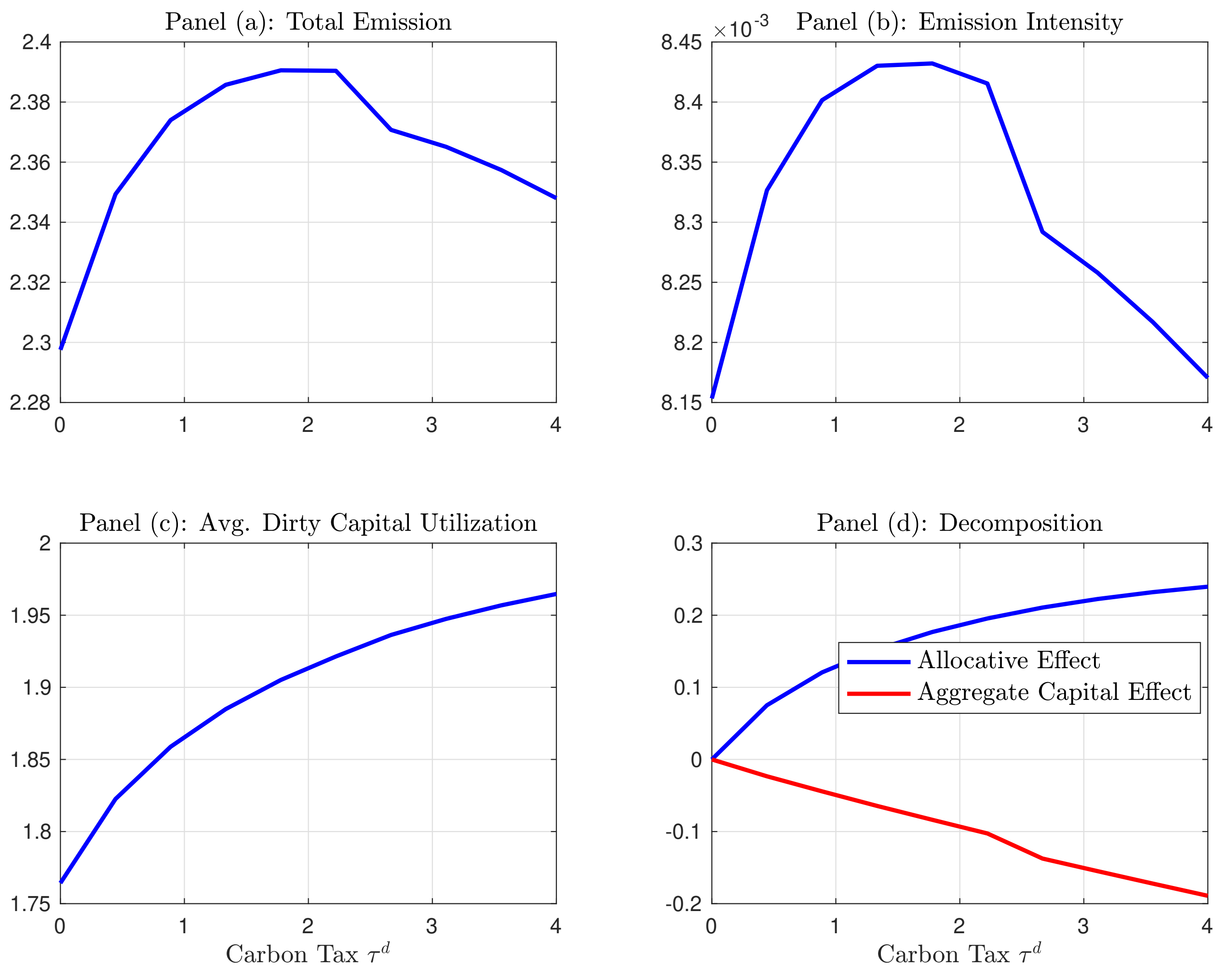
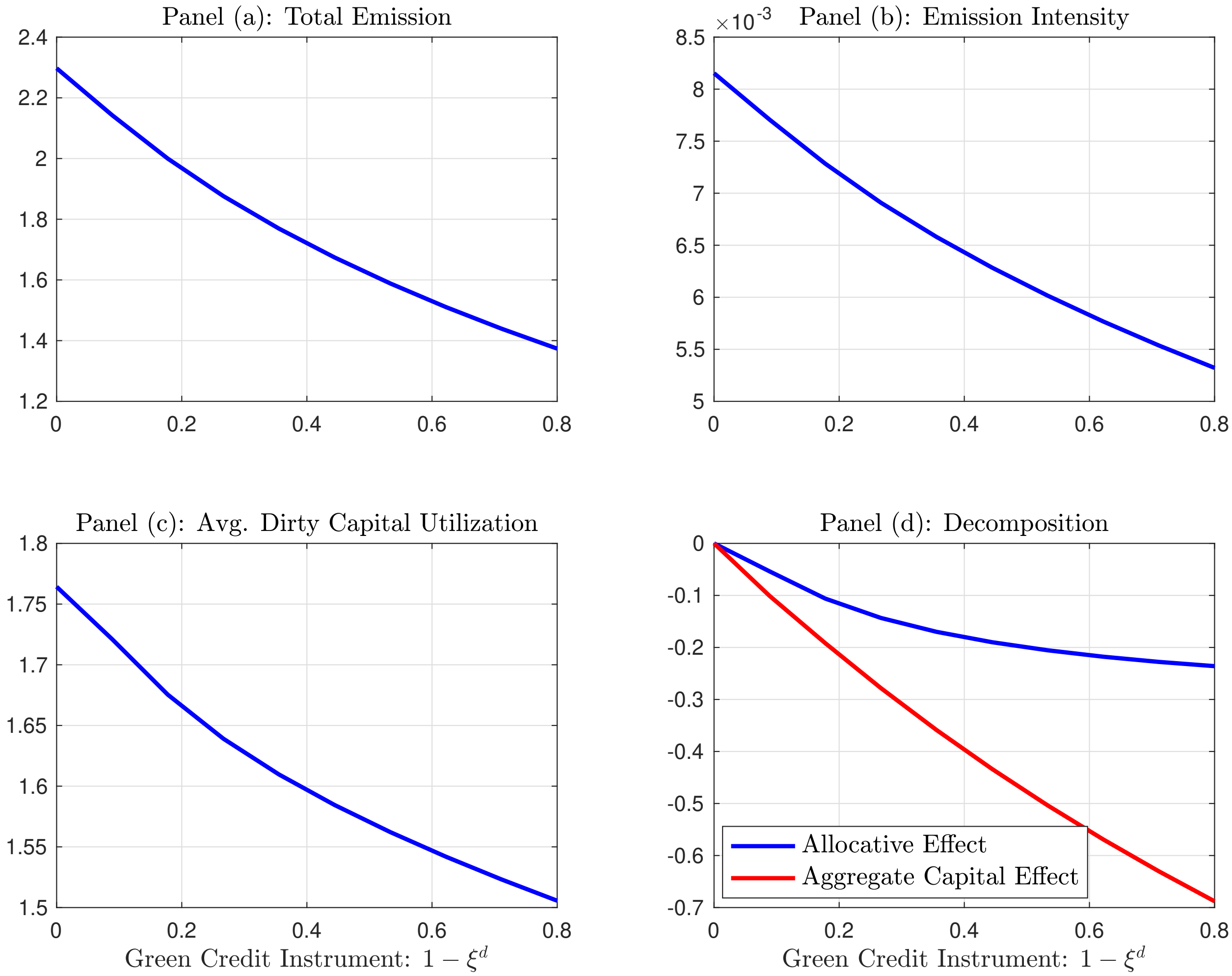


Figure 7: **Aggregate Effect, Ex-ante Instrument**

This figure illustrates the aggregate impact of a dirty capital lending ban, represented by $1-\xi^d$, where a higher value indicates lower collateralizability of dirty capital. It is divided into four panels: Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E=\int h_i k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output; Panel (c) illustrates average dirty capital utilization intensity, $H^d=\frac{\int h_i k_i^d}{\int k_i^d}$; Panel (d) decompose the change in total emissions relative to the benchmark into two components: the allocative effect, calculated as $K^d H^d - K^d H^d_{bench}$, and the level effect, calculated as $K^d H^d_{bench} - K^d_{bench} H^d_{bench}$. Each variable is plotted against the green credit instrument, $1-\xi^d$. Detailed parameters for the underlying model are provided in Table 3.



A Capital Utilization

Denote $\underline{\nu}$ and $\overline{\nu}$ as the lower and upper bounds of capital utilization intensity, the first order condition incorporating variable capital utilization are:

$$\begin{aligned}
d_0 &: \mu = 1 + \eta, && (39)\\
k^d &: \mu q^d - \lambda \xi^d \theta q^d = \beta \left[F' h g_1 - c^d(h) - \tau^d h\right], && (40)\\
k^g &: \mu q^g - \lambda \xi^g \theta q^g = \beta \left[F' h g_2 - c^g(h) - \tau^g h\right], && (41)\\
h &: F' k = c^{d\prime}(h) k^d + c^{g\prime}(h) k^g + \overline{\nu} - \underline{\nu}, && (42)\\
b &: \mu = 1 + \lambda., && (43)
\end{aligned}$$

Proof of Lemma 1. Consider unconstrained firms ($\lambda = 0$), which operate under the first-order conditions (40)-(42) for k^d, k^g, and h. These conditions yield:

$$q^d k^d + q^g k^g = \beta \left[c^{d\prime}(h)h - c^d(h) - \tau^d h\right] k^d + \beta \left[c^{g\prime}(h)h - c^g(h) - \tau^g h\right] k^g + h\overline{\nu} - h\underline{\nu} \quad (44)$$

Rearranging, we get:

$$h\overline{\nu} - h\underline{\nu} = q^d k^d + q^g k^g - \beta \left[c^{d\prime}(h)h - c^d(h) - \tau^d h\right] k^d - \beta \left[c^{g\prime}(h)h - c^g(h) - \tau^g h\right] k^g \quad (45)$$

The right-hand side (RHS) is a weakly decreasing function of h because its first-order derivative, $-[\beta c^{d\prime\prime}(h)h + \tau^d]k^d - [\beta c^{g\prime\prime}(h)h + \tau^g]hk^g$, is non-positive. Therefore, the RHS is bounded above by:

$$\begin{aligned}
&q^d k^d + q^g k^g - \beta \left[c^{d\prime}(1)k^d + c^{g\prime}(1)k^g - c^d(1)k^d - c^g(1)k^g\right] \\
&= U^d(1)k^d + U^g(1)k^g - \beta c^{d\prime}(1)k^d - \beta c^{g\prime}(1)k^g < 0
\end{aligned} \quad (46)$$

which implies $\underline{\nu} > 0$. Therefore, unconstrained firms will choose $h = \underline{h} = 1$. □

Proof of Lemma 2. Consider highly constrained firms, characterized by a large value of λ (i.e., $\lambda \gg 0$), which operate under the first-order conditions (40)-(42) for k^d, k^g, and h.

These conditions yield:

$$
\begin{aligned}
&\left[q^d + (1-\theta)q^d\lambda\right] k^d + \left[q^g + (1-\theta)q^g\lambda\right] k^g \\
=&\beta \left[c^{d\prime}(h)h - c^d(h) - \tau^d h\right] k^d + \beta \left[c^{g\prime}(h)h - c^g(h) - \tau^g h\right] k^g + h\overline{\nu} - h\underline{\nu}
\end{aligned}
\tag{47}
$$

Rearranging, we obtain:

$$
\begin{aligned}
h\overline{\nu} - h\underline{\nu} = &\left[q^d + (1-\theta)q^d\lambda\right] k^d + \left[q^g + (1-\theta)q^g\lambda\right] k^g \\
&- \beta \left[c^{d\prime}(h)h - c^d(h) - \tau^d h\right] k^d - \beta \left[c^{g\prime}(h)h - c^g(h) - \tau^g h\right] k^g
\end{aligned}
\tag{48}
$$

The right-hand side (RHS) of this equation is a weakly decreasing function of h because its first-order derivative, $-[\beta c^{d\prime\prime}(h)h + \tau^d]k^d - [\beta c^{g\prime\prime}(h)h + \tau^g]hk^g$, is non-positive. As a result, the RHS is bounded below by its value at $\overline{h}$:

$$
\begin{aligned}
&\left[q^d + (1-\theta)q^d\lambda\right] k^d + \left[q^g + (1-\theta)q^g\lambda\right] k^g \\
&- \beta \left[c^{d\prime}(\overline{h})\overline{h} - c^d(\overline{h}) - \tau^d \overline{h}\right] k^d - \beta \left[c^{g\prime}(\overline{h})\overline{h} - c^g(\overline{h}) - \tau^g \overline{h}\right] k^g
\end{aligned}
\tag{49}
$$

For firms with sufficiently small initial net worth, where λ is large, it follows that RHS is greater than zero, implying that $\overline{\nu} > 0$. Consequently, highly constrained firms will choose the utilization intensity $h = \overline{h}$. □