

Resource Reallocation with Carbon Emission Policies

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- Key policies: carbon pricing, renewable subsidies to curb emissions.
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 - Limitation in fossil fuel usage.
 - Adoption of renewable technologies.
 - **Reallocation of resources to greener firms/industries.**

- **What is the Economic Outcomes of environmental policies due to resources reallocation?**
 - Industry output
 - Firm-level productivity
 - Sector size
 - Emission intensity
 - Total Emission

- Effectiveness of Carbon policies:
 - Martinsson et al. (2024); Shapiro and Walker (2018); Ahmadi, Yamazaki, and Kabore (2022); Andersson (2019)
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Contribution: **Quantify substitution between green and brown capital**
- Misallocation:
 - Whited and Zhao (2021); Hsieh and Klenow (2009); Ai, Li, and Yang (2020); Asker, Collard-Wexler, and De Loecker (2014)
Contribution: **Misallocation (Reallocation) in the context of environmental policies**

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Contribution: **Misallocation (Reallocation) in the context of environmental policies**
- Climate Policy Design:
 - Acemoglu, Gancia, and Zilibotti (2012); Acemoglu et al. (2016); Oehmke and Opp (2023)
Contribution: **Assess alternative instruments in Emission Intensity / resource reallocation**

Road map

- ① Develop Economic model with Emission
- ② Characterize the allocation of resources
- ③ Estimate the model by Swedish data
- ④ Compare the optimal Policy with resource reallocation
- ⑤ Discuss the cost of the environmental policies

Road map

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- 1 Develop Economic model with Emission ✓
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- 3 Estimate the model by Swedish data ⚠
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- 5 Discuss the cost of the environmental policies ⚠

Standard Framework

Hsieh and Klenow (2009)

- Heterogeneous monopolistic competitive firms
- Partial equilibrium
- Cobb-Douglas Production function
- CES aggregator for output
- Normal aggregation of emissions

Extension

Production functions

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}$$

- \hat{A}_{si} : total factor of productivity

Firm's profit

Extension

Production functions

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}, \quad \hat{K} = (\alpha_s G_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1 - \alpha_s) B_{si}^{\frac{\gamma_s-1}{\gamma_s}})^{\frac{\gamma_s}{\gamma_s-1}}$$

$$E_{si} = \tilde{A}_{si} B_{si}$$

Emission General Model

- \hat{A}_{si} : total factor of productivity
- α_s : importance of Green capital in the production
- γ_s : elasticity of substitution between Green and Brown capital
- \tilde{A}_{si} : emission inefficiency
- Firms maximize over G, B, and L

Firm's profit

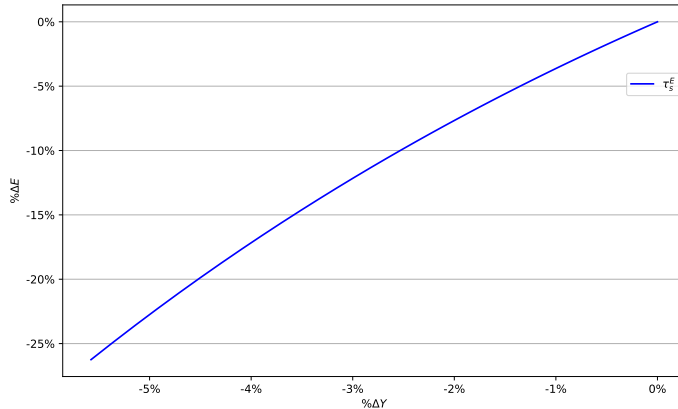
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment
Panel A: Estimated Value		
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
\tilde{A}	0.018	E/PY
Panel B: Inputs		
σ	∞	Fully competitive
r	5%	-
w	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
Panel C: Calibrated Value		
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B , Wiedemann (2023)

Sensitivity of α

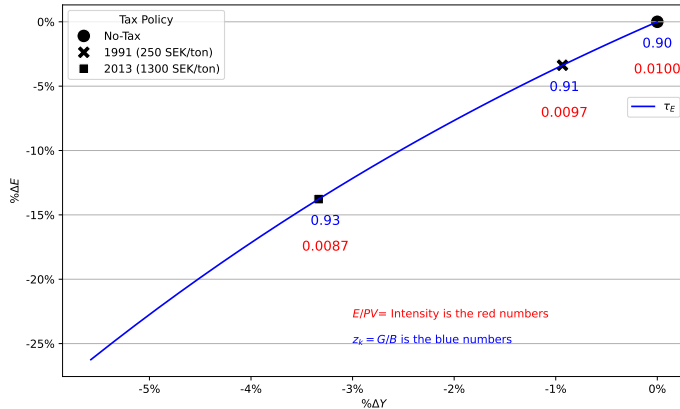
Emission and Production

Results



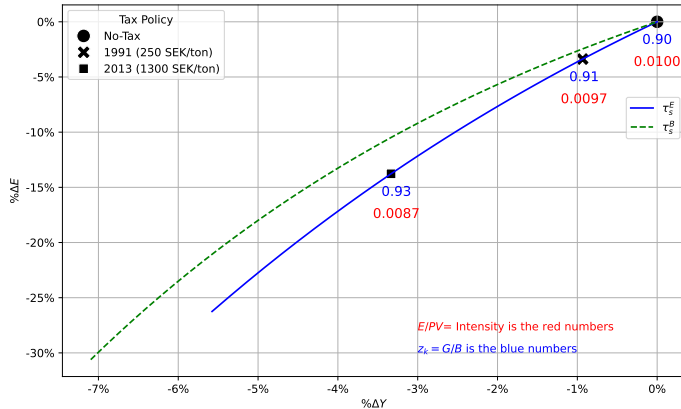
Emission and Production

Results



Emission and Production

Results



Future Steps

- ① Develop Economic model with Emission
 - Firms could R&D
 - Add Household and Government
 - Firms could enter and exit the market
- ② Characterize the allocation of resources
- ③ Provide a definition of Green and Brown capital
- ④ Estimate the model by Swedish data
- ⑤ Compare the optimal Policy with resource reallocation
- ⑥ Discuss the cost of the environmental policies

Thank you!

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- The firm's emission is:

$$E_{si} = \tilde{A}_{si} \tilde{K}_{si}^{\theta_s} L_{si}^{1-\theta_s} \quad , \quad \tilde{K} = \left(\mu_s G_{si}^{\frac{\eta_s-1}{\eta_s}} + (1 - \mu_s) B_{si}^{\frac{\eta_s-1}{\eta_s}} \right)^{\frac{\eta_s}{\eta_s-1}}$$

- The nominal profit for firms:

$$\pi_{si} = (1 + \tau_{si}^p) P_{si} Y_{si} - ([(1 + \tau_{G_s}) r_{si} G_{si} + (1 + \tau_{B_s}) r_{si} B_{si} + (1 + \tau_{l_s}) w_{si} l_{si}] + \tau_E E_{si})$$

Back

Firm's profit

- The nominal profit for firms:

$$\pi_{si} = (1 + \tau_s^P) P_{si} Y_{si}$$

- where

- τ_s^P is the tax / Demand preference for the firm

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- τ_s^P is the **tax** / **Demand preference** for the firm
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- τ_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)

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 - τ_s^W is the **Labor market preference** to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)
- The firm chooses the optimal capital and labor to minimize the cost of production and then chooses the price level to maximize the profit

Back

$$\max_{G_{si}, B_{si}, L_{si}} -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

General Model Solution

$$\max_{G_{si}, B_{si}, L_{si}} -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = z_{si}^k$$

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$$\frac{G_{si}}{B_{si}} = z_{si}^k = \left(\frac{\alpha_s}{1 - \alpha_s} \frac{(1 + \tau_s^B) r_s + \tau_s^E \tilde{A}}{(1 + \tau_s^G) r_s} \right)^{\frac{1}{\gamma_s}}$$

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$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k \gamma_s - 1} + (1 - \alpha_s) \right)^{\frac{\gamma_s}{1-\gamma_s}} z_{si}^l^{1-\beta} \bar{Y}_{si}$$

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$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k \gamma_s - 1} + (1 - \alpha_s) \right)^{\frac{\gamma_s}{1-\gamma_s}} z_{si}^{l 1-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}$$

- Firm will then charge markup over the marginal cost

$$\max \quad -Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$z_{si}^k \equiv \frac{G_{si}}{B_{si}} = \left[\frac{\alpha_s}{1 - \alpha_s} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial G} Cost_{si}} \right]^{\gamma_s}$$

$$\begin{aligned} z_{si}^l \equiv \frac{L_{si}}{\hat{K}_{si}} &= \frac{1 - \beta_s}{\beta_s} \frac{1}{1 - \alpha_s} (\alpha_s z_{si}^{k(\gamma_s-1)} + (1 - \alpha_s))^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}} \\ &= \frac{1 - \beta_s}{\beta_s} \frac{1}{\alpha_s} (\alpha_s + (1 - \alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial G} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}} \end{aligned}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\frac{\phi_{si}}{z_{si}^l} \right)^{\theta_s} z_{si}^{l\beta_s} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si}, \quad \text{where} \quad \phi_{si} = \frac{(\mu_s + (1 - \mu_s) z_{si}^{k(1-\eta_s)})^{\frac{\eta_s}{\eta_s-1}}}{(\alpha_s + (1 - \alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{\gamma_s}{\gamma_s-1}}}$$

Model

Optimal firm level price

- Now Firm need to choose the price level to maximize the profit:

$$\max_{P_{si}} \pi_{si} = P_{si}F_{si} - C_{si}F_{si}$$

- Firm-level real output is a function of the sector price, firm price, and sector real output (i.e. $F_{si} = (\frac{P_s}{P_{si}})^{\sigma_s} F_s$)
- Therefore, because the optimal ratio does not depend on the price, the ratio can be maximized out of the problem of the optimal determination of the price, leaving the firm's real output as just a function of price

$$P_{si} = \frac{1}{1 + \tau_{si}^p} \frac{\sigma_s}{\sigma_s - 1} C_{si}$$

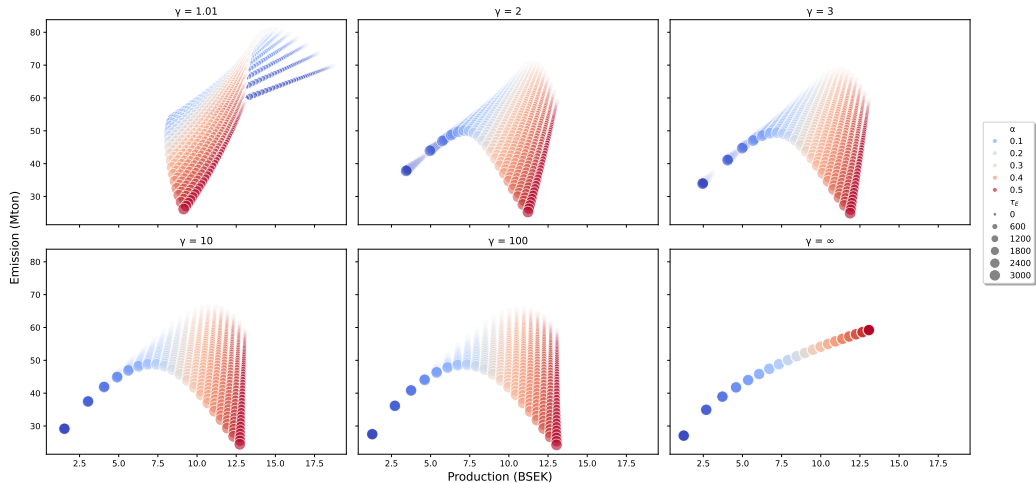
Estimation / Calibration

- My goal is to estimate the parameters sector by sector for Sweden
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment
Panel A: Inputs		
σ	∞	Fully competitive
r	5%	-
w	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
Panel B: Calibrated Value		
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B , Wiedemann (2023)
Panel C: Estimated Value		
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
\tilde{A}	0.018	E/PY

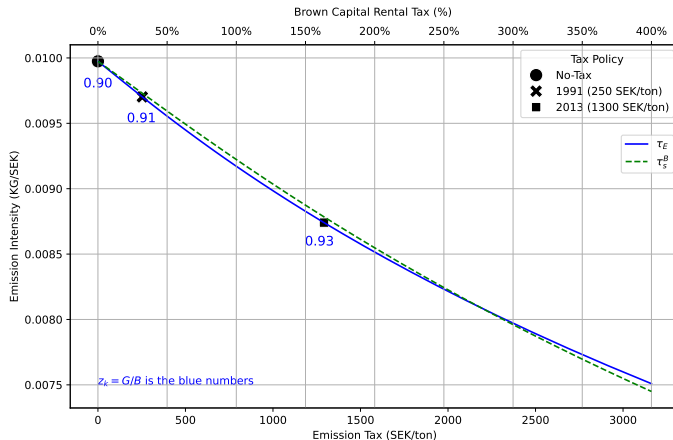
Sensitivity analysis

Production vs Emission with different Carbon Tax on different α and γ



Carbon Intensity and Tax

Counterfactual



τ_E	τ_S^B
100	14%
250	36%
500	66%
1300	171%
3000	360%

Reallocation

Resources allocation

- Now, we need to find the optimal allocation of resources in the economy under two scenarios:

Back

Reallocation

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$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} L_s$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{z_s^k}{1 + z_s^k} K_s$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{1}{1 + z_s^k} K_s$$

Back

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$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_j \hat{A}_{sj}^{\sigma-1}} \frac{1}{1 + z_s^k} K_s$$

$$\tilde{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} L_s$$

Back

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$$\begin{aligned}\tilde{L}_{si} &= \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} L_s \\ \tilde{G}_{si} &= \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} \frac{z_s^k}{1 + z_s^k} K_s \\ \tilde{B}_{si} &= \frac{\hat{A}_{si}^{\sigma-1} / \tilde{A}_{si}^{\sigma}}{\sum_j \hat{A}_{sj}^{\sigma-1} / \tilde{A}_{sj}^{\sigma}} \frac{1}{1 + z_s^k} K_s\end{aligned}$$

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