Speculative Betas[1]

Harrison Hong David A.Sraer

S.M. Aghajanzadeh

Tehran Institute for Advanced Studies

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Asset pricing theory

- Higher beta , higher expected return
  1. Aim to maximize economic utilities.
  2. Are rational and risk-averse.
  3. Are broadly diversified across a range of investments.
  4. Are price takers, i.e., they cannot influence prices.
  5. Can lend and borrow unlimited amounts under the risk free rate of interest.
  6. Trade without transaction or taxation costs. Deal with securities that are all highly divisible into small parcels.
  7. Assume all information is available at the same time to all investors.

High-risk, low-return puzzle

- Black (1972) by relaxing the assumption of borrowing at the risk-free rate or noise traders in Delong et al. (1990) or liquidity shocks as in Campbell, Grossman, and Wang (1993) reconcile a flat Security Market Line
- We show that by relaxing the other CAPM assumptions of
  1. homogeneous expectations
  2. costless short-selling

  can deliver rich patterns in the Security Market Line, including an inverted-U shape or even a downward-sloping line

Main Results

- High-beta assets are overpriced compared to low beta assets when disagreement is high:
  - beta amplifies disagreement about the macroeconomy. Because of short-sales constraints, high-beta stocks are only held in equilibrium by optimists, as pessimists are sidelined.
- Testable implications:
  - Macrodisagreement is low, all investors are long and short-sales constraints do not bind
  - For high enough aggregate disagreement, the relationship between risk and return takes on an inverted-U shape
- Stocks' cash flow process is heteroskedasticity
  - large idiosyncratic variance makes optimist investors reluctant to demand of a stock

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Model

Assets

- $\forall i \in \{1, \ldots, N\}, \tilde{d}_i = d + b_i \tilde{z} + \tilde{\epsilon}_i$
  - $E[\tilde{z}] = 0, Var[\tilde{z}] = \sigma_z^2$
  - $E[\tilde{\epsilon}_i] = 0, Var[\tilde{\epsilon}_i] = \sigma_\epsilon^2$
- $s_i = \frac{1}{N}$
- $0 < b_1 < b_2 < \cdots < b_N$

Invesots

- Heterogeneous investors (fractio α)
  - Cannot short
  - Two groups:
    - A : $E^A[\tilde{z}] = \lambda$
    - B : $E^B[\tilde{z}] = -\lambda$
- Homogeneous investors (fractio $1-\alpha$)
  - No short-sales constraint
  - $E^a[\tilde{z}] = 0$
- Investor's utility:

$$U(\tilde{w}^k) = E^k[\tilde{w}^k] - \frac{1}{2\gamma} Var(\tilde{w}^k) \qquad k \in \{a, A, B\}$$

Equilibrium

- Asset prices:

$$P_i(1+r) = \begin{cases} d - \frac{1}{\gamma}(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}) & \text{for } i < \bar{i} \\ d - \frac{1}{\gamma}(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}) + \underbrace{\frac{\theta}{\gamma}(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N})}_{\pi^i = \text{speculative premimum}} & \text{for } i \geq \bar{i} \end{cases}$$

$$\omega(\lambda) = \frac{\lambda\gamma - \frac{\sigma_z^2}{N}(\sum_{i\geq\bar{i}} b_i)}{\sigma_z^2(1+\sigma_z^2(\sum_{i<\bar{i}} \frac{b_i^2}{\sigma_\epsilon^2}))} \qquad \theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$$

- If $\alpha = 0$ then $\theta = 0$, As a result $\pi^i =$ speculative premimum
- $\lambda \uparrow \pi^i \uparrow \qquad b_i \uparrow \pi^i \uparrow \qquad \alpha \uparrow \pi^i \uparrow$

Equilibrium

Beta and Expected Return

- Expected excess return :

$$E[\tilde{R}_i^e] = \begin{cases} \beta_i \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} & \text{for } i < \bar{i} \\ \beta_i \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}(1 - \theta\omega(\lambda)) + \theta \frac{\sigma_\epsilon^2}{\gamma N}(1 + \omega(\lambda)) & \text{for } i \geq \bar{i} \end{cases}$$

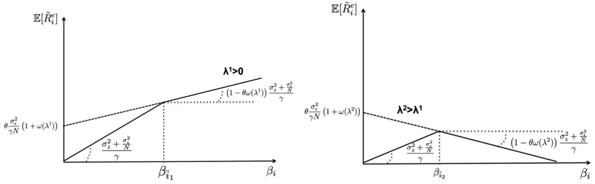
- Expected excess return (Heteroskedastic Idiosyncratic Variance) :

$$E[\tilde{R}_i^e] = \begin{cases} \beta_i \frac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N}}{\gamma} & \text{for } \frac{\beta_i}{\sigma_i^2} < \frac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \\ \beta_i \frac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N}}{\gamma}(1 - \theta\omega(\lambda)) + \theta \frac{\sigma_i^2}{\gamma N}(1 + \omega(\lambda)) & \text{for} \frac{\beta_i}{\sigma_i^2} \geq \frac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \end{cases}$$

Proof

Security Market Line

different levels of aggregate disagreement



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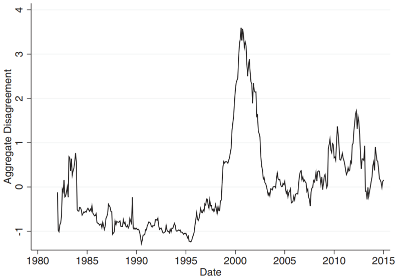
β-Sorted Portfolios

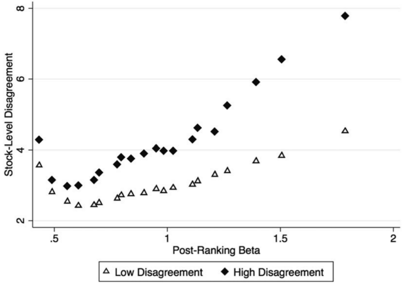
- At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month
- The ranked stocks are assigned to 1 of 20 value-weighted portfolios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
β	0.43	0.49	0.55	0.60	0.67	0.69	0.77	0.79	0.83	0.89	0.95	0.98	1.02	1.11	1.13	1.21	1.26	1.39	1.50	1.78
Median Vol.	1.56	1.14	1.10	1.11	1.13	1.14	1.15	1.17	1.22	1.23	1.28	1.33	1.34	1.43	1.50	1.58	1.70	1.88	2.26	3.16
$R^{(1)}_{i,t}$	0.12	0.39	0.63	0.61	0.61	0.73	0.87	0.61	0.79	0.60	0.67	0.64	0.53	0.57	0.73	0.54	0.58	0.44	−0.17	−0.72
$R^{(12)}_{i,t}$	3.21	5.58	7.98	7.74	8.19	7.94	8.10	8.15	8.92	6.88	8.28	8.51	7.46	7.82	8.08	7.86	8.23	6.62	−0.55	−11.69
Stock Disp.	2.97	2.76	2.81	2.78	2.89	3.02	3.24	3.35	3.44	3.38	3.54	3.56	3.60	3.72	3.94	3.93	4.29	4.71	5.03	6.91
% Mkt. Cap.	2.77	3.58	4.15	4.73	4.86	5.00	5.32	5.44	5.49	5.84	5.60	5.58	5.62	5.63	5.36	5.45	5.31	5.09	5.21	7.48
N stocks	171	143	145	147	149	154	157	160	161	169	164	166	167	167	167	168	175	186	210	365

Measuring Aggregate Disagreement

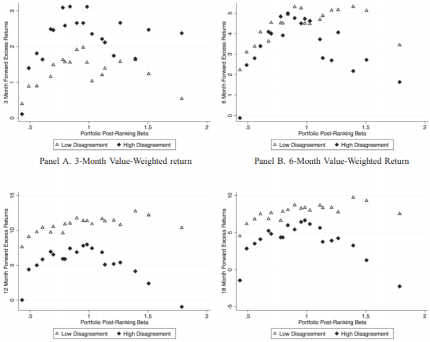
- Measure stock-level disagreement as the dispersion in analyst forecasts
- Aggregate this stock-level disagreement measure, weighting each stock by its preranking β





Concavity of the Security Market Line

- Average excess returns-to-β relationship is mostly upward-sloping
- Inverted-U shape predicted by the theory



Panel A. 3-Month Value-Weighted return

Panel B. 6-Month Value-Weighted Return

Panel C. 12-Month Value-Weighted Return

Panel D. 18-Month Value-Weighted Return

Formally test

- two-stage Fama and MacBeth regression
- First Stage:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}$$

- Second Stage:

$$\begin{cases} \phi_t = c_1 + \psi_1.Dis_{t-1} + \delta_1^m.R_{m,t}^{(12)} + \delta_1^{HML}.HML_t^{(12)} + \delta_1^{SMB}.SMB_t^{(12)} + \delta_1^{UMD}.UMD_t^{(12)} + \sum_{x \in X} \delta_1^x.x_{t-1} + \zeta_t \\ \pi_t = c_2 + \psi_2.Dis_{t-1} + \delta_2^m.R_{m,t}^{(12)} + \delta_2^{HML}.HML_t^{(12)} + \delta_2^{SMB}.SMB_t^{(12)} + \delta_2^{UMD}.UMD_t^{(12)} + \sum_{x \in X} \delta_2^x.x_{t-1} + \zeta_t \\ \kappa_t = c_3 + \psi_3.Dis_{t-1} + \delta_2^m.R_{m,t}^{(12)} + \delta_3^{HML}.HML_t^{(12)} + \delta_3^{SMB}.SMB_t^{(12)} + \delta_3^{UMD}.UMD_t^{(12)} + \sum_{x \in X} \delta_2^x.x_{t-1} + \zeta_t \end{cases}$$

Estimation ressults

	ϕ_t				π_t				κ_t			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						Panel B: Equal-Weighted Portfolios						
Agg. Disp.$_{t-1}$	−6.80**	−4.85***	−6.48***	−6.32***	9.81**	10.20**	10.17**	9.04**	−3.43*	−3.12	−0.67	0.25
	(−2.55)	(−2.62)	(−3.13)	(−3.12)	(2.25)	(2.52)	(2.34)	(2.20)	(−1.95)	(−1.48)	(−0.32)	(0.13)
$R_{m,t}^{(12)}$		−0.22**	−0.30***	−0.30***		1.09***	1.24***	1.26***		0.16	0.11	0.10
		(−2.54)	(−3.53)	(−3.41)		(6.35)	(6.51)	(6.14)		(1.62)	(1.12)	(0.88)
$HML_t^{(12)}$		−0.69***	−0.65***	−0.66***		0.87***	0.89***	0.96***		0.22	0.13	0.08
		(−4.41)	(−5.01)	(−5.32)		(2.71)	(3.08)	(3.29)		(1.39)	(0.88)	(0.51)
$SMB_t^{(12)}$		0.01	0.12	0.12		0.69**	0.58*	0.58*		−0.11	−0.16	−0.15
		(0.05)	(0.86)	(0.86)		(2.51)	(1.85)	(1.81)		(−0.69)	(−0.88)	(−0.87)
$UMD_t^{(12)}$		−0.08	−0.03	−0.03		0.06	−0.01	0.02		0.03	0.04	0.01
		(−1.12)	(−0.55)	(−0.62)		(0.39)	(−0.07)	(0.14)		(0.28)	(0.34)	(0.13)
D/P$_{t-1}$			1.31	1.74			−6.12	−9.14*			4.80**	7.26***
			(0.61)	(0.67)			(−1.36)	(−1.76)			(2.37)	(3.28)
Inflation$_{t-1}$			−4.68***	−4.44***			7.13**	5.44			−0.90	0.48
			(−3.60)	(−3.20)			(2.07)	(1.41)			(−0.48)	(0.22)
Ted Spread$_{t-1}$				−0.75				5.22				−4.25**
				(−0.43)				(1.36)				(−2.23)
Constant	−9.73***	−4.51**	−4.41**	−4.32**	21.57***	7.30	6.43	5.81	−0.92	−3.27	−2.47	−1.97
	(−4.75)	(−2.31)	(−2.34)	(−2.20)	(4.97)	(1.59)	(1.38)	(1.18)	(−0.46)	(−1.39)	(−1.00)	(−0.76)
N	385	385	385	385	385	385	385	385	385	385	385	385

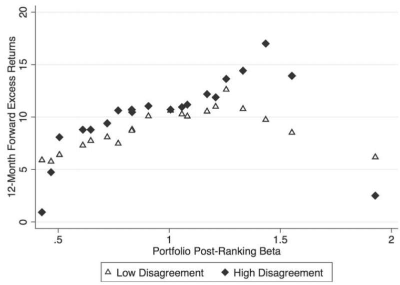
β-Sorted Portfolios

- Rank stocks based on preranking ratio of β to σ^2 and define as speculative stocks all stocks with a ratio above the median ratio
- Then, within each of these two groups creat 20 β-sorted portfolios

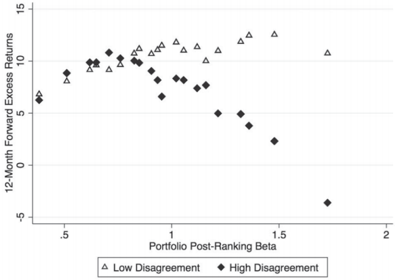
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Panel A: Nonspeculative Stocks																				
β	0.42	0.46	0.50	0.61	0.64	0.72	0.77	0.83	0.83	0.90	1.00	1.05	1.08	1.17	1.20	1.25	1.33	1.43	1.55	1.92
Median Vol.	1.85	1.35	1.29	1.40	1.38	1.46	1.54	1.62	1.71	1.80	1.86	1.93	2.01	2.15	2.26	2.33	2.43	2.70	2.92	4.09
$\frac{\beta}{\sigma^2}$	0.12	0.25	0.30	0.31	0.33	0.33	0.32	0.31	0.28	0.27	0.29	0.28	0.26	0.25	0.23	0.22	0.22	0.19	0.18	0.11
$R_{i,t}^{(1)}$	0.05	0.21	0.34	0.35	0.57	0.46	0.79	0.59	0.66	0.82	0.82	0.61	0.61	0.93	0.43	0.90	0.75	0.51	0.76	−0.02
$R_{i,t}^{(12)}$	1.89	3.01	3.93	5.70	5.88	7.57	6.77	9.75	9.07	9.03	8.83	8.34	7.90	10.35	10.15	12.73	10.80	11.21	11.58	−2.78
Stock Disp.	3.12	2.85	3.06	3.56	3.34	3.36	3.54	3.80	3.87	4.16	4.74	4.73	4.83	4.95	5.17	5.66	5.96	6.06	6.24	7.99
% Mkt. Cap.	4.31	3.95	4.76	5.15	5.56	5.62	5.90	5.65	5.47	5.57	5.19	5.00	5.06	4.74	4.81	4.61	4.58	4.55	5.14	8.70
N stocks	96	73	71	75	81	84	87	89	93	96	97	98	100	101	108	111	118	126	151	339
Panel B: Speculative Stocks																				
β	0.38	0.51	0.61	0.64	0.70	0.76	0.82	0.84	0.90	0.93	0.95	1.02	1.05	1.11	1.15	1.21	1.32	1.36	1.47	1.72
Median Vol.	0.78	0.86	0.93	0.98	1.01	1.02	1.05	1.09	1.13	1.17	1.17	1.24	1.27	1.34	1.39	1.47	1.56	1.75	1.94	2.60
$\frac{\beta}{\sigma^2}$	0.62	0.68	0.70	0.66	0.68	0.71	0.73	0.70	0.69	0.68	0.68	0.65	0.65	0.62	0.59	0.56	0.54	0.44	0.39	0.25
$R_{i,t}^{(1)}$	0.62	0.81	0.54	0.78	0.76	0.66	0.47	0.61	0.63	0.55	0.83	0.48	0.46	0.73	0.43	0.65	0.49	−0.24	0.00	−0.91
$R_{i,t}^{(12)}$	8.51	9.47	8.71	8.24	7.38	7.10	6.49	7.43	7.96	7.58	7.41	7.90	7.31	7.06	7.55	6.54	4.57	−1.52	−2.38	−11.62
Stock Disp.	1.95	2.17	2.33	2.66	2.76	2.90	2.98	3.07	3.16	3.24	3.23	3.17	3.31	3.49	3.50	3.75	4.21	4.55	5.03	6.57
% Mkt. Cap.	4.16	4.48	4.98	5.05	5.19	5.37	5.88	5.84	5.73	5.50	5.35	5.24	5.30	5.17	4.98	4.68	5.01	5.15	4.81	6.64
N stocks	57	58	58	60	63	64	65	67	68	67	68	69	67	66	65	66	65	66	68	90

Concavity of the Security Market Line

- For nonspeculative stocks, the Security Market Line is not related with aggregate disagreement.
- For speculative stocks when aggregate disagreement is high, the Security Market Line exhibits an inverted-U shape



(a) Nonspeculative Stocks



(b) Speculative Stocks

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Conclusion

- High-beta assets are more speculative because they are more sensitive to disagreement about common cash flows
- As aggregate disagreement rises, the slope of the Security Market Line is piecewise constant, higher in the low-beta range and potentially negative for the highbeta range

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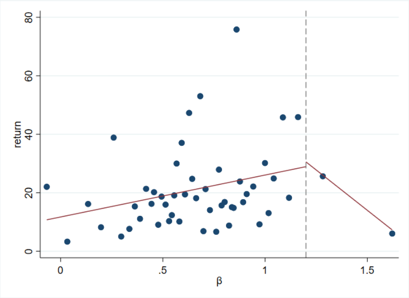
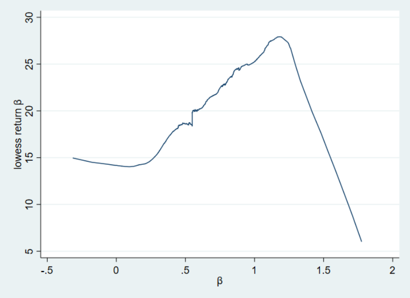
Return & β

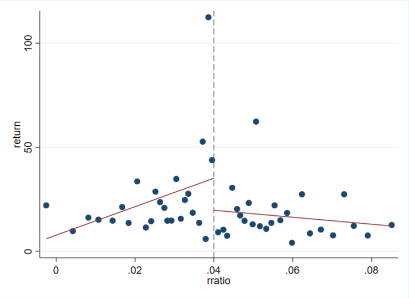


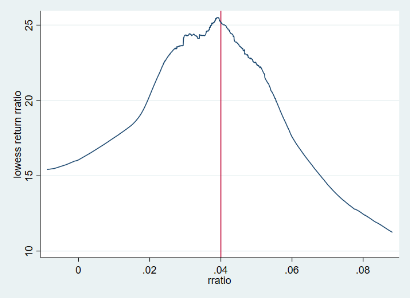
return
0
20
40
60
80
0
.5
1
1.5
β



lowess return β
5
10
15
20
25
30
-.5
0
.5
1
1.5
2
β

Return & β/σ





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Equilibrium

Beta and Expected Return

- $\tilde{R}_i^e = d + b_i\tilde{z} + \tilde{\epsilon}_i - (1+r)P_i$
- $\beta_i = \frac{Cov(\tilde{R}_i^e, \tilde{R}_m^e)}{Var(\tilde{R}_m^e)} = \frac{b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}$
- $E[\tilde{R}_i^e] = d - (1+r)P_i \rightarrow (1+r)P_i = d - E[\tilde{R}_i^e]$
- $d - E[\tilde{R}_i^e] = d - \frac{1}{\gamma}(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}) + \frac{\theta}{\gamma}(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N})$
- $E[\tilde{R}_i^e] = \frac{1}{\gamma}(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}) - \frac{\theta}{\gamma}(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N})$
- $$\begin{aligned} E[\tilde{R}_i^e] &= \frac{b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}(1 - \theta\omega(\lambda)) + \theta\frac{\sigma_\epsilon^2}{\gamma N}(1 + \omega(\lambda))^\circ \\ &= \beta_i\frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}(1 - \theta\omega(\lambda)) + \theta\frac{\sigma_\epsilon^2}{\gamma N}(1 + \omega(\lambda)) \end{aligned}$$