

Machine learning

K-MEANS

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K - MEANS

1

Clustering K-Means

How does the K-Means Algorithm Work?

Step-1: Select random K points or centroids as the center of clusters.

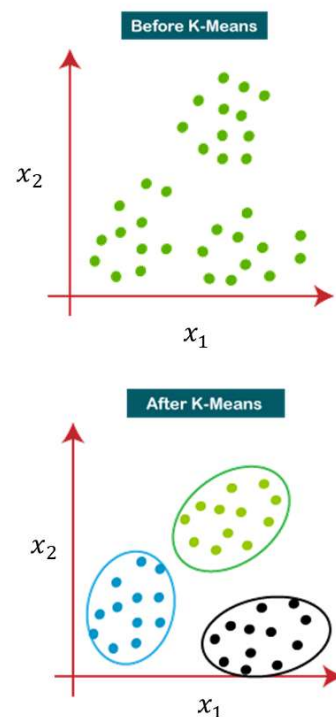
Step-2: Assign each data point to its closest centroid, which will form the predefined K clusters.

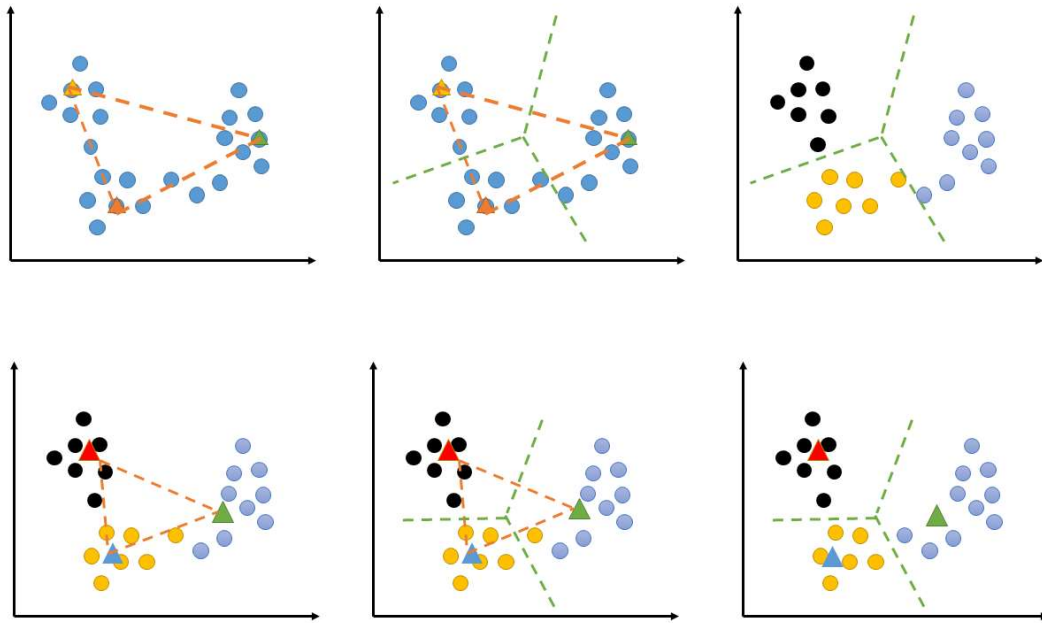
Step-3: Calculate the variance and place a new centroid of each cluster.

Step-4: Repeat the third step, which means reassigning each data point to the new closest centroid of each cluster.

Step-4: If any reassignment occurs, then go to step-4 else go to FINISH.

Step-6: The model is ready.





Pseudo code

randomly initiate K cluster centroid ($\mu_1, \mu_2, \dots, \mu_k \in R^n$)

Repeat

{

for $i = 1$ to m

$c^{(i)} = \operatorname{argmin} \|x^{(i)} - \mu_k\|$

for $k = 1$ to k

$\mu_k = \text{average of points assigned to cluster } K$

}

Centroids = `np.random.random((k, n))`

While True:

for i in `range(m)`:

`c[i] = np.argmin(np.linalg.norm(x[i] - centroids, axis = 1))`

for k in `range(k)`:

`centroids[k] = np.mean(X[c==k], axis = 0)`

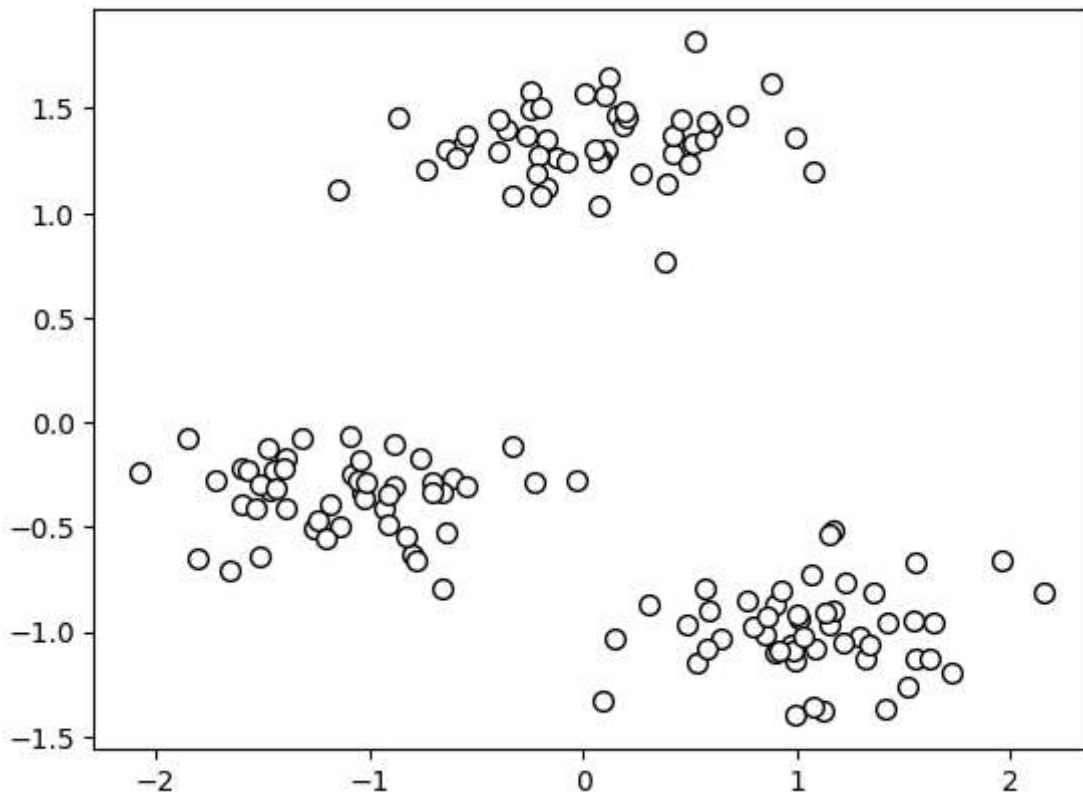
Termination condition

```
In [10]: import os
os.environ["OMP_NUM_THREADS"] = '4'
```

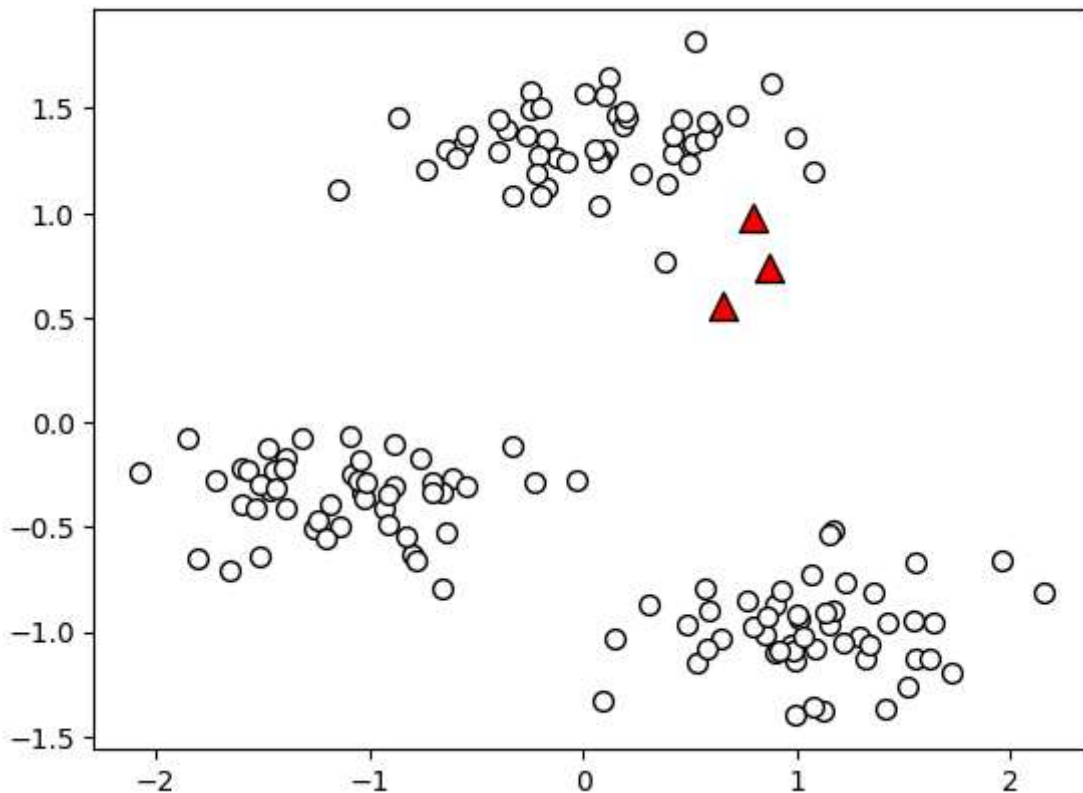
```
#C:\ProgramData\Anaconda3\lib\site-packages\sklearn\cluster\_kmeans.py:1036: UserWarning  
#warnings.warn
```

```
In [11]: #import libraries and data  
import numpy as np  
import matplotlib.pyplot as plt  
from sklearn.datasets import make_blobs  
# create random data  
X , y = make_blobs(n_samples=150, centers=3, cluster_std=1.2, random_state=10)
```

```
In [12]: # Normalize X  
mu = X.mean(axis=0)  
std = X.std(axis=0)  
X = (X - mu) / std  
  
# plot data  
plt.scatter(X[:, 0], X[:, 1], edgecolors='k', s=50, c='w')  
plt.show()
```



```
In [4]: m , n= X.shape  
K=3  
initial_centroids= np.random.rand(K,n)  
  
#plot centeroids  
plt.scatter(initial_centroids[:, 0], initial_centroids[:, 1], edgecolors='k', s=100, c='k')  
plt.scatter(X[:, 0], X[:, 1], edgecolors='k', s=50, c='w' )  
plt.show()
```



```
In [13]: centroids=initial_centroids.copy()
print(centroids)
```

```
[[0.87297331 0.73952191]
 [0.65928464 0.56123203]
 [0.79516162 0.97372791]]
```

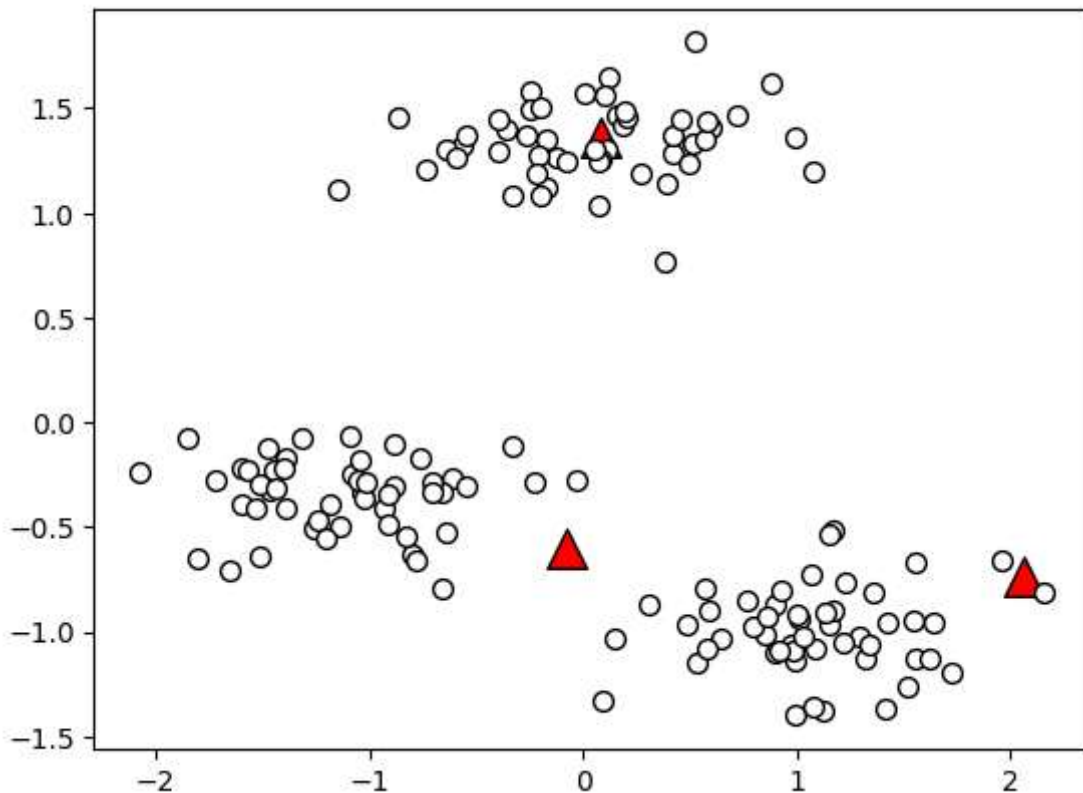
```
In [6]: cluster_ids=np.array([np.argmin(np.linalg.norm(X[i] - centroids , axis = 1)) for i in
print(cluster_ids)
```

```
[1 2 2 1 1 2 1 2 1 1 2 2 1 1 2 2 1 2 1 1 1 1 1 1 1 1 1 2 2 1 1 1 2 1 1 1
 2 0 2 1 1 2 2 1 1 2 1 1 2 2 1 1 1 1 1 1 2 2 2 1 2 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 0 1 2 1 1 1 1 2 1 2 1 1 1 1 1 1 2 2 1 1 2 1 1 1 1 1 1 2 2 1 2 1 2 2
 1 2 2 1 1 1 2 1 1 1 1 1 1 2 2 1 1 2 1 1 1 1 1 1 1 1 1 2 1 1 2 1 2 2 2 1 1
 1 1]
```

```
In [14]: for k in range(K):
centroids[k] = np.mean(X[cluster_ids==k], axis=0)    #ریس ایکس رویکشیم معلوم میشه
centroids
```

```
Out[14]: array([[ 2.05851469, -0.73481985],
 [-0.0775947 , -0.60107719],
 [ 0.08255718,  1.36477202]])
```

```
In [15]: plt.scatter(centroids[:, 0], centroids[:, 1], edgecolors='k', s=200, c='red' , marker
plt.scatter(X[:, 0], X[:, 1], edgecolors='k', s=50, c='w' )
plt.show()
```



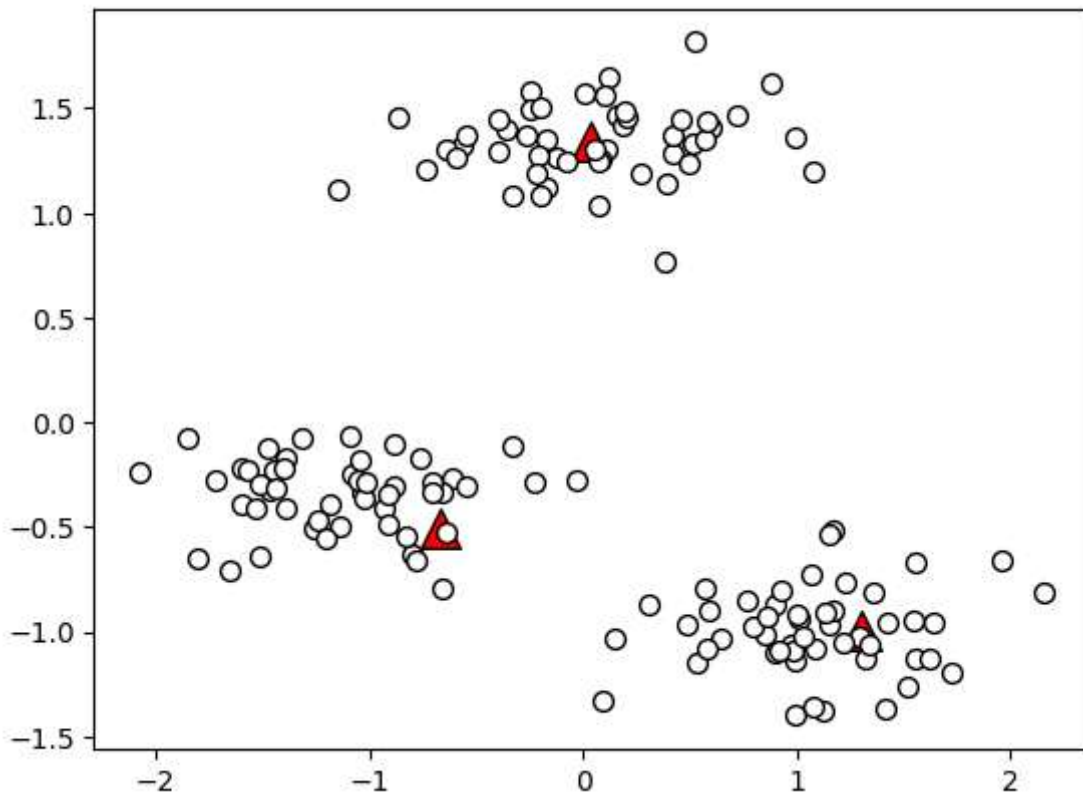
```
In [16]: cluster_ids=np.array([np.argmin(np.linalg.norm(X[i] - centroids , axis = 1)) for i in
print(cluster_ids)

[1 2 2 1 1 2 1 2 0 0 2 2 1 1 2 2 2 0 2 1 1 0 1 1 1 1 0 1 2 2 0 1 1 2 0 1 0
 2 0 2 1 1 2 2 0 1 2 2 1 2 2 1 1 0 0 0 1 1 2 2 2 0 2 0 1 1 1 0 1 1 1 0 1 0
 1 0 1 0 1 2 0 1 0 1 2 1 2 1 2 1 1 1 0 2 2 2 1 2 1 1 1 0 1 1 2 2 1 2 1 2 2
 1 2 2 0 0 0 2 0 1 1 1 1 0 2 2 1 0 2 1 1 1 1 2 0 1 1 1 2 1 0 2 1 2 2 2 1 1
 0 1]
```

```
In [17]: for k in range(K):
centroids[k] = np.mean(X[cluster_ids==k], axis=0)      #ریس ایکس رویکشیم معلوم میشه
centroids
```

```
Out[17]: array([[ 1.30323512, -0.99493691],
                [-0.67151137, -0.50937076],
                [ 0.03969005,  1.33921518]])
```

```
In [18]: plt.scatter(centroids[:, 0], centroids[:, 1], edgecolors='k', s=200, c='red' , marker
plt.scatter(X[:, 0], X[:, 1], edgecolors='k', s=50, c='w' )
plt.show()
```



K - MEANS

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μ_k : center of clusters

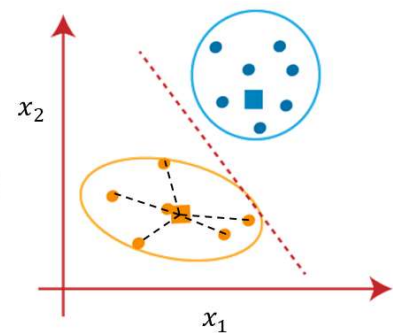
$c^{(i)}$: index of cluster assigned to $x^{(i)}$

$\mu_c^{(i)}$: center of cluster assigned to $x^{(i)}$

Goal function : $J(c^{(1)}, c^{(2)}, c^{(3)} \dots c^{(m)}, \mu_1, \mu_2, \mu_3 \dots \mu_k)$

$$WCSS = \frac{1}{m} \sum_{i=1}^m \|x^{(i)} - \mu_k\|^2$$

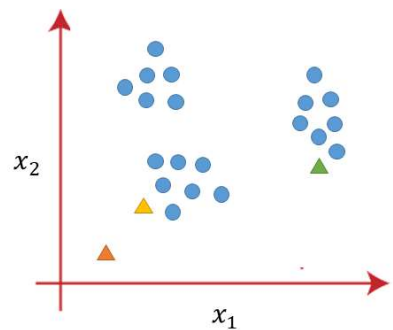
$$WCSS = \sum_{P_i \text{ in Cluster1}} \text{distance}(P_i, C_1)^2 + \sum_{P_i \text{ in Cluster2}} \text{distance}(P_i, C_2)^2 + \sum_{P_i \text{ in Cluster3}} \text{distance}(P_i, C_3)^2$$



Problem

Select from existing data

`C = np.random.permutation(x)[:, k]`



```
In [1]: import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_blobs
```

```
In [2]: def kmeans(X, initial_centroids):
    m = X.shape[0]
    K = initial_centroids.shape[0]
    centroids = initial_centroids
    initial_cluster_ids = np.zeros((m,))
    cost=0

    while True:
        cluster_ids = np.array([np.argmin(np.linalg.norm(X[i] - centroids, axis=1)) for i in range(m)])

        # update cluster centers
        for j in range(K):
            centroids[j] = np.mean(X[cluster_ids==j], axis=0) # بگیر بنابر به عنوان مرکز

        # stop
        if np.all(cluster_ids == initial_cluster_ids):
            for z in range(K):
                cost+=1/m * (np.linalg.norm(X[cluster_ids==z] - centroids[z])**2)
            return cost , initial_cluster_ids , centroids
        else:
            initial_cluster_ids = cluster_ids
```

```
In [3]: X, y = make_blobs(n_samples=1000, centers=3, cluster_std=1.5, random_state=10)
# plot data
plt.scatter(X[:, 0], X[:, 1], edgecolors='k', s=50, c='w');

costs=[]
best_centeroids=[]
final_ids=[]
for i in range(1 , 10):
    initial_centroids= np.random.permutation(X[ : i])
    A= kmeans(X , initial_centroids)
    costs.append(A[0])
    final_ids.append(A[1])
    best_centeroids.append(A[2])

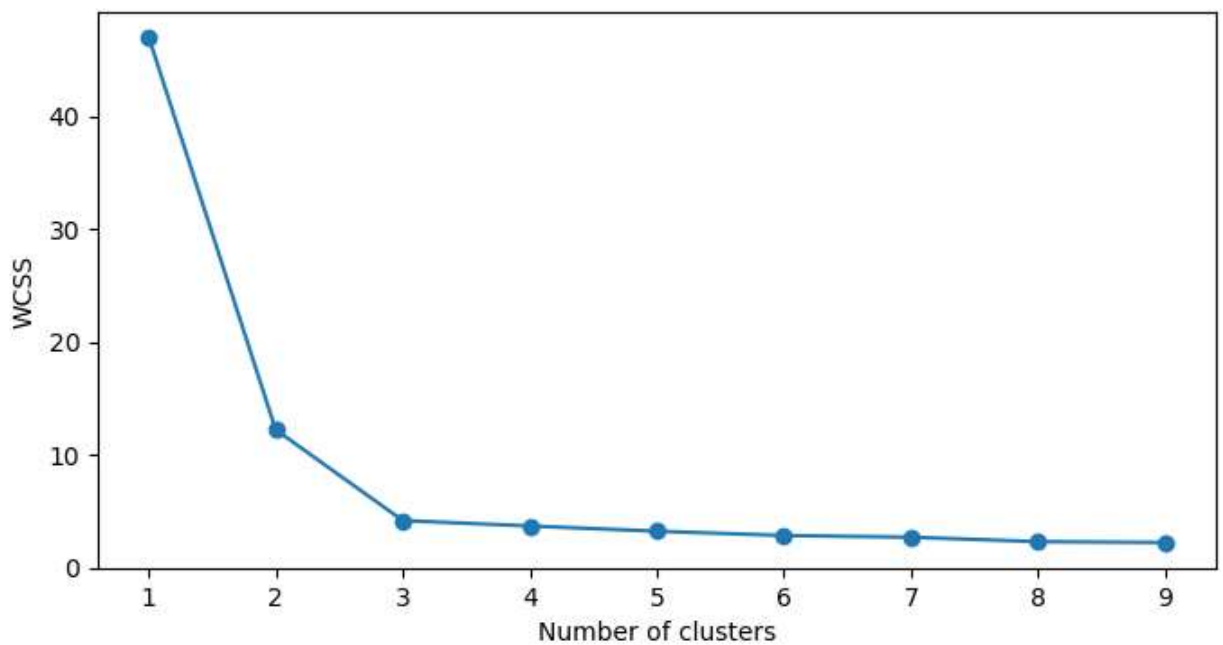
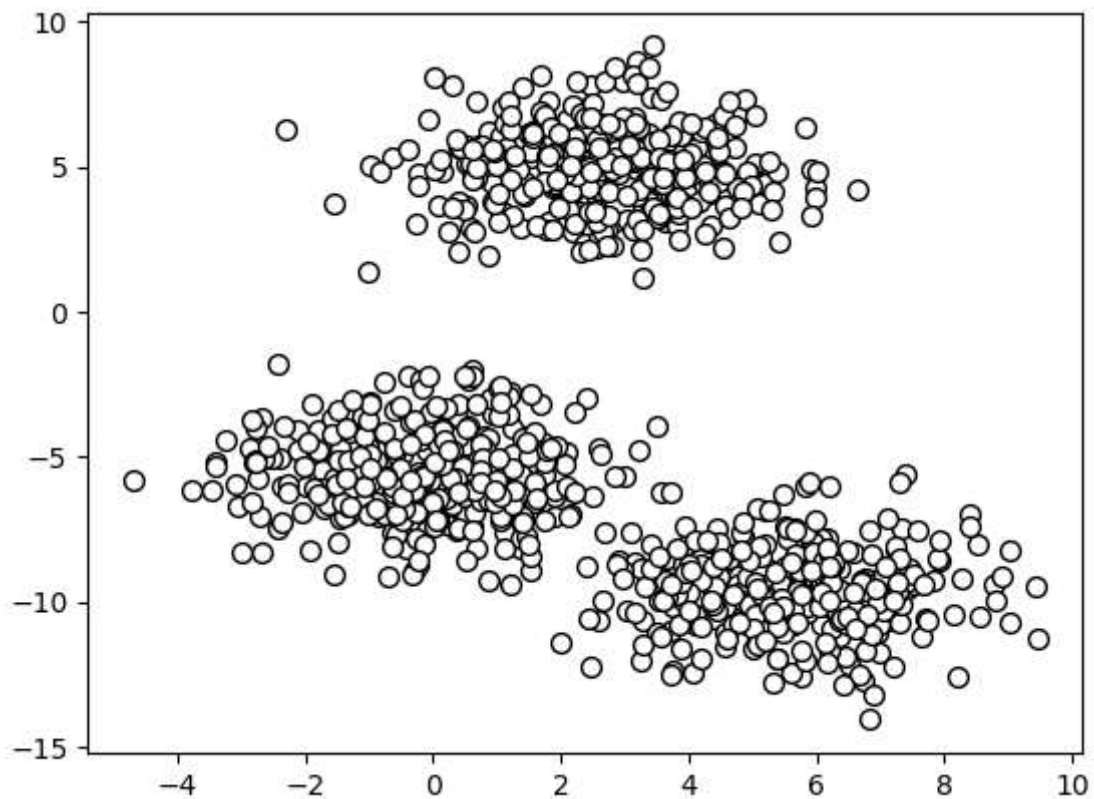
print(costs)
print("-----")
print(best_centeroids[2])
print("-----")
print(final_ids[2])

plt.figure(figsize=(8, 4))
plt.plot(range(1, 10), costs, marker='o')
plt.xticks(range(1, 10))
plt.xlabel('Number of clusters')
plt.ylabel("WCSS")
plt.show()
```

[47.029062199797174, 12.21527718761187, 4.167450222191264, 3.6894595293854398, 3.224286583361303, 2.843484126290323, 2.6948171538429073, 2.3008401012827138, 2.2250155257616107]

[[2.61164305 4.95788186]
[5.51998791 -9.57304198]
[-0.13936238 -5.54381213]]

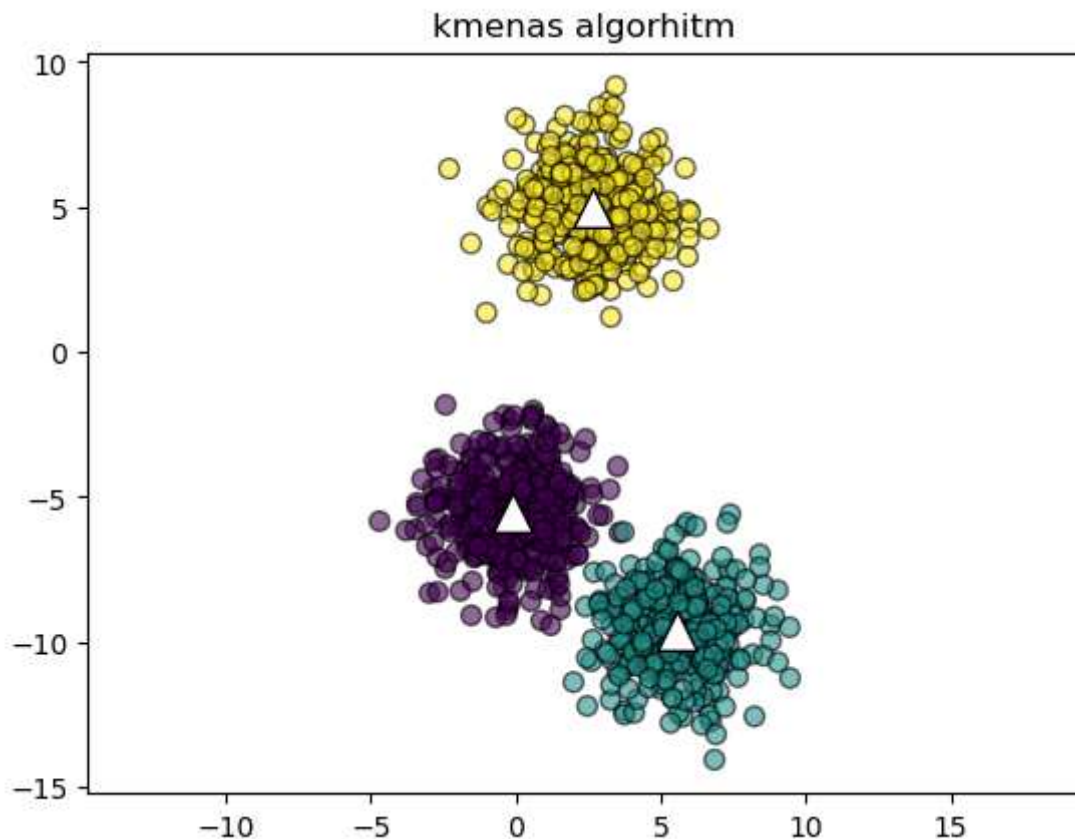
[1 0 2 2 0 2 2 2 2 2 1 0 1 1 2 2 2 1 1 1 2 1 2 2 0 0 1 0 1 1 1 1 1 1 1 0 0
1 0 0 0 2 0 0 1 0 2 2 1 1 1 2 0 2 0 2 2 1 0 0 2 2 1 0 2 1 0 2 0 0 0 2 2 1
2 2 1 1 1 2 2 2 0 2 0 1 0 2 2 0 0 1 1 0 2 1 0 1 1 0 0 2 0 0 2 1 2 0 1 2 2
0 1 0 2 1 0 2 2 0 1 1 1 1 1 1 0 2 0 0 0 2 0 2 2 2 2 1 2 2 1 0 2 1 0 1 2 0
0 2 0 1 1 2 2 0 2 2 0 2 0 2 1 1 0 0 0 0 1 2 0 2 2 2 1 1 2 1 1 1 2 2 1 2 0
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1 1 2 0 2 0 1 2 0 1 0 1 2 2 1 0 2 2 0 2 0 1 1 0 2 2 2 0 2 0 2 0 1 0 0 0 2 0
0 1 1 0 2 0 1 1 1 0 1 1 0 2 1 2 1 0 0 1 1 1 0 2 0 0 1 1 1 1 2 2 0 0 0 2 0
2 1 1 2 2 1 2 1 1 1 2 2 1 0 1 1 2 1 1 0 2 1 2 2 2 0 0 1 2 1 0 2 1 2 0 2 1
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2 2 1 0 2 2 1 1 2 1 1 0 2 0 0 0 1 2 1 1 2 2 0 2 2 0 1 2 2 2 0 1 2 2 2 1 0
2 0 2 1 1 2 2 2 1 2 1 1 0 2 1 2 1 2 1 0 0 0 0 2 1 2 1 1 0 0 2 0 2 1 0 1 1
0 0 1 1 2 0 0 0 2 1 1 0 0 1 0 1 0 2 1 1 2 0 1 2 2 0 2 1 2 2 1 1 1 2 1 1 0
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0 0 0 1 2 2 2 2 2 0 0 1 0 0 1 1 2 0 2 0 2 0 2 0 1 0 1 2 2 1 1 2 0 2 2 1 0
1 1 2 0 0 0 1 0 1 0 2 0 1 0 0 1 1 2 2 0 2 2 0 1 2 2 1 0 1 0 1 0 1 2 2 2 2
0 2 2 2 2 1 1 0 0 0 0 1 2 2 0 1 2 2 0 1 1 0 2 0 1 1 1 1 1 1 1 0 1 2 0 2 1
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1 0 2 2 2 1 0 1 0 1 0 0 1 1 0 1 0 2 0 0 0 1 2 0 1 2 1 1 2 0 2 0 1 0 0 0 2
0 2 0 2 0 0 0 0 1 2 1 2 1 2 1 2 0 0 2 0 2 1 0 2 0 1 2 2 0 1 0 0 1 1 1 0 2
0 1 2 0 2 0 1 0 1 0 1 0 1 0 0 2 2 1 0 1 0 1 2 0 2 1 2 1 2 2 1 2 2 1 1 1 0
1 1 2 1 2 1 1 2 2 0 2 0 1 2 0 0 1 2 2 1 2 1 1 1 0 2 1 1 1 0 2 1 2 2 2 1 0
0 1 1 1 0 0 2 2 0 0 2 2 1 1 0 0 2 0 1 1 2 1 0 0 0 1 2 0 0 1 1 0 1 1 1 2 2
2 1 0 1 0 0 0 1 0 1 1 1 2 0 2 1 1 0 1 2 0 2 2 2 1 0 0 0 2 0 0 0 2 0 1 2 2
0 1 1 2 0 1 1 0 2 1 1 2 2 0 1 2 0 1 0 1 2 0 0 0 2 0 2 2 1 1 2 0 1 2 1 0 0
2]



```
In [22]: centroids= best_centeroids[2]
ids=final_ids[2]

K = centroids.shape[0]

plt.figure()
plt.scatter(X[:, 0], X[:, 1], s=50,c=ids , edgecolors='k', alpha=0.6)
plt.scatter(centroids[:, 0], centroids[:, 1], marker='^', s=200, c="white", edgecolors='k')
plt.title("kmenas algorhitm")
plt.axis('equal')
plt.show()
```



sklearn

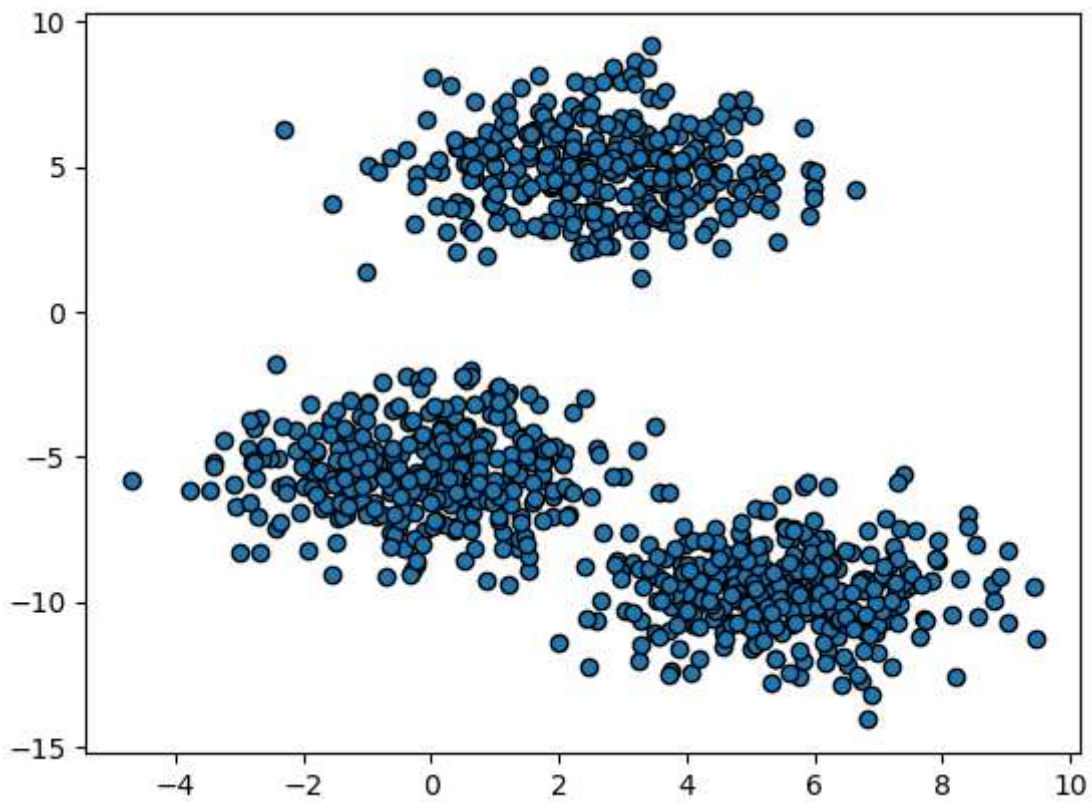
```
In [23]: from sklearn.cluster import KMeans
import pandas as pd
#the first two one(MinMaxScaler,StandardScaler) can scale and fit the data , they are
#the third one(scale) scale the data, that is a method.
from sklearn.preprocessing import MinMaxScaler
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import scale

import matplotlib.pyplot as plt
```

```
In [24]: X, y = make_blobs(n_samples=1000, centers=3, cluster_std=1.5, random_state=10)

plt.scatter(X[:, 0], X[:, 1], cmap=plt.cm.Set1, edgecolor="k")
```

```
Out[24]: <matplotlib.collections.PathCollection at 0x1c40e2b7fa0>
```



```
In [26]: X1_normalized= scale(X[:, 0] , axis= 0 , with_mean= True , with_std= True)
X2_normalized= scale(X[:, 1] , axis= 0 , with_mean= True , with_std= True)
```

```
In [27]: d = {'first_value': X1_normalized, 'second_value':X2_normalized}
df=pd.DataFrame(d)
df
```

Out[27]:

	first_value	second_value
0	0.541071	-1.070662
1	-0.173577	0.981308
2	-0.983675	-0.465678
3	-0.904900	-0.067413
4	-0.567329	1.491656
...
995	-1.446594	0.053757
996	1.100001	-0.660024
997	1.224302	1.304300
998	-0.868375	1.108034
999	-0.439211	-0.174590

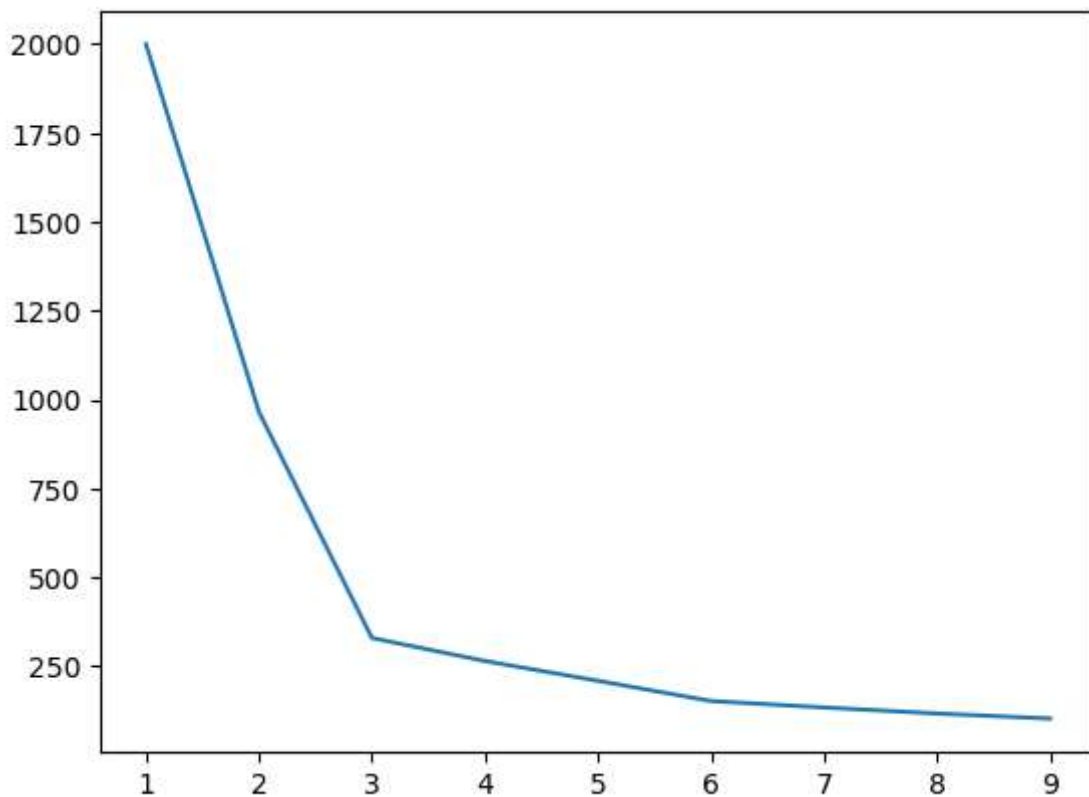
1000 rows × 2 columns

```
In [28]: wcss = []
for i in range(1,10):
    km= KMeans(n_clusters= i )
    km.fit(df[["first_value","second_value"]])
    wcss.append(km.inertia_)

wcss
```

```
Out[28]: [1999.9999999999995,
964.9482561631145,
329.0090988040914,
268.1966585182072,
203.78400010045277,
151.65696339152393,
133.62066616440416,
116.5614373429577,
102.31137820719674]
```

```
In [18]: plt.plot(range(1, 10),wcss)
plt.show()
```



```
In [29]: km= KMeans(n_clusters= 3 ) #constructing the model
y_predict=km.fit_predict (df[["first_value","second_value"]])

df["predict"] = y_predict
df
```

```
Out[29]:
```

	first_value	second_value	predict
0	0.541071	-1.070662	1
1	-0.173577	0.981308	0
2	-0.983675	-0.465678	2
3	-0.904900	-0.067413	2
4	-0.567329	1.491656	0
...
995	-1.446594	0.053757	2
996	1.100001	-0.660024	1
997	1.224302	1.304300	0
998	-0.868375	1.108034	0
999	-0.439211	-0.174590	2

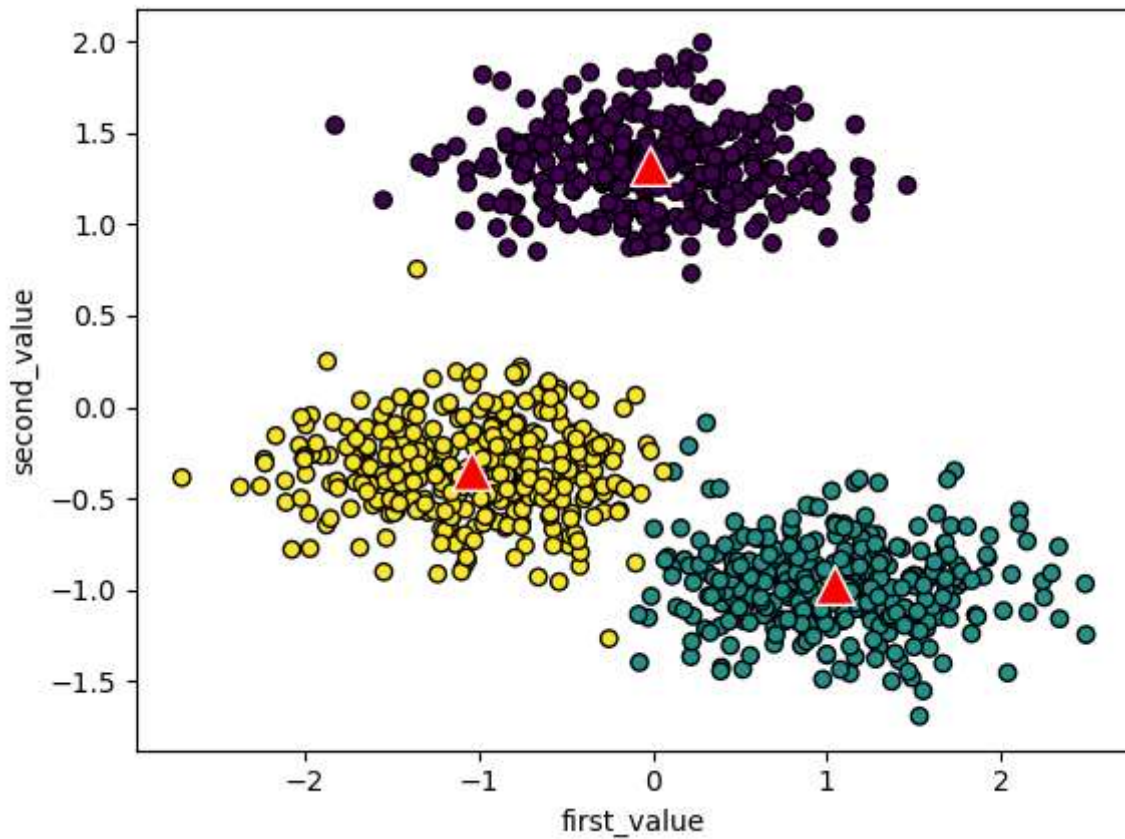
1000 rows × 3 columns

```
In [30]: km.cluster_centers_
```

```
Out[30]: array([[ -0.0162935 ,  1.32842067],
               [ 1.04554222, -0.97308104],
               [-1.04184561, -0.34361576]])
```

```
In [31]: plt.scatter(df["first_value"], df["second_value"], c=df["predict"], edgecolor="k")
plt.scatter(km.cluster_centers_[ :,0] ,km.cluster_centers_[ :,1] , marker = "^" , s=200)
plt.xlabel("first_value")
plt.ylabel("second_value")
```

```
Out[31]: Text(0, 0.5, 'second_value')
```



```
In [35]: from sklearn.datasets import load_iris
```

```
In [36]: iris=load_iris()
```

```
In [37]: kmn=KMeans(n_clusters= 3)
kmn.fit(iris.data)
labels=kmn.predict(iris.data)
centroids=kmn.cluster_centers_
```

```
In [38]: plt.scatter(iris.data[ : , 0] , iris.data[:,1] , c= labels)
plt.scatter(centroids[ : , 0] , centroids[:,1] , marker= "*" , c= "red" , s=150)
plt.show()
```

