Internet Appendix to "Speculative Betas"*

This internet appendix has two sections. Section I contains all the proofs for the paper. Section II contains additional tables.

I. Proofs of the Model

A. *Proof of Theorem 1*

Proof. We solve the model here allowing for heteroskedastic dividends σ_i^2. Theorem 1 can then be proved as the special case $\sigma_\epsilon^2 = \sigma_i^2$. We assume that assets are ranked in ascending order of β/σ^2.

We first posit an equilibrium structure. We then check ex-post that it is indeed an equilibrium and that it is a unique equilibrium. Let $\bar{i} \in [2, N]$ and let μ_i^m be the share holdings of asset k by group m, where $m \in \{a, A, B\}$. Consider an equilibrium where group B investors are long assets $i < \bar{i}$ and hold no position (i.e., $\mu_i^B = 0$) for assets $i \geq \bar{i}$, and group A investors are long all assets $i \in [1, N]$. Since group A investors are long all assets, their holdings satisfy the following first-order condition:

$$\forall i \in [1, N]: \quad d + \lambda b_i - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^A\right) b_i \sigma_z^2 + \mu_i^A \sigma_i^2\right).$$

Since group B investors are long only assets $i < \bar{i}$, their holdings for these assets must also satisfy the following first-order condition:

$$\forall i \in [1, \bar{i} - 1], \qquad d - \lambda b_i - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2 + \mu_i^B \sigma_i^2\right).$$

For assets $i \geq \bar{i}$, group B investors have zero holdings and so $\mu_i^B = 0$. For these assets, it must be the case that the group B investors' marginal utility of holding the asset, taken at the equilibrium holdings, is strictly negative (otherwise, group B investors would have an incentive to increase their holdings). This is equivalent to

$$\forall i \geq \bar{i}, \qquad d - \lambda b_i - P_i(1 + r) - \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2\right) < 0.$$

Finally, since arbitrageurs are not short-sales constrained, their holdings always satisfy the following first-order condition:

$$\forall i \in [1, N], \quad d - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^a\right) b_i \sigma_z^2 + \mu_i^a \sigma_i^2\right).$$

The market clearing condition for asset i is simply $\alpha\frac{\mu_i^A + \mu_i^B}{2} + (1 - \alpha)\mu_i^a = \frac{1}{N}$. We sum the first-order conditions of investors a, A, and B for assets $i < \bar{i}$, and of investors a and A for assets $i \geq \bar{i}$, weighting the sum by the size of each investor group ($\frac{\alpha}{2}$ for groups A and B, and $1 - \alpha$ for group a). This results in the following system of equations:

$$\begin{cases} d - P_i(1 + r) = \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_i^2}{N}\right) \quad \text{for } i < \bar{i} \\ \left(1 - \dfrac{\alpha}{2}\right)(d - P_i(1 + r)) + \dfrac{\alpha}{2}\lambda b_i = \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_i^2}{N} - \dfrac{\alpha}{2}\sigma_z^2 b_i \displaystyle\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) \quad \text{for } i \geq \bar{i}. \end{cases} \tag{1}$$

Let $S^B = \sum_{k=1}^{\bar{i}-1} b_k \mu_k^B$, where S^B represents the exposure of group B investors to the aggregate factor $\tilde{z}$. We look for an expression for S^B. We start by using the first-order condition of group B investors on assets $k < \bar{i}$ and plug in the equilibrium price of assets $k < \bar{i}$

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found in the first equation of system (1):

$$\forall k < \bar{i}, \qquad -\lambda\gamma b_k + b_k\sigma_z^2 + \frac{\sigma_k^2}{N} = S^B b_k \sigma_z^2 + \mu_k^B \sigma_k^2.$$

We can now multiply the previous equation by b_k, divide it by σ_k^2 for all $k < \bar{i}$, and then sum up the resulting equations for $k < \bar{i}$. This results in

$$S^B = -\lambda\gamma \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2} \right) - S\sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2} \right) + \sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2} \right) + \sum_{k<\bar{i}} \frac{b_k}{N}. \tag{2}$$

From the previous expression, we can now derive S^B as

$$S^B = 1 - \frac{\left(\sum_{k\geq\bar{i}} \frac{b_k}{N}\right) + \lambda\gamma \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}{1 + \sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}.$$

Now that we have a closed-form expression for S^B, we simply plug it into the second equation of system 1. Define $\theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$. The price of assets $i \geq \bar{i}$ is then given by

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\sigma_z^2 \underbrace{\frac{\lambda\gamma - \sigma_z^2 \sum_{k\geq\bar{i}} \frac{b_k}{N}}{\sigma_z^2\left(1 + \sigma_z^2\left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_i^2}\right)\right)}}_{=\omega(\lambda)} - \frac{\sigma_i^2}{N} \right)}_{\pi^i = \text{speculative premium}}. \tag{3}$$

The first equation of system 1 provides us with a simple expression for the price of assets $i < \bar{i}$:

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right). \tag{4}$$

To derive the conditions under which the proposed equilibrium is indeed an equilibrium (i.e., $\bar{i}$ is indeed the marginal asset), we need to derive the equilibrium holdings of group B investors:

$$\mu_i^{B,\star} = \begin{cases} \dfrac{1}{N} + \dfrac{b_i}{\sigma_i^2}\left(\dfrac{\sigma_z^2\left(\sum_{i\geq\bar{i}} \frac{b_i}{N}\right) - \lambda\gamma}{1 + \sigma_z^2\left(\sum_{i<\bar{i}} \frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i < \bar{i} \\ 0 \text{ for } i \geq \bar{i}. \end{cases}$$

We are now ready to derive the conditions under which the proposed equilibrium is indeed an equilibrium. The marginal asset is asset $\bar{i}$ if and only if $\frac{\partial U^B}{\partial \mu_{\bar{i}}^B}(\mu^{B,\star}) < 0$ and $\mu_{\bar{i}-1}^B \geq 0$, where $\mu^{B,\star}$ is group B investors' holdings derived above. The condition that the marginal utility of investing in asset $\bar{i}$ for pessimist agents is equivalent to $\pi_{\bar{i}} > 0$, so that $\bar{i}$ is the marginal asset if and only if

$$\frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}} b_k + \frac{1}{\gamma N \frac{b_{\bar{i}-1}}{\sigma_{\bar{i}-1}^2}}\left(1 + \sigma_z^2 \sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right) \geq \lambda > \frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}} b_k + \frac{1}{\gamma N \frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}}\left(1 + \sigma_z^2 \sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right).$$

Let $u_k = \frac{1}{\gamma N \frac{b_k}{\sigma_k}}\left(1 + \sigma_z^2\left(\sum_{i<k} \frac{b_i^2}{\sigma_i^2}\right)\right) + \frac{\sigma_z^2}{\gamma}\left(\sum_{i\geq k} \frac{b_i}{N}\right)$. Clearly, u_k is a strictly decreasing sequence as

$$
\begin{aligned}
u_{i-1}-u_i &= \frac{1}{\gamma N\frac{b_{i-1}}{\sigma_{i-1}^2}}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)+\frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i-1}\frac{b_j}{N}\right)-\frac{1}{\gamma N\frac{b_i}{\sigma_i^2}}\left(1+\sigma_z^2\left(\sum_{j<i}\frac{b_j^2}{\sigma_j^2}\right)\right)-\frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i}\frac{b_j}{N}\right)\\
&= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)-\frac{1}{\gamma N\frac{b_i}{\sigma_i^2}}\sigma_z^2\frac{b_{i-1}^2}{\sigma_{i-1}^2}+\frac{\sigma_z^2}{\gamma N}b_{i-1}\\
&= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)+\frac{\sigma_z^2}{\gamma N}\frac{b_{i-1}^2}{\sigma_{i-1}^2}\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)\\
&= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)>0.
\end{aligned}
$$

Define $u_0=+\infty$ and $u_{N+1}=0$. Then the sequence $(u_i)_{i\in[0,N+1]}$ spans $\mathbb{R}^+$ and the marginal asset is simply defined as $\bar{i}=\min\{k|\lambda>u_k\}$. We know that $\bar{i}>0$ since $u_0=+\infty$. If $\bar{i}=N+1$, then group B investors are long all assets and all the previous formulas apply except that there is no asset such that $i\geq\bar{i}$. If $\bar{i}\in[1,n]$, then the equilibrium has the proposed structure, that is, investors B are long only assets $i<\bar{i}$.

We have so far assumed that $\bar{i}>1$. The equilibrium is easily derived when $\bar{i}=1$, that is, when all assets are overpriced. In this case, $S^B=0$ and we have

$$
d-(1+r)P_i=\frac{1}{\gamma}(1+\theta)\left(b_i\sigma_z^2+\frac{\sigma_\epsilon^2}{N}\right)-\theta\lambda b_i.
$$

This corresponds to the formula derived in Theorem 1 where we define $\sum_{i<1}b_i^2=0$. Moreover, $\bar{i}=1$ is an equilibrium if and only if $\mu_1^{B,\star}<0$, which is equivalent to $\lambda<u_N$, as stated in the theorem.

We now show that the equilibrium is unique. Let $J=j|\mu_j^B>0-$J be the set of assets that pessimists are long. It is straightforward to show that prices are given by in this case

$$
P_i(1+r)=\begin{cases}
d-\dfrac{1}{\gamma}\left(b_i\sigma_z^2+\dfrac{\sigma_\epsilon^2}{N}\right) & \text{for } i\in J\\[2ex]
d-\dfrac{1}{\gamma}\left(b_i\sigma_z^2+\dfrac{\sigma_i^2}{N}\right)+\underbrace{\dfrac{\theta}{\gamma}\left(b_i\left(\dfrac{\lambda\gamma-\frac{\sigma_z^2}{N}\left(\sum_{i\notin J}b_i\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right)-\dfrac{\sigma_i^2}{N}\right)}_{\pi^i=\text{speculative premium}} & \text{for } i\notin J\cdot
\end{cases}
\tag{5}
$$

The holdings of the pessimists can then be written as

$$
\mu_i^{B,\star}=\begin{cases}
\dfrac{1}{N}+\dfrac{b_i}{\sigma_i^2}\left(\dfrac{\sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right)-\lambda\gamma}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i\in J\\[2ex]
0 \quad \text{for } i\notin J
\end{cases}
$$

This implies that for $j\in J$, we need to have $\frac{b_j}{\sigma_j^2}\left(\frac{\lambda\gamma-\sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right)<\frac{1}{N}$, and for $i\notin J$ that $b_i\left(\frac{\lambda\gamma-\frac{\sigma_z^2}{N}\left(\sum_{i\notin J}b_i\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right)>\frac{\sigma_i^2}{N}$.

Thus, for all $j\in J$ and for all $i\notin J$,

$$
\frac{b_i}{\sigma_i^2}>\frac{1}{N}\left(\frac{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}{\lambda\gamma-\sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right)}\right)>\frac{b_j}{\sigma_j^2}.
$$

The equilibrium structure is necessarily in the form of a cutoff and hence our equilibrium is unique.

■

B. Proof of Corollary 1

Proof. Corollary 1 characterizes overpricing. Overpricing for assets $i\geq\bar{i}$ is defined as the difference between the equilibrium price and the price that would prevail in the absence of heterogeneous beliefs and short-sales constraints ($\alpha=0$). Overpricing is simply equal to the speculative premium:

$$
\forall i\geq\bar{i},\quad \text{Overpricing}^i=\pi^i=\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda)-\frac{\sigma_i^2}{N}\right).
$$

By definition of the equilibrium, $\lambda > u_{\bar{i}}$, which is equivalent to $\frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}\sigma_z^2\omega(\lambda) > \frac{1}{N}$. Since assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$, this implies that for $i \geq \bar{i}$, $\pi^i > 0$ and assets $i \geq \bar{i}$ are in fact overpriced. That mispricing is increasing with the fraction of short-sales-constrained investors α follows as θ is a strictly increasing function of α. That mispricing increases with b_i and decreases with σ_i^2 also follows from the definition of mispricing:[1]

$$\forall j > i \geq \bar{i}, \quad \text{Overpricing}^j - \text{Overpricing}^i = \frac{\theta}{\gamma}\sigma_z^2\omega(\lambda)(b_j - b_i).$$

■

C. Proof of Corollary 2

Proof. Corollary 2 characterizes the amount of shorting in the equilibrium. We first need to derive the equilibrium holdings of arbitrageurs. Group a holdings need to satisfy the following first-order condition:

$$\forall i \in [1, N], \quad d - P_i(1+r) = \frac{1}{\gamma}\left(b_i\sigma_z^2\left(\sum_{k=1}^{N}\mu_k^a b_k\right) + \mu_i^a\sigma_i^2\right)$$

Define $S^a = \sum_{k=1}^{N}\mu_k^a b_k$. Using the equilibrium pricing equation in equation 3 and equation 4, this first-order condition can be rewritten as

$$\forall k \in [1, N], \quad b_k\sigma_z^2 + \frac{\sigma_k^2}{N} - \gamma\pi^k 1_{k\geq\bar{i}} = b_k\sigma_z^2 S^a + \mu_k^a\sigma_k^2.$$

We multiply each of these equations by b_k, divide them by σ_k^2, and sum up the resulting equations for all $i \in [1, N]$ to obtain

$$S^a = 1 - \frac{\gamma\sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2\left(\sum_{k=1}^{N}\frac{b_k^2}{\sigma_k^2}\right)}$$

We can now plug this expression for S^a in group a investors' first-order conditions derived above. This yields the following expression for group a investors' holdings of assets $i \in [1, N]$:

$$\forall i \in [1, N], \quad \mu_i^a\sigma_i^2 = \frac{\sigma_i^2}{N} - \gamma\pi^i 1_{\{i\geq\bar{i}\}} + b_i\sigma_z^2\frac{\gamma\sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2\left(\sum_{k=1}^{N}\frac{b_k^2}{\sigma_k^2}\right)}.$$

First note that if $i < \bar{i}$, $\mu_i^a > 0$, so that arbitrageurs are long assets $i < \bar{i}$. Now consider the case $i \geq \bar{i}$. Notice from the expression of the speculative premium that

$$\forall k, i \geq \bar{i}, \quad \pi_k + \frac{\theta\sigma_k^2}{\gamma N} = \frac{b_k}{b_i}\left(\pi^i + \frac{\theta\sigma_i^2}{\gamma N}\right).$$

Thus, multiplying the previous expression by b_k, dividing by σ_k^2, and summing over all $k \geq \bar{i}$, we have

$$\sum_{k\geq\bar{i}} b_k\frac{\pi_k}{\sigma_k^2} + \frac{\theta}{\gamma N}\left(\sum_{k\geq\bar{i}} b_k\right) = \left(\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\pi^i + \frac{\theta\sigma_i^2}{\gamma N}}{b_i}\right).$$

Thus, for $i \geq \bar{i}$,

$$
\begin{aligned}
\gamma\pi^i - b_i\sigma_z^2 \frac{\gamma\sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} &= \gamma\pi^i - \frac{b_i\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\left(\left(\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\gamma\pi^i+\frac{\theta}{N}\sigma_i^2}{b_i}\right) - \frac{\theta}{N}\left(\sum_{k\geq\bar{i}} b_k\right)\right) \\
&= \gamma\pi^i \frac{1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\frac{\theta}{N}\left(\sum_{k\geq\bar{i}}\sigma_i^2\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right) \\
&= \theta\left[b_i\left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\geq\bar{i}} b_i\right)}{1+\sigma_z^2\left(\sum_{i=1}^N\frac{b_i^2}{\sigma_i^2}\right)}\right) - \frac{\sigma_i^2}{N}\frac{1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N\frac{b_k^2}{\sigma_k^2}\right)}\right. \\
&\qquad \left. - \frac{1}{N}\frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N\frac{b_k^2}{\sigma_k^2}\right)}\left(\sigma_i^2\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right)\right] \\
&= \theta\left[b_i\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N\frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_i^2}{N}\right].
\end{aligned}
$$

We can now derive the actual holdings of arbitrageurs on assets $i \geq \bar{i}$:

$$
\forall i \geq \bar{i}, \quad \mu_i^a = \frac{1+\theta}{N} - \theta\frac{b_i}{\sigma_i^2}\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N\frac{b_k^2}{\sigma_k^2}\right)}.
$$

Notice that arbitrageurs' holdings are decreasing with i since $\frac{b_i}{\sigma_i^2}$ increases strictly with i. There is at least one asset shorted by group a investors provided that $\mu_N^a < 0$, which is equivalent to $\lambda > \hat{\lambda} = \frac{1+\theta}{\theta}\frac{1+\sigma_z^2\left(\sum_{k=1}^N\frac{b_k^2}{\sigma_k^2}\right)}{N}\frac{\sigma_N^2}{\gamma b_N}$. If this holds, there exists a unique $\tilde{i} \in [1, N]$ such that $\mu_i^a < 0 \Leftrightarrow i \geq \tilde{i}$. We already know that $\tilde{i} \geq \bar{i}$ since for $i < \bar{i}$, group a investors holdings are strictly positive. It follows from the expression for group a investors' holdings that provided $i \geq \tilde{i}$, we have

$$
\frac{\partial|\mu_i^a|}{\partial\lambda} > 0, \quad \frac{\partial|\mu_i^a|}{\partial\frac{b_i}{\sigma_i^2}} > 0, \quad \text{and} \quad \frac{\partial^2|\mu_i^a|}{\partial\lambda\partial b_i} > 0.
$$

There is more shorting of assets with a larger ratio of cash-flow beta to idiosyncratic variance, there is more shorting the larger is aggregate disagreement, and the effect of aggregate disagreement on shorting is larger for assets with a high ratio of cash-flow beta to idiosyncratic variance. ■

D. Proof of Formula (3) for Expected Excess Returns

Proof. From Theorem 1, we know that

$$
P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_I^2}{N}\right) + \mathbf{1}_{i\geq\bar{i}}\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_I^2}{N}\right).
$$

Denote by $\tilde{R}_i^e$ the percentage excess return per share on asset i. Define $\tilde{R}_i^e = d + b_i\tilde{z} + \tilde{\epsilon}_i - (1+r)P_i$ and $\mathbb{E}[\tilde{r}_i^e] = d - (1+r)P_i$. Further, define the market portfolio as the portfolio of all assets in the market. Since all assets have a supply of 1/N, the excess return per share on the market portfolio is $\tilde{R}_m^e = \sum_{j=1}^N \frac{\tilde{R}_j^e}{N}$. Let $P_m = \sum_{j=1}^N \frac{P_j}{N}$ be the price of the market portfolio. Stock i's beta is defined as $\beta_i = \frac{\mathrm{Cov}\left(\tilde{R}_i^e, \tilde{R}_m^e\right)}{Var(\tilde{R}_m^e)}$ and can be written as

$$
\beta_i = \frac{b_i\sigma_z^2 + \frac{\sigma_i^2}{N}}{\sigma_z^2 + \sum_{k=1}^N\frac{\sigma_k^2}{N^2}}, \quad \text{so that} \quad b_i\sigma_z^2 = \beta_i\left(\sigma_z^2 + \sum_{k=1}^N\frac{\sigma_k^2}{N^2}\right) - \frac{\sigma_i^2}{N}.
$$

We can thus substitute b_i by β_i in the price formula and derive an expression for expected excess returns per share as a function of β_i:

$$
\mathbb{E}[\tilde{R}_i^e] = \beta_i\frac{\sigma_z^2 + \sum_{k=1}^N\frac{\sigma_k^2}{N^2}}{\gamma}\left(1 - \mathbf{1}_{i\geq\bar{i}}\theta\omega(\lambda)\right) + \theta\frac{\sigma_i^2}{\gamma N}\mathbf{1}_{i\geq\bar{i}}(1+\omega(\lambda)).
$$

$\blacksquare$

E. Proof of Corollary 3

Proof. We make this proof in the context of homoskedastic dividends: $\sigma_i^2 = \sigma_\epsilon^2$. We can write actual excess returns as

$$\tilde{R}_i^e = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} + \tilde{\eta}^i & \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} (1 - \theta\omega(\lambda)) + \dfrac{\sigma_\epsilon^2}{\gamma N}\theta(1+\omega(\lambda)) + \tilde{\eta}^i & \text{for } i \geq \bar{i} \end{cases},$$

where $\tilde{\eta_i} = b_i \tilde{z} + \tilde{\epsilon_i}$.

Using the fact that, by definition, $\sum_{i=1}^N b_i = \sum_{i=1}^N \beta_i = N$, a cross-sectional regression of realized excess returns per share $\left(\tilde{R}_i^e\right)_{i\in[1,N]}$ on $(\beta_i)_{i\in[1,N]}$ and a constant would deliver the following coefficient estimate:

$$\begin{aligned} \hat{\mu} &= \frac{\sum_{i=1}^N \beta_i \tilde{R}_i - \sum_{i=1}^N \tilde{R}_i}{\sum_{i=1}^N \beta_i^2 - N} \\ &= \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1 + \frac{\gamma}{\sigma_z^2}\tilde{z} - \left(\frac{\sum_{i\geq\bar{i}} \beta_i^2 - \sum_{i\geq\bar{i}}\beta_i}{\sum_{i=1}^N \beta_i^2 - N}\right)\theta\omega(\lambda)\right) + \frac{\sum_{i\geq\bar{i}}(\beta_i - 1)}{\sum_{i=1}^N \beta_i^2 - N}\frac{\sigma_\epsilon^2}{\gamma N}\theta\left(1+\omega(\lambda)\right). \end{aligned}$$

Let $\frac{u_{\bar{i}-1}}{\gamma} > \lambda_1 > \lambda_2 > \frac{u_{\bar{i}}}{\gamma}$, and let $\bar{i}_1$ ($\bar{i}_2$) the threshold associated with disagreement λ_1 (λ_2). We have that $\bar{i}_1 = \bar{i_2} = \bar{i}$. Thus,

$$\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) = -\frac{1}{\gamma}\frac{\theta\left(\omega(\lambda_1) - \omega(\lambda_2)\right)}{\sum_{i=1}^N \beta_i^2 - N}\left(\sigma_z^2\left(\sum_{i\geq\bar{i}}^N \beta_i^2 - \sum_{i\geq\bar{i}}^N \beta_i\right) + \frac{\sigma_\epsilon^2}{N}\sum_{i\geq\bar{i}}^N(\beta_i - 1)^2\right).$$

We show that $\sum_{i\geq\bar{i}}\beta_i^2 \geq \sum_{i\geq\bar{i}}\beta_i$. Since average β is one, we can write β_i as $\beta_i = 1 + y_i$ with y_i such that $\sum y_i = 0$. Using this decomposition, we have that

$$\sum_{i=1}^N \beta_i^2 = N + 2\underbrace{\sum_{i=1}^N y_i}_{=0} + \sum_{i=1}^N y^2 > N = \sum_{i=1}^N \beta_i.$$

Thus, the relationship is true for $\bar{i} = 1$. Now assume it is true for $\bar{i} = k > 1$. We have $\sum_{i\geq k+1}\beta_i^2 - \sum_{i\geq k+1}\beta_i = \sum_{i\geq k}\beta_i^2 - \sum_{i\geq k}\beta_i + \beta_k - \beta_k^2$. Either $\beta_k > 1$, in which case it is evident that $\sum_{i\geq k+1}\beta_i^2 - \sum_{i\geq k+1}\beta_i > 0$ as $\beta_k > 1$ implies that $\beta_i > 1$ for $i \geq k$, or $\beta_k \leq 1$, in which case $\beta_k - \beta_k^2 > 0$ and using the recurrence assumption, $\sum_{i\geq k+1}\beta_i^2 - \sum_{i\geq k+1}\beta_i > 0$. This proves that $\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) < 0$.

We show now that for all $i \in [1,N]$, $\hat{\mu}(\lambda)$ is continuous in u_i where u_i is the sequence defined in Theorem 1 and calculated as $b_i\sigma_z^2\omega(u_i) = \frac{\sigma_\epsilon^2}{N}$. When $\lambda = u_i^+$, we have $\bar{i} = i$. When $\lambda = u_i^-$, we have $\bar{i} = i+1$. Notice that $\omega(\lambda)$ is continuous in u_i and

$$\omega(u_i^-) = \omega(u_i^+) = \omega(u_i) = \frac{\sigma_\epsilon^2}{\sigma_z^2}\frac{1}{N\gamma}\frac{1}{b_i}.$$

Thus,

$$\begin{aligned} \hat{\mu}\left(u_i^+\right) - \hat{\mu}\left(u_i^-\right) &= -\frac{\theta}{\gamma}\frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N}\left(-\beta_k\omega(u_k)\left(\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) + \frac{\sigma_\epsilon^2}{N}\left(1+\omega(u_k)\right)\right) \\ &= -\frac{\theta}{\gamma}\frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N}\left(-b_i\sigma_z^2\omega(u_i) + \frac{\sigma_\epsilon^2}{N}\right) = 0 \quad \text{by definition of } u_i. \end{aligned}$$

Therefore, $\hat{\mu}$ is continuous and strictly decreasing for λ in $]u_{i+1}, u_i[$ and is continuous at $\lambda = u_i$, so that it is overall strictly decreasing in aggregate disagreement λ. Since the derivative of the slope of the Security Market Line w.r.t. λ is linear in θ, it is trivial to show that $\frac{\partial^2\hat{\mu}}{\partial\lambda\partial\theta} < 0$, that is, the negative effect of λ on the slope of the Security Market Line is stronger when there is a larger fraction of short-sales-constrained agents, that is, when θ is larger.

We can show that the slope of the Security Market Line, $\hat{\mu}$, is strictly decreasing with θ, the fraction of short-sales-constrained investors in the model. Since the marginal asset $\bar{i}$ is independent of θ and since we have already shown that $\sum_{i\geq\bar{i}}\beta_i^2 - \sum_{i\geq\bar{i}}\beta_i$, we have that

$$\frac{\partial\hat{\mu}}{\partial\theta} = -\frac{\omega(\lambda)}{\gamma\left(\sum_{i=1}^N \beta_i^2 - N\right)}\left(\frac{\sigma_\epsilon^2}{N}\sum_{i\geq\bar{i}}(\beta_i - 1)^2 + \sigma_z^2\left(\sum_{i\geq\bar{i}}\beta_i^2 - \sum_{i\geq\bar{i}}\beta_i\right)\right) < 0.$$

$\blacksquare$

F. Proof of Theorem 2

Proof. We first consider the case in which $\tilde{\lambda}_t = 0$. There is no disagreement among investors, so all investors are long all assets $i \in [1, N]$. There is thus a unique first-order-condition for all investor types – for all $j \in [1, N]$ and $k = a$, A or B:

$$d - (1+r)P_t^j(0) + \mathbb{E}_t[P_{t+1}^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = 0] = \frac{1}{\gamma}\left(b_j\sigma_z^2\sum_{i\leq N}\mu_i^k(0)b_i + \mu_j^k(0)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^k(0)\left(\Delta P_{t+1}^i\right)\right)\right).$$

Summing this equation across investor types, using the market-clearing condition, and dropping the time subscript leads to[2]

$$d - (1+r)P^j(0) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1})|\tilde{\lambda}_t = 0] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\left(\sum_{i\leq N}\frac{(\Delta P^i)}{N}\right)\right). \tag{6}$$

Consider now the case in which $\lambda_t = \lambda$. Importantly, investors disagree on the expected value of the aggregate factor $\tilde{z}_{t+1}$, but they agree on the expected value of asset i's resale price $\mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})]$. This is because investors agree to disagree, so they recognize the existence of the next generation of investors with heterogeneous beliefs – and in particular, with beliefs different from theirs. However, they nevertheless evaluate the $t+1$ expected dividend stream differently. We proceed as in the static model. We assume there is a marginal asset $\bar{i}$ such that there are no binding short-sales constraints for assets $j < \bar{i}$ and strictly binding short-sales constraints for assets $j \geq \bar{i}$. We check ex post the conditions under which this is indeed an equilibrium. Under the proposed equilibrium structure, the first-order condition of the three groups of investors born at date t for assets $j < \bar{i}$ is easily written since, in the proposed equilibrium structure, these assets do not experience binding short-sales constraints:

$$\begin{gathered} d + b_j\lambda_t^k - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda] = \\ \tfrac{1}{\gamma}\left(b_j\sigma_z^2\left(\textstyle\sum_{i\leq N}\mu_i^k(\lambda)b_i\right) + \mu_j^k(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P^j\left(\textstyle\sum_{i\leq N}\mu_i^k(\lambda)\left(\Delta P^i\right)\right)\right). \end{gathered}$$

Summing across investor types (using the weight of each investor group) and using the market-clearing condition leads to

$$\forall j < \bar{i}, \quad d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1})|\tilde{\lambda}_t = \lambda] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\sum_{i\leq N}\frac{\Delta P^i}{N}\right). \tag{7}$$

Subtracting equation (6) – prices in the low-disagreement state – from equation (7) leads to

$$\forall j < \bar{i}, \quad -(1+r)\Delta P^j + \rho\Delta P^j - (1-\rho)\Delta P^j = 0 \Leftrightarrow P^j(\lambda) = P^j(0),$$

since $\rho < 1$.

Thus, for all $j < \bar{i}$, $\Delta P^j = 0$. The payoff of assets below $\bar{j}$ is not sufficiently exposed to aggregate disagreement to make pessimist investors willing to go short. Hence, even in the high-disagreement state, these assets experience no mispricing. In particular, their price is independent of the realization of aggregate disagreement. Aggregate disagreement thus creates resale price risk only for assets that experience binding short-sales constraints in high-aggregate disagreement states, that is, high $\frac{b}{\sigma^2}$ assets with $i \geq \bar{i}$.

We now turn to the assets with binding short-sales constraints in high-disagreement states, that is, assets $j > \bar{i}$. For these assets, we know that under the proposed equilibrium, $\mu_j^B(\lambda) = 0$. We thus have the following first-order conditions for hedge funds and optimist mutual funds respectively:

$$\begin{cases} d + b_j\lambda - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^A(\lambda)b_i + \mu_j^A(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^A(\lambda)\Delta P_{t+1}^i\right)\right) \\ d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^a(\lambda)b_i + \mu_j^a(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^a(\lambda)\Delta P_{t+1}^i\right)\right) \end{cases}.$$

Define $\Gamma = \sum_{i\geq\bar{i}}\frac{\Delta P^i}{N}$, the average price difference between high- and low-aggregate disagreement states across all assets. Summing these equations across investor types (using the weight of each investor group) and using the market-clearing condition leads to:

$$\begin{gathered} \tfrac{\alpha}{2}b_j\lambda + (1-\tfrac{\alpha}{2})\left(d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda]\right) = \\ \tfrac{1}{\gamma}\left(b_j\sigma_z^2 + \tfrac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\Gamma - \tfrac{\alpha}{2}b_j\sigma_z^2\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)b_i}_{S^1} - \tfrac{\alpha}{2}\rho(1-\rho)\Delta P^j\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)\left(\Delta P^i\right)}_{S^2=0}\right). \end{gathered}$$

In the previous equation, $S^2 = 0$ since for all $i < \bar{i}$, $\Delta P^i = 0$. To recover S^1, we use B-investors' first-order condition on assets $j < \bar{i}$, the equilibrium prices derived above for assets $j < \bar{i}$, and the fact that for all $i < \bar{j}$, $\Delta P^j = 0$. This leads to the following equation:

$$\forall j < \bar{i}, \quad b_j \sigma_z^2 \underbrace{\sum_{i \leq N} \mu_i^B b_i}_{S^1} + \mu_j^B \sigma_j^2 = -\lambda\gamma b_j + b_j \sigma_z^2 + \frac{\sigma_j^2}{N}.$$

Multiplying the previous expression by b_j, dividing by σ_j^2, and summing the equations over j gives the following formula for S^1:

$$S^1 = 1 - \frac{\left(\sum_{i \geq \bar{i}} \frac{b_i}{N}\right) + \lambda\gamma \left(\sum_{i < \bar{i}} \frac{b_i^2}{\sigma_i^2}\right)}{1 + \sigma_z^2 \left(\sum_{i < \bar{i}} \frac{b_i^2}{\sigma_i^2}\right)}.$$

This allows us to derive the excess return on assets $j \geq \bar{i}$:

$$\underbrace{d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1}) | \tilde{\lambda}_t = \lambda]}_{\text{Excess Return}} =$$

$$\underbrace{\frac{1}{\gamma}\left(b_j \sigma_z^2 + \frac{\sigma_j^2}{N} + (1+\theta)\rho(1-\rho)\Delta P^j \Gamma\right)}_{\text{Risk Premium}} - \underbrace{\frac{\theta}{\gamma}\left(b_j \frac{\lambda\gamma - \frac{\sigma_z^2}{N}\sum_{k \geq \bar{i}} b_k}{1 + \sigma_z^2 \left(\sum_{i < \bar{i}} \frac{b_i^2}{\sigma_i^2}\right)} - \frac{\sigma_j^2}{N}\right)}_{\text{Speculative Premium}=\pi^j}.$$

Note that the risk premium embeds a term that reflects the resale price risk of high-b assets. Subtracting equation (6) from the previous equation yields, for all $j \geq \bar{i}$,

$$-(1+r)\Delta P^j + (2\rho - 1)\Delta P^j = -\pi^j + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma \Delta P^j \Rightarrow \left((1+r) - (2\rho - 1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Delta P^j = \pi^j. \tag{8}$$

Remember that $\Gamma = \sum_{i \geq \bar{i}} \frac{\Delta P^i}{N}$. We can thus obtain a formula for Γ by adding up the previous equations for all $j \geq \bar{i}$ and dividing by N:

$$\left((1+r) - (2\rho - 1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Gamma = \frac{1}{N}\sum_{j \geq \bar{i}} \pi^j.$$

There is a unique $\Gamma^+ > 0$ which satisfies the previous equation. Call it Γ^+:

$$\Gamma^+ = \frac{-(1+r) + (2\rho - 1) + \sqrt{((1+r) - (2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j \geq \bar{i}} \pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}.$$

There is also a unique $\Gamma^- < 0$ that satisfies equation (8):

$$\Gamma^- = \frac{-(1+r) + (2\rho - 1) - \sqrt{((1+r) - (2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j \geq \bar{i}} \pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}.$$

Let $\Gamma^\star$ be the actual value of Γ, the average price difference between high- and low-aggregate disagreement states across all assets. $\Gamma^\star \in \{\Gamma^-, \Gamma^+\}$. For $j \geq \bar{i}$, the price difference is simply expressed as a function of the speculative premium π^j and $\Gamma^\star$:

$$\Delta P^j = \frac{\pi^j}{1 + r - (2\rho - 1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}.$$

For the equilibrium to exist, it needs to be the case that for each asset $j \geq \bar{i}$, pessimists do not want to hold asset j, that is, the marginal utility of holding assets $j \geq \bar{i}$ at the optimal holding is zero. This is equivalent to

$$\forall j \geq \bar{i}, \quad d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma} b_j \sigma_z^2 \underbrace{\sum_{j < \bar{i}} \mu_i^B b_i}_{=S^1} < 0.$$

We have that

$$\begin{aligned}
& d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -b_j\lambda + \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)(1+\theta)\Delta P^j\Gamma^\star\right) - \pi^j - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -\frac{\pi^j}{\theta} - \pi^j + (1+\theta)\frac{\rho(1-\rho)}{\gamma}\Gamma^\star\Delta P^j \\
= \quad & \frac{1+\theta}{\theta}\pi^j\left(\frac{\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star} - 1\right) \\
= \quad & -\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star} \times \pi^j.
\end{aligned}$$

Assume $\Gamma^\star = \Gamma^- < 0$. We know that

$$\theta\rho(1-\rho)\frac{\Gamma^-}{\gamma} + (1+r) - (2\rho-1) = \frac{(1+r)-(2\rho-1) - \sqrt{((1+r)-(2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}} < 0.$$

Thus, if $\Gamma^\star = \Gamma^-$, then $-\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma} > 0$, so that it has to be the case that for all $j \geq \bar{i}$, $\pi^j < 0$. Thus $\sum_{j\geq\bar{i}}\pi^j < 0$, so that

$$\left((1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^-\right)\Gamma^- < 0.$$

However, the previous expression is strictly positive since $\Gamma^- < 0$ and $(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^- < 0$. Thus, we can't have $\Gamma^\star = \Gamma^-$ and it has to be the case that $\Gamma^\star = \Gamma^+$.

Since $\Gamma^\star > 0$, we have from the previous equilibrium condition that, for all $j \geq \bar{i}$, $\pi^j > 0$. Similarly, it is straightforward to show that for pessimists to have strictly positive holdings of assets $\bar{j}-1$, a necessary and sufficient condition is that $\pi^{\bar{j}-1} < 0$. Overall, this leads to the following equilibrium condition:

$$\frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}}b_k\right) + \frac{1}{N}\frac{\sigma_{\bar{j}-1}^2}{b_{\bar{j}-1}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right) \geq \lambda\gamma \geq \frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}}b_k\right) + \frac{1}{N}\frac{\sigma_{\bar{j}}^2}{b_{\bar{j}}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right).$$

We can define a sequence v_i, analogous to the sequence u_i defined in Theorem 1, as

$$\forall i \in [1,N], \quad v_i = \frac{\sigma_z^2}{N}\left(\sum_{k\geq i}b_k\right) + \frac{1}{N}\frac{\sigma_i^2}{b_i}\left(1+\sigma_z^2\sum_{k<i}\frac{b_k^2}{\sigma_k^2}\right), \quad v_{N+1} = 0 \text{ and } v_0 = +\infty.$$

It can be easily shown that this sequence is strictly decreasing since, for all $i \in [2,N]$,

$$v_i - v_{i-1} = \frac{1}{N}\left(\frac{\sigma_i^2}{b_i} - \frac{\sigma_{i-1}^2}{b_{i-1}}\right)\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right),$$

and assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$.

The equilibrium condition can thus be written as $v_{\bar{i}-1} \geq \lambda\gamma \geq v_{\bar{i}}$, with $\bar{i}$ defined as the smallest $i \in [1,N]$ such that $\lambda\gamma \geq v_i$.

We now move on to the expression for expected excess returns. Since $\Delta P^j = 0$ for $j < \bar{i}$, we have that for all $j < \bar{i}$,

$$\mathbb{E}[R^j(\lambda)] = \mathbb{E}[R^j(0)] = d - rP^j(\lambda) = d - rP^j(0) = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N}\right).$$

For $j \geq \bar{i}$, however,

$$\begin{aligned}
\mathbb{E}[R^j(0)] \quad &= \quad d - (1+r)P^j(0) + \rho P^j(0) + (1-\rho)P^j(\lambda) \\
&= \quad \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right).
\end{aligned}$$

The extra term is the risk premium required by investors for holding stocks that are sensitive to disagreement and thus are exposed to changes in prices coming from changes in the aggregate disagreement state variable. Of course, in the data, since ρ is very close to one, this risk premium is going to be quantitatively small. Nevertheless, the intuition here is that high $\frac{b}{\sigma^2}$ stocks have low prices in low-disagreement states for two reasons: they are exposed to aggregate risk $\tilde{z}$, and they are exposed to changes in aggregate disagreement

$\tilde{\lambda}$. Finally,

$$\begin{aligned}
\mathbb{E}[R^j(\lambda)] &= d-(1+r)P^j(\lambda)+\rho P^j(\lambda)+(1-\rho)P^j(0)\\
&= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\pi^j\\
&\quad +\theta\frac{\rho(1-\rho)}{\gamma}\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\\
&= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\frac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j.
\end{aligned}$$

Thus, for assets $j\geq\bar{i}$, the expected return is strictly lower in high-disagreement states than in low-disagreement states.

The proof for the unicity of the equilibrium is similar to the proof of Theorem 1 and is thus omitted. ■

G. Proof of Corrolary 4

Proof. Part (i) is a direct consequence of the formula for expected excess returns in Theorem 2. For (ii), we do a Taylor expansion around $\rho=1$ for $\Gamma^\star$: $\Gamma^\star\approx\frac{1}{r}\sum_{j\geq\bar{i}}\frac{\pi^j}{N}>0$, so in the vicinity of $\rho=1$ and for $j\geq\bar{i}$,

$$\mathbb{E}[R^j(\lambda)]\approx\frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}\right)-\frac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j.$$

The slope of the Security Market Line for assets $i<\bar{i}$ (expressed as a function of b_i – it would be equivalent as a function of β_i) is thus strictly lower for $i<\bar{i}$ than for $i\geq\bar{i}$ in the vicinity of $\rho=1$, which proves (ii). (iii) can also be seen from the previous Taylor expansion and making λ grow to infinity. (iv) is also a direct consequence of the formula for expected excess returns in Theorem 2. ■

II. Additional Tables

Table IAI
Disagreement and Concavity of the Security Market Line: Monthly βs

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20-beta sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r^{(12)}_{P,t} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r^{(12)}_{P,t}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R^{(12)}_{m,t} + \delta_1^{HML} \cdot HML^{(12)}_t + \delta_1^{SMB} \cdot SMB^{(12)}_t + \delta_1^{UMD} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R^{(12)}_{m,t} + \delta_2^{HML} \cdot HML^{(12)}_t + \delta_2^{SMB} \cdot SMB^{(12)}_t + \delta_2^{UMD} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R^{(12)}_{m,t} + \delta_3^{HML} \cdot HML^{(12)}_t + \delta_3^{SMB} \cdot SMB^{(12)}_t + \delta_3^{UMD} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

Table IAI (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-3.69	-7.13**	-11.98***	-11.94***	3.16	13.80**	17.80**	17.44**	-2.80	-5.49*	-4.37	-4.11
	(-1.32)	(-2.46)	(-3.58)	(-3.58)	(0.66)	(2.29)	(2.42)	(2.40)	(-1.06)	(-1.75)	(-1.15)	(-1.09)
$R_{m,t}^{(12)}$		-0.32**	-0.34**	-0.34**		1.24***	1.21***	1.25***		0.12	0.17	0.15
		(-2.31)	(-2.41)	(-2.41)		(4.59)	(4.17)	(4.34)		(0.80)	(1.15)	(1.00)
$HML_t^{(12)}$		-0.34*	-0.23	-0.23		-0.18	-0.28	-0.23		0.68***	0.66***	0.63***
		(-1.79)	(-1.40)	(-1.42)		(-0.50)	(-0.77)	(-0.61)		(3.19)	(2.98)	(2.66)
$SMB_t^{(12)}$		0.98***	1.18***	1.19***		-1.45**	-1.59***	-1.63***		0.43	0.36	0.38
		(3.02)	(4.60)	(4.57)		(-2.38)	(-2.73)	(-2.84)		(1.28)	(1.04)	(1.16)
$UMD_t^{(12)}$		0.20	0.23*	0.23		-0.64**	-0.65**	-0.62**		0.40**	0.38**	0.36**
		(1.25)	(1.70)	(1.62)		(-2.21)	(-2.28)	(-2.17)		(2.54)	(2.41)	(2.39)
D/P_{t-1}			-4.50	-4.18			5.38	2.37			-0.80	1.31
			(-1.24)	(-1.06)			(0.71)	(0.30)			(-0.21)	(0.30)
Inflation$_{t-1}$			-4.49*	-4.30*			2.06	0.29			2.84	4.08
			(-1.93)	(-1.80)			(0.43)	(0.06)			(1.16)	(1.45)
Ted Spread$_{t-1}$				-0.62				5.79				-4.07
				(-0.24)				(1.05)				(-1.25)
Constant	-4.90	-3.37	-4.16*	-4.09	10.73*	6.92	7.83	7.18	3.94	-3.27	-3.36	-2.91
	(-1.58)	(-1.48)	(-1.70)	(-1.62)	(1.74)	(1.39)	(1.53)	(1.31)	(1.21)	(-1.13)	(-1.22)	(-0.98)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.75**	-5.99***	-9.50***	-9.35***	8.09**	10.14**	12.09**	11.52**	-2.36	-2.49	-0.13	0.25
	(-2.28)	(-2.76)	(-3.54)	(-3.51)	(2.15)	(2.51)	(2.33)	(2.30)	(-1.50)	(-1.28)	(-0.05)	(0.11)
$R_{m,t}^{(12)}$		-0.27**	-0.38***	-0.39***		1.12***	1.31***	1.36***		0.17	0.12	0.08
		(-2.04)	(-3.03)	(-3.10)		(5.14)	(5.46)	(5.68)		(1.58)	(1.04)	(0.75)
$HML_t^{(12)}$		-0.78***	-0.71***	-0.73***		0.92***	0.90***	0.97***		0.24	0.17	0.12
		(-4.34)	(-5.32)	(-5.62)		(2.75)	(3.23)	(3.73)		(1.64)	(1.29)	(1.00)
$SMB_t^{(12)}$		0.47	0.67**	0.69**		-0.07	-0.26	-0.32		0.15	0.09	0.12
		(1.52)	(2.53)	(2.58)		(-0.14)	(-0.57)	(-0.69)		(0.97)	(0.48)	(0.75)
$UMD_t^{(12)}$		0.01	0.06	0.05		-0.27	-0.35	-0.30		0.26**	0.27**	0.24***
		(0.05)	(0.58)	(0.45)		(-1.19)	(-1.62)	(-1.44)		(2.50)	(2.41)	(2.64)
D/P_{t-1}			0.44	1.68			-5.33	-10.05*			4.78*	7.94***
			(0.14)	(0.54)			(-0.90)	(-1.75)			(1.95)	(3.51)
Inflation$_{t-1}$			-6.87***	-6.14**			8.81*	6.03			-0.35	1.51
			(-2.61)	(-2.30)			(1.83)	(1.25)			(-0.18)	(0.73)
Ted Spread$_{t-1}$				-2.39				9.09**				-6.10***
				(-1.16)				(2.35)				(-3.44)
Constant	-9.71***	-4.85**	-4.87**	-4.60*	20.52***	9.01**	8.25*	7.23	-0.18	-4.66**	-3.88*	-3.20
	(-3.66)	(-2.36)	(-2.12)	(-1.92)	(4.35)	(2.04)	(1.73)	(1.45)	(-0.08)	(-2.08)	(-1.72)	(-1.40)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAII
Disagreement and Concavity of the Security Market Line: Different Horizons

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression:

$$r_{P,t}^{(k)} = \kappa_t^{(k)} + \pi_t^{(k)} \times \beta_P + \phi_t^{(k)} \times (\beta_P)^2 + \epsilon_{P,t}^{(k)}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t^{(k)} = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t^{(k)} = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t^{(k)} = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_3^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Panel A uses k=1 months, Panel B uses k=3 months, Panel C uses k=6 months, and Panel D uses k=18 months. Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long run EPS growth. Columns (2) and (6) add the factor $z \in Z$, where Z contains the k-month excess market return from t to $t+k-1$ and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

Dep. Var:	$\phi_t^{(k)}$				$\pi_t^{(k)}$				$\kappa_t^{(k)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 months												
Agg. Disp.$_{t-1}$	-0.21	-0.31	-0.57	-0.66*	-0.01	0.53	0.89	0.96	0.04	-0.08	-0.16	-0.15
	(-0.38)	(-0.93)	(-1.58)	(-1.81)	(-0.01)	(0.75)	(1.15)	(1.22)	(0.11)	(-0.21)	(-0.42)	(-0.38)
Panel A: k=3 months												
Agg. Disp.$_{t-1}$	-0.98	-0.99	-1.89**	-2.09***	0.48	1.60	2.80*	2.99*	-0.02	-0.08	-0.30	-0.32
	(-0.81)	(-1.34)	(-2.34)	(-2.60)	(0.24)	(1.06)	(1.69)	(1.77)	(-0.02)	(-0.10)	(-0.36)	(-0.37)
Panel A: k=6 months												
Agg. Disp.$_{t-1}$	-2.81	-2.47*	-4.40***	-4.68***	2.47	4.47	6.93**	6.94**	-1.09	-0.92	-1.21	-1.04
	(-1.31)	(-1.68)	(-2.87)	(-3.08)	(0.76)	(1.60)	(2.31)	(2.30)	(-0.85)	(-0.74)	(-0.87)	(-0.74)
Panel A: k=18 months												
Agg. Disp.$_{t-1}$	-6.39**	-7.41**	-11.77***	-11.70***	7.65*	12.17**	17.55***	16.57***	-3.97*	-3.24	-3.99	-3.51
	(-2.02)	(-2.27)	(-3.49)	(-3.45)	(1.71)	(2.05)	(2.70)	(2.65)	(-1.88)	(-1.32)	(-1.47)	(-1.36)

Table IAIII
Disagreement and Concavity of the Security Market Line: Alternative Measures of Disagreement

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \qquad P = 1, ..., 20$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

In Panel A, Agg. Disp. is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth, where the pre-ranking β has been compressed to one ($\beta^i = 0.5\hat{\beta} + 0.5$). In Panel B, Agg. Disp. is the monthly β- and value-weighted average of stock level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. In Panel C, Agg. Disp. is the principal component of the monthly standard deviation of forecasts on GDP, IP, corporate profit and the unemployment rate in the SPF and is taken from Li and Li (2014). In Panel D, Agg. Disp is the "top-down" measure of market disagreement used in Yu (2011) and is measured as the standard deviation of analysts' forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. Columns (1) and (5) control only for Agg. Disp.$_{t-1}$. Columns (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance.

Table IAIII (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Compressed betas												
Agg. Disp.$_{t-1}$	-5.50*	-4.53	-8.71**	-8.64**	6.34	8.70*	13.52**	12.83**	-4.02*	-2.80	-2.91	-2.48
	(-1.71)	(-1.59)	(-2.49)	(-2.50)	(1.38)	(1.67)	(2.07)	(2.03)	(-1.86)	(-1.33)	(-1.12)	(-1.00)
Panel B: Value and beta-weighted												
Agg. Disp.$_{t-1}$	-5.31	-4.50	-6.90**	-6.84**	5.49	7.81	10.09*	9.31*	-3.43	-1.88	-1.51	-1.00
	(-1.58)	(-1.64)	(-2.37)	(-2.39)	(1.07)	(1.49)	(1.79)	(1.73)	(-1.50)	(-0.85)	(-0.63)	(-0.45)
Panel C: SPF disagreement												
Agg. Disp.$_{t-1}$	-2.68	-2.41	-5.13*	-4.88*	6.38	2.51	10.40*	9.41	0.90	-0.10	-4.35	-3.85
	(-1.06)	(-1.09)	(-1.90)	(-1.70)	(1.33)	(0.57)	(1.91)	(1.56)	(0.49)	(-0.05)	(-1.45)	(-1.21)
Panel D: Top-down measure												
Agg. Disp.$_{t-1}$	0.17	0.10	-3.55**	-3.72**	2.41	0.27	5.69*	6.23*	-0.11	-0.10	-1.45	-1.71
	(0.15)	(0.08)	(-2.48)	(-2.52)	(1.17)	(0.12)	(1.72)	(1.87)	(-0.11)	(-0.11)	(-0.77)	(-0.92)

Table IAIV
Disagreement and Concavity of the Security Market Line: Controlling for Stock-Level Disagreement

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < $5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\text{Disp}_{P,t}) + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio, $\text{Disp}_{P,t}$ is the value-weighted average of the stock-level dispersion in analysts' forecasts for stocks in portfolio P in month t, and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parenthesis. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAIV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.90*	-5.49*	-9.64***	-9.67***	6.45	11.28**	17.02***	16.62***	-3.17	-2.26	-1.45	-0.51
	(-1.92)	(-1.92)	(-2.94)	(-2.98)	(1.50)	(2.16)	(2.69)	(2.68)	(-1.46)	(-0.85)	(-0.45)	(-0.17)
$R_{m,t}^{(12)}$		-0.16	-0.22*	-0.22		0.99***	1.07***	1.07***		0.29*	0.26*	0.25
		(-1.25)	(-1.65)	(-1.63)		(3.66)	(3.77)	(3.74)		(1.86)	(1.67)	(1.49)
$HML_t^{(12)}$		-0.50**	-0.38**	-0.37**		0.47	0.30	0.32		0.31*	0.28	0.22
		(-2.56)	(-2.17)	(-2.18)		(1.25)	(0.81)	(0.87)		(1.65)	(1.51)	(1.30)
$SMB_t^{(12)}$		0.34	0.52**	0.52**		-0.33	-0.58	-0.58		-0.19	-0.20	-0.20
		(1.39)	(2.33)	(2.32)		(-0.65)	(-1.10)	(-1.11)		(-0.73)	(-0.72)	(-0.75)
$UMD_t^{(12)}$		0.02	0.07	0.07		0.02	-0.06	-0.05		0.06	0.07	0.04
		(0.16)	(0.67)	(0.68)		(0.07)	(-0.24)	(-0.20)		(0.55)	(0.57)	(0.38)
D/P_{t-1}			-2.73	-2.79			3.49	2.43			1.76	4.27
			(-1.22)	(-1.07)			(0.71)	(0.46)			(0.60)	(1.35)
Inflation$_{t-1}$			-4.83***	-4.87***			7.00*	6.40			-0.51	0.90
			(-2.96)	(-3.13)			(1.74)	(1.50)			(-0.25)	(0.40)
Ted Spread$_{t-1}$				0.11				1.84				-4.34*
				(0.06)				(0.52)				(-1.88)
Constant	-5.69**	-2.83	-3.42	-3.43	12.95***	2.69	3.47	3.25	3.26	-0.69	-0.40	0.12
	(-2.42)	(-1.16)	(-1.46)	(-1.43)	(2.77)	(0.50)	(0.65)	(0.59)	(1.34)	(-0.24)	(-0.13)	(0.04)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-6.15**	-4.53**	-6.39***	-6.26***	8.91**	9.90**	10.72**	10.06**	-1.77	-1.83	0.86	2.20
	(-2.33)	(-2.35)	(-3.09)	(-3.12)	(2.10)	(2.41)	(2.50)	(2.46)	(-0.93)	(-0.70)	(0.31)	(0.89)
$R_{m,t}^{(12)}$		-0.22**	-0.30***	-0.30***		1.07***	1.23***	1.24***		0.14	0.14	0.12
		(-2.51)	(-3.50)	(-3.39)		(6.48)	(6.73)	(6.40)		(1.23)	(1.30)	(0.99)
$HML_t^{(12)}$		-0.61***	-0.56***	-0.57***		0.77***	0.76***	0.80***		0.41**	0.32*	0.24
		(-4.15)	(-4.80)	(-5.11)		(2.60)	(2.97)	(3.08)		(2.17)	(1.79)	(1.45)
$SMB_t^{(12)}$		-0.03	0.09	0.09		0.62**	0.48	0.48		-0.32*	-0.41**	-0.41**
		(-0.20)	(0.64)	(0.64)		(2.34)	(1.64)	(1.60)		(-1.80)	(-2.10)	(-2.07)
$UMD_t^{(12)}$		-0.09	-0.04	-0.05		0.07	-0.00	0.02		0.03	0.01	-0.02
		(-1.32)	(-0.82)	(-0.89)		(0.51)	(-0.01)	(0.11)		(0.24)	(0.11)	(-0.22)
D/P_{t-1}			0.89	1.24			-5.21	-6.96			3.17	6.75***
			(0.47)	(0.52)			(-1.34)	(-1.48)			(1.32)	(2.63)
Inflation$_{t-1}$			-4.64***	-4.45***			7.70**	6.71*			1.48	3.48
			(-3.38)	(-3.04)			(2.32)	(1.79)			(0.69)	(1.52)
Ted Spread$_{t-1}$				-0.61				3.03				-6.19***
				(-0.36)				(0.80)				(-2.68)
Constant	-9.11***	-4.08**	-4.05**	-3.98**	20.11***	6.35	5.66	5.30	0.23	-2.45	-1.87	-1.14
	(-4.77)	(-2.19)	(-2.20)	(-2.09)	(5.04)	(1.46)	(1.27)	(1.14)	(0.10)	(-0.89)	(-0.66)	(-0.39)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAV

Disagreement and Concavity of the Security Market Line: Controlling for Idiosyncratic Volatility

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\sigma_{P,t}) + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio, $\sigma_{P,t}$ is the value-weighted median of the idiosyncratic volatility of stocks in portfolio P in month t and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns(1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-7.02*	-5.26	-8.74**	-8.55**	7.78	9.44	13.51*	12.60*	-6.00*	-3.38	-3.40	-2.58
	(-1.85)	(-1.62)	(-2.36)	(-2.35)	(1.38)	(1.51)	(1.86)	(1.78)	(-1.76)	(-1.05)	(-0.92)	(-0.73)
$R^{(12)}_{m,t}$		-0.11	-0.18	-0.18		0.75***	0.91***	0.93***		0.36**	0.29*	0.28*
		(-0.83)	(-1.30)	(-1.31)		(2.77)	(3.02)	(3.01)		(2.35)	(1.82)	(1.67)
$HML^{(12)}_t$		-0.63***	-0.53***	-0.54***		0.63	0.52	0.57		0.08	0.07	0.02
		(-3.00)	(-2.78)	(-2.91)		(1.48)	(1.27)	(1.43)		(0.37)	(0.33)	(0.11)
$SMB^{(12)}_t$		0.20	0.37	0.37		0.01	-0.24	-0.24		-0.25	-0.20	-0.20
		(0.85)	(1.62)	(1.64)		(0.02)	(-0.44)	(-0.46)		(-0.87)	(-0.68)	(-0.69)
$UMD^{(12)}_t$		-0.02	0.04	0.03		0.14	0.04	0.07		0.01	0.04	0.02
		(-0.13)	(0.35)	(0.31)		(0.55)	(0.17)	(0.27)		(0.11)	(0.33)	(0.17)
D/P$_{t-1}$			-1.31	-0.81			-1.44	-3.87			2.54	4.72
			(-0.55)	(-0.29)			(-0.29)	(-0.75)			(0.91)	(1.58)
Inflation$_{t-1}$			-5.19***	-4.91***			9.56**	8.20*			-3.02	-1.80
			(-2.98)	(-3.01)			(2.20)	(1.81)			(-1.18)	(-0.64)
Ted Spread$_{t-1}$				-0.87				4.20				-3.77
				(-0.45)				(1.09)				(-1.60)
Constant	-6.61***	-3.30	-3.66	-3.56	13.10***	3.01	3.01	2.51	2.29	-0.98	-0.63	-0.18
	(-2.63)	(-1.39)	(-1.54)	(-1.44)	(2.75)	(0.60)	(0.59)	(0.46)	(0.90)	(-0.34)	(-0.21)	(-0.06)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.68**	-3.82*	-5.68***	-5.50***	6.03	5.38	8.22*	7.05	-1.32	-0.99	0.89	1.92
	(-2.09)	(-1.96)	(-2.65)	(-2.64)	(1.15)	(1.15)	(1.67)	(1.51)	(-0.72)	(-0.46)	(0.37)	(0.90)
$R^{(12)}_{m,t}$		-0.22***	-0.27***	-0.28***		0.77***	0.86***	0.88***		0.20**	0.23**	0.21**
		(-2.63)	(-3.05)	(-2.98)		(4.48)	(4.46)	(4.24)		(2.04)	(2.57)	(2.13)
$HML^{(12)}_t$		-0.67***	-0.62***	-0.63***		0.90***	0.82***	0.88***		0.27*	0.21	0.15
		(-4.67)	(-5.24)	(-5.73)		(3.10)	(3.17)	(3.59)		(1.83)	(1.41)	(1.06)
$SMB^{(12)}_t$		-0.01	0.09	0.09		0.37	0.21	0.21		-0.11	-0.19	-0.18
		(-0.07)	(0.60)	(0.61)		(1.16)	(0.64)	(0.61)		(-0.62)	(-1.00)	(-0.98)
$UMD^{(12)}_t$		-0.08	-0.05	-0.05		0.11	0.05	0.08		0.03	0.01	-0.02
		(-1.12)	(-0.79)	(-0.88)		(0.52)	(0.25)	(0.45)		(0.24)	(0.06)	(-0.21)
D/P$_{t-1}$			-0.17	0.32			-0.23	-3.33			1.33	4.07*
			(-0.09)	(0.13)			(-0.06)	(-0.75)			(0.59)	(1.84)
Inflation$_{t-1}$			-3.39**	-3.12**			5.75*	4.02			2.07	3.61
			(-2.50)	(-2.23)			(1.72)	(1.06)			(0.93)	(1.51)
Ted Spread$_{t-1}$				-0.84				5.36				-4.74**
				(-0.50)				(1.33)				(-2.31)
Constant	-8.50***	-3.27*	-3.38*	-3.28*	16.31***	4.72	4.83	4.19	1.73	-1.18	-0.89	-0.33
	(-4.24)	(-1.77)	(-1.85)	(-1.76)	(4.33)	(1.16)	(1.19)	(1.01)	(0.85)	(-0.53)	(-0.39)	(-0.14)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAVI
Disagreement and the Slope of the Security Market Line

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20 beta-sorted portfolios using the same market model. We estimate every month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \pi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_2^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add the factor $z \in Z$, where Z contains the k-month excess market return from t to $t+k-1$ and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12 months inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAVI (Continued):

Dep. Var:	π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Value-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-6.06**	-0.86	-4.90**	-5.68***	1.80	1.94	6.07***	6.62***
	(-2.11)	(-0.39)	(-2.30)	(-2.66)	(0.70)	(0.89)	(2.79)	(2.98)
$R_{m,t}^{(12)}$		0.60***	0.58***	0.59***		0.40***	0.43***	0.42***
		(5.12)	(6.59)	(6.09)		(3.55)	(4.70)	(4.24)
$HML_t^{(12)}$		-0.61***	-0.48***	-0.44***		0.77***	0.64***	0.60***
		(-3.22)	(-3.03)	(-2.88)		(4.02)	(4.07)	(4.05)
$SMB_t^{(12)}$		0.48**	0.63***	0.62***		-0.44*	-0.60***	-0.59***
		(2.13)	(3.37)	(3.54)		(-1.91)	(-3.23)	(-3.34)
$UMD_t^{(12)}$		-0.00	0.03	0.05		-0.00	-0.04	-0.05
		(-0.03)	(0.31)	(0.58)		(-0.03)	(-0.45)	(-0.65)
D/P$_{t-1}$			-3.92*	-6.01**			3.93*	5.39*
			(-1.79)	(-2.29)			(1.66)	(1.80)
Inflation$_{t-1}$			-3.22*	-4.39**			3.38**	4.20**
			(-1.90)	(-2.12)			(2.16)	(2.31)
Ted Spread$_{t-1}$				3.62*				-2.54
				(1.77)				(-1.28)
Constant	0.87	-2.58	-3.34	-3.77*	8.54***	2.50	3.26*	3.56*
	(0.31)	(-1.16)	(-1.60)	(-1.75)	(3.24)	(1.21)	(1.67)	(1.75)
Panel B: Equal-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-5.11*	-0.46	-4.06**	-4.84***	3.85	2.08	6.27***	7.02***
	(-1.85)	(-0.26)	(-2.47)	(-2.99)	(1.60)	(1.05)	(3.30)	(3.60)
$R_{m,t}^{(12)}$		0.60***	0.58***	0.59***		0.40***	0.43***	0.42***
		(5.18)	(6.59)	(6.14)		(3.30)	(4.54)	(4.12)
$HML_t^{(12)}$		-0.66***	-0.54***	-0.50***		0.96***	0.83***	0.79***
		(-3.97)	(-3.72)	(-3.55)		(5.38)	(5.65)	(5.87)
$SMB_t^{(12)}$		0.70***	0.84***	0.84***		-0.11	-0.28*	-0.28*
		(3.93)	(5.37)	(5.44)		(-0.55)	(-1.70)	(-1.75)
$UMD_t^{(12)}$		-0.11	-0.08	-0.06		0.11	0.07	0.05
		(-0.93)	(-0.78)	(-0.67)		(0.95)	(0.71)	(0.57)
D/P$_{t-1}$			-3.25*	-5.32**			3.41	5.40**
			(-1.81)	(-2.56)			(1.60)	(2.03)
Inflation$_{t-1}$			-3.15*	-4.31**			4.11***	5.23***
			(-1.77)	(-2.16)			(2.62)	(3.13)
Ted Spread$_{t-1}$				3.58**				-3.45*
				(2.13)				(-1.93)
Constant	0.20	-2.61	-3.25*	-3.67*	9.50***	1.56	2.25	2.66
	(0.07)	(-1.37)	(-1.71)	(-1.91)	(3.47)	(0.83)	(1.20)	(1.41)

Table IAVII

Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative Stocks; Monthly βs

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full-sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \dots, 20$ and s $\in$ {speculative, nonspeculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln(\sigma_{P,s,t-1}) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P,s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the equal-weighted 12-month excess return of portfolio (P,s). We then estimate second-stage regressions in the time-series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{3,s} x_{t-1} + \nu_{t,s}. \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth, where the β are the pre-ranking β computed above. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table VI (Continued):

Dep. Var:	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-6.50*	-3.26	-9.63***	-9.87***	0.29	0.33	1.17	1.13	2.68	4.55	13.08**	13.18**
	(-1.92)	(-1.64)	(-4.83)	(-4.95)	(0.24)	(0.27)	(0.77)	(0.74)	(0.52)	(1.00)	(2.28)	(2.28)
$R_{m,t}^{(12)}$		0.49***	0.46***	0.48***		0.17**	0.11	0.11		1.01***	0.87***	0.86***
		(3.71)	(5.12)	(4.72)		(2.28)	(1.39)	(1.44)		(3.50)	(2.94)	(2.87)
$HML_t^{(12)}$		-0.81***	-0.66***	-0.62***		0.16	0.13	0.13		1.32**	1.09*	1.07*
		(-4.47)	(-4.08)	(-3.57)		(1.17)	(0.84)	(0.87)		(2.58)	(1.91)	(1.84)
$SMB_t^{(12)}$		0.39	0.66***	0.64***		0.12	0.13	0.12		-0.15	-0.40	-0.39
		(1.20)	(2.80)	(2.94)		(0.84)	(0.86)	(0.82)		(-0.22)	(-0.67)	(-0.66)
$UMD_t^{(12)}$		-0.38**	-0.33**	-0.32**		0.14*	0.16**	0.16**		0.73**	0.74**	0.73**
		(-2.11)	(-2.40)	(-2.52)		(1.93)	(2.23)	(2.16)		(2.17)	(2.40)	(2.32)
D/P$_{t-1}$			-5.80**	-7.77**			3.20*	2.93			15.02**	15.84*
			(-2.12)	(-2.36)			(1.85)	(1.50)			(2.07)	(1.88)
Inflation$_{t-1}$			-6.00***	-7.16***			-1.61	-1.76			0.93	1.41
			(-3.03)	(-3.19)			(-1.24)	(-1.31)			(0.18)	(0.27)
Ted Spread$_{t-1}$				3.80*				0.52				-1.58
				(1.65)				(0.37)				(-0.29)
Constant	-2.93	-1.79	-2.82	-3.25	2.33	-0.94	-0.44	-0.50	18.69***	-0.47	2.00	2.18
	(-0.91)	(-0.91)	(-1.30)	(-1.45)	(1.44)	(-0.54)	(-0.26)	(-0.29)	(2.82)	(-0.08)	(0.31)	(0.33)
Panel B: Non speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ ≤ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-1.66	0.57	-5.18	-5.44	-1.44	-2.04	-1.88	-1.93	-3.65	-4.51	1.08	1.17
	(-0.65)	(0.18)	(-1.54)	(-1.58)	(-0.66)	(-0.99)	(-0.82)	(-0.84)	(-0.50)	(-0.62)	(0.13)	(0.14)
$R_{m,t}^{(12)}$		0.27	0.21	0.23		0.28***	0.31**	0.31**		1.43***	1.56***	1.55***
		(1.32)	(1.17)	(1.23)		(2.70)	(2.56)	(2.50)		(3.53)	(3.61)	(3.46)
$HML_t^{(12)}$		-1.06***	-0.92***	-0.89***		0.02	0.02	0.03		1.31**	1.19**	1.18**
		(-3.18)	(-3.35)	(-3.31)		(0.17)	(0.18)	(0.22)		(2.35)	(2.32)	(2.24)
$SMB_t^{(12)}$		0.76	1.03**	1.00**		0.55**	0.53**	0.52**		0.99	0.69	0.70
		(1.63)	(2.39)	(2.41)		(2.43)	(2.25)	(2.15)		(0.99)	(0.69)	(0.70)
$UMD_t^{(12)}$		-0.05	0.00	0.02		0.02	0.01	0.01		0.17	0.10	0.09
		(-0.23)	(0.00)	(0.12)		(0.14)	(0.06)	(0.10)		(0.39)	(0.23)	(0.22)
D/P$_{t-1}$			-3.76	-5.95			-0.74	-1.18			1.03	1.77
			(-1.13)	(-1.44)			(-0.27)	(-0.37)			(0.11)	(0.16)
Inflation$_{t-1}$			-6.86**	-8.15**			1.01	0.75			9.24	9.67
			(-2.03)	(-2.05)			(0.62)	(0.43)			(1.34)	(1.27)
Ted Spread$_{t-1}$				4.22				0.84				-1.42
				(1.01)				(0.35)				(-0.15)
Constant	-1.70	-0.45	-1.16	-1.63	3.61	0.23	0.12	0.02	20.30**	0.02	0.31	0.47
	(-0.41)	(-0.12)	(-0.31)	(-0.45)	(1.56)	(0.12)	(0.06)	(0.01)	(2.31)	(0.00)	(0.04)	(0.06)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAVIII

Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative stocks; Other Horizons

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, nonspeculative} and t is a month:

$$r^{(k)}_{P,s,t} = \iota^{(k)}_{s,t} + \chi^{(k)}_{s,t} \times \beta_{P,s} + \varrho^{(k)}_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon^{(k)}_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(k)}_{P,s,t}$ is the value-weighted k-month excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West (1987) adjusted standard errors allowing for $k-1$ lags:

$$\begin{cases} \chi^{(k)}_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{1,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(k)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^x_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \pi^{(k)}_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{2,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(k)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^x_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \kappa^{(k)}_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{3,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(k)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^x_{3,s} x_{t-1} + \nu_{t,s}. \end{cases}$$

Panel A use k=1 months, Panel B uses k=3 month, Panel C uses k=6 month, Panel D uses k=18 month. Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) control for the k-month excess market return from t to $t+k-1$, and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAVIII (Continued):

Dep. Var:	$\chi_{s,t}^{(k)}$				$\varrho_{s,t}^{(k)}$				$\iota_t^{(k)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-1.06	-0.74	-0.78	-0.99*	0.49	0.55	0.31	0.42	0.54	0.57**	0.75**	0.89***
	(-1.39)	(-1.63)	(-1.55)	(-1.95)	(1.27)	(1.47)	(0.73)	(0.99)	(1.31)	(2.06)	(2.43)	(2.85)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	0.15	0.48*	-0.00	-0.01	-0.57	-0.68**	-0.34	-0.41	0.39	0.34	0.53*	0.64**
	(0.47)	(1.81)	(-0.01)	(-0.05)	(-1.23)	(-2.59)	(-1.07)	(-1.25)	(0.80)	(1.19)	(1.67)	(1.97)
N	396	396	396	396	396	396	396	396	396	396	396	396
Panel B: k=3 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-3.62**	-2.34**	-3.02***	-3.35***	1.51*	1.39**	1.38*	1.25	1.77*	1.74**	2.48***	2.81***
	(-2.09)	(-2.53)	(-3.04)	(-3.38)	(1.78)	(2.06)	(1.76)	(1.55)	(1.75)	(2.49)	(3.35)	(3.81)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	-0.26	0.78	-0.71	-0.57	-0.80	-0.88	0.04	-0.20	1.20	0.94	1.63**	1.72**
	(-0.42)	(1.20)	(-0.94)	(-0.78)	(-0.68)	(-1.14)	(0.05)	(-0.24)	(1.12)	(1.41)	(2.20)	(2.40)
N	394	394	394	394	394	394	394	394	394	394	394	394
Panel C: k=6 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-7.59**	-4.19**	-5.63***	-6.31***	2.53	1.70	1.28	1.04	3.55*	3.45***	5.15***	5.85***
	(-2.39)	(-2.48)	(-3.04)	(-3.49)	(1.65)	(1.39)	(0.82)	(0.67)	(1.88)	(2.64)	(3.73)	(4.36)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	-0.07	2.24	-0.96	-1.18	-2.91	-2.69**	-1.10	-1.30	2.42	1.84	3.63***	3.97***
	(-0.06)	(1.60)	(-0.64)	(-0.85)	(-1.43)	(-2.25)	(-0.82)	(-0.99)	(1.21)	(1.50)	(2.81)	(3.26)
N	391	391	391	391	391	391	391	391	391	391	391	391
Panel D: k=18 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-12.60**	-11.07***	-14.91***	-15.69***	2.90	4.42*	3.76	3.35	6.02*	8.78***	12.89***	13.49***
	(-2.35)	(-3.08)	(-3.27)	(-3.39)	(1.10)	(1.77)	(1.33)	(1.17)	(1.89)	(2.93)	(3.45)	(3.52)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	0.04	0.94	-5.59**	-6.01**	-4.21	-5.59*	-3.57	-3.98	3.05	5.37**	9.23***	9.49***
	(0.02)	(0.30)	(-2.09)	(-2.29)	(-1.12)	(-1.91)	(-1.02)	(-1.14)	(1.06)	(2.01)	(3.33)	(3.32)
N	379	379	379	379	379	379	379	379	379	379	379	379

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