Internet Appendix for
The Misallocation of Finance

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This appendix contains four sections. Section I presents a model that extends our baseline model by allowing labor to be part of the production function, along with debt and equity. It also presents reallocation estimates using this alternative model. Section II demonstrates that our empirical methods can recover financial frictions and misallocation, using a standard dynamic capital structure model as a laboratory. Section III outlines our estimation of the elasticity of substitution between debt and equity. Section IV contains supplementary tables and figures.

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I. Model with Labor

In this appendix, we extend our basic static model to encompass labor, where all notation has been defined in the main text. Specifically, we reformulate the model under the assumption that debt and equity are used to back capital accumulation, so a CES aggregation of debt and equity results in productive capital. As we show below, this notion of productive capital is different from the actual amount of physical capital. We then assume that productive capital and labor are combined in a Cobb-Douglas production function.

A. Assumptions

First, we assume that the real benefit for firm i in sector s is given by

$$F_{si} = A_{si} \left(\alpha_s D_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1 - \alpha_s) E_{si}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\beta_s \gamma_s}{\gamma_s - 1}} L_{si}^{1-\beta_s}, \tag{IA.1}$$

where $\alpha_s = D_s^{1/\gamma_s} / (D_s^{1/\gamma_s} + E_s^{1/\gamma_s})$. Next, we assume that the firm minimizes the cost of production,

$$\min_{\{D_{si}, E_{si}, L_{si}\}} \{(1 + \tau_{D_{si}}) r_{si} D_{si} + (1 + \tau_{E_{si}}) \lambda_{si} E_{si} + (1 + \tau_{L_{si}}) w L_{si}\}, \tag{IA.2}$$

subject to a certain level of output,

$$A_{si} \left(\alpha_s D_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1 - \alpha_s) E_{si}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\beta_s \gamma_s}{\gamma_s - 1}} L_{si}^{1-\beta_s} = \bar{F}_{si}. \tag{IA.3}$$

Note that r_{si} and λ_{si} are firm-specific flexible prices, while w is an economy-wide fixed price.

B. Analysis

First, we derive the optimal debt-equity ratio and let ν_{si} denote the Lagrange multiplier. The first-order condition with respect to D_{si} is given by

$$\begin{aligned} 0 &= \frac{\partial}{\partial D_{si}}\left[\left(1+\tau_{D_{si}}\right) r_{si} + \left(1+\tau_{E_{si}}\right)\lambda_{si} E_{si}\right] \\ &\quad + \nu_{si}\alpha_s\beta_s D_{si}^{-\frac{1}{\gamma_s}} A_{si}\left(\alpha_s D_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1-\alpha_s) E_{si}^{\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{\beta_s\gamma_s-(\gamma_s-1)}{\gamma_s-1}} L_{si}^{1-\beta_s}. \end{aligned} \tag{IA.4}$$

Similarly, the first-order condition with respect to E_{si} is given by

$$\begin{aligned} 0 &= \frac{\partial}{\partial E_{si}}\left[\left(1+\tau_{D_{si}}\right) r_{si} + \left(1+\tau_{E_{si}}\right)\lambda_{si} E_{si}\right] \\ &\quad + \nu_{si}(1-\alpha_s)\beta_s E_{si}^{-\frac{1}{\gamma_s}} A_{si}\left(\alpha_s D_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1-\alpha_s) E_{si}^{\frac{\gamma_s-1}{\gamma_s}}\right)^{\frac{\beta_s\gamma_s-(\gamma_s-1)}{\gamma_s-1}} L_{si}^{1-\beta_s}. \end{aligned} \tag{IA.5}$$

To find the optimal debt-equity ratio, we combine (IA.4) and (IA.5) to arrive at

$$Z_{si} \equiv \frac{D_{si}}{E_{si}} = \left[\frac{\alpha_s}{1-\alpha_s}\frac{\frac{\partial}{\partial E_{si}}\left[\left(1+\tau_{D_{si}}\right) r_{si} D_{si} + \left(1+\tau_{E_{si}}\right)\lambda_{si} E_{si}\right]}{\frac{\partial}{\partial D_{si}}\left[\left(1+\tau_{D_{si}}\right) r_{si} D_{si} + \left(1+\tau_{E_{si}}\right)\lambda_{si} E_{si}\right]}\right]^{\gamma_s}. \tag{IA.6}$$

A comparison with the benchmark model shows that we have the same optimal debt-equity ratio as before.

Next, we derive the optimal capital-labor ratio. We start with the original minimization problem given by (IA.2), but use (IA.6) to substitute D_{si} with a function of E_{si}, which yields

$$\min_{\{E_{si}, L_{si}\}} \left\{(1+\tau_{D_{si}}) r_{si} Z_{si} E_{si} + (1+\tau_{E_{si}})\lambda_{si} E_{si} + (1+\tau_{L_{si}}) w L_{si}\right\}. \tag{IA.7}$$

We define actual, as opposed to productive, capital, K_{si}, as

$$K_{si} \equiv D_{si} + E_{si} = \left(Z_{si}+1\right) E_{si}. \tag{IA.8}$$

Substituting K_{si} for E_{si} in (IA.7) thus yields

$$\min_{\{K_{si},L_{si}\}} \left\{ \frac{(1+\tau_{D_{si}})r_{si}Z_{si} + (1+\tau_{E_{si}})\lambda_{si}}{Z_{si}+1} K_{si} + (1+\tau_{L_{si}})wL_{si} \right\}. \tag{IA.9}$$

To simplify exposition, we define θ_{si} as the term multiplying K_{si} in (IA.9):

$$\theta_{si} \equiv \frac{(1+\tau_{D_{si}})r_{si}Z_{si} + (1+\tau_{E_{si}})\lambda_{si}}{Z_{si}+1}. \tag{IA.10}$$

Substituting (IA.6) into (IA.10), we obtain

$$\theta_{si} = \frac{(1+\tau_{D_{si}})r_{si}D_{si} + (1+\tau_{E_{si}})\lambda_{si}E_{si}}{D_{si}+E_{si}},$$

which demonstrates that θ_{si} is the weighted average cost of capital.

We now turn to the constraint in (IA.1), rewriting it by using (IA.6) to replace D_{si} with a function of E_{si}

$$F_{si} = A_{si} \left(\alpha_s Z_{si}^{\frac{\gamma_s - 1}{\gamma_s}} E_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha_s) E_{si}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\beta_s \gamma_s}{\gamma_s - 1}} L_{si}^{1-\beta_s}. \tag{IA.11}$$

As before, we can use the definition in (IA.8) to replace E_{si} with K_{si} in (IA.11), which yields

$$F_{si} = A_{si} \frac{\left(\alpha_s Z_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha_s) \right)^{\frac{\beta_s \gamma_s}{\gamma_s - 1}}}{(Z_{si}+1)^{\beta_s}} K_{si}^{\beta_s} L_{si}^{1-\beta_s}. \tag{IA.12}$$

To interpret (IA.12), we define

$$\psi_{si} \equiv \frac{\left(\alpha_s Z_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha_s) \right)^{\frac{\beta_s \gamma_s}{\gamma_s - 1}}}{(Z_{si}+1)^{\beta_s}} = \left[\frac{\left(\alpha_s D_{si}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha_s) E_{si}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\gamma_s}{\gamma_s - 1}}}{D_{si}+E_{si}} \right]^{\beta_s}, \tag{IA.13}$$

which allows us to simplify (IA.12) as

$$F_{si} = A_{si}\psi_{si}K_{si}^{\beta_s}L_{si}^{1-\beta_s}. \tag{IA.14}$$

A comparison of (IA.1) with the representation in (IA.14) shows that ψ_{si} can be interpreted as the efficiency of debt and equity in generating capital. Put differently, the term in parentheses in (IA.1) does not represent the actual capital stock, but (approximately) the capital stock times its efficiency. The expression for ψ_{si} in (IA.13) is also interesting in that it shows that debt and equity are more efficient if their ratio is closer to the optimal ratio.

We can now use (IA.9) and (IA.14) to derive the optimal levels of capital and labor. The first-order condition with respect to K_{si} is

$$\theta_{si} - \phi_{si}\beta_s A_{si}\psi_{si}K_{si}^{\beta_s-1}L_{si}^{1-\beta_s} = 0, \tag{IA.15}$$

where ϕ_{si} is the Lagrange multiplier. The first-order condition with respect to L_{si} is

$$(1+\tau_{L_{si}})w - \phi_{si}(1-\beta_s)A_{si}\psi_{si}K_{si}^{\beta_s}L_{si}^{-\beta_s} = 0. \tag{IA.16}$$

Combining these two first-order conditions gives the optimal capital-labor ratio

$$\frac{K_{si}}{L_{si}} = \frac{(1+\tau_{L_{si}})w}{\theta_{si}}\frac{\beta_s}{1-\beta_s}. \tag{IA.17}$$

C. Reallocation

Finally, we can use this model to compute the gains from reallocation. Let $\hat{K}_{si}$ and $\hat{L}_{si}$ denote the levels of capital and labor after reallocation. We set the wedges to zero so that the optimal capital-to-labor ratio becomes

$$\frac{\hat{K}_{si}}{\hat{L}_{si}} = \frac{w}{u_s}\frac{\beta_s}{1-\beta_s} = \frac{K_s}{L_s}, \tag{IA.18}$$

where u_s is a constant sector-level weighted average cost of capital. The weighted average cost of capital comes from the simplification of θ_{si}, and we assume that all firms in the sector face the same undistorted cost. The total amount of labor and capital within a sector are the same before and after reallocation. Therefore, β_s is the capital share of income, that is,

$$\beta_s = \frac{u_s K_s}{u_s K_s + w L_s}. \tag{IA.19}$$

Capital and labor thus sum to

$$\hat{K}_{si} + \hat{L}_{si} = \bar{Y}_{si}, \tag{IA.20}$$

where $\bar{Y}_{si}$ is the fixed total amount of inputs.

From the firm first-order conditions, we find that the optimal capital-labor ratio after reallocation is

$$\frac{\hat{K}_{si}}{\hat{L}_{si}} = \left(\frac{\beta_s}{1-\beta_s}\right) = \frac{K_s}{L_s}, \tag{IA.21}$$

since we assume that $\beta_s = (D_s + E_s)/(D_s + E_s + L_s)$.

The efficient level of production after reallocation, $\hat{F}_{si}$, can be written as a function of labor after reallocation, $\hat{L}_{si}$:

$$\hat{F}_{si} = A_{si}\psi_s \left(\frac{D_s + E_s}{L_s}\right)^{\beta_s} \hat{L}_{si}. \tag{IA.22}$$

At an optimal allocation, in every sector the following expression is maximized:

$$\max_{\hat{L}_{si}} \left[\sum_i \left(A_{si}\psi_s \left(\frac{D_s + E_s}{L_s}\right)^{\beta_s} \hat{L}_{si}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{IA.23}$$

subject to

$$\sum_i \hat{L}_{si} = L_s. \tag{IA.24}$$

The first-order conditions with respect to $\hat{L}_{si}$ and $\hat{L}_{sj}$ of firm i and j, respectively, combine to give

$$\left(\frac{\hat{L}_{si}}{\hat{L}_{sj}}\right)^{-\frac{1}{\sigma}} = \left(\frac{A_{sj}}{A_{si}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{IA.25}$$

This last condition has to hold for all firms i and j, so after aggregation we get the expression

$$\hat{L}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} L_s. \tag{IA.26}$$

Similarly, we can derive

$$\hat{K}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} K_s. \tag{IA.27}$$

Finally, we can split reallocated capital back into reallocated debt,

$$\hat{D}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} D_s, \tag{IA.28}$$

and reallocated equity,

$$\hat{E}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} E_s. \tag{IA.29}$$

D. Estimation

We can use this alternative framework to construct an analysis directly analogous to the computations in Table IV. Here, we use the same calibration as for computing Table IV, with one addition: we set labor's share of income to two-thirds for the United States and one-half for China. The results are in Table IA.V. Not surprisingly, given the similarity between the solution for this alternative model with labor and our original model in Section I, the results are quite similar to those in Table IV.

II. Dynamic Model

This section outlines a simple dynamic capital structure model that closely follows Bazdresch, Kahn, and Whited (2018), except that we augment this structure to allow for two types of capital, one that can be used as collateral and one that cannot. The purpose is to provide a laboratory in which to determine whether our static model can detect any frictions that are present in a stochastic, dynamic environment. Specifically, the goal is to see whether the estimation methods laid out in Section I can detect misallocation when frictions are actually present, and whether the measures of misallocation are increasing in the magnitude of modeled financial frictions. All notation in this section is internal to this section.

The environment is an equilibrium economy, with a representative consumer and a unit continuum of firms. Each of these infinitely lived firms uses two types of capital and labor in a stochastic, decreasing returns-to-scale technology to generate output, Y, according to

$$Y = z^{\nu} \left(\left(K_t^{\beta} K_r^{1-\beta} \right)^{\phi} L_d^{1-\phi} \right)^{\theta}, \tag{IA.30}$$

where K_t is the stock of tangible capital, K_r is the stock of intangible capital, L_d is labor demand, and z is a profitability shock. The parameter $0 < \theta < 1$ governs returns to scale. As is standard, we normalize the parameter ν to be $1 - (1 - \phi)\theta$. The profitability shock, z, is lognormally distributed and follows a process given by

$$\ln(z') = \rho \ln(z) + \sigma_z \varepsilon', \qquad \varepsilon' \sim \mathcal{N}(0, 1), \tag{IA.31}$$

where a prime indicates the subsequent period, no prime indicates the current period, and $\mathcal{N}$ indicates the standard normal distribution.

Investment in tangible and intangible capital, I_t and I_r, are defined by standard capital stock accounting identities

$$K_t' \equiv (1-\delta_t)K_t + I_t, \tag{IA.32}$$

$$K_r' \equiv (1-\delta_r)K_r + I_r, \tag{IA.33}$$

where δ_t is the rate of tangible capital depreciation and δ_r is the rate of intangible capital depreciation. The prices of both capital goods have been normalized to one. Adjusting both capital stocks incurs quadratic costs that take the form

$$\psi(K_t, K_t') = \frac{\psi_t(K_t' - (1-\delta_t)K_t)^2}{2K_t} \tag{IA.34}$$

$$\psi(K_r, K_r') = \frac{\psi_r(K_r' - (1-\delta_r)K_r)^2}{2K_r}, \tag{IA.35}$$

where ψ_t and ψ_r are the parameters that govern the magnitude of adjustment costs.

Let P_s denote the outstanding stock of debt and τ denote the corporate tax rate. The firm's cash flow, $E^*(K_t, K_r, P_s, K_t', K_r', P_s', z)$, is then its operating income plus net debt issuance, minus net expenditure on investment, and minus tax-deductible interest payments on debt, as follows:

$$\begin{aligned} E^*(K_t, K_r, P, K_t', K_r', P', z) &= z^\nu \left(\left(K_t^\beta K_r^{1-\beta} \right)^\phi L_d^{1-\phi} \right)^\theta \\ &- (K_t' - (1-\delta_t)K_t) - (K_r' - (1-\delta_r)K_r) \\ &- \psi(K_t, K_t') - \psi(K_r, K_r') + P_s' - P_s(1 + r(1-\tau)). \end{aligned} \tag{IA.36}$$

Note that we model the tax benefit as a nonpecuniary wedge between the discount rate and the rate on debt. This simplification matters little for the model solution and has the advantage of not requiring the modeling of government transfers. Next, motivated by the dynamic contracting literature (Rampini and Viswanathan (2013)), we assume that this debt is secured by tangible capital, that is, we allow a fraction ξ of the stock of tangible capital to be used as collateral. The collateral constraint can thus be expressed as

$$P_s' \leq \xi(1-\delta)K_T'. \tag{IA.37}$$

Cash flows to shareholders, $E(K_t, K_r, P_s, K_t', K_r', P_s', z)$, are defined in terms of the firm's cash flows, $E^*(K_t, K_r, P_s, K_t', K_r', P_s', z)$. A positive firm cash flow is distributed to its only shareholder, the consumer, while a negative cash flow implies that the firm instead obtains funds from the consumer. In this case, the firm pays a linear cost, λ. Thus, shareholder cash flows are given by

$$\begin{aligned} E^* \geq 0 \quad &\Rightarrow \quad E = E^* \\ E^* < 0 \quad &\Rightarrow \quad E = E^*(1+\lambda). \end{aligned} \tag{IA.38}$$

Because labor is costlessly adjustable, we can solve for it analytically and plug the solution into the firm's problem,

$$\Pi(K_t, K_r, P_s, z) = \max_{K_t', K_r', P_s'} \left\{ E(K_t, K_r, P_s, K_t', K_r', P_s', z) + \frac{1}{1+r} \mathbb{E}\Pi(K_t', K_r', P', z') \right\} \tag{IA.39}$$

subject to (IA.32), (IA.33), and (IA.37). Given easily verifiable restrictions on the parameters, this model satisfies the conditions in Stokey, Lucas, and Prescott (1989) for the existence of a solution.

The firm's problem contains three financial frictions. The first is the tax benefit, which makes the firm impatient relative to the interest rate charged by lenders. This friction would incentivize the firm to use an unlimited amount of debt if not for the next two frictions. The second friction is the collateral constraint. With only these two frictions, the model has a simple solution for optimal debt. In each period, to satisfy its impatience, the firm borrows up to the limit given by (IA.37) and therefore has a constant ratio of debt to tangible capital. Any extra funds needed for its optimal hiring and investment policies come from deep-pocket shareholders. An interior solution for optimal debt comes from the third friction, which is the equity issuance cost, λ. In this case, facing the possibility of having to raise costly external equity, especially in those states in which productivity, z, is high, the firm preserves debt capacity, thus choosing optimal debt at a level less than the constraint given by $\xi(1-\delta)K_t$.

The economy also contains an infinitely lived representative consumer who chooses consumption and labor each period to maximize the expected present value of her utility, discounted at the risk-free rate r. Her one-period utility function is given by $\ln(C)+\chi(1-L_s)$, where C is consumption, L_s is the supply of labor, and χ is a parameter that governs the utility of leisure. Her budget constraint is given by

$$C + P_d' - P_d(1+r) = wL_s + E(\cdot), \tag{IA.40}$$

where w is the wage and P_d is the amount of savings demanded. Equation (IA.40) says that consumption plus saving (the left-hand side) equals labor income plus net distributions from firms. We set $\chi = 2.5$, which implies that consumers devote approximately one-third of their time to supplying labor. It is slightly higher than the value of 2 used in Bloom et al. (2018). Households save by buying the firm's bonds, and the competitive financial intermediary sector acts as a pass-through agent between firms and the consumer.

In summary, output is the numeraire and can be used in three ways. The consumer can consume it if she receives it as a dividend, or the firm can use it as either tangible or intangible capital after paying adjustment costs.

Let ζ be the stationary distribution over the firm's states, (z, K_t, K_r, P_s). We define the equilibrium in this economy as follows.

Definition 1 *A competitive equilibrium consists of (i) optimal firm policies for both types* *of capital and debt,* $\{K_t'(z, K_t, K_r, P_s), K_r'(z, K_t, K_r, P_s), P_s'(z, K_t, K_r, P_s)\}$*, (ii) allocations to the consumer of consumption* C *and labor* L_s*, and (iii)* (w, r) *such that:*

1. *All firms solve the problem given by* (IA.39)*.*
2. *The consumer maximizes her utility, subject to* (IA.40)*.*
3. *The labor, bond, and output markets clear:*

$$L_s = \int L_d d\zeta \tag{IA.41}$$

$$P_d = \int P_s'(z, K_t, K_r, P_s) d\zeta = P \tag{IA.42}$$

$$C = \int (Y - I_t - I_r) d\zeta. \tag{IA.43}$$

To solve for the equilibrium, we note that in this setting, constant marginal utility of leisure implies that the consumer supplies labor elastically, clearing the labor market. Also, from the consumer's first-order conditions, consumption is given by $C = w/\chi$. Thus, for any real wage w, we can solve for the interest rate i that clears the output market.

We solve the model and simulate 20 years of data for 2,000 firms using the following parameterization: $\theta = 0.9$, $\rho = 0.7$, $\sigma_z = 0.1$, $\xi = 1$, $\delta_t = \delta_r = 0.15$, and $\psi_t = \psi_r = 0.1$. We also let λ take 20 evenly spaced values in $(0, 0.15)$. Finally, we set the risk-free rate at 0.02, labor's share $1 - \phi$ at 0.5, and $\beta = 0.5$ so that in a frictionless optimum, the shares of intangible and tangible capital should be equal. These parameter values are close to many of the estimated values used in the dynamic corporate finance literature (Bazdresch, Kahn, and Whited (2018)).

We then compute measures of misallocation using the simulated data. The first is the percentage reallocation gains, which we obtain as follows. First, we compute actual value-added as $z^{\nu}\left(\left(K_t^{\beta} K_r^{1-\beta}\right)^{\phi} L_d^{1-\phi}\right)^{\theta}$. Next, the accumulated stock of equity is given by $K_t - P$, and the stock of debt is given by P. It is natural to think of $K_t - P$ as equity as it represents accumulated forgone distributions to shareholders. We then use $K_t - P$ and P in (16) to compute optimal value-added, given the static model in the main text. Here, we set $\sigma = 1.77$, and $\gamma = 1.52$, which is the average of the two-digit estimates in Table III for our Chinese sample. The second and third statistics are the percentages of these gains that come from scale and type.

We also use the simulated data to calculate the elasticity of substitution between debt and equity, γ_s, the variance of leverage, P/K_t, and the average market-to-book ratio, $(Pi+P)/K_t$.

III. CES Estimation

We use the method in Kmenta (1967) to estimate the CES function

$$F_{it} = A_{it} \left(\alpha D_{it}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha) E_{it}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\gamma_s}{\gamma_s - 1}}. \tag{IA.44}$$

We initially assume that $A_{it} \equiv e^a e^{u_{it}}$, where a is the common component of productivity and ϵ_{it} is an unpredictable component with mean zero. Next, we take logs of both sides of (IA.44) to obtain

$$\ln F_{it} = a + u_{it} + \frac{\gamma_s}{\gamma_s - 1} \ln \left(\alpha D_{it}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha) E_{it}^{\frac{\gamma_s - 1}{\gamma_s}} \right). \tag{IA.45}$$

The method in Kmenta (1967) produces an estimating equation from (IA.45) by taking a first-order approximation around $\gamma_s = 1$. This approximation requires taking the limit of the third term on the right-hand side of (IA.45) as $\gamma_s \to 1$:

$$\lim_{\gamma_s \to 1} \left[\frac{\gamma_s}{\gamma_s - 1} \ln \left(\alpha D_{it}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha) E_{it}^{\frac{\gamma_s - 1}{\gamma_s}} \right) \right] = \alpha \ln D_{it} + (1-\alpha) \ln E_{it}. \tag{IA.46}$$

The approximation also requires the limit of the derivative of this same term as $\gamma_s \to 1$:

$$\lim_{\gamma_s \to 1} \frac{\partial}{\partial \gamma_s} \left[\frac{\gamma_s}{\gamma_s - 1} \ln \left(\alpha D_{it}^{\frac{\gamma_s - 1}{\gamma_s}} + (1-\alpha) E_{it}^{\frac{\gamma_s - 1}{\gamma_s}} \right) \right] = \frac{\alpha(1-\alpha)}{2} \left(\ln D_{it} - \ln E_{it} \right)^2. \tag{IA.47}$$

Combining the two limits from above, the first-order approximation becomes

$$\ln F_{it} = a + \alpha \ln D_{it} + (1-\alpha)\ln E_{it} + \frac{\alpha(1-\alpha)(\gamma_s - 1)}{2}\left(\ln D_{it} - \ln E_{it}\right)^2 + u_{it}. \tag{IA.48}$$

The approximation in (IA.48) can be estimated as the following linear regression, with the coefficients appropriately defined:

$$\ln F_{it} = \beta_A + \beta_D \ln D_{it} + \beta_E \ln E_{it} + \beta_{DE}\left(\ln D_{it} - \ln E_{it}\right)^2 + u_{it}. \tag{IA.49}$$

However, because the error term, u_{it}, is essentially the productivity shock, the regression in (IA.49) has a correlated error-regressor problem that precludes the use of OLS. We take two different approaches to estimate (IA.49). First we assume that the error term u_{it} can be decomposed as

$$u_{it} = f_i + \epsilon_{it}, \tag{IA.50}$$

where f_i varies across firms, but not over time, and ϵ_{it} is uncorrelated with the regressors in (IA.49).[2] Under this assumption, we can estimate (IA.49) using OLS with firm fixed effects.

Alternatively, we can assume that the error term u_{it} can be decomposed as

$$u_{it} = z_{it} + \epsilon_{it}, \tag{IA.51}$$

where z_{it} is a predictable component that follows an $AR(1)$ process given by

$$z_{it} = \rho z_{it-1} + \nu_{it}. \tag{IA.52}$$

We assume further that ϵ_{it} and ν_{it} are independent of D_{it} and E_{it}, while we impose no such

[2]This assumption requires an assumption of independence between ϵ_{it} and (D_{it}, E_{it}). We need an independence assumption because the regressors in (IA.49) are nonlinear functions of D_{it} and E_{it}.

restrictions on z_{it}. Under this assumption, we can apply a ρ-difference to (IA.49), as in Blundell and Bond (2000) or Davis et al. (2014), to obtain

$$\begin{aligned}
\log F_{it} - \rho \log F_{it-1} &= \beta_A(1-\rho) + \beta_D(\log D_{it} - \rho \log D_{it-1}) + \beta_E(\log E_{it} - \rho \log E_{it-1}) \\
&+ \beta_{DE}\left[(\log D_{it} - \log E_{it})^2 - \rho\left(\log D_{it-1} - \log E_{it-1}\right)^2\right] \\
&+ (z_{it} - \rho z_{it-1}) + (\epsilon_{it} - \rho \epsilon_{it-1}). \qquad \text{(IA.53)}
\end{aligned}$$

Note that $(z_{it} - \rho z_{it-1}) + (\epsilon_{it} - \rho \epsilon_{it-1}) = \nu_{it} + (\epsilon_{it} - \rho \epsilon_{it-1})$ equals zero in expectation and is independent of all lagged variables. Thus, we can use GMM to estimate the three regression coefficients with four instruments, namely, $\ln F_{it-1}$, $\ln D_{it-1}$, $\ln E_{it-1}$, and $(\ln D_{it-1} - \ln E_{it-1})^2$.

Under either estimation strategy, the elasticity of substitution, γ_s, can be computed from the regression coefficients as

$$\gamma_s = 1 + \frac{2\beta_{DE}}{\beta_D \beta_E}. \qquad \text{(IA.54)}$$

IV. Supplementary Tables and Figures

This section contains supplementary tables and figures. Table IA.I presents sector-level estimates of the elasticity of substitution between debt and equity, γ_s. Table IA.II replicates Table IV, except that the parameter γ is estimated at the country level, using the GMM method described in Section III of this Internet Appendix. Table IA.III replicates Table IV in the main text, except we substitute net debt for debt. Table IA.IV presents estimates of the unit costs of finance for under- and overlevered firms. Table IA.V presents reallocation gains from our alternative model that includes labor in the production function. Table IA.VI shows the relation between the parameters of the dynamic model in this Internet Appendix and CES elasticities of substitution between debt and equity, which are calculated from data simulated from this dynamic model. Panel A of Figure IA.1 graphs the reallocation gains calculated from data simulated from the dynamic model. Panel B of Figure IA.1 plots the variance of leverage for different issuance costs. Panel A of Figure IA.2 graphs the average firm market-to-book ratio, the fractional benefit, and the aggregate equilibrium output as a fraction of output in an economy with no financial frictions. Panel B of Figure IA.2 plots the estimate of the elasticity of substitution between debt and equity.

Table IA.I
Sector Estimates of the Elasticity of Substitution

Calculations are based on two samples of firms. One is a sample of U.S. firms from Compustat, and the other is a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is 1999 to 2007. Column (1) lists the two-digit sectors, column (2) reports the number of observations, and column (3) contains the γ_s estimates for the United States, while columns (4) to (6) shows the corresponding information for China.

United States			China		
Two-Digit Sector	Observations	γ_s	Two-Digit Sector	Observations	γ_s
20	809	1.95	13	68,093	1.49
21	30	1.28	14	26,848	1.39
22	141	1.34	15	16,100	1.47
23	378	1.79	16	323	2.05
24	179	1.68	17	117,117	1.53
25	225	1.65	18	68,968	1.49
26	284	1.73	19	34,800	1.53
27	417	1.51	20	26,494	1.61
28	3,156	1.59	21	16,251	1.41
29	193	3.38	22	35,589	1.51
30	370	1.67	23	18,880	1.53
31	149	1.66	24	18,571	1.48
32	165	1.59	25	8,465	1.45
33	462	1.67	26	89,218	1.52
34	496	1.52	27	23,901	1.44
35	2,281	1.52	28	7,071	1.47
36	3,226	1.55	29	15,017	1.48
37	688	1.67	30	62,864	1.50
38	2,704	1.35	31	93,722	1.51
39	368	2.37	32	29,033	1.44
			33	24,852	1.46
			34	73,249	1.53
			35	94,783	1.52
			36	48,842	1.44
			37	53,063	1.56
			39	61,034	1.97
			40	53,997	1.55
			41	25,892	1.48
			42	24,710	1.38
			43	10,982	1.48

Table IA.II
Reallocation Gains by Year with Country-Level Elasticity of Substitution

Calculations are based on two samples of firms. One is a sample of U.S. firms from Compustat, and the other is a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is 1999 to 2007. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma_{\text{US}} = 1.519$ and $\gamma_{\text{China}} = 1.584$, and the elasticity of substitution between the real benefit of firms in a sector is $\sigma = 1.77$. Column (1) shows the observed U.S. allocation of real value-added as a fraction of the optimal U.S. allocation, $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3) and (4) present analogous calculations for Chinese firms. Columns (5) and (6) show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio, $(F_{\text{China}}/\hat{F}_{\text{China}})/(F_{\text{US}}/\hat{F}_{\text{US}})$, and the corresponding percentage gains. Columns (7) and (8) provide a breakdown of total misallocation into the misallocations due to the scale of finance and due to the mix of debt and equity, holding scale fixed. Standard errors are in parentheses below each estimate.

	United States		China		United States vs. China			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Percent Scale	Percent Type
1999	0.900	11.2	0.585	71.1	0.650	53.9	45.0	8.9
	(0.006)	(0.7)	(0.008)	(2.4)	(0.010)	(2.3)	(1.9)	(0.7)
2000	0.889	12.5	0.577	73.4	0.649	54.2	46.3	7.9
	(0.009)	(1.1)	(0.013)	(3.7)	(0.016)	(3.7)	(3.2)	(0.7)
2001	0.895	11.7	0.591	69.2	0.660	51.5	43.4	8.1
	(0.009)	(1.1)	(0.007)	(2.1)	(0.011)	(2.6)	(2.1)	(0.7)
2002	0.890	12.4	0.583	71.5	0.655	52.6	44.5	8.1
	(0.008)	(1.0)	(0.011)	(3.2)	(0.013)	(3.0)	(2.7)	(0.7)
2003	0.889	12.4	0.594	68.5	0.667	49.8	41.3	8.6
	(0.009)	(1.1)	(0.007)	(1.9)	(0.009)	(2.1)	(1.5)	(0.8)
2004	0.900	11.1	0.559	79.0	0.620	61.2	50.7	10.5
	(0.008)	(1.0)	(0.005)	(1.6)	(0.008)	(2.2)	(1.7)	(0.7)
2005	0.901	11.0	0.546	83.3	0.606	65.1	53.3	11.8
	(0.009)	(1.1)	(0.005)	(1.7)	(0.009)	(2.4)	(1.8)	(0.8)
2006	0.895	11.7	0.549	82.1	0.613	63.0	50.8	12.2
	(0.009)	(1.0)	(0.004)	(1.4)	(0.007)	(1.9)	(1.5)	(0.7)
2007	0.894	11.8	0.533	87.6	0.596	67.7	54.4	13.3
	(0.008)	(1.0)	(0.004)	(1.3)	(0.007)	(1.9)	(1.4)	(0.8)

Table IA.III
Misallocation of Net Debt and Equity

This table is identical to Table IV, except that all debt measures are calculated as debt minus cash. In both tables, calculations are based on two samples of firms. One is a sample of U.S. firms from Compustat, and the other is a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is 1999 to 2007. This table presents potential reallocation gains when the substitutability between debt and equity varies at the two-digit sector level, and the elasticity of substitution between the real benefit of firms in a sector is $\sigma = 1.77$. Column (1) shows the observed U.S. allocation of real value-added as a fraction of the optimal U.S. allocation, $F_{\mathrm{US}}/\hat{F}_{\mathrm{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3) and (4) present analogous calculations for Chinese firms. Columns (5) and (6) show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio, $(F_{\mathrm{China}}/\hat{F}_{\mathrm{China}})/(F_{\mathrm{US}}/\hat{F}_{\mathrm{US}})$, and the corresponding percentage gains. Columns (7) and (8) provide a breakdown of total misallocation into the misallocations due to the scale of finance and due to the mix of debt and equity, holding scale fixed. Standard errors are in parentheses below each estimate.

	United States		China		United States vs. China			
Year	(1) Fractional Benefit	(2) Percent Gain	(3) Fractional Benefit	(4) Percent Gain	(5) Fractional Benefit	(6) Percent Gain	(7) Percent Scale	(8) Percent Type
1999	0.875	14.3	0.493	102.9	0.563	77.6	62.2	15.3
	(0.010)	(1.2)	(0.014)	(5.4)	(0.017)	(5.3)	(3.8)	(1.9)
2000	0.876	14.1	0.515	94.3	0.587	70.3	56.9	13.3
	(0.012)	(1.5)	(0.008)	(3.1)	(0.012)	(3.4)	(2.4)	(1.6)
2001	0.889	12.5	0.515	94.3	0.579	72.8	56.0	16.8
	(0.012)	(1.5)	(0.008)	(2.8)	(0.012)	(3.5)	(2.8)	(1.6)
2002	0.872	14.7	0.519	92.7	0.595	68.0	56.0	12.0
	(0.012)	(1.5)	(0.009)	(3.2)	(0.014)	(3.9)	(2.8)	(1.6)
2003	0.874	14.4	0.530	88.9	0.606	65.0	51.5	13.6
	(0.013)	(1.7)	(0.009)	(3.1)	(0.013)	(3.7)	(2.8)	(1.2)
2004	0.881	13.4	0.497	101.4	0.563	77.5	63.2	14.3
	(0.012)	(1.5)	(0.007)	(2.8)	(0.011)	(3.4)	(2.7)	(1.2)
2005	0.884	13.2	0.488	104.9	0.552	81.0	63.8	17.3
	(0.013)	(1.6)	(0.007)	(3.1)	(0.011)	(3.6)	(2.8)	(1.2)
2006	0.867	15.3	0.489	104.4	0.564	77.3	61.3	16.0
	(0.013)	(1.7)	(0.007)	(2.8)	(0.010)	(3.3)	(2.8)	(1.2)
2007	0.865	15.7	0.489	104.7	0.565	77.0	63.2	13.8
	(0.013)	(1.6)	(0.007)	(3.0)	(0.011)	(3.7)	(2.9)	(1.5)

Table IA.IV
Unit Costs for Over- and Underlevered Firms

Calculations are based on two samples of firms. One is a sample of U.S. firms from Compustat, and the other is a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is 1999 to 2007. This table presents the unit cost of finance for firms that are either under- or overleveraged, where underleveraged is defined as leverage below the industry mean or industry median, and overleverage is defined analogously. The unit cost is defined as $[(1+\tau_{D_{si}})r_{si}D_{si}+(1+\tau_{E_{si}})\lambda_{si}E_{si}]/(D_{si}+E_{si})$, and all model parameters are set as in Table IV. Standard errors are in parentheses below each estimate.

	United States				China			
Firm Size	Over Mean	Under Mean	Over Median	Under Median	Over Mean	Under Mean	Over Median	Under Median
0-5	0.192	0.218	0.168	0.184	0.337	0.432	0.243	0.306
	(0.019)	(0.012)	(0.022)	(0.008)	(0.0022)	(0.0031)	(0.0014)	(0.0012)
5-15	0.219	0.179	0.171	0.160	0.201	0.260	0.157	0.197
	(0.012)	(0.005)	(0.008)	(0.004)	(0.0007)	(0.0016)	(0.0005)	(0.0006)
15-30	0.205	0.165	0.166	0.151	0.150	0.200	0.117	0.149
	(0.008)	(0.003)	(0.006)	(0.003)	(0.0004)	(0.0006)	(0.0003)	(0.0004)
30-50	0.190	0.159	0.167	0.147	0.118	0.157	0.092	0.117
	(0.005)	(0.002)	(0.003)	(0.002)	(0.0003)	(0.0006)	(0.0002)	(0.0003)
50-70	0.165	0.159	0.153	0.145	0.096	0.127	0.075	0.095
	(0.003)	(0.002)	(0.002)	(0.002)	(0.0002)	(0.0003)	(0.0002)	(0.0002)
70-85	0.160	0.148	0.147	0.144	0.081	0.106	0.064	0.082
	(0.002)	(0.002)	(0.002)	(0.002)	(0.0002)	(0.0004)	(0.0002)	(0.0003)
85-95	0.145	0.138	0.141	0.132	0.069	0.094	0.057	0.076
	(0.002)	(0.003)	(0.002)	(0.003)	(0.0002)	(0.0003)	(0.0002)	(0.0003)
95-100	0.126	0.136	0.123	0.134	0.062	0.088	0.051	0.074
	(0.003)	(0.003)	(0.003)	(0.005)	(0.0003)	(0.0004)	(0.0003)	(0.0004)

Table IA.V
Reallocation Gains by Year: Model with Labor

Calculations are based on two samples of firms. One is a sample of U.S. firms from Compustat, and the other is a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is 1999 to 2007. This table presents potential reallocation gains when the substitutability between debt and equity varies at the two-digit sector level, and the elasticity of substitution across goods in a sector is $\sigma = 1.77$. Column (1) shows the observed U.S. allocation of real value-added as a fraction of the optimal U.S. allocation, $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3) and (4) present analogous calculations for Chinese firms. Columns (5) and (6) show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio, $(F_{\text{China}}/\hat{F}_{\text{China}})/(F_{\text{US}}/\hat{F}_{\text{US}})$, and the corresponding percentage gains. Columns (7) and (8) provide a breakdown of total misallocation into the misallocation due to the scale of finance and due to the mix of debt and equity, holding scale fixed. Standard errors are in parentheses below each estimate.

	United States		China		United States vs. China			
Year	(1) Fractional Benefit	(2) Percent Gain	(3) Fractional Benefit	(4) Percent Gain	(5) Fractional Benefit	(6) Percent Gain	(7) Percent Scale	(8) Percent Type
1999	0.934	7.0	0.551	81.4	0.590	69.4	63.7	5.8
	(0.006)	(0.7)	(0.007)	(2.1)	(0.008)	(2.1)	(2.0)	(0.3)
2000	0.925	8.1	0.528	89.4	0.571	75.1	69.5	5.6
	(0.006)	(0.7)	(0.011)	(3.7)	(0.013)	(3.7)	(3.6)	(0.3)
2001	0.939	6.5	0.540	85.2	0.575	73.9	68.2	5.8
	(0.005)	(0.6)	(0.009)	(2.9)	(0.010)	(2.9)	(2.7)	(0.3)
2002	0.935	6.9	0.536	86.7	0.573	74.5	68.8	5.7
	(0.005)	(0.5)	(0.011)	(3.8)	(0.012)	(3.7)	(3.5)	(0.3)
2003	0.932	7.3	0.543	84.0	0.583	71.6	66.1	5.5
	(0.007)	(0.8)	(0.008)	(2.7)	(0.010)	(2.8)	(2.7)	(0.3)
2004	0.930	7.5	0.525	90.6	0.564	77.3	70.8	6.5
	(0.007)	(0.8)	(0.007)	(2.4)	(0.009)	(2.8)	(2.6)	(0.3)
2005	0.928	7.7	0.546	83.1	0.588	70.0	63.3	6.6
	(0.006)	(0.7)	(0.006)	(1.9)	(0.007)	(2.1)	(2.0)	(0.3)
2006	0.932	7.2	0.554	80.4	0.595	68.2	61.3	6.9
	(0.005)	(0.6)	(0.004)	(1.4)	(0.005)	(1.5)	(1.4)	(0.2)
2007	0.928	7.8	0.541	84.9	0.583	71.5	64.1	7.4
	(0.005)	(0.6)	(0.004)	(1.4)	(0.006)	(1.7)	(1.6)	(0.3)

Table IA.VI
Dynamic Model Parameters and the Elasticity of Substitution

This table presents two sets of estimates. Both are based on data where each observation corresponds to a single solution of the model in Section II of this Internet Appendix. We solve and simulate the model 400 times, with each solution corresponding to a different set of randomly chosen model parameters. θ is the curvature of the production function; β is the share of intangible capital; ρ is the serial correlation of the shock process; σ is the standard deviation of the innovation to the shock process; δ_t and δ_r are the depreciation rates for tangible and intangible capital; ψ_t and ψ_r are the quadratic adjustment cost parameters for tangible and intangible capital; ξ is the fraction of tangible capital that can be collateralized; and λ is the cost of equity issuance. The upper and lower bounds for the parameters are in Panel A. In Panel B, we present results from two regressions. The first set of estimates is from an OLS regression of our estimate of the elasticity of substitution between debt and equity, γ, on the model parameters, with standard errors in parentheses to the right. The second set of estimates is the set of best LASSO predictors, calculated with ten-fold cross-validation.

Panel A. Parameter Bounds

Parameter		Lower Bound	Upper Bound
θ	(returns to scale)	0.70	0.90
β	(labor share)	0.30	0.70
ρ	(shock serial correlation)	0.40	0.90
σ	(shock standard deviation)	0.05	0.40
δ_t	(tangible capital depreciation)	0.05	0.20
δ_r	(intangible capital depreciation)	0.05	0.20
ψ_t	(tangible capital adjustment costs)	0	0.50
ψ_r	(intangible capital adjustment costs)	0	0.50
ξ	(collateral)	0	1
λ	(equity issuance cost)	0	0.15

Panel B. Regressions

Parameter		OLS		LASSO predictors
θ	(returns to scale)	0.7844	(0.3553)	0.6666
β	(labor share)	0.1243	(0.1783)	0.0912
ρ	(shock serial correlation)	0.5473	(0.1532)	0.4909
σ	(shock standard deviation)	0.4483	(0.2097)	0.3647
δ_t	(tangible capital depreciation)	0.0828	(0.4797)	——
δ_r	(intangible capital depreciation)	0.4890	(0.4574)	0.3593
ψ_t	(tangible capital adjustment costs)	-0.2503	(0.1412)	-0.2112
ψ_r	(intangible capital adjustment costs)	-0.4520	(0.1388)	-0.4042
ξ	(collateral)	0.2456	(0.0685)	0.2250
λ	(equity issuance cost)	0.7305	(0.4761)	0.5936

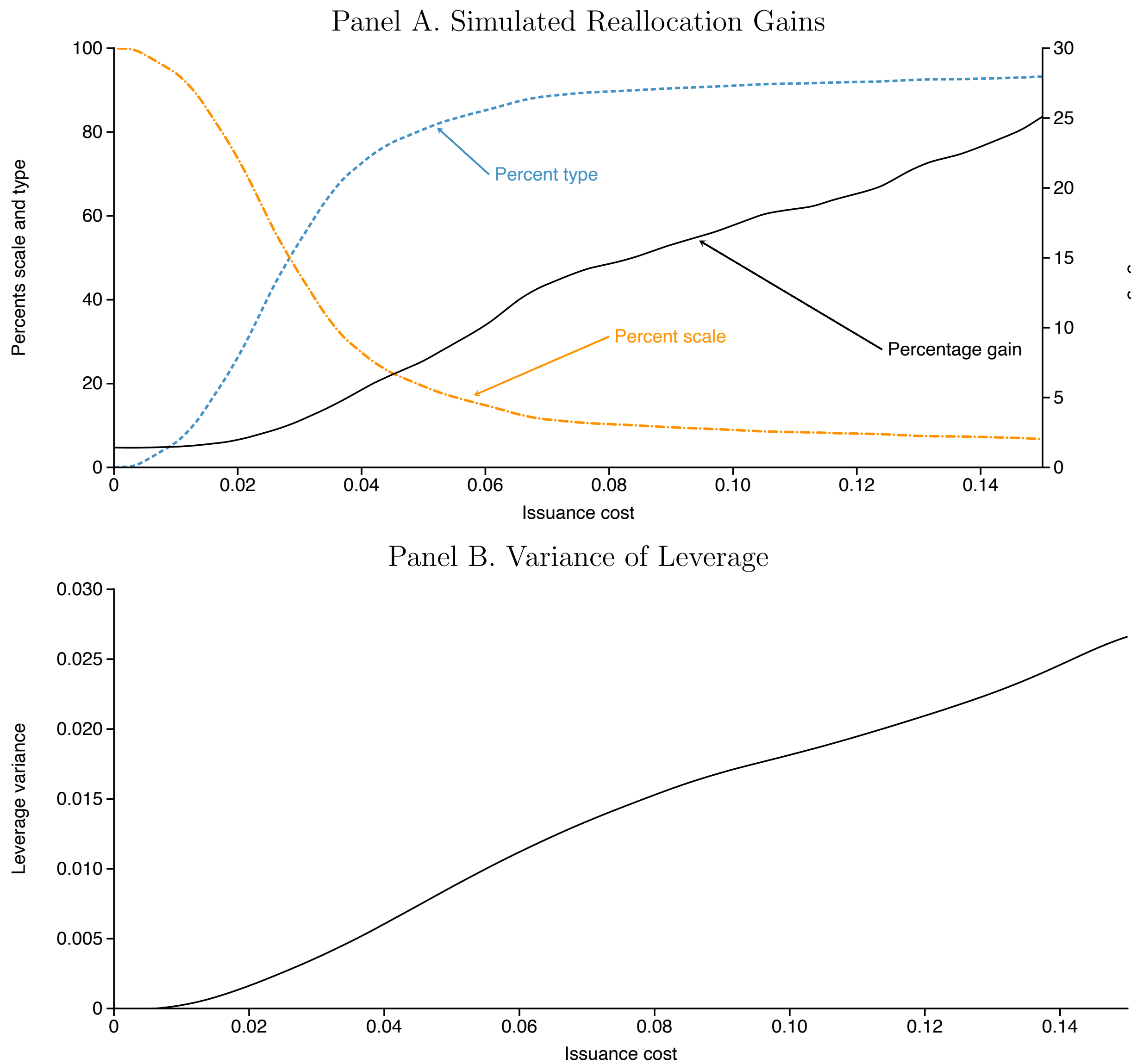

Figure IA.1. Reallocation gains and the variance of leverage. Panel A graphs the reallocation gains calculated from our dynamic model in Section II of this Internet Appendix as a function of the cost of issuing equity in this model. The solid black line represents these gains. The dashed blue line represents the percent of these gains that stem from the type of finance, and the dashed and dotted orange line represents the percent of these gains that stem from the scale of finance. Panel B plots the variance of leverage for different issuance costs.

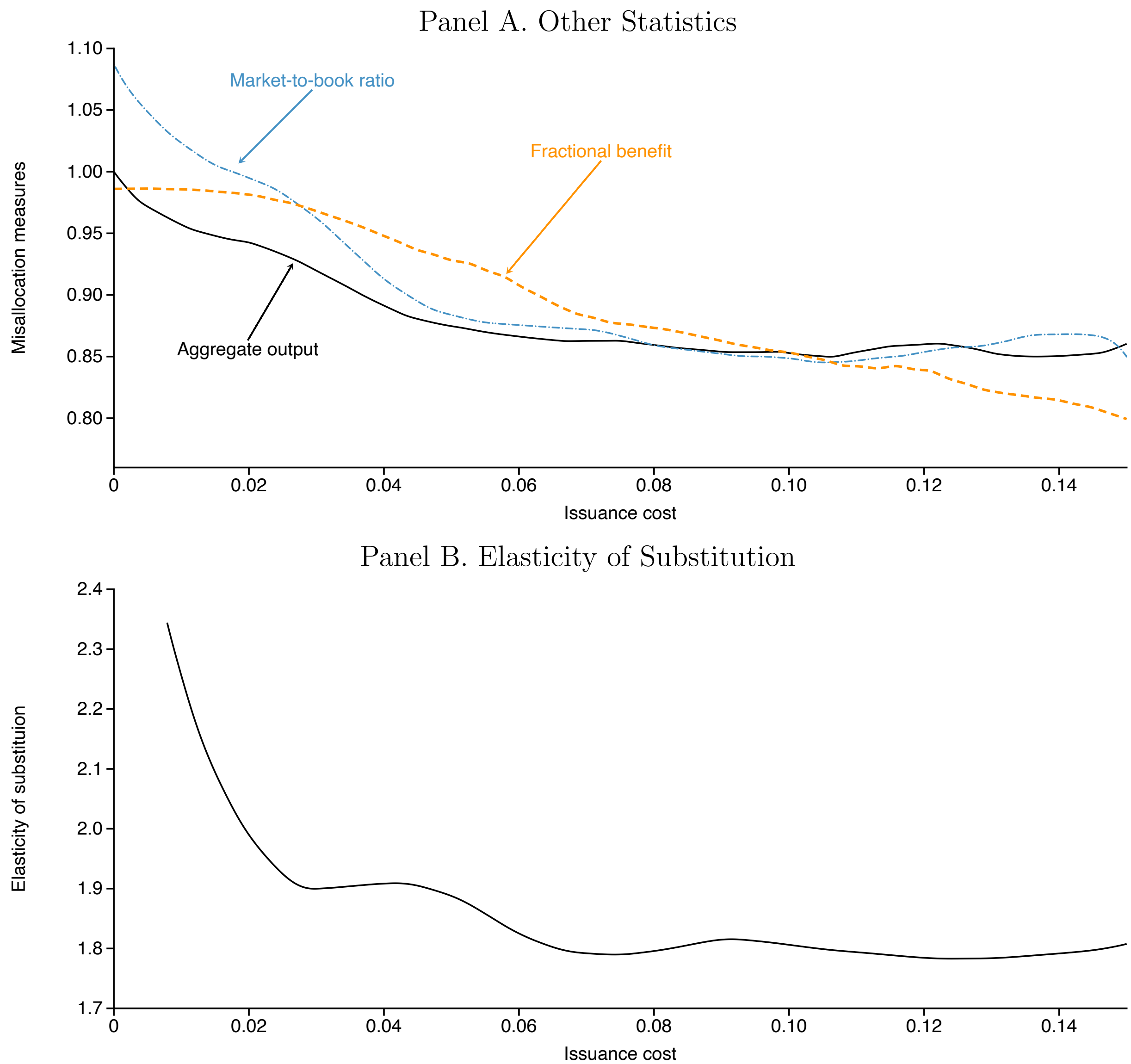

Figure IA.2. Other statistics and the elasticity of substitution. Panel A graphs the average firm market-to-book ratio, the fractional benefit, and the aggregate equilibrium output as a fraction of output in an economy with no financial frictions. Panel B plots the estimate of the elasticity of substitution between debt and equity.

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