Math Intro

Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Derivative

By chainrule:

$$\frac{\partial \sigma}{\partial x} = \frac{\partial \frac{1}{1+e^{-x}}}{\partial 1+e^{-x}} \cdot \frac{\partial 1+e^{-x}}{\partial e^{-x}} \cdot \frac{\partial e^{-x}}{\partial -x} \cdot \frac{\partial -x}{\partial x}$$

Indivudual derivatives:

$$\frac{\partial \frac{1}{1+e^{-x}}}{\partial 1+e^{-x}} = \frac{-1}{(1+e^{-x}) \cdot (1+e^{-x})}$$

$$\frac{\partial 1+e^{-x}}{\partial e^{-x}} = 1$$

$$\frac{\partial e^{-x}}{\partial -x} = e^{-x}$$

$$\frac{\partial -x}{\partial x} = -1$$

Combining derivatives:

$$\frac{\partial \sigma}{\partial x} = \frac{-1}{(1+e^{-x}) \cdot (1+e^{-x})} \cdot 1 \cdot e^{-x} \cdot (-1)$$

Simplifing:

$$\frac{\partial \sigma}{\partial x} = \frac{e^{-x}}{(1+e^{-x}) \cdot (1+e^{-x})}$$

$$\frac{\partial \sigma}{\partial x} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

Inspecting single term:

$$\frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^{x}+1} = \sigma(-x)$$

$$\frac{e^{-x}}{1+e^{-x}} = \frac{1+e^{-x}-1}{1+e^{-x}} = \frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}} = 1-\sigma(x)$$

Substituting for $\frac{e^{-x}}{1+e^{-x}}$:

$$\frac{\partial \sigma}{\partial x} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \sigma(x) \cdot \sigma(-x) = \sigma(x)(1-\sigma(x))$$

Task

$$f(x,z,a,b) := y = (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b))^2$$

$$\frac{\partial y}{\partial x} = 8ax$$

$$\frac{\partial y}{\partial z} = \sigma'(z)$$

$$\frac{\partial y}{\partial a} = 4x^2 + 1$$

$$\frac{\partial y}{\partial b} = 2\sigma(b)\sigma'(b)$$

$$\nabla y = \begin{bmatrix} 8ax \\ \sigma(z) \\ 4x^2+1 \\ 2\sigma(b)\sigma'(b) \end{bmatrix}$$