Resource Reallocation with Carbon Emission Policies

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Motivation

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- Government interventions steer markets towards sustainability.
- Key policies: carbon pricing, renewable subsidies to curb emissions.
- Economic impacts:
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- Economic impacts:
 - Limitation in fossil fuel usage.
 - Adoption of renewable technologies.
 - Reallocation of resources to greener firms/industries.

Research Question

- What is the Economic Outcomes of environmental policies due to resources reallocation?
 - Industry output
 - Firm-level productivity
 - Sector size
 - Emission intensity
 - Total Emission

Literature and Contribution

- Effectiveness of Carbon policies
 - Contribution: Quantify substitution between green and brown capital (Martinsson et al., 2024; Shapiro and Walker, 2018; Ahmadi, Yamazaki, and Kabore, 2022; Andersson, 2019)

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- Climate Policy Design
 - Contribution: Assess alternative instruments in Emission Intensity / resource reallocation
 - Acemoglu, Gancia, and Zilibotti (2012); Acemoglu et al. (2016); Oehmke and Opp (2023)

Road map

- Develop Economic model with Emission
- Characterize the allocation of resources
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Oiscuss the cost of the environmental policies

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Standard Framework

Hsieh and Klenow (2009)

- Heterogeneous monopolistic competitive firms
- Partial equilibrium
- Cobb-Douglas Production function
- CES aggregator for output
- Normal aggregation of emissions

Extension

Production functions

$$Y_{si} = \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s}$$

• \hat{A}_{si} : total factor of productivity



Extension

Production functions

$$Y_{si} = \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} \quad , \qquad \hat{K} = (\alpha_s G_{si}^{\frac{\gamma_s-1}{\gamma_s}} + (1-\alpha_s)B_{si}^{\frac{\gamma_s-1}{\gamma_s}})^{\frac{\gamma_s}{\gamma_s-1}}$$

$$E_{si} = \tilde{A}_{si}B_{si}$$

Emission General Model

- \hat{A}_{si} : total factor of productivity
- α_s : importance of Green capital in the production
- ullet γ_s : elasticity of substitution between Green and Brown capital
- \tilde{A}_{si} : emission inefficiency
- Firms maximize over G, B, and L

Firm's profit



Estimation / Calibration

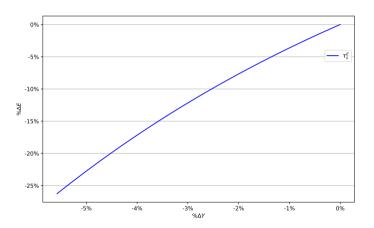
• I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment
	Panel A: Estim	nated Value
${\gamma}$	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
$ ilde{ ilde{A}}$	0.018	E/PY
	Panel B:	Inputs
σ	∞	Fully competitive
r	5%	-
W	500 TSEK	-
L	$250 \; (sd = 900)$	Martinsson et al. (2024)
	Panel C: Calibi	rated Value
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B, Wiedemann (2023)

Sensitivity of α

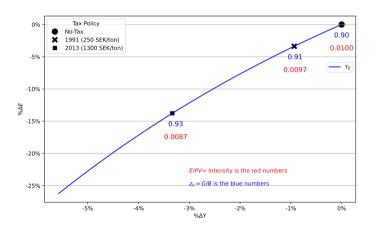
Emission and Production

Results



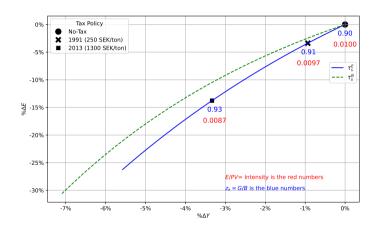
Emission and Production

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Emission and Production

Results



Future Steps

- Develop Economic model with Emission
 - Firms could R&D
 - Add Household and Government
 - Firms could enter and exit the market
- Characterize the allocation of resources
- Provide a definition of Green and Brown capital
- Estimate the model by Swedish data
- Ompare the optimal Policy with resource reallocation
- Objective the cost of the environmental policies

Thank you!

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Emission General Model

• The firm's emission is:

$$E_{si} = ilde{A}_{si} ilde{K}_{si}^{ heta_s}L_{si}^{1- heta_s} \quad , \qquad ilde{K} = (\mu_sG_{si}^{rac{\eta_s-1}{\eta_s}} + (1-\mu_s)B_{si}^{rac{\eta_s-1}{\eta_s}})^{rac{\eta_s}{\eta_s-1}}$$

$$\pi_{si} = (1 + \tau_{si}^p) P_{si} Y_{si} - \left(\left[(1 + \tau_{G_{si}}) r_{si}^G G_{si} + (1 + \tau_{B_{si}}) r_{si}^B B_{si} + (1 + \tau_{I_{si}}) w_{si} I_{si} \right] + \tau_E E_{si} \right)$$





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si}$$

- where
 - τ_s^p is the tax / Demand preference for the firm





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left((1 + \tau_s^G) r_s G_{si} \right)$$

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 - τ_s^p is the tax / Demand preference for the firm
 - ullet au_s^G is the Green capital subsidy / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s})P_{si}Y_{si} - \left((1 + \frac{\tau_s^G}{s})r_sG_{si} + (1 + \frac{\tau_s^B}{s})r_sB_{si}\right)$$

- where
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 - τ_s^G is the Green capital subsidy / ESG preference of Financier
 - τ_s^B is the Brown capital tax / ESG preference of Financier





$$\pi_{si} = (1 + \frac{\tau_s^p}{s}) P_{si} Y_{si} - \left((1 + \frac{\tau_s^G}{s}) r_s G_{si} + (1 + \frac{\tau_s^B}{s}) r_s B_{si} + (1 + \frac{\tau_s^W}{s}) w_{si} I_{si} \right)$$

- where
 - τ_s^p is the tax / Demand preference for the firm
 - \bullet au_s^G is the Green capital subsidy / ESG preference of Financier
 - τ_s^B is the Brown capital tax / ESG preference of Financier
 - \bullet au_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)





$$\pi_{si} = (1 + \tau_s^p) P_{si} Y_{si} - \left((1 + \tau_s^G) r_s G_{si} + (1 + \tau_s^B) r_s B_{si} + (1 + \tau_s^W) w_{si} I_{si} \right) - \tau_s^E E_{si}$$

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 - τ_s^W is the Labor market preference to work in the green/brown sector (Krueger, Metzger, and Wu, 2023)
- The firm chooses the optimal capital and labor to minimize the cost of production and then chooses the price level to maximize the profit





$$\max_{G_{si},B_{si},L_{si}} - Cost$$
 s.t. $\hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si}$





$$\max_{\substack{G_{si},B_{si},L_{si}}} \quad -\textit{Cost} \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^{k}$$





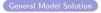
$$\max_{G_{si},B_{si},L_{si}} -Cost \quad \text{s.t.} \qquad \hat{A}_{si}\hat{K}_{si}^{\beta_s}L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^{k} = \left(\frac{\alpha_{s}}{1 - \alpha_{s}} \frac{(1 + \tau_{s}^{B})r_{s} + \tau_{s}^{E}\tilde{A}}{(1 + \tau_{s}^{G})r_{s}}\right)^{\frac{1}{\gamma_{s}}}$$



$$\begin{aligned} & \max_{G_{si},B_{si},L_{si}} & -Cost \quad \text{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ & \frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\frac{1}{\gamma_s}} \end{aligned}$$

$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{l}$$





$$\max_{G_{si},B_{si},L_{si}}$$
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$$\frac{L_{si}}{\hat{K}_{si}} = \mathbf{z}_{si}^{I} = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)\mathbf{z}_{si}^{k^{1-\gamma_s}}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^{\mathbf{G}})r_s}{(1+\tau_s^{\mathbf{W}})w_{si}}$$



$$\max_{G_{si},B_{si},L_{si}} \quad -Cost \quad ext{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{eta_s} \, L_{si}^{1-eta_s} = ar{Y}_{si}$$

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$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k \gamma_s - 1} + (1 - \alpha_s) \right)^{\frac{\gamma_s}{1 - \gamma_s}} z_{si}^{\prime 1 - \beta} \bar{Y}_{si}$$



$$\max_{m{G}_{si},m{B}_{si},m{L}_{si}} \quad -Cost \quad ext{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{eta_s} \, L_{si}^{1-eta_s} = ar{Y}_{si}$$

$$\frac{G_{si}}{B_{si}} = \mathbf{z}_{si}^{k} = \left(\frac{\alpha_{s}}{1 - \alpha_{s}} \frac{(1 + \tau_{s}^{B})r_{s} + \tau_{s}^{E} \tilde{A}}{(1 + \tau_{s}^{G})r_{s}}\right)^{\frac{1}{\gamma_{s}}}$$

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$$\begin{split} \max_{G_{si},B_{si},L_{si}} &- Cost \quad \text{s.t.} \quad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si} \\ \frac{G_{si}}{B_{si}} &= z_{si}^k = \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_s^B)r_s + \tau_s^E \tilde{A}}{(1+\tau_s^G)r_s}\right)^{\frac{1}{\gamma_s}} \\ \frac{L_{si}}{\hat{K}_{si}} &= z_{si}^l = \frac{1-\beta}{\beta} \frac{1}{\alpha_s} \left(\alpha_s + (1-\alpha_s)z_{si}^{k1-\gamma_s}\right)^{\frac{1}{1-\gamma_s}} \frac{(1+\tau_s^G)r_s}{(1+\tau_s^W)w_{si}} \\ E_{si} &= \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\alpha_s z_{si}^{k\gamma_s-1} + (1-\alpha_s)\right)^{\frac{\gamma_s}{1-\gamma_s}} z_{si}^{l1-\beta} \bar{Y}_{si} = \psi_{si} \bar{Y}_{si} \end{split}$$

• Firm will then charge markup over the marginal cost



Optimal Allocation

$$\max \quad -Cost \quad \text{s.t.} \qquad \hat{A}_{si} \hat{K}_{si}^{\beta_s} L_{si}^{1-\beta_s} = \bar{Y}_{si}$$

$$z_{si}^k \equiv \frac{G_{si}}{B_{si}} = \left[\frac{\alpha_s}{1-\alpha_s} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial G} Cost_{si}} \right]^{\gamma_s}$$

$$z_{si}^l \equiv \frac{L_{si}}{\hat{K}_{si}} = \frac{1-\beta_s}{\beta_s} \frac{1}{1-\alpha_s} (\alpha_s z_{si}^{k(\gamma_s-1)} + (1-\alpha_s))^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial B} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}}$$

$$= \frac{1-\beta_s}{\beta_s} \frac{1}{\alpha_s} (\alpha_s + (1-\alpha_s) z_{si}^{k(1-\gamma_s)})^{\frac{1}{1-\gamma_s}} \frac{\frac{\partial}{\partial G} Cost_{si}}{\frac{\partial}{\partial L} Cost_{si}}$$

$$E_{si} = \frac{\tilde{A}_{si}}{\hat{A}_{si}} \left(\frac{\phi_{si}}{z_{si}^{l}}\right)^{\theta_{s}} z_{si}^{l} \overset{\beta_{s}}{\tilde{Y}}_{si} = \psi_{si} \tilde{Y}_{si}, \quad \text{where} \quad \phi_{si} = \frac{\left(\mu_{s} + (1 - \mu_{s}) z_{si}^{k(1 - \eta_{s})}\right)^{\frac{\eta_{s}}{\eta_{s} - 1}}}{\left(\alpha_{s} + (1 - \alpha_{s}) z_{si}^{k(1 - \gamma_{s})}\right)^{\frac{\gamma_{s}}{\gamma_{s} - 1}}}$$





April. 2024

Model

Optimal firm level price

• Now Firm need to choose the price level to maximize the profit:

$$\max_{P_{si}} \quad \pi_{si} = P_{si}F_{si} - C_{si}F_{si}$$

- Firm-level real output is a function of the sector price, firm price, and sector real output (i.e. $F_{si} = (\frac{P_s}{P_{ci}})^{\sigma_s} F_s$)
- Therefore, because the optimal ratio does not depend on the price, the ratio can be maximized out of the problem of the optimal determination of the price, leaving the firm's real output as just a function of price

$$P_{si} = \frac{1}{1 + \tau_{si}^p} \frac{\sigma_s}{\sigma_s - 1} C_{si}$$



Estimation / Calibration

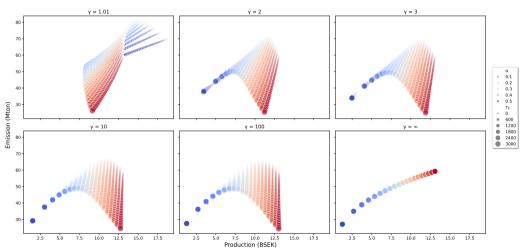
- My goal is to estimate the parameters sector by sector for Sweden
- I just reasonably calibrate the model to match the summary statistics of Martinsson et al. (2024)

Parameter	Value	Source/Moment
	Panel A:	Inputs
σ	∞	Fully competitive
r	5%	-
w	500 TSEK	-
L	250 (sd = 900)	Martinsson et al. (2024)
	Panel B: Calib	rated Value
β_s	0.6	Martinsson et al. (2024)
α_s	0.25	G/B, Wiedemann (2023)
Panel C: Estimated Value		
γ	10.34	$\Delta(\frac{E}{PY})/\Delta(\frac{C}{PY})$
$ ilde{ ilde{A}}$	0.018	E/PY

8/11

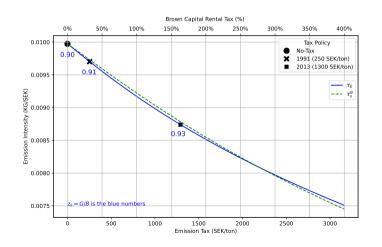
Sensitivity analysis

Production vs Emission with different Carbon Tax on different α and γ



Carbon Intensity and Tax

Counterfactual



$ au_{ extsf{E}}$	$ au_s^B$
100	14%
250	36%
500	66%
1300	171%
3000	360%

Resources allocation





Resources allocation

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$

$$\hat{G}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} \frac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$$

$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} \frac{1}{1 + z_{s}^{k}} K_{s}$$





Resources allocation

$$\hat{L}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} L_{s}$$

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$$\hat{B}_{si} = \frac{\hat{A}_{si}^{\sigma - 1}}{\sum_{j} \hat{A}_{sj}^{\sigma - 1}} \frac{1}{1 + z_{s}^{k}} K_{s}$$

$$\tilde{L}_{si} = \frac{A_{si}^{\sigma-1}/A_{si}^{\sigma}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s}$$





Resources allocation

$$\hat{L}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} L_{s}$$
 $\hat{G}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{z_{s}^{k}}{1 + z_{s}^{k}} K_{s}$
 $\hat{B}_{si} = rac{\hat{A}_{si}^{\sigma-1}}{\sum_{j} \hat{A}_{sj}^{\sigma-1}} rac{1}{1 + z_{s}^{k}} K_{s}$

$$\begin{split} \tilde{L}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} L_{s} \\ \hat{G}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{sj}^{\sigma}} \frac{z_{s}^{k}}{1+z_{s}^{k}} K_{s} \\ \hat{B}_{si} &= \frac{\hat{A}_{si}^{\sigma-1}/\tilde{A}_{si}^{\sigma}}{\sum_{j}\hat{A}_{sj}^{\sigma-1}/\tilde{A}_{si}^{\sigma}} \frac{1}{1+z_{s}^{k}} K_{s} \end{split}$$



