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Course: Computational Neuroscience

Project 1

University of Tehran

What is Lif ?

The LIF neuron is modeled as **a parallel RC circuit that charges in response to its input**. It has a voltage threshold and when this is reached, the circuit emits a spike and then resets to its resting level, which is usually zero.

$$\tau_m \frac{\mathrm{d}u}{\mathrm{d}t} = -\left[u\,(t) - u_{\mathrm{rest}}\right] + R\,I\,(t)$$

ELIF:

The exponential integrate-and-fire model (EIF) is **a biological neuron model, a simple modification of the classical leaky integrate-and-fire model describing how neurons produce action potentials**. In the EIF, the threshold for spike initiation is replaced by a depolarizing non-linearity.

$$\tau_m \frac{dV}{dt} = RI(t) + [E_m - V + \Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right)]$$

AELIF:

The adaptive exponential integrate-and-fire neuron is a spiking neuron model where the above exponential nonlinearity of the voltage equation is combined with an adaptation variable w. where w denotes an adaptation current with time scale.

$$\tau_m \frac{dV}{dt} = RI(t) + [E_m - V + \Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right)] - Rw$$

$$\tau \frac{dw(t)}{dt} = -a[V_{\mathrm{m}}(t) - E_{\mathrm{m}}] - w + b\tau\delta(t - t^f)$$

Big mistake! :

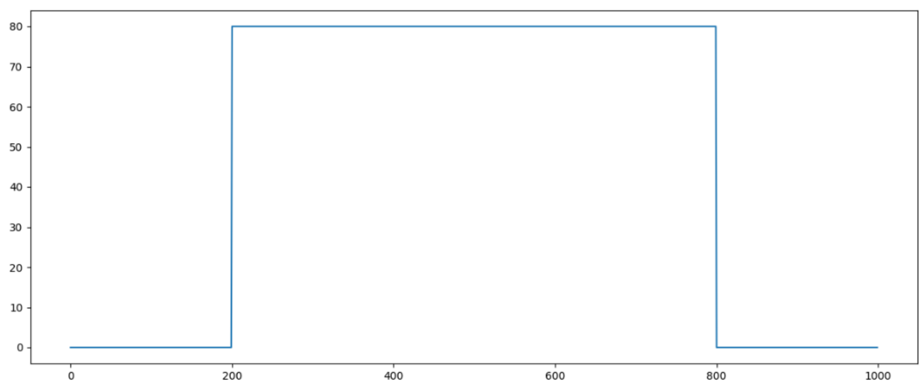
for AELIF model, its behavior was a little bit weird for me but by changing the parameters I got the desire output but I was suspect about it. During preparing this report, in one part I decided to record first term of adaptation and plot it, and I realized that I was updating “w” term after updating “v (potential)”. It in the next iteration I was using “w” in update process of “v”, and it was a mistake that had changed the AELIF behavior. Wrong:

```
ng.v += ((leakage + currents + exp_term + (-(self.R * self.w))) / self.tau) * ng.network.dt
leakge_w = self.a * (ng.v - self.u_rest) - self.w
```

Edited:

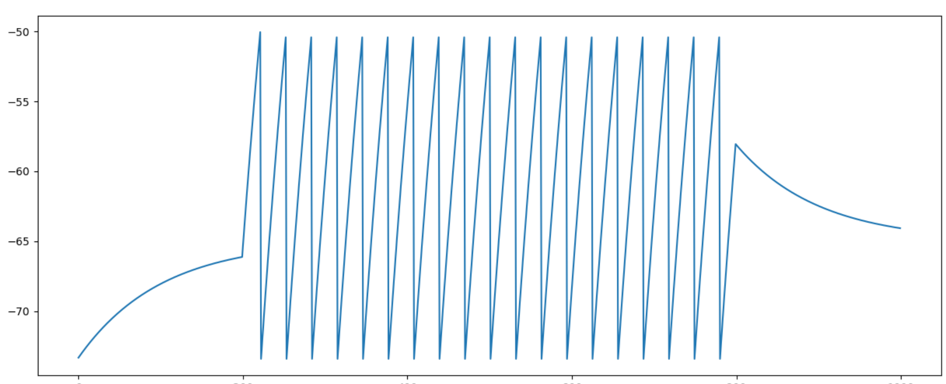
```
leakge_w = self.a * (ng.v - self.u_rest) - self.w
ng.v += ((leakage + currents + exp_term + (-(self.R * self.w))) / self.tau) * ng.network.dt
```

1 - Constant Current (80) :



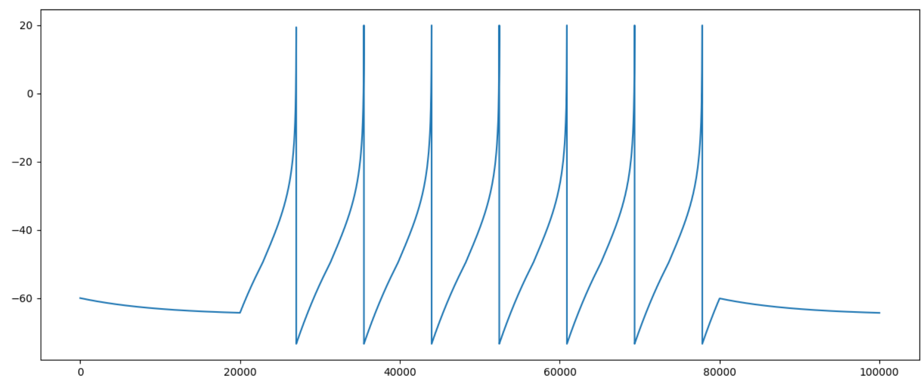
1.1 – LIF:

- Tau: 10
- u_rest: -65
- u_back: -73.42
- threshold: -50
- R: 1
- v_init: "normal(-40, 10)"



1.2 - ELIF:

- tau: 10
- u_rest: -65
- u_reset: 20
- u_back: -73.42
- threshold: -50
- R: 1
- v_init: "normal(-60, 0)"
- delta_T: 30
- refractory_T: 0 (Refractory is disable)



- Note: For this behavior, I changed dt to 0.001 (which is was 0.1 for LIF) to capture the graph more accurately in the spike time.

1.3 - AELIF:

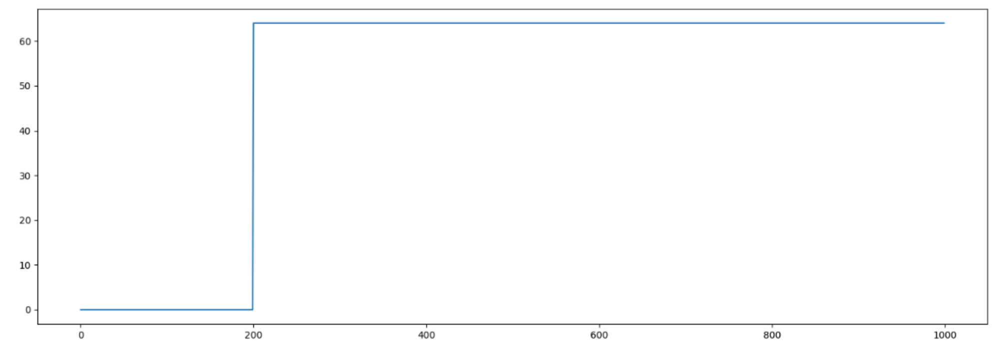
$$\tau_m \frac{\mathrm{d}u}{\mathrm{d}t} = -\left(u - u_{\text{rest}}\right) + \Delta_T \exp\left(\frac{u - \vartheta_{rh}}{\Delta_T}\right) - R\,w + R\,I\left(t\right)$$
$$\tau_w \frac{\mathrm{d}w}{\mathrm{d}t} = a\,\left(u - u_{\text{rest}}\right) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)\ .$$

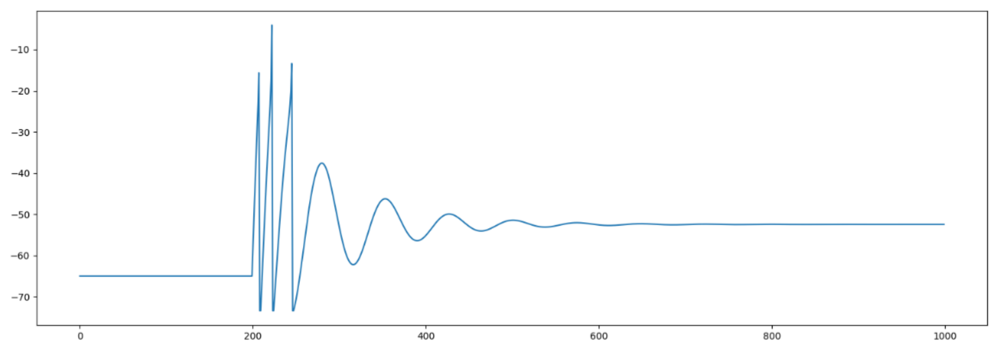
- tau=10,
- u_rest=-65,
- u_reset=0,
- u_back=-73.42,
- threshold=-40,
- R=10,
- v_init="normal(-40, 10)",
- delta_T=4,
- refractory_T=0,
- a=0,

- b=10,
- tau_w=20

Note : If I set tau_w too little, the memberance potential will change so fast after one time step and if I set it too high, the adaptation effect will disappear very late.

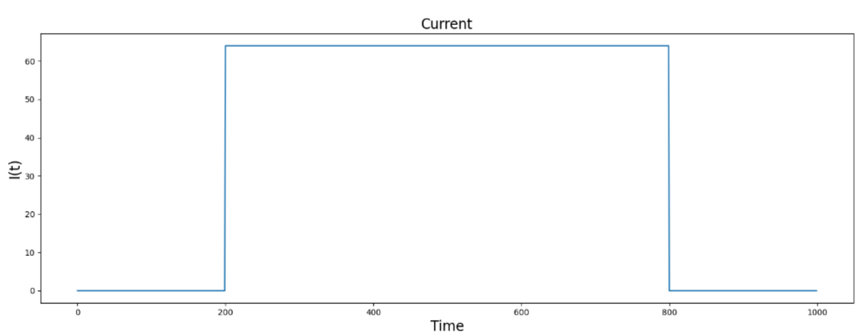
In this case, I used an extra current pattern at first:

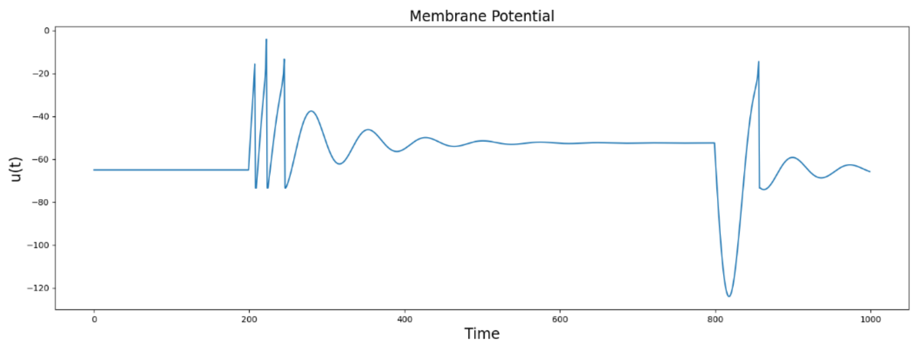




As is obvious, after some spikes, neuron witnessed adaptation behavior.

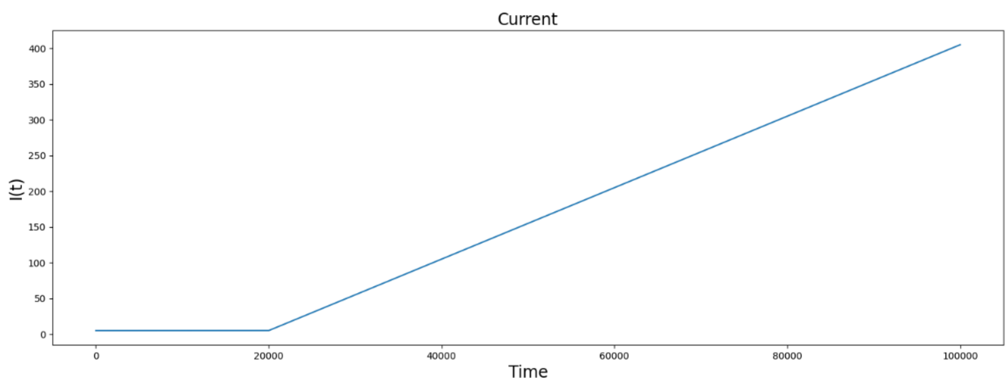
And this is for the current with end in iteration 800 :





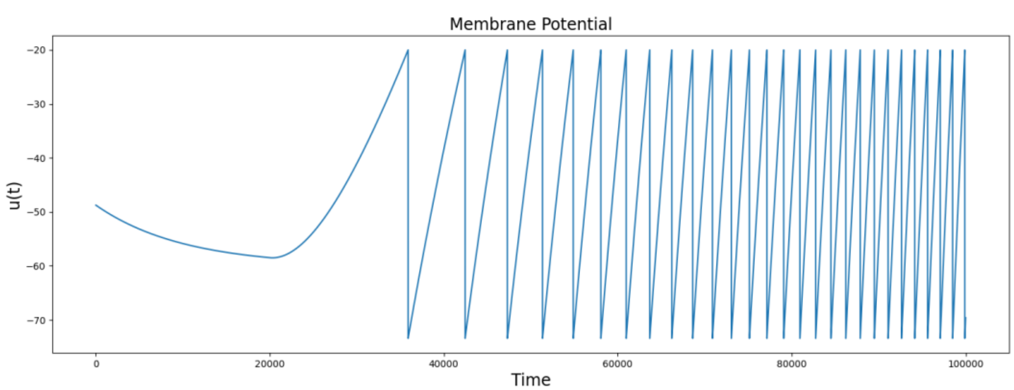
As its evident, because adoptability term (w) have a high amount after some spikes to prevent rise in potential, when the current suddenly cut off, the potential fall into -120, and after a while, the potential come back to the rest potential (because "w" term dacay).

2 - Ascending current with constant slope: (slope is 5)

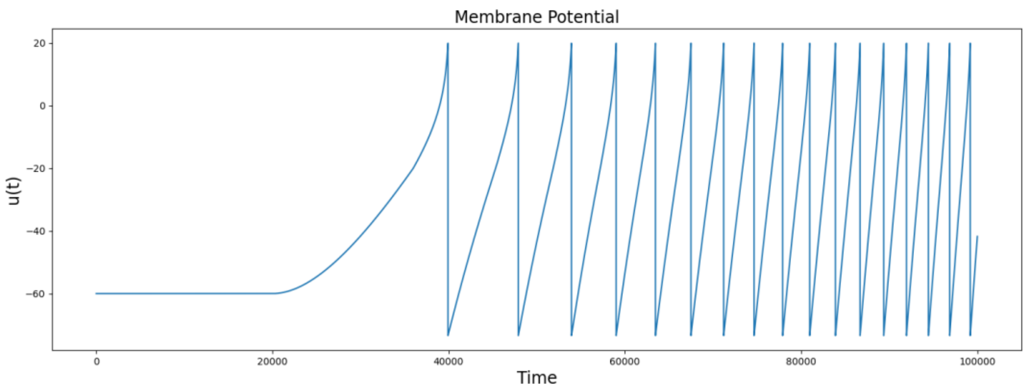


Because I needed more temporal resolution to capture all firing in the flow of time, I changed "dt" to 0.001 and iterations are 100000 (like before, the model is perform in 100 second)

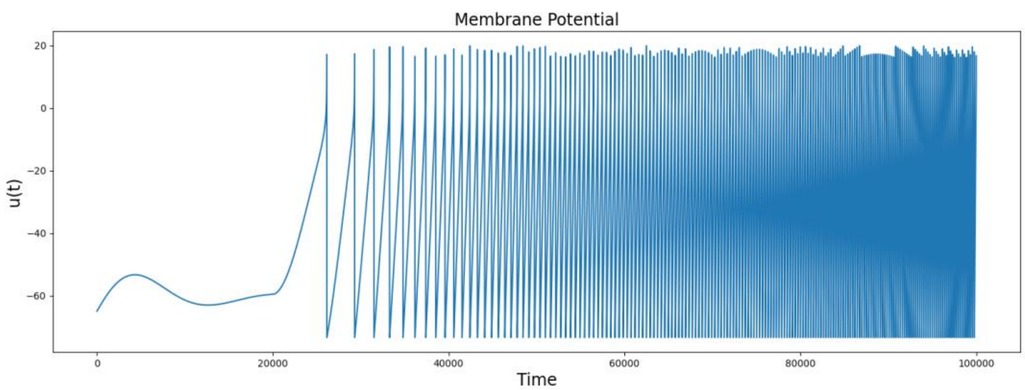
2.1 – LIF:



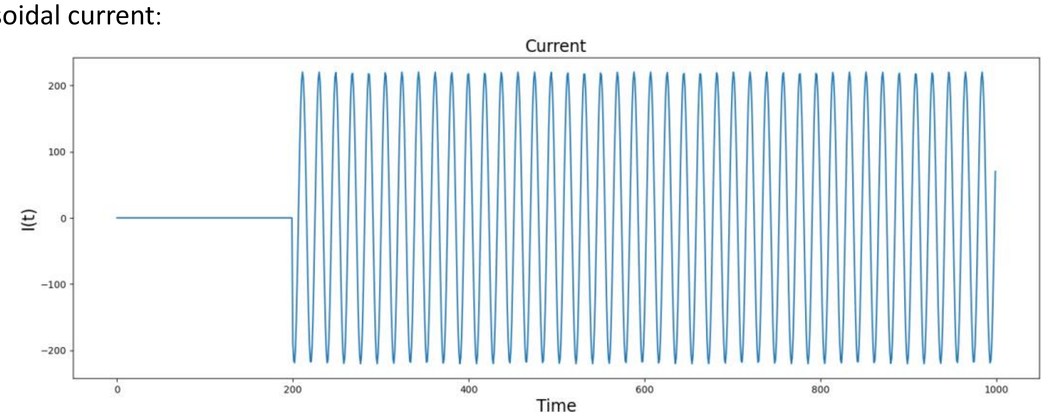
2.2 – ELIF:



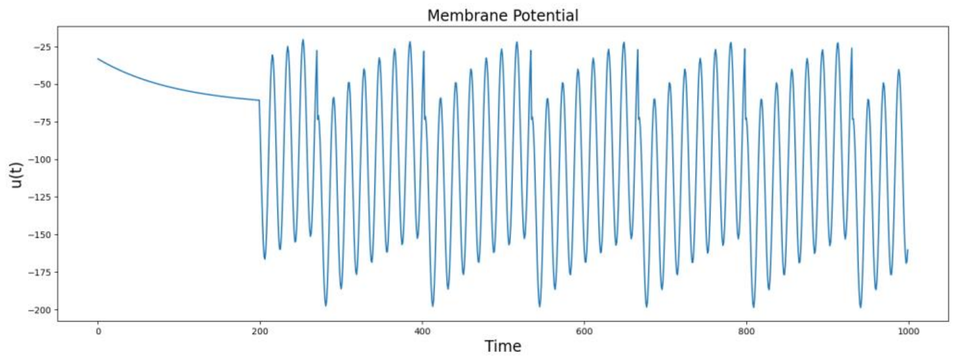
2.3 – AELIF:



3 - Sinusoidal current:



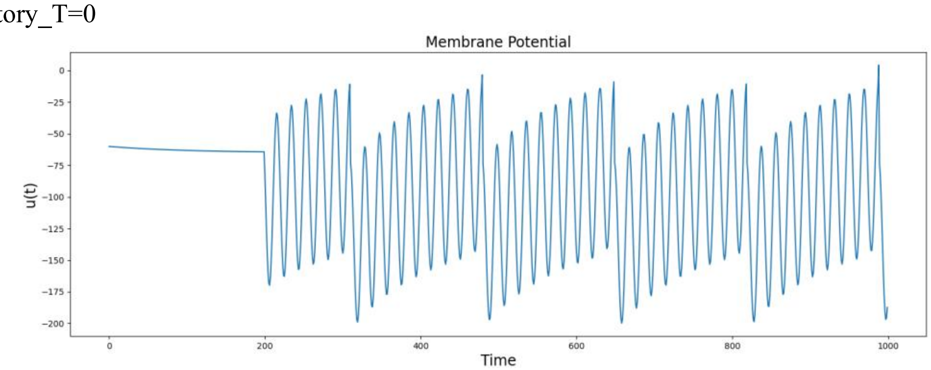
3.1 – LIF



Note : As the current is sinusoidal, the potential initially rises, then decays. However, upon reaching a potential higher than the previous low point, the current reconnects, resulting in an ascent to an even higher point in potential.

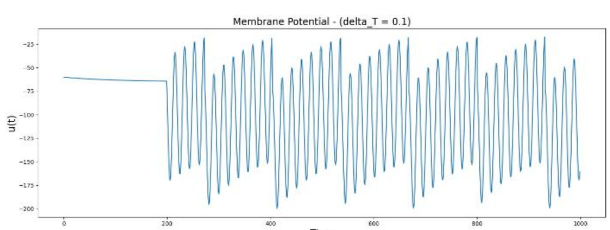
3.2 – ELIF:

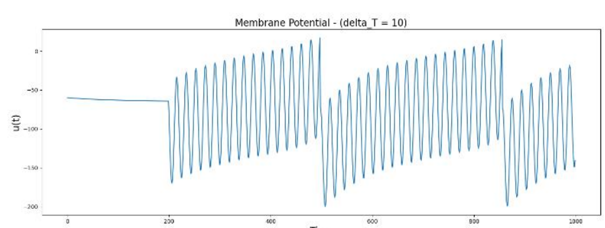
- tau=10,
- u_rest=-65,
- u_reset=20,
- u_back=-73.42,
- threshold=-20,
- R=10,
- v_init="normal(-60, 0)",
- delta_T=1,
- refractory_T=0



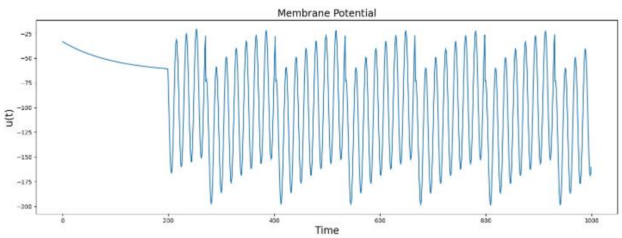
AS you can see, its behavior is not exactly like "LIF". Of course with changing delta_T, it would witness different behaviors. How much "delta_T" reduce, the time between reaching threshold (rh) and reaching the reset threshold would be faster and its behavior would be more like LIF.

For example for delta_T=0.1 and delta_T=10 :



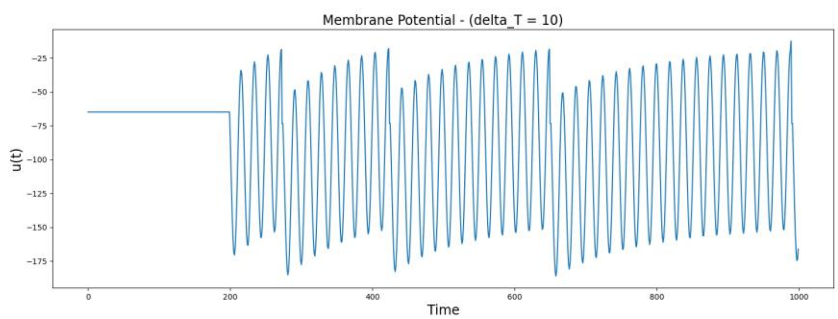


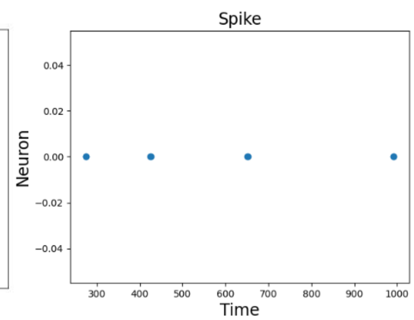
LIF:



3.3 AELIF:

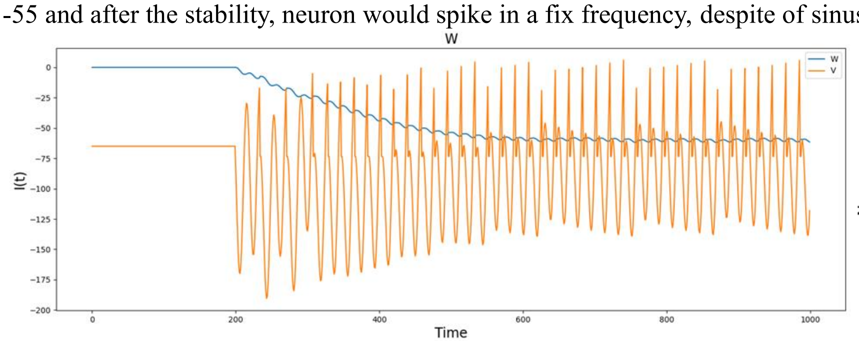
- tau=10,
- u_rest=-65,
- u_reset=20,
- u_back=-73.42,
- threshold=-20,
- R=10,
- v_init=-65,
- delta_T=0.1,
- refractory_T=0,
- a=0, (decay term is disable)
- b=10,
- tau_w=100

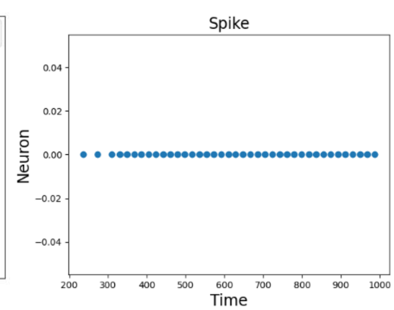




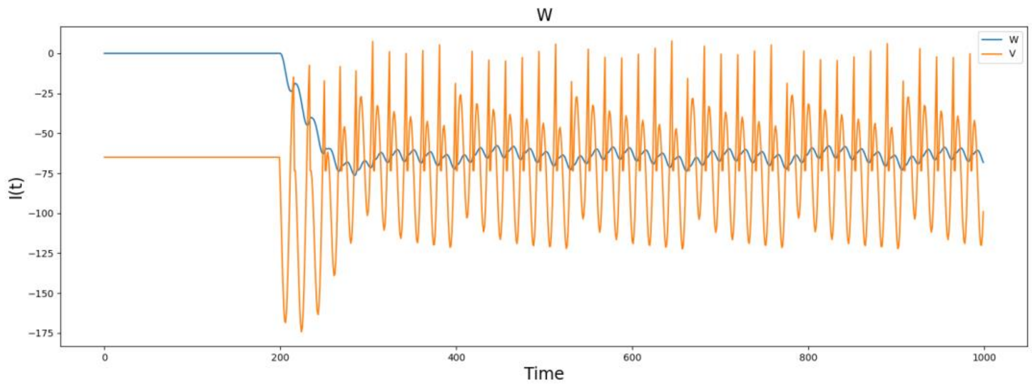
As you can see, frequency of spiking has reduced after a while due to adaptation term.

Changing “a” : with changing “a”, neuron would show very different behaviors. Choosing “a” high positive, would change the point and behavior of stability of “w”. for example for “a=7” you can see the stable point of “w” is about -55 and after the stability, neuron would spike in a fix frequency, despite of sinusoidal current.



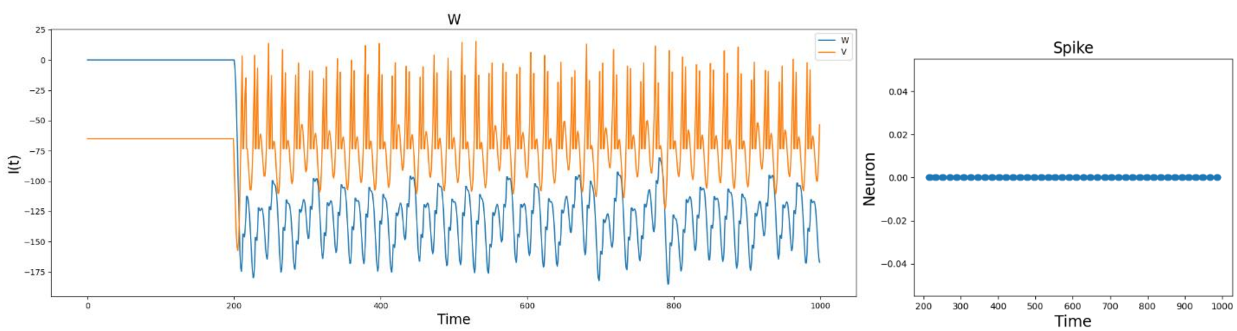


Or for bigger “a” the time to reach this stable point would reduce. For example “a=30”:



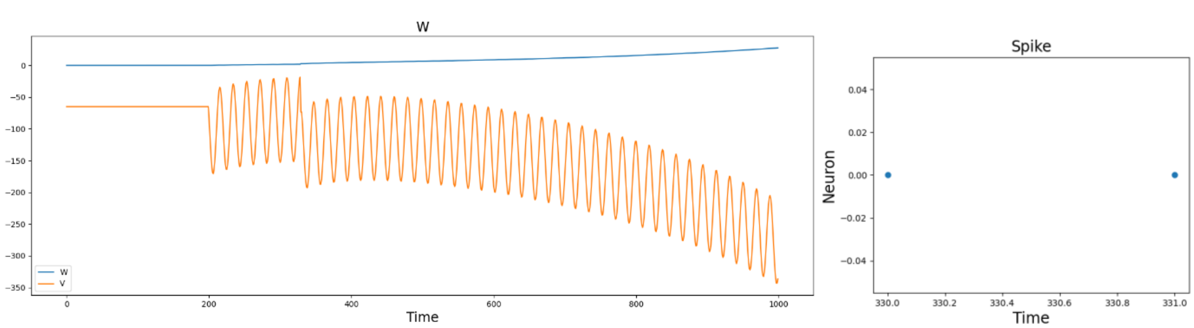
As you can see, the noise more noise around of stable point due to higher value of “a(u – u_rest)” and it have more impact on changing w.

If we increase a more, the noise would so higher and stable point would change due to high changes of first term in update “w” equation. For example “a=300”:



And its side-effect on changing “u” would lead to other behaviors like the above. (in the above example, spike frequency has been increased)

And for “a<0” the pattern would be like this:

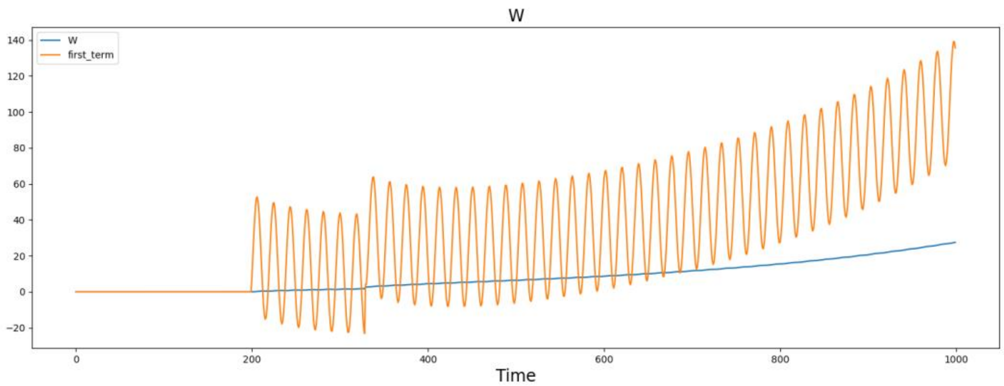


The reason of this behavior for updating “W” is this:

“first_term” : a(u – u_rest)

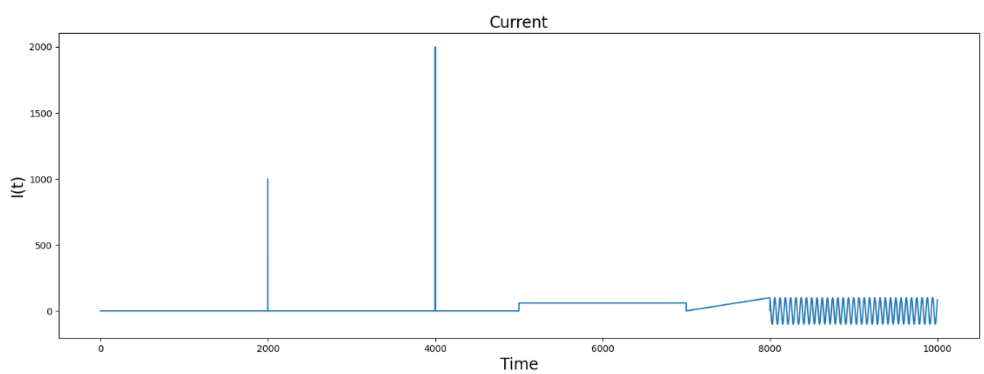
W : w

Update equation when there is no spike: a(u – u_rest) - w



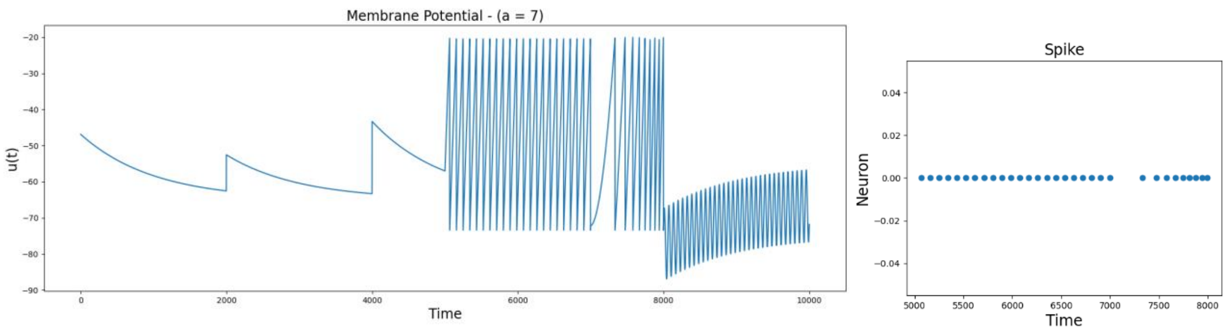
from the beginning, in more times, first_term is bigger that w, so the update equation would be positive often and generally, w would increase without reaching the stable point, and after a iteration, the first would be higher that w in all times. And w would increase with higher speed. So in the update “v” equation, “- R * w” would increase iteration by iteration, and the potential would decrease although it should reach the u_rest.

4 – In a nutshell:



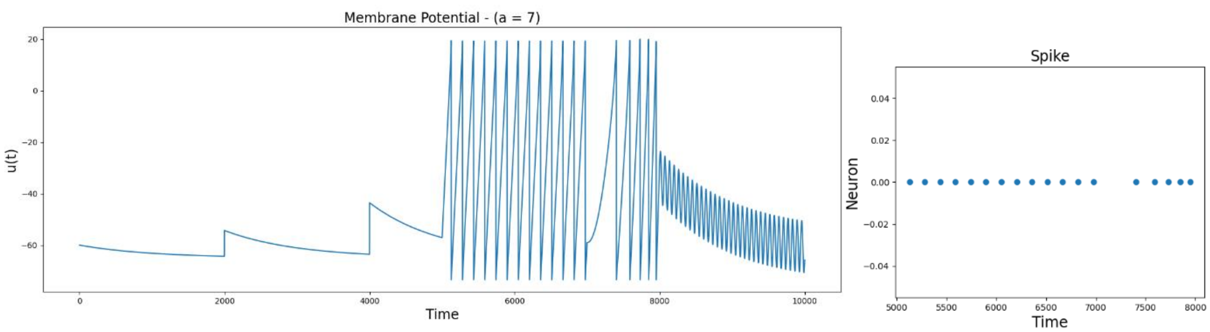
4.1 – LIF:

tau=10, u_rest=-65, u_back=-73.42, threshold=-20, R=10, v_init="normal(-60, 0)"



4.2 – ELIF :

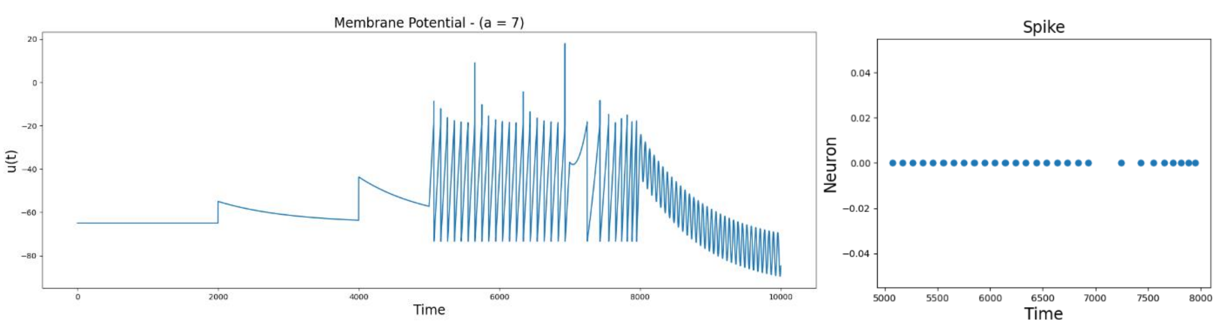
tau=10, u_rest=-65, u_back=-73.42, threshold=-20, R=10, v_init="normal(-60, 0)", delta_T=10



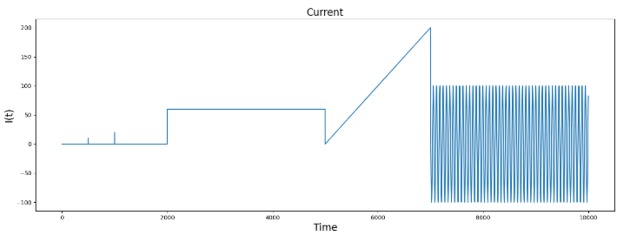
Note: as you can see, the frequency of spiking is less than simple LIF, due to a little high "delta_T".

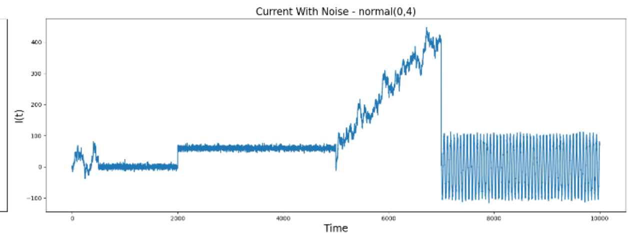
4.3 – AELIF:

tau=10, u_rest=-65, u_reset=20, u_back=-73.42, threshold=-20, R=10, v_init=-65, delta_T=0.1, refractory_T=0, a=0.01, b=10, tau_w=100



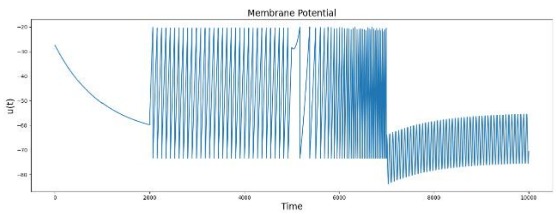
5 – Current with noise :

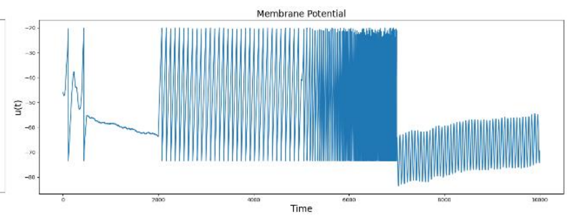


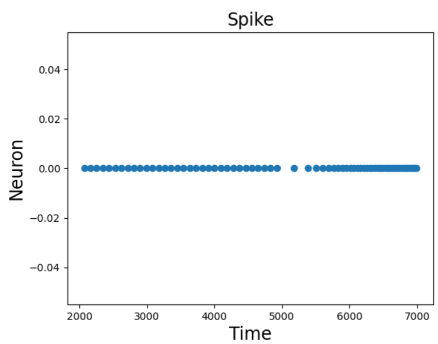


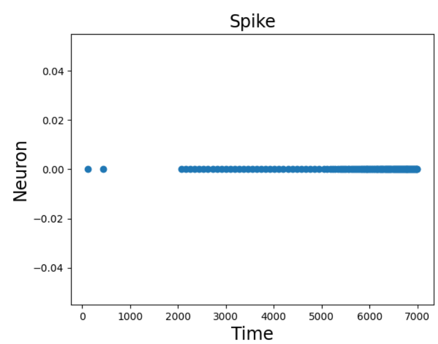
5.1 – LIF:

Without noise | with noise

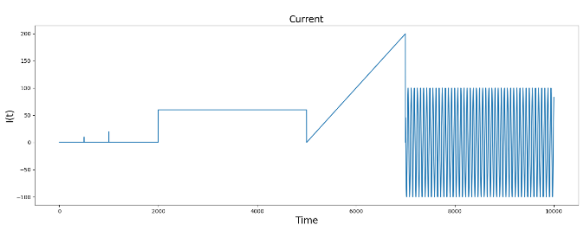


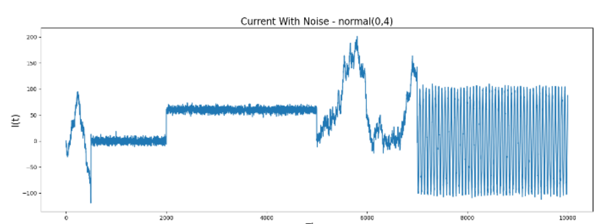




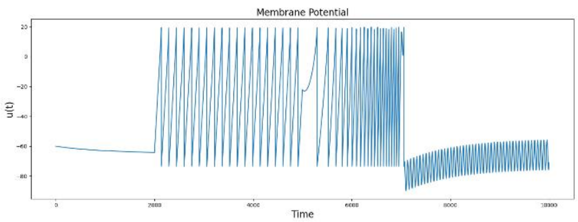


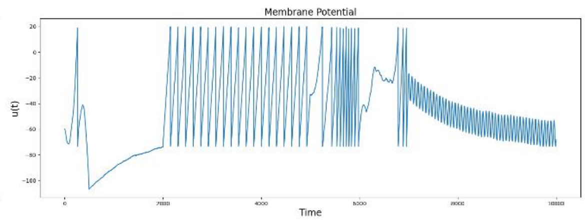
5.2 – ELIF:

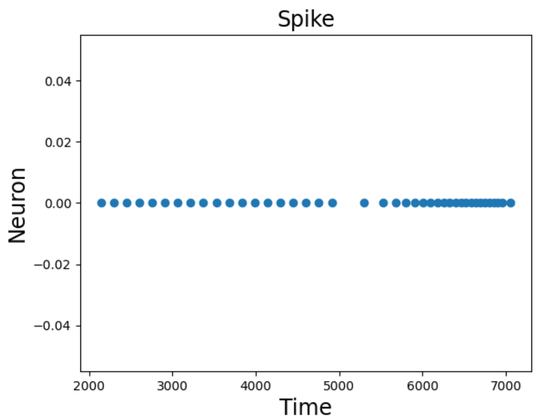


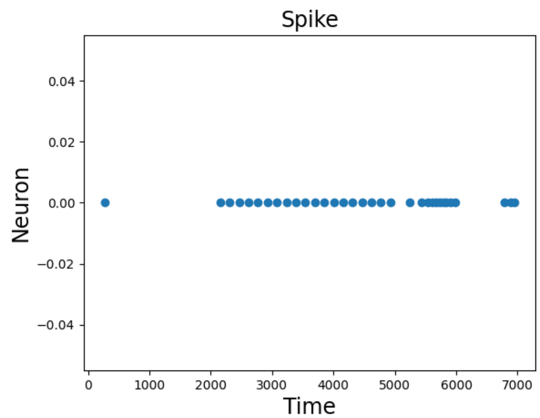


Without noise | with noise

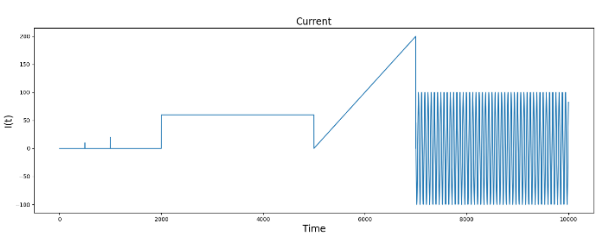


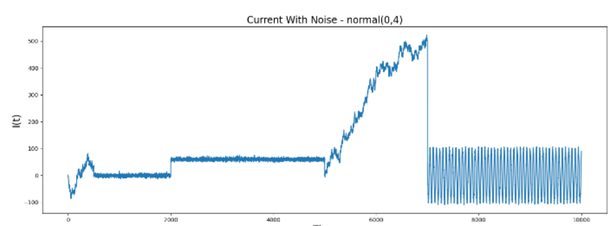






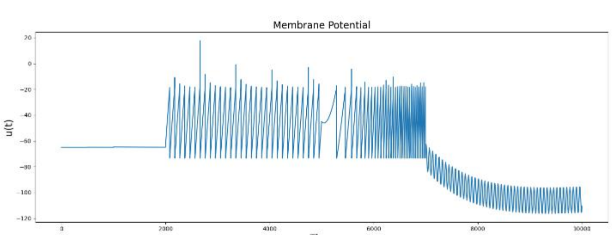
5.3 – AELIF:

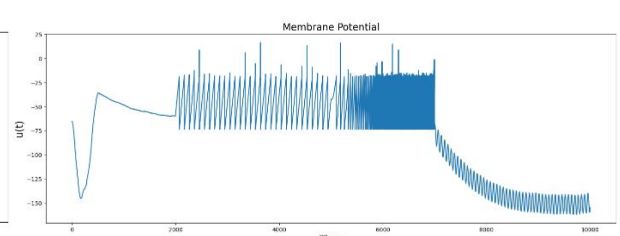


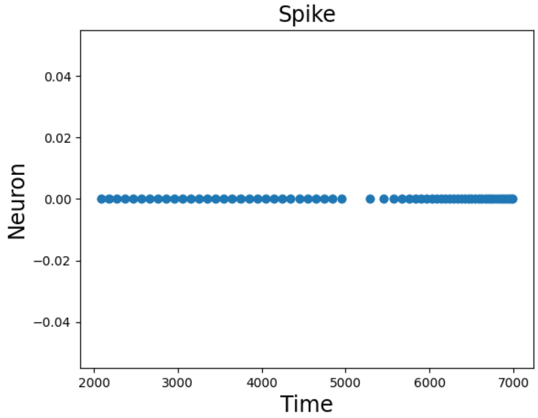


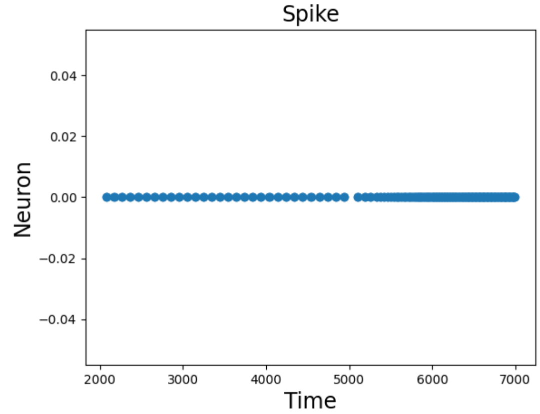
Without noise

With noise

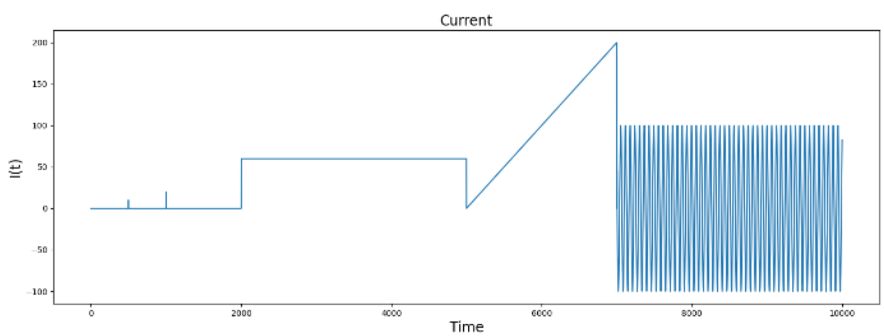








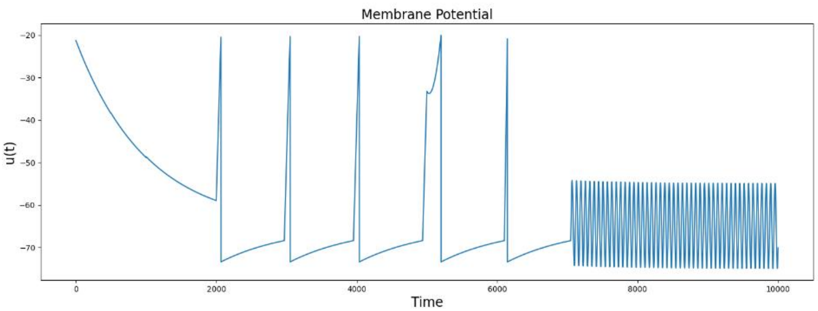
6 – Refractory: dt = 0.01

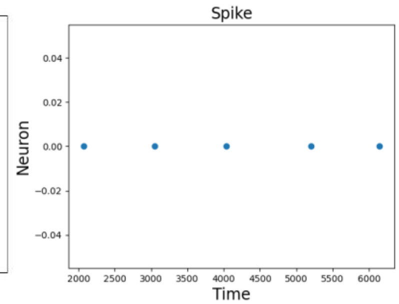


The refractory term is indicating the period of time that the current would not allow to have impact on neuron potential. When dt is 0.01, if I set refractory term to 1, it will block current for 100

6.1 - Lif:

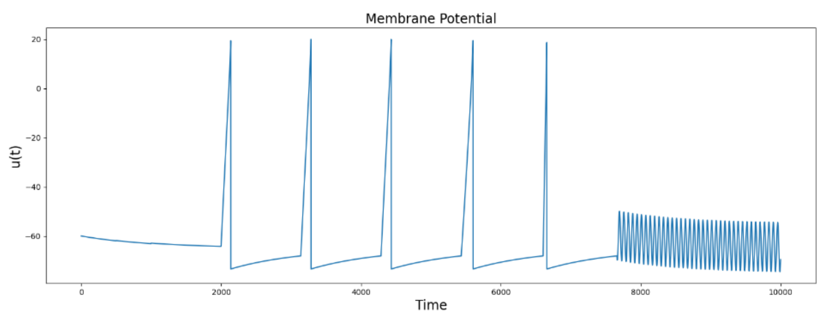
tau=10, u_rest=-65, u_back=-73.42, threshold=-20, R=10, v_init="normal(-40, 10)", refractory_T=10 (1000 iteration)

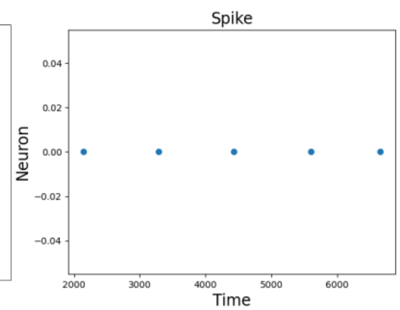




6.2 – ELIF:

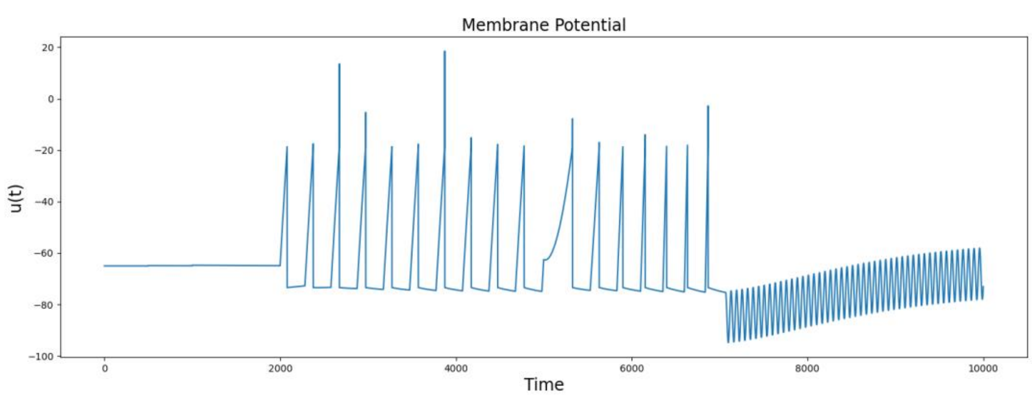
tau=10, u_rest=-65, u_reset=20, _back=-73.42, threshold=-20, R=10, v_init="normal(-60, 0)", delta_T=10, refractory_T=10





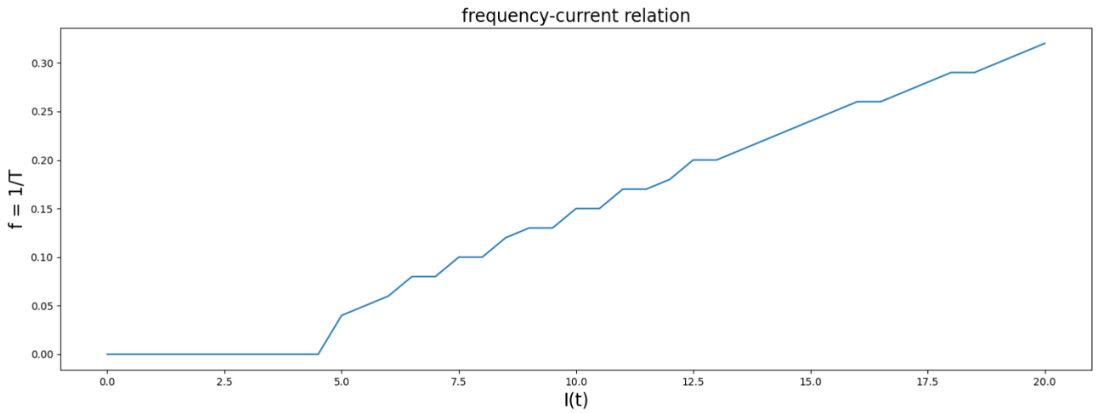
6.3 – AELIF:

tau=10, u_rest=-65, u_reset=20, u_back=-73.42, threshold=-20, R=10, v_init=-65, delta_T=0.1, refractory_T=2, a=0.01, b=50, tau_w=10

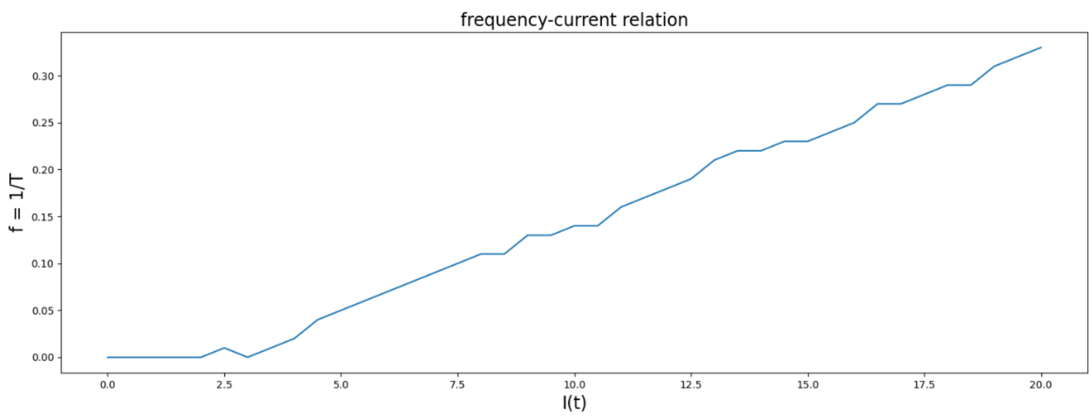


7 - frequency-current relation:

7.1 – LIF:

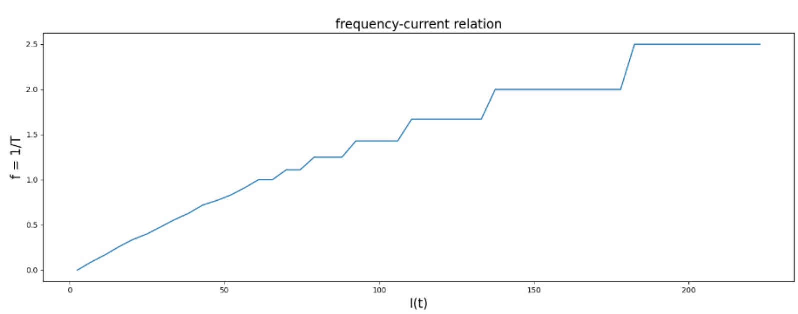


with noise:

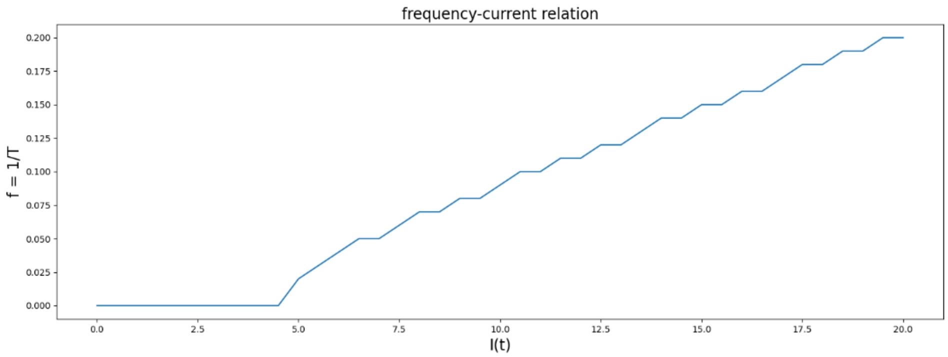


Note: noise does not have so much on the relation in big picture.

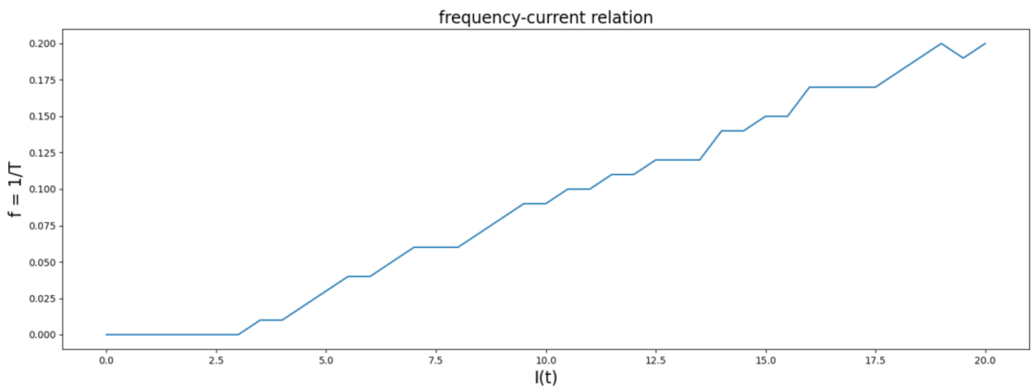
the relation between these two terms are approximately logarithmic.(bigger time resolution)



7.2 - ELIF:

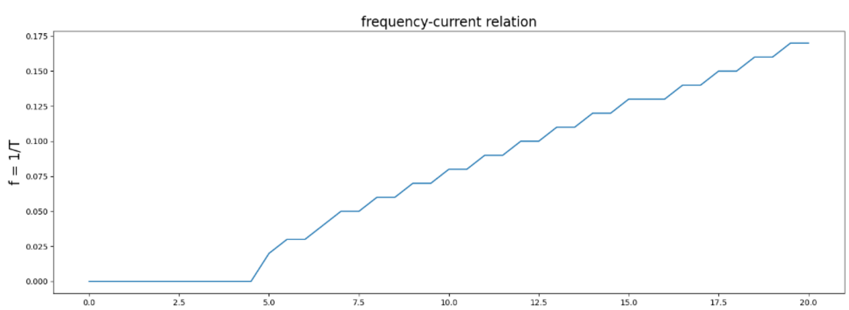


With noise:

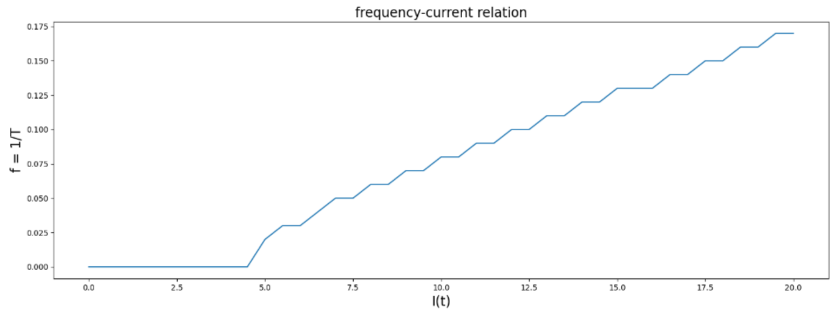


Note: as it's evident, with "delta_T = 10", the frequency is less than simple LIF.

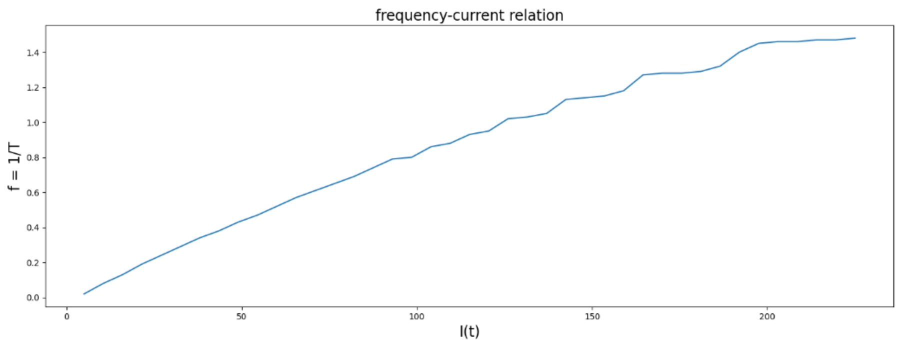
7.3 – AELIF:



With noise:

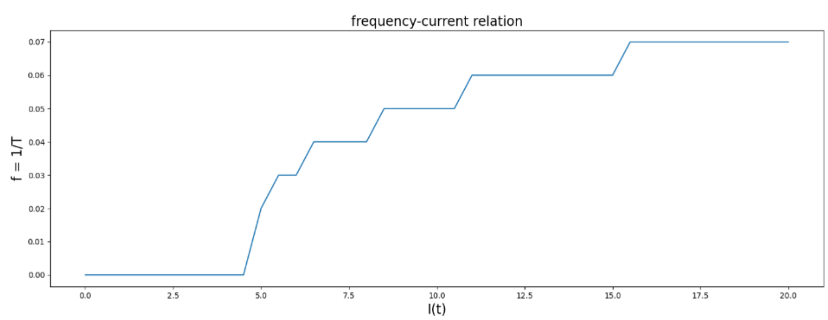


For high current’s, you can see the adaptation. (compare with the frequency of LIF with high current)



7.4 – with refractory:

Above AELIF model with refractory term = 10 (10 seconds)



And refractory term = 2 second.