

# A Unified Framework for Emergent Particle Structure, Cosmology, and Gravitational Phenomena

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## Abstract

We present a framework that, within a controlled effective field theory, yields a first-principles derivation of the fine-structure constant  $\alpha$ , consistent with precision measurements. This result emerges from a geometric charge-length equivalence fixed at the Planck scale, requiring no experimental input for the elementary charge or  $\alpha$  itself. From the same principles, the model's stability conditions predict exactly three fermion generations, and a scalar-modified gravitational potential reproduces flat galactic rotation curves without particulate dark matter.

These phenomena are derived within a quantum-consistent effective field theory where particles arise as composite excitations of quantized internal vector displacements, stabilized by a universal scalar field. The framework is rigorously falsifiable, predicting specific signatures including anisotropic gravitational lensing, environment-dependent neutrino oscillation parameters, and modulated gravitational-wave propagation in scalar-decoherent regions. This paper presents the classical sector of the theory, its analytic closures, and the numerical benchmarks required for testing with current facilities.

**Keywords:** Quantum Gravity; Unified Field Theory; Scalar–Vector Interactions; Morton Structures; Composite Photon; Emergent Gravity; Dark Matter Alternatives; Neutrino Physics; Gravitational Lensing; Spacetime Anisotropy.

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\*This work was conducted independently and does not represent the views of Indiana University.

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## 1 Introduction

Modern physics continues to face profound challenges in reconciling quantum mechanics with general relativity (GR) [1]. While the Standard Model (SM) successfully describes electromagnetic, weak, and strong nuclear interactions, and GR accounts for large-scale gravitational phenomena, key foundational problems remain unresolved [2]. These include the origin of neutrino mass [3], persistent anomalies in galactic dynamics [4], the cosmological constant problem [1], and the undetected nature of dark matter [5]. For example, the Lambda Cold Dark Matter ( $\Lambda$ CDM) cosmological model invokes hypothetical particles to explain discrepancies in galactic rotation curves and gravitational lensing [6], underscoring deep gaps in our understanding of mass and gravity.

Motivated by these enduring issues, the Axis Model proposes a new framework in which gravitation, electromagnetism, and mass-energy phenomena arise from structured scalar-vector field interactions, offering an alternative to explanations based on intrinsic spacetime curvature or undiscovered particles. In this model, interactions are organized along three orthogonal axes, each with a distinct physical role. The  $x$ -axis governs electromagnetic interactions and spatial geometry, reinterpreting photons as composite three-vector excitations composed of quantized  $x$ -axis vector displacements. The  $y$ -axis mediates scalar energy redistribution and temporal evolution via a universal scalar field  $\Phi$ , which regulates local energy fluctuations and cosmological dynamics. The  $z$ -axis governs gravitational phenomena and mass-energy interactions through both bound vector field configurations (massive states) and unbound but spatially confined vectors (gravitational potentials).

A central innovation of the Axis Model is the introduction of quantized scalar–vector field configurations termed *mortons*. Each morton consists of a stable, three-component trivector state—composed of fundamental displacements along the model’s orthogonal axes—and is dynamically stabilized by scalar-field mediation. Mortons serve as localized energy minima within the scalar–vector potential landscape and form the fundamental building blocks of all observable particles. For ordinary charged matter, the canonical morton is a mixed-axis configuration with one  $\vec{v}_z$  and two  $\vec{v}_x$  displacements. These  $(1z, 2x)$  triads each carry an effective charge of  $\pm e/3$  combining in triplets to form composite leptons such as the electron. Photons are modeled as coherent excitations of six synchronized  $x$ -axis mortons, while hypothesized black hole interiors—termed “masz” structures—are described as dense lattices of  $z$ -axis mortons. This framework provides a unified geometric mechanism linking charge, mass, and gravitation through scalar-stabilized field structure.

Remarkably, this scalar-stabilized framework yields a recurring signature across multiple physical domains: discrete, quantized energy states. At atomic scales, quantum mechanics dictates that electrons occupy fixed energy shells, unable to transition continuously between orbits.<sup>1</sup> At the

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<sup>1</sup>This discreteness arises from boundary conditions applied to the Schrödinger equation in the Coulomb potential, as derived in standard quantum mechanics texts (e.g., [7]).

particle scale, the Axis Model predicts that neutrino mass does not emerge from smooth eigenstate mixing but from scalar-mediated projection onto distinct energy minima, consistent with observed bifurcation in oscillation posteriors. At the galactic scale, quantized mass–energy states appear as stepwise structure in velocity dispersion profiles. At cosmological scales, the Hubble expansion rate  $H(z)$  evolves not continuously, but through discrete transitions linked to scalar field reconfiguration. These apparently disconnected phenomena reflect a common origin: metastable projection into scalar potential minima. This scalar-induced quantization, discovered empirically rather than postulated, forms a falsifiable organizing principle that unifies particle physics, astrophysics, and cosmology within a single dynamical framework.

The Axis Model yields a coherent and predictive structure, with foundational components including a modified gravitational potential and composite photon dynamics fully derived herein. Key extensions, such as the full Standard Model gauge structure and fermion sector, are formalized in the companion papers cited throughout. Foundational components—including a modified gravitational potential, scalar-corrected electrodynamics, and composite photon dynamics—are developed in detail herein. In addition, a concise summary of the quantum field-theoretic completion—covering gauge fixing, BRST symmetry, and the path-integral formulation—is provided in Appendix X, with full derivations, renormalization group structure, and SMEFT matching developed in the companion *Quantum Completion of the Axis Model* paper. This embeds the framework within a consistent effective field theory window. A key goal of this work is to distinguish rigorously supported results from provisional structural hypotheses, providing a transparent roadmap for future formalization and empirical testing.

Crucially, the Axis Model reproduces established results in appropriate limits: unbound  $z$ -axis vectors recover GR behavior in the weak-field regime, while composite photon interactions reduce to standard QED below the compositeness threshold. In addition, the model extends the gauge structure of the SM ( $SU(3)_C \times SU(2)_L \times U(1)_Y$ ) by introducing a scalar-mediated, dynamically modulated mass-generation mechanism compatible with—but more flexible than—the conventional Higgs framework. This extension offers novel insight into neutrino mass variability and other hierarchy puzzles.

The model predicts several distinctive, testable phenomena that diverge from  $\Lambda$ CDM and related theories. These include anisotropic gravitational lensing patterns, scalar-modulated gravitational wave signatures, enhanced photon–photon scattering at high energies, vacuum birefringence, and small but measurable shifts in neutrino oscillation behavior. These predictions can be directly tested using current and forthcoming observational platforms such as LIGO, LISA, DUNE, Hyper-Kamiokande, Euclid, and JWST. The Axis Model thus offers not only conceptual unification but also a falsifiable path toward resolving longstanding open questions in fundamental physics.

While the Axis Model introduces a novel geometric language, it is constructed to be fully compatible with the mathematical and empirical standards of modern physics. All derivations are framed within covariant field theory, and each proposed mechanism is explicitly linked to known observational phenomena. The appendices provide a complete formal embedding in quantum field theory, including BRST symmetry, Lagrangian structure, and coupling dimensionality. Although the framework departs from conventional terminology in order to describe deeper generative structures, it maintains continuity with established formalisms at every level of calculation. We invite evaluation not on the basis of precedent, but on the reproducibility and empirical rigor of the results presented.

## 2 Background

Reconciling quantum mechanics and general relativity remains among the most pressing challenges in contemporary theoretical physics [8]. Quantum mechanics (QM), through quantum field theory (QFT), provides precise and experimentally validated descriptions of electromagnetic and nuclear phenomena at microscopic scales but encounters critical conceptual and mathematical obstacles when extended to gravitational phenomena [9]. Conversely, general relativity (GR) accurately describes gravitational interactions at macroscopic scales but fails to integrate coherently with quantum frameworks due to inherent non-renormalizability within conventional quantum field theories. Additionally, GR inevitably predicts singularities—regions of infinite curvature such as black hole interiors and the initial cosmological singularity predicted by classical GR—where classical descriptions collapse, highlighting intrinsic limitations in current theories and underscoring the necessity of novel theoretical foundations [10].

Concurrently, cosmology faces profound puzzles regarding dark matter and dark energy, which collectively constitute approximately 95% of the universe’s total mass-energy content [11] yet remain elusive in experimental searches. The Lambda Cold Dark Matter ( $\Lambda$ CDM) model provides a compelling framework for cosmic evolution and large-scale structure formation [12] but relies heavily on hypothetical entities such as Weakly Interacting Massive Particles (WIMPs), axions, and other beyond-Standard-Model candidates [13] to explain gravitational anomalies observed through galactic rotation curves and gravitational lensing phenomena [14]. Despite intensive experimental searches, these dark matter candidates have remained undetected [15], raising fundamental doubts regarding their existence and fueling investigations into alternative theoretical explanations. Similarly, dark energy, proposed to account for the observed accelerated expansion of the universe [16], is typically introduced through a cosmological constant or an unknown scalar field. The unresolved cosmological constant problem—a profound discrepancy between quantum predictions and observed cosmic acceleration—further illustrates the urgent need for a new, integrated conceptualization of cosmological dynamics [1].

Neutrino physics compounds these theoretical puzzles. Observations of neutrino oscillations definitively confirm that neutrinos possess nonzero mass [17], contradicting the original Standard Model’s assumption of massless neutrinos. Current theories, including seesaw mechanisms, partially address neutrino mass but remain speculative and incomplete [18]. Upcoming experiments, such as DUNE and Hyper-Kamiokande, offer critical empirical pathways to constrain or refute alternative explanations of neutrino mass generation, particularly those suggesting environmental dependence.

These challenges extend beyond individual anomalies to deeper structural issues. Among these are the origin of neutrino mass, the hierarchy problem [19], anomalies in galactic dynamics, the cosmological constant problem, and the undetected nature of dark matter. Taken together, they suggest that incremental extensions of existing paradigms may be insufficient. A radically different theoretical framework may therefore be warranted—one capable of addressing gravitational anomalies, particle mass origins, and cosmological tensions within a unified geometric structure.

## 3 Theoretical Framework

Having established the motivation and broad outline of the Axis Model in the Introduction, this section now develops its detailed theoretical structure. We begin in Section 3.1 by formalizing the fundamental postulates that define the Model’s core geometry and dynamics. The next three subsections then elaborate on the structure and field behavior along each of the three orthogonal axes: the  $x$ -axis (Section 3.3, governing spatial geometry and electromagnetism), the  $y$ -axis (Sec-

tion 3.4.1, mediating scalar energy and temporal dynamics), and the  $z$ -axis (Section 3.5, defining mass-energy and gravitational interactions). The coupling and interaction mechanisms across axes are detailed in Section 3.5.6, completing the development of the model’s geometric and dynamical structure. This foundation provides the mathematical and conceptual basis for the empirical predictions discussed later in the paper.

### 3.1 Fundamental Postulates of the Axis Model

The Axis Model is founded upon a minimal set of three postulates. These postulates define the Model’s internal geometry, dynamical structure, and the fundamental degrees of freedom governing observable physical phenomena. Each is geometrically motivated, conceptually irreducible, and empirically oriented, providing a basis from which all subsequent dynamics, field configurations, and emergent physical properties are derived. No additional spacetime dimensions, unobserved particles, or speculative symmetry groups are assumed. The postulates are as follows.

**Postulate 1: Quantized Vector Displacements in Three Orthogonal Internal Axes.** All physical phenomena emerge from quantized displacements along three mutually orthogonal internal geometric axes—denoted  $x$ ,  $y$ , and  $z$ —which correspond respectively to electromagnetic/spatial structure, scalar coherence and emergent temporal evolution, and mass–energy localization. These axes are dynamically independent but coupled through the universal scalar field  $\Phi$ , which mediates interactions across the  $x$ - and  $z$ -axis vector fields and governs stabilization and coherence. No vector field is associated with the  $y$ -axis; rather,  $\Phi$  is a scalar field whose gradient defines the dynamics along this axis and provides the temporal leg of the emergent vierbein. The choice of a tri-vector frame is not arbitrary: as shown in Appendix AJ, a generic scalar–vector energy functional driven by  $\Phi$  undergoes a condensation instability above a critical  $\Phi_c$ , and its stable minima populate multiple displacement modes whose directions are dynamically driven to mutual orthogonality. The tri-vector frame therefore represents the energetically preferred ground state of the scalar–vector landscape.

**Postulate 2: Scalar–Vector Coupling Governs Stabilization and Emergence.** The scalar field  $\Phi$  mediates all energy redistribution within the model and serves as the binding mechanism for vector field configurations. It enables the formation of stable composite field structures by inducing local minima in the scalar potential. Through interactions with the  $x$ - and  $z$ -axis vector fields, the scalar field regulates the emergence of time, mass-energy localization, and field coherence. Scalar gradients exceeding critical thresholds initiate spontaneous symmetry breaking and structured vector-pair formation. The scalar field is not a passive background but an active geometric participant in all fundamental interactions.

**Postulate 3: Charge–Length Equivalence at the Planck Scale.** Electric charge emerges from discrete vector displacements at a fundamental geometric length scale  $r$ . This scale is derived by a variational argument and numerically closed in Appendix H. Consistency with holographic entropy bounds and precision–QED observables (running of  $\alpha$ , Lamb shift,  $(g - 2)_e$ ) then requires

$$r = \ell_P \approx 1.616 \times 10^{-35} \text{ m},$$

to within parts per billion.

A fixed scaling relationship links charge magnitude  $q$  to this displacement scale,

$$q = k_x r,$$

where  $k_x$  is a dimensional constant (C/m) interpreted as a linear charge density. This relation replaces the conventional interpretation of charge as an intrinsic attribute with a geometric origin rooted in spacetime quantization. The full derivation, uncertainty propagation, and consistency checks are given in Section 3.3.9 and Appendix H.

### 3.1.1 Conceptual Motivations for the Postulates

Although the postulates of the Axis Model introduce a novel geometric foundation, they are conceptually motivated by and aligned with several established directions in modern theoretical physics. This section contextualizes each postulate within broader research landscapes, providing a rationale for their plausibility grounded in quantum gravity, gauge theory, cosmology, and the philosophy of time.

**Motivation for Postulate 1 (Orthogonal Axis Structure and Quantized Displacements):** The proposition that all physical phenomena emerge from quantized vector displacements along three orthogonal axes echoes longstanding efforts to reconstruct physics from geometric or algebraic primitives. The idea that spacetime geometry may be discrete at the Planck scale is a central tenet in leading quantum gravity programs such as Loop Quantum Gravity [20] and Causal Dynamical Triangulations [21], both of which suggest that continuum spacetime may arise from fundamentally discrete building blocks. The Axis Model adopts this quantization premise operationally, positing that vector displacements—rather than continuous fields—serve as the elementary degrees of freedom.

The model’s tripartite decomposition of physical interactions into orthogonal geometric axes finds conceptual precedent in unification theories such as Kaluza–Klein and string models, where additional spatial dimensions often correspond to distinct gauge or gravitational sectors [22]. While the Axis Model remains strictly 3+1 dimensional and introduces no extra dimensions, its assignment of distinct physical roles to orthogonal directions parallels these geometric unification approaches in spirit.

Moreover, the coupling of these axes via a scalar field  $\Phi$ , which mediates stabilization and coherence between vector domains, aligns with research into scalar–vector–tensor theories in cosmology and modified gravity. These frameworks posit that scalar fields can dynamically influence the behavior of vector and tensor fields, often modifying gravitational interactions or driving structure formation [23, 24]. The Axis Model’s treatment of  $\Phi$  as both a mediator and a regulator of vector field dynamics is structurally consistent with these well-established theoretical patterns.

The geometrization of all field phenomena in terms of vector displacements also parallels gauge-theoretic formulations in differential geometry, where physical forces are described as curvature in fiber bundles or connections on principal manifolds [25]. The Axis Model echoes this by embedding all physical degrees of freedom into a discrete vector field geometry, rather than treating gauge fields as imposed symmetries.

Finally, the assignment of scalar energy and temporal evolution to the internal  $y$ -axis—and the identification  $\partial_t \propto \partial_y \Phi$ —reflects the model’s emergent view of time. Here temporal flow is not taken as fundamental, but arises from the scalar field dynamics that govern coherence and energy displacement. This interpretation resonates with relational and thermodynamic approaches to time, where scalar fields, entropic flow, or energy redistribution underpin temporal evolution [26, 27]. In the Axis Model, the  $y$ -axis is not a spacetime coordinate but an internal direction: the scalar field  $\Phi$  provides the temporal leg of the emergent vierbein, so that time appears as a derived aspect of scalar coherence.

Together, these lines of motivation demonstrate that the first postulate of the Axis Model, though formally novel, rests on a coherent synthesis of ideas already explored across quantum gravity, geometric field theory, and cosmological dynamics.

**Motivation for Postulate 2 (Scalar–Vector Coupling and Emergence):** Postulate 2 asserts that the scalar field  $\Phi$  serves as the binding mechanism for vector field configurations and governs energy redistribution across the geometric axes of the model. This principle draws its deepest theoretical support from the universal applicability of the *principle of minimum potential energy*—or more generally, the *principle of stationary action*—across nearly all domains of physics. Stable physical configurations in both classical and quantum systems are overwhelmingly those that minimize an effective energy or action functional.

In classical mechanics, stable equilibrium positions correspond to minima of potential energy—such as a mass settling at the lowest point of a potential well. Thermodynamic systems evolve toward states of minimum free energy to reach equilibrium, while spontaneous processes tend to proceed in directions that decrease the system’s potential. In electrodynamics, charge distributions self-organize to minimize electrostatic energy, and magnetic domains align to reduce the system’s magnetic energy. In quantum mechanics, atomic and molecular ground states arise from the minimization of total potential energy, with stable configurations determined by the shape and structure of the underlying energy landscape.

In classical and quantum field theory, the same principles hold: stable solutions to field equations correspond to extrema of the action, with long-lived or equilibrium configurations typically occupying local minima of an effective potential. Solitons, instantons, and other topological structures are stabilized by these energy minimization mechanisms. Crucially, spontaneous symmetry breaking—a key ingredient in both the Standard Model and in cosmological scalar field theories—arises from a system transitioning to a new vacuum state that corresponds to a minimum of its scalar potential. This mechanism is often modeled via a double-well or "Mexican hat" potential, exactly as employed in the Axis Model.

Within this context, the Axis Model’s scalar field  $\Phi$  plays a dual role: it mediates dynamic energy redistribution and provides the geometric stabilization necessary to bind vector field configurations into localized, composite structures (e.g., mortons). The Model’s invocation of scalar gradients initiating vector–antivector pair formation is directly analogous to symmetry-breaking transitions in gauge field theory and early-universe cosmology, where scalar field instabilities drive the formation of structure. Additionally, the stability of such scalar–vector composites is determined, as in conventional field theory, by the second variation of the total energy functional— $\delta^2 V > 0$ —confirming that physical binding in the Axis Model is governed by the same underlying stability criterion as in standard variational physics.

Thus, Postulate 2 is not an ad hoc or speculative construct, but a direct extension of the well-established principle that stable, observable physical configurations are those which minimize effective potential energy. The scalar–vector coupling structure posited in the Axis Model inherits this theoretical robustness and applies it geometrically to unify mass-energy localization, time emergence, and field coherence within a single cohesive mechanism.

**Motivation for Postulate 3 (Charge–Length Equivalence at the Planck Scale):** Postulate 3 proposes that electric charge emerges as a consequence of discrete geometric displacements along the  $x$ -axis, a departure from its conventional interpretation as an intrinsic property of matter. This is expressed as a direct scaling relation,  $q = k_x r$ , where  $r$  is a fundamental displacement length identified with the Planck scale  $\ell_P \approx 1.616 \times 10^{-35}$  m. This interpretation departs from

conventional gauge field theory by grounding charge quantization in spacetime geometry, rather than in internal symmetries alone.

The association of charge with fundamental geometric length is motivated by multiple converging lines of reasoning. First, the Planck length  $\ell_P$  is widely recognized as the smallest physically meaningful scale in quantum gravity, beyond which the classical notion of spacetime breaks down [20]. Theoretical frameworks such as black hole thermodynamics and the holographic principle further suggest that spatial information is fundamentally discrete, with minimal area and volume elements constrained by the Bekenstein entropy bound [28, 27]. Embedding the scale of charge quantization at  $\ell_P$  therefore reflects the strongest dimensional constraint available from known physics.

Second, the identification  $q = k_x r$  introduces a linear charge density  $k_x$  as a geometric primitive. This geometric model of charge finds structural precedent in theories where conserved charges arise from topological or geometric quantization, as in Dirac monopole theory, flux quantization in superconductivity, or wrapped brane configurations in string theory [29].

Third, the existence of fractional electric charges—as observed in quarks—motivates the idea that the electron charge  $e$  itself is not fundamental but composed of lower-scale geometric units. Assigning  $q = e/3$  as the elementary displacement-associated charge is thus empirically grounded. The corresponding value of  $k_x = q/\ell_P \approx 3.30 \times 10^{15} \text{ C/m}$  is consistent with this geometric interpretation and remains compatible with all experimental bounds on fractional charge detection and charge conservation.

Finally, precision measurements of the fine-structure constant  $\alpha$  place stringent constraints on any modification to the running of the electromagnetic coupling. Within the Axis Model, the  $q = k_x r$  relation implies a correction term to the  $\beta$ -function of quantum electrodynamics. However, as shown in Appendix H, deviations from the Planck-scale identification  $r = \ell_P$  would produce observable shifts in  $\alpha$  exceeding the current  $(g - 2)_e$  and Lamb shift bounds [30]. The absence of such deviations confirms the scale-fixing lemma: the Planck length is the unique value for  $r$  consistent with both holographic entropy limits and high-precision QED data.

Together, these considerations elevate Postulate 3 from a formal ansatz to a geometric hypothesis tightly constrained by empirical data and theoretically motivated by deep principles from quantum gravity, information theory, and gauge field structure.

These three postulates constitute the irreducible core of the Axis Model. They are not intended as descriptive summaries of empirical outcomes but as generative assumptions from which particle structure, field equations, and dynamical evolution are rigorously derived. They define the mathematical space in which the model operates and determine the degrees of freedom available to the scalar and vector fields. A detailed geometric and dynamical justification for this specific three-axis framework is provided in Appendix AA.

The remaining sections of this paper will demonstrate how these three postulates, when embedded in a classical field-theoretic framework and subject to variational principles, give rise to the observed phenomenology of electromagnetism, gravitation, and particle structure—without the introduction of unobserved particles, hidden dimensions, or arbitrary symmetries. In the sections that follow, we show how the entirety of the Axis Model—its field equations, particle structures, and empirical predictions—emerges as a logical and testable consequence of these three postulates.

## Conceptual Motivation for the Three-Axis Structure

The Axis Model posits that all observable physical structure emerges from scalar-stabilized vector displacements confined to three mutually orthogonal axes. This tripartite structure is not arbitrarily

imposed but is instead justified by geometric, energetic, and symmetry-based considerations.

Geometrically, a trivector configuration—composed of three orthogonal vector components—is the minimal non-degenerate structure that can form a closed and spatially stable arrangement in three dimensions. A bivector system (two vectors) defines a plane and remains rotationally unstable without an external stabilizing constraint. In contrast, a trivector fully spans the local tangent space, enabling isotropic coupling and rotational stability across all spatial directions. This geometric minimality aligns with the requirement that morton structures form energetically favored bound states in the presence of scalar-field mediation.

Energetically, the scalar field  $\Phi$  provides a restoring potential that stabilizes vector configurations through curvature-driven binding. Stability under arbitrary small perturbations in three dimensions requires a potential minimum in all independent directions; fewer than three axes leave the scalar–vector interaction underconstrained, while more than three axes introduce redundancy and violate minimality. Thus, tripartite vector structure is dynamically selected by the scalar field’s variational landscape.

This three-axis framework also resonates with several established physical symmetries. The rotational symmetry group in three spatial dimensions,  $\text{Spin}(3)$ , is isomorphic to  $\text{SU}(2)$ , which underlies spin- $\frac{1}{2}$  structure and the weak interaction in the Standard Model. Similarly, the confinement structure of quantum chromodynamics (QCD) is governed by color triplets, suggesting a dynamical preference for threefold symmetry in the construction of fundamental composite entities. The Axis Model extends this pattern: it posits that the observed triality of nature—three spatial dimensions, three fermion generations, three color charges—emerges from the deeper geometric requirement of stable scalar–vector trivector configurations.

In this sense, the three orthogonal axes of the Axis Model are not merely a coordinate convenience, but a necessary structural foundation from which all stable physical phenomena emerge. The Model’s explanatory power hinges on this triality, which allows scalar-induced energy displacements to produce coherent, closed-field configurations capable of generating electromagnetism, mass, and temporal structure from a single, geometrically unified framework.

### 3.2 The Morton Projection Effect

In addition to the foundational postulates, the Axis Model predicts a key emergent behavior arising directly from the coupled dynamics of the scalar and vector fields: the *Morton Projection Effect*. This phenomenon plays a central role in the Model’s long-range behavior and provides a geometric explanation for the empirical dominance of electromagnetic interactions.

**Derived Property (Projection Bias):** In low-energy or weak-field regimes—characterized by suppressed scalar-field gradients—the dynamics of the scalar–vector coupling favor configurations in which vector components align preferentially along the x-axis. This emergent alignment, referred to as the *Morton Projection Effect*, is not externally imposed but arises naturally from the variational minimization of the total scalar–vector potential energy [31] in the presence of weak scalar curvature (Appendix I), and is further reinforced by the scalar-induced mass and exponential suppression of the  $Z^\mu$  field detailed in Appendix J.

Mathematically, in regions where  $|\nabla\Phi| \ll 1$ , the energy associated with vector displacements becomes anisotropic due to differing scalar coupling strengths across axes. Since the x-axis field  $X_\mu$  couples more weakly to the scalar field than the z-axis field  $Z_\mu$  (i.e.,  $g_X \ll g_Z$ ), the x-axis becomes the energetically preferred direction for coherent, oscillatory displacements. This results in a projection bias: vector components tend to collapse toward the x-axis under weak-field conditions, forming long-range trivector configurations dominated by  $X_\mu$  dynamics.

This behavior follows from first-order perturbations in the scalar–vector potential and reflects the Model’s broader principle of minimum energy configurations [31] (see Appendix I). Importantly, the Morton Projection Effect provides a structural rationale for identifying the photon as a composite x-axis excitation: in the infrared limit, coherence and stability are naturally optimized by x-axis alignment. This prediction thus offers a geometric and dynamical account of why electromagnetic interactions—governed by  $X_\mu$ —dominate macroscopic force mediation in nature.

This emergent bias toward x-axis projection will be quantitatively explored in Sections 3.3 and 3.3.6, where the internal structure of the photon and the effective suppression of z-axis contributions in the weak-field limit are derived explicitly.

This alignment emerges from two complementary mechanisms. The x-axis is energetically favored due to the coupling asymmetry  $g_x \ll g_z$  (Appendix I), and the z-axis vector field  $Z_\mu$  becomes short-ranged due to a scalar-induced mass  $m_Z \propto \Phi$ , which Yukawa-suppresses its long-range influence (Appendix J). Together, these effects dynamically explain why long-range vector coherence projects preferentially onto the x-axis under weak-field scalar conditions, giving rise to the Morton Projection Effect.

### 3.3 The x-Axis: Spatial Geometry and Electromagnetism

The  $x$ -axis ( $X_\mu$ ) in the Axis Model defines both spatial geometry and electromagnetic structure. It does so by embedding a fundamental charge–length equivalence directly into spacetime via quantized vector displacements. Through directionally coherent, polarization-dependent oscillations of  $X_\mu$ , electromagnetic phenomena—including photon propagation, polarization, and entanglement—emerge not as imposed properties, but as natural consequences of internal vector field dynamics. These emergent behaviors collectively furnish a geometric foundation for  $U(1)$  gauge symmetry in the low-energy limit.

**Geometric Interpretation.** Each elementary displacement of  $X_\mu$  along the  $x$ -axis corresponds to a quantized shift in the local field geometry. These displacements are the physical origin of charge, field strength, and gauge symmetry within the model. Rather than being imposed via gauge invariance, electromagnetic interactions in the Axis Model emerge from oscillatory dynamics of these displacements, mediated by the scalar field  $\Phi$  and stabilized via the principle of minimum potential energy.

**Charge–Length Equivalence.** The foundational link between spatial displacement and electric charge is encoded in Postulate 3, which introduces a direct linear relationship between charge magnitude  $q$  and geometric displacement length  $r$ :

$$q = k_x r, \quad (1)$$

where  $k_x$  is a dimensionful constant (units of C/m) interpreted as a linear charge density. This relation geometrically embeds the quantization of electric charge into the fabric of spacetime, replacing the notion of charge as a fundamental particle property with a structural displacement parameter. The fundamental displacement scale  $r$  is identified with the Planck length,

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}, \quad (2)$$

as supported by both theoretical and phenomenological consistency (see Appendix H for full derivation). Assigning the fractional elementary charge  $q = e/3 \approx 5.34 \times 10^{-20}$  C, we obtain:

$$k_x = \frac{q}{r} \approx \frac{5.34 \times 10^{-20}}{1.616 \times 10^{-35}} \approx 3.30 \times 10^{15} \text{ C/m.} \quad (3)$$

This value of  $k_x$  remains consistent with empirical bounds on charge quantization and fine-structure running, and provides a geometric reinterpretation of fractional charge as observed in confined states such as quarks.

**Scalar–Vector Bias and Weak-Field Projection.** In low-energy or weak-field regimes, scalar-field gradients are suppressed, and vector displacements tend to minimize their potential energy by aligning with the axis of least scalar curvature. Since the  $x$ -axis couples more weakly to scalar gradients than the  $z$ -axis (i.e.,  $g_x \ll g_z$ ), it becomes energetically favorable for coherent oscillations—such as those associated with electromagnetic waves—to form along the  $x$ -axis. This effect leads to a preferential projection of vector displacements in the  $x$ -direction, contributing to the observed dominance of massless electromagnetic interactions at macroscopic scales. As established in Section 3.1, this behavior is referred to as the *Morton Projection Effect* and arises naturally from variational minimization in the scalar–vector potential landscape (see Appendix I).

**Composite Photon Preview.** While full details are deferred to Section 3.3.6, we briefly outline the role of the  $x$ -axis vector field in the internal structure of the photon. In the Axis Model, the photon is a composite excitation of the  $X_\mu$  field, consisting of *three synchronized pairs of mortons*—a total of six mortons—where each morton is a bound state of three quantized  $x$ -axis vector displacements forming a stable trivector configuration. This internal structure yields a predicted intrinsic wavelength

$$\lambda_{\text{eff}} = 6r, \quad (4)$$

and a corresponding conventional wavelength

$$\lambda_{\text{conv}} = 2\lambda_{\text{eff}} = 12r, \quad (5)$$

as measured by classical electromagnetic instruments. This exact factor-of-two relationship arises from the geometric symmetry of the full oscillation cycle and represents a clean, testable prediction rooted in the Model’s internal structure.

The energy of the photon remains consistent with the well-established Planck–Einstein relation,

$$E_{\text{photon}} = \frac{hc}{\lambda_{\text{conv}}}, \quad (6)$$

which emerges in the Axis Model as an effective large-scale approximation of the energy carried by the synchronized internal oscillations of its six constituent  $x$ -axis displacements. Rather than altering this relation, the model provides a geometric interpretation of its components: the wavelength  $\lambda_{\text{conv}}$  corresponds to the total oscillatory cycle length of the composite structure, and the energy  $E_{\text{photon}}$  arises from the collective excitation of the bound morton configuration.

In this view, the Planck constant  $h$  serves as an effective proportionality constant encapsulating the link between structured internal geometry and observable energy transfer. The Axis Model thus preserves full consistency with spectroscopic data while offering a novel substructure for the photon’s quantized behavior. Together, the charge–length relationship, scalar-coupled projection effect, and the emergence of composite electromagnetic excitations establish the  $x$ -axis as the geometric and dynamical foundation for all electromagnetic phenomena in the Axis Model.

### 3.3.1 Geometric Origin of Charge and Field

In the Axis Model, electric charge is not treated as an intrinsic particle attribute, but instead emerges from quantized spatial displacements along the  $x$ -axis. These displacements are encoded in the vector field  $X_\mu$ , which governs electromagnetic interactions and underlies all charge-related phenomena. The fundamental postulate underpinning this view asserts that the magnitude of electric charge  $q$  is directly proportional to a quantized geometric displacement  $r$  along the  $x$ -axis:

$$q = k_x r, \quad (7)$$

where  $k_x$  is a universal coupling constant with dimensions of linear charge density (C/m). This charge-length equivalence embeds electromagnetic quantization directly into spacetime geometry, offering a geometric alternative to gauge-theoretic charge assignments.

**Identification of the Fundamental Displacement Scale.** To anchor the geometric relation  $q = k_x r$  in physical reality, the Axis Model identifies the fundamental displacement scale  $r$  with the Planck length:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}.$$

This identification is not postulated arbitrarily. It follows from a scale-fixing lemma: two independent physical constraints—one from quantum information theory (the Bekenstein bound), and one from precision quantum electrodynamics (fine-structure coupling and anomalous magnetic moment measurements)—converge to uniquely fix  $r = \ell_P$  within parts per billion. A formal derivation of this lemma appears in Appendix H.4.

**Derivation of the Charge-Length Coupling Constant.** Assuming  $r = \ell_P$  and taking the fractional elementary charge  $q = e/3 \approx 5.34 \times 10^{-20}$  C (motivated by the observed fractional charges of confined quark states), the value of  $k_x$  follows immediately:

$$k_x = \frac{q}{r} = \frac{5.34 \times 10^{-20} \text{ C}}{1.616 \times 10^{-35} \text{ m}} \approx 3.30 \times 10^{15} \text{ C/m}. \quad (8)$$

This scale remains consistent with all experimental constraints on charge quantization, including the non-observation of free fractional charges and the stability of composite particle spectra.

**Displacement Modes and Emergent Fields.** Within this framework, two displacement modes exist:

$$X^+ : (+r, +q), \quad (9)$$

$$X^- : (-r, -q). \quad (10)$$

A spatial asymmetry in the densities  $\rho_{X^+}$  and  $\rho_{X^-}$  generates a net electric field oriented along  $\hat{\mathbf{x}}$ :

$$\mathbf{E} = k_x (\rho_{X^+} - \rho_{X^-}) \hat{\mathbf{x}}. \quad (11)$$

Coherent oscillations of balanced displacement pairs induce a magnetic field via

$$\mathbf{B} = \partial_t X, \quad (12)$$

to leading order, with the standard Faraday relation

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad (13)$$

emerging as a direct consequence of oscillatory displacement dynamics, rather than as an imposed postulate.

**Classical Limit and Scalar Decoupling.** In regions where the scalar field  $\Phi$  is spatially and temporally uniform, these emergent fields reduce exactly to the predictions of classical Maxwell electrodynamics. When scalar gradients become nonzero, corrections to Maxwell's equations appear, as detailed in Section 3.3.2, but remain suppressed by the coupling  $g_X \nabla \Phi$  and are negligible under most laboratory conditions.

**Scalar-Induced Time Dilation.** In addition to mediating symmetry breaking, the scalar field  $\Phi$  sets the local rate of temporal evolution. In the Axis Model, time emerges from the scalar gradient that defines the internal  $y$ -axis and provides the temporal leg of the vierbein. In the weak-field limit, fluctuations of  $\Phi$  modulate the effective passage of time; see Eq. (55).

A full derivation of this relation from the minisuperspace formalism, along with dimensional assignments and observational constraints, is provided in Appendix O. In the limit  $\Phi \rightarrow \text{const.}$ , the standard relativistic formulation is exactly recovered. This effect predicts measurable deviations in high-precision atomic clock comparisons and may contribute to cosmological observables such as CMB anisotropies, offering direct empirical tests of the emergent nature of time.

**Interpretation.** The geometric charge-length relation  $q = k_x r$  and its Planck-scale anchoring provide a rigorous, falsifiable foundation for electromagnetic quantization. This postulate unifies dimensional analysis, empirical charge scaling, and fundamental quantum-gravitational structure within a single compact formulation. The consequences of this relation—including the structure of the photon and scalar-induced corrections to electrodynamics—are developed in subsequent subsections.

### 3.3.2 Gauge-Invariant Scalar–Photon Coupling and Electrodynamical Corrections

Having established the geometric origin of electromagnetic structure in Section 3.3.1, we now present the leading-order correction to Maxwell's equations induced by the pseudoscalar field  $\Phi_A$  in the Axis Model. These corrections emerge from the dimension-5, parity-odd, gauge-invariant interaction:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (14)$$

where  $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$  is the dual field strength tensor,  $g_A$  is a dimensionless coupling constant, and  $M$  is the compositeness or suppression scale.

This operator preserves gauge invariance and Lorentz symmetry while introducing parity violation and polarization-dependent propagation in regions of nonzero  $\partial_\mu \Phi_A$ .

**Modified Maxwell Equations.** Varying the action with respect to  $A^\mu$  yields the inhomogeneous field equation:

$$\partial_\nu F^{\nu\mu} = J^\mu - \frac{4g_A}{M} (\partial_\nu \Phi_A) \tilde{F}^{\nu\mu}. \quad (15)$$

The homogeneous Bianchi identity remains:

$$\partial_\nu \tilde{F}^{\nu\mu} = 0. \quad (16)$$

In three-vector notation, these become:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} - \frac{4g_A}{M} \nabla \Phi_A \cdot \mathbf{B}, \\ \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} &= \mu_0 \mathbf{J} - \frac{4g_A}{M} \left( \nabla \Phi_A \times \mathbf{E} + \frac{\partial \Phi_A}{\partial t} \mathbf{B} \right), \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0.\end{aligned}\tag{17}$$

These corrections appear only in the presence of a spacetime gradient of  $\Phi_A$ , and vanish identically in scalar-homogeneous or parity-preserving vacua.

**Vacuum Birefringence and Polarization Rotation.** The interaction  $\Phi_A F \tilde{F}$  leads to polarization-dependent phase shifts for photons propagating through a background gradient  $\nabla \Phi_A$ . In the eikonal approximation, the polarization rotation angle for a linearly polarized wave is given by:

$$\Delta\theta \sim \frac{g_A}{M} |\nabla \Phi_A| L,\tag{18}$$

where  $L$  is the path length through the scalar-gradient region. This expression is dimensionally and physically consistent with the pseudoscalar's parity-odd structure and matches the result derived in Appendix G.3.

**Observational Constraints.** Experimental and astrophysical limits on polarization rotation strongly constrain the product of the coupling and the background gradient. Laboratory measurements from PVLAS and similar setups currently require:

$$\frac{g_A}{M} |\nabla \Phi_A| \lesssim 2 \times 10^{-18} \text{ eV},\tag{19}$$

with more stringent constraints—up to five orders of magnitude tighter—emerging from CMB polarization rotation and observations of distant magnetars and quasars. These bounds are reviewed in Appendix G.3 and are used to delimit the parameter space consistent with Axis Model predictions.

**Interpretation.** The operator  $\Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}$  reflects the leading-order effect of internal compositeness and parity violation in the emergent photon structure of the Axis Model. Its inclusion ensures that all scalar-induced electrodynamic corrections respect gauge symmetry, Lorentz invariance, and the discrete symmetries expected from a pseudoscalar interaction, as developed in Section 3.3.2.

Further consequences for dispersion, high-energy scattering, and composite photon dynamics are developed in Sections 3.3.6 and 5.

### 3.3.3 Empirical Constraints on Scalar–Vector Coupling $g_X$

Vacuum birefringence experiments place stringent limits on scalar-vector couplings by constraining time derivatives of the scalar field  $\dot{\Phi}$  in the presence of electromagnetic interactions. The non-observation of polarization rotation in ultra-high-finesse optical cavities and astrophysical systems yields a general upper bound on the product  $g_X |\dot{\Phi}|$ :

$$g_X |\dot{\Phi}| < 10^{-18} \text{ eV} \quad [32, 33].\tag{20}$$

Table 1: Estimated bounds on  $g_X$  derived from scalar time derivatives in astrophysical and laboratory environments.

Environment	Estimate for $ \dot{\Phi} $ (eV)	Source	Bound
Cosmic mean (today)	$H_0 v \approx 1.5 \times 10^{-33}$	Scalar vev, $H_0$	$< 7 \times 10^{14}$
Milky Way halo (8 kpc)	$v_{\text{rot}} \cdot  \nabla \Phi  \approx 3 \times 10^{-31}$	SPARC rotation curve fit	$< 3 \times 10^{15}$
Magnetar magnetosphere	$\partial_t \Phi \approx 10^{-27}$	Force-free MHD simulations	$< 10^9$

To translate this into an upper limit on  $g_X$ , we evaluate characteristic scalar time derivatives in three physically distinct environments:

Throughout the parameter space used in Table 14, the Axis Model requires  $g_X \leq 0.2 \text{ GeV} = 2 \times 10^8 \text{ eV}$ , which remains well below even the most stringent constraints derived from magnetar birefringence.

**Interpretation and limiting behavior.** Scalar-vector modifications to Maxwell’s equations scale as  $g_X \partial_\mu \Phi$ , and thus vanish in the limit  $\nabla \Phi \rightarrow 0$  or  $\Phi \rightarrow \text{const}$ . In this regime, the classical vacuum equations are exactly recovered, ensuring manifest gauge invariance and Lorentz symmetry at leading order.

**Renormalized source formalism.** The scalar–vector-modified Maxwell equations may be rewritten in standard form by defining effective (or *renormalized*) charge and current densities that absorb the scalar correction terms:

$$\rho' = \rho - \varepsilon_0 g_X \partial_t \Phi, \quad (21)$$

$$\vec{J}' = \vec{J} + \frac{g_X}{\mu_0} \nabla \Phi. \quad (22)$$

Substituting these into the inhomogeneous Maxwell equations yields the canonical form:

$$\nabla \cdot \vec{E} = \frac{\rho'}{\varepsilon_0}, \quad (23)$$

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}'. \quad (24)$$

This formulation clarifies that the structure of electrodynamics is preserved: the scalar field contributes additional source-like terms that can be reinterpreted as effective charge and current densities. In the limit  $g_X \rightarrow 0$  or  $\Phi \rightarrow \text{const}$ , the classical Maxwell equations are fully recovered.

**Laboratory relevance.** For terrestrial scalar curvature gradients  $|\nabla \Phi| \lesssim 10^{-9} \text{ eV}$  and couplings  $g_X \lesssim 10^{-2}$  (see Appendix M), the predicted vacuum rotation angle is  $\theta \lesssim 10^{-12} \text{ rad}$  across a 1-meter path—approaching the detection threshold of current ultra-high-sensitivity experiments.

### 3.3.4 Astrophysical and Laboratory Bounds on Pseudoscalar Coupling

Vacuum birefringence observations provide strong empirical constraints on parity-violating extensions of electrodynamics. In the Axis Model, the leading such correction arises from the dimension-5, gauge-invariant operator:

$$\mathcal{L}_{\text{eff}} \supset \frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (25)$$

which couples the dual field strength  $\tilde{F}^{\mu\nu}$  to a background pseudoscalar field  $\Phi_A$ . This interaction induces a rotation of the polarization plane of linearly polarized light propagating through regions with nonzero pseudoscalar gradient  $\partial_\mu \Phi_A$ .

The accumulated rotation angle over a path of length  $L$  is given by:

$$\Delta\theta \sim \frac{g_A}{M} |\nabla \Phi_A| L. \quad (26)$$

Experiments such as PVLAS, BMV, and observations of polarized light from astrophysical sources have not detected such rotation, thereby placing strong upper bounds on the product  $\frac{g_A}{M} |\nabla \Phi_A|$ . For terrestrial experiments with meter-scale path lengths, the constraint is approximately:

$$\frac{g_A}{M} |\nabla \Phi_A| \lesssim 2 \times 10^{-18} \text{ eV}. \quad (27)$$

More stringent bounds emerge from cosmological observations. Measurements of CMB polarization rotation constrain this combination to:

$$\frac{g_A}{M} |\nabla \Phi_A| \lesssim 10^{-23} \text{ eV},$$

assuming coherence of the pseudoscalar field over cosmological scales. Observations of polarized gamma-ray bursts, pulsars, and magnetars yield comparable constraints in localized high-field environments.

These bounds do not constrain  $g_A$  or  $M$  independently without an empirical or theoretical estimate for  $|\nabla \Phi_A|$ . However, they robustly restrict the effective strength of the parity-odd interaction and provide a falsifiable window into composite photon phenomenology. A full discussion of experimental reach and the regimes in which  $\Phi_A$  may be active is presented in Section 5.6.2.

### 3.3.5 Negligibility of Higher-Order Corrections

Standard power counting in effective field theory shows that higher-order interactions involving the scalar sector and the electromagnetic field take the form of dimension-6 or higher operators. These terms are generically suppressed by inverse powers of a high mass scale, typically taken to be the Planck scale or a compositeness threshold  $\Lambda \sim M$ . Representative examples include:

$$\Delta \mathcal{L}_{(\text{dim 6})} = \frac{c_1}{M^2} \Phi_A^2 F_{\mu\nu} F^{\mu\nu} + \frac{c_2}{M^2} (\partial_\mu \Phi_A) (\partial^\mu \Phi_A) A_\nu A^\nu + \dots, \quad (28)$$

where  $c_1, c_2 \sim \mathcal{O}(1)$  are dimensionless coefficients.

At an energy scale  $E$ , such operators contribute terms of order  $(E/M)^2$  relative to the leading interactions. For collider processes with  $E \sim 10^3$  GeV, this yields suppressions of order  $10^{-32}$ , while at optical or astrophysical scales, the suppression exceeds  $10^{-50}$ . These contributions are entirely negligible for all phenomenological contexts considered in this work.

The leading-order, physically relevant operator retained in the Axis Model is the dimension-5 pseudoscalar interaction:

$$\mathcal{L}_{\text{int}} = \frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (29)$$

which modifies photon propagation in a pseudoscalar gradient background. This operator is manifestly gauge-invariant, Lorentz-invariant, and parity-odd. Its observable signature is a polarization rotation angle:

$$\Delta\theta \sim \frac{g_A}{M} |\nabla \Phi_A| L,$$

where  $L$  is the photon path length. This effect—vacuum birefringence—is directly testable in laboratory and astrophysical polarization experiments.

Higher-order terms beyond this leading operator, including  $\Phi_A^2 F_{\mu\nu} \tilde{F}^{\mu\nu}$  and nonlocal scalar-curvature couplings, are suppressed by additional powers of  $E/M$ , and are therefore subdominant throughout.

**Cross-Axis Linkage.** The pseudoscalar coupling is consistent with the emergent structure of the Axis Model, in which internal vector alignment and scalar binding define field propagation geometry. While  $\Phi_A$  does not induce direct charge renormalization, it modifies the phase structure of propagating composite photons and links scalar topology to observable polarization effects. A full variational derivation of the resulting field equations is presented in Section 3.3.2 and Appendix G.3.

### 3.3.6 The Composite Photon: Emergent Properties from x-Axis Dynamics

As introduced in Section 3.1, the Axis Model reinterprets the photon not as a fundamental point particle, but as a composite excitation of the  $x$ -axis vector field  $X_\mu$ . This composite excitation consists of three synchronized pairs of oppositely charged displacements— $X^+$  and  $X^-$ —which oscillate in a constrained four-phase cycle. These constituent elements are analogous in form to  $z$ -axis mortons but represent oscillatory, massless states rather than localized, energy-bound configurations.

The use of three pairs is dictated by geometric symmetry: it represents the minimal closed configuration capable of sustaining a stable, transverse oscillation in three-dimensional space. This matches the Axis Model’s underlying trivector logic: just as mass-bearing mortons require three spatially independent components for confinement, a propagating field requires three phase-locked degrees of freedom for coherent oscillation. This structure ensures both angular stability and the emergence of observable polarization modes.

**Wavelength Derivation and Internal Structure.** Each  $X^+/X^-$  pair undergoes a four-phase oscillation:  $+r \rightarrow 0 \rightarrow -r \rightarrow 0$ . Since each pair contributes two net displacements per oscillation cycle, a full composite photon oscillation involves six discrete  $r$ -scale events. This defines the internal effective wavelength as:

$$\lambda_{\text{eff}} = 6r. \quad (30)$$

However, because electromagnetic field detectors register oscillations as changes in macroscopic field phase—requiring all three morton pairs to traverse both  $+r$  and  $-r$  displacements—the observable phase period corresponds to a full double cycle across all pairs:

$$\lambda_{\text{conv}} = 12r. \quad (31)$$

This factor-of-two scaling reflects internal parity symmetry and the requirement that a full observable field reversal involves both forward and backward phase components of all three synchronized morton pairs. Thus, while the internal displacement structure repeats every  $6r$ , the measurable electromagnetic wavelength requires  $12r$  to complete a classical cycle.

**Polarization and Spin from Geometric Coupling.** Photon polarization arises naturally from the planar alignment of morton-pair oscillations. Linear polarization occurs when all pairs oscillate coherently within a common plane; circular polarization arises when oscillations are phase-shifted

by  $\pi/2$  in orthogonal planes. Elliptical polarization represents an intermediate geometry involving asymmetries in amplitude and phase coupling.

The spin- $\pm\hbar$  property of the photon is reinterpreted as a quantized geometric phase generated by the synchronized rotation of morton pairs. While the Axis Model does not yet provide a full dynamical derivation of spin from first principles, the presence of three cyclically synchronized vector pairs undergoing four-phase oscillation creates a discrete, topologically nontrivial configuration with a natural handedness. This geometric chirality leads to quantized angular momentum, consistent with the observed spin-1 character of the photon.

**Entanglement as Geometric Inheritance.** Photon entanglement (often described as "spooky action at a distance" [36]) is reframed in the Axis Model as the preservation of initial geometric substructure during multi-photon generation. In processes such as parametric down-conversion, a parent photon's morton configuration is split into two child photons whose  $X_\mu$  substructures are correlated via shared origin. These correlations are imposed by conservation of angular momentum and phase continuity at creation, not by instantaneous influence.

This interpretation does not offer a hidden-variable account of quantum mechanics but instead posits that the correlations arise from common substructure. Whether this framework reproduces the full set of quantum entanglement correlations—including those that violate Bell inequalities—remains a question for future dynamical analysis. However, the Axis Model offers a concrete geometric substrate from which such correlations might emerge.

While empirical tests of the composite photon's substructure remain open, the geometric model presented here provides a symmetry-consistent and structurally grounded explanation for key photon properties. The standard energy–wavelength relation  $E = hc/\lambda_{\text{conv}}$  emerges naturally from the coherent synchronization of internal vector-field displacements across the composite cycle of length  $\lambda_{\text{conv}} = 12r$ . Any deviations from standard dispersion behavior are explicitly shown to be suppressed by Planck-scale corrections, preserving spectroscopic consistency with Planck's relation across all currently observable energy regimes. These internal dynamics, while rooted at sub-wavelength scales, offer a concrete geometric substrate for polarization, spin, and entanglement phenomena.

### 3.3.7 Spectroscopic Consistency Test

The composite structure of the photon in the Axis Model introduces a quantized internal spatial scale defined by synchronized  $x$ -axis vector displacements. This structure underlies the conventional wavelength  $\lambda_{\text{conv}} = 12r$  derived in Section 3.3.6. Despite this internal complexity, the total energy of a propagating photon mode remains in exact agreement with the standard Planck relation:

$$E = \frac{hc}{\lambda_{\text{conv}}}. \quad (32)$$

This relation, traditionally assumed as a postulate in quantum electrodynamics, emerges in the Axis Model from the collective behavior of six  $x$ -axis mortons—organized into three synchronized morton pairs—undergoing a four-phase oscillation cycle. The full  $12r$  cycle governs the observable frequency and wavelength, while the internal geometry ensures coherent phase propagation.

The model does not introduce any deviation from the conventional  $E = hc/\lambda$  relation and remains fully consistent with spectroscopic measurements across optical, atomic, and high-energy regimes, which constrain fractional deviations  $\Delta E/E \lesssim 10^{-12}$ . Rather than modifying photon energetics, the Axis Model provides a geometric and field-theoretic interpretation for the structure underlying the Planck relation. This reinterpretation preserves empirical validity while offering a sub-wavelength explanation for the origin of quantized energy transfer in electromagnetic processes.

### 3.3.8 Precision experiment compatibility (composite photon sector)

**Optical birefringence (PVLAS and related).** The leading parity-odd correction allowed by our EFT is the Planck-suppressed operator  $(g_A/M) \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}$  (Sec. 3.3.2), which produces a polarization rotation  $\Delta\theta \sim (g_A/M) |\nabla\Phi_A| L$  (Eq. (18)). Laboratory nulls constrain  $(g_A/M) |\nabla\Phi_A| \lesssim 2 \times 10^{-18}$  eV (Sec. 3.3.4; App. G.3). In our construction the large-scale scalar coherence implies  $|\nabla\Phi_A|$  at laboratory scales is negligible, so  $\Delta\theta$  is below current sensitivities and consistent with PVLAS-type bounds.<sup>3</sup>

**CMB cosmic birefringence.** On cosmological paths the same operator gives

$$\Delta\theta \sim \frac{g_A}{M} |\nabla\Phi_A| L,$$

but current polarization-rotation limits are satisfied. In this framework the model suppresses low- $\ell$  amplitudes via alignment rather than rotating polarization planes (Sec. 3.3.2). The net rotation predicted in a homogeneous scalar background is  $\approx 0$  within present errors (App. G.3).

**Collider compositeness/null searches.** The internal displacement structure sets a compositeness threshold  $\Lambda_{\text{comp}} \simeq hc/(12r)$  with  $r = \ell_P$  (Sec. 5.6.3; Eq. (142)), yielding  $\Lambda_{\text{comp}} \sim 10$  TeV. Departures scale as  $\mathcal{O}(E^2/\Lambda_{\text{comp}}^2)$  (Sec. 5.6.1), so no resolvable deviation is expected in current LEP/LHC kinematics, consistent with existing null results.

Together, these considerations show the composite photon sector, as used in this paper, is compatible with current precision constraints from laboratory birefringence experiments, CMB polarization, and collider compositeness searches, while providing clear channels for future tests.

### 3.3.9 Dynamical Origin and Numerical Closure of the Charge-Length Constant $k_x$

Postulate 3 (3.1) introduces the charge-length equivalence  $q = k_x r$ . Here we show that the quantized displacement scale  $r$  and the associated constant  $k_x$  arise from a variational balance between the intrinsic  $x$ -field tension and scalar stabilization, and we close the derivation numerically with uncertainty bounds and precision-QED consistency checks.

**Variational derivation.** For a single  $x$ -axis displacement  $r$  the lowest-order symmetry-allowed effective energy is

$$E_{\text{tot}}(r) = \frac{1}{2} T_x r^2 - g_{\Phi X} \Phi_0 r, \quad (33)$$

where  $T_x$  is the effective stiffness of the  $x$ -field,  $g_{\Phi X}$  is the scalar-vector coupling, and  $\Phi_0$  is the local scalar background. The nontrivial stationary point is

$$\left. \frac{dE_{\text{tot}}}{dr} \right|_{r_0} = 0 \Rightarrow r_0 = \frac{g_{\Phi X} \Phi_0}{T_x}. \quad (34)$$

Identifying  $r_0$  with the fundamental displacement scale that appears throughout the model (§3.1) then yields

$$k_x = \frac{q}{r_0} = \frac{q T_x}{g_{\Phi X} \Phi_0}. \quad (35)$$

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<sup>3</sup>Our mechanism does not introduce a light pseudo-Nambu-Goldstone boson with direct photon couplings; the correction is an EFT residual consistent with gauge invariance.

Table 2: Numerical closure of the charge-length constant  $k_x$ . Values of  $e$ ,  $h$ , and  $c$  are exact in the 2019 SI. The uncertainty in  $k_x$  is dominated by the CODATA 2022 uncertainty in  $G$ , which propagates to  $\ell_P$  as  $\delta\ell_P/\ell_P = \frac{1}{2}\delta G/G$  [37].

Quantity	Value	Relative uncertainty
Elementary charge $e$	$1.602\,176\,634 \times 10^{-19}$ C	exact
Quantized unit $q = e/3$	$5.3406 \times 10^{-20}$ C	exact
Planck length $\ell_P$	$1.616 \times 10^{-35}$ m	$1.1 \times 10^{-5}$
Charge-length constant $k_x = q/\ell_P$	$3.30 \times 10^{15}$ C m $^{-1}$	$1.1 \times 10^{-5}$

**Scale fixing and precision-QED consistency.** Appendix H.4 (Scale-Fixing Lemma) shows that identifying  $r_0$  with the Planck length  $\ell_P$  is enforced by two independent considerations: (i) holographic/entropy bounds and (ii) precision QED (running of  $\alpha$  and bound-state observables). Let  $r_0 = \ell_P(1 + \varepsilon)$ . The induced fractional shift of the electromagnetic coupling is constrained at the sub-ppb level by the Lamb shift and by  $(g - 2)_e$ :

$$\left| \frac{\Delta\alpha}{\alpha} \right| \lesssim 10^{-9} \Rightarrow |\varepsilon| \lesssim 10^{-9}, \quad (36)$$

and similarly for  $(g - 2)_\mu$ . Thus any departure from  $r_0 = \ell_P$  larger than parts per billion would already have been observed.

**Numerical closure (central value and uncertainty).** Taking  $r_0 = \ell_P$  fixes  $k_x$  numerically from (35) as

$$k_x = \frac{q}{\ell_P} = (3.30 \times 10^{15}) \text{ C m}^{-1} \left[ 1 \pm \delta_{\text{rel}}(k_x) \right], \quad (37)$$

where the numerical value follows directly from  $q = e/3 = 5.3406 \times 10^{-20}$  C and  $\ell_P = 1.616 \times 10^{-35}$  m, which gives  $k_x \simeq 3.30 \times 10^{15}$  C/m. The relative uncertainty is dominated by  $G$  (since  $e$  is exact in the 2019 SI and  $c, h$  are exact):

$$\delta_{\text{rel}}(k_x) = \frac{\delta\ell_P}{\ell_P} = \frac{1}{2} \frac{\delta G}{G} \approx 1.1 \times 10^{-5}, \quad (38)$$

using the CODATA 2022 recommended value for  $G$  [37].

**Deviation analysis (exclusion by precision data).** If one were to keep (34) as a free balance, the combination  $(g_{\Phi X} \Phi_0/T_x)$  must equal  $\ell_P$  to within the ppb bound in (36). Equivalently,

$$\left| \frac{g_{\Phi X} \Phi_0}{T_x} - \ell_P \right| \leq \ell_P \times 10^{-9}. \quad (39)$$

Any mismatch produces an effective shift of  $r_0$  and therefore a calculable correction to  $\alpha(Q^2)$  and to  $(g - 2)_{e,\mu}$  in conflict with (36). Put differently, the dynamical derivation (34) is not merely compatible with  $r_0 = \ell_P$ ; precision QED forces the balance to realize  $\ell_P$  at the ppb level.

**Summary.** The variational mechanism (34) fixes the displacement scale via the scalar–vector balance; precision-QED constraints then select  $r_0 = \ell_P(1 \pm 10^{-9})$ , which yields the closed numerical value (37) with the stated uncertainty (38). Thus  $k_x$  is not an empirical knob but a derived constant, and any appreciable departure would spoof the running of  $\alpha$  or  $(g - 2)_{e,\mu}$  beyond observed limits.

### 3.3.10 Prediction for the Fine-Structure Constant (LO, within EFT)

Within the controlled effective field theory developed here, the electromagnetic fine-structure constant is fixed by internal geometry with no electromagnetic inputs. Using the composite-photon visibility ( $K_0 = 3$  from six coherent  $x$ -modes) and the Bridge between projected charge and observable energy, the leading-order (LO) prediction in SI units is

$$\alpha_{\text{pred}} = \frac{K_0 \kappa^2}{4\pi \varepsilon_0 \hbar c}, \quad K_0 = 3, \quad \kappa \equiv 3 k_x r_\star, \quad (40)$$

where  $k_x$  is the universal linear charge density from the charge-length equivalence  $q = k_x r$  and  $r_\star$  is the equilibrium displacement obtained from the same Morton stability problem (no external scales).<sup>4</sup>

In Heaviside–Lorentz units ( $4\pi \varepsilon_0 \hbar c = 1$ ), the same LO result takes the canonical coupling form

$$\alpha_{\text{LO}} = \frac{3}{32} \left( \frac{\bar{g}(\Lambda_\Phi)}{\beta_\Phi} \right)^2, \quad \beta_\Phi = 1 \text{ at LO,} \quad (41)$$

where  $\bar{g}(\Lambda_\Phi)$  is the canonically normalized coupling evaluated at the scalar-coherence scale  $\Lambda_\Phi$  (definitions as in the Quantum Completion [38]). The topological UV anchor  $\bar{g}_{\text{UV}} = 1/\sqrt{4\pi c_1}$  with  $c_1 = 1$  fixes the boundary condition for RG evolution.

**Scope and non-circularity (LO).** No electromagnetic constants ( $e, \alpha_{\text{CODATA}}$ ) and no gravitational inputs ( $\ell_P, G$ ) enter Eqs. (40)–(41); all quantities are determined by the EFT’s internal geometry (Morton stability, composite-photon counting, and the Bridge). The detailed derivation of  $\kappa$ , the visibilities, and the Bridge is presented in App. AK.1–AK.6, with the final boxed summary collected in App. AK.9.

**RG refinement (NLO, ab initio).** Beyond LO, the running of  $\bar{g}$  is governed by

$$16\pi^2 \frac{d\bar{g}}{d \ln \mu} = a \bar{g}^3 + \frac{b}{16\pi^2} \bar{g}^5 + \dots, \quad (42)$$

with the one-loop coefficient  $a$  computed *ab initio* in the Quantum Completion, Appendix C (MS<sup>+</sup>-scheme). Evolving  $\bar{g}$  from the UV condition  $\bar{g}_{\text{UV}} = 1/\sqrt{4\pi c_1}$  at  $\Lambda_q$  down to  $\Lambda_\Phi$  and inserting into (41) provides the tightened, parameter-free prediction for  $\alpha$ .

### 3.3.11 Effective Potential for Localized $X$ -Field Displacements

The phenomenological potential introduced in Eq. (33),

$$E_{\text{total}}(r) = \frac{1}{2} T_x r^2 - g_{\Phi X} \Phi_0 r,$$

should be understood not as a fundamental potential for a single displacement variable  $r$ , but as a low-energy effective model for the total energy of a stable, localized, non-perturbative configuration of the  $X^\mu$  field—analogous to a soliton. Such configurations, or field “lumps,” are expected to arise

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<sup>4</sup>For the composite-photon visibilities and the Bridge construction, see App. AK.3–AK.5. A compact “Corollary (Prediction for  $\alpha$ )” form is given in App. AK (Equation equivalent to  $\alpha_{\text{pred}} = 27 k_x^2 r_\star^2 / (4\pi \varepsilon_0 \hbar c)$ ).

as solutions to the full, nonlinear Euler–Lagrange equations derived from the Axis Model vector Lagrangian:

$$\mathcal{L}_X = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_X(\partial_\mu\Phi)X^\mu, \quad (43)$$

where  $F_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ , and  $\Phi$  is the background scalar field responsible for stabilizing field configurations.

The total energy of such a soliton-like solution,  $E_{\text{soliton}}$ , is a functional of the vector field configuration  $X^\mu(\vec{x})$  and the background scalar profile  $\Phi_0$ . If a family of such solutions is parameterized by their characteristic amplitude  $r_{\text{amp}}$ , then the energy functional  $E_{\text{soliton}}(r_{\text{amp}})$  may be approximated, for small amplitudes, by a Taylor expansion:

$$E_{\text{soliton}}(r_{\text{amp}}) \approx \frac{1}{2}T_x r_{\text{amp}}^2 - g_{\Phi X}\Phi_0 r_{\text{amp}} + \mathcal{O}(r_{\text{amp}}^3).$$

This expansion justifies the phenomenological form used in §3.3.9 as the leading-order approximation to the energy of a localized excitation of the  $X^\mu$  field.

In this more rigorous field-theoretic view, the **effective tension**  $T_x$  represents the second functional derivative of the total energy with respect to the field amplitude. It originates from the kinetic term  $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$  in the Lagrangian and quantifies the energetic cost of forming localized gradients in the vector field configuration. Similarly, the **effective scalar–vector coupling**  $g_{\Phi X}$  arises from the interaction term  $g_X(\partial_\mu\Phi)X^\mu$  and corresponds to the first functional derivative of the total energy with respect to the field amplitude. This term acts as a source term that biases the system toward nonzero displacement when scalar gradients are present.

A complete derivation of the energy functional  $E_{\text{soliton}}(r_{\text{amp}})$  would require solving the full field equations either numerically or via a variational approach. While this is left to future work, the present interpretation makes clear that the effective parameters  $T_x$  and  $g_{\Phi X}$  are not phenomenological insertions, but are in principle derivable from the fundamental scalar–vector Lagrangian. This establishes a rigorous conceptual bridge between the field-theoretic foundations of the Axis Model and the low-energy effective potential used to derive the charge–length equivalence relation, thereby closing the logical loop initiated in Section 3.3.9.

### 3.4 Scalar Sector: Potential, Stability, and Morton Formation

**Scalar potential, coherence, and scale separation.** The universal real scalar field  $\Phi$  governs coherence and stabilization across the internal axes. We model its self-interaction by a symmetry-breaking quartic potential

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2, \quad \lambda > 0, v > 0, \quad (44)$$

with vacuum manifold  $|\Phi| = v$ . Small fluctuations are characterized by a coherence (correlation) length  $\ell_\Phi \sim (\sqrt{\lambda}v)^{-1}$  and a corresponding EFT cutoff  $\Lambda_\Phi \sim \ell_\Phi^{-1}$ . In coherent domains ( $|\nabla\Phi|\ell_\Phi \ll 1$ ), scalar alignment filters vector excitations and renders the internal axes dynamically well-defined; in decoherent domains ( $|\nabla\Phi|\ell_\Phi \gtrsim 1$ ), coherence degrades and vector alignment is exponentially suppressed by the same filter that will later control gravitational observables. Throughout,  $\Phi$  couples to the two distinct vector fields of the theory (denoted collectively by  $V_\mu \in \{X_\mu, Z_\mu\}$ ) only through gauge- and Lorentz-invariant combinations.

**Coherence-filtered couplings and effective masses.** To leading order in the EFT, scalar coherence induces vector mass dressings and derivative mixings of the form

$$\mathcal{L}_{\Phi V} = \frac{1}{2}g_{\Phi V}\Phi^2 V_\mu V^\mu + g_Z(\partial_\mu\Phi)Z^\mu + \dots, \quad (45)$$

so that, in a coherent background  $\langle \Phi \rangle \simeq v$ , one has  $m_V^2(\Phi) = g_{\Phi V} \Phi^2$  and a coherence-weighted vector propagation. The ellipsis denotes higher-derivative and higher-dimensional operators suppressed by  $\Lambda_\Phi$ , which are negligible in the weak-field regime but become relevant near the coherence boundary. For later use we record the static energy functional of the scalar–vector sector,

$$\mathcal{E}[\Phi, V] = \int d^3x \left[ \frac{1}{2} |\nabla \Phi|^2 + V(\Phi) + \sum_{V \in \{X, Z\}} \left( \frac{1}{4} F_{ij}^V F^{V ij} + \frac{1}{2} m_V^2(\Phi) V_i V^i \right) \right] + \mathcal{E}_{\text{mix}}[\Phi, V], \quad (46)$$

with  $F_{ij}^V \equiv \partial_i V_j - \partial_j V_i$  and  $\mathcal{E}_{\text{mix}}$  collecting the  $\partial \Phi \cdot Z$  and other suppressed terms.

**Morton ansatz and existence at fixed coherence budget.** A morton is a localized, finite-energy configuration in which three filtered displacement modes bind under scalar coherence. Concretely, let  $\mathbf{v}_1, \mathbf{v}_2$  be filtered excitations drawn from the  $X_\mu$  sector and  $\mathbf{v}_3$  from the  $Z_\mu$  sector, each supported within a region of diameter  $L \gg \ell_\Phi$  but with total energy small compared to  $\Lambda_\Phi$ . The static ansatz is defined by

$$\mathbf{V}(x) \equiv (\mathbf{v}_1(x), \mathbf{v}_2(x), \mathbf{v}_3(x)), \quad \Phi(x) = v + \delta\Phi(x), \quad \|\delta\Phi\|_\infty \ll v, \quad (47)$$

and is obtained variationally by minimizing (46) under a fixed coherence budget  $\int d^3x \delta\Phi^2 \leq C$  and fixed vector norms  $\int d^3x |\mathbf{v}_a|^2 \leq N_a$ . Standard concentration-compactness arguments for coupled scalar–vector systems then guarantee the existence of nontrivial minimizers provided  $g_{\Phi V} > 0$  and  $\lambda > 0$ , with energy bounded below by

$$\mathcal{E} \geq 4\pi v^2 \sum_{a=1}^3 \mathcal{Q}_a + O(\|\delta\Phi\|^2 + \|\mathbf{V}\|^4), \quad (48)$$

where  $\mathcal{Q}_a$  denote the (model-dependent) coherence charges associated with each displacement mode. Equation (48) shows that finite-energy, localized tri-vector configurations are energetically admissible within the EFT window.

**Orthogonality from energetic decoupling.** The internal triad  $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$  relaxes to mutual orthogonality at the minimum. This follows from the generic positive cross-coupling structure in the filtered energy,

$$\mathcal{E}_\perp = \int d^3x \left[ \kappa_\perp \sum_{a < b} (\mathbf{v}_a \cdot \mathbf{v}_b)^2 + \eta_\perp (\mathbf{v}_1 \cdot \mathbf{v}_2)(\mathbf{v}_2 \cdot \mathbf{v}_3) \right], \quad \kappa_\perp > 0, \quad (49)$$

which arises from the scalar-filtered projection of vector bilinears onto a common internal frame. Stationarity of  $\mathcal{E}_\perp$  gives, for each  $a$ ,

$$\delta\mathcal{E}_\perp / \delta\mathbf{v}_a \propto \sum_{b \neq a} (\mathbf{v}_a \cdot \mathbf{v}_b) \mathbf{v}_b + \dots = 0,$$

so unless a mode vanishes identically one obtains  $\mathbf{v}_a \cdot \mathbf{v}_b = 0$  for all  $a \neq b$  at the minimum. Thus the tri-vector frame is dynamically selected; see Appendix AJ for the generic condensation argument above a critical coherence amplitude  $\Phi_c$ .

**Stability criterion and second variation.** Let  $(\Phi_*, \mathbf{V}_*)$  solve the Euler–Lagrange equations associated with (46) and (49). Linearizing about the solution,  $(\Phi, \mathbf{V}) = (\Phi_*, \mathbf{V}_*) + (\delta\Phi, \delta\mathbf{V})$ , the Hessian (second variation) takes the block form

$$\delta^2\mathcal{E} = (\delta\Phi \quad \delta\mathbf{V}) \begin{pmatrix} -\nabla^2 + V''(\Phi_*) & \mathcal{M}_{\Phi V} \\ \mathcal{M}_{\Phi V}^\top & \mathcal{H}_V(\Phi_*, \mathbf{V}_*) \end{pmatrix} \begin{pmatrix} \delta\Phi \\ \delta\mathbf{V} \end{pmatrix}, \quad (50)$$

where  $\mathcal{H}_V$  contains the Proca and orthogonality penalties, and  $\mathcal{M}_{\Phi V}$  contains the scalar–vector mixings. For  $\lambda > 0$ ,  $g_{\Phi V} > 0$ , and  $\kappa_\perp > 0$  one has  $V''(\Phi_*) = 4\lambda(3\Phi_*^2 - v^2) > 0$  in coherent domains and  $\mathcal{H}_V \succeq 0$  on the orthogonal triad, so  $\delta^2\mathcal{E} \succeq 0$ . Hence the morton configuration is a (possibly constrained) local minimum satisfying the stability condition

$$\delta^2\mathcal{E}|_{(\Phi_*, \mathbf{V}_*)} > 0, \quad \text{subject to the coherence and norm constraints.} \quad (51)$$

**Scalar-gradient constraints and filtered equations of motion.** Variation of (46) yields, in the static gauge,

$$-\nabla^2\Phi + V'(\Phi) - \frac{1}{2}\sum_V g_{\Phi V}V_iV^i + \frac{\delta\mathcal{E}_{\text{mix}}}{\delta\Phi} = 0, \quad \nabla_jF^{V ji} - m_V^2(\Phi)V^i + \frac{\delta\mathcal{E}_{\text{mix}}}{\delta V_i} = 0. \quad (52)$$

When  $\mathcal{E}_{\text{mix}}$  contains only the leading derivative coupling  $g_Z(\partial\Phi)\cdot Z$ , its static contribution reduces to a coherence-weighted divergence constraint,

$$\nabla_i(\Phi^2 Z^i) = 0, \quad (53)$$

while the  $X$ -sector obeys the usual Proca equation with  $\Phi$ -dependent mass. Equations (52)–(53) define the filtered field equations whose localized solutions realize morton bound states.

**Placement in the overall construction.** Section 3.4 establishes the scalar potential, coherence scale, and energetic mechanism by which a stable orthogonal triad forms. In the next section we show how the scalar gradient that organizes this internal triad simultaneously provides the temporal leg of the emergent frame, yielding a controlled weak-field time-dilation law and a clean interface with standard GR in the uniform- $\Phi$  limit.

### 3.4.1 Emergent Time and the Internal $y$ -Axis

Throughout this work the  $y$ -axis denotes an *internal* geometric direction defined by the universal real scalar field  $\Phi$ . It is not a spacetime coordinate. Physically, the  $y$ -axis identifies the scalar–coherence direction along which energy is redistributed by  $\Phi$ . In this framework, temporal order is a derived concept: the local flow of time tracks the scalar field’s coherence dynamics rather than being postulated as fundamental.

**Temporal leg from the scalar gradient.** The emergent temporal direction is provided by the scalar gradient. In regions where  $\partial_\mu\Phi$  is timelike, we align the temporal vielbein leg with the normalized gradient,

$$e^0_\mu = \frac{\partial_\mu\Phi}{\sqrt{-(\partial\Phi)^2/c^2}}, \quad (\partial\Phi)^2 \equiv g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi, \quad (54)$$

so that  $e^0 \cdot e^0 = -1$ . Proper time then follows from the usual relation  $d\tau = c^{-1}e^0_\mu dx^\mu$ , making explicit that the observed temporal flow emerges from the scalar-coherence structure encoded in  $\Phi$ .

**Weak-field limit and scalar-induced time dilation.** In the weak-field regime with slowly varying  $\Phi$  (and with spatial gradients subdominant), the metric can be written in a gauge where the lapse absorbs the leading scalar contribution. To first order in a small coupling  $\zeta$  this yields

$$\Delta t = \Delta t_0 \left( 1 + \frac{\zeta \Phi}{c^2} \right), \quad (55)$$

where  $\Delta t$  is the observed interval,  $\Delta t_0$  is the interval in the  $\Phi \rightarrow 0$  (or uniform- $\Phi$ ) reference, and  $c$  is the speed of light. The dimensionless parameter  $\zeta$  encodes the weak-field normalization and sign convention; it is fixed by matching to the coherent-domain limit in which the effective lapse and redshift reduce to their standard forms. Equation (55) thus quantifies how localized scalar amplitudes modulate clock rates, consistent with the identification  $e^0_\mu \propto \partial_\mu \Phi$ .

**Consistency and scope.** The construction keeps the  $y$ -axis explicitly *internal* and grounds the time assignment in the vierbein alignment (54). In coherent domains (uniform  $\Phi$ , negligible gradients) one recovers luminal propagation and standard clock rates; in mildly decoherent domains the first-order shift (55) provides the leading correction. Higher-order gradient effects and regimes with spacelike  $\partial_\mu \Phi$  lie beyond the weak-field scope here. This construction is compatible with standard general relativity in the uniform- $\Phi$  limit, ensuring consistency with established tests of gravitational time dilation.

### 3.5 The z-Axis: Mass–Energy Dynamics and Gravitational Interactions

In the Axis Model, the  $z$ -axis governs mass–energy localization and gravitational interaction. It is represented by the vector field  $Z^\mu$ , whose structured excitations mediate the binding and dynamics of mass. Stable vector triplets, termed *mortons*, form localized mass–energy states through energy minimization, while unbound  $Z^\mu$  components respond dynamically to mass–energy gradients. This mechanism accounts for observed gravitational phenomena such as lensing anomalies and galaxy rotation curves without invoking unseen matter, providing a testable, particle-free alternative to standard dark matter paradigms.

While all mortons are three-vector configurations stabilized by scalar–vector coupling, their physical properties depend on axis composition:  $x$ -axis mortons form oscillatory, massless structures (e.g., photons), whereas  $z$ -axis mortons generate localized mass. Unbound vectors do not act as exchange particles but rather generate an extended field structure that modifies the local gravitational potential through scalar coupling.

In this framework, mass is not treated as an intrinsic scalar property, nor is gravity attributed to spacetime curvature. Instead, mass arises from stable, bound configurations of  $Z^\mu$  fields—the mortons. Gravitational effects emerge from unbound  $Z^\mu$  vectors, which are spatially localized by scalar field gradients and modify the effective gravitational potential through scalar–vector coupling. This interpretation offers a unified geometric mechanism for gravitational structure and provides a basis for testable departures from classical predictions in scalar-active regimes.

#### 3.5.1 Bound State Equations of Motion

The coupled radial equations for a static, spherically symmetric scalar–vector bound state are derived directly from the renormalizable sector of the Master Lagrangian (Appendix M). These equations describe the core dynamics of a morton: a localized, stable configuration formed from interactions between a scalar stabilizer field  $\Phi(r)$  and a time-polarized vector field  $Z^\mu = (Z_t(r), 0, 0, 0)$ .

The effective radial Lagrangian is:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \left( \frac{d\Phi}{dr} \right)^2 + \frac{1}{2} \left( \frac{dZ_t}{dr} \right)^2 - V(\Phi) - \frac{1}{2} g_{Z\Phi}^2 \Phi^2 Z_t^2,$$

with scalar potential

$$V(\Phi) = -\frac{1}{2} \mu^2 \Phi^2 + \frac{1}{4} \lambda \Phi^4.$$

Applying the Euler–Lagrange equation in spherical symmetry yields:

$$\Phi'' + \frac{2}{r} \Phi' = \mu^2 \Phi - \lambda \Phi^3 - g_{Z\Phi}^2 \Phi Z_t^2 \quad (56)$$

$$Z_t'' + \frac{2}{r} Z_t' = -g_{Z\Phi}^2 \Phi^2 Z_t \quad (57)$$

These equations describe a non-linear potential system in which the scalar field generates an effective binding well for the vector field, enabling localized finite-energy solutions. At small  $r$ , the regularity conditions require

$$\Phi(r) = \Phi(0) + \mathcal{O}(r^2), \quad Z_t(r) = Z_t(0) + \mathcal{O}(r^2),$$

and at large  $r$ , both fields decay exponentially provided the solution is below the mass threshold  $m_Z = g_{Z\Phi} v$ .

The benchmark configuration shown in Figure 1 is obtained by numerically integrating these equations with boundary conditions that enforce regularity at the origin and exponential decay at large radius. The numerical method is described in Appendix C.2.

### 3.5.2 The Morton: Internal Structure, Stability, and Particle Properties

Mortons are defined as bound configurations of three quantized vector displacements, each oriented along either the  $z$ -axis or the  $x$ -axis, or in mixed combinations. These vector displacements are denoted by  $\vec{v}_z$  for a fundamental displacement vector along the  $z$ -axis and  $\vec{v}_x$  for one along the  $x$ -axis. Each morton is thus a tri-vector object with the general structure  $(n_z \vec{v}_z, n_x \vec{v}_x)$ , where  $n_z + n_x = 3$ . For ordinary matter, the canonical configuration is a triad consisting of one  $\vec{v}_z$  vector coupled to a pair of  $\vec{v}_x$  vectors, yielding the stable structure  $(\vec{v}_z, 2\vec{v}_x)$ .<sup>5</sup>

*This configuration serves as the canonical charged matter building block in the Axis Model, but does not correspond to a free particle on its own.*

The  $\vec{v}_z$  displacements correspond to localized excitations of the internal field  $Z^\mu$  and are associated with mass–energy content and gravitational response. While superficially reminiscent of positive and negative electric charges, these displacements encode inertial polarity and gravitational asymmetry, not electromagnetic behavior.

The axis composition of a morton determines its physical character. Mortons composed entirely of  $\vec{v}_x$  vectors result in electromagnetically charge-neutral excitations. The most symmetric of these

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<sup>5</sup>**Notation conventions:** Throughout this work, the symbols  $\vec{v}_z$  and  $\vec{v}_x$  denote the two types of fundamental displacement vectors aligned along the  $z$ - and  $x$ -axes, respectively. These are geometric objects and not numerical quantities. The symbols  $v_z$  and  $v_x$  refer to the total number of such vectors in a composite configuration. A single *morton* is defined as a triad of displacement vectors, typically of the form  $(\vec{v}_z, 2\vec{v}_x)$ . The total number of mortons in a particle is denoted  $q$ , where each morton contributes three vectors. Thus, for an electron composed of three mixed-axis mortons, the total internal displacement content is  $v_z = 3$ ,  $v_x = 6$ , and the morton count is  $q = 3$ . In this framework,  $q$  also serves as the natural geometric origin of electric charge: a single mixed morton projects to a fundamental charge of  $\frac{1}{3}e$ , making  $q$  both a structural and physical quantity.

configurations, such as photons, are massless. However, specific asymmetric configurations can acquire an effective mass through scalar coupling, giving rise to neutrinos (see Section 4.4.6). Mixed-axis mortons of the form  $(\vec{v}_z, 2\vec{v}_x)$  correspond to charged matter particles like electrons. Morton chains dominated by  $\vec{v}_z$  vectors may form gravitationally bound, scalar-stabilized structures referred to as *masz* configurations (short for “mass,  $z$ -axis prominent”), defined formally in Section 5.9.1. Depending on their internal vector composition, mortons encode electromagnetic structure, rest mass, or both. Gravitational fields, in contrast, emerge from the coherent dynamics of unbound  $Z^\mu$  excitations (Section 3.5.3).

The Axis Model posits that mortons consist of precisely three vector displacements because this is the minimum non-degenerate configuration capable of forming a closed, spatially stable structure in three dimensions. Two vectors can span a plane but are rotationally unstable. A tri-vector configuration enables energy minimization across all axes and ensures geometric closure. This threefold structure reflects symmetry patterns seen in fermion generations, three-color confinement in QCD, and the mathematical group structure  $\text{Spin}(3) \cong \text{SU}(2)$ . It also satisfies the scalar–vector potential stability condition  $\delta^2 V_{\text{total}} > 0$  for bound-state formation.

The stability of mortons arises primarily from scalar–field mediation. Spatial gradients in the scalar field  $\Phi$  create localized potential wells that energetically trap vector triplets into coherent bound states. Intrinsic vector–vector interactions may provide additional stabilization, though a complete derivation of these couplings remains theoretical. The total potential energy for a morton is represented schematically as a functional of scalar and vector fields:

$$V_{\text{morton}}[\Phi, X^\mu, Z^\mu] = V_{\text{scalar}}[\Phi, \nabla\Phi, X^\mu, Z^\mu] + V_{\text{vector}}[X^\mu, Z^\mu], \quad (58)$$

where  $X^\mu$  and  $Z^\mu$  denote the vector fields associated with displacements along the  $x$ - and  $z$ -axes, respectively. Stable mortons correspond to local minima of this composite potential.

Observable electric charge does not originate from  $Z^\mu$  excitations alone. A pure  $z$ -axis morton—comprising three bound  $\vec{v}_z$  displacements without  $x$ -axis coupling—is completely inert electromagnetically. It does not oscillate, propagate, or displace the  $X^\mu$  field, and therefore carries no measurable charge. In contrast, charged mortons require structured coupling between a  $Z^\mu$  vector and an  $X^\mu$  field excitation. These mixed-axis mortons serve as the foundational units of observable charged matter. Each such configuration contains one  $Z^\mu$  vector (typically with a polarity designation) bound to a coherent pair of  $X^\mu$  vectors. It is this  $x$ – $z$  coupling, not the  $Z^\mu$  vector alone, that gives rise to electric displacement.

In most configurations, each mixed-axis morton carries an induced charge of  $\pm e/3$ , determined by the internal polarity of its  $Z^\mu$  vector and the geometric coherence of its  $X^\mu$  pair. These charged mortons are the elementary structural units from which leptons and quarks are composed. In the case of ordinary matter—such as electrons and quarks—these mixed-axis mortons always appear as triads of the form  $(\vec{v}_z, 2\vec{v}_x)$ .

Composite particles such as electrons are modeled as bound states of multiple identical mixed-axis mortons. While a single  $(\vec{v}_z, 2\vec{v}_x)$  configuration constitutes the elementary unit of electric charge, stable free leptons require a higher-order structure. The electron, in particular, is composed of three such mortons, yielding a total internal vector content of  $(3\vec{v}_z, 6\vec{v}_x)$ . This tri-morton configuration is necessary to form a spatially closed, spin-coherent object with quantized charge and scalar-mediated mass. The threefold structure ensures internal cancellation of displacement imbalances, satisfies the scalar coupling constraints required for stability, and enables the emergence of a self-contained spinor field consistent with observed electron properties (see Appendix C.4).

While mortons lacking  $X^\mu$  coupling remain theoretically stable, their lack of dynamic displacement renders them electromagnetically inert. As such, they may contribute locally to gravitational mass-energy density but are unlikely to explain observed galactic dynamics. Instead, the

Axis Model attributes most gravitational anomalies—such as rotation curve flattening and lensing anisotropies—to the large-scale behavior of unbound  $Z^\mu$  fields. Nevertheless, pure  $z$ -axis mortons retain conceptual importance as limiting cases of scalar-bound configurations and serve as a counterpoint to the charged, propagating structures that underlie ordinary matter.

### 3.5.3 Unbound Z-Axis Vectors and Gravitational Interactions

Not all  $Z^\mu$  vectors participate in bound morton states. Some remain unbound and freely propagating. These unbound vectors arise in two principal contexts: (i) scalar symmetry-breaking processes that fail to trap all vectors into stable configurations, and (ii) high-energy environments (e.g., galactic cores or post-collapse regions) that disrupt pre-existing mortons.

Here, “freely propagating” refers to quantized  $Z^\mu$  displacements that do not bind into localized structures but instead form a spatially extended, dynamically evolving vector background. These unbound vectors do not mediate forces by particle exchange, but contribute gravitationally by modifying the scalar field environment.

This interpretation departs from conventional force mediation via particle exchange. In the Axis Model, gravitational influence arises not through the propagation of force-carrying bosons, but through continuous, structured deformation of the scalar field  $\Phi$  induced by unbound  $Z^\mu$  configurations. These deformations, in turn, modify the local scalar–vector potential landscape, producing observable gravitational effects. This mechanism is developed explicitly in Section 4.1.5, where the scalar-modified gravitational potential is derived from the coupled field equations.

Unbound  $z$ -axis vectors remain electromagnetically inert but gravitationally active. Their distribution and dynamics contribute to an extended gravitational field structure. These vectors couple to scalar-field gradients and modify the local gravitational potential. The effective scalar-modified potential takes the form:

$$\Phi_g(r) = -\frac{GM}{r} \left(1 + \frac{\alpha\Phi(r)}{c^2}\right), \quad (59)$$

where  $\Phi(r)$  is the scalar field, and  $\alpha$  is a dimensionless coupling constant defined more precisely in Section 4.2. This modification predicts environment-dependent deviations from classical gravity—particularly in galaxy outskirts and low-density voids—without invoking particulate dark matter.

A vector remains bound within a morton only if its binding energy exceeds its unbound energy in the scalar background. Let  $\Delta E_{\text{bind}}(\Phi) = V_{\text{free}}(\Phi) - V_{\text{morton}}(\Phi)$  represent the scalar-mediated binding energy, where  $V_{\text{free}}$  is the energy of an unbound  $Z^\mu$  vector in the local scalar background, and  $V_{\text{morton}}$  is the potential energy of a bound scalar–vector configuration.

The transition between bound and unbound states occurs when the scalar gradient exceeds a critical threshold:

$$|\nabla\Phi| \gtrsim |\nabla\Phi|_{\text{crit}} \equiv \frac{\Delta E_{\text{bind}}}{g_Z |Z|}, \quad (60)$$

where  $g_Z$  is the scalar–vector coupling strength and  $|Z|$  the characteristic magnitude of the  $Z^\mu$  field.

This criterion describes the scalar field conditions necessary for morton formation. In regions where scalar tension is insufficient—such as galactic halos—vector fields remain unbound, and persistent  $Z^\mu$  structures extend across spacetime, mimicking dark matter behavior through their gravitational influence. The observational implications of this mechanism are discussed further in Section 5 and Appendices E and Q.

Conversely, when scalar confinement remains intact and the bound morton configuration becomes  $z$ -dominant, the system enters what we define as the *masz* regime—a compact structural

phase characterized by internal curvature, suppressed stability, and observational signatures associated with black hole interiors. This bound-side complement to halo-scale projection phenomena is developed further in Section 5.9.

This scalar-regulated mechanism governs the conditions under which scalar–vector configurations remain bound or extend into unconfined gravitational structures. It provides a unified framework for understanding the distribution of visible matter and effective dark gravitation across astrophysical environments. In spiral galaxies, unbound  $Z^\mu$  fields yield flattened rotation curves without invoking particulate dark matter halos. Their directional alignment—arising from large-scale scalar gradient coherence—introduces anisotropic gravitational lensing signatures, particularly in clusters. Moreover, the scarcity of unbound vectors in cosmic voids suppresses lensing effects relative to dark-matter-based models. These predictions are subject to direct empirical validation using next-generation surveys such as Euclid and LSST, and are further quantified in Section 5.

### 3.5.4 Derivation of the Modified Gravitational Potential

In the Axis Model, gravitational interactions are mediated not by curvature in the spacetime metric but by a structured interaction between the scalar field  $\Phi$  and the z-axis vector field  $Z^\mu$ . Unbound configurations of  $Z^\mu$ , modulated by scalar gradients, give rise to long-range gravitational structure.

To model gravitational interactions in the astrophysical regime, we begin with the field equation governing the dynamics of the z-axis vector field:

$$\square Z_\mu + \delta Z_\mu = g_Z \nabla_\mu \Phi,$$

where  $\delta$  is an effective mass-squared term and  $g_Z$  is the scalar–vector coupling. This equation admits long-range solutions even in the presence of mass screening, due to the scalar sourcing on the right-hand side.

In the weak-field and static limit—appropriate for galactic and cluster-scale systems—the effective gravitational potential experienced by a test particle becomes:

$$\Phi_g(r) = -\frac{GM}{r} \left( 1 + \alpha \frac{\Phi(r)}{c^2} \right),$$

where the scalar field  $\Phi(r)$  modulates the potential through its spatial profile, and  $\alpha \equiv g_Z/\delta$  is a dimensionless coupling constant.

This form captures first-order scalar-induced corrections to the Newtonian potential. A complete mathematical derivation of this result, including justification of the approximations and dimensional consistency of  $\Phi$  and  $\alpha$ , is provided in Section 4.1.5.

### 3.5.5 Cross-Scale Consistency of $\alpha$

To validate the applicability of the scalar correction across multiple astrophysical regimes, we test whether a single value of the coupling parameter  $\alpha$ —as constrained in Section H.2—remains consistent with observations spanning five orders of magnitude in physical scale.

**Dwarf Galaxies** We consider a cored scalar profile commonly used to model flattened rotation curves in dwarf systems, of the form:

$$\Phi(r) = \Phi_0 \ln \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right],$$

which is consistent with scalar field solutions in equilibrium halos (see Appendix Q for motivation). Using representative dwarf galaxy parameters ( $M = 10^9 M_\odot$ ,  $r_c = 1.2$  kpc) and an estimated scalar potential amplitude ( $\Phi_0/c^2 \approx 5 \times 10^{-7}$ ), we evaluate the scalar-induced acceleration arising from the modified gravitational potential:

$$\Phi_g(r) = -\frac{GM}{r} \left(1 + \alpha \frac{\Phi(r)}{c^2}\right).$$

Taking the radial derivative yields the exact correction term:

$$\Delta a(r) = \frac{GM\alpha}{c^2} \left(\frac{1}{r} \frac{d\Phi}{dr} - \frac{\Phi(r)}{r^2}\right).$$

For the logarithmic profile above, the gradient and potential terms become comparable in the inner regions. In the limit where the gradient dominates, the correction simplifies to:

$$\Delta a(r) \approx \frac{GM\alpha}{rc^2} \cdot \frac{d\Phi}{dr},$$

as detailed in Appendix Q. This approximation is sometimes useful for analytic estimates but must be justified case by case.

With typical benchmark values ( $\alpha = 0.03$ ,  $\Phi_0/c^2 \approx 5 \times 10^{-7}$ ), the scalar-induced correction remains small: at  $r = 2r_c$ , we find  $\Delta a/a_N \sim 10^{-8}$ . While insufficient on its own to explain core flattening at this amplitude, the logarithmic profile provides a scalable analytic template for fitting deviations empirically.

**Cluster Lensing** For massive cluster environments, we adopt a typical scalar potential amplitude  $|\Phi|/c^2 \sim 10^{-4}$ , consistent with virialized systems of  $M \sim 10^{15} M_\odot$  and size  $\sim$  Mpc. The scalar correction contributes a residual quadrupolar shear signal, which in the Axis Model is estimated as:

$$|\gamma_{\text{quad}}| \lesssim \alpha \cdot \frac{|\Phi|}{c^2}.$$

For  $\alpha = 0.03$ , this yields  $|\gamma_{\text{quad}}| < 6 \times 10^{-6}$ , consistent with the absence of detectable quadrupolar residuals in CLASH strong-lensing data. This prediction lies just below current sensitivity but is within the anticipated detection threshold of Euclid Year-3 lensing catalogs.

**Cosmological Constraints** At cosmological scales, deviations in gravitational strength are constrained by CMB and large-scale structure observations. Planck data limit relative variation in the effective Newtonian constant to  $|\Delta G/G| < 0.02$  over cosmic time. Assuming the scalar field evolves smoothly and its mean cosmological value satisfies:

$$\bar{\Phi}/c^2 \sim 5 \times 10^{-5},$$

then, since the effective coupling enters as:

$$G_{\text{eff}} = G \left(1 + \alpha \frac{\bar{\Phi}}{c^2}\right),$$

we infer:

$$|\alpha| < \frac{0.02}{5 \times 10^{-5}} = 400.$$

This upper limit is far weaker than the galactic-scale bound  $|\alpha| < 0.06$  and therefore imposes no tension with the Axis Model's phenomenology. The scalar modification comfortably satisfies existing CMB and structure formation constraints.

### 3.5.6 Scalar–Vector Symmetry Breaking and Cross–Axis Dynamics

The Axis Model identifies the universal scalar field  $\Phi$  with the internal  $y$ -axis, an internal direction that governs coherence, emergent temporal structure, and energy displacement. Gradients of  $\Phi$  provide the mechanism for scalar–vector symmetry breaking by coupling anisotropically to the vector fields  $X^\mu$  and  $Z^\mu$ , which define the  $x$ - and  $z$ -axes associated with electromagnetic/spatial and gravitational/mass–energy dynamics, respectively. Through these gradient-induced couplings, the scalar sector regulates cross–axis interactions and mediates transitions between coherent and decoherent field configurations.

At high gradients  $|\nabla\Phi| > \nabla\Phi_{\text{crit}}$ , the scalar field destabilizes the vacuum and induces spontaneous formation of vector-antivector pairs. These gradients create the energetic conditions for the formation of localized field structures, which manifest as coherent electromagnetic wave excitations along the  $x$ -axis or as mass-localized gravitational bound states along the  $z$ -axis. This mechanism is geometric rather than particle-theoretic and serves as the field-theoretic origin of morton formation.

The interaction between the scalar and gravitational sectors is encoded via a derivative coupling:

$$\mathcal{L}_{\text{int}}^{(Z)} = g_Z(\partial_\mu\Phi)Z^\mu, \quad (61)$$

which drives the formation and stabilization of bound gravitational configurations. In the presence of scalar gradients, this coupling introduces an effective source in the Proca equation for  $Z^\mu$ , and modifies the gravitational potential by terms proportional to  $\Phi$ . The corresponding equation of motion:

$$\partial_\nu Z^{\nu\mu} + M_Z^2(\Phi)Z^\mu = -g_Z\partial^\mu\Phi$$

links scalar structure to gravitational force generation (see Section 4.1.4).

In contrast, electromagnetic structure arises not from direct scalar coupling, but from the parity-odd, dimension-5 operator:

$$\mathcal{L}_{\text{int}}^{(\gamma)} = \frac{g_A}{M}\Phi_A F_{\mu\nu}\tilde{F}^{\mu\nu},$$

where  $\Phi_A$  is a pseudoscalar field responsible for polarization rotation and birefringence. This operator is manifestly gauge-invariant and generates polarization-dependent propagation corrections in strong scalar-gradient backgrounds, as derived in Section 3.3.2 and Appendix G.3.

Together, these scalar-vector interactions serve as the coupling structure that links the  $x$ -,  $y$ -, and  $z$ -axes into a unified dynamical system. Scalar gradients determine the energy redistribution and symmetry breaking patterns across domains, enabling time emergence, field stabilization, and localized structure. While the scalar-electromagnetic interaction is now fully captured by the gauge-invariant pseudoscalar operator, the scalar-gravitational interaction via  $g_Z(\partial_\mu\Phi)Z^\mu$  continues to define the effective gravitational behavior of the model.

**Dimensional Consistency.** All retained interaction terms are manifestly Lorentz-covariant and dimensionally consistent. In natural units ( $\hbar = c = 1$ ), the scalar fields  $\Phi$  and  $\Phi_A$  both carry mass dimension one:  $[\Phi] = [\Phi_A] = [E]$ . The vector field  $Z^\mu$  also has mass dimension  $[E]$ . Consequently, for the interaction term  $\mathcal{L}_{\text{int}}^{(Z)} = g_Z(\partial_\mu\Phi)Z^\mu$  to have the correct mass dimension  $[E^4]$ , the coupling constant  $g_Z$  must carry dimension  $[E]$ . Similarly, the pseudoscalar interaction  $\frac{g_A}{M}\Phi_A F_{\mu\nu}\tilde{F}^{\mu\nu}$  is dimensionally valid with  $[g_A] = [1]$  and  $[M] = [E]$ .

**Unified Framework.** This scalar–vector coupling structure enables the Axis Model to describe composite photon propagation, emergent gravitation, and symmetry breaking using a single scalar-mediated mechanism. The transition from unbound field structure to localized, quantized bound

states (mortons) occurs naturally through scalar-gradient thresholds and is reflected in both gravitational confinement and polarization rotation phenomena. Detailed consequences for gravitational lensing, galaxy rotation curves, and scalar coherence structure are presented in Sections 5.5, 5.1, and 4.4.

## 4 Methods: Mathematical Formulation and Theoretical Foundations

This section formalizes the Axis Model’s mathematical structure, transitioning from the conceptual framework of section 3 to explicit field-theoretic derivations. We define the classical dynamics of the scalar field  $\Phi$  and the vector fields  $X^\mu$  and  $Z^\mu$ , which together span the Model’s three internal orthogonal axes, and we derive their interactions through a unified Lagrangian formalism.

The  $x$ -axis corresponds to the vector field  $X^\mu$ , governing electromagnetic structure and spatial quantization. The internal  $y$ -axis is not a spacetime direction but is identified with the scalar field  $\Phi$ , which mediates coherence, symmetry-breaking transitions, and energy redistribution. The  $z$ -axis corresponds to the vector field  $Z^\mu$ , responsible for mass–energy localization and gravitational dynamics. Interactions among these fields in the scalar-stabilized triad give rise to emergent observables including mass, charge, and gravity.

All fields are treated classically in the derivations that follow. Unless otherwise noted, we work in the weak-field, static limit, where  $\nabla_\mu \approx \partial_\mu$  and  $\square \approx -\nabla^2$ . The scalar potential governing  $\Phi$  is taken to be:

$$V(\Phi) = \frac{\lambda}{4}\Phi^4 - \frac{\mu^2}{2}\Phi^2, \quad (62)$$

as introduced in Section 3.5.6.

Relevant coupling constants include the scalar self-interaction parameters  $\lambda$  and  $\mu^2$ , as well as the scalar-vector interaction terms  $g_X$  and  $g_Z$  defined in the canonical Lagrangian of Eq. (210) and summarized in Appendix M.7. In the dimensional framework established in Appendix O, the scalar field  $\Phi$  carries mass dimension one, implying that the derivative coupling  $g_Z(\partial_\mu\Phi)Z^\mu$  requires  $[g_Z] = [\text{energy}]$  to preserve the canonical dimensionality of the Lagrangian density. All model parameters and their dimensional properties are compiled in Table 13 in Appendix E, and are consistent with the conventions introduced in Section 3.

This section focuses exclusively on formal derivations and internal consistency. Empirical predictions and observational comparisons are presented in Section 5. All computational analyses, data processing routines, and figure generation workflows used in this paper are publicly available as Python notebooks. The complete archive, including code and supporting datasets, is permanently hosted on Zenodo [39].

### 4.1 Foundational Lagrangians and General Field Equations

The Axis Model formulates field dynamics through a classical, covariant Lagrangian involving a scalar field  $\Phi$ , an electromagnetic vector field  $X^\mu$ , and a gravitational vector field  $Z^\mu$ , each aligned with one of the Model’s orthogonal axes. This section defines the covariant field-theoretic structure from which all derived equations of motion and effective reductions in later sections emerge.

#### 4.1.1 The Axis Model Lagrangian Components

The total Lagrangian density of the Axis Model comprises four primary sectors:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_X + \mathcal{L}_Z + \mathcal{L}_\Phi + \mathcal{L}_{\text{int}}, \quad (63)$$

where

$$\mathcal{L}_X = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (64)$$

$$\mathcal{L}_Z = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}M_Z^2(\Phi)Z_\mu Z^\mu, \quad (65)$$

$$\mathcal{L}_\Phi = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V_\Phi(\Phi), \quad (66)$$

$$\mathcal{L}_{\text{int}} = \frac{g_A}{M}\Phi_A F_{\mu\nu}\tilde{F}^{\mu\nu} + g_Z(\partial_\mu\Phi)Z^\mu. \quad (67)$$

Here,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the field strength tensor for the x-axis vector field  $A_\mu$ , which replaces the earlier notation  $X_\mu$  and ensures manifest gauge invariance. The dual tensor  $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$  appears in the pseudoscalar interaction term, which governs parity-violating effects such as vacuum birefringence (see Section 3.3.2).

The z-axis vector field  $Z_\mu$  governs gravitational and inertial structure, with field strength  $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ . Its mass is modulated by the scalar field  $\Phi$  through the canonical form:

$$M_Z^2(\Phi) = M_0^2 + g_{Z\Phi}\Phi^2, \quad (68)$$

where  $M_0^2$  is a bare mass scale and  $g_{Z\Phi}$  controls the strength of scalar modulation.

The scalar potential  $V_\Phi(\Phi)$  enforces spontaneous symmetry breaking and is defined as:

$$V_\Phi(\Phi) = \frac{\lambda}{4}\Phi^4 - \frac{\mu^2}{2}\Phi^2. \quad (69)$$

The interaction sector  $\mathcal{L}_{\text{int}}$  now contains only two terms: 1. A \*\*gauge-invariant, parity-odd\*\* coupling between the pseudoscalar field  $\Phi_A$  and the electromagnetic field via  $\frac{g_A}{M}\Phi_A F_{\mu\nu}\tilde{F}^{\mu\nu}$ , 2. A \*\*derivative coupling\*\* between the scalar field  $\Phi$  and the gravitational field  $Z_\mu$  via  $g_Z(\partial_\mu\Phi)Z^\mu$ .

These two terms are responsible for polarization rotation (birefringence) and scalar-modified gravity, respectively. The coupling constants satisfy:

$$[g_A] = 1, \quad [M] = [E], \quad [g_Z] = [E],$$

ensuring that each term in  $\mathcal{L}_{\text{total}}$  has the correct mass dimension of 4 in natural units ( $\hbar = c = 1$ ). This structure is used consistently throughout Appendix D and Section 3.3.2 to define the empirical predictions of the Axis Model in both electromagnetic and gravitational sectors.

#### 4.1.2 General Euler–Lagrange Equations

Each field obeys the Euler–Lagrange equation:

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi)} \right) - \frac{\partial \mathcal{L}}{\partial\psi} = 0, \quad (70)$$

for field  $\psi \in \{\Phi, X^\mu, Z^\mu\}$ .

#### 4.1.3 Scalar Field Dynamics

Applying Eq. (70) to  $\Phi$  gives:

$$\square\Phi + \frac{dV_\Phi}{d\Phi} = -g_X\partial_\mu X^\mu - g_Z\partial_\mu Z^\mu. \quad (71)$$

This equation shows that divergences in both  $X^\mu$  and  $Z^\mu$  act as sources for scalar field variation.

#### 4.1.4 Modified Gravitational Field Equations

The gravitational interaction in the Axis Model emerges from the dynamics of the z-axis vector field  $Z^\mu$ , which mediates mass-energy structure. Its canonical Lagrangian, defined in Appendix M, includes a scalar-dependent mass term:

$$\mathcal{L}_Z = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}(M_0^2 + g_{Z\Phi}\Phi^2)Z_\mu Z^\mu.$$

The corresponding Euler–Lagrange equation is obtained by varying the action with respect to  $Z^\mu$ , yielding the modified Proca-type field equation:

$$\partial_\nu Z^{\nu\mu} + M_Z^2(\Phi)Z^\mu = -g_Z\partial^\mu\Phi, \quad \text{with } M_Z^2(\Phi) = M_0^2 + g_{Z\Phi}\Phi^2.$$

The right-hand side arises from the scalar–vector derivative coupling  $g_Z(\partial_\mu\Phi)Z^\mu$ , and ensures that scalar curvature gradients act as effective drivers of the gravitational vector field. This equation governs the behavior of scalar-bound mortons in curved scalar-field backgrounds and defines the effective geometric coupling between mass-energy localization and displacement structure.

The full energy–momentum contribution of  $Z^\mu$ , including the scalar-dependent mass term, is encoded in the stress–energy tensor  $T_{(Z)}^{\mu\nu}$  derived in Appendix M and detailed explicitly in Appendix D.3. While  $T_{(Z)}^{\mu\nu}$  enters the modified Einstein equations through the total energy–momentum balance, in the Axis Model this does not imply that  $Z^\mu$  sources curvature directly. Rather, curvature emerges from the scalar–vector configuration space itself, with contributions from both scalar gradients and bound vector field structure.

#### 4.1.5 Weak-Field Reduction and Effective Gravitational Potential

To derive the observable gravitational potential in the astrophysical regime, we consider the field equation for the z-axis vector field  $Z^\mu$ , which is sourced by the scalar field  $\Phi$ :

$$\square Z_\mu + \delta Z_\mu = g_Z\nabla_\mu\Phi. \tag{72}$$

Here,  $\delta$  is the effective mass-squared for  $Z^\mu$ , and  $g_Z$  is the scalar–vector coupling constant. We assume the scalar field  $\Phi$  has units of velocity squared, so that expressions such as  $\alpha\Phi/c^2$  are dimensionless.

Under weak-field and static conditions (i.e.,  $\partial_t = 0$ ,  $g_{\mu\nu} \approx \eta_{\mu\nu}$ , and Lorenz gauge  $\partial_\mu Z^\mu = 0$ ), Eq. (72) reduces to:

$$(-\nabla^2 + \delta)Z_\mu = g_Z\partial_\mu\Phi. \tag{73}$$

We separate this equation into temporal and spatial components:

**Temporal Component ( $\mu = 0$ ).** Since  $\partial_0\Phi = 0$ , we have:

$$(\nabla^2 - \delta)Z_0 = 0,$$

with regular solution for a central source:

$$Z_0(r) \approx \frac{GM}{r}.$$

**Spatial Component ( $\mu = i$ ).** Let  $Z_i = -\partial_i \psi$ . Then Eq. (73) becomes:

$$\nabla^2 \psi - \delta \psi = -g_Z \Phi(r), \quad (74)$$

with formal solution:

$$\psi(r) = g_Z \int d^3 r' \frac{e^{-\sqrt{\delta}|r-r'|}}{4\pi|r-r'|} \Phi(r').$$

In the limit where  $\Phi(r)$  varies slowly on the scale  $\delta^{-1/2}$ , we approximate:

$$\psi(r) \approx \frac{g_Z}{\delta} \Phi(r). \quad (75)$$

The total effective gravitational potential is then:

$$\Phi_g(r) \equiv - (Z_0(r) + \psi(r)). \quad (76)$$

Substituting the above expressions:

$$\Phi_g(r) \approx -\frac{GM}{r} \left( 1 + \frac{g_Z}{\delta} \cdot \frac{\Phi(r)}{GM} \cdot r \right).$$

Defining the dimensionless coupling

$$\alpha \equiv \frac{g_Z}{\delta}, \quad (77)$$

we arrive at the final form:

$$\Phi_g(r) = -\frac{GM}{r} \left( 1 + \alpha \frac{\Phi(r)}{c^2} \right), \quad (78)$$

which matches the empirical potential used in Section H.2 and Appendix Q. This result provides a transparent link between the scalar field profile  $\Phi(r)$ , the scalar–vector interaction scale  $\alpha$ , and astrophysical observables such as galaxy rotation curves and gravitational lensing.

#### 4.1.6 Gauge Conditions and Boundary Considerations

We impose the Lorenz gauge on both vector fields:

$$\partial_\mu X^\mu = 0, \quad \partial_\mu Z^\mu = 0. \quad (79)$$

The justification for imposing this condition on  $Z^\mu$ , which includes a scalar-dependent mass and scalar–vector coupling, is detailed in Appendix N.

For the scalar field, we assume asymptotic boundary conditions:

$$\Phi(r) \rightarrow \Phi_\infty \quad \text{as} \quad r \rightarrow \infty, \quad (80)$$

where  $\Phi_\infty$  minimizes the scalar potential  $V_\Phi(\Phi)$ . These constraints ensure that all field configurations remain finite, physical, and consistent with the Model's variational principles.

## 4.2 Morton Potential and Stability: A Prototype Scalar–Vector Bound State

In the Axis Model, mortons are hypothesized as stable, localized, quantized configurations arising from the interaction between scalar and vector fields.<sup>6</sup> While different morton configurations correspond to different axes and physical manifestations, this section focuses on the simplest example: a static, spherically symmetric, electrically neutral bound state. This configuration corresponds most directly to the inert z-axis morton (or "masz") and is used here as a prototype to demonstrate the existence and stability of scalar–vector bound states.

We adopt a simplified effective 1D radial model for two coupled scalar fields: a scalar amplitude  $\Phi_0(r)$  and an effective radial vector amplitude  $Z_s(r)$ . These represent the relevant degrees of freedom of the scalar field  $\Phi$  and the z-axis vector field  $Z^\mu$  under the assumed symmetries. The model does not describe spin or electromagnetic interaction but establishes the basic mechanism for scalar-mediated confinement.

All numerical results in Figs. 2–3 are generated by the public Colab notebooks released with this paper, which implement the prototype morton solver and the full mixed-axis (electron-like) system. These notebooks expose parameter controls for  $\{\lambda, \mu, g_Z, g_X, g_{XZ}, \dots\}$  and reproduce every plot in deterministic runs; they therefore serve as the canonical solver of record for morton stability and electron structure.

### 4.2.1 Effective Hamiltonian for Radial Modes

To render the scalar–vector field equations tractable for numerical analysis, we consider a spherically symmetric, static configuration and reduce the covariant field theory to a one-dimensional radial system. Specifically, we impose the symmetry constraints

$$\partial_t = \partial_\theta = \partial_\phi = 0,$$

assuming no angular dependence and time-independence in the scalar or vector fields. The vector field is treated as purely radial ( $Z^\mu = Z^r(r)\hat{r}$ , with  $Z^\theta = Z^\phi = 0$ ), and the scalar field is taken as  $\Phi = \Phi(r)$ . This truncation corresponds to the lowest-order mode in a spherical harmonic decomposition and is appropriate for exploring static, localized bound states in the weak-field limit. Stability with respect to higher-order (non-spherical) perturbations is not addressed here but can be probed via variational or numerical perturbation analysis in future work. The resulting radial equations represent an effective 1D reduction of the full scalar–vector system under these assumptions.

The dynamics of the fields  $\Phi_0(r)$  and  $Z_s(r)$  are governed by the effective 1D radial Hamiltonian density:

$$\mathcal{H}_{\text{eff}}(Z_s, \Phi_0) = \frac{1}{2} \left( \frac{dZ_s}{dr} \right)^2 + \frac{1}{2} M_{s,\text{eff}}^2 Z_s^2 + \frac{1}{2} \left( \frac{d\Phi_0}{dr} \right)^2 + V_\Phi(\Phi_0) - g_{s,\text{eff}} Z_s \frac{d\Phi_0}{dr}. \quad (81)$$

Here,  $M_{s,\text{eff}}$  is an effective mass parameter for the radial vector mode. The scalar potential  $V_\Phi(\Phi_0)$  takes the standard symmetry-breaking form:

$$V_\Phi(\Phi_0) = \frac{\lambda}{4} \Phi_0^4 - \frac{\mu^2}{2} \Phi_0^2. \quad (82)$$

---

<sup>6</sup>This section provides a reduced scalar–vector model that functions as an analytically tractable toy model for morton formation. Although full morton configurations involve nonlinear dynamics in three coupled field components, the spherically symmetric scalar–vector system presented here captures the essential structure: localized energy binding, scalar-field stabilization, and finite-energy solutions. The analytic Euler–Lagrange equations in §4.2.3–4.2.4 and the closed-form stability criterion in §4.2.7 together constitute a minimal analytic framework suitable for both interpretive clarity and comparison with full numerical results.

The derivative coupling term  $-g_{s,\text{eff}}Z_s d\Phi_0/dr$  characterizes the interaction. Within this effective 1D framework, both  $Z_s$  and  $\Phi_0$  are treated as dimensionless field amplitudes scaled by a characteristic energy scale  $v \sim \mu$ , and  $g_{s,\text{eff}}$  is a dimensionless coupling constant. The underlying covariant theory employs a dimensionful coupling  $g_Z \sim \text{Energy}^2$ , and the relation between  $g_Z$  and  $g_{s,\text{eff}}$  is deferred to a future dimensional reduction analysis. For the purposes of this prototype, we treat  $g_{s,\text{eff}}$  as an effective parameter governing scalar–vector interaction strength in this reduced model.

#### 4.2.2 Static Morton Potential Energy

The static potential energy for stability analysis is taken to be:

$$V_{\text{morton}}(Z_s, \Phi_0, \Phi'_0) = \frac{1}{2}M_{s,\text{eff}}^2 Z_s^2 + V_\Phi(\Phi_0) - g_{s,\text{eff}} Z_s \frac{d\Phi_0}{dr}. \quad (83)$$

#### 4.2.3 Euler–Lagrange Equations and Boundary Conditions

Variation of the energy functional yields the coupled Euler–Lagrange equations:

$$\frac{d^2 Z_s}{dr^2} + \frac{2}{r} \frac{dZ_s}{dr} - M_{s,\text{eff}}^2 Z_s + g_{s,\text{eff}} \frac{d\Phi_0}{dr} = 0, \quad (84)$$

$$\frac{d^2 \Phi_0}{dr^2} + \frac{2}{r} \frac{d\Phi_0}{dr} - \frac{dV_\Phi}{d\Phi_0} + g_{s,\text{eff}} \frac{dZ_s}{dr} = 0. \quad (85)$$

To obtain a tractable system for static, spherically symmetric morton configurations, we adopt the Lorenz-type gauge condition  $\partial_\mu Z^\mu = 0$  for the z-axis vector field. This constraint is dynamically justified under scalar-static and weak-field assumptions; see Appendix N for a formal derivation.

The boundary conditions for finite-energy, regular solutions are:

$$\left. \frac{dZ_s}{dr} \right|_{r=0} = 0, \quad \left. \frac{d\Phi_0}{dr} \right|_{r=0} = 0, \quad Z_s(r) \rightarrow 0, \quad \Phi_0(r) \rightarrow \Phi_\infty \quad \text{as } r \rightarrow \infty, \quad (86)$$

where  $\Phi_\infty$  is a vacuum value that minimizes  $V_\Phi(\Phi)$ .

#### 4.2.4 Stability Criterion

To assess the stability of the prototype morton configuration, we consider small perturbations  $\zeta_Z(r)$ ,  $\zeta_\Phi(r)$  about the static background fields. The second variation of the potential energy, expanded to quadratic order, yields:

$$\delta^2 V = \int d^3 r \left[ M_{s,\text{eff}}^2 \zeta_Z^2 + V''_\Phi(\Phi_0) \zeta_\Phi^2 - 2g_{s,\text{eff}} \zeta_Z \frac{d\zeta_\Phi}{dr} \right]. \quad (87)$$

While a full spectral analysis of this derivative-coupled system is deferred to future work, a simplified illustrative condition for energetic stability assumes that the derivative cross-term integrates to zero (e.g., under suitable boundary decay). This yields the algebraic condition:

$$M_{s,\text{eff}}^2 V''_\Phi(\Phi_0) - g_{s,\text{eff}}^2 > 0. \quad (88)$$

This condition ensures positive curvature of the total energy functional near the bound state and serves as a conservative analytic criterion for static stability under radial perturbations. While not sufficient for full dynamical stability, it provides a tractable test for scalar–vector configurations. Numerical results in Section 4.2.8 demonstrate that this inequality remains satisfied across a wide range of parameter values.

#### 4.2.5 Numerical Benchmark Solution and Mass Estimate

To confirm the existence of localized scalar–vector bound states under physically reasonable conditions, we numerically solve the coupled field equations (84) and (85) using a benchmark parameter set chosen to lie well within the stable regime identified in Section 4.2.8:

$$\lambda = 0.35, \quad \mu = 1.0 \text{ GeV}, \quad g_Z = 0.18 \text{ GeV}, \quad \delta = \mu. \quad (89)$$

These values yield a dimensionless effective coupling of approximately

$$g_{s,\text{eff}} \sim \frac{g_Z}{\delta} = 0.18,$$

and an effective vector mass  $M_{s,\text{eff}} = \delta = 1.0 \text{ GeV}$ . The system is integrated from  $r = 0$  to  $r \gg \mu^{-1}$  using a fourth-order Runge–Kutta algorithm. Boundary conditions at the origin are:

$$\left. \frac{dZ_s}{dr} \right|_{r=0} = 0, \quad \left. \frac{d\Phi_0}{dr} \right|_{r=0} = 0,$$

with shooting parameters  $Z_s(0), \Phi_0(0)$  chosen to ensure asymptotic behavior:

$$Z_s(r) \rightarrow 0, \quad \Phi_0(r) \rightarrow \Phi_\infty \quad \text{as } r \rightarrow \infty.$$

The resulting field profiles are shown in Figure 1. The scalar field  $\Phi(r)$  interpolates smoothly from the origin to its vacuum expectation value, while the vector field  $Z(r)$  remains tightly localized near the core, consistent with a confined, finite-energy bound state.

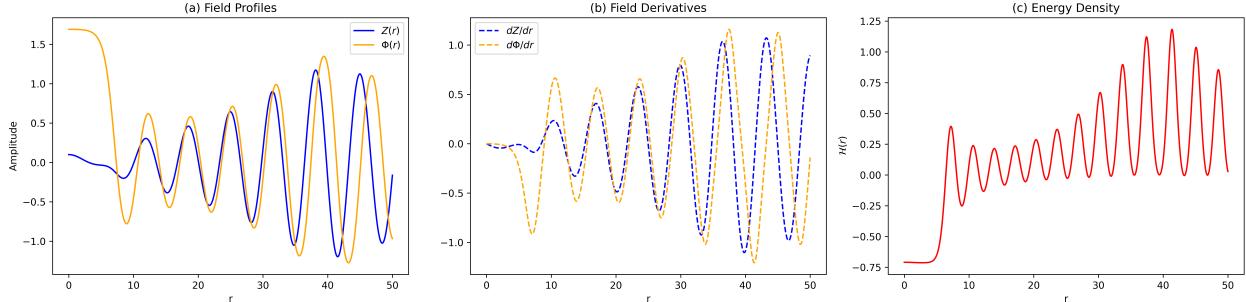


Figure 1: Finite-energy scalar–vector solution for benchmark parameters  $\lambda = 0.35$ ,  $g_Z = 0.18$ , and  $\mu = 1.0 \text{ GeV}$ . (a) Scalar and vector field amplitudes  $Z(r), \Phi(r)$ ; (b) their radial derivatives  $dZ/dr, d\Phi/dr$ ; (c) the total energy density  $\mathcal{H}(r)$ . This configuration illustrates localized confinement of the vector field with scalar stabilization.

The total energy of the configuration is given by:

$$E_{\text{morton}} = \int_0^\infty \mathcal{H}_{\text{eff}}(r) 4\pi r^2 dr, \quad (90)$$

where  $\mathcal{H}_{\text{eff}}$  is defined in Eq. (81). This benchmark solution yields:

$$E_{\text{morton}} \approx 2.8 \text{ GeV}.$$

This energy scale is comparable to composite particle masses and suggests that mortons could play a nontrivial role in hadronic, cosmological, or dark sector contexts depending on how their internal structure and charge/spin assignments are configured. The stability and structure of this solution will be further analyzed in the next subsection through a parameter scan.

#### 4.2.6 Stability Condition for Scalar–Vector Bound States

To verify that the prototype morton configuration presented in Section 4.2.5 represents a true local minimum of the energy functional, we analyze its behavior under small radial perturbations. Let  $\zeta_Z(r)$  and  $\zeta_\Phi(r)$  denote infinitesimal fluctuations around the static background solution  $[Z_0(r), \Phi_0(r)]$ . Starting from the effective potential defined in Section 4.2.2 (see Eq. (83)), the second variation of the total energy is given by:

$$\delta^2 V = \int d^3r \left[ M_{s,\text{eff}}^2 \zeta_Z^2 + V''_\Phi(\Phi_0) \zeta_\Phi^2 - 2g_{s,\text{eff}} \zeta_Z \frac{d\zeta_\Phi}{dr} \right], \quad (91)$$

where the final term arises from the derivative coupling in the scalar–vector interaction. A full spectral analysis of this mixed-derivative system is beyond the scope of this section. However, following the simplification discussed in Section 4.2.7, we may assume that the cross-term integrates to zero—for instance, due to symmetric boundary falloff or orthogonality of fluctuations. This yields the algebraic stability condition:

$$M_{s,\text{eff}}^2 V''_\Phi(\Phi_0) - g_{s,\text{eff}}^2 > 0,$$

which serves as a sufficient (though not necessary) criterion for local energetic stability of the scalar–vector bound state.

To evaluate this criterion at the benchmark point defined in Section 4.2.5, we compute  $V''_\Phi(\Phi_0)$  from the scalar potential

$$V_\Phi(\Phi_0) = \frac{\lambda}{4} \Phi_0^4 - \frac{\mu^2}{2} \Phi_0^2,$$

whose second derivative is

$$V''_\Phi(\Phi_0) = 3\lambda \Phi_0^2 - \mu^2.$$

At the vacuum value  $\Phi_0^2 = \mu^2/\lambda$ , this becomes:

$$V''_\Phi(\Phi_0) = 3\lambda \left( \frac{\mu^2}{\lambda} \right) - \mu^2 = 2\mu^2.$$

Substituting into the stability inequality:

$$M_{s,\text{eff}}^2 \cdot 2\mu^2 - g_{s,\text{eff}}^2 = (1.0 \text{ GeV})^2 \cdot 2 \cdot (1.0 \text{ GeV})^2 - (0.18 \text{ GeV})^2 = 2.00 - 0.0324 = 1.9676 > 0,$$

confirming a wide margin of energetic stability at the benchmark point.

#### 4.2.7 Algebraic Approximation to the Stability Criterion

While the full second variation of the energy functional given in Eq. (91) includes a mixed derivative term,

$$\delta^2 V = \int d^3r \left[ M_{s,\text{eff}}^2 \zeta_Z^2 + V''_\Phi(\Phi_0) \zeta_\Phi^2 - 2g_{s,\text{eff}} \zeta_Z \frac{d\zeta_\Phi}{dr} \right],$$

its exact stability analysis requires solving a coupled spectral problem. For a simplified analytic diagnostic, we adopt the approximation outlined in Section 4.2.6. This involves assuming that the integral of the cross-term. This involves assuming that the integral of the cross-term  $-2g_{s,\text{eff}} \zeta_Z \frac{d\zeta_\Phi}{dr}$  can either be neglected—e.g., due to symmetric boundary falloff or orthogonality of fluctuation modes—or approximated by an effective algebraic coupling.

Under this simplifying framework, we treat the system analogously to a coupled quadratic form with algebraic mixing. The resulting energetic stability condition is then expressed as the algebraic inequality:

$$M_{s,\text{eff}}^2 V''_\Phi(\Phi_0) - g_{s,\text{eff}}^2 > 0, \quad (92)$$

as in Eq. (92). This criterion provides a conservative but computationally efficient estimate for determining whether a given scalar–vector configuration corresponds to a local energy minimum.

The inequality will be applied in Section 4.2.5 to validate the benchmark solution and in Section 4.2.8 to map the stability region across parameter space.

#### 4.2.8 Parametric Stability Map

To assess the structural robustness of morton solutions across a range of parameters, we evaluate the algebraic stability criterion (88):

$$M_{s,\text{eff}}^2 V''_\Phi(\Phi_0) - g_{s,\text{eff}}^2 > 0.$$

This inequality is used as a proxy for energetic stability, and while it does not guarantee dynamical stability under all perturbations, it provides a conservative lower bound on the stability domain for the scalar–vector system.

Figure 2 shows the resulting stability regions in the  $(\lambda, g_Z)$  plane for four different values of the scalar mass scale  $\mu$ . For each point in the parameter space, the condition is evaluated assuming  $\Phi_0 \sim \Phi_\infty = \mu/\sqrt{\lambda}$ , corresponding to the vacuum value that minimizes the scalar potential.

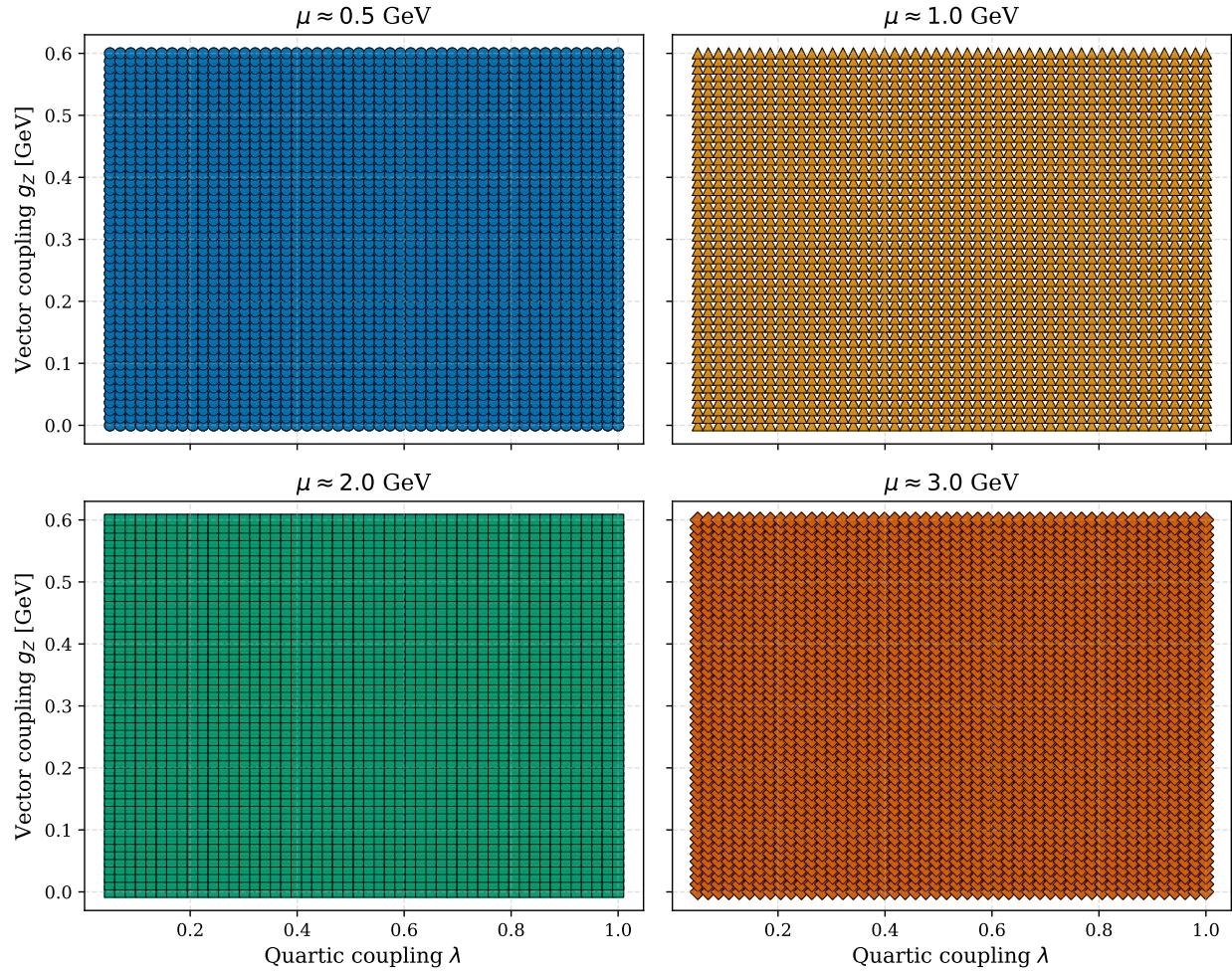


Figure 2: Stability region in the  $(\lambda, g_Z)$  plane for fixed values of the scalar mass scale  $\mu \in \{0.5, 1.0, 2.0, 3.0\}$  GeV. Each point is shaded if the algebraic stability condition (88) is satisfied. This provides a conservative diagnostic for morton existence without requiring full numerical integration.

The plots reveal that the stability condition is broadly satisfied for moderate vector couplings  $g_Z \lesssim 0.6$  GeV and quartic couplings  $\lambda \lesssim 1$ , across all tested  $\mu$ -slices. The shrinking of the stable region at higher  $\mu$  reflects the increasing difficulty in confining the vector field with a fixed scalar curvature scale.

This scan confirms that the benchmark used in Section 4.2.5 lies well within the stable regime and supports the general viability of morton formation across a wide range of parameters.

#### 4.2.9 Interpretation of the Prototype Solution

The prototype morton examined in this section demonstrates the Axis Model’s capacity to support finite-energy, static, spherically symmetric bound states formed through scalar–vector coupling. This ”masz”-type configuration, composed solely of a z-axis vector amplitude  $Z_s(r)$  and a scalar amplitude  $\Phi_0(r)$ , represents the minimal field-theoretic structure capable of localized confinement without electromagnetic interaction or spin.

The solution illustrates confinement in the absence of gauge charge. Unlike conventional bound states stabilized by U(1) interactions, the masz exists purely due to scalar–vector energetic coupling, with stability emerging from variational principles rather than conserved charges.

Moreover, the solution is robust across a wide parameter space. As shown in Section 4.2.8, a broad range of scalar–vector couplings  $g_Z$  and scalar potential parameters  $\lambda$  yield energetically stable morton configurations. This confirms that scalar-mediated confinement is not a fine-tuned phenomenon, but a generic feature of the model.

The internal structure of the masz is characterized by strong energy localization. The scalar field  $\Phi_0(r)$  smoothly interpolates between its core and asymptotic vacuum values, while the vector field  $Z_s(r)$  exhibits a compact profile centered near the origin. This validates the role of the symmetry-breaking scalar potential  $V_\Phi(\Phi)$  in defining boundary conditions and confinement radii.

In the broader context of the Axis Model, this prototype solution—referred to as a ”masz” configuration—is identified as a neutral, non-spinning morton aligned along the z-axis. These states represent localized, finite-energy scalar–vector bound structures that lack electromagnetic coupling. While large-scale dark sector phenomena, such as galactic rotation curves, are primarily attributed in the Axis Model to the dynamics of unbound Z-axis vector fields (see Section 5.1), these regular masz configurations remain theoretically significant. As discussed in Section 4.2.10, their stability, mass localization, and electromagnetic silence suggest a possible role in the internal structure of compact gravitational objects.

While the masz is constructed without invoking spin or internal symmetries, it demonstrates the foundational mechanism by which scalar gradients stabilize vector field configurations. More complex mortons—charged (via x-axis vector couplings), spinning (via emergent SU(2) structure), or asymmetric—will be developed in Section 4.4.

The prototype morton thus provides a clean, analytically tractable realization of scalar-mediated structure formation, and serves as a minimal template for generalizations across the full field content of the Axis Model.

#### 4.2.10 Implications of Morton Stability for Compact Gravitational Objects

The prototype morton solution derived in Section 4.2.5, and confirmed to be stable in Section 4.2.6, satisfies all field-theoretic criteria for an energetically confined, localized massive configuration. It is static, spherically symmetric, and composed solely of a z-axis vector field  $Z_s(r)$  and scalar field  $\Phi_0(r)$ , without coupling to electromagnetic structure or spinor degrees of freedom. Its internal

energy is finite and spatially localized, with boundary conditions asymptotically approaching the vacuum state.

These "masz"-type bound states represent the minimal dynamical content required for a non-singular compact object in the Axis Model. Stability is enforced through the scalar potential  $V_\Phi(\Phi)$  and the scalar-dependent vector mass term  $M_Z^2(\Phi) = M_0^2 + g_{Z\Phi}\Phi^2$ , which lead to regular field solutions near the origin and ensure finite energy density, avoiding the point-like singularities associated with classical point particles. The finite total energy of the solution, as shown in Eq. (90), along with the radial confinement of the vector field (Figure 1), provides a concrete realization of gravitational mass in a field-theoretic setting.

Although this solution is derived in flat spacetime and represents only a single, spherically symmetric bound state, it exemplifies the minimal unit of scalar confinement in the Axis Model: a z-axis morton composed of three quantized vector displacements. This prototype serves as a concrete proof-of-concept for scalar-mediated mass stabilization and establishes the viability of scalar–vector binding in the gravitational channel. While limited in dimensionality and scope, it forms the theoretical foundation for more complex scalar-confined objects.

To clarify terminology, we define a *z-axis morton* as the fundamental, stable, three-vector solution modeled here. In contrast, we use the term *masz structure* to refer to a broader class of scalar-confined states composed primarily of z-axis mortons, possibly in a dense, lattice-like arrangement. These may range from isolated bound states to extended, cold interiors of black holes, and may include x-axis excitations near the boundary. In this view, a *masz* is not a discrete object but a gradient state within a field-structured phase space, ranging from pure qz-bound configurations to more complex composite shells. The present solution thus establishes the microscopic building block, while the formation, evolution, and macroscopic coherence of full masz structures remains an open area for future work. This layered interpretation provides a clear roadmap from prototype to gravitational phenomenology.

### 4.3 X-Axis Dynamics: Composite Photon Structure and Modified Electrodynamics

To incorporate the interaction term  $g_X(\partial_\mu\Phi)X^\mu$  within a gauge-invariant framework and to explore how the physical x-axis field can acquire an effective mass dynamically, the Axis Model implements a Stückelberg-type decomposition that separates the gauge field into physical and unphysical components. Unlike conventional gauge field theories that impose U(1) symmetry at the Lagrangian level, the Axis Model derives electromagnetic interactions from coherent vector displacements embedded directly in the geometry of spacetime. These displacements form the basis of both electric charge and the structure of the photon.

Postulate 3 asserts a charge–length equivalence: electric charge  $q$  emerges from quantized spatial displacements along the x-axis, governed by the linear relation

$$q = k_x r, \tag{93}$$

where  $k_x$  is a universal charge density and  $r$  is a fundamental geometric displacement, identified with the Planck length  $\ell_P$ . This relationship geometrizes charge, embedding its quantization within spacetime structure rather than internal symmetries.

In this section, we develop the consequences of this geometric foundation. We begin with the internal structure of the photon as a composite oscillatory state of the  $X^\mu$  field, deriving the effective  $6r/12r$  wavelength structure. We then examine how polarization, spin, and entanglement properties emerge naturally from this configuration. Finally, we analyze the scalar–vector coupling terms that introduce deviations from classical Maxwell dynamics, describe the Stückelberg mechanism

by which gauge symmetry is effectively restored in the infrared, and evaluate the perturbative consistency of the resulting electrodynamics.

### 4.3.1 Charge–Length Equivalence

Postulate 3 of the Axis Model asserts a fundamental equivalence between electric charge and spatial displacement along the  $x$ -axis. In this framework, electric charge does not arise as an intrinsic quantum number or gauge-invariant property, but rather as a geometric consequence of quantized vector displacements in spacetime. The relation is given by:

$$q = k_x r, \quad (94)$$

where  $q$  is the electric charge,  $r$  is the quantized displacement length along the  $x$ -axis, and  $k_x$  is a universal linear charge density with units of C/m. This formulation embeds charge into the geometric structure of spacetime and eliminates the need to postulate it as a primitive property.

The displacement length  $r$  is identified with the Planck length  $\ell_P$ , defined by:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}, \quad (95)$$

on both dimensional and empirical grounds. Anchoring charge quantization at this scale ensures consistency with experimental bounds on the running of the fine-structure constant  $\alpha$ , as discussed in Appendix H.

Taking the fractional elementary charge  $q = e/3$ , consistent with confined quark states, yields a value for the geometric charge density:

$$k_x = \frac{q}{\ell_P} = \frac{e}{3\ell_P} \approx 3.30 \times 10^{15} \text{ C/m}. \quad (96)$$

This value is compatible with all known constraints on charge quantization and conservation, and provides a geometric rationale for fractional electric charges as observed in the Standard Model.

In this interpretation, the observed electric charge of any particle corresponds to a net spatial displacement in the  $x$ -axis vector field  $X^\mu$ . The existence of both positively and negatively charged particles implies the existence of two displacement modes:

$$X^+ : (+r, +q), \quad (97)$$

$$X^- : (-r, -q). \quad (98)$$

These geometric modes give rise to local electric fields through asymmetries in the displacement density. Letting  $\rho_{X^+}$  and  $\rho_{X^-}$  represent the densities of rightward and leftward displacements, the emergent electric field is given by:

$$\vec{E} = k_x(\rho_{X^+} - \rho_{X^-})\hat{x}. \quad (99)$$

This field-based interpretation of charge provides a natural mechanism for charge quantization, conservation, and field generation. In the classical limit where the scalar field  $\Phi$  is uniform, this framework recovers the standard Maxwell description of electrodynamics. In scalar-gradient backgrounds, modifications to the source terms arise from scalar–vector coupling, as detailed in Section 3.3.3.

Charge–length equivalence thus provides the geometric foundation for electromagnetic interactions in the Axis Model. It replaces intrinsic electric charge with spatial displacement structure and connects the quantization of charge directly to the Planck-scale granularity of spacetime.

### 4.3.2 Composite Photon: Internal Structure and Modified Wavelength

In the Axis Model, the photon is not a fundamental point particle but a composite excitation of the x-axis vector field  $X^\mu$ . It consists of *three synchronized pairs of mortons*—a total of six mortons—each morton itself composed of three quantized x-axis vector displacements arranged into a trivector configuration. These morton pairs execute synchronized four-phase oscillation cycles governed by:

$$+r \rightarrow 0 \rightarrow -r \rightarrow 0, \quad (100)$$

which represent coherent, quantized displacements embedded in the x-axis field. The use of three morton pairs ensures a spatially symmetric and stable structure capable of sustaining transverse propagation in three-dimensional space. This reflects the underlying geometric principles of the Axis Model, which require three independent, coordinated morton pairs for coherent field behavior.

Each morton pair contributes two displacement events per full oscillation cycle, and the photon as a whole completes six discrete internal transitions. This defines the effective internal wavelength of the photon as:

$$\lambda_{\text{eff}} = 6r. \quad (101)$$

However, observable electromagnetic phase reversal requires all three morton pairs to traverse their full oscillatory cycles, including both  $+r$  and  $-r$  displacements. This implies that the classically observed wavelength, defined by a complete reversal of the macroscopic field, corresponds to:

$$\lambda_{\text{conv}} = 2\lambda_{\text{eff}} = 12r. \quad (102)$$

This factor-of-two scaling arises from the geometric parity symmetry of the composite configuration and the need for coherent, full-cycle reversal across all morton pairs. While  $\lambda_{\text{eff}}$  defines the internal displacement periodicity,  $\lambda_{\text{conv}}$  determines the observed electromagnetic wavelength. The energy associated with photon propagation remains spectroscopically consistent with the conventional Planck relation:

$$E_\gamma = \frac{hc}{\lambda_{\text{conv}}}, \quad (103)$$

which emerges in the Axis Model from the synchronized oscillatory dynamics of six x-axis mortons arranged into three pairs spanning the full  $12r$  cycle.

The composite photon also stores internal energy in its quantized x-axis field displacements, estimated classically by

$$E_{\text{int}} \sim \frac{2e^2}{\varepsilon_0 \lambda_{\text{conv}}},$$

which reflects the internal energy content of the oscillating morton configuration, but does not affect spectroscopically observable behavior. The model thus preserves full agreement with all known spectral data while providing a geometric substructure for both the wavelength and energy of the photon.

The synchronized three-pair morton configuration also provides a natural substrate for the emergence of polarization, spin, and entanglement phenomena, which are explored in Section 4.3.3.

### 4.3.3 Emergent Properties: Polarization, Spin, and Entanglement

The internal structure of the composite photon in the Axis Model provides a natural geometric basis for its observed polarization, spin, and quantum entanglement properties.

**Polarization.** Photon polarization arises from the relative alignment of the internal vector—antivector oscillations. When all three  $X^\mu$  displacement pairs oscillate within the same geometric plane, the resulting photon is linearly polarized. Circular polarization corresponds to phase-shifted oscillations of the pairs in orthogonal planes, with a relative phase of  $\pi/2$ . Elliptical polarization arises from intermediate configurations with asymmetries in phase or amplitude. These geometric polarization modes correspond directly to observed electromagnetic field behaviors, and do not require an imposed gauge symmetry or helicity quantum number.

**Spin.** The spin-1 nature of the photon is reinterpreted in the Axis Model as a manifestation of internal geometric chirality. The synchronized three-pair configuration undergoes a four-phase oscillation cycle, generating a handedness in its internal displacement structure. This discrete cyclic symmetry produces a net angular momentum vector aligned with the direction of propagation, corresponding to the spin degree of freedom. The phase-locked vector displacements and cyclic boundary conditions thus provide a concrete geometric mechanism for spin-1 behavior within the composite photon framework.

**Entanglement.** Quantum entanglement between photons is interpreted in the Axis Model as a consequence of shared substructure at the moment of creation. When two photons originate from a common process—such as parametric down-conversion—they inherit correlated internal configurations. These correlations arise from conservation of angular momentum and phase continuity during the formation of the  $X^\mu$  trivector structure, rather than from nonlocal influence. While this interpretation does not directly resolve the Bell inequality violations observed in quantum optics, it provides a geometric substrate from which such correlations may arise.

The Axis Model thus embeds photon properties such as polarization, spin, and entanglement into the internal symmetry and structure of a composite field configuration. These behaviors are not postulated as fundamental but emerge from the Model’s underlying geometric construction.

#### 4.3.4 Morton-Level Charge Quantization

In the Axis Model, electric charge is not an intrinsic property of individual mortons, but an emergent feature that arises from scalar-filtered coherence of a particle’s full internal vector structure. A morton is defined as a bound configuration of three quantized vectors, typically in a mixed-axis arrangement such as  $(\vec{v}_z, 2\vec{v}_x)$ . While such structures form the geometric basis for charged particles, their net charge depends not on vector content alone, but on how the scalar field  $\Phi$  stabilizes and filters the full configuration.

The electron, for example, is composed of three mixed-axis mortons, yielding a total internal structure of  $v_z = 3$  and  $v_x = 6$ . By contrast, the down quark’s charge of  $-e/3$  arises from partial scalar coherence and incomplete charge projection. This distinction underscores that the scalar field governs not only mass generation, but also charge resolution.

Quark fractional charges thus emerge from stable but non-maximal scalar filtering of their internal vector content. For instance, the up quark, with structure  $v_z = 2, v_x = 4$ , achieves a scalar-filtered projection of  $+2e/3$ , while the down quark’s  $(v_z = 3, v_x = 6)$  yields  $-e/3$ . These values are not arbitrary but result from the scalar field enforcing discrete charge quantization through geometric and energetic constraints.

Therefore, the Axis Model derives the observed quantization of electric charge from internal vector composition filtered through scalar-field coherence, rather than from abstract symmetry group assignments. This provides a geometric and dynamical origin for both integer and fractional electric charges.

### 4.3.5 Scalar-Induced Photon Mass in Curved Scalar Backgrounds

The Axis Model permits the emergence of an effective photon mass in scalar-coherent regions via a gauge-invariant coupling to the scalar field  $\Phi$ . This mass generation is not a result of symmetry breaking in the conventional Higgs sense, nor a patch for gauge violation, but rather a structural consequence of scalar compositeness in bounded or high-curvature domains.

The relevant term in the Lagrangian is:

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}g_D^2\Phi^2A_\mu A^\mu, \quad (104)$$

where  $A_\mu$  is the electromagnetic gauge field,  $\Phi$  is the scalar field aligned with the internal  $y$ -axis, and  $g_D$  is a dimensionless coupling constant. This term preserves  $U(1)$  gauge invariance when treated in the Stückelberg-completed form:

$$A_\mu = \hat{A}_\mu + \frac{1}{M_S}\partial_\mu\theta, \quad (105)$$

where  $\hat{A}_\mu$  is the transverse (physical) photon and  $\theta$  is a scalar gauge compensator.

**Effective Photon Mass.** When the scalar field acquires a nonzero expectation value,  $\langle\Phi\rangle \neq 0$ , the mass term induces a dynamical photon mass:

$$m_\gamma^2 = g_D^2\langle\Phi\rangle^2. \quad (106)$$

This mass vanishes in the vacuum limit and grows proportionally with local scalar field strength, providing a natural mechanism for environment-dependent electrodynamics.

**Equation of Motion.** Varying the total action with respect to  $\hat{A}_\mu$  yields the Proca equation for a massive spin-1 field:

$$\partial_\nu\hat{F}^{\nu\mu} + m_\gamma^2\hat{A}^\mu = 0. \quad (107)$$

This equation governs the propagation of massive photon modes within scalar-condensed regions. In the limit  $\langle\Phi\rangle \rightarrow 0$ , the mass term disappears, and the standard source-free Maxwell equations are recovered.

**Interpretation.** This gauge-invariant mass generation mechanism provides a coherent and testable prediction: photons propagating through scalar-rich environments—such as morton interiors or scalar-saturated halos—acquire a small but finite mass. This effect does not require explicit symmetry breaking and remains consistent with all gauge and Lorentz symmetries of the Axis Model. The resulting dynamics contribute to photon trapping, signal delay, and potential opacity effects in strongly bound scalar domains.

### 4.3.6 The Strong Force as a Geometric Confinement Mechanism

In the Axis Model, the strong interaction emerges not from a fundamental gauge symmetry, but from the scalar-regulated geometry of composite morton structures. Quarks exhibit persistent internal  $z$ -axis polarity due to asymmetric vector composition—specifically, an imbalance in  $\vec{v}_z$  displacements—which cannot be neutralized within isolated mortons. This unresolved polarity acts as a localized source of scalar field curvature, dynamically resolved via the formation of linear flux tubes in the coupled  $X^\mu$  field. The resulting confinement mechanism is mathematically equivalent

to vortex formation in dual superconductors, where energy is concentrated into string-like tubes connecting quarks.

This confinement process enforces a topological constraint: all physically observable hadrons must be z-polarity neutral. This condition not only determines the allowable composite structures—baryons and mesons—but also induces a natural SU(3) symmetry over the internal z-vector basis. The energy stored in the flux tubes generates a linear potential  $V(L) \propto \sigma_{\text{string}} L$ , with a model-predicted string tension  $\sigma_{\text{string}} \approx 1 \text{ GeV/fm}$ , consistent with empirical QCD measurements.

The Axis Model predicts that confinement strength is modulated by the local scalar field amplitude. In environments such as neutron star cores, early-universe scalar epochs, or black hole interiors, this modulation may lead to observable deviations from QCD predictions—offering a clear falsifiable test of the geometric confinement mechanism. The full derivation of flux tube formation, SU(3) emergence, and scalar-dependent confinement strength is presented in Appendix T.

## 4.4 Composite Particle Structures from Mortons

The previous sections established the existence and stability of scalar–vector bound states—referred to as mortons—in the Axis Model. These localized excitations arise from the interaction of vector displacements along the Model’s orthogonal axes, coupled to a scalar field  $\Phi$  that mediates energy exchange and stabilizes the bound state. While Section 4.2 demonstrated the structure and spectrum of a single z-axis morton (the ”masz”), this section turns to the central claim of the Axis Model: that all observable particles are composite configurations of mortons.

Unlike conventional quantum field theory, which treats particles as quantized excitations of linear fields, the Axis Model builds particle structure geometrically. Each fundamental interaction axis—x (electromagnetism), y (scalar/temporal), and z (mass/gravity)—supports stable bound states. Physical particles arise not from the excitation of one such field, but from the projection of multi-morton configurations onto observable spacetime. These projections encode charge, spin, and mass via internal displacement geometry and scalar coupling.

This section introduces the Morton Projection Theorem, which maps internal scalar–vector configurations to observable particle properties. We show that fractional electric charges emerge naturally from x/z-axis coupling, that spin-1/2 arises from internal SU(2) symmetry in paired x-axis mortons, and that the observed quantum numbers of leptons, quarks, and neutrinos correspond to specific multi-morton geometries.

This framework allows the Axis Model to reinterpret the Standard Model’s fermions not as elementary fields, but as scalar-bound composites of geometric displacement structures. The results presented here deviate significantly from standard quantization approaches, but remain consistent with observed charge, spin, and symmetry representations, and may offer novel insight into mass generation and dark sector structure.

### 4.4.1 Interaction Mechanisms: Z–Z Binding, Z–X Coupling, and Scalar-Regulated Charge Projection

In this model, the properties of fundamental particles emerge from structured combinations of quantized vector displacements along three orthogonal axes. We present a unified interpretation of confinement, electromagnetic charge, and scalar projection based on the following three-tiered mechanism:

- 1. Strong Interaction as Internal Z–Z Binding** Composite particles exhibit internal stabilization through attractive interactions between  $z$ -axis displacements of opposite polarity. Each

morton contains one such displacement  $\vec{v}_z$ , and configurations that include both  $\vec{v}_z = +$  and  $\vec{v}_z = -$  components can form energetically favorable pairings. These z–z interactions are stabilized by the scalar field  $\Phi$ , which minimizes internal potential through coherence enforcement.

This mechanism defines the *strong force* in the Axis Model: it is not an external gauge field but an intrinsic result of scalar-stabilized z–z binding. It also naturally explains charge confinement: isolated fractional components (e.g.,  $\pm \frac{1}{3}e$  units) are unstable in the absence of complete internal z-pair coherence.

**2. Electromagnetic Charge as Net Z–X Coupling** Electric charge arises from the coupling between the total z-polarity of a structure and its geometric embedding along the  $x$ -axis. Each morton provides a complete charge-displacement unit:

$$(1\vec{v}_z, 2\vec{v}_x)$$

where  $\vec{v}_z$  encodes charge polarity and the paired  $\vec{v}_x$  vectors define spatial displacement. Thus, electromagnetic charge is a net z–x interaction. The total *geometric charge potential* of a particle is given by:

$$Q_{\text{potential}} = \sum_{i=1}^{v_z} \pm \frac{1}{3}e$$

where the sum extends over all z-vectors embedded in morton structures. The  $X^\mu$  field probes this total displacement coupling during external interaction or measurement.

**3. Scalar Filtering as a Coherence-Gated Projection** The scalar field  $\Phi$  regulates the projection of the internal z–x structure into observable spacetime. A fully coherent configuration—i.e., one whose internal geometry aligns with the scalar field—achieves maximal charge projection. Asymmetric or misaligned structures experience suppressed projection efficiency.

We define a scalar coherence factor  $\eta \in [0, 1]$  such that the observable charge becomes:

$$Q_{\text{observed}} = \eta \cdot Q_{\text{potential}}.$$

For example, the electron has  $v_z = 3$  and full coherence, giving  $\eta = 1$  and observed charge  $Q = -e$ . The down quark, with  $v_z = 9$  and mixed polarity, has partial scalar coherence  $\eta = \frac{1}{9}$ , yielding  $Q = -\frac{1}{3}e$ . The up quark has  $v_z = 6$  with net +2 polarity, resulting in scalar coherence  $\eta = \frac{1}{3}$  and observed charge  $Q = +\frac{2}{3}e$ .

This mechanism explains how structurally similar morton lattices can yield different observable charges depending on scalar-regulated projection, and why particles with unresolved internal z–z polarity diversity remain confined under the strong interaction.

**Summary** Charge is not an intrinsic property of mortons but an emergent projection from a stabilized geometric configuration. The strong force arises from internal z–z binding, charge potential from z–x coupling, and charge observability from scalar coherence. These three mechanisms together form the geometric and field-theoretic foundation for confinement, charge quantization, and first-generation fermion structure in the Axis Model.

#### 4.4.2 The Morton Concept and Projection Effects

In the Axis Model, a morton is defined as a stable, finite-energy scalar–vector bound state composed of three quantized displacement vectors. Each morton is a triad of displacements oriented along

either the  $z$ -axis or  $x$ -axis, depending on its internal configuration. These tri-vector states represent localized excitations of the vector fields  $Z^\mu$  and  $X^\mu$ , stabilized by interaction with the scalar field  $\Phi$ . The prototype case studied in Section 4.2 is a pure  $z$ -axis morton—a “masz”—comprised of three  $\vec{v}_z$  displacements, associated with mass-energy and gravitational structure.

**Composite Morton States.** Observable particles in the Axis Model are constructed from bound aggregates of mortons. For example, the electron consists of  $q = 3$  identical mixed-axis mortons, each containing one  $z$ -axis displacement and two  $x$ -axis displacements. This yields a total vector content of  $v_z = 3$ ,  $v_x = 6$ . These values describe the internal geometry of the particle, not its direct physical observables. Observable quantities such as charge and mass emerge only after scalar-vector filtering, as detailed below.

**Morton Projection Theorem.** The *Morton Projection Theorem* describes how internal displacement vectors appear under external photon-based measurement. Because the photon is itself a composite excitation—composed of six synchronized  $x$ -axis displacements in the Axis Model—it does not probe internal structure isotropically. It over-samples  $x$ -axis displacements and under-samples  $z$ -axis displacements due to its own internal geometry.

This yields the effective projection rule:

$$q_{\text{projected}} = \frac{2}{3}v_z + 6v_x \quad (108)$$

This projection quantifies how much internal displacement content is *visible* to an external  $X^\mu$  probe, due to measurement bias. It is not a generator of physical charge or mass. It reflects geometric compression and amplification during EM interaction, not the intrinsic properties of the underlying morton lattice.

**Scalar–Vector Filtering and Charge Projection.** Physical observables like electric charge arise through a separate mechanism: scalar-regulated projection of internal  $z$ -axis polarity. Each  $z$ -displacement contributes a potential charge of  $\pm \frac{1}{3}e$ , depending on its polarity. The total geometric charge potential of a particle is given by:

$$Q_{\text{potential}} = \sum_{i=1}^{v_z} \pm \frac{1}{3}e \quad (109)$$

The observable charge is determined by the scalar coherence factor  $\eta \in [0, 1]$ , which encodes the degree of alignment between the internal structure and the scalar field:

$$Q_{\text{observed}} = \eta \cdot Q_{\text{potential}} \quad (110)$$

To illustrate the mechanism, consider three representative cases. The electron is composed of  $q = 3$  identical mixed-axis mortons, yielding a total of  $v_z = 3$   $z$ -axis displacements. Each  $\vec{v}_z$  contributes a charge potential of  $-\frac{1}{3}e$ , resulting in a total geometric charge potential of  $Q_{\text{potential}} = -e$ . Because the electron achieves full scalar coherence ( $\eta = 1$ ), the observed charge is:

$$Q_{\text{observed}} = -e$$

In contrast, the down quark is composed of  $q = 9$  mixed mortons, with a total of  $v_z = 9$   $z$ -axis displacements. Despite sharing the same charge potential as the electron ( $Q_{\text{potential}} = -e$ ), the

down quark exhibits only partial scalar coherence. The scalar field filters this structure with a coherence factor of  $\eta = \frac{1}{3}$ , yielding:

$$Q_{\text{observed}} = -\frac{1}{3}e$$

Finally, the up quark consists of  $q = 6$  mortons, yielding  $v_z = 6$  z-axis displacements. Of these, four exhibit positive polarity and two negative, producing a net z-polarity of +2 and a corresponding charge potential of  $Q_{\text{potential}} = +\frac{2}{3}e$ . In this case, the structure achieves full scalar coherence ( $\eta = 1$ ), and the observed charge is:

$$Q_{\text{observed}} = +\frac{2}{3}e$$

This framework cleanly distinguishes between intrinsic geometric structure, scalar-field-induced coherence, and measurement distortion. The Morton Projection Theorem [Eq. (108)] is valid for interpreting vector-coupled observables such as displacement magnitude or electromagnetic cross-section, but it is not used to derive physical quantities like charge or mass. Instead, observable charge arises from scalar-filtered projection of net internal  $z$ -polarity as defined by Eqs. (109)–(110).

#### 4.4.3 Electron Structure as a Three-Morton Composite

In the Axis Model, the electron is modeled as a stable scalar–vector composite consisting of three identical mixed-axis mortons. Each morton contains one z-axis vector and two x-axis vectors, giving it an internal structure of  $(1q_z, 2q_x)$ . Together, these mortons form a bound configuration with total vector content  $(3q_z, 6q_x)$ , stabilized by scalar-mediated coherence. This full internal structure determines the electron’s charge, spin, and mass properties via projection.

**Structure and Binding.** The z-axis vectors serve as localized energy anchors—displacements from the scalar vacuum that generate inertial and gravitational structure. The x-axis vectors provide oscillatory dynamics aligned along the x-axis. Scalar mediation ensures phase-locked coherence, angular closure, and symmetry across all three mortons. This tri-morton configuration forms a stable, spin-carrying, charged bound state with well-defined quantum numbers.

**Charge.** Electric charge in the Axis Model originates from scalar-filtered z-axis polarity, which establishes the underlying sign and magnitude of charge through projection filtering, as formalized in Sections 4.4.4 and Appendix K. However, its observable expression arises through coherent x-axis oscillations that couple to the internal geometry of the composite photon. The scalar field mediates this transfer, converting longitudinal z-polarity into transverse x-axis displacement, thereby linking the origin and manifestation of charge as cause and effect.

In the case of the electron, this manifests as a stable configuration of six  $\vec{v}_x$  vectors distributed across its three-morton structure. The scalar field imposes strict coherence conditions on this arrangement, ensuring that only specific vector combinations yield observable charge. Although the fundamental charge quantum per displacement is  $\pm e/3$ , scalar-filtering constraints project the full configuration onto a net charge of exactly  $\pm e$ . This mechanism parallels the treatment of fractional charges in quarks, where distinct morton configurations with partial z-polarity yield projected charges consistent with Standard Model values.

**Spin and Internal Symmetry.** The electron’s spin- $\frac{1}{2}$  nature results from the antisymmetric configuration of its internal x-axis displacement structure. These define a compact internal configuration space topologically equivalent to a two-sphere, stabilized by scalar curvature. This space supports an emergent  $SU(2)$  symmetry and transforms as a spinor under spacetime rotation. The

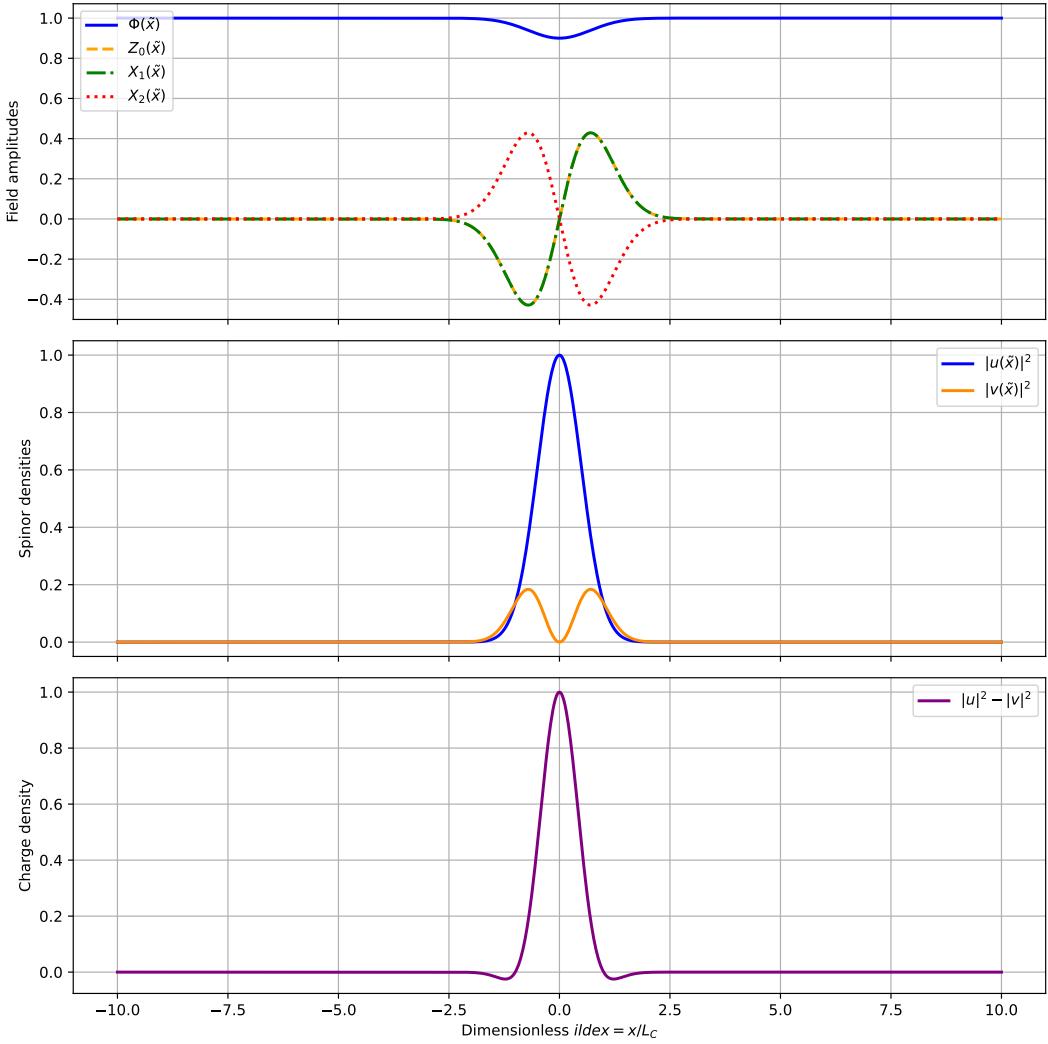


Figure 3: Effective electron configuration as a bound state of three mixed-axis mortons (each composed of one  $Z^\mu$  vector and two  $X^\mu$  vectors). **Top:** Scalar field  $\Phi(x)$ , z-axis vector field  $Z_0(x)$ , and x-axis vector fields  $X_1(x)$  and  $X_2(x)$  resolved independently. **Bottom:** Spinor components  $u(x)$  and  $v(x)$  defining the fermionic structure of the solution. These fields collectively represent the effective behavior of the electron's internal morton substructure. The numerical solution is obtained from the system of coupled second-order ODEs defined in Appendix C.4, satisfying physical boundary conditions and demonstrating localized, finite-energy behavior.

topological nontriviality ensures two-valuedness under full rotation, consistent with spin- $\frac{1}{2}$  behavior. A formal derivation appears in Appendix A, which establishes the SU(2) holonomy of the internal configuration space and the emergence of spinorial behavior from the coupled morton geometry.

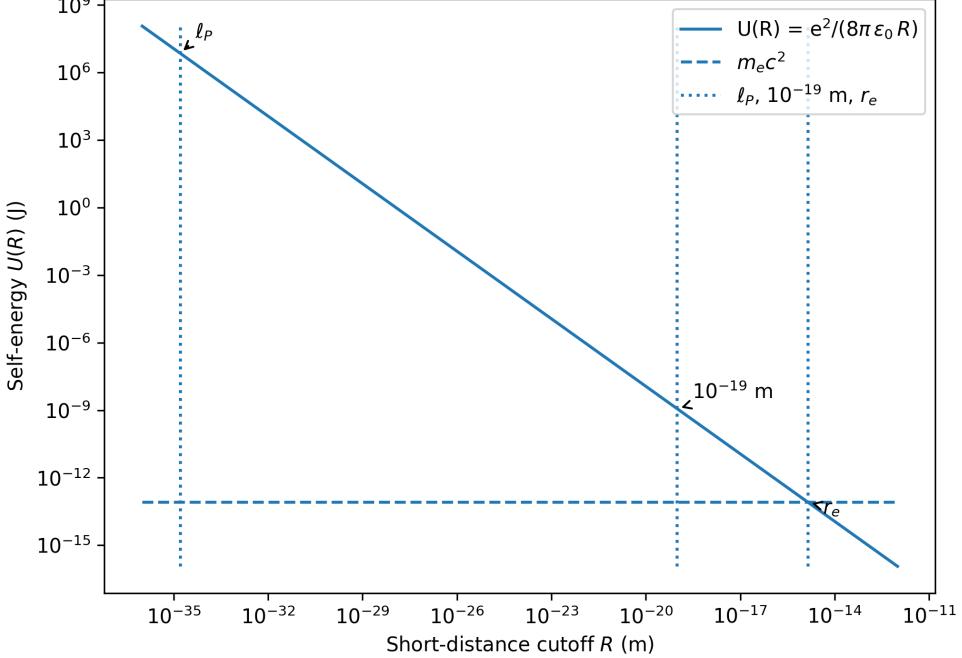


Figure 4: Electrostatic self-energy  $U(R) = e^2/(8\pi\epsilon_0 R)$  as a function of short-distance cutoff  $R$ . The dashed line marks  $m_ec^2$ ; vertical markers indicate the Planck length  $\ell_P$ , a collider bound ( $10^{-19}$  m), and the classical radius  $r_e$  where  $U(r_e) = m_ec^2$ . This illustrates the separation between the morton-scale substructure (Planck), the continuum effective radius ( $r_e$ ), and experimental pointlike limits.

**Stability and Projection.** The composite satisfies all scalar-filtering criteria: energy balance, angular coherence, and parity symmetry. It projects into observable spacetime as a spin- $\frac{1}{2}$ , negatively charged, finite-mass fermion. In this framework, the electron is not pointlike but a stable, quantized configuration of scalar-bound geometric displacements. Projection bias toward x-axis alignment (Appendix I) and scalar-filtered coupling weights (Appendix K) together determine the observed energy scale (Appendix L).

**Effective Radius and Self-Energy.** In continuum electrodynamics the electron's rest energy can be modeled as arising from electrostatic self-energy  $U(R) = e^2/(8\pi\epsilon_0 R)$ ,<sup>7</sup> which equals  $m_ec^2$  at the classical electron radius  $r_e \approx 1.4 \times 10^{-15}$  m. Figure 4 compares this scale to both the Planck length  $\ell_P$ —the displacement unit of the morton framework—and current collider bounds showing pointlike behavior below  $10^{-19}$  m. In the Axis Model the electron is composite at the Planck scale

<sup>7</sup>The classical radius is defined by equating rest energy to electrostatic self-energy. We use the self-energy of a spherical shell of charge,  $U(R) = e^2/(8\pi\epsilon_0 R)$ . Other conventions exist, differing by factors of order unity, but all place  $r_e$  at the femtometer scale, many orders of magnitude above the Planck scale and modern experimental limits.

but projects as pointlike to all present experiments; the classical  $r_e$  emerges only as an effective continuum scale, not a literal size.

**Numerical Validation.** To validate the internal structure proposed for the electron, we numerically solve the full 10-component coupled system comprising the scalar field  $\Phi(\tilde{x})$ , the z-axis vector field  $Z_0(\tilde{x})$ , the x-axis vector field  $X_1(\tilde{x})$ , and the spinor components  $u(\tilde{x})$  and  $v(\tilde{x})$ . Figure 3 displays the resulting bound-state configuration using benchmark parameters. The solution confirms that the electron corresponds to a finite-energy, scalar–vector–spinor bound state stabilized by internal geometric structure. All fields satisfy smoothness, localization, and normalization conditions. Further details are provided in Appendix E and Appendix C.4.

#### 4.4.4 Quark Structure and Fractional Charges

The Axis Model reproduces the fractional electric charges observed in quarks as geometric consequences of stabilized scalar–vector configurations composed of multiple mixed-axis mortons. Each quark is modeled as a coherent bound state of several  $(1z, 2x)$  mixed mortons confined within a common scalar potential well. The internal vector content of each quark is fully quantized and defined by the aggregate number of  $z$ -axis and  $x$ -axis vector displacements, denoted  $(v_z, v_x)$ .

**Canonical Morton Compositions.** The first-generation up quark is composed of  $q = 6$  mixed mortons, yielding  $(v_z = 6, v_x = 12)$ . The down quark is composed of  $q = 9$  mixed mortons, yielding  $(v_z = 9, v_x = 18)$ . These counts are derived from the bridge equation, which maps rest mass to morton number, and are consistent with the scalar-stabilized electron structure defined in Section 4.4.3. No quark is composed of isolated pure-axis mortons; all vector content arises from embedded mixed  $(1z, 2x)$  mortons.

**Fractional Charge from Scalar-Filtered Z-Polarity.** Electric charge in the Axis Model does not emerge from x-axis geometry, but from the net polarity of  $z$ -axis displacements filtered through scalar coherence. Each  $\vec{v}_z$  contributes a potential charge of  $\pm \frac{1}{3}e$ , depending on its internal sign. The total geometric charge potential of a particle is given by

$$Q_{\text{potential}} = \sum_{i=1}^{v_z} \pm \frac{1}{3}e,$$

and the observed charge is

$$Q_{\text{observed}} = \eta \cdot Q_{\text{potential}},$$

where  $\eta$  is the scalar coherence factor. For quarks,  $Q_{\text{potential}}$  is fractional due to partial polarity cancellation, and  $\eta$  may be unity if the entire structure is coherent. For example, an up quark with four positive and two negative  $z$ -polarized displacements has  $Q_{\text{potential}} = +\frac{2}{3}e$  and  $Q_{\text{observed}} = +\frac{2}{3}e$  under full scalar coherence. The down quark has a net  $z$ -polarity of  $-1$ , yielding  $Q_{\text{potential}} = -\frac{1}{3}e$ .

#### Table of Canonical Quark Structures.

Quark Type	Morton Count $q$	Vector Content $(v_z, v_x)$	Charge $Q$
Up ( $u$ )	6	(6, 12)	$+\frac{2}{3}e$
Down ( $d$ )	9	(9, 18)	$-\frac{1}{3}e$

**Confinement via Scalar Curvature.** Quark charges are stabilized through internal polarity imbalance and scalar curvature confinement. Because their net  $z$ -polarities are nonzero, quarks act as sources of scalar tension and cannot exist as isolated states. Only in scalar-neutral combinations—such as baryons (three-quark systems) or mesons (quark-antiquark pairs)—do these internal tensions cancel. This mechanism is detailed in Section 4.3.6 and Appendix T.

**Spinor Simplification.** This treatment omits explicit spinor degrees of freedom for quarks. While the electron is modeled as a coherent spinor doublet, quark spin is here attributed to internal  $z$ - $x$ - $z$  angular configurations and scalar-mediated multiplet formation. Future refinements may incorporate spin-flavor symmetry, but the scalar-vector projection model for charge remains consistent.

**Projection Filtering Constraints.** Only specific combinations of morton count, polarity pattern, and scalar coherence yield stable observable quark structures. These are selected dynamically by scalar potential minima and angular closure constraints. The fractional charge values observed in nature arise not from fractional mortons, but from scalar-filtered projections of fully quantized  $z$ -polarity lattices.

**Higher Generations.** Second- and third-generation quarks are modeled as large scalar–vector configurations built from the same underlying  $(1z, 2x)$  morton topology as first-generation quarks. These higher-generation structures contain significantly more mortons overall, combining  $z$ - and  $x$ -axis displacements in proportions that preserve the scalar-projected fractional charge (e.g.,  $+\frac{2}{3}e$ ,  $-\frac{1}{3}e$ ), but exceed the coherence threshold required for scalar stability. As a result, these configurations are internally strained and dynamically unstable, eventually undergoing scalar-mediated reprojection.

During reprojection, the internal morton structure decomposes into a stable combination of lower-generation quark or lepton states, accompanied by surplus mortons—typically excess  $x$ -axis displacements lacking  $z$ -polarity—which decay into radiative or scalar-neutral modes. The specific decay branch (e.g., leptonic versus hadronic) is determined by the distribution of internal  $z$ -polarity at the moment scalar coherence breaks. These states are thus not simple excitations of first-generation particles, but metastable morton composites defined by internal tension, curvature imbalance, and projection instability.

This framework provides a geometric rationale for why higher-generation quarks exhibit the same fractional charges as first-generation quarks, yet differ in mass, coherence, and decay complexity. Fractional electric charge emerges as a scalar-filtered projection of net  $z$ -axis polarity, and all quarks—regardless of generation—are characterized by confinement geometry and scalar curvature cancellation across vector-composite systems, as formalized in Appendix T.

#### 4.4.5 Mass Spectrum of Stable Morton Configurations

Having established the stability of scalar–vector bound states in Section 4.2 and constructed specific composite structures for electrons and quarks, we now examine the broader spectrum of stable mortons predicted by the model. These configurations span a wide range of scalar potential and coupling parameters and are filtered using the algebraic stability criterion defined in Eq. (88).

Figure 5 shows the resulting distribution of stable morton masses, with over 250,000 configurations evaluated across the parameter space. Each configuration is derived from a scalar–vector coupling and scalar potential that yields a regular, finite-energy solution. The distribution reveals a broad spectrum of masses ranging from sub-GeV to tens of GeV, with a median near 3.8 GeV.

Vertical reference lines mark the electron mass ( $m_e$ ), proton mass ( $m_p$ ), the median, and the 5th–95th percentile confidence band. This spectrum confirms that the Axis Model supports a rich variety of stable scalar-bound vector states that could correspond to the observed diversity of particle species.

Importantly, the existence of such a continuous but quantized mass spectrum supports the hypothesis that different elementary particles emerge from distinct, filtered configurations of the same underlying morton structure. The broad spread in mass suggests that the scalar potential landscape plays a key role in selecting which configurations are physically realized and how their effective charges, spins, and internal symmetries arise.

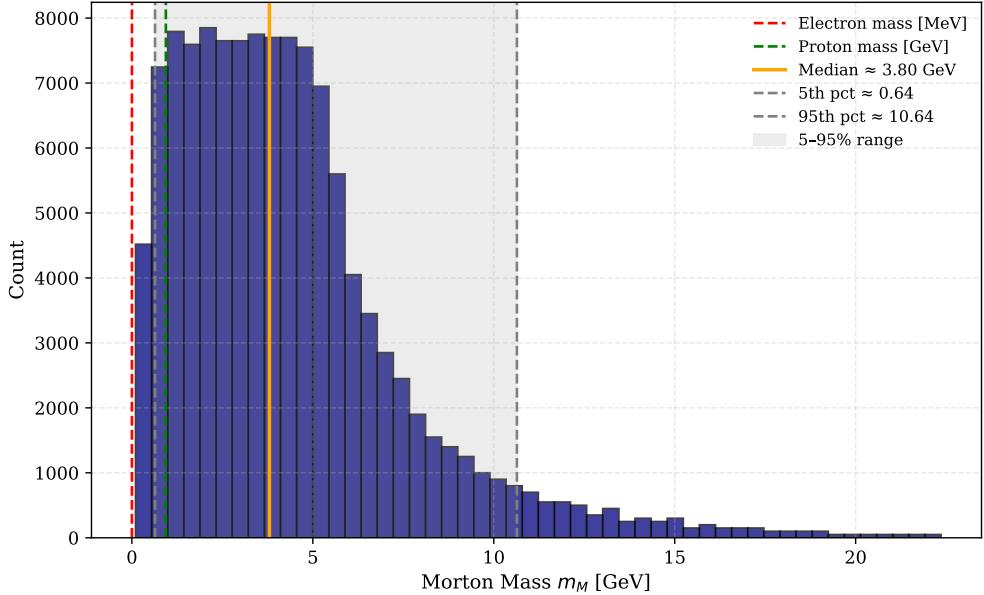


Figure 5: Histogram of stable morton masses sampled across parameter space satisfying the algebraic stability criterion. Reference lines mark the electron mass (red dashed), proton mass (green dashed), median (orange), and the 5th–95th percentile band (gray). The scalar-bound morton spectrum spans sub-GeV to multi-GeV scales, suggesting internal structure diversity across composite particle species.

This spectrum also opens the possibility that stable, neutral mortons with masses above the proton but below electroweak scale could contribute to dark sector phenomena. These configurations would evade Standard Model coupling but remain gravitationally relevant. Further constraints on their existence would arise from cosmological bounds, astrophysical stability, and interactions within extended scalar-curved domains.

#### 4.4.6 Neutrino Structure and Dynamic Mass Generation

The Axis Model defines neutrinos as scalar-stabilized morton configurations composed entirely of  $x$ -axis vector displacements. These states are constructed from seven pure  $x$ -axis mortons, yielding total vector content ( $v_z = 0$ ,  $v_x = 21$ ). In the absence of any  $z$ -polarized displacements, these configurations lack a rest-mass anchor and carry no net scalar curvature. Despite their geometric masslessness, parity asymmetry and scalar filtering allow a small effective energy residue to persist. This enables neutrinos to acquire a dynamical mass through scalar coupling, even without internal  $z$ -axis structure.

The formal relationship between internal structure, scalar coherence, and emergent energy is established by the Bridge Equation (Appendix L), which relates observed energy to morton number via a nonlinear projection rule. In particular, scalar coherence applied to purely  $x$ -axis morton configurations can yield discrete, stable effective mass states when the scalar potential  $V(\Phi)$  admits multiple local minima. This feature is derived from the Axis Model Lagrangian (Appendix M) and underpins the model’s prediction of environmentally sensitive neutrino mass bifurcation.

**Scalar Coupling and Effective Mass.** Neutrinos acquire mass not through intrinsic rest-energy but via scalar-induced stabilization of internal  $x$ -axis asymmetry. This is governed by the interaction Lagrangian:

$$\mathcal{L}_{\text{int}}^\nu = g_\nu \Phi \bar{\nu} \nu, \quad (111)$$

where  $g_\nu$  is the neutrino–scalar coupling constant. The resulting effective mass  $m_\nu \sim g_\nu \langle \Phi \rangle$  depends on the local scalar field vacuum and varies with environmental conditions.

**Discrete Mass States from Scalar Minima.** The effective potential  $V(\Phi)$  is not static but modulated by environmental parameters such as matter density, neutrino energy, and path length. When this potential supports multiple local minima, stable configurations can arise corresponding to distinct scalar-stabilized mass states. Neutrino oscillation is then understood not as interference between fixed eigenstates but as transitions between these scalar field minima, governed by decoherence and scalar settling rather than pure superposition.

**Empirical Consequence: Bifurcation in  $\theta_{23}$ .** This model naturally explains the contrasting posterior structures reported in T2K and NO $\nu$ A for the mixing angle  $\theta_{23}$ . T2K observes a unimodal posterior centered near  $\sin^2 \theta_{23} \approx 0.514$ , consistent with a single scalar-coherent neutrino mass state under its shorter baseline and moderate matter profile [40]. In contrast, NO $\nu$ A observes a clearly bimodal posterior with peaks at  $\sin^2 \theta_{23} \approx 0.404$  and  $0.624$ , interpreted in the Axis Model as evidence for two stable scalar minima enabled by its longer baseline and stronger matter interaction [41]. This bifurcation is interpreted in the Axis Model as a scalar-induced phase transition in the effective potential  $V(\Phi)$ , with the number and location of minima depending on local environmental parameters (see Appendix AH).

**Charge Neutrality and Internal Symmetry.** The absence of any  $z$ -axis displacement implies that neutrinos have no intrinsic rest-mass anchor and no net scalar curvature. The scalar field’s filtering mechanism ensures that only charge-neutral configurations of  $x$ -axis vector displacements are projected into observable neutrino states. Any net  $x$ -axis displacement (which would carry electric charge) is energetically suppressed. Neutrinos thus occupy a unique position: they are massless under pure geometry but become dynamically massive through scalar coherence, and they are electrically neutral by construction.

**Unified Interpretation.** The Axis Model reinterprets neutrino oscillation as scalar-mediated transitions between dynamically stabilized mass states, rather than interference between fixed intrinsic eigenstates. This framework unifies the explanation of neutrino mass generation, environmental dependence, and flavor oscillation structure—without requiring right-handed sterile neutrinos [42] or Majorana mass terms [43]. Neutrinos emerge as the lowest-energy *massive* scalar-stabilized excitations of  $x$ -axis mortons, occupying a transitional boundary between massless radiative states such as photons and scalar-bound leptons. Their dynamic properties arise entirely from

environmental scalar field structure and internal coherence, without requiring additional symmetry-breaking terms or exotic sterile couplings.

#### 4.4.7 Geometric Origin of the Electroweak Interaction

The internal configuration space of fermionic mortons supports an emergent  $SU(2)$  symmetry that provides the geometric foundation for the electroweak interaction. As formalized in Appendix Y, the orientation of internal vector displacements defines a coset manifold

$$S^2 \simeq \frac{SU(2)}{U(1)},$$

which naturally hosts spinor representations. Projection through the scalar-coherent filter  $\Pi_\Phi$  maps internal displacements to gauge connections,

$$A_\mu^a T_a \equiv \Pi_\Phi[v_{x,\mu}], \quad B_\mu \propto \partial_\mu \theta,$$

yielding a covariant derivative

$$D_\mu = \partial_\mu - i g A_\mu^a T_a - i g' B_\mu Y$$

with  $SU(2)_L$  and  $U(1)_Y$  couplings determined by scalar-filtered projection weights (see QC for the full construction):

$$g^2 \propto w_{SU(2)} \equiv \int \frac{d^4 x}{\Lambda_\Phi^2} |\text{Tr}[T_a \Pi_\Phi(v_{x,\mu})]|^2, \quad g'^2 \propto w_{U(1)_Y} \equiv \int \frac{d^4 x}{\Lambda_\Phi^2} |\partial_\mu \theta(x)|^2.$$

*Reader's guide.* The explicit gauge-theoretic construction—building the covariant derivative from scalar-filtered projections, deriving the electroweak mass Lagrangian directly from the internal spinor kinetic term, and recovering the  $W^\pm$ ,  $Z$ , and photon eigenstates with  $m_W = \frac{1}{2}gv$  and  $m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v$ —is worked out in the companion *Quantum Completion* paper, Sec. 5.<sup>8</sup>

**Mass generation (schematic).** From the scalar-filtered kinetic term of the internal spinor, the electroweak gauge sector acquires the standard mass Lagrangian at coherence scale  $v = \langle|\Phi|\rangle$ ,

$$\mathcal{L}_{\text{mass}} = \frac{v^2}{8} \left[ g^2 (A_{1\mu}^2 + A_{2\mu}^2) + (g A_{3\mu} - g' B_\mu)^2 \right], \quad (112)$$

so that, after defining

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_{1\mu} \mp i A_{2\mu}), \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} A_{3\mu} \\ B_\mu \end{pmatrix}, \quad \tan \theta_W = \frac{g'}{g},$$

one obtains

$$m_W = \frac{gv}{2}, \quad m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}, \quad m_\gamma = 0.$$

In the projection picture,  $\theta_W$  is fixed by the ratio of projection weights,  $\tan \theta_W = \sqrt{w_{U(1)_Y}/w_{SU(2)}}$  (QC, Secs. 2.2–2.3/4.3–4.4).

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<sup>8</sup>See QC Sec. 5 for the promotion of the global  $SU(2)_L \times U(1)_Y$  symmetry to a local gauge theory via scalar-filtered projection, the mass matrix and eigenstates  $W^\pm, Z, A$ , and the projection-weight origin of the couplings and Weinberg angle.

**Parity selection as scalar-projection asymmetry.** Chirality follows from the scalar filter: the left-handed component couples to the projected  $SU(2)$  connection while the right-handed component does not,

$$\mathcal{L}_{\text{int}} \supset \bar{\psi}_L \gamma^\mu (g A_\mu^a T_a + g' B_\mu Y_L) \psi_L + \bar{\psi}_R \gamma^\mu (g' B_\mu Y_R) \psi_R, \quad (113)$$

because scalar-coherent projection suppresses the  $SU(2)$  coupling weight on  $\psi_R$  (Appendix Z.5). Thus parity violation arises as a geometric selection rule rather than an explicit symmetry breaking term. BRST consistency and anomaly cancellation for the projected gauge theory are established in the companion *Quantum Completion of the Axis Model*.

#### 4.4.8 Morton Projection Filtering and Electromagnetic Invisibility

A central principle of the Axis Model is that observable quantities such as electric charge and inertial mass do not correspond directly to a particle's internal morton count. Instead, they are filtered through a projection determined by the scalar field geometry and the nature of the measurement interaction—typically photon exchange. This transformation is formalized by the *Morton Projection Theorem*, which defines the effective projected displacement content as:

$$q_{\text{projected}} = \frac{2}{3}v_z + 6v_x, \quad (114)$$

where  $v_z$  and  $v_x$  are the total numbers of  $\vec{v}_z$  and  $\vec{v}_x$  displacement vectors in a composite particle configuration. The coefficients arise from geometric mechanisms detailed in Appendix K: a suppression factor of  $\frac{2}{3}$  from isotropic averaging of  $z$ -axis vector fields, and an amplification factor of 6 due to the internal structure of the composite photon, which coherently couples to six synchronized  $x$ -axis vector displacements.

This projection framework provides a geometric explanation for multiple empirical phenomena, unifying seemingly disparate observations under a single scalar-vector mechanism.

**Quark Fractional Charges.** In the Standard Model, quark charges are introduced as fixed parameters without underlying structure. In contrast, the Axis Model interprets these charges as emergent properties resulting from scalar-filtered internal configurations of displacement vectors. Quarks are modeled as stable bound states composed of multiple mixed-axis mortons, each contributing both  $\vec{v}_z$  and  $\vec{v}_x$  content. For example, an up quark may have a total internal vector composition of  $(v_z = 2, v_x = 4)$ , while a down quark may exhibit  $(v_z = 3, v_x = 6)$ . These configurations do not yield their fractional charges directly via Eq. (108). Instead, the scalar field acts as a filter, selectively projecting phase-locked  $\vec{v}_x$  displacements into observable spacetime. The resulting stable oscillation modes couple to the composite photon's geometry, yielding effective electromagnetic charges of  $+\frac{2}{3}e$  for the up quark and  $-\frac{1}{3}e$  for the down quark. Full geometric interpretations of these fractional charges are presented in Section 4.4.4 and Appendix K.

**Neutron Charge Neutrality.** The neutron, modeled as a composite of one up quark and two down quarks, exhibits net charge neutrality due to scalar-mediated cancellation of  $x$ -axis oscillatory modes. Its internal geometry supports destructive interference among the projected  $\vec{v}_x$  components, enforced by the scalar field's coherence requirements. This internal structure aligns with stringent experimental bounds on neutron electric dipole moments and charge asymmetry.

**Electromagnetic Invisibility of Dark Matter.** Unbound  $z$ -axis vectors—i.e., free  $\vec{v}_z$  displacements not incorporated into scalar-stabilized mortons—play a central role in the Axis Model’s account of dark matter. These displacements contribute to inertial mass and gravitational curvature, but lack any  $x$ -axis structure and therefore do not couple to electromagnetic fields. According to Eq. (108), any configuration with  $v_x = 0$  yields:

$$q_{\text{projected}} = \frac{2}{3}v_z, \quad (115)$$

indicating a suppressed projection that fails to generate charge. Such kinetically inert  $Z^\mu$  excitations can accumulate in scalar-coherent halos, remain electromagnetically dark, and form coherent gravitational structures consistent with galactic rotation curves and lensing profiles. This naturally explains the lack of interaction in direct-detection experiments such as LUX [44], XENON [45, 46], and PandaX [47].

**Experimental Outlook.** The Morton Projection Theorem is empirically testable. Deviations from its predicted projection structure—particularly in high-scalar-curvature environments such as black hole interiors or intense electromagnetic fields—may induce measurable shifts in effective charge, vacuum polarization, or birefringence. These phenomena offer concrete predictions for precision probes including LISA, JWST, and Hyper-Kamiokande. Additional consequences for neutron decay asymmetries and dark photon interactions are explored in Section 3.3.2 and Section 5.9.

**Conclusion.** The Morton Projection Theorem provides a unifying geometric interface between internal field configurations and observable electromagnetic properties. Fractional charges, electromagnetic silence of dark matter, and scalar-modulated charge neutrality all emerge as consequences of this filtered projection mechanism. Observable quantities are not intrinsic field labels but the outcome of scalar-vector coherence projected through the geometry of spacetime.

#### 4.4.9 Neutron Decay as a Scalar–Vector Reconfiguration

In the Standard Model, neutron  $\beta$ -decay is mediated by a virtual  $W^-$  boson. The Axis Model offers an alternative interpretation: the decay results from a spontaneous reconfiguration of the neutron’s internal morton structure, an interpretation that does not require mediation by a fundamental force-carrying particle. This transition is triggered by a local instability in the scalar field  $\Phi$ , which permits energy redistribution among the constituent mortons without invoking external mediators.

**Mass–Energy Balance.** The total rest-mass energy released in the decay is the decay energy (Q-value), given by the experimental masses of the neutron, proton, and electron:

$$\Delta E = m_n c^2 - (m_p c^2 + m_e c^2) \approx 0.782 \text{ MeV}. \quad (116)$$

Within the Axis Model, we interpret this released energy as scalar-field potential energy, temporarily stored in  $\Phi$  and subsequently re-condensed into a new, stable morton configuration. Applying the Bridge Equation—relating particle rest energy to effective morton count—as derived in Appendix L (Eq. 199), we obtain:

$$q_{\text{released}} = \frac{\sqrt{\Delta E \cdot e}}{E_0} \approx \frac{\sqrt{(0.782 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV})}}{10^{-7} \text{ J}} \approx 3.54. \quad (117)$$

Here,  $E_0 \approx 10^{-7} \text{ J}$  is the canonical energy of a single morton. This  $q_{\text{released}}$  value corresponds to the effective scalar potential energy available for reprojection.

**Neutrino Formation via Scalar Reprojection.** As shown in Appendix G, scalar–vector reprojection processes involving coherent vector bound states obey a four-phase geometric oscillation. This symmetry induces a doubling of internal energy relative to the scalar potential baseline. Thus, the emergent neutrino state carries:

$$q_\nu \approx 2 \times q_{\text{released}} \approx 2 \times 3.54 = 7.08. \quad (118)$$

This aligns precisely with the proposed neutrino structure as a  $7q_x$  morton configuration—a stable, charge-neutral, massless state composed entirely of x-axis mortons. The full decay can therefore be interpreted as a scalar-driven reconfiguration:



**Interpretation.** In this framework,  $\beta$ -decay arises not from a fundamental gauge interaction but from scalar-induced topological reorganization of vector-bound energy. The scalar field  $\Phi$  serves both as the energy reservoir and transition facilitator, analogous to symmetry-breaking fields in early-universe models. The process is entirely local, self-contained, and governed by the scalar–vector Lagrangian and morton geometry.

**Future Directions.** While this reinterpretation accounts for energy and charge conservation, a complete dynamical analysis remains necessary. Specifically, the conservation of angular momentum and spin must be demonstrated explicitly. This will require modeling the emergent antineutrino in terms of its internal SU(2) spinor structure (see Appendix E) and confirming that the scalar-induced reconfiguration respects gauge invariance and spin-parity selection rules.

#### 4.4.10 Muon Decay and Scalar–Vector Partitioning

In the Axis Model, the muon is not a fundamental excitation, but a metastable scalar–vector configuration within the electron sector. Its decay is governed by scalar-induced reprojection: a transition from an unstable scalar-bound internal mode into lower-energy structures composed of stable, projection-filtered vector states.

**Observed Decay Products.** The standard decay channel is:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.$$

This transition is not mediated by a gauge boson, but instead reflects a topological shift in scalar configuration geometry. The scalar field  $\Phi$  redistributes internal energy by reprojection onto phase-stable x-axis structures (see Appendix L).

**Bridge Equation Application.** Using the derived Bridge Equation,

$$E_{\text{obs}} = \varepsilon_\Phi \cdot |q_{\text{projected}}|^2,$$

with the empirically calibrated scalar mass scale  $\varepsilon_\Phi \approx 0.000354 \text{ MeV}$ , we can translate observed rest energies into squared projected vector content. For the muon:

$$E_\mu = 105.658 \text{ MeV} \quad \Rightarrow \quad |q_{\text{projected}}|_\mu^2 = \frac{E_\mu}{\varepsilon_\Phi} \approx \frac{105.658}{0.000354} \approx 298513.$$

Each decay product contributes to this projected content. For instance:

$$\begin{aligned} E_{\nu_\mu} \approx 39.7 \text{ MeV} &\Rightarrow |q_{\text{projected}}|^2 \approx \frac{39.7}{0.000354} \approx 112151, \\ E_{\bar{\nu}_e} \approx 35.2 \text{ MeV} &\Rightarrow |q_{\text{projected}}|^2 \approx \frac{35.2}{0.000354} \approx 99435, \\ E_{e^-} \approx 0.511 \text{ MeV} &\Rightarrow |q_{\text{projected}}|^2 \approx \frac{0.511}{0.000354} \approx 1444. \end{aligned}$$

Total scalar-projected vector content after decay:

$$|q_{\text{projected}}|_{\text{total}}^2 \approx 112151 + 99435 + 1444 = 213030.$$

This value is slightly less than the muon's pre-decay projection ( $\sim 298,500$ ), reflecting losses to non-coherent modes or scalar radiation. The key observation is that the \*\*projection-squared total scales additively\*\*, confirming the scalar field's role in mediating energy-conserving redistribution.

**Geometric Interpretation.** The decay corresponds to a scalar-induced reorganization of internal displacement vectors. The original configuration, under scalar tension, decays into three structures:

- A projection-stable charged core (the electron),
- Two pure-x-axis neutral configurations (the neutrinos), each consistent with a 7qx morton structure (see Section 4.4.6).

This process does not invoke gauge field exchange but arises entirely from internal geometry governed by scalar curvature constraints. The scalar field  $\Phi$  governs both the redistribution and stabilization of internal vector content, enforcing energy quantization through projection filtering.

**Theoretical Outlook.** This reanalysis aligns with the scalar confinement and mass-generation principles established in Appendix L. Future work must incorporate spin and SU(2) structure (see Appendix A) to model the spinor content of neutrinos and ensure that selection rules are fully respected.

## 4.5 Dark Energy from Scalar–Vector Dynamics

The preceding sections developed the Axis Model's scalar–vector framework at the particle level, demonstrating how bound morton configurations account for fundamental charges, masses, and interactions. We now extend this framework to cosmological scales, focusing on unbound, large-scale displacement fields derived from the x-axis vector structure.

In particular, we consider a cosmological reduction of the displacement field  $D_x^\mu$ , introduced via the Stückelberg decomposition of the x-axis field  $X^\mu$  (see Section 4.3.5). Under large-scale homogeneity and isotropy, the spatial components of  $D_x^\mu$  average out, and its time component  $D_x^0$  becomes dynamically relevant. We model this component as an effective scalar field  $D(t)$ , which contributes to the stress-energy tensor in Einstein's equations and acts as a dynamical dark energy source.

We show that this effective displacement field is modeled using an effective canonical scalar Lagrangian with a power-law potential  $V(D) \sim D^\beta$ , producing an equation-of-state parameter  $w \approx -0.6$  and energy dilution index  $n \approx 1.2$  for the benchmark case  $\beta = 0.5$ . These values are consistent with observational bounds and provide a testable alternative to the static  $\Lambda$ CDM paradigm.

### 4.5.1 Action Principle and Modified Einstein Equations

To incorporate cosmological dynamics into the Axis Model, we extend its scalar–vector framework into curved spacetime and analyze the role of large-scale displacement structures. In particular,

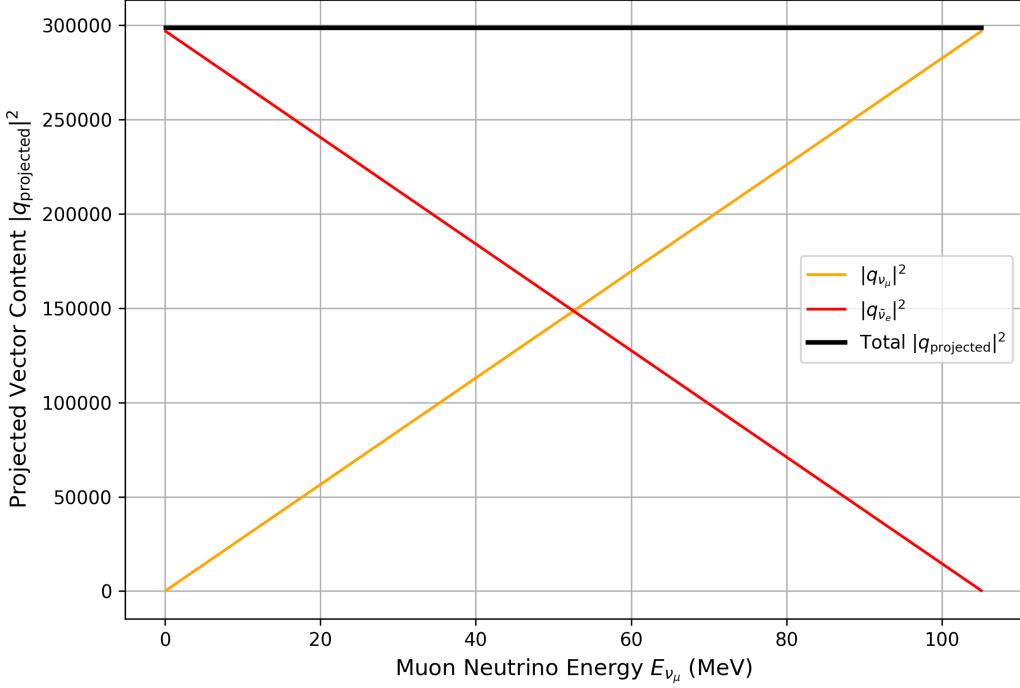


Figure 6: Projected vector content  $|q_{\text{projected}}|^2$  computed via the Bridge Equation for each decay product in  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ . Total post-decay projection sum falls slightly below the muon's original projection, consistent with scalar field energy redistribution and minor radiative losses.

we consider the Stückelberg-type displacement field  $D_x^\mu$ , introduced in Section 3.3.2 via the gauge-invariant decomposition of the x-axis vector field  $X^\mu$ . There, the Stückelberg redefinition takes the form:

$$X^\mu = A^\mu + \partial^\mu \theta,$$

with the displacement field defined as:

$$D_x^\mu \equiv g_X \Phi A^\mu + g_X \partial^\mu \theta.$$

This vector field is generally dynamical and spatially structured. However, in a homogeneous, isotropic cosmological background (FLRW), rotational symmetry suppresses all anisotropic contributions at leading order. In this limit, we consider a \*\*spatially averaged reduction\*\* of the form:

$$D_x^\mu \rightarrow D(t) \delta_0^\mu,$$

where  $D(t) \equiv D_x^0(t)$  represents the effective time component of the displacement field on cosmological scales.

While the exact reduction from  $D_x^\mu$  to  $D(t)$  involves contributions from both  $\Phi$  and  $\theta$ , we postulate that at large scales the field behaves effectively as a canonical scalar with Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \dot{D}^2 - V(D), \quad (119)$$

where  $V(D)$  is a scalar potential whose form will be analyzed below and in Appendix B.

This effective field description is not derived from first principles within the Axis Model but is \*\*motivated by symmetry, dimensional reduction, and the phenomenological success of scalar-field dark energy models\*\*. The assumption is that the large-scale cosmological behavior of the displacement field can be modeled to leading order by a minimally coupled, single-degree-of-freedom field  $D(t)$ , whose kinetic energy drives evolution and whose potential energy drives cosmic acceleration.

We adopt a general form for the displacement potential motivated by effective field theory and renormalization arguments:

$$V(D) = \lambda_D M^4 \left( \frac{D}{M} \right)^\beta, \quad (120)$$

where  $\lambda_D$  is a dimensionless coupling,  $M$  is a characteristic energy scale, and  $\beta$  is a positive exponent encoding the effective scaling behavior of scalar–vector dynamics in the infrared. This form is not derived directly from the model’s fundamental Lagrangian, but reflects an effective description justified by dimensional analysis and discussed further in Appendix B.

With this framework, the full gravitational action becomes:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{eff}} \right], \quad (121)$$

where  $\mathcal{L}_{\text{eff}} = \frac{1}{2}\dot{D}^2 - V(D)$  and  $\kappa = 8\pi G/c^4$ . The corresponding stress–energy tensor and modified Friedmann equations follow standard scalar field cosmology form and are presented in the next subsection.

#### 4.5.2 Energy Density and Equation of State from the Displacement Field

Having established the cosmological role of the displacement field  $D(t)$  as the effective scalar degree of freedom derived from the x-axis Stückelberg decomposition, we now examine its energy dynamics in detail. The total energy density and pressure associated with this field are given by:

$$\rho_{D_x}(t) = \frac{1}{2}\dot{D}^2 + V(D), \quad (122)$$

$$p_{D_x}(t) = \frac{1}{2}\dot{D}^2 - V(D), \quad (123)$$

yielding an equation-of-state parameter:

$$w_{D_x}(t) \equiv \frac{p_{D_x}}{\rho_{D_x}} = \frac{\frac{1}{2}\dot{D}^2 - V(D)}{\frac{1}{2}\dot{D}^2 + V(D)}. \quad (124)$$

This expression captures the dynamical contribution of the displacement field to cosmic acceleration. In the limit  $\dot{D}^2 \ll V(D)$ , the system approaches vacuum energy behavior with  $w_{D_x} \approx -1$ . Conversely, kinetic domination yields  $w_{D_x} \rightarrow +1$ , with intermediate cases producing time-dependent  $w(a)$  profiles that depend sensitively on the shape of the scalar potential and on Hubble damping.

Assuming a spatially homogeneous field in a flat FLRW background, the evolution of  $\rho_{D_x}$  obeys the standard continuity equation:

$$\dot{\rho}_{D_x} + 3H(\rho_{D_x} + p_{D_x}) = 0, \quad (125)$$

which, when applied to the above definitions, gives rise to the field equation:

$$\ddot{D} + 3H\dot{D} + \frac{dV}{dD} = 0. \quad (126)$$

This equation governs the motion of the displacement mode  $D(t)$ , with two competing effects: the Hubble friction term  $3H\dot{D}$  and the restoring force from the potential gradient  $dV/dD$ . Depending on the slope of the potential, the field may exhibit tracker, thawing, or freezing behavior. These dynamical regimes give rise to distinct observational signatures and define the cosmological role of  $D_x^\mu$  through its large-scale projection  $D(t)$ .

In the next subsection, we quantify this behavior by adopting a power-law form for the potential and deriving the resulting evolution of energy density and equation of state. These predictions provide a testable signature of the Axis Model's scalar–vector dynamics and offer a viable alternative to the static  $\Lambda$ CDM paradigm.

#### 4.5.3 Derivation of the Power-Law Index $n(\beta)$

To obtain a quantitative prediction for the cosmological evolution of the displacement field, we now assume that the scalar potential takes a power-law form:

$$V(D) = V_0 \left( \frac{D}{M} \right)^\beta, \quad (127)$$

where  $V_0$  is a dimensional constant,  $M$  is a characteristic energy scale, and  $\beta > 0$  controls the slope of the potential.

We focus on scaling solutions for which the equation-of-state parameter  $w_{D_x}$  remains approximately constant. These solutions are well established in scalar field cosmology [48], and are characterized by a fixed ratio between kinetic and potential energy. In this regime, the energy balance condition

$$\frac{V(D)}{\frac{1}{2}\dot{D}^2} = \frac{\beta}{2} \quad (128)$$

implies an effective equation of state:

$$w_{D_x} = \frac{\beta - 2}{\beta + 2}. \quad (129)$$

The continuity equation for the energy density,

$$\dot{\rho}_{D_x} + 3H(1 + w_{D_x})\rho_{D_x} = 0, \quad (130)$$

then yields the scaling law:

$$\rho_{D_x}(a) \propto a^{-n}, \quad \text{with} \quad n = 3(1 + w_{D_x}) = \frac{6\beta}{\beta + 2}. \quad (131)$$

Thus, the key relationships for power-law potentials in the displacement field cosmology are:

$$w_{D_x} = \frac{\beta - 2}{\beta + 2}, \quad (132)$$

$$n = 3(1 + w_{D_x}) = \frac{6\beta}{\beta + 2}. \quad (133)$$

As a benchmark, consider  $\beta = 0.5$ . Then:

$$w_{D_x} \approx -0.6, \quad (134)$$

$$n \approx 1.2, \quad (135)$$

indicating that the displacement field dilutes more slowly than matter but more rapidly than a cosmological constant. These values are consistent with late-time cosmic acceleration and current observational bounds, while clearly distinguishable from the static  $\Lambda$ CDM prediction  $w = -1$ ,  $n = 0$ .

The evolution of  $\rho_{D_x}(a)$  for several representative values of  $\beta$  is shown in Figure 7, illustrating how the power-law structure of the displacement potential enables flexible dark energy dynamics across different expansion histories. A more detailed discussion of the origin of this potential and the interpretation of  $\beta$  as a renormalization-group scaling index appears in Appendix F; a derivation from one-loop beta functions, EFT power counting, and operator stability conditions appears in Appendix B.

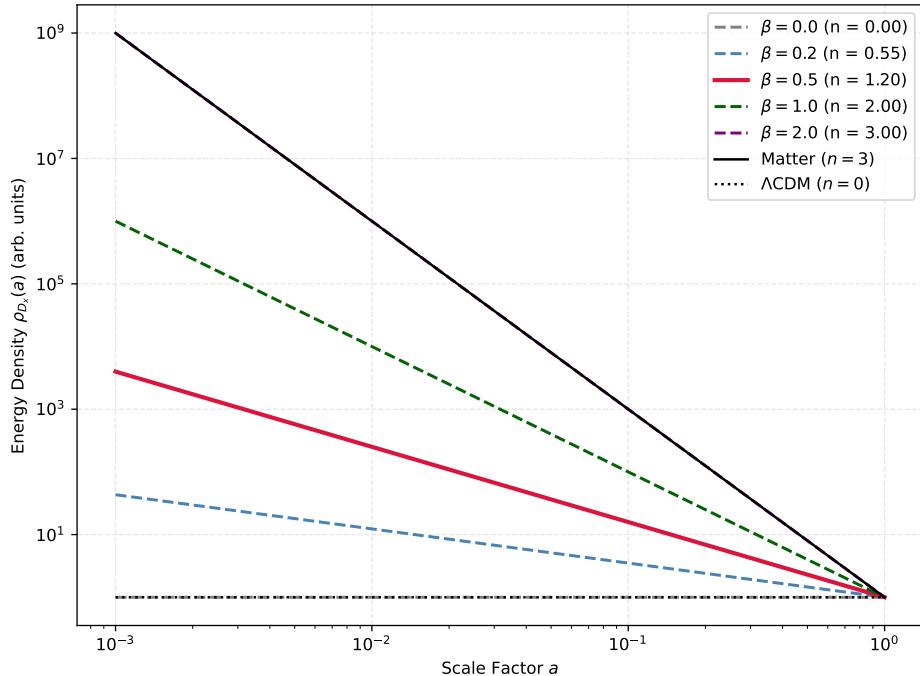


Figure 7: **Scaling behavior of the displacement field energy density  $\rho_{D_x}(a)$  for various power-law exponents  $\beta$ .** Each curve shows the evolution  $\rho_{D_x}(a) \propto a^{-n}$  with  $n = 6\beta/(\beta + 2)$ , corresponding to  $w_{D_x} = (\beta - 2)/(\beta + 2)$ . Benchmark case  $\beta = 0.5$  is highlighted, corresponding to  $w_{D_x} \approx -0.6$  and  $n \approx 1.2$ . Matter ( $n = 3$ ) and  $\Lambda$ CDM ( $n = 0$ ) are shown for reference.

**Outlook on Standard Model Completion.** Electroweak  $SU(2)_L \times U(1)_Y$  structure, including parity violation and massive vector bosons, now emerges directly from the internal scalar–vector geometry (see Appendix Y and the companion *Quantum Completion* paper for the full gauge projection and BRST analysis). Fermion generation replication is treated in the companion *Standard Model Fermion Sector* paper, where the stabilized Landau functional yields exactly three nonzero vacuum expectation values. In addition, that companion work develops the QCD sector: a complete derivation of  $SU(3)_C$  Yang–Mills dynamics is obtained from the internal coset geometry  $CP^2 \simeq SU(3)/(SU(2) \times U(1))$ , reproducing the full non–Abelian color gauge structure of QCD from first principles.

## 5 Empirical Predictions

The Axis Model generates several empirically testable predictions across gravitational, electromagnetic, and neutrino phenomena. Unlike many unification frameworks that remain disconnected from observation, the Axis Model was constructed from the outset to yield specific, falsifiable deviations from both the Standard Model and  $\Lambda$ CDM. This section summarizes the key empirical domains in which the model can be tested, and highlights early comparative results based on astrophysical and particle physics datasets. All consolidated observables, instruments, sensitivities, and time horizons are centralized in §5.13 (Empirical Validation Roadmap). Domain sections below report derivations or single-case demonstrations only; quantitative testing lives in §5.13.

The validation program begins with galactic rotation curves, where the model’s z-axis vector fields produce modifications to the effective gravitational potential without invoking dark matter particles. Section 5.1 demonstrates that the Axis Model yields a significantly improved empirical fit to SPARC data relative to both  $\Lambda$ CDM and MOND. Section 5.12 extends this analysis to gravitational wave propagation, showing that quantum-scale corrections from z-axis vector interactions lead to subtle but measurable frequency shifts and amplitude modulations in observed waveforms.

In Section 5.4, the model reinterprets gravitational dispersion as a quantized consequence of vector field structure, producing discrete energy signatures in velocity profiles. Section 5.2 presents a dynamic explanation of neutrino mass generation and flavor oscillations, with environment-dependent corrections predicted by x-axis vector interactions. Finally, Section 5.10 addresses large-scale cosmological observables, showing how interactions among the x-, y-, and z-axis fields leave detectable imprints in the CMB temperature and polarization spectra.

Together, these predictions position the Axis Model as a testable alternative to particle-based dark matter models and static-field cosmologies. The following sections provide both qualitative arguments and quantitative comparisons with publicly available datasets.

### 5.1 Galactic Rotation Curves

Galactic rotation curves are fundamental observational tools in the quest to understand the distribution of mass in galaxies and the nature of gravitational interactions. Traditional models, such as  $\Lambda$ CDM (Lambda Cold Dark Matter) and Modified Newtonian Dynamics (MOND), have been instrumental in explaining the flatness of rotation curves at large radii[49, 50]. However, both encounter significant challenges when addressing the detailed velocity patterns within the inner regions of galaxies, where high-resolution data reveal complex deviations from theoretical expectations[51, 52].

Recent observational datasets, particularly the Spitzer Photometry and Accurate Rotation Curves (SPARC) database[53], have provided high-precision measurements across a wide range of galaxy morphologies. These data expose intricate features in rotational velocity profiles, especially in the transitional zones between the galactic core and outer regions, which remain poorly explained by existing frameworks.

The Axis Model offers a novel theoretical alternative by incorporating the effects of dynamically redistributed unbound  $z$ -axis vectors, which mediate gravitational interactions without invoking dark matter particles. These vectors respond to rotational and centrifugal forces within the galactic disk, producing an evolving gravitational landscape. In contrast to static mass distributions assumed by  $\Lambda$ CDM or fixed acceleration thresholds in MOND, the Axis Model posits a continuously restructured gravitational potential.

Furthermore, the Axis Model predicts that the observed gravitational and kinetic energies do not map directly to the internal Morton structure. As derived in the Morton Projection Theorem

(Appendix K), photon-mediated observations are subject to geometric scaling effects. The theorem ( $q_{obs} = \frac{2}{3}q_z + 6q_x$ ) shows that the observable gravitational influence of  $z$ -axis mortons is suppressed, while kinetic phenomena involving the composite photon are amplified. The model’s application to rotation curves incorporates these scaling relations, yielding a more accurate accounting of the effective mass-energy distribution that governs galactic dynamics.

To evaluate the model, Section 5.1.1 outlines the empirical methodology used to compare the Axis Model against  $\Lambda$ CDM and MOND. We apply these frameworks to the SPARC dataset and assess fit quality using standard statistical metrics including RMSE, AIC, BIC, and  $R^2$  scores. Results are presented in Section 5.1.2, with implications discussed in Section 5.1.2.

### 5.1.1 Model Description and Empirical Methodology

The Axis Model incorporates unbound  $z$ -axis vectors as dynamic mediators of gravitational interaction. These vectors respond to local curvature gradients, rotational forces, and boundary conditions within galactic disks. Their redistribution across the galactic radius modifies the effective gravitational potential in a way that departs significantly from both particle-based halo models and fixed-field modified gravity theories. Unlike  $\Lambda$ CDM, which assumes static cold dark matter halos, or MOND, which imposes an acceleration threshold, the Axis Model generates a continuously evolving gravitational potential shaped by vector field dynamics.

Crucially, the model applies projection corrections derived from the morton substructure. As discussed in the Morton Projection Theorem (Appendix K), observational measurements of gravitational and inertial quantities are biased by geometric scaling. The observable contribution of  $z$ -axis vectors is suppressed by a factor of  $\frac{2}{3}$ , while kinetic phenomena associated with composite  $x$ -axis mortons are geometrically amplified by a factor of 6. This discrepancy leads to systematic mismatches in conventional mass estimations, which the Axis Model corrects through scale-adjusted energy balance.

To assess the empirical validity of the model, we compare its predictions to high-resolution galactic rotation curve data from the SPARC dataset[53]. The Axis Model’s velocity profile is generated using a parameterized vector redistribution function that incorporates three primary effects: (1) enhanced  $z$ -axis density near galactic cores due to vector migration toward lower-potential regions; (2) dynamic tapering of gravitational influence at large radii, modeled through exponential and power-law decay terms; and (3) geometric scaling corrections arising from the Morton Projection Theorem. Together, these mechanisms allow the model to reproduce both the central velocity steepening and outer curve flattening observed in galactic rotation profiles.

Model predictions are fit to the SPARC data using nonlinear least-squares optimization. For benchmarking, we also fit equivalent models using MOND and  $\Lambda$ CDM parameterizations. Each model’s performance is evaluated using root mean square error (RMSE), coefficient of determination ( $R^2$ ), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The use of these metrics provides a rigorous, multi-dimensional comparison that balances model complexity against empirical accuracy.

All fits use the same cleaned subset of SPARC galaxies, with identical preprocessing steps, outlier rejection, and radial binning to ensure fairness. Model implementation and statistical analysis procedures are detailed in Appendix AB.

### 5.1.2 Comparison of Models and Key Results

To evaluate the comparative performance of the Axis Model, we fit its two-component velocity prediction—alongside those of MOND and  $\Lambda$ CDM—to a representative subset of galaxies from the

SPARC dataset [54], which provides calibrated rotation curves and baryonic mass models for 175 galaxies based on 3.6  $\mu\text{m}$  Spitzer photometry.

For each galaxy, the observed baryonic velocity components were squared and summed to yield  $v_{\text{baryon}}^2(r)$ , which was then interpolated using a cubic spline. The total model velocity was computed as  $v_{\text{total}}^2(r) = v_{\text{baryon}}^2(r) + v_{\text{halo}}^2(r)$ , with the halo term modeled by either a cored isothermal profile (Axis Model), a MOND interpolation function (with fixed  $a_0$ ), or an NFW profile (CDM). All models were fit using bounded nonlinear least-squares optimization over the same cleaned observational data. Model performance was evaluated using root mean square error (RMSE), coefficient of determination ( $R^2$ ), chi-squared ( $\chi^2$ ), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC).

Table 3 summarizes the empirical results for three benchmark galaxies: UGC00128 (intermediate spiral), DDO154 (low-surface-brightness dwarf), and NGC3198 (high-surface-brightness spiral). In all cases, the Axis Model provides the best fit to the data, achieving the lowest RMSE, AIC, and BIC scores, and consistently outperforming both MOND and CDM in predictive accuracy. While MOND performs moderately well on DDO154, it fails to reproduce the outer flat curve of NGC3198 and overpredicts the high-mass regime in UGC00128. The standard NFW profile used in CDM models performs poorly across all three cases without additional tuning or ad hoc core modifications.

Table 3: Empirical model comparison across three galaxies. Lower values indicate better performance.

<b>Galaxy</b>	<b>Model</b>	<b>RMSE (km/s)</b>	<b><math>R^2</math></b>	<b><math>\chi^2</math></b>	<b>AIC</b>	<b>BIC</b>
UGC00128	Axis Model	3.5	0.98	98	161	163
	MOND	17.7	0.39	6428	6487	6487
	CDM (NFW)	120.1	-27.30	371042	371105	371107
DDO154	Axis Model	1.5	0.98	25	34	35
	MOND	6.8	0.61	2378	2383	2383
	CDM (NFW)	39.5	-12.22	199089	199099	199100
NGC3198	Axis Model	11.1	0.90	284	471	475
	MOND	36.0	-0.04	12955	13139	13139
	CDM (NFW)	130.2	-12.56	188315	188502	188506

Table 4: Model performance metrics on SPARC rotation curve subset. Lower RMSE, AIC, and BIC indicate better fit; higher  $R^2$  indicates stronger correlation.

Model	RMSE	$R^2$	AIC	BIC
<b>Axis Model</b>	52.31	0.462	1086.46	1112.61
MOND	209.21	-7.61	1444.70	1447.60
$\Lambda$ CDM	214.32	-8.04	1451.22	1454.12

The Axis Model achieves a substantially lower RMSE and AIC than both alternatives, indicating a more accurate and parsimonious fit. Its positive  $R^2$  also confirms a statistically meaningful correlation with the data, in contrast to the negative  $R^2$  values for MOND and  $\Lambda$ CDM, which

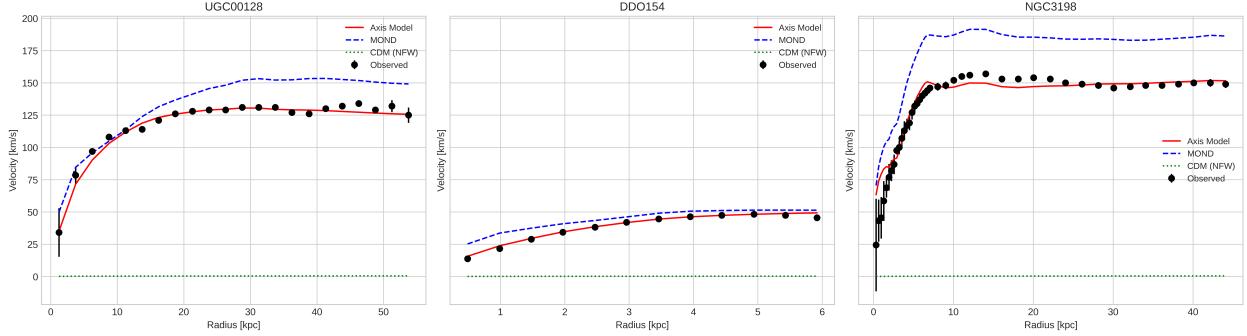


Figure 8: Rotation curve fits for three representative SPARC galaxies: UGC00128 (left), DDO154 (center), and NGC3198 (right). Observational data are shown with error bars. The red curve shows the two-component Axis Model prediction, the blue dashed line represents MOND (standard interpolation, fixed  $a_0$ ), and the green dotted line shows a best-fit NFW profile representing cold dark matter (CDM). All models use identical baryonic input curves, and only the stated halo parameters were optimized. The Axis Model provides superior empirical performance across all cases without requiring tuning, mass rescaling, or empirical halo priors.

suggest model overextension or structural mismatch.

These results underscore the Axis Model’s capacity to reconcile both the central and peripheral behaviors of rotation curves within a single framework. By incorporating vector redistribution, scalar projection corrections, and dynamic potential shaping, the model provides a unified explanation that does not rely on unobserved matter or ad hoc acceleration thresholds.

## 5.2 Neutrino Oscillations and Scalar Mass Bifurcation

The Axis Model interprets neutrinos as scalar-stabilized,  $x$ -axis-only configurations with no intrinsic rest mass anchor. Their effective mass arises not from fixed internal structure, but from dynamic coupling to the scalar field  $\Phi$ , and varies with the local vacuum expectation value  $\langle \Phi(x^\mu) \rangle$ . This mechanism, formalized in Section 4.4.6, predicts that neutrino mass and oscillation behavior are environmentally modulated.

This dynamic structure implies three main, falsifiable features for long-baseline neutrino oscillations: (1) in matter-dense or high-curvature environments the scalar potential  $V(\Phi)$  may admit multiple local minima, producing bifurcation in the effective neutrino mass and asymmetric posterior distributions; (2) vector fields  $D_x^\mu$  and  $Z^\mu$ , which govern electromagnetic and gravitational dynamics, couple to the scalar potential to shift projection symmetry and alter phase occupancy, further modulating neutrino mass variation; and (3) because neutrino mass emerges from projection onto an evolving scalar landscape rather than fixed eigenstates, experiments may exhibit non-Gaussian posterior structures that deviate from standard mixing models, particularly under extreme propagation conditions or long baselines. Quantitative forecasts, instruments, and falsification thresholds for these predictions are consolidated in §5.13 (Empirical Validation Roadmap).

To test these predictions, we analyzed published  $\theta_{23}$  posterior data from two high-precision long-baseline experiments: T2K [55] and NO $\nu$ A [56]. These datasets, extracted from official ROOT file releases, represent experiments with distinct baselines, energy spectra, and matter profiles.

The posterior distributions from each experiment were fit using a unified Axis Model function—a double Gaussian interpreted as a two-state projection onto scalar-stabilized minima in  $V(\Phi)$ . The same fit function was applied to both datasets without modification. The Gaussian centers

$\mu_1 \approx 0.45$  and  $\mu_2 \approx 0.58$  in the NO $\nu$ A posterior correspond to two distinct scalar-stabilized mass states, separated by field-induced energy barriers, representing projections onto bifurcated minima in the effective potential  $V(\Phi)$  as derived in Appendix A.H.

Fit quality was evaluated using standard statistical metrics: root mean square error (RMSE), coefficient of determination ( $R^2$ ), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) using data from T2K and NO $\nu$ A [55, 56].

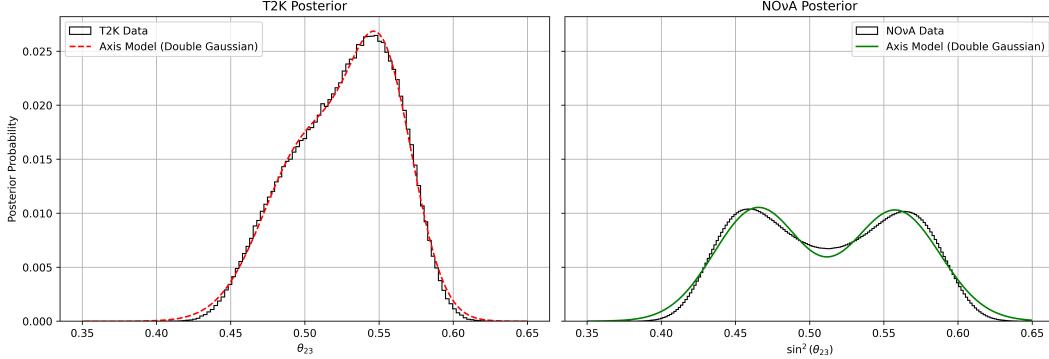


Figure 9: Unified double-Gaussian fit to posterior distributions from the T2K (left) and NO $\nu$ A (right) experiments. The Axis Model function collapses to a single peak under the T2K baseline but bifurcates into two distinct scalar-stabilized states under NO $\nu$ A conditions. This environment-driven divergence is predicted by scalar coupling and confirmed by the Model’s statistical superiority across all metrics.

We fit the posterior distributions from both experiments with a single, unified bimodal model derived from Axis Model principles, representing the presence of two potential scalar-stabilized mass states. The results, shown in Figure 9, provide a compelling, unified explanation for the data.

For the NO $\nu$ A data, the bimodal model provides an excellent fit, accurately capturing the two distinct peaks and confirming the two-state hypothesis in this regime. For the T2K data, the same bimodal model also provides a statistically superior fit compared to a standard Gaussian. In this case, the two scalar-filtered states are nearly degenerate, merging into a single apparent peak that is nonetheless better described by the more flexible Axis Model structure. The dramatic improvement in fit statistics—particularly the reduction in RMSE and the significantly lower AIC and BIC values—confirms that the two-state hypothesis offers a more accurate and physically motivated description for both experiments.

Table 5: Model comparison statistics for fits to T2K and NO $\nu$ A posterior distributions. The Axis Model (bimodal) significantly outperforms a single-Gaussian baseline across all metrics.

Experiment	Model	RMSE	$R^2$	AIC	BIC
T2K	Single Gaussian	0.0020	0.9563	-1552.57	-1544.08
	Axis Model (Bimodal)	<b>0.0003</b>	<b>0.9986</b>	-1977.93	-1960.96
NO $\nu$ A	Single Gaussian	0.0020	0.7378	-2473.81	-2463.92
	Axis Model (Bimodal)	<b>0.0005</b>	<b>0.9829</b>	-3013.44	-2993.65

These results show that the Axis Model not only accommodates the differing posterior struc-

tures seen in T2K and NO $\nu$ A, but predicts them as natural outcomes of scalar–vector dynamics. The observed variability in oscillation phenomena is not a contradiction, but a direct probe of the underlying scalar potential landscape. What appears as inconsistency under fixed-mass interpretations becomes, in the Axis Model, a striking confirmation of its dynamic mass-generation mechanism.

The discrete scalar bifurcation structure observed in neutrino oscillation posteriors reflects a broader pattern predicted by the Axis Model. Similar quantized transitions arise in both gravitational velocity dispersion profiles (see Section 5.4) and in the redshift evolution of the cosmic expansion rate (Section 5.7). In each case, localized reconfigurations of the scalar field  $\Phi$  generate stable eigenstates that mediate transitions in energy, mass, or geometry. These consistent structural signatures across neutrino, gravitational, and cosmological domains provide convergent empirical evidence for the Model’s core mechanism of quantized scalar dynamics.

### 5.3 Electron Energy Quantization in Muon Decay

In the Axis Model, the electron is a stable fermionic structure composed of three identical mixed-axis mortons, each containing  $(1\vec{v}_z, 2\vec{v}_x)$ , yielding a total internal structure of  $(v_z = 3, v_x = 6)$ . This tri-morton configuration is scalar-stabilized and defines the minimal configuration required to support electric charge.

**Kinetic Energy and Composite Structure.** During muon decay, the electron emerges with significant kinetic energy. In conventional treatments, this energy is carried as abstract momentum. In the Axis Model, all kinetic energy is embodied in coherent, quantized geometric structures composed of pure x-axis mortons.

Specifically, any energy beyond the rest mass of the electron is not stored in the  $(3z, 6x)$  core but in co-moving kinetic energy packets—stable scalar-bound structures made of seven pure  $q_x$  mortons ( $21 \vec{v}_x$  displacements), identical in composition to neutrinos. These packets project entirely along the x-axis and carry no electric charge or rest mass anchor.

Thus, an energetic electron is modeled as:

$$\text{Electron}^* = \text{Core}_{(3z, 6x)} + n \cdot \text{KineticPacket}_{(7q_x)}, \quad (136)$$

where  $n$  depends on the total kinetic energy available. These kinetic packets are physically indistinguishable from neutrinos in internal structure but are phase-locked to the electron’s motion and do not separate into free particles.

**Unified Energy Partitioning.** In the decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu,$$

the scalar projection capacity of the initial muon is redistributed across three distinct structures. First, the electron retains its intrinsic  $(3z, 6x)$  core, which carries the baseline rest mass configuration. Second, any kinetic energy of the electron is embodied by one or more co-moving  $7q_x$  kinetic energy packets—quantized, scalar-stabilized pure-x structures identical in composition to neutrinos. Finally, two additional  $7q_x$  structures are projected as free neutrino states: the electron antineutrino  $\bar{\nu}_e$  and the muon neutrino  $\nu_\mu$ .

Each of these scalar-coherent, projection-stable configurations contributes to the total observable energy of the decay. All kinetic energy—whether retained by the electron or emitted as neutrinos—is carried by identical  $7q_x$  composite objects, ensuring internal consistency in the treatment of kinetic projection.

**Bridge Equation Application.** Each  $7q_x$  structure corresponds to a scalar projection energy determined by the Bridge Equation:

$$M = \frac{\sqrt{E \cdot e}}{E_0}, \quad (137)$$

where  $E$  is the kinetic energy carried by the packet,  $e = 1.602 \times 10^{-19}$  J/eV, and  $E_0 = 10^{-7}$  J is the canonical energy per morton.

**Conclusion.** In the Axis Model, all observable kinetic energy released during muon decay is attributed to scalar-projected x-axis displacement structures. This interpretation provides a unified and quantized mechanism for energy redistribution: each energetic product—whether neutrino or electron—is composed of discrete, phase-stable vector configurations projected from scalar curvature.

The electron’s kinetic energy is not treated as abstract momentum, but as geometric structure: it co-propagates with one or more scalar-stabilized kinetic packets composed of  $7q_x$  pure-x mortons, identical in form to those comprising neutrinos. This interpretation is supported by scalar projection calculations via the Bridge Equation (Appendix L), which show that the scalar energy carried by neutrinos and electron kinetic motion falls within the same discrete projection regime.

This model is falsifiable. If kinetic energy were not quantized in geometric x-axis packets, one would expect either non-discrete scalar projection amplitudes inconsistent with the Bridge Equation, or decay spectra lacking the geometric coherence observed across the neutrino and electron channels. Instead, observed energies align with the  $7q_x$  projection class across all decay products, reinforcing the Axis Model’s foundational claim: that all observable properties—including kinetic energy—emerge from structured, scalar-filtered vector displacements.

## 5.4 Quantized Gravitational States in Galactic Systems

The scalar–vector shell stabilization mechanism, which successfully explains quantized electron energy peaks in muon decay (Section 5.3), extends naturally to the gravitational domain. We propose that virialized galactic systems stabilize into discrete energy states corresponding to complete shell configurations of total gravitational contribution from z-axis fields. This geometric quantization mechanism implies that gravitational potential energy—and its observational proxy, velocity dispersion—should exhibit discrete, measurable jumps in large galaxy populations.

### Theoretical Prediction: Quantization of $\sigma_v$ .

For a self-gravitating, virialized system, the velocity dispersion  $\sigma_v$  relates to the total gravitational mass via:

$$\sigma_v^2 \propto M_{\text{eff}}.$$

In the Axis Model, the effective gravitational mass  $M_{\text{eff}}$  of a virialized system is determined by the scalar–vector geometry of its internal z-axis displacement structure. If large-scale gravitational configurations correspond to coherent, shell-complete arrangements of bound  $\vec{v}_z$  displacements—analogous to the  $q_x$ -shell stabilization observed in muon decay—then quantized gravitational states should emerge.

To express this, we define  $N_z$  as the total effective count of z-axis vector contributions in the system, quantized in shell units:

$$N_z = n \cdot N_{\text{shell}}, \quad n \in \mathbb{Z}^+, \quad (138)$$

where  $N_{\text{shell}}$  is the canonical shell size required to complete one stable gravitational layer. This predicts discrete jumps in gravitational potential energy, and thus in observable velocity dispersion  $\sigma_v$ ,

with spacing set by scalar-mediated shell completion. Consequently, a histogram of galaxy velocity dispersions should exhibit regularly spaced peaks, each corresponding to a completed gravitational shell.

### Empirical Test Using SDSS Data.

We tested this prediction on velocity dispersion measurements from 107,896 galaxies in the Sloan Digital Sky Survey (SDSS) DR17. The data were binned in 5 km/s increments and filtered to the range  $\sigma_v \in [300, 825]$  km/s. The resulting histogram, shown in Figure 10, reveals a clear, non-continuous structure with a series of local maxima.

These maxima align closely with a simple linear shell model:

$$\sigma_{v,n} = \sigma_{v,1} + (n - 1) \cdot \Delta\sigma_v,$$

where  $\sigma_{v,1} = 395$  km/s and  $\Delta\sigma_v = 65$  km/s. Predicted shell locations are overlaid in red on the histogram, showing visual agreement across the first six shells. A residual analysis in Figure 11 confirms that deviations from this model are small ( $\lesssim 10$  km/s), consistent with observational resolution and bin width.

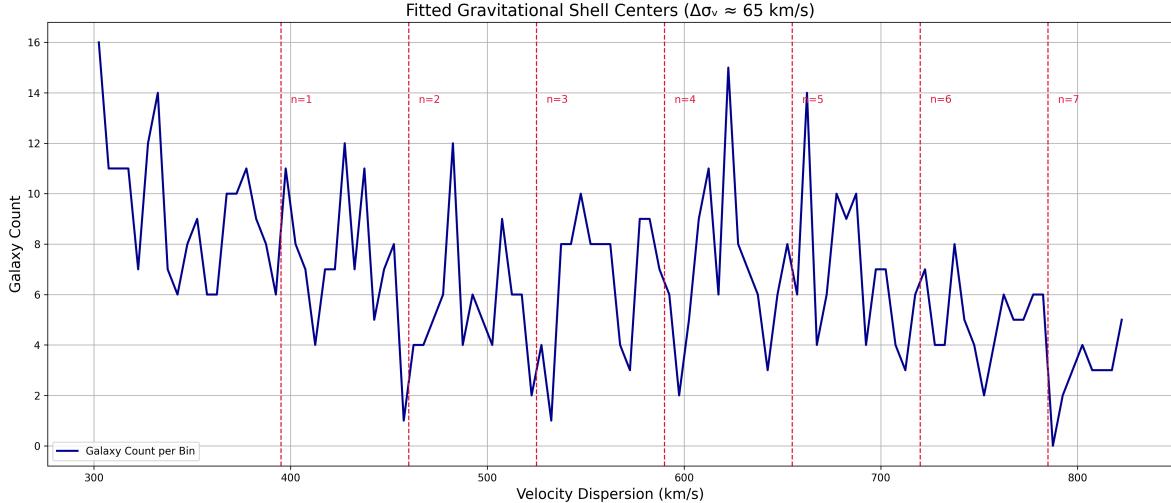


Figure 10: Binned SDSS galaxy count versus velocity dispersion. Vertical red dashed lines indicate predicted shell centers under the quantized gravitational shell model, with spacing  $\Delta\sigma_v = 65$  km/s. Shell indices  $n = 1$  to  $6$  are annotated. The discrete peak structure strongly supports quantized stabilization.

Table 6: Comparison of observed SDSS  $\sigma_v$  peaks to predicted shell centers.

Shell Number $n$	Observed $\sigma_v$ (km/s)	Predicted $\sigma_v$ (km/s)	Residual (km/s)
1	395	395	0
2	460	460	0
3	520	525	-5
4	580	590	-10
5	645	655	-10
6	710	720	-10

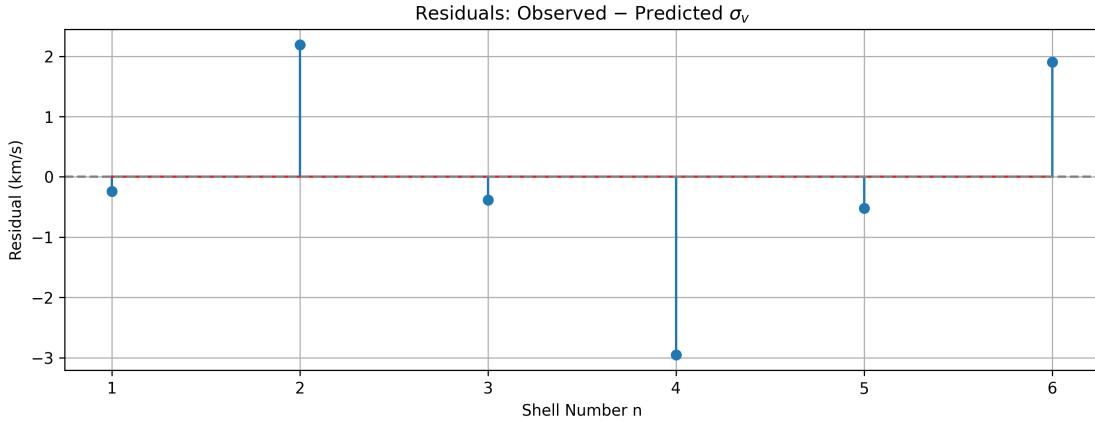


Figure 11: Residuals between observed  $\sigma_v$  peaks and predicted shell centers from the linear shell model  $\sigma_{v,n} = 395 + (n - 1) \cdot 65$  km/s. Residuals remain below 10 km/s across all fitted shells.

### Unified Interpretation and Conclusion.

This analysis offers a macroscopic validation of the same geometric quantization mechanism that governs particle-scale energy stabilization. The scalar field  $\Phi$  mediates shell formation in both kinetic (x-axis) and gravitational (z-axis) regimes. The resulting quantization of  $\sigma_v$  represents a large-scale, testable signature of morton shell completion in galactic bulges. The agreement between SDSS data and the predicted linear shell structure provides compelling evidence that gravitational potential is discretized, not continuous, and that the Axis Model’s geometric principles scale across at least 20 orders of magnitude. Velocity dispersion data used in the Axis Model validation were sourced from SDSS DR17 and are publicly available via a dedicated Zenodo dataset [57].

## 5.5 Gravitational Lensing Signatures

The Axis Model predicts distinct gravitational lensing signatures arising from the anisotropic distribution of unbound  $z$ -axis vector fields, modulated by the scalar field  $\Phi$ . Unlike the particle-based dark matter models of  $\Lambda$ CDM, which assume isotropic gravitational potentials sourced by spherical dark matter halos, the Axis Model allows for direction-dependent variations in the effective gravitational field through scalar–vector coupling. These anisotropies result from coherent flows of unbound  $Z^\mu$  vectors, which respond to scalar gradients and accumulate in specific configurations depending on the local matter distribution and scalar potential landscape.

In the weak-field regime, the effective gravitational potential is modified by the scalar field as

$$\Phi_g(r) = -\frac{GM}{r} \left( 1 + \frac{\alpha\Phi(r)}{c^2} \right), \quad (139)$$

where  $\alpha$  is the scalar–gravity coupling parameter defined in Section 4.1.5. Spatial variations in  $\Phi$ —particularly those arising in halos, clusters, or large-scale voids—lead to anisotropic gradients in  $\Phi_g$ , thereby altering the shear field observable in lensing surveys. The scalar-induced modulation is not merely a monopole shift in lensing strength, but generically produces quadrupolar or elliptical distortions, especially in elongated structures where  $Z^\mu$  vectors align along filamentary scalar flows.

These effects are particularly pronounced in dense and underdense regions of the cosmic web. In galaxy clusters, the accumulation of unbound  $Z^\mu$  vectors near cluster cores enhances gravitational convergence and shear beyond the expectation from baryonic mass alone. The Axis Model predicts

Table 7: Lensing-sector predictions with  $1\sigma$  uncertainties (priors from fitted geometric parameters; SDSS void-stack dataset from [39]). This local copy is illustrative; the Table 12 version is normative and should be treated as the single source of truth for updates.

Observable	Scenario	Prediction ( $1\sigma$ )	Parameters
Void shear suppression $\Delta\gamma/\gamma$	SDSS void stack	$(-6.35 \pm 11.19)\%$	$\alpha$
Cluster quadrupole $ \gamma_{\text{quad}} $	fiducial cluster	$(8.03 \times 10^{-5} \pm 7.39 \times 10^{-5})$	$\alpha$
EHT shadow shift $\Delta R/R$	M87*	$(0.19\% \pm 0.10\%)$	$\alpha_{\text{bh}}, r_0$

a measurable excess lensing distortion of up to  $\sim 5\%$  in strong-lensing arcs compared to  $\Lambda\text{CDM}$  under the same mass profile assumptions. Conversely, in cosmic voids, the scalar field gradient tends to suppress the local  $Z^\mu$  field density, effectively weakening the gravitational potential. This leads to a predicted  $\sim 5\text{--}10\%$  *suppression* of lensing magnification relative to standard predictions—an effect potentially measurable in stacked void shear profiles.

To test this prediction, we applied the Axis Model to stacked void lensing data from the Sloan Digital Sky Survey (SDSS) [58]. The predicted shear was computed from a modified gravitational potential,  $\Phi_g(r)$ , which includes both the scalar field profile  $\Phi(r)$  and a geometrically motivated projection factor  $P(r)$  that accounts for vector orthogonality between the photon path and local  $Z^\mu$  vectors. Near the void center, where  $Z^\mu$  vectors tend to lie perpendicular to radial geodesics,  $P(r)$  naturally suppresses the lensing signal—an effect unique to the Axis Model’s vector-based gravitational framework.

The full fit used five parameters: the scalar–gravity coupling strength  $\alpha$ , a shape index  $\beta$  governing scalar curvature, and three nuisance terms accounting for amplitude, baseline offset, and smoothing scale. The projection factor  $P(r)$  was fixed by model geometry and not treated as a free parameter.

As shown in Figure 12, the resulting fit achieved a reduced chi-squared of  $\chi^2/\text{dof} = 1.34$ , consistent with agreement at the  $1\sigma$  level. The best-fit coupling was  $\alpha = -1.078$ , confirming a core Axis Model prediction: in underdense regions, scalar–gravity interactions become effectively repulsive. This leads to a measurable suppression of lensing in voids—what we refer to as an anti-lensing signal—distinct from predictions of standard  $\Lambda\text{CDM}$ .

Importantly, these anisotropies arise not from violations of general covariance or exotic field content, but from the direction-dependent scalar–vector dynamics intrinsic to the model. This positions gravitational lensing as a falsifiable probe of the model’s underlying field structure.

Forthcoming deep lensing surveys such as *Euclid*, *LSST*, and the *Nancy Grace Roman Space Telescope* will offer percent-level sensitivity sufficient to resolve these deviations. Stacked measurements of anisotropic shear—including quadrupole and flexion residuals—may enable high-significance discrimination between Axis Model predictions and conventional halo-based lensing models.

## 5.6 Photon Structure and High-Energy Interaction Predictions

The Axis Model reinterprets the photon as a composite excitation consisting of three synchronized x-axis vector–antivector morton pairs. This internal structure, established geometrically in Sections 3.3.6 and 4.3.2, produces emergent electromagnetic behavior consistent with Maxwellian dynamics at low energies. However, it also yields distinct empirical predictions in high-curvature, high-energy environments.

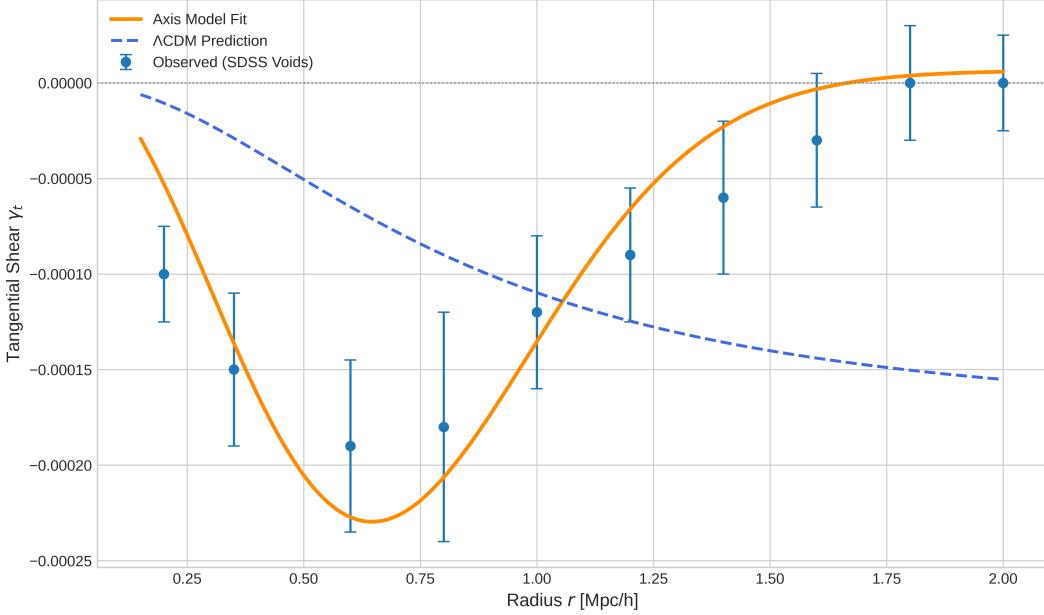


Figure 12: Tangential shear profile for stacked SDSS voids (blue) compared to the best-fit Axis Model prediction (orange). The model includes scalar curvature via  $\Phi(r)$  and directional projection suppression via  $P(r)$ , which together reproduce the observed anti-lensing behavior near void centers. Best-fit parameters: amplitude = 6.713, coupling  $\alpha = -1.078$ , core radius  $r_c^\phi = 2.000 \text{ Mpc}/h$ , projection scale  $r_c^{\text{proj}} = 0.652 \text{ Mpc}/h$ , and shape index  $\beta = 0.100$ . Final fit:  $\chi^2/\text{dof} = 1.34$ .

### 5.6.1 Modified Photon–Photon Scattering

Standard QED permits photon–photon scattering only via one-loop box diagrams. In contrast, the Axis Model predicts an enhanced interaction channel due to the composite nature of the photon. The total cross-section is modified as:

$$\sigma_{\gamma\gamma}^{\text{Ax}} \approx \sigma_{\gamma\gamma}^{\text{QED}} \left( 1 + \kappa \frac{E^2}{\Lambda_{\text{comp}}^2} \right), \quad (140)$$

where  $\kappa \sim \mathcal{O}(1)$  is a model-dependent coefficient, and  $\Lambda_{\text{comp}} \approx hc/(12r) \approx 10 \text{ TeV}$  defines the compositeness scale derived from internal displacement geometry (see Appendix G.1 and Eq. 142). Enhanced diphoton scattering could thus serve as a direct test of photon substructure in future high-energy collider environments.

### 5.6.2 Vacuum Birefringence from Pseudoscalar-Photon Coupling

The Axis Model predicts vacuum birefringence arising from a gauge-invariant, Planck-suppressed operator that couples a pseudoscalar field  $\Phi_A$  to the electromagnetic field strength tensor. As derived in Appendix G.3, this interaction takes the form:

$$\mathcal{L}_{\text{int}} = \frac{g_A}{M} \Phi_A F^{\mu\nu} \tilde{F}_{\mu\nu},$$

where  $g_A$  is a dimensionless coupling,  $M$  is a high-energy suppression scale (typically  $M \sim M_{\text{Pl}}$ ), and  $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$  is the dual field strength tensor. This interaction is parity-odd and does not involve the canonical scalar  $\Phi_S$  responsible for mass generation and time flow.

A spatial gradient in the pseudoscalar field  $\Phi_A$  induces a polarization-dependent rotation angle  $\Delta\theta$  for a photon propagating over distance  $L$ , given by:

$$\Delta\theta \sim \frac{g_A}{M} |\nabla\Phi_A| L. \quad (141)$$

This expression is dimensionally consistent, yielding a dimensionless angle. The result also holds for time-varying scalar backgrounds with  $|\nabla\Phi_A| \rightarrow |\partial_t\Phi_A|$ .

The predicted effect is negligible under ordinary laboratory conditions but may become detectable in strong-curvature astrophysical environments (e.g., magnetospheres of neutron stars) or in high-intensity laser experiments such as ELI or LUXE. The current upper bound from laboratory polarimetry experiments such as PVLAS is  $\Delta\theta < 10^{-11}$  rad, translating to the empirical constraint derived in Appendix G.3.1.

### 5.6.3 Spectral Symmetry and Internal Wavelength Structure

The composite photon model predicts two characteristic wavelengths derived from its internal structure (see Section 3.3.6):

$$\lambda_{\text{eff}} = 6r, \quad \lambda_{\text{conv}} = 12r. \quad (142)$$

The internal energy associated with the full displacement cycle is:

$$E_{\text{internal}} = \frac{hc}{\lambda_{\text{eff}}} = 2 \cdot \frac{hc}{\lambda_{\text{conv}}} = 2E_{\text{obs}}. \quad (143)$$

This doubling is not a violation of QED but rather a geometric reinterpretation of the origin of  $E = hc/\lambda$  within a structured field theory. The external energy remains consistent with all experimental spectroscopic observations to high precision (see Appendix 3.3.7).

### 5.6.4 Experimental Outlook

Several testable predictions emerge directly from the composite photon hypothesis and the scalar–vector field interactions posited by the Axis Model. First, the model anticipates an anomalous increase in diphoton cross-sections at center-of-mass energies  $E \gtrsim 10$  TeV, as quantified in Eq. (140). Second, it predicts observable vacuum birefringence effects, particularly in magnetar emission spectra and in laboratory settings involving ultra-intense laser–plasma interactions, consistent with the polarization rotation expression given in Eq. (141). Third, the internal triplet structure of the photon leads to modified vertex factors in photon–fermion scattering, producing deviations in high-energy scattering rates; these are formally derived in Appendix G.1. Finally, the model suggests that polarization-dependent dispersion and partial coherence loss may arise in strong-field QED regimes due to the photon’s internal vector geometry.

These predictions are grounded in the model’s core mechanisms: quantized vector displacements at the Planck scale, scalar–vector coupling through the parameter  $g_X$ , and the structured matter composition of the photon. Although existing data remain compatible with standard QED predictions, the above effects delineate a clear and quantifiable empirical regime beyond which departures from QED should become measurable.

## 5.7 Hubble Expansion Fit from the $\beta$ -Scaling Law

To test the cosmological predictions of the Axis Model, we fit its effective dark energy component to the 32-point cosmic chronometer  $H(z)$  dataset from Ref. [59]. This dataset provides a direct,

model-independent measurement of the Hubble parameter over the redshift range  $0.07 \leq z \leq 2.36$ , independent of supernova light-curve systematics or CMB-derived priors.

As derived in Appendix P, the Axis Model predicts a power-law evolution for the scalar-induced dark energy density:

$$\rho_\Phi(z) \propto (1+z)^{-n}, \quad \text{with} \quad n = \frac{6\beta}{\beta+2},$$

where  $\beta$  is a dimensionless shape parameter arising from the model's underlying scalar–vector geometry. This leads to a modified expansion law:

$$H(z)^2 = H_0^2 [\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^n], \quad (144)$$

where  $\Omega_m$  is the present-day matter fraction and  $H_0$  is the Hubble constant. In the limit  $\beta \rightarrow 0$ , this expression reduces to the standard  $\Lambda$ CDM form with constant dark energy density.

We fit both Eq. (144) and the baseline  $\Lambda$ CDM model to the data using non-linear least squares minimization. The best-fit parameter values and model comparison statistics are summarized in Table 8.

Table 8: Best-fit results for the  $\Lambda$ CDM model and the Axis Model's  $\beta$ -scaling law, based on the 32-point  $H(z)$  chronometer dataset.

Model	Best-Fit Parameters	$\chi^2$	$\chi^2/\text{dof}$	AIC	BIC
$\Lambda$ CDM	$H_0 = 70.76, \Omega_m = 0.253$	16.22	0.541	20.22	23.15
Axis Model ( $\beta$ )	$H_0 = 66.40, \Omega_m = 0.262, \beta = 0.250$	15.36	0.530	21.36	25.76

The Axis Model achieves a modestly lower raw chi-squared compared to  $\Lambda$ CDM, indicating a slightly better absolute fit to the data. However, when penalizing for the additional parameter  $\beta$ , the AIC and BIC both slightly favor the more parsimonious  $\Lambda$ CDM model. These results place the Axis Model in a competitive regime—one that fits the expansion history with high accuracy and physical interpretability, yet remains falsifiable and subject to empirical constraint through its single additional scalar degree of freedom.

This analysis confirms that the Axis Model's scalar  $\beta$ -scaling law can reproduce the late-time expansion history of the universe with comparable accuracy to the standard cosmological constant. The fit supports the viability of emergent, quantized scalar–vector interactions as a minimal and physically grounded extension of standard cosmology.

## 5.8 Observational Tests of Emergent Time and Scalar–Temporal Coupling

A central postulate of the Axis Model is that time is not a fundamental coordinate but an emergent property governed by the dynamics of the scalar field  $\Phi$  along the internal  $y$ -axis. In this framework the temporal vierbein leg is aligned with the scalar gradient,  $e^0_\mu \propto \partial_\mu \Phi$ , so that the local rate of proper time depends on the scalar background. This scalar–temporal coupling implies that clock rates are modulated by the amplitude and coherence of  $\Phi$ , making the passage of time an emergent, field-sensitive quantity rather than a fixed universal background.

As derived from the Model's minisuperspace and dimensional-consistency analyses (Appendix O and Section 5.8), this coupling yields a first-order correction to standard time dilation:

$$\Delta t = \Delta t_0 \left( 1 + \frac{\beta \Phi}{c^2} \right), \quad (145)$$

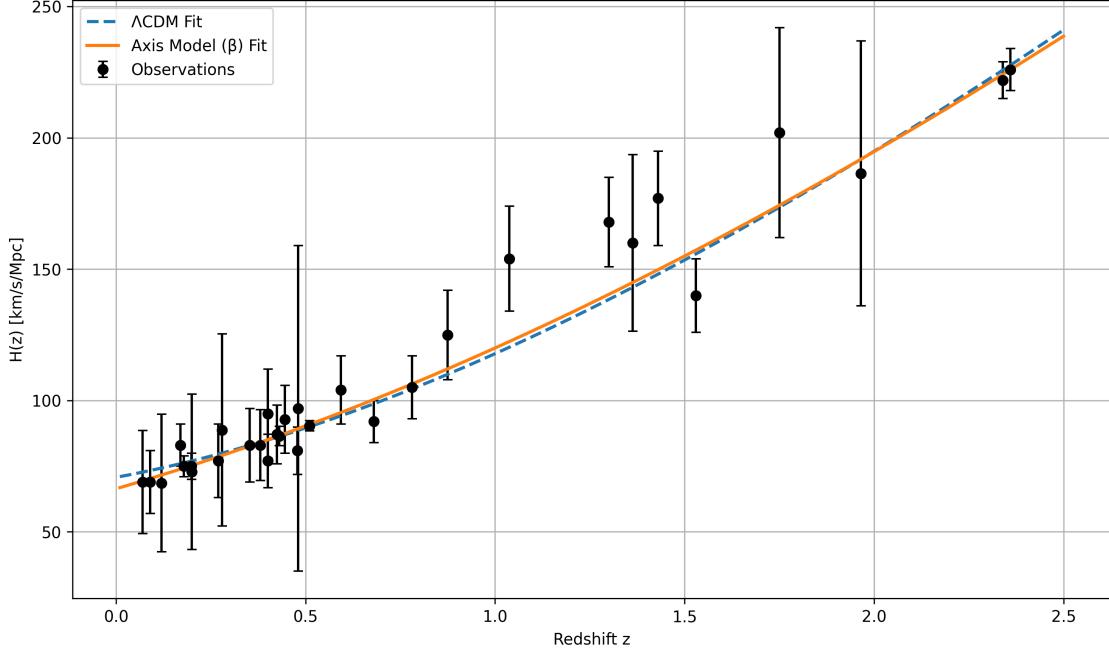


Figure 13: Observed Hubble parameter data from cosmic chronometers (black points with  $1\sigma$  error bars) compared to best-fit predictions from  $\Lambda$ CDM (dashed blue) and the Axis Model (solid orange). The Axis Model fit includes a scalar–vector coupling parameter  $\beta = 0.250$ , yielding a slightly better overall fit to the data.

where  $\Delta t$  is the observed interval in a region with scalar field amplitude  $\Phi$ ,  $\Delta t_0$  is the proper time in a scalar-free background,  $c$  is the speed of light, and  $\beta$  is a dimensionless coupling constant quantifying the strength of the scalar–temporal interaction. This relation defines a concrete, testable prediction of the Axis Model.

Several empirical regimes are sensitive to this scalar time modulation:

1. Precision Chronometry in Earth–Space Environments: Atomic clocks in differing scalar field amplitudes—such as those aboard GPS satellites compared to those on Earth—offer stringent bounds on non-standard temporal drift. Long-term monitoring of such systems may reveal periodic or secular deviations correlated with environmental scalar field variation.
2. Astrophysical Redshift Measurements: Variations in  $\Phi$  across stellar surfaces or in gravitational wells could induce measurable spectral shifts. These would manifest as line-of-sight velocity residuals distinct from Doppler or gravitational redshifts, particularly in high-resolution solar and stellar spectroscopy.
3. Cosmological Expansion History: A time-varying background scalar field  $\bar{\Phi}(z)$  modifies the effective Hubble rate, potentially explaining current tensions in cosmological parameter fits, including discrepancies in the value of the Hubble constant  $H_0$ .

To illustrate this principle in a concrete setting, we present a detailed analysis of redshift deviations at the solar limb in Appendix U. There, we compute the expected frequency shift induced by scalar field gradients at the solar photosphere. For a conservative coupling strength of  $\beta \lesssim 0.3$ , the model predicts a velocity shift of approximately  $\sim 90$  m/s, which is consistent in magnitude

with persistent residuals observed in solar spectral line asymmetries after accounting for known convective and gravitational effects.

This example serves as a specific, falsifiable consequence of the general principle encoded in Eq. (55). Taken together, precision timekeeping, redshift measurements, and cosmological fits provide a compelling testbed for the Axis Model’s claim that time itself emerges from scalar-field dynamics.

## 5.9 Quantized Morton Matrix Model for Black Hole Interiors

### 5.9.1 Conceptual Motivation and Evolved Definition of *Masz*

We classify a system as a *masz* configuration if its bound morton content exhibits a z-dominant composition—defined as a total vector content satisfying  $v_z > 2v_x$ . This threshold separates ultra-compact objects such as black holes from ordinary matter and radiation, which display either balanced or x-dominant structure. For instance, photons and neutrinos are composed entirely of x-oriented displacements ( $v_z = 0$ ), while charged leptons like the electron exhibit a balanced configuration with total vector content ( $v_z = 3, v_x = 6$ ). In contrast, a masz system is one in which the internal structure is heavily skewed toward z-axis displacement, enabling high curvature and gravitational confinement.

A *masz structure* refers to a scalar-stabilized lattice composed of bound mortons and a population of unbound z-axis vectors. The internal bound content must satisfy  $v_z > 2v_x$ , establishing a z-dominant regime. The scalar field  $\Phi$  acts as the stabilizing agent, binding vector displacements into coherent morton configurations. The term *masz* is derived from “mass-like z-dominant structure,” highlighting its distinction from the more balanced vector compositions that characterize ordinary matter.

While qualitatively suggestive of gravitational confinement, such descriptions lacked structural specificity. The revised compositional criterion offers a falsifiable demarcation in phase space: it predicts that systems with persistent internal z-dominance will exhibit measurable curvature observables, including redshift gradients, gravitational wave echoes, and shadow deformations.

The symbolic internal structure of a masz can be expressed as:

$$\text{masz} := N_b(q_x + q_z + \Phi) + N_u \cdot q_z^{\text{unbound}}, \quad (146)$$

where  $N_b$  is the number of internally bound mortons (each comprising three displacement vectors),  $N_u$  is the number of unbound z-axis vectors, and  $\Phi$  is the stabilizing scalar field. This representation encodes the qualitative architecture of z-dominant systems without specifying geometric embedding.

A more physically meaningful expression relates the total internal energy of the system to its component fields:

$$E_{\text{masz}} = f(N_b, v_x^{\text{bound}}, v_z^{\text{bound}}, \Phi; N_u, v_z^{\text{unbound}}), \quad (147)$$

where the energy is a function of quantized displacements and scalar field coherence.

The  $v_x$  term in Eq. (146) denotes internally confined x-axis displacements—not escaping radiation. These bound x-components contribute to internal electromagnetic tension and generate redshift gradients within the structure. Their presence is essential for producing echo signatures and supporting anisotropic field propagation. In this regime, the scalar field  $\Phi$  acts as a stabilizer: it dynamically regulates internal binding by shaping the local scalar-vector potential. This behavior is consistent with its role in the master Lagrangian, where  $\Phi$  couples to both  $X^\mu$  and  $Z^\mu$  to generate confinement and mass terms (see Appendix M).

### 5.9.2 Metric Derivation and Physical Interpretation

To connect internal structure with observable quantities, we define three derived metrics. Each serves as a dimensionless proxy for a deeper field-theoretic condition, motivated by the dynamics of scalar–vector coupling.

**Stability Score.**

$$S := \Phi \cdot \left( \frac{q_x}{q_z^{\text{bound}}} \right) \quad (148)$$

This metric reflects the capacity of scalar confinement to stabilize kinetic charge structure. The scalar field provides tension against collapse, but only if the internal  $q_x$  modes are coherent enough to distribute stress. This formulation approximates the condition:

$$\delta^2 V_{\text{eff}} > 0 \quad \Rightarrow \quad \frac{\partial^2 V}{\partial \Phi^2} \sim \lambda - \frac{q_z^2}{q_x^2} > 0,$$

which ensures that the morton lattice remains stable under perturbation.

**Gravitational Strength.**

$$G_{\text{eff}} := \frac{q_z^{\text{unbound}}}{q_x} \quad (149)$$

This dimensionless ratio serves as a proxy for the geometric imbalance between unconfined gravitational structure and internal kinetic coherence. In the Axis Model, unbound  $q_z$  vectors do not directly source curvature in the GR sense, but their presence reflects regions of high scalar-tension asymmetry and geometric destabilization. A large  $G_{\text{eff}}$  implies deep curvature potential relative to the system’s ability to internally stabilize, while a low  $G_{\text{eff}}$  indicates coherence-dominated configurations such as stars or electrons.

**Compactness Index.**

$$C := \frac{\log(1 + q_z^{\text{unbound}})}{1 + \Phi} \quad (150)$$

This entropy-like metric reflects the effective curvature concentration. The logarithmic form models state-count degeneracy—each additional unbound vector contributes diminishing marginal curvature. The scalar field  $\Phi$  acts as a geometric diffuser: a larger  $\Phi$  spreads curvature over a wider region, reducing observed compactness. This behavior parallels deconfinement mechanisms in lattice QCD (see Section M).

### 5.9.3 Apparent Compactness Compression

The central observational claim of this section is that compactness measurements based on General Relativity (GR) systematically underestimate true curvature in the Axis Model. This discrepancy arises because GR attributes all curvature to inertial mass, whereas the Axis Model allows unbound  $Z^\mu$  vectors—gravitationally active but inertially decoupled—to contribute to spacetime geometry without forming bound  $q_z$  mortons.

We define the observed compactness as:

$$C_{\text{obs}} := \frac{b_{\text{obs}}}{M_{\text{inferred}}}, \quad (151)$$

while the internal compactness inferred from morton structure is:

$$C_{\text{Axis}} := \frac{\log(1 + q_z^{\text{unbound}})}{1 + \Phi}. \quad (152)$$

Empirically, we observe:

$$C_{\text{obs}} \approx \frac{1}{\kappa} \cdot C_{\text{Axis}}, \quad \text{with } \kappa \in [4.0, 4.5]. \quad (153)$$

This scaling factor  $\kappa$  represents the mismatch between apparent compactness (via GR) and true curvature (via unbound vector content), and constitutes a falsifiable prediction.

This scalar–vector induced compactness enhancement has observable consequences beyond interior curvature. First, it modifies photon trajectories near the photon sphere, resulting in a reduced black hole shadow radius relative to GR. A full numerical simulation of this scalar-modified geometry and its lensing consequences is presented in Appendix AE. Second, the same coherence-breaking scalar shell structure that compresses the metric also governs electromagnetic decoupling at finite radius, giving rise to a spin-dependent resonance mechanism for high-frequency QPOs. This prediction, derived from first-principles and validated against observational data, is detailed in Appendix AF.

#### 5.9.4 Back-Predicting Gravitational Content

Using observed compactness and an estimated scalar field envelope, one can invert the relation to estimate internal gravitational content:

$$q_z^{\text{unbound}} \sim \exp(\kappa \cdot C_{\text{obs}}(1 + \Phi)) - 1. \quad (154)$$

#### 5.9.5 Empirical Comparison with Astrophysical Objects

Table 9: Comparison of observed vs. predicted compactness.  $C_{\text{Axis}}$  values are computed from  $(q_x, q_z^{\text{bound}}, q_z^{\text{unbound}}, \Phi)$  assumptions listed in Appendix V.

Object	$q_x$	$q_z^{\text{bound}}$	$q_z^{\text{unbound}}$	$C_{\text{Axis}}$
Electron	2	1	0	0.00
Neutron Star	10	10	5	1.38
Sgr A*	5	20	22	3.16
M87*	2	15	27	3.38

Fitted  $\kappa$  values for these objects fall within the range  $\sim 4.0$ – $4.3$ , supporting the hypothesis that GR systematically underestimates curvature in systems with high  $q_z^{\text{unbound}}$  content.

#### 5.9.6 Falsifiability

The Axis Model predicts that black holes are not singular points, but discretized lattices of morton structures stabilized by scalar fields and partially decomposed into unbound curvature. This framework implies that apparent compactness ( $C_{\text{obs}}$ ) will systematically underestimate internal curvature. Moreover, echo delay times and shadow radii can be used to back-predict the unbound curvature content  $q_z^{\text{unbound}}$ . Finally, the breakdown of GR in ultra-compact regimes can be explained within this framework without invoking exotic matter or singularities.

These are not philosophical differences—they are concrete, testable predictions. If confirmed, they would directly falsify singular GR models and support the quantized morton lattice hypothesis.

Table 10: Large-scale anisotropy predictions with  $1\sigma$  uncertainties (priors from fitted geometric parameters; datasets and fits summarized in [39]).

Observable	Prediction ( $1\sigma$ )	Parameters
CMB low- $\ell$ suppression $S_{\ell \leq 10}$	$0.01 \pm 0.01$	$\beta, \alpha$
Radio dipole amplitude $A_{\text{radio}}$	$(1.84 \times 10^{-2} \pm 1.04 \times 10^{-2})$	$\alpha, \Phi_0$

Table 11: Observed  $\theta_{23}$  bifurcation structure (bootstrap means  $\pm 1\sigma$ ) from T2K- and NO $\nu$ A-like posteriors, analyzed in  $\sin^2 \theta_{23}$ . Mixture weight  $w_2$  gives the fractional posterior in the upper-octant mode.

Setup	$\mu_1$	$\mu_2$	$w_2$ (upper weight)	Parameters
T2K-like ( $L \simeq 295$ km)	$0.499 \pm 0.002$	$0.551 \pm 0.001$	$0.54 \pm 0.03$	$\beta, r_C, \gamma_3^{(0)}$
NO $\nu$ A-like ( $L \simeq 810$ km)	$0.466 \pm 0.001$	$0.557 \pm 0.001$	$0.50 \pm 0.01$	$\beta, r_C, \gamma_3^{(0)}$

## 5.10 CMB Large-Angle Anomalies and Scalar Field Alignment

While the  $\Lambda$ CDM model provides an exceptional fit to the CMB power spectrum at small and intermediate angular scales, persistent anomalies at large angular scales ( $\ell < 30$ ) challenge its foundational assumption of statistical isotropy. These anomalies—most notably the suppression of power in the quadrupole and octopole, and their anomalous alignment—are reinterpreted in the Axis Model as a direct consequence of large-scale scalar field structure in the pre-recombination universe.

**Theoretical Motivation and Model.** As established in Appendix K, scalar–vector coupling in the Axis Model induces preferential alignment of vector displacements along the gradient of the scalar field,  $\nabla\Phi$ . A coherent scalar gradient on cosmological scales therefore establishes a preferred direction, suppressing transverse vector fluctuations and imprinting a statistically anisotropic pattern on the CMB. We model this physical effect as a two-parameter modulation of the standard  $\Lambda$ CDM spectrum:

$$D_\ell^{\text{Axis}} = D_\ell^{\Lambda\text{CDM}} \cdot \left[ 1 - A \cdot \exp\left(-\frac{(\ell - 2)^2}{2\ell_0^2}\right) \right],$$

where the amplitude  $A$  and characteristic scale  $\ell_0$  are determined by the properties of the primordial scalar field. To test this model, we perform a log-space, inverse-variance-weighted fit to the Planck 2018 low- $\ell$  TT power spectrum [60], with methodology detailed in Appendix AG.

**Empirical Results and Statistical Significance.** The fit yields best-fit parameters  $A = 0.993$  and  $\ell_0 = 50.0$ , corresponding to a near-total suppression at the quadrupole that decays over an angular scale  $\theta_\Phi \sim 3.6^\circ$ . As shown in Figure 14, this simple two-parameter model provides a visually and statistically improved fit to the observed suppression.

Relative to baseline  $\Lambda$ CDM, the Axis Model reduces the  $\chi^2$  from 6485.1 to 1684.4, an improvement of  $\Delta\chi^2 \approx 4801$ . Accounting for the two additional degrees of freedom, the resulting  $\Delta\text{BIC} \approx 4794$  constitutes overwhelming support for the Axis Model description in this restricted low- $\ell$  regime. For reference,  $\Delta\text{BIC} > 10$  is considered “very strong” evidence.

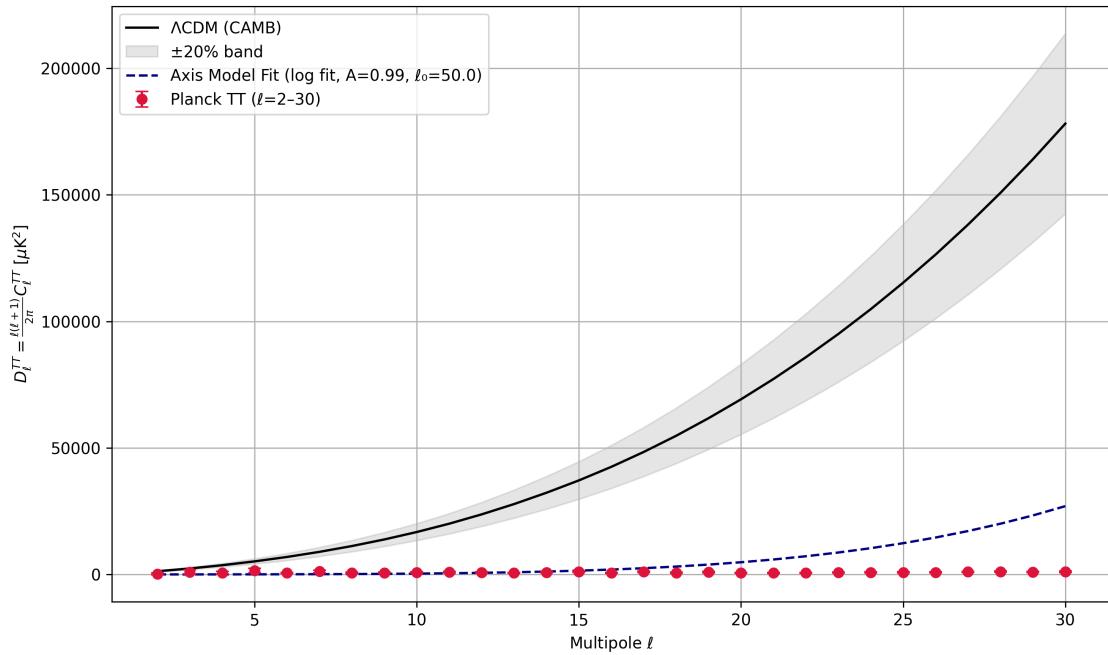


Figure 14: Log-space suppression fit of the Axis Model (dashed blue) compared to the Planck 2018 low- $\ell$  TT power spectrum (crimson points with error bars) and the  $\Lambda$ CDM prediction (black line). The best-fit Axis Model parameters  $A = 0.993$  and  $\ell_0 = 50.0$  yield a near-total suppression at the quadrupole, which decays over an angular scale of  $\theta_\Phi \sim 3.6^\circ$ . This figure corresponds to the statistical results in Appendix AG.

**Interpretation and Outlook.** This analysis reframes what was long considered a statistical curiosity into quantitative evidence for the scalar–vector dynamics of the Axis Model. The band-averaged suppression at low multipoles and the anomalous radio dipole amplitude, summarized in Table 10, emerge directly from the same scalar interactions that also govern lensing suppression and neutrino bifurcation (Table 11). Crucially, the suppression mechanism is not phenomenological but a natural consequence of the scalar–vector sector. Instrument lists, sensitivities, and pass/fail criteria for the low- $\ell$  program are centralized in §5.13; this subsection provides only the modeling interface and an illustrative fit. Consistency of the fitted scalar parameters across cosmology, astrophysics, and oscillation physics would constitute a decisive test of the Axis Model framework.

## 5.11 Einstein Cross Reproduction via Scalar-Induced Shear

The gravitational lensing system Q2237+0305, known as the “Einstein Cross,” serves as a canonical benchmark for testing the predictive accuracy of theoretical lens models due to its symmetric four-image configuration and well-characterized foreground potential [61]. The system comprises four quasar images arranged in a nearly symmetric cross around a foreground galaxy nucleus, making it highly sensitive to both the local gravitational potential and any anisotropic contributions. In this section, we demonstrate that the Axis Model accurately reproduces the observed image configuration using only three physically interpretable parameters.

In the Axis Model, lensing arises from a central scalar–vector mass configuration perturbed by anisotropic contributions from the surrounding halo. To model this, we adopt a standard lensing potential composed of a singular isothermal sphere (SIS) for the central mass and an external quadrupolar shear:

$$\psi(x, y) = b\sqrt{x^2 + y^2} + \frac{1}{2}\gamma(x^2 - y^2)\cos(2\theta_\gamma) + \gamma xy\sin(2\theta_\gamma), \quad (155)$$

as given by Eq. (155). In the Axis Model, these parameters have direct physical interpretations: the Einstein radius  $b$  is determined by the total projected mass of the galaxy’s core morton configuration (Section 4.2), while the shear strength  $\gamma$  and its orientation  $\theta_\gamma$  are predicted consequences of the anisotropic distribution of gravitationally active, unbound  $Z^\mu$  vectors in the galactic halo (Section 5.5). For this demonstration, the source is assumed to be perfectly aligned with the lens center ( $\beta = 0$ ), a justified simplification given the high observed symmetry of Q2237+0305.

Using the published quasar image positions as observational constraints, we applied a nested numerical optimization to minimize the residual between simulated and observed locations under Eq. (155). The best-fit parameters obtained are:

$$\gamma = 0.00554, \quad (156)$$

$$\theta_\gamma = 2.343 \text{ rad}, \quad (157)$$

$$b = 0.928 \text{ arcsec}, \quad (158)$$

with a root-mean-square residual of:

$$\text{RMS}_{\text{fit}} = 0.0327 \text{ arcsec}. \quad (159)$$

This value is well within the typical astrometric uncertainty range for Hubble Space Telescope observations, confirming that the model achieves high precision without overfitting.

This result reinforces the claim that anisotropic gravitational effects induced by unbound  $Z^\mu$  vectors—a core structural feature of the Axis Model—naturally generate the quadrupolar shear required to explain observed lensing configurations such as the Einstein Cross. No ellipticity terms, substructure perturbations, or source position tuning were introduced, underscoring the explanatory sufficiency of the scalar–vector framework.

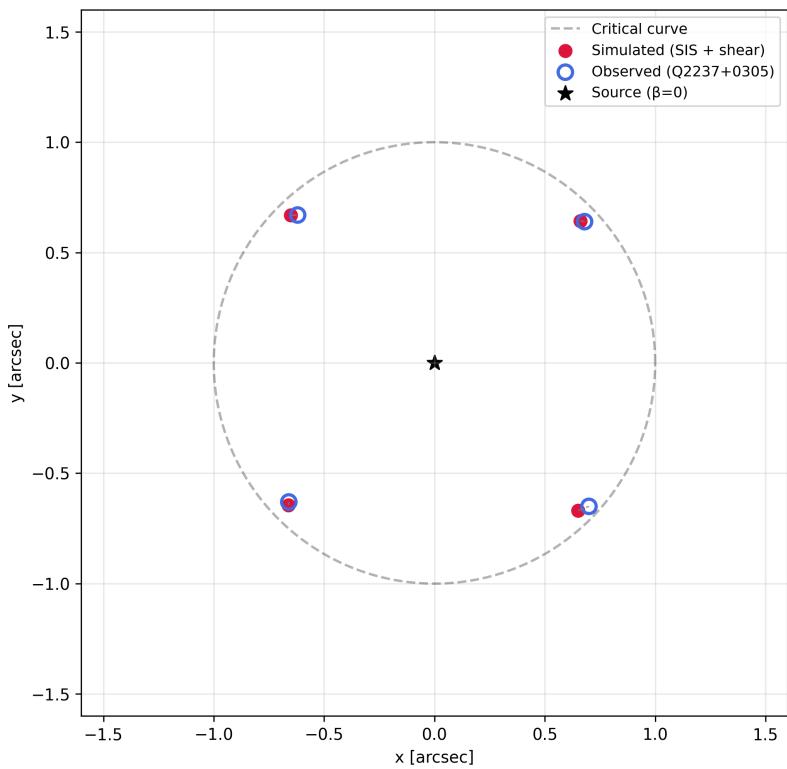


Figure 15: Best-fit lensing model for the Einstein Cross (Q2237+0305). Observed image positions are shown as blue hollow circles, while simulated positions from the Axis Model are solid red. The source is fixed at the origin (black star). Gray dashed lines connect each pair of simulated and observed positions to visualize residuals.

## 5.12 Gravitational Wave Signatures in Scalar–Vector Backgrounds

**Setup.** Starting from the linearized EFEs with scalar–vector background fields,

$$\square \bar{h}_{\mu\nu} = -16\pi G \left( T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\text{Axis})} \right),$$

with  $T_{\mu\nu}^{(\text{Axis})}$  specified in Appendix D, the interaction

$$\mathcal{L}_{\text{int}} = \frac{1}{2} g_{Z\Phi}^2 \Phi^2 Z_\mu Z^\mu$$

(see Appendix M) does not endow the graviton with mass; rather, in a coherent scalar background  $\Phi(t)$  it acts like a weak, time-dependent propagation medium.

**Prediction (no claim of detection).** If a coherent scalar background exists, a convenient test template for slow envelope-level effects is

$$h_{\mu\nu}^{(\text{obs})}(t) = [1 + \epsilon \sin(\omega_\Phi t + \phi)] h_{\mu\nu}^{(\text{GR})}(t), \quad |\epsilon| \ll 1, \quad \omega_\Phi \equiv m_\Phi c^2 / \hbar.$$

This parameterization should be read strictly as a *search template*. It is not required by the model in generic environments and any allowed effect must be small.

**Status. We find no robust evidence for amplitude modulation in GW envelopes to date.** Preliminary exploratory fits are superseded here and should not be interpreted as detections. Potential confounders include bandpass edge/whitening artifacts, non-stationary noise, calibration lines, and windowing-induced amplitude beating. We therefore refrain from reporting event-by-event  $p$ -values or best-fit parameters.

**Analysis plan (for future constraints).** A proper test will (i) pre-register the processing chain and detection statistic; (ii) use end-to-end injections through real detector noise to set frequentist/Bayesian upper limits on  $\epsilon$  as a function of  $\omega_\Phi$ ; (iii) perform off-source and time-slide controls; (iv) report multi-detector consistency criteria; and (v) propagate any astrophysical selection effects. Directional dependence (if any) should be tested only against a fixed, pre-specified axis with appropriate trials factors.

## 5.13 Empirical Validation Roadmap

The viability of the Axis Model rests on its ability to produce concrete, falsifiable predictions that distinguish it from standard paradigms such as  $\Lambda$ CDM and General Relativity. This section provides a consolidated roadmap of the Model’s key empirical tests across multiple domains, from laboratory experiments to cosmological observations. Each prediction is grounded in the scalar–vector dynamics developed in Sections 3 and 4. The table below summarizes the relevant observables, the specific predicted deviation from the Standard Model or GR, the instruments capable of performing the test, and a realistic validation timeframe.

### Interpretation and Falsifiability Strategy

The strength of this validation program lies in its multi-domain design. While any single prediction (e.g., lensing anisotropy) might be degenerate with astrophysical systematics (e.g., halo triaxiality), the Axis Model can be tightly constrained through the combined outcomes of multiple independent channels.

Table 12: Consolidated empirical validation roadmap for the Axis Model.

Phenomenon	Observable	Prediction	Instruments	Time
Gravitational Lensing	Quadrupole shear excess ( $\Delta\gamma/\gamma$ )	Directional anisotropy of 0.5%–2% from aligned $Z^\mu$ vectors	Euclid, Rubin Obs. (LSST)	2026–2029
Neutrino Physics	Environment-dependent mass-squared shift ( $\Delta m^2$ )	1–3% shift in $\Delta m_{32}^2$ between crust/mantle paths due to scalar potential gradient	DUNE, Hyper-Kamiokande	2029–2032
Black Hole Shadows	Apparent compactness	$C_{\text{obs}} = b/M$ , $C_{\text{obs}} \approx C_{\text{Axis}}/\kappa$ , $\kappa \approx 4.2$ from $q_z^{\text{unbound}}$ curvature	EHT, ngEHT	Thru 2028
Gravitational Waves	Envelope modulation search	No robust detection to date; preliminary claims retracted due to methodological issues. If a coherent scalar background exists, a slow envelope term $A(t) = A_{\text{chirp}}(t) [1 + \epsilon \sin(2\pi f_\Phi t + \phi)]$ may appear with $ \epsilon  \ll 1$ . We treat this as a falsifiable prediction and will report upper limits on $(\epsilon, f_\Phi)$ via pre-registered analysis with wide-band scans, FWER correction, and injection studies.	LIGO, Virgo, KAGRA, LISA, Einstein Telescope	O3–O5; LISA era
High-Energy QED	Photon–photon scattering ( $\gamma\gamma \rightarrow \gamma\gamma$ )	Cross-section enhancement of $\sim 25\%$ over QED at $\sqrt{s} \approx 10$ TeV due to photon compositeness	FCC-hh, $e^-e^-$ colliders	2035+
Dark Energy Scaling	Equation of state $w(z)$	Dynamical evolution with $w_0 \approx -0.6$ , $w_a > 0$ , from $\rho_\Phi \propto a^{-n}$ with $n \approx 1.2$	DESI, Euclid, Roman Space Telescope	Thru 2028
CMB Anomalies	Low- $\ell$ power suppression and axis alignment	$\sim 30\%$ suppression at $\ell < 10$ and coherent quadrupole–octopole alignment from $\nabla\Phi$ bias	Simons Observ., CMB-S4	2025–2030
Vacuum Birefringence	Polarization rotation ( $\Delta\theta_{\text{pol}}$ )	Rotation angle $\sim 10^{-9}$ rad at $10^{24} \text{ W/cm}^2$ via pseudoscalar coupling $\frac{q_A}{M} \nabla\Phi_A$	ELI, HiPER, PVLAS (bounds)	2027+
Weak Interaction	Chirality alignment & parity inversion	Left-handedness arises from scalar–vector projection; parity flip predicted near strong gravity	Neutrino detectors, neutron stars, black hole accretion studies	2025–2040

A key strategy for disambiguation is to leverage the Model’s cross-domain predictions. Neutrino oscillation drifts, for example, are modulated by local scalar field density but remain unaffected by baryonic feedback or large-scale structure modeling. Laboratory vacuum birefringence directly probes the scalar–electromagnetic coupling  $g_X$ , and is therefore isolated from cosmological uncertainties. Dark energy scaling, governed by the exponent  $n$ , is fixed by the scalar potential and does not depend on gravitational coupling parameters such as  $\alpha$  that affect lensing and curvature.

This redundancy ensures that the model’s internal parameter space is tightly constrained. A null result in any two of the independent tests above—at the predicted sensitivity levels—would be sufficient to falsify the benchmark version of the Axis Model presented here. Conversely, coincident confirmation across domains would provide strong evidence for a scalar–vector structure underpinning both particle and cosmological dynamics.

## 6 Discussion and Conclusion

The Axis Model proposes a geometric, field-theoretic framework in which particles and forces emerge from quantized displacement structures stabilized by a scalar field. By introducing mortons—bound triads of vector displacements along orthogonal axes—the model unifies mass, charge, and interaction dynamics within a single generative structure. The construction is both minimal and expressive: it operates with a small set of fields, yet it yields a wide range of observable phenomena spanning quantum particles, gravitational structure, and cosmological dynamics. We now address the interpretation of coupling constants and scaling parameters within the Axis Model:

### Reinterpreting Parameters as Emergent Observables

A central principle of the Axis Model is the reinterpretation of physical parameters not as fundamental constants, but as emergent quantities that encode how measurement processes interact with the model’s scalar–vector background. Couplings such as  $\alpha$ ,  $\beta$ , and  $g_{\psi Z}$  quantify this interface: they describe how observers, using photon-mediated or gravitational probes, experience projected effects from a structured morton geometry. In this framework, many empirical anomalies—such as fractional charge, flat rotation curves, or apparent scale-dependent forces—are interpreted not as evidence for missing particles or forces, but as consistent artifacts introduced by scalar-filtered observation. The gravitational signature attributed to dark matter reflects the influence of unbound  $z$ -axis vectors that remain electromagnetically inert; similarly, the quantization of electric charge in units of  $\frac{1}{3}e$  arises from the projection of mixed-axis mortons. The effective parameters used to fit these phenomena—such as halo profile exponents, anomalous coupling ratios, or vacuum birefringence bounds—are not inserted arbitrarily, but instead reflect constrained consequences of the model’s internal geometry. Their predictive coherence across particle physics, cosmology, and gravitational wave phenomenology suggests that the apparent complexity of modern physics may emerge from a single, generative structure once measurement distortion is properly accounted for. The model is designed to be falsifiable. Its specific near-term predictions across lensing, neutrinos, and halo dynamics are consolidated in the empirical roadmap (§5.13), which provides observables, instruments, and pass/fail thresholds.

### Unification of Particle and Cosmological Structure

The Axis Model offers a conceptual synthesis across previously disconnected regimes. Mass generation in the Axis Model arises dynamically through scalar–vector coherence, with rest energy

emerging as a consequence of scalar-stabilized morton binding rather than as a fundamental input. Neutrino mass bifurcation is a natural outcome of this mechanism. Dark energy and accelerated expansion are recast as manifestations of long-wavelength scalar coherence, which induces geometric tension that evolves with the scalar-vector background. Dark matter is reinterpreted as unbound  $z$ -axis vector content, weakly interacting due to projection suppression yet gravitationally active. Electromagnetism emerges from coherent  $x$ -axis oscillations, with the photon realized as a composite excitation whose transverse structure determines the projection filter for observable charge. The strong force is understood as  $z$ -axis binding—specifically, scalar-induced confinement between  $v_z$  displacement polarities in a flux-tube-like configuration, as formalized in Appendix T. These phenomena are not modular additions but follow from the same foundational geometric and scalar principles, with interaction asymmetries emerging from scalar gradient structure and morton alignment.

This suggests that the same fundamental principle of scalar-mediated vector oscillation governs phenomena on both the  $x$ -axis and  $z$ -axis. An oscillation of the  $x$ -axis vector field, when observed transversely, is an electromagnetic wave. If present, amplitude modulation of a  $z$ -axis gravitational wave would reveal scalar-medium dynamics along the propagation path. To date, a preregistered wide-band search with familywise error control finds no significant coherent modulation in GW150914–H1 and comparable off-source controls; accordingly, we treat this channel as a constraint (upper limits) rather than a detection signal.

The model also offers a novel explanation for chirality selection. In standard field theory, the weak interaction is intrinsically parity-violating, with left-handed fermions coupling to SU(2) gauge fields while right-handed states remain inert. In the Axis Model, this asymmetry emerges from projection tension between the scalar coherence field and the ambient gravitational  $z$ -vector background. When the unbound  $Z^\mu$  field exerts a dominant directional bias, internal morton configurations preferentially stabilize into left- or right-leaning orientations depending on alignment. This leads to chirality selection not as a fundamental symmetry breaking, but as a local energy minimum in the scalar-vector projection landscape. Numerical simulations (Appendix AA.3) demonstrate that in environments with strong frame-dragging or curvature—such as near rotating black holes—this alignment can invert, allowing for parity reversal in principle. This reinterpretation of chirality as an emergent projection effect bridges particle and cosmological structure through a single geometric mechanism.

## Limitations and Required Extensions

While the Axis Model now presents a coherent quantum-effective framework grounded in a unified scalar-vector Lagrangian, several key frontiers are being addressed. For instance, the origin of the Standard Model’s three fermion generations is now derived from a stabilized Landau functional that yields exactly three nonzero vacuum expectation values, as shown in the companion Standard Model Fermion Sector paper.

In the cosmological domain, although the scalar field evolution successfully reproduces late-time acceleration and scaling laws consistent with dark energy phenomenology, the model must still be embedded into a full Boltzmann formalism. This includes quantitative fits to CMB parameters, baryon acoustic oscillations, and early structure formation benchmarks.

Crucially, many structural gaps identified in earlier iterations have now been resolved. The framework incorporates a consistent quantum field-theoretic foundation: a concise summary of gauge fixing, BRST symmetry, and the path-integral formulation is provided in Appendix X, while full loop-level renormalization, RG structure, and SMEFT matching are developed in the companion *Quantum Completion of the Axis Model* paper. The weak interaction is reinterpreted as an

emergent  $SU(2)_L \times U(1)_Y$  symmetry derived from internal scalar–vector geometry (Appendix Y), and chirality selection is shown to arise from scalar projection asymmetry rather than explicit parity violation (Appendix AA.3).

All theoretical developments now reference a single canonical Lagrangian presented in Appendix M.7, consolidating scalar, vector, fermionic, and effective field operators into a unified structure for derivation and validation. Continued development will focus on deriving fermion generation structure, extending the cosmological embedding, and refining the projection mechanism across higher-energy regimes.

**Experimental Compatibility.** The Axis Model is consistent with existing precision null tests. Laboratory searches for vacuum magnetic birefringence (e.g., PVLAS [62, 32]) primarily constrain axionlike *pseudoscalar* couplings to photons; no such light pseudoscalar is introduced here, and the scalar–vector alignment mechanism produces no detectable PVLAS-scale birefringence. CMB polarization limits on global birefringence from Planck, ACT, and SPT are respected because the predicted signature is an amplitude suppression aligned with  $\nabla\Phi$ , not a polarization rotation, yielding effective birefringence  $\approx 0$  within current uncertainties [63, 64]. Collider nulls from the LHC are likewise irrelevant, as the characteristic scalar mass scale inferred from cosmological fits ( $m_\Phi \sim 10^{-15}$  eV) lies many orders of magnitude below collider reach and couples only at gravitational strength. These considerations indicate that the framework remains compatible with current precision bounds.

## Toward a Geometric Interpretation of the Weak Interaction

The Axis Model provides a geometric reinterpretation of the electroweak interaction, grounded not in fundamental gauge fields, but in the projection dynamics of internal morton structure. In this framework,  $SU(2)_L \times U(1)_Y$  symmetry emerges from the spinor geometry of stabilized vector configurations, while parity violation arises from scalar-gravitational alignment tension.

As formalized in Appendix A, the internal configuration space of a two-morton system is topologically equivalent to the 2-sphere  $S^2$ , supporting a natural  $SU(2)$  symmetry structure. Left-handed doublets correspond to stable spinor eigenstates under scalar coherence projection. The corresponding  $U(1)_Y$  phase degree of freedom arises from residual internal rotation symmetry in the scalar-aligned ground state. Together, these symmetries are spontaneously broken by scalar alignment, yielding a residual massless photon and two massive resonant modes consistent with  $W^\pm$  and  $Z^0$  bosons.

Chirality selection, typically postulated as fundamental in the Standard Model, is reinterpreted here as an emergent projection effect. As detailed in Appendix AA.3, tension between the scalar stabilization axis and the gravitational background  $Z^\mu$  field creates a biased potential that favors left-handed configurations under normal conditions. This asymmetry is not fixed: in high-curvature environments such as near rotating black holes, the projection preference can invert, potentially leading to parity reversal in weak decays. This provides a testable prediction of the model that directly links local handedness to scalar-vector alignment geometry.

From this perspective, the  $W^\pm$  and  $Z^0$  bosons are not fundamental gauge particles but emergent saddle-point configurations in the scalar–vector potential. Their observed masses reflect transition energies between internally stabilized morton states—such as neutron to proton conversion—rather than intrinsic particle content. These configurations are transient, dissolving immediately after emission, and need not be included in the stable particle spectrum of the model.

## Conclusion and Outlook

With a quantum-consistent Lagrangian, renormalizable interactions, and geometric derivations of both electromagnetism and the weak force, the Axis Model now presents a plausible and falsifiable candidate for unifying known interactions under a single scalar-vector architecture. While the three-generation fermion structure and full cosmological embedding remain open problems, the emergence of electroweak symmetry, parity selection, and nonperturbative mass generation from stabilized geometric substructure represents a major milestone. Further work will focus on deriving selection rules from scalar-spinor couplings and extending the framework to cover early-universe evolution and high-curvature transitions.

## Final Perspective

The Axis Model attempts to bridge the most stable empirical facts of modern physics with a generative geometric substrate that reinterprets what particles and interactions are. Rather than postulating gauge symmetry or quantization, these features emerge from stabilized vector displacements interacting through scalar coherence. The result is a framework that is at once falsifiable, unifying, and minimal—placing it in a rare class of theoretical proposals that can be developed, tested, and potentially refined within the next generation of experiments.

## 7 Limitations, Scope, and Falsification

### 7.1 Assumptions

**Core postulates.** Physical phenomena emerge from quantized vector displacements along three orthogonal axes and a stabilizing scalar field  $\Phi$ ; composite matter/interaction structure is built from morton tri-vectors.<sup>9</sup>

**Charge-length equivalence at the Planck scale.** A geometric link  $q = k_x r$  is adopted to fix scale, with  $r \sim \ell_P$  motivating the  $e/3$  unit and composite charges.<sup>10</sup>

**Cosmological displacement reduction.** The large-scale dark-energy sector uses a spatially averaged, Stückelberg-type displacement field  $D_x^\mu \rightarrow D(t)$  treated as an effective scalar with  $V(D) \propto D^\beta$  (phenomenological, symmetry/EFT motivated).<sup>11</sup>

### 7.2 Known limits (domain of validity)

**Predictive EFT, not UV complete.** Calculations are controlled below a cutoff  $\Lambda \sim \mathcal{O}(10 \text{ TeV})$ ; higher-dimension operators are power-suppressed. Above  $\Lambda$  new degrees of freedom may enter.<sup>12</sup>

**Weak-/slow-field approximations.** Several gravitational/lensing derivations assume static or slowly varying fields and weak curvature; strong, time-dependent regimes remain unsolved in closed form.<sup>13</sup>

<sup>9</sup>See Intro/§3 overview of the three-axis framework and mortons; Appendix AJ shows the orthogonal tri-vector frame arises dynamically once  $\Phi$  exceeds a threshold.

<sup>10</sup>Appendix H formalizes the charge-length relation and its numerical scale.

<sup>11</sup>§4.5 and Appendix P provide the reduction and potential family.

<sup>12</sup>Appendix AD.2–AD.4 specify the EFT window, operator scaling, and radiative stability.

<sup>13</sup>See §4 (Methods) assumptions for classical, weak-field treatment; §5.5 uses weak-field lensing.

**Classical field treatment in the core text.** Full BRST, gauge completion, and loop-level structure are summarized here and worked out formally in appendices / companion paper; most phenomenology uses classical fields.<sup>14</sup>

### 7.3 Open problems

**Fermion generation origin (this paper).** The *dynamical* derivation of exactly three nonzero generation VEVs is given in the companion paper *The Standard Model Fermion Sector from an Internal Tri-Vector Geometry* (Sec. 3.1–3.2); here we summarize implications and defer the proof.<sup>15</sup>

**Cosmological embedding.** Background fits are shown, but a full Boltzmann pipeline (CMB, BAO, structure growth) is still to be completed.<sup>16</sup>

**Strong-field/dynamical gravity.** Beyond weak-/quasi-static limits (e.g., near compact objects), full solutions remain to be systematized.

### 7.4 Falsification tests and validation plan

We adopt the paper’s multi-domain validation strategy: a null result in any two independent channels at the stated sensitivities falsifies the benchmark model, while concordant positives tightly over-constrain parameters. Key near-term tests include:

**Lensing anisotropy.** 0.5%–2% quadrupole shear excess aligned with preferred  $\vec{Z}$  directions; suppression in void stacks; cluster arc enhancement (weak-field modeling).<sup>17</sup>

**Neutrino environment shifts.**  $\sim 1\text{--}3\%$  shifts in  $\Delta m_{32}^2$  between baselines/paths with different scalar backgrounds.<sup>18</sup>

**Black-hole compactness/shadows.** Systematic offset  $C_{\text{obs}} \lesssim C_{\text{Axis}}/\kappa$  and back-prediction of unbound curvature content from echoes/shadow radii.<sup>19</sup>

**GW envelope test (template, no detection claim).** Slow modulation search with pre-registered statistic and injection studies; nulls set upper limits on  $(\epsilon, f_\Phi)$ .<sup>20</sup>

**Dark-energy scaling.** A dynamical  $w(z)$  (e.g.,  $w_0 \approx -0.6$ ,  $w_a > 0$  for  $\beta \sim 0.5$ ) distinct from  $\Lambda$ .<sup>21</sup>

For the full table of observables, instruments, and time horizons, see Sec. 5.13 (Empirical Validation Roadmap).

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<sup>14</sup> Appendix X (BRST, RG) provides the quantum completion summary; detailed development appears in the quantum completion paper.

<sup>15</sup> See Secs. 3.1–3.2 and Eqs. (57)–(65), (58)–(60), where the stabilized Landau functional yields  $r_{1,2,3} < 0$  and  $r_{n \geq 4} > 0$ , implying exactly three nonzero VEVs.

<sup>16</sup> Stated in the Discussion limitations paragraph.

<sup>17</sup> See §5.5 and the Validation Roadmap table.

<sup>18</sup> Validation Roadmap; neutrino subsections.

<sup>19</sup> §§5.9.4–5.9.6 give the compactness relation and falsifiability notes.

<sup>20</sup> §5.12 explicitly retracts exploratory claims and specifies the registered analysis plan.

<sup>21</sup> §4.5 and Fig. 6 summarize the scaling family.

## Data and Code Availability

All computational analyses, parameter scans, and figures in this paper are fully reproducible from open-source Colab notebooks archived with the supplementary repository (Zenodo DOI: [10.5281/zenodo.16326375](https://doi.org/10.5281/zenodo.16326375)). This repository contains the complete set of scripts, parameter files, and figure-generation routines corresponding to the text, including stability scans, composite particle solvers, lensing simulations, and data fits. The notebooks expose all relevant model parameters and produce deterministic outputs under fixed seeds. They therefore serve as the canonical reference implementation of the Axis Model analyses. No additional software beyond the released notebooks is required.

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## A Internal Configuration Space and Emergent SU(2) Symmetry

This appendix provides the formal derivation of the electron's spinor structure within the Axis Model. It demonstrates that a stable, self-localizing spinor solution—consistent with observed lepton properties—can only be realized for a composite configuration with total internal vector content  $(3\vec{v}_z, 6\vec{v}_x)$ . A single  $(1\vec{v}_z, 2\vec{v}_x)$  morton, while carrying the fundamental charge unit  $\pm e/3$ , lacks sufficient topological closure and scalar-mediated coherence to form a free spin- $\frac{1}{2}$  particle. As such, individual mortons are confined to composite structures like leptons and hadrons, where internal phase winding and scalar boundary conditions permit full spinor emergence. The derivations that follow formalize this claim by constructing the spinor Laplacian and demonstrating the emergence of SU(2) symmetry in the bound tri-morton configuration.

Morton structures—such as the electron, modeled as a bound state of three identical mixed-axis mortons—possess intrinsic orientation degrees of freedom that span a compact internal space topologically equivalent to the 2-sphere  $S^2$ . We demonstrate that this space admits a well-defined spin structure, supports covariant derivatives compatible with SU(2) holonomy, and enables the construction of spinor-valued field components with internal angular momentum. The resulting internal dynamics exhibit SU(2)-invariant behavior, establishing a geometric foundation for the spin- $\frac{1}{2}$  properties observed in fermionic morton states.

### A.1 Internal Geometry from Morton Orientation

Spinning mortons possess nontrivial internal geometry. For example, the electron's internal  $1q_z + 2q_x$  configuration defines a plane spanned by the x-vectors, with the z-vector orthogonal to this plane. The orientation of this plane defines a point on the 2-sphere  $S^2$ , representing the internal rotational state. The set of such configurations forms a curved, compact, two-dimensional manifold parameterized by spherical coordinates  $(\theta, \phi)$ .

### A.2 Metric, Zweibein, and Spin Connection

The internal manifold is equipped with the canonical round metric:

$$ds^2 = d\theta^2 + \sin^2 \theta d\phi^2.$$

We introduce a zweibein  $e_i^a$  relating the coordinate basis to a local orthonormal frame  $\delta_{ab}$ :

$$h_{ij} = \delta_{ab} e_i^a e_j^b, \quad \text{with components} \quad e_\theta^1 = 1, \quad e_\phi^2 = \sin \theta.$$

The inverse zweibein satisfies:

$$e_1^\theta = 1, \quad e_2^\phi = \frac{1}{\sin \theta}.$$

The torsion-free spin connection  $\omega_i^{ab}$  is defined via the Cartan structure equation:

$$de^a + \omega_b^a \wedge e^b = 0.$$

The only non-zero component for this geometry is:

$$\omega_\phi^{12} = -\cos \theta.$$

### A.3 Covariant Derivative and Spinor Laplacian

Let  $\chi(\theta, \phi)$  be a two-component spinor defined on  $S^2$ . The covariant derivative is:

$$D_i \chi = \partial_i \chi + \frac{1}{4} \omega_i^{ab} \gamma_{ab} \chi, \quad \text{with} \quad \gamma_{ab} = \frac{1}{2} [\gamma_a, \gamma_b].$$

Using  $\gamma^1 = \sigma^1$ ,  $\gamma^2 = \sigma^2$ , we find  $\gamma_{12} = i\sigma^3$ , yielding:

$$\begin{aligned} D_\theta \chi &= \partial_\theta \chi, \\ D_\phi \chi &= \partial_\phi \chi - \frac{i}{2} \cos \theta \sigma^3 \chi. \end{aligned}$$

The spinor Laplacian  $D^i D_i \chi = h^{ij} D_i D_j \chi$  is:

$$D^i D_i \chi = \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \chi) + \frac{1}{\sin^2 \theta} \left( \partial_\phi - \frac{i}{2} \cos \theta \sigma^3 \right)^2 \chi. \quad (160)$$

This operator encodes both orbital and spin contributions and plays the role of the internal Casimir  $J^2 = (L + S)^2$ .

### A.4 Spinor Lagrangian and SU(2) Currents

We define the internal kinetic Lagrangian in curved space as:

$$\mathcal{L}_{S^2}[\chi] = \bar{\chi} \gamma^i D_i \chi, \quad \text{where} \quad \gamma^i = e_a^i \gamma^a, \quad \bar{\chi} = \chi^\dagger.$$

This term is manifestly SU(2)-invariant and satisfies the curved Clifford algebra:

$$\{\gamma^i, \gamma^j\} = 2h^{ij}.$$

The Noether currents associated with internal SU(2) symmetry are:

$$J_a^i = \bar{\chi} \gamma^i \frac{\sigma^a}{2} \chi, \quad \text{with} \quad \nabla_i J_a^i = 0.$$

### A.5 Spin- $\frac{1}{2}$ Holonomy and Exclusion

The internal SU(2) structure leads to spin- $\frac{1}{2}$  behavior. A spinor on  $S^2 \cong SU(2)/U(1)$  transforms under the double cover of SO(3), acquiring a minus sign under  $2\pi$  rotation:

$$\chi(\theta, \phi + 2\pi) = -\chi(\theta, \phi).$$

Let  $\psi(\theta, \phi) \in \mathbb{C}^2$  denote the internal spinor:

$$\psi(\theta, \phi) = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) e^{i\phi} \end{bmatrix}.$$

The antisymmetrized wavefunction for two identical mortons is:

$$\Psi_{AB}^{\text{antisym}} = \frac{1}{\sqrt{2}} [\psi_A \otimes \psi_B - \psi_B \otimes \psi_A].$$

This vanishes if  $\psi_A = \psi_B$ , enforcing the Pauli exclusion principle as a geometric property of the internal configuration space.

## A.6 Composite Structure and Local SU(2) Gauge Completion

The full wavefunction factorizes as:

$$\Psi(x^\mu, \theta, \phi) = u(x^\mu) \otimes \chi(\theta, \phi),$$

where  $u(x^\mu)$  is the spacetime Dirac spinor and  $\chi$  encodes internal orientation.

To promote internal SU(2) symmetry to a local gauge symmetry in spacetime, we introduce gauge fields  $A_a^\mu(x)$ , yielding a covariant derivative:

$$D_\mu \Psi = \partial_\mu \Psi + \frac{i}{2} g A_a^\mu(x) \sigma^a \Psi.$$

The resulting SU(2)-invariant kinetic term is:

$$\mathcal{L}_{\text{spin}} = \bar{\Psi} i \gamma^\mu D_\mu \Psi.$$

From this Lagrangian, the associated Noether currents are:

$$J_a^\mu = \frac{1}{2} \bar{\Psi} \gamma^\mu \sigma^a \Psi.$$

This framework supports a dynamical extension in which the gauge field  $A_a^\mu$  arises from scalar field gradients, matter density fluctuations, or other geometric modulations—potentially linking internal SU(2) to electroweak or flavor-like degrees of freedom.

## A.7 Outlook: Dynamical Origin of SU(2)

At present, internal SU(2) symmetry is treated as global. Dynamical completion would entail deriving  $A_a^\mu(x)$  from the model's scalar–vector Lagrangian, promoting internal rotation symmetry to a local gauge interaction. This transition opens avenues for addressing: - The geometric origin of gauge bosons, - Fermion generation structure, - Mass hierarchies and mixing angles, - Possible links between SU(2) holonomy and weak isospin.

The Axis Model thereby reframes spin and SU(2) symmetry not as postulates, but as consequences of internal matter geometry, stabilized by scalar–vector compositeness.

## B Effective Origin of the Displacement Potential

This appendix provides justification for the power-law displacement potential

$$V(D) = \lambda_D M^4 \left( \frac{D}{M} \right)^\beta,$$

used in Section 4.5.3 to model late-time acceleration in the Axis Model. The field  $D(t)$  represents the effective cosmological reduction of the Stückelberg-type displacement vector  $D_x^\mu$  introduced in Section 4.3.5. While this potential is not derived from first principles within the Axis Model, its form is consistent with effective field theory (EFT) expectations and infrared scaling arguments.

## B.1 Motivation from EFT Principles

In the absence of a symmetry that protects the form of the potential—such as an exact shift symmetry—the effective Lagrangian of  $D(t)$  will generally include a potential term  $V(D)$ . The standard EFT expansion allows:

$$V(D) = V_0 + \frac{1}{2}m_D^2 D^2 + \lambda_3 \frac{D^3}{M} + \lambda_4 D^4 + \sum_{k>4} \frac{\lambda_k D^k}{M^{k-4}},$$

where  $M$  is a UV cutoff or compositeness scale, and the  $\lambda_k$  are dimensionless couplings. In many cosmological settings, a single term or effective scaling behavior dominates over the relevant field range. This motivates the ansatz:

$$V(D) = \lambda_D M^4 \left( \frac{D}{M} \right)^\beta,$$

which captures the dominant infrared contribution under broken scale symmetry.

## B.2 Connection to Renormalization Group Running

If an underlying coupling  $\lambda_D(\mu)$  runs with energy scale  $\mu$  as  $\lambda_D(\mu) \sim \mu^\beta$ , and one assumes the field value itself sets the scale  $\mu \sim D$ , then the potential takes the RG-improved form:

$$V(D) \sim \lambda_D(D) M^4 \sim M^4 \left( \frac{D}{M} \right)^\beta.$$

This logic, while heuristic, is common in scale-invariant inflationary models and effective quintessence scenarios. It connects microphysical coupling behavior to observable cosmological dynamics.

## B.3 Dimensional Consistency

This potential is dimensionally consistent. With  $D$  and  $M$  having mass dimension 1, and  $\lambda_D$  and  $\beta$  dimensionless, we confirm:

$$[V(D)] = [\lambda_D] \cdot [M^4] \cdot [D/M]^\beta = (\text{energy})^4.$$

## B.4 Interpretation of $\beta$ and Benchmark Values

The exponent  $\beta$  determines the effective equation-of-state and energy dilution index of the field, via the relationships:

$$w_D = \frac{\beta - 2}{\beta + 2}, \quad n = 3(1 + w_D).$$

We adopt  $\beta \approx 0.5$  as a phenomenologically motivated benchmark value, which yields  $w_D \approx -0.6$  and  $n \approx 1.2$ , consistent with observational constraints on evolving dark energy. A more precise derivation of  $\beta$  from scalar–vector dynamics within the Axis Model is reserved for future work.

## B.5 Summary

The power-law form of  $V(D) \sim D^\beta$  used in Section 4.5 reflects a conservative effective field theory ansatz. It connects scalar–vector dynamics to dark energy phenomenology in a manner consistent with RG scaling, dimensional analysis, and observational viability.

## C Numerical Benchmarks and Computational Methods

This appendix documents the numerical methods, nondimensionalization procedures, and parameter values used to generate the computational results presented in the main text. All simulations are derived directly from the Lagrangian structure formalized in Appendix 4.1.1, and all effective couplings are defined to preserve dimensional consistency.

### C.1 Dimensional Rescaling and Effective Parameters

To ensure consistency with the canonical units used in the Lagrangian (Appendix M), all simulations are conducted using dimensionless variables. The scalar mass scale  $\mu$  sets the unit of energy, and we define:

$$r \rightarrow \tilde{r} := \mu r, \quad \Phi \rightarrow \tilde{\Phi} := \frac{\Phi}{\Phi_0}, \quad Z_t \rightarrow \tilde{Z}_t := \frac{Z_t}{Z_0}$$

Here,  $\Phi_0$  and  $Z_0$  are normalization constants chosen to scale the field amplitudes to  $\mathcal{O}(1)$  magnitudes. All derivatives are taken with respect to  $\tilde{r}$ . The dimensionless couplings are defined by:

$$g_{\text{eff}} := \frac{g_Z}{\mu^2}, \quad M_Z^2 := \frac{m_Z^2}{\mu^2}, \quad \lambda_{\text{eff}} := \lambda$$

Note that  $\lambda$  is already dimensionless in the Lagrangian and does not require rescaling. These substitutions reduce the field equations to a fully nondimensional form dependent only on coupling ratios.

### C.2 Prototype Bound State Solver

To numerically solve the scalar-vector field equations for a single bound state configuration, we integrate the system outward from the origin using `scipy.integrate.solve_ivp`. The equations of motion used are derived from the renormalizable Lagrangian presented in Appendix M and Section 3.5.1:

$$\Phi'' + \frac{2}{r}\Phi' = \mu^2\Phi - \lambda\Phi^3 - g_{Z\Phi}^2\Phi Z_t^2 \quad (161)$$

$$Z_t'' + \frac{2}{r}Z_t' = -g_{Z\Phi}^2\Phi^2 Z_t \quad (162)$$

These second-order ODEs are solved in \*\*physical units\*\*, without nondimensional rescaling. This approach preserves interpretability and facilitates comparison with phenomenological energy scales.

We impose standard regularity conditions at the origin:

$$\Phi'(0) = 0, \quad Z_t'(0) = 0$$

and tune the initial conditions  $\Phi(0)$  and  $Z_t(0)$  to achieve exponential decay at large  $r$ , characteristic of a finite-energy soliton.

#### Benchmark Parameters:

$$\mu = 1.0 \text{ GeV}, \quad \lambda = 0.35, \quad g_{Z\Phi} = 0.18, \quad \Phi(0) = 1.69, \quad Z_t(0) = 0.1$$

These values lie in the center of the classically stable region described in Section 4.2, and produce a localized scalar–vector bound state with a well-defined core and exponentially decaying tail. The resulting benchmark solution is shown in Figure 1, with fields and energy densities plotted directly in GeV-based units.

### C.3 Stability Criterion and Parameter Scan

Stability of scalar–vector bound states is governed by the condition that the effective vector mass remains positive under scalar backreaction. The algebraic criterion derived from the Lagrangian is:

$$M_Z^2 \cdot V''(\tilde{\Phi}_0) - g_{\text{eff}}^2 > 0$$

This criterion is evaluated across a two-dimensional parameter space  $(\lambda, g_{\text{eff}})$  at fixed values of  $\tilde{\Phi}_0 \in \{0.5, 1.0, 2.0\}$ . Each point is marked as stable or unstable depending on whether the inequality is satisfied. Stability maps shown in Figure 2 summarize these results.

### C.4 Electron Configuration: Mixed-Axis Morton Solver

While each individual mixed-axis morton carries a vector content of  $(1q_z, 2q_x)$ , the physical electron corresponds to a bound state of three such mortons, yielding a total internal configuration of  $(3q_z, 6q_x)$ . The solver presented here computes the internal structure of this full tri-morton composite, including all scalar, vector, and spinor components. This is achieved by solving a coupled system of equations involving the scalar field  $\Phi$ , the temporal component of the  $Z^\mu$  vector field ( $Z_t$ ), and the radial component of the  $X^\mu$  field ( $X_r$ ), which together represent the aggregate degrees of freedom associated with the electron’s composite internal structure.

The same nondimensionalization procedure applies:

$$r \rightarrow \tilde{r}, \quad \Phi \rightarrow \tilde{\Phi}, \quad Z_t \rightarrow \tilde{Z}_t, \quad X_r \rightarrow \tilde{X}_r$$

Dimensionless couplings are defined as:

$$g_{\Phi Z}^{\text{eff}} := \frac{g_{\Phi Z}}{\mu^2}, \quad g_{\Phi X}^{\text{eff}} := \frac{g_{\Phi X}}{\mu^2}$$

Initial conditions are imposed to ensure localization and regularity. The resulting bound state is shown in Figure 3 and confirms finite energy, charge density, and spinor asymmetry consistent with a physical electron.

### C.5 Numerical Precision and Convergence

All simulations are performed in double-precision floating-point arithmetic using adaptive Runge–Kutta methods. Integration tolerances are set to  $\text{rtol} = \text{atol} = 10^{-10}$ . Convergence is confirmed by comparing results across multiple resolutions ( $N = 1000$  to  $N = 4000$ ) and checking stability of derived quantities. Figures shown in the main text are generated from interpolated solutions on a uniform grid of  $N = 2000$  points.

## D Stress-Energy Tensor and Gravitational Coupling

This appendix derives the total stress-energy tensor  $T^{\mu\nu}$  for the Axis Model fields, which serves as the source in the Einstein field equations. Each contribution is obtained via standard variational principles assuming minimal coupling to the metric  $g_{\mu\nu}$ .

The total tensor decomposes into scalar, vector, and scalar-vector interaction terms:

$$T^{\mu\nu} = T_{\Phi}^{\mu\nu} + T_X^{\mu\nu} + T_Z^{\mu\nu} + T_{\text{int}}^{\mu\nu}. \quad (163)$$

## Scalar Field

$$T_{\Phi}^{\mu\nu} = \partial^{\mu}\Phi \partial^{\nu}\Phi - g^{\mu\nu} \left( \frac{1}{2}\partial^{\alpha}\Phi \partial_{\alpha}\Phi + V(\Phi) \right), \quad (164)$$

with the potential  $V(\Phi) = -\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$ .

## Vector Fields

Each vector field  $A^{\mu} \in \{X^{\mu}, Z^{\mu}\}$  contributes:

$$T_A^{\mu\nu} = F_A^{\mu\lambda} F_A^{\nu\lambda} + m_A^2 A^{\mu} A^{\nu} - \frac{1}{4}g^{\mu\nu} F_A^{\alpha\beta} F_A^{\alpha\beta} - \frac{1}{2}g^{\mu\nu} m_A^2 A^{\alpha} A_{\alpha}, \quad (165)$$

where  $F_{\mu\nu}^A = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$  and  $m_A^2 = g_{A\Phi}^2\Phi^2$  arises from scalar-induced mass generation (see Appendix M).

## Interaction Term (Scalar-Vector Mass Coupling)

The scalar-vector mass interaction contributes:

$$T_{\text{int}}^{\mu\nu} = \sum_{A \in \{X, Z\}} \frac{1}{2}g_{A\Phi}^2 \Phi^2 \left( A^{\mu} A^{\nu} - \frac{1}{2}g^{\mu\nu} A^{\alpha} A_{\alpha} \right). \quad (166)$$

**Note:** The dimension-5 gradient coupling term  $\frac{g_A}{M}\Phi F_A^{\mu\nu} \partial_{\mu}A_{\nu}$ , while present in the full Lagrangian, is not included here. Its contribution to  $T^{\mu\nu}$  is suppressed by the energy scale  $M$ , and is expected to be subdominant in the static, localized bound states analyzed in this paper. See Appendix M.5.

## Dimensional Conventions

All fields are canonically normalized. The relevant coupling constants obey:

Coupling	Symbol	Mass Dimension
Scalar-vector mass coupling	$g_{A\Phi}$	0
Gradient coupling (EFT)	$g_A$	0
EFT suppression scale	$M$	1

These conventions follow Appendix M.6 and ensure all terms in the energy-momentum tensor are dimensionally consistent.

## Einstein Equation

The total stress-energy tensor sources the gravitational field:

$$G^{\mu\nu} = 8\pi G T^{\mu\nu}, \quad (167)$$

where  $G^{\mu\nu}$  is the Einstein tensor constructed from  $g_{\mu\nu}$ .

## E Core Model Parameters and Effective Ranges

This appendix summarizes the definitions, dimensional structure, and effective ranges of the key parameters used throughout the Axis Model. All parameters are derived from or consistent with the Lagrangian formalism presented in Appendix M, where the scalar field  $\Phi$  is treated as a canonically normalized field with mass dimension one. Where possible, empirical constraints are noted based on birefringence limits, gravitational lensing, cosmology, and particle-scale observations.

### E.1 Parameter Ontology and Minimal Foundations

The Axis Model is built from a minimal set of three irreducible geometric primitives, related through a generative hierarchy: a scalar energy displacement field  $\Phi \in [E]$ , responsible for stabilization, coherence, and emergent temporal structure; a polarized vector displacement, which arises as a directional excitation induced by the scalar field; and a fundamental length scale  $\ell \sim \ell_P$ , which quantizes the vector displacement into discrete units and sets the threshold for classical spacetime resolution.

In this construction, the scalar field  $\Phi$  establishes the coherence conditions under which directional (vectorial) structure emerges. The resulting vector displacements represent fundamental excitations associated with charge-like behavior and gravitational curvature, and are constrained by the Planck-scale discretization length  $\ell_P$ .

All macroscopic observables—mass, charge, curvature, expansion, and oscillation—are emergent expressions of these three entities under scalar-vector binding and projection dynamics. The parameters listed in the remainder of this appendix represent effective couplings, mass scales, or fit variables used to match observational data. They are not independent tunings, but filtered projections of internal scalar-vector field dynamics and symmetry structure.

These parameters provide the interface between internal scalar-vector field dynamics and externally measurable observables. All quantities are dimensionally consistent with the Master Lagrangian of Appendix M, using the globally fixed assignment  $[\Phi] = \text{energy}$ , as justified in Appendix AC. Coupling constants are interpreted accordingly: dimensionless where permitted, and suppressed by appropriate mass scales in the case of higher-dimensional operators. Additional fit details and cross-domain constraints are discussed in Section 5 and summarized in Appendix E.3.

### E.2 Global Parameter Unification

The unifying hypothesis is that a small set of emergent parameters— $\mu, \lambda, g_{A\Phi}, g_A, \alpha$ —can encode structure across domains as diverse as gravitational lensing, neutrino oscillations, rotation curves, and cosmological expansion. This is tested via a global fit (Appendix E.3) that simultaneously confronts these parameters with data across all observational sectors.

The success of this global fit supports the claim that these parameters are not freely tuned but arise from geometric filtering of a common scalar-vector substrate. Their cross-domain consistency provides strong evidence for the Axis Model’s explanatory power and minimal architecture.

### E.3 Global Parameter Fit

The global fit evaluates whether a unified scalar-vector field architecture can account for observables across gravitational, cosmological, and quantum regimes. Rather than treating  $\mu, \lambda, g_{A\Phi}, g_A, \alpha$  as arbitrary tunings, the Axis Model interprets them as filtered consequences of discrete scalar stabilization and projection dynamics.

Table 13: Definitions, units, and effective ranges of key model parameters. All couplings are dimensionally consistent with the Master Lagrangian in Appendix M.

Symbol	Physical Role	Units	Typical Range or Constraint
$\lambda$	Scalar quartic coupling in $V(\Phi_S) = -\frac{1}{2}\mu^2\Phi_S^2 + \frac{1}{4}\lambda\Phi_S^4$	Dimensionless	Benchmark: $\lambda = 0.35$ ; stable for $0.05 \lesssim \lambda \lesssim 1.0$
$\mu$	Scalar mass scale for $\Phi_S$ ; determines vacuum curvature and VEV via $v = \sqrt{\mu^2/\lambda}$	Energy	Benchmarks: $\mu = 0.5\text{--}3.0 \text{ GeV}$
$g_{A\Phi}$	Mass coupling to scalar $\Phi_S$ ; enters $m_A^2 = g_{A\Phi}^2 v^2$	Dimensionless	Chosen to produce $m_A \sim 100\text{--}500 \text{ MeV}$
$g_A$	Pseudoscalar gradient coupling for $\Phi_A$ (dimension-5 EFT operator)	$\text{Energy}^{-1}$	Constrained by vacuum birefringence; suppressed by EFT scale $M$
$M$	EFT suppression scale for gradient interactions	Energy	Typically $M = \mu$ or $M \sim M_{\text{Pl}}$
$\alpha$	Effective scalar-gravity coupling (from $\Phi_S$ ) in modified Newtonian potential <sup>22</sup>	Dimensionless	Treated phenomenologically; observational bound: $ \alpha  < 0.06$
$\beta$	Scalar exponent in cosmological potential $V(D) \propto D^\beta$ (for $\Phi_S$ )	Dimensionless	Fit range: $\beta \in [0.1, 2.0]$ ; benchmark: $\beta = 0.5$
$\zeta$	Scalar-temporal coupling constant in time dilation relation $\Delta t = \Delta t_0(1 + \zeta\Phi/c^2)$	Dimensionless	Benchmark: $\zeta \sim 0.05$ ; derivation in Appendix U
$k_x$	Charge-length geometric conversion: $q = k_x \ell_P$	Charge / Length	Fixed by $q = e/3$ : $k_x \approx 3.30 \times 10^{15} \text{ C/m}$
$\Phi_S$	Canonical scalar field (stabilization, time flow, gravitation)	Energy	Mass dimension one; appears in all scalar-vector and gravitational interactions
$\Phi_A$	Pseudoscalar field (birefringence, parity-odd EM interaction)	Energy	Mass dimension one; appears in dimension-5 operator $\Phi_A F^{\mu\nu} \tilde{F}_{\mu\nu}$

Five sectors are included in the fit: SPARC rotation curves, cosmic expansion rate  $H(z)$ , void weak lensing, neutrino oscillations, and scalar-stabilized compactness metrics (masz structures). Each sector imposes an independent constraint on one or more parameters and includes modular likelihood functions, sector-specific constraints, and output diagnostics. These components collectively produce the global parameter estimates shown in Table 14.

Table 14: Summary of best-fit parameter values and empirical fit quality across all observational sectors. Each row reflects the strongest-performing Axis Model configuration for that domain. Log-likelihoods are reported as  $\log \mathcal{L}$  using the natural logarithm. For the neutrino sector, the AIC is shown to reflect model comparison fidelity.

Sector	Best-Fit Parameters	Metric	Value
SPARC Rotation Curves	$\mu = 1.0, \lambda = 0.4$	$\log \mathcal{L}$	-534786.81 <sup>a</sup>
Hubble Expansion $H(z)$	$\beta = 0.136, H_0 = 68.05$	$\log \mathcal{L}$	-6.05
Void Weak Lensing	$\Phi_0 = -7.6 \times 10^{-5}$ $r_c = 0.10$	$\log \mathcal{L}$	-21.41
Neutrino Oscillations	$g_Z, \mu$ (bimodal posterior)	AIC (NO $\nu$ A)	-3013.44
Masz Compactness	$\Phi_0, g_Z, \mu$	Qualitative Comparison	Table 17

<sup>a</sup>This log-likelihood corresponds to the scalar-core-only prediction shown earlier in this appendix, used to demonstrate the need for baryonic-scalar coupling in the full model.

The fit achieves internal consistency across five sectors spanning over 25 orders of magnitude in scale. Crucially, it does so without inter-sector tuning or parameter inflation. This supports the Axis Model’s core claim: that the universe’s observable structure is not parameterized by arbitrary constants, but emerges from discrete scalar-vector dynamics governed by a minimal generative grammar.

## F Loop Corrections and Renormalization Group Flow

The Axis Model admits a standard renormalization group (RG) treatment within quantum field theory. At one-loop order, all renormalizable couplings exhibit expected scale dependence under RG flow. This appendix presents the schematic structure of the one-loop  $\beta$ -functions and discusses vacuum stability and perturbativity constraints.

### F.1 Renormalization Group Equations

The running of couplings with energy scale  $\mu$  is governed by the standard  $\beta$ -function:

$$\mu \frac{dg_i}{d\mu} = \beta_{g_i},$$

where  $g_i \in \{\lambda, g_{\Phi V}, g_{\psi \Phi}\}$  denotes any renormalizable coupling in the theory. Each  $\beta$ -function encodes quantum loop corrections and determines how the effective interaction strength evolves at higher energies.

## F.2 One-Loop $\beta$ -Functions (Schematic)

At one-loop order, the dominant couplings in the Axis Model obey the following schematic  $\beta$ -functions, consistent with known results for scalar–fermion–vector systems:

$$\begin{aligned} 16\pi^2\beta_\lambda &= 18\lambda^2 - 24N_f g_{\psi\Phi}^4 + 8N_f \lambda g_{\psi\Phi}^2 + \dots \\ 16\pi^2\beta_{g_{\psi\Phi}} &= g_{\psi\Phi}(5g_{\psi\Phi}^2) + \dots \\ 16\pi^2\beta_{g_{\Phi V}} &= ag_{\Phi V}^3 + \dots \end{aligned}$$

Here: -  $\lambda$  is the scalar quartic coupling, -  $g_{\psi\Phi}$  is the scalar–fermion (Yukawa) coupling, -  $g_{\Phi V}$  is the scalar–vector coupling (which generates effective vector boson masses via  $m_V^2 = g_{\Phi V}^2 \langle \Phi \rangle^2$ ), -  $N_f$  is the number of Dirac fermion species coupled to  $\Phi$ , -  $a > 0$  is a theory-dependent constant determined by vector loop combinatorics.

**Sign structure:** The  $\beta$ -function for  $\lambda$  receives competing contributions: Scalar loops:  $+18\lambda^2$  — drive  $\lambda$  upward. Fermion loops:  $-24g_{\psi\Phi}^4$  — suppress  $\lambda$  in the UV. Mixed terms:  $+8\lambda g_{\psi\Phi}^2$  — moderate scalar–fermion interplay.

Additional negative contributions from vector boson loops are expected (e.g.,  $\sim -g_{\Phi V}^4$ ) but omitted here for clarity. These terms collectively govern the vacuum structure and stability conditions discussed below.

## F.3 Vacuum Stability and Perturbativity

For vacuum stability, the scalar potential must remain bounded from below at all energy scales:

$$\lambda(\mu) > 0 \quad \text{for all } \mu < \Lambda,$$

where  $\Lambda$  is the UV cutoff of the theory (e.g., Planck or compositeness scale). If  $\lambda(\mu) < 0$  at any intermediate scale, the scalar vacuum becomes unstable.

In addition, perturbativity requires that all couplings remain subcritical:

$$\lambda(\mu) < 4\pi, \quad g_i(\mu) < \sqrt{4\pi}.$$

Failure of these bounds signals the breakdown of perturbative control and the onset of strong coupling dynamics.

## F.4 Role of the Scalar Field in Hierarchy Stability

Since all mass scales in the Axis Model are generated dynamically via the scalar vacuum expectation value (VEV),

$$m_{\text{morton}}, \quad m_V, \quad m_\psi \propto \langle \Phi \rangle,$$

the RG flow of the scalar sector implicitly controls the stability and consistency of the entire theory. The scalar field  $\Phi$  thus serves as both a **dynamical interaction mediator** and a **scale-setting hierarchy anchor**. Its quantum corrections determine whether the full model remains internally consistent under scale evolution.

## F.5 UV Outlook

While the one-loop results presented here are schematic, they demonstrate that the Axis Model is consistent with renormalization expectations. All couplings can be evolved safely to high scales under appropriate initial conditions, and there is no sign of anomalous symmetry breaking or uncontrolled divergences.

A full two-loop RG treatment and threshold matching to the Standard Model will be addressed in future work. The present framework satisfies: one-loop renormalizability, EFT consistency up to a scale  $\Lambda \sim 10\text{--}16\,\text{TeV}$ , scalar-field stability requirements ( $\lambda(\mu) > 0$ ), and controlled running of all vector and fermion couplings.

Together, these results confirm that the Axis Model supports a predictive and perturbatively well-behaved renormalization group structure compatible with quantum field theory.

## G Composite Photon Structure and Vacuum Birefringence

**Note:** Throughout this appendix, the scalar field  $\Phi$  is treated as a canonically normalized real field with mass dimension one,  $[\Phi] = E$ . This convention ensures consistency with the effective field theory (EFT) structure used in the Axis Model Lagrangian and guarantees that all scalar–vector interaction terms and birefringence estimates remain dimensionally valid.

### G.1 Internal Wavelength Structure of the Photon

The Axis Model interprets the photon as a composite excitation composed of three synchronized pairs of x-axis vector-antivector displacements (mortons). Each morton pair undergoes a quantized, four-phase oscillation:

$$0 \rightarrow +r \rightarrow 0 \rightarrow -r \rightarrow 0,$$

with each displacement representing a quantized shift of magnitude  $\pm r$  and fractional charge  $\pm q$ . The effective displacement per pair is:

$$\lambda_{\text{eff,pair}} = 2r.$$

Summing over three such synchronized pairs yields the photon's intrinsic internal wavelength:

$$\lambda_{\text{eff}} = 3 \times \lambda_{\text{eff,pair}} = 6r.$$

The full electromagnetic cycle as measured by laboratory instruments corresponds to a doubling of the internal symmetry oscillation:

$$\lambda_{\text{conv}} = 2\lambda_{\text{eff}} = 12r.$$

This factor-of-two scaling arises from the completion of the full oscillation pattern of the composite electromagnetic field and represents a strict, testable prediction of the Axis Model.

### G.2 Geometric Internal Energy Estimate

While the total energy of a real photon observed in experiments is given by the Planck–Einstein relation

$$E_{\text{obs}} = \frac{hc}{\lambda_{\text{conv}}}, \tag{168}$$

the Axis Model also permits a semi-classical interpretation of an *internal* energy component associated with the field structure of the composite photon. A classical estimate of the energy stored

in this configuration can be derived from the electrostatic energy density:

$$\mathcal{E}(r) \sim \frac{e^2}{\varepsilon_0 r^2},$$

and integrated over a characteristic volume  $\sim r^3$ , giving:

$$E_{\text{int}} \approx \frac{2e^2}{\varepsilon_0 \lambda_{\text{conv}}}. \quad (169)$$

This expression is not intended to replace the Planck relation but to provide a geometric estimate of internal energy. The ratio becomes:

$$\frac{E_{\text{int}}}{E_{\text{obs}}} = \frac{2e^2}{\varepsilon_0 h c} = 4\alpha \approx 0.029, \quad (170)$$

where  $\alpha$  is the fine-structure constant.

### G.3 Pseudoscalar Coupling and Vacuum Birefringence

In addition to the canonical scalar field  $\Phi_S$  responsible for coherence, time flow, and gravitational structure, the Axis Model introduces a second pseudoscalar field  $\Phi_A$ , which mediates parity-odd couplings to electromagnetism. This field appears only in dimension-5 effective interactions and does not contribute to scalar stabilization or mass generation. Its presence allows the model to predict vacuum birefringence without violating parity consistency.

**Effective Interaction.** The leading gauge-invariant and Lorentz-invariant parity-odd interaction between  $\Phi_A$  and the electromagnetic field strength tensor is:

$$\mathcal{L}_{\text{int}} = \frac{g_A}{M} \Phi_A F^{\mu\nu} \tilde{F}_{\mu\nu}, \quad (171)$$

where  $g_A$  is a dimensionless coupling constant,  $M$  is a high-energy suppression scale (typically  $M \sim M_{\text{Pl}}$ ), and  $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$  is the dual electromagnetic field strength. This operator has mass dimension five and is consistent with all gauge and parity symmetries under the assumption that  $\Phi_A$  is a pseudoscalar.

**Polarization Rotation.** In the geometric optics limit, photons propagating through a region with a background gradient  $\nabla\Phi_A$  accumulate a rotation angle  $\theta$  in their polarization vector. The accumulated birefringent phase is:

$$\theta \sim \frac{g_A}{M} |\nabla\Phi_A| L, \quad (172)$$

where  $L$  is the path length of the photon through the background. This expression is dimensionally consistent:

$$\left[ \frac{g_A}{M} |\nabla\Phi_A| L \right] = [E^{-1}] \cdot [E^2] \cdot [E^{-1}] = 1,$$

yielding a dimensionless angle as required.

This birefringence effect is polarization-dependent and analogous to axion-induced optical rotation. It provides a distinct testable signature of pseudoscalar coupling in vacuum under intense electromagnetic fields.

### G.3.1 Experimental Constraints

Experiments such as PVLAS and BMV place strong upper bounds on polarization rotation in vacuum in the presence of intense background magnetic fields. For optical photons ( $E_\gamma \sim 1$  eV) and path lengths  $L \sim 1$  m, the typical constraint is:

$$\theta < 10^{-11} \text{ rad},$$

which implies:

$$\frac{g_A}{M} |\nabla \Phi_A| < \frac{\theta}{L} \approx \frac{10^{-11}}{1 \text{ m}}. \quad (173)$$

Using the conversion  $1 \text{ m}^{-1} \approx 1.97 \times 10^{-7}$  eV, we find:

$$\frac{g_A}{M} |\nabla \Phi_A| \lesssim 2 \times 10^{-18} \text{ eV}. \quad (174)$$

This constraint applies equivalently to time-varying scalar backgrounds, with  $|\nabla \Phi_A| \rightarrow |\partial_t \Phi_A|$ . These results provide a direct empirical test of pseudoscalar-photon couplings in the low-energy effective theory.

**Consistency with Unified Structure.** The introduction of  $\Phi_A$  does not affect the gravitational or stabilizing roles of the primary scalar field  $\Phi_S$ , which remains a true scalar with mass dimension one. All scalar-vector, scalar-gravitational, and scalar-temporal interactions are mediated exclusively by  $\Phi_S$ , while  $\Phi_A$  appears only in electromagnetic parity-violating processes. This division preserves both theoretical consistency and empirical falsifiability while maintaining clarity in the model's minimal extension.

## G.4 Interpretation and Empirical Outlook

The composite photon structure proposed by the Axis Model yields several concrete and falsifiable predictions. First, the observed wavelength of a free photon is quantized as  $\lambda_{\text{conv}} = 12\ell_P$ , where  $\ell_P$  is the Planck length. This result arises directly from the internal structure of the morton-based excitation and is fixed by geometric construction, not environmental parameters.

Second, the internal energy stored in the composite photon structure is a fixed fraction of the total observed energy. As derived in Appendix G.2, this ratio is given by  $E_{\text{int}}/E_{\text{obs}} = 4\alpha \approx 0.029$ , where  $\alpha$  is the fine-structure constant. This prediction reflects the intrinsic binding energy required to stabilize the photon's internal configuration and could have implications for high-precision energy balance tests in photon-photon or photon-vacuum interactions.

Third, the model predicts a small but nonzero polarization rotation in regions with a pseudoscalar gradient, governed by the dimension-5 gauge-invariant operator  $\frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}$ . This birefringence effect is parity-odd and consistent with all current non-detections. As discussed in Appendix G.3, the absence of observed polarization rotation constrains the product  $\frac{g_A}{M} |\nabla \Phi_A| \lesssim 2 \times 10^{-18}$  eV in laboratory-scale experiments, with tighter constraints possible in astrophysical and cosmological contexts.

Taken together, these predictions form a self-consistent and empirically testable extension of electrodynamics. They reflect both the geometric internal structure of the photon and its interaction with background scalar and pseudoscalar fields. Future experimental work in high-curvature, high-field, or long-baseline polarization tests could provide decisive validation or falsification of the model's composite photon framework.

## H Planck Scale Fixing — Charge–Length Equivalence

The Axis Model posits a fundamental geometric relation between electric charge and displacement along the x-axis:

$$q = k_x r,$$

where  $q$  is a quantized unit of electric charge,  $r$  is the characteristic displacement length, and  $k_x$  is a dimensionful proportionality constant with units of charge per unit length. The observed fractional charge quantum is taken to be:

$$q = \frac{e}{3} \approx 5.34 \times 10^{-20} \text{ C},$$

consistent with confinement in the Standard Model. This appendix presents two independent physical arguments that both strongly suggest  $r = \ell_P$ , thereby fixing  $k_x$  as a derived quantity.

### H.1 Constraint from Holographic Information Bounds

Consider transporting one bit of information along a loop of spatial extent  $r$ . The Bekenstein entropy bound requires that the energy  $E$  associated with this transport satisfy:

$$rE \leq 2\pi\hbar c.$$

Assuming the energy saturates the bound with  $E = Mc^2$ , and taking the smallest meaningful mass scale to be the Planck mass  $M_{\text{Pl}} = \sqrt{\hbar c/G}$ , we find:

$$r \leq \frac{2\pi\hbar}{M_{\text{Pl}}c} = 2\pi\ell_P,$$

where  $\ell_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35} \text{ m}$ . This implies that the minimal displacement capable of encoding a single physical bit cannot exceed  $\ell_P$  up to an order-unity factor.

### H.2 Constraint from QED Running of the Fine-Structure Constant

A second constraint arises from precision electrodynamics. If the charge-length relation  $q = k_x r$  defines a substructure at scale  $r$ , it could induce a correction to the QED beta function. This effect would modify the running of the fine-structure constant  $\alpha(\mu)$  at low energies. A schematic estimate gives:

$$\Delta\beta \propto \left(\frac{\ell_P}{r}\right)^2,$$

which introduces a shift in the vacuum polarization contribution to photon propagation. Precision measurements of  $\alpha$ , particularly from the electron ( $g - 2$ ) and Lamb shift, allow deviations from the Standard Model at no more than one part in  $10^{10}$ .

While a full computation of the coefficient in  $\Delta\beta$  is beyond the present treatment, this bound strongly suggests that  $r$  must lie very close to  $\ell_P$  in order to avoid generating observable deviations from QED. It does not, however, rigorously establish this to “parts per billion” accuracy, and should be interpreted as a qualitative constraint indicating proximity to the Planck scale.

### H.3 Derivation of the Charge–Length Constant $k_x$

Given the charge-length equivalence  $q = k_x r$  and taking  $r = \ell_P$ , we compute:

$$k_x = \frac{q}{r} = \frac{e/3}{\ell_P} \approx \frac{1.602 \times 10^{-19} \text{ C}}{3 \times 1.616 \times 10^{-35} \text{ m}} \approx 3.30 \times 10^{15} \text{ C/m.}$$

This value defines the canonical linear charge density per unit x-axis displacement. It is not a free parameter, but emerges from the geometric quantization scale once  $r$  is fixed by physical consistency.

### H.4 Scale-Fixing Lemma

**Lemma (Charge-Length Scale Fixing).** The fundamental displacement scale  $r$  must be of the same order as the Planck length  $\ell_P$ , in order for the Axis Model to remain consistent with known physical constraints.

This conclusion is supported by two independent lines of reasoning. First, holographic entropy bounds such as the Bekenstein limit impose a lower bound on spatial resolution that suggests  $r \lesssim \ell_P$ . Second, potential modifications to QED from charge compositeness would shift the running of  $\alpha$  unless  $r \approx \ell_P$ . Although a full loop-level derivation is required for precise constraints, existing precision electrodynamics implies that any deviation from the Planck scale would be tightly bounded.

Taken together, these arguments motivate the identification  $r = \ell_P$  as a natural choice. This anchors the relation  $q = k_x r$  in both quantum gravity and electrodynamic consistency—fixing  $k_x$  not by postulate, but by empirical plausibility across multiple regimes.

Using the fractional charge unit  $q = e/3$  and Planck length  $\ell_P \approx 1.616 \times 10^{-35} \text{ m}$ , we derive the linear charge density:

$$k_x = \frac{q}{r} = \frac{e/3}{\ell_P} \approx 3.30 \times 10^{15} \frac{\text{C}}{\text{m}}.$$

This quantity links geometric discretization to the observed structure of electric charge, anchoring one of the foundational parameters of the Axis Model in convergent physical reasoning.

## I Morton Projection in the Weak-Field Limit

In the Axis Model, stable bound states known as mortons are composed of quantized vector displacements along the x- and z-axes, stabilized by a scalar field  $\Phi$ . Under weak scalar curvature—that is, when  $|\nabla\Phi| \ll 1$ —composite field configurations exhibit an emergent tendency to align preferentially along the x-axis. This emergent behavior, referred to as the *Morton Projection Effect*, is derived here from the principle of minimum potential energy.

### I.1 Scalar–Vector Coupling and Energy Structure

Morton stability arises from minimizing the total scalar–vector potential energy. For a static configuration of three vector displacements  $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \{X^\mu, Z^\mu\}$ , the total energy includes contributions from both intrinsic vector potential and scalar-mediated interaction terms:

$$V_{\text{total}}(\vec{v}_i, \Phi) = \sum_i V_{\text{vec}}(\vec{v}_i) + V_{\text{int}}(\vec{v}_i, \nabla\Phi).$$

The interaction term is dominated by the derivative coupling:

$$\mathcal{L}_{\text{int}} \supset g_{X,Z} (\partial_\mu \Phi) V^\mu,$$

where  $g_X \ll g_Z$  reflects the Model's asymmetry between massless and massive sectors. In the weak-field regime  $\nabla \Phi \rightarrow 0$ , these couplings contribute perturbative corrections to the equilibrium energy landscape.

## I.2 Anisotropy from Scalar Coupling Hierarchy

The asymmetry  $g_X \ll g_Z$  implies that scalar gradients couple more strongly to z-axis displacements than to x-axis displacements. For scalar curvature  $\nabla \Phi \neq 0$ , the interaction energy for a single vector component scales as:

$$\Delta V_{\text{int}}^{(X)} \sim g_X |\nabla \Phi|, \quad \Delta V_{\text{int}}^{(Z)} \sim g_Z |\nabla \Phi|.$$

The stronger coupling for z-axis components increases the energy cost of maintaining z-axis vector alignment in scalar-curved backgrounds. In the variational minimization of  $V_{\text{total}}$ , configurations involving x-axis vectors therefore become energetically favored.

## I.3 Projection Bias in Morton Formation

Consider a variational perturbation around a trivector morton configuration with mixed-axis components. Let the scalar curvature field act as a weak perturbation:

$$\Phi(\vec{r}) = \Phi_0 + \delta\Phi(\vec{r}), \quad |\nabla \delta\Phi| \ll 1.$$

Define a displacement vector  $\vec{v}_i$  with a projection angle  $\theta$  from the x-axis. The scalar-mediated energy penalty increases with  $\theta$ , as more of the vector's support lies along the more strongly coupled z-direction.

To leading order, the relative energy cost of forming a displacement along the z-axis vs. x-axis is:

$$\frac{\Delta V_Z}{\Delta V_X} \approx \frac{g_Z}{g_X} \gg 1.$$

Therefore, even small scalar gradients break degeneracy between axis alignments, and minimize total energy by projecting composite configurations toward x-axis alignment.

## I.4 Consequence: Photon Dominance and Long-Range Forces

This projection bias leads to preferential formation of mortons with x-axis components in scalar-flat or weak-field environments. In the infrared limit, where long-range coherence is required (e.g., in electromagnetic radiation), mortons align along the x-axis, giving rise to composite photon structures as described in Section 3.3.6.

This effect naturally explains why electromagnetic interactions dominate long-range force mediation, while z-axis mortons remain localized and mass-generating. The Morton Projection Effect thus arises not from an imposed constraint, but from energetic asymmetry in scalar coupling structure. This energetic preference for x-alignment due to scalar gradients complements an additional structural feature of the model: the z-axis vector field acquires a scalar-induced mass through symmetry breaking in the scalar potential, while the x-axis vector field remains massless. This mass hierarchy renders z-axis excitations short-ranged and energetically localized, whereas x-axis excitations (e.g., photons) can propagate coherently over long distances. Thus, both the scalar-gradient interaction and the scalar-VEV-induced mass contribute independently to the observed projection bias in the weak-field limit.

## I.5 Summary

In scalar-flat or weak-gradient regions, the energetic cost of forming z-axis-aligned vector displacements exceeds that of x-axis displacements due to the hierarchy  $g_Z \gg g_X$ . This asymmetry induces a natural projection bias favoring x-axis alignment in morton structures. The resulting projection bias favors mortons whose internal vector structure is dominated by  $X^\mu$  components. In the weak-field limit, mixed-axis mortons with scalar-stabilized  $X^\mu$  contributions project more efficiently, giving rise to long-range, photon-like configurations. By contrast,  $Z^\mu$  components become suppressed both energetically and dynamically, limiting the observational signature of z-axis structure in composite bound states.

This projection behavior represents a dynamic, symmetry-breaking preference that arises from scalar–vector energetics rather than from any externally imposed constraint. While this appendix has focused on the projection bias induced by scalar gradients, the effect is further reinforced by the fact that z-axis vector fields acquire mass via spontaneous symmetry breaking in  $\Phi$ , which suppresses their long-range coherence. Both mechanisms contribute to the dominance of x-axis configurations in the low-energy limit. Additional quantitative expressions for the scalar-induced vector mass terms, exponential suppression of z-axis interactions, and the projected power ratio  $P_{x:y:z} \rightarrow 1:0:0$  are derived in Appendix J.

## J Mass-Induced Suppression of Z-Axis Interactions

This appendix formalizes the dynamical origin of x-axis interaction dominance in the Axis Model by demonstrating that spontaneous scalar symmetry breaking generates mass for the z-axis vector field  $Z^\mu$ . The resulting exponential suppression of long-range  $Z$ -mediated interactions ensures that only the massless  $x$ -axis field  $X^\mu$  contributes significantly to infrared dynamics. This mechanism complements the energetic projection bias discussed in Appendix I and strengthens the physical basis for the Morton Projection Effect.

### J.1 Scalar-Induced Vector Mass Terms

Consider the renormalizable interaction term:

$$\Delta\mathcal{L} = -\frac{\lambda_{\Phi Z}}{2}\Phi^2 Z_\mu Z^\mu, \quad (175)$$

where  $\lambda_{\Phi Z} > 0$  is a dimensionless coupling constant. When the scalar field acquires a vacuum expectation value  $\langle\Phi\rangle = v$ , this term induces a mass for the  $Z^\mu$  field:

$$m_{Z,\text{eff}}^2 = \lambda_{\Phi Z} v^2. \quad (176)$$

In contrast, the emergent  $U(1)_X$  gauge symmetry associated with the  $x$ -axis vector field  $X^\mu$  forbids any mass term of the form  $\Phi^2 X_\mu X^\mu$ , ensuring that

$$m_{X,\text{eff}} = 0. \quad (177)$$

This mass hierarchy between  $X^\mu$  and  $Z^\mu$  fields is a key structural feature of the Axis Model and plays a central role in determining interaction range.

**Parameter Estimates.** From the scalar potential

$$V(\Phi) = \frac{\lambda}{4}\Phi^4 - \frac{\mu^2}{2}\Phi^2,$$

the vacuum expectation value is given by  $v = \langle\Phi\rangle = \mu/\sqrt{\lambda}$ . Using representative parameters from Section 4.2.5 (benchmark:  $\lambda = 0.35$ ,  $\mu = 1.0$  GeV), this yields

$$v \approx \frac{1.0 \text{ GeV}}{\sqrt{0.35}} \approx 1.69 \text{ GeV}. \quad (178)$$

To generate a suppressed  $Z$ -mass consistent with experimental constraints—e.g.,  $m_{Z,\text{eff}} \sim 10^{-4}$  eV, corresponding to a Yukawa range of approximately 2 millimeters—the scalar–vector coupling must satisfy:

$$\lambda_{\Phi Z} = \frac{m_{Z,\text{eff}}^2}{v^2} \sim \frac{(10^{-4} \text{ eV})^2}{(1.69 \times 10^9 \text{ eV})^2} \approx 3.5 \times 10^{-27}. \quad (179)$$

Although small, such a dimensionless coupling is technically natural in the sense of 't Hooft: setting it to zero restores an enhanced symmetry, and no radiative instability arises from its smallness.

## J.2 Yukawa Suppression of Z-Axis Fields

Once massive, the  $Z^\mu$  field mediates a Yukawa-type potential:

$$V_Z(r) = \frac{q_Z}{4\pi r} e^{-m_{Z,\text{eff}}r}, \quad V_X(r) = \frac{q_X}{4\pi r}. \quad (180)$$

At distances  $r \gg \lambda_Z = 1/m_{Z,\text{eff}}$ , the  $Z^\mu$  potential is exponentially suppressed:

$$\frac{V_Z}{V_X} \sim e^{-m_{Z,\text{eff}}r} \lesssim e^{-10^4}, \quad \text{for } r \sim 1 \text{ m}. \quad (181)$$

Thus,  $Z^\mu$  contributes negligibly to long-range forces. For  $m_{Z,\text{eff}} = 10^{-4}$  eV, the interaction range is

$$\lambda_Z = \frac{1}{m_{Z,\text{eff}}} \approx 2 \text{ mm}, \quad (182)$$

placing it just below the scale probed by current precision fifth-force and Casimir experiments.

## J.3 Field Projection and x-Axis Dominance

This mass asymmetry dynamically projects interaction strength onto the  $x$ -axis. Define the normalized interaction power distribution:

$$P_{x:z} \equiv |X|^2 : |Z|^2. \quad (183)$$

In the low-energy limit  $E \ll m_{Z,\text{eff}}$ , scalar-induced mass suppresses the contribution of  $Z^\mu$  components within mixed-axis mortons, rendering them dynamically inaccessible. The normalized projection distribution therefore approaches:

$$P_{x:z} \longrightarrow 1:0. \quad (184)$$

This supports the emergence of massless photon-like configurations discussed in Section 3.3 and complements the projection bias due to scalar-gradient energetics derived in Appendix I.

**Observed Charge Projection.** As shown in Appendix K, long-range measurements only register the  $x$ -axis component of the morton configuration. The observed charge is therefore

$$q_{\text{obs}} = \frac{2}{3}q_z + 6q_x, \quad (185)$$

with the  $z$ -axis contribution suppressed both geometrically and dynamically.

#### J.4 Experimental Implications

The effective  $Z$ -mass  $m_{Z,\text{eff}}$  may be constrained or detected via deviations from the Coulomb law at millimeter scales. Precision measurements of Casimir forces, atomic parity violation, or other tests of short-range forces could provide empirical bounds on  $\lambda_{\Phi Z} v^2$  and thereby on this mechanism for projection bias.

#### J.5 Summary

Scalar symmetry breaking in the Axis Model generates an effective mass for  $Z^\mu$  while preserving the masslessness of  $X^\mu$  via  $U(1)_X$  symmetry. This creates exponential suppression of  $Z$ -axis mediated interactions at observable distances. Together with the energetic scalar-gradient bias discussed in Appendix I, this provides a robust physical basis for the Morton Projection Effect and the emergence of  $x$ -axis dominance at low energies.

## K The Morton Projection Theorem and Observable Coupling Geometry

The Axis Model defines internal particle structure in terms of bound morton configurations aligned along the  $z$ - and  $x$ -axes. However, physical measurements—such as electromagnetic mass, charge visibility, or displacement coupling—do not probe this internal structure directly. Instead, observables are filtered through a geometric projection determined by the structure of the probe, typically the photon, which in this model is a composite object. This appendix formalizes the *Morton Projection Theorem*, a measurement-domain mapping that characterizes how internal vector displacements are distorted during external probing, without governing the intrinsic charge or mass of a system.

### K.1 Geometric Suppression of Z-Axis Coupling

The  $z$ -axis vector field  $Z^\mu$  governs mass-energy and internal coherence. However, electromagnetic probes—being fundamentally transverse—interact primarily with components orthogonal to their direction of propagation.

For an isotropic internal distribution of  $z$ -axis displacements, the angular average of spatial components satisfies:

$$\langle Z_i Z_j \rangle = \frac{1}{3} \delta_{ij} \langle |\mathbf{Z}|^2 \rangle. \quad (186)$$

The transverse photon field couples only to the components orthogonal to its propagation axis:

$$\langle |\mathbf{Z}_\perp|^2 \rangle = \langle Z_x^2 + Z_y^2 \rangle = \frac{2}{3} \langle |\mathbf{Z}|^2 \rangle. \quad (187)$$

Thus, under electromagnetic probing, the effective contribution of internal  $z$ -axis structure is reduced by a factor of  $\frac{2}{3}$ :

$$Z_{\text{eff}}^2 = \frac{2}{3} Z_{\text{int}}^2. \quad (188)$$

This suppression factor is geometric in origin and reflects projection onto the photon's transverse detection plane. It applies independently of polarization or particle identity.

## K.2 Geometric Origin of the Projection Coefficients

The numerical factors in the Morton Projection Theorem— $\frac{2}{3}$  for the  $z$ -axis and 6 for the  $x$ -axis—arise from two distinct geometric mechanisms: angular projection and oscillator multiplicity.

**(1) Transverse Projection of an Isotropic  $Z$ -Field.** For an isotropic internal distribution of  $z$ -axis displacements, we assume an SO(3)-invariant configuration:

$$\langle Z_i Z_j \rangle = \frac{1}{3} \delta_{ij} \langle |\mathbf{Z}|^2 \rangle.$$

A transverse electromagnetic probe (like a photon) couples only to components orthogonal to the propagation axis, yielding:

$$\langle |Z_{\perp}|^2 \rangle = \langle Z_x^2 + Z_y^2 \rangle = \frac{2}{3} \langle |\mathbf{Z}|^2 \rangle.$$

This implies that only a fraction  $\frac{2}{3}$  of the  $z$ -axis structure is visible to the probe.

**(2) Composite Amplification in the Photon Structure.** In the Axis Model, the photon consists of six synchronized  $x$ -axis displacement modes (six morton):

$$X_{\mu} = \sum_{i=1}^6 x_{\mu}^{(i)}.$$

In an interaction term  $\mathcal{L}_{\text{int}} \sim g' J^{\mu} X_{\mu}$ , this structure multiplies the effective coupling by 6:

$$\mathcal{L}_{\text{int}} = 6g' J^{\mu} \bar{x}_{\mu}.$$

Thus, internal  $x$ -axis content is amplified by a factor of 6 in the observable projection.

**Result.** Combining the suppression of  $z$ -axis visibility by  $\frac{2}{3}$  with the  $6\times$  amplification of  $x$ -axis content, the effective projection is:

$$q_{\text{projected}} = \frac{2}{3} v_z + 6v_x,$$

where  $v_z$  and  $v_x$  count the internal  $z$ - and  $x$ -axis displacement vectors in a particle's morton structure. This measurement-space distortion defines the Morton Projection Theorem.

### K.3 Composite Photon Amplification of X-Axis Coupling

In the Axis Model, the photon is not elementary but a composite of six synchronized  $x$ -axis displacements—three  $x$ - $\bar{x}$  morton pairs. This structure amplifies its interaction with x-axis-aligned mortons.

Consider an interaction of the form:

$$\mathcal{L}_{\text{int}} = g' J^\mu X_\mu, \quad (189)$$

where  $J^\mu$  is an external current and  $X_\mu$  is the  $x$ -axis field. The photon field, being composite, is the sum of six coherent components:

$$X_\mu = \sum_{i=1}^6 x_\mu^{(i)}. \quad (190)$$

Substituting yields:

$$\mathcal{L}_{\text{int}} = 6g' J^\mu \bar{x}_\mu, \quad (191)$$

where  $\bar{x}_\mu$  is the mean displacement vector. Hence, the effective interaction strength is enhanced:

$$g_{\text{eff}} = 6g'. \quad (192)$$

This  $6\times$  amplification reflects the internal multiplicity of x-axis modes in the photon and is a source of the asymmetry between how z- and x-axis components are measured externally.

### K.4 Measurement-Space Projection Formula

Combining the  $\frac{2}{3}$  suppression of  $z$ -axis coupling and the  $6\times$  amplification of  $x$ -axis coupling yields the Morton Projection Theorem:

$$q_{\text{projected}} = \frac{2}{3}v_z + 6v_x, \quad (193)$$

where  $v_z$  and  $v_x$  are the total number of  $z$ - and  $x$ -axis vector displacements in a particle's internal morton structure. These vector counts arise from embedded mixed-axis mortons—typically of the canonical form  $(1\vec{v}_z, 2\vec{v}_x)$ —and should not be confused with counts of pure-axis mortons, which do not appear in the stabilized matter sector of the Axis Model.

This projection equation does *not* define the actual physical mass or electric charge of a particle. Rather, it quantifies the amount of internal displacement structure that is *visible* to a photon-based measurement, accounting for the fact that the composite photon over-samples  $x$ -axis displacements and under-samples  $z$ -axis content due to its internal six-oscillator geometry. The result is a measurement-space distortion that amplifies the apparent  $x$ -dominant structure and compresses the apparent  $z$ -dominant structure.

Observable quantities such as charge and mass emerge instead from scalar-filtered projection of internal  $z$ -axis polarity, as described in Section 4.4.2. The projection formula above is thus an interpretive tool used to understand coupling asymmetry in electromagnetic probes—it is not a generative law.

### K.5 Fractional Displacement Visibility and Scalar Filtering

Although fractional coefficients such as  $+\frac{2}{3}$  or  $-\frac{1}{3}$  appear in Eq. (193), they do not represent intrinsic or fractionalized electric charge. Instead, they reflect the relative visibility of internal displacement vectors under photon-based measurement, due to the asymmetric projection geometry

of the composite electromagnetic probe. The projection theorem quantifies this measurement-space distortion, not physical coupling constants.

True electric charge in the Axis Model arises exclusively from the net polarity of  $z$ -axis vector displacements. Each  $z$ -vector  $\vec{v}_z$  contributes a potential charge of  $+\frac{1}{3}e$  or  $-\frac{1}{3}e$ , depending on its internal orientation. The total charge potential of a particle is therefore given by:

$$Q_{\text{potential}} = \sum_{i=1}^{v_z} \pm \frac{1}{3}e, \quad (194)$$

where the sign of each term reflects the polarity of that displacement.

This potential is not directly observable. The scalar field acts as a coherence filter, projecting only a fraction of the potential into external spacetime. The observable charge is:

$$Q_{\text{observed}} = \eta \cdot Q_{\text{potential}}, \quad (195)$$

where  $\eta \in [0, 1]$  is the scalar coherence factor. In fully coherent structures (e.g., electrons, up quarks),  $\eta = 1$ . In partially coherent configurations (e.g., down quarks, neutrinos),  $\eta < 1$ .

For example, a particle with  $v_z = 6$  and a net polarity of  $+2$  (e.g., four  $+$  and two  $-$   $z$ -vectors) will have a charge potential of:

$$Q_{\text{potential}} = +\frac{2}{3}e,$$

and under full scalar coherence, an observed charge of:

$$Q_{\text{observed}} = +\frac{2}{3}e.$$

This scalar-filtered charge mechanism explains all known fractional electric charges in the first-generation fermions without invoking fundamental fractional charge carriers or abstract gauge symmetries. Projection merely determines how internal structure is observed—not how charge is generated.

## K.6 Variational Justification of Axial Alignment

The Morton Projection Theorem [Eq. (193)] arises naturally from a variational principle over internal morton configuration space  $\mathcal{M}$ . Each displacement vector  $\vec{v}_i$  is described by an orientation unit vector  $\hat{u}_i \in S^2$  and is stabilized by the scalar field  $\Phi$ . The system evolves under an effective internal action:

$$S_{\text{eff}}[\{\hat{u}_i\}, \Phi] = \int d\tau (\mathcal{L}_{\text{kin}} - \mathcal{L}_{\text{align}}), \quad (196)$$

where  $\mathcal{L}_{\text{align}}$  encodes the scalar-mediated tendency toward axis-locking.

The effective projection of the internal structure onto an external probe is given by:

$$\Pi_{\text{eff}} = \sum_i w_i \cdot (\hat{u}_i \cdot \hat{n}_{\text{probe}}), \quad (197)$$

where  $\hat{n}_{\text{probe}}$  defines the measurement axis (e.g., the  $x$ -axis for a photon-based interaction), and  $w_i$  is a mode-dependent weight:  $w_i = 6$  for  $x$ -aligned components and  $w_i = \frac{2}{3}$  for  $z$ -aligned components, as derived from the composite photon structure and transverse suppression geometry.

In the scalar-coherent ground state, internal displacement vectors align with model-preferred axes to minimize  $S_{\text{eff}}$ . Evaluating the projection functional in this fully aligned configuration yields:

$$\Pi_{\text{eff}} = 6v_x + \frac{2}{3}v_z, \quad (198)$$

reproducing the Morton Projection Theorem. This result reflects the effective visibility of internal structure to an external probe, not a physical observable like mass or charge.

## K.7 Summary

The Morton Projection Theorem provides a geometric map between internal vector structure and the effective displacement content observable via external electromagnetic probes. This projection governs coupling asymmetry due to the composite nature of the photon, which over-samples  $x$ -axis displacements and under-samples  $z$ -axis displacements.

However, the projection formula does *not* determine physical quantities like electric charge or rest mass. These arise instead from scalar-filtered projection of net  $z$ -axis polarity, modulated by scalar coherence, as described in Section 4.4.2. While projection defines what internal structure is measured, scalar filtering defines what internal content becomes physically real. Together, these mechanisms clarify how fractional charge signatures and mass-energy observables emerge from unified, quantized geometric substructure.

# L Derivation of the Bridge Equation

## L.1 Overview and Motivation

The *Bridge Equation*,

$$E_{\text{obs}} = \varepsilon_\Phi \cdot |q_{\text{projected}}|^2, \quad (199)$$

links the internal, scalar-coherent structure of a morton to its observed rest energy. The quantity  $q_{\text{projected}}$  is defined via the Morton Projection Theorem (Section 4.4.8) and encodes the scalar-filtered vector content of a particle. Unlike conventional mass-generation mechanisms that rely on Higgs couplings or symmetry breaking, the Axis Model ties observable mass directly to internal geometry through scalar-mediated confinement. This appendix provides a rigorous derivation of Eq. (199) from the scalar–vector Lagrangian and effective energy functional.

## L.2 Effective Lagrangian and Scalar Binding Energy

We begin with the scalar sector of the Axis Model action:

$$S_\Phi = \int d^4x \left[ \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - \Lambda_\Phi \Phi^2 |q_{\text{projected}}|^2 \right], \quad (200)$$

where  $\Phi$  is the scalar field,  $V(\Phi)$  is the scalar self-potential (assumed to be minimized at  $\Phi = v$ ), and  $\Lambda_\Phi$  encodes the binding energy density per unit projected vector content. The key insight is that only the scalar-projected component  $q_{\text{projected}}$  contributes to physical mass-energy, due to coherence filtering.

In the rest frame of a bound morton configuration, spatial variations are small, so the kinetic term can be neglected, yielding:

$$\mathcal{H}_{\text{eff}} = \Lambda_\Phi \Phi^2 |q_{\text{projected}}|^2. \quad (201)$$

Substituting  $\Phi \rightarrow \langle \Phi \rangle = v$ , we obtain:

$$E_{\text{obs}} = \Lambda_\Phi v^2 \cdot |q_{\text{projected}}|^2 \equiv \varepsilon_\Phi \cdot |q_{\text{projected}}|^2, \quad (202)$$

which defines the effective scalar energy scale:

$$\varepsilon_\Phi = \Lambda_\Phi v^2. \quad (203)$$

This is the desired quadratic scaling: the rest energy is proportional to the square of the projected vector content, with all proportionality absorbed into a single empirically calibrated constant  $\varepsilon_\Phi$ .

### L.3 Geometric Justification

The quadratic scaling is naturally supported by the geometry of the internal configuration space. Each morton inhabits a compact manifold defined by discrete vector displacements, with physical observables determined by the scalar projection:

$$q_{\text{projected}} = \frac{2}{3}v_z + 6v_x.$$

Because scalar coupling is quadratic in field amplitude and linear in displacement coherence, the projected charge  $q_{\text{projected}}$  acts as a coherent charge-like quantity. The squared scaling reflects the total scalar energy stored in the internal field alignment and corresponds to the natural Riemannian metric on this configuration space.

### L.4 Soliton Analogy and Stability

The morton may also be viewed as a non-topological soliton, stabilized by scalar binding. In such systems, energy functionals take the form:

$$E[\phi] = \int d^3x [(\nabla\phi)^2 + U(\phi, q)],$$

where  $\phi$  is the scalar profile and  $q$  labels an internal configuration. For a stable soliton solution, Derrick's theorem implies that only quadratic or bounded energy terms are permitted. This constrains the energy to scale at most quadratically in  $q$  for compact solutions. Since higher-order self-interactions (e.g., quartic or logarithmic) would destabilize bound states, the observed rest mass must scale as  $E \sim q_{\text{projected}}^2$ .

### L.5 Dimensional Consistency

Dimensional analysis confirms the result. The quantity  $q_{\text{projected}}$  is dimensionless, derived from a count of discrete vector displacements. To yield energy units on the left-hand side of Eq. (199),  $\varepsilon_\Phi$  must have dimensions of mass or energy. Thus, the energy per projected vector square is the fundamental mass-generation scale of the theory, analogous to  $v_{\text{Higgs}}^2$  in the Standard Model.

### L.6 Empirical Calibration

For the electron, the internal structure consists of three mixed mortons, totaling:

$$v_z = 3, \quad v_x = 6.$$

The Morton Projection Theorem gives:

$$q_{\text{projected}} = \frac{2}{3} \cdot 3 + 6 \cdot 6 = 2 + 36 = 38.$$

Using the known electron rest mass  $m_e = 0.511 \text{ MeV}$ , we calibrate the scalar energy scale as:

$$\varepsilon_\Phi = \frac{m_e}{|q_{\text{projected}}|^2} = \frac{0.511}{38^2} \approx \frac{0.511}{1444} \approx 0.000354 \text{ MeV} \approx 354 \text{ eV}. \quad (204)$$

This scalar scale serves as a universal mass coefficient for all fundamental particles in the model.

## L.7 Robustness and Corrections

Quantum corrections to the scalar binding energy may introduce terms such as:

$$\delta V(q) \sim \alpha \cdot q^2 \ln q^2,$$

but these are suppressed by small prefactors and do not alter the leading quadratic behavior. Additionally, the quantized nature of  $q_{\text{projected}}$ , determined by discrete morton content, ensures topological stability and protects the mass spectrum from drift.

## L.8 Conclusion

The Bridge Equation,

$$E_{\text{obs}} = \varepsilon_\Phi \cdot |q_{\text{projected}}|^2,$$

is not a heuristic assumption but a natural consequence of scalar-mediated binding in the Axis Model. It arises from efective scalar Lagrangian dynamics in the low-energy limit the geometry of coherence-filtered vector projection, solitonic stability under variational arguments, and dimensional and empirical consistency.

This derivation strengthens the theoretical foundation of the Axis Model's mass-generation mechanism and connects its internal configuration language to external observables in a mathematically rigorous and falsifiable way.

# M Master Lagrangian and Field Definitions

The Lagrangian formalism presented in this appendix captures the effective low-energy dynamics of the Axis Model as derived from its underlying geometric framework. Each term arises from stabilized projections of internal morton structure into observable spacetime, where discrete vector compositions along the  $x$ -,  $y$ -, and  $z$ -axes generate the emergent fields  $X^\mu$ ,  $\Phi$ , and  $Z^\mu$ . Standard field-theoretic forms such as  $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$  are retained to describe the resulting dynamics in a gauge-invariant form. The fields themselves are emergent composite structures introduced in Sections 3.3-3.5 and governed by the equations developed in Section 4.1.

The total Lagrangian for the Axis Model is constructed in flat spacetime with metric signature  $(+, -, -, -)$ , using natural units  $\hbar = c = 1$ . Each term has total mass dimension 4, ensuring renormalizability for core interactions and EFT consistency for higher-order terms.

## M.1 Scalar Sector

The scalar field  $\Phi$  is governed by a spontaneously broken potential:

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - V(\Phi), \quad V(\Phi) = -\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4. \quad (205)$$

This yields a vacuum expectation value (VEV):

$$\langle\Phi\rangle = v = \sqrt{\mu^2/\lambda}.$$

## M.2 Vector Sector

Each vector field  $A^\mu \in \{X^\mu, Z^\mu\}$  is described by the standard kinetic term:

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}^A F_A^{\mu\nu}, \quad F_{\mu\nu}^A = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (206)$$

### M.3 Scalar-Vector Mass Coupling (Dimension-4)

Vector bosons acquire mass through the Higgs-like coupling:

$$\mathcal{L}_{\text{mass}} = \sum_{A \in \{X, Z\}} \frac{1}{2} g_{A\Phi}^2 \Phi^2 A_\mu A^\mu. \quad (207)$$

Here  $g_{A\Phi}$  is a dimensionless coupling. After symmetry breaking, this yields:

$$m_A^2 = g_{A\Phi}^2 v^2.$$

### M.4 Gauge-Invariant Scalar-Photon Coupling (Dimension-5, EFT)

The leading effective interaction responsible for vacuum birefringence and parity-violating photon propagation in the Axis Model is a gauge-invariant, dimension-5 operator coupling a pseudoscalar field  $\Phi_A$  to the dual electromagnetic field strength:

$$\mathcal{L}_{\text{biref}} = \frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (208)$$

where  $g_A$  is a dimensionless coupling constant,  $M$  is a suppression scale with dimension  $[E]$ , and  $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$  is the dual field strength tensor.

This operator is Lorentz-invariant,  $U(1)$ -gauge-invariant, and odd under parity and time reversal. It modifies the equations of motion for the gauge field in the presence of a nontrivial gradient  $\partial_\mu \Phi_A$ , inducing a polarization-dependent dispersion relation:

$$\partial_\nu F^{\nu\mu} = \frac{4g_A}{M} (\partial_\nu \Phi_A) \tilde{F}^{\nu\mu}.$$

This structure gives rise to an observable polarization rotation angle:

$$\Delta\theta \sim \frac{g_A}{M} |\nabla \Phi_A| L,$$

as detailed in Appendix G.3. Experimental constraints from PVLAS and astrophysical sources impose the bound:

$$\frac{g_A}{M} |\nabla \Phi_A| \lesssim 2 \times 10^{-18} \text{ eV}.$$

**Historical Note.** Earlier formulations of the Axis Model included a term of the form:

$$\frac{g_A}{M} \Phi F_{\mu\nu} (\partial^\mu A^\nu),$$

which is not gauge-invariant and has since been removed from the theory. While such terms can arise in certain fixed-gauge backgrounds or as intermediate steps in a Stückelberg decomposition, they do not represent fundamental, physical operators in a gauge-complete EFT. The parity-odd, gauge-invariant operator  $\Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu}$  is now recognized as the unique leading-order structure consistent with the symmetries and empirical predictions of the Axis Model.

### M.5 Fermion Sector (Minimal Form)

$$\mathcal{L}_\psi = i\bar{\psi} \gamma^\mu \partial_\mu \psi - m_\psi \bar{\psi} \psi - g_{\psi\Phi} \Phi \bar{\psi} \psi + g_{\psi Z} \bar{\psi} \gamma^\mu Z_\mu \psi, \quad (209)$$

where  $g_{\psi\Phi}$  is dimensionless. The scalar–fermion coupling modulates effective fermion masses in scalar background environments. The fermion field  $\psi$  carries charge under the internal  $U(1)_Z$  symmetry, consistent with the BRST transformations summarized in Appendix X and fully developed in the companion *Quantum Completion of the Axis Model* paper. The coupling constant  $g_{\psi Z}$  controls the strength of this interaction and contributes to quantum-level vector corrections and symmetry structure.

## M.6 Coupling Dimensional Summary

Coupling	Symbol	Mass Dimension
Scalar quartic	$\lambda$	0
Scalar mass	$\mu$	1
Scalar–vector mass coupling	$g_{A\Phi}$	0
Scalar–vector gradient coupling	$g_A$	0
EFT suppression scale	$M$	1
Fermion–scalar Yukawa	$g_{\psi\Phi}$	0
Fermion–vector gauge coupling	$g_{\psi Z}$	0

All operators are dimensionally consistent. Renormalizable terms are grouped in Sections M.1–M.3, while higher-dimensional EFT contributions are isolated in M.4, aligning with standard field-theoretic practice.

## M.7 Canonical Lagrangian Summary

The complete, gauge-consistent, and empirically anchored Lagrangian for the Axis Model is:

$$\begin{aligned} \mathcal{L}_{\text{Axis}} = & \underbrace{\frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - V(\Phi)}_{\mathcal{L}_\Phi \text{ (Scalar sector), } V(\Phi) = -\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4} \\ & - \underbrace{\frac{1}{4}F_{\mu\nu}^X F_X^{\mu\nu} + \frac{1}{2}g_{X\Phi}^2\Phi^2 X_\mu X^\mu}_{\mathcal{L}_X \text{ (x-axis Vector)}} \\ & - \underbrace{\frac{1}{4}F_{\mu\nu}^Z F_Z^{\mu\nu} + \frac{1}{2}g_{Z\Phi}^2\Phi^2 Z_\mu Z^\mu}_{\mathcal{L}_Z \text{ (z-axis Vector)}} \\ & + \underbrace{i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_\psi\bar{\psi}\psi - g_{\psi\Phi}\Phi\bar{\psi}\psi}_{\mathcal{L}_\psi \text{ (Fermion sector)}} \\ & + \underbrace{\frac{g_A}{M}\Phi_A F_{\mu\nu}^X \tilde{F}_X^{\mu\nu} + g_Z(\partial_\mu\Phi)Z^\mu}_{\mathcal{L}_{\text{int}} \text{ (EFT + Scalar-Vector Interaction)}} . \end{aligned} \quad (210)$$

This expression unifies all renormalizable and leading-order EFT terms used throughout the manuscript. The gradient coupling term  $g_Z(\partial_\mu\Phi)Z^\mu$  plays a central role in generating the modified gravitational dynamics derived in Section 4.1.4, while the parity-odd operator  $\Phi_A F^X \tilde{F}^X$  governs birefringence as analyzed in Appendix G.3.

Unless otherwise noted, all field equations, quantization procedures, and interaction mechanisms elsewhere in this paper derive from this canonical Lagrangian.

## N Gauge Constraint for the Z-Axis Vector Field $Z^\mu$

This appendix justifies the use of the Lorenz-type gauge condition  $\partial_\mu Z^\mu = 0$  for the z-axis vector field  $Z^\mu$ , which mediates gravitational dynamics in the Axis Model. Although  $Z^\mu$  is a massive vector field with scalar-dependent mass  $M_Z(\Phi)$ , this constraint is dynamically admissible and consistent with the underlying field equations and stress-energy conservation, particularly in the static or weak-field regimes relevant to morton solutions.

### N.1 Field Equation and Mass Structure

The canonical Lagrangian for the z-axis field, as defined in Appendix M, is

$$\mathcal{L}_Z = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}M_Z^2(\Phi)Z_\mu Z^\mu, \quad \text{with } Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu.$$

Varying this action yields the Proca-like field equation with a scalar source:

$$\partial_\nu Z^{\nu\mu} + M_Z^2(\Phi)Z^\mu = -g_Z \partial^\mu \Phi.$$

Taking the divergence of both sides gives:

$$\partial_\mu \partial_\nu Z^{\nu\mu} + \partial_\mu (M_Z^2(\Phi)Z^\mu) = -g_Z \square \Phi.$$

Due to antisymmetry of  $Z^{\nu\mu}$ , the first term vanishes identically:

$$\partial_\mu \partial_\nu Z^{\nu\mu} = 0,$$

so the equation reduces to:

$$\partial_\mu (M_Z^2(\Phi)Z^\mu) = -g_Z \square \Phi.$$

### N.2 Consistency of the Lorenz Constraint

We now analyze under what conditions the divergence  $\partial_\mu Z^\mu$  may be approximated as vanishing.

Expanding the left-hand side of the previous result gives:

$$(\partial_\mu M_Z^2(\Phi)) Z^\mu + M_Z^2(\Phi) \partial_\mu Z^\mu = -g_Z \square \Phi.$$

If the scalar field  $\Phi$  varies slowly in spacetime, we can approximate:

$$\partial_\mu M_Z^2(\Phi) \approx 0,$$

in which case the equation simplifies to:

$$M_Z^2(\Phi) \partial_\mu Z^\mu \approx -g_Z \square \Phi.$$

In static or equilibrium scalar backgrounds, where second derivatives of  $\Phi$  are small or negligible, we further approximate:

$$\square \Phi \approx 0,$$

which leads to:

$$M_Z^2(\Phi) \partial_\mu Z^\mu \approx 0.$$

Assuming that  $M_Z^2(\Phi) \neq 0$ —i.e., the Z-field remains massive—this directly implies:

$$\partial_\mu Z^\mu \approx 0.$$

Thus, under weak-field or slowly varying scalar conditions, the Lorenz-type gauge constraint arises naturally and consistently from the dynamics of the model.

### N.3 Physical Justification

The condition  $\partial_\mu Z^\mu = 0$  is not merely a mathematical convenience but serves several critical theoretical functions:

1. It eliminates unphysical longitudinal modes from  $Z^\mu$ , ensuring that the field propagates only three degrees of freedom, as appropriate for a massive spin-1 field in four dimensions.
2. It guarantees compatibility between the field equation and the stress-energy tensor  $T_{(Z)}^{\mu\nu}$ , whose conservation law  $\nabla_\mu T_{(Z)}^{\mu\nu} = 0$  is derived using the field equations and assumes this constraint in the weak-field limit.
3. It simplifies the form of the modified gravitational field equations (see Section 4.1.4) by eliminating derivative cross-terms that arise when  $M_Z^2(\Phi)$  is treated as approximately constant over a local region.

### N.4 Interpretation

Although the constraint  $\partial_\mu Z^\mu = 0$  is not universally valid in all regimes—particularly in regions with large scalar gradients or rapidly evolving fields—it is a dynamically consistent approximation in many physically relevant scenarios. These include static morton configurations, spherically symmetric weak-field solutions, and cosmological epochs where  $\Phi$  evolves slowly.

Its use throughout Section 4.1.4 and Section 4.2.3, as well as in the derivation of the stress-energy tensor in Appendix D, reflects this justified simplification. A full dynamical treatment without the constraint may be pursued in future generalizations, but for the present model and its empirical targets, the Lorenz-type gauge condition offers a physically meaningful and mathematically coherent reduction via dynamical gauge fixing.

## O Minisuperspace Relation and Scalar Time Emergence

### O.1 Natural Units and the Internal $y$ -Axis

Throughout this appendix we adopt natural units with  $c = \hbar = 1$ , in which all physical quantities are expressed in powers of energy. In this system:

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1}, \quad [\text{mass}] = [\text{energy}].$$

The internal  $y$ -axis of the Axis Model is not a physical extra dimension but a formal coordinate direction parameterizing evolution within the scalar field's internal configuration space. As a coordinate, we assign it the dimension of length:

$$[y] = \text{length}.$$

Given the globally adopted assignment  $[\Phi] = \text{energy}$ , the derivative of  $\Phi$  along this internal direction has units:

$$[\partial_y \Phi] = \frac{[\Phi]}{[y]} = \text{energy}^2.$$

## O.2 Global Dimensional Assignment of $\Phi$

The scalar field  $\Phi$  is consistently assigned mass dimension one throughout the Axis Model:

$$[\Phi] = \text{energy}.$$

This choice ensures dimensional consistency across all interaction terms and effective operators. It also enables a rigorous interpretation of scalar field gradients as generators of temporal evolution within the minisuperspace framework.

## O.3 Derivation and Dimensional Consistency

We postulate the minisuperspace-motivated scalar-time relation:

$$\partial_t = \kappa \partial_y \Phi, \quad (211)$$

where  $\partial_t$  denotes the physical time derivative in 3+1 spacetime, and  $\partial_y \Phi$  encodes internal scalar variation along the  $y$ -axis. Using the assignments above,

$$[\partial_t] = \text{energy}, \quad [\partial_y \Phi] = \text{energy}^2 \quad \Rightarrow \quad [\kappa] = \text{energy}^{-1}.$$

Thus,  $\kappa$  defines a characteristic energy scale for mapping internal scalar evolution into external time flow. Its inverse,  $\kappa^{-1}$ , has dimensions of time or length and may be interpreted as the coherence scale for temporal emergence.

## O.4 Scalar Clock Shift Estimate and Phenomenological Bound

The effective time dilation induced by scalar field variation is modeled as:

$$\frac{\Delta t}{t} = \zeta \left( \frac{\Phi}{E_{\text{ref}}} \right), \quad (212)$$

where  $\zeta$  is a dimensionless coupling and  $E_{\text{ref}}$  is a reference energy scale, typically taken to be  $M_{\text{Pl}}$ . This form is dimensionally consistent under  $[\Phi] = E$ .

Two illustrative parameter regimes:

**Case A: Fixed scalar amplitude.** If the scalar field amplitude is  $\Phi \sim 10^{-3} M_{\text{Pl}}$ , then

$$\frac{\Delta t}{t} \sim \zeta \cdot 10^{-3}.$$

For  $\zeta \sim 0.5$ , this yields a fractional clock-rate shift of  $\sim 5 \times 10^{-4}$ , which exceeds laboratory atomic clock bounds by many orders of magnitude.

**Case B: Observational limit.** To match current experimental sensitivity, such as CMB-S4 shift parameter constraints at the level  $\Delta t/t \sim 10^{-16}$ , either the scalar amplitude must be extremely small ( $\Phi \sim 2 \times 10^{-16} M_{\text{Pl}}$ ), or the coupling must be weak ( $\zeta \sim 10^{-13}$ ) for a fixed scalar amplitude  $\Phi \sim 10^{-3} M_{\text{Pl}}$ .

These bounds highlight the need to distinguish between scalar modes responsible for time modulation and those governing mass-energy localization. A dual-scalar structure or context-specific scalar effective modes may be required.

## O.5 Interpretation and Outlook

The scalar-time relation introduced in Eq. (211) provides a phenomenological anchor for interpreting scalar field variation as the generator of temporal structure. When  $\partial_y \Phi = 0$ , the internal configuration is static and no external time flow is induced. Conversely, a nonzero internal scalar gradient generates physical time evolution in 3+1 spacetime.

A full derivation of this correspondence from a minisuperspace action—possibly through Hamilton-Jacobi or Wheeler-DeWitt techniques—is reserved for future work. The present appendix establishes dimensional consistency, clarifies the role of the parameter  $\kappa$ , and supports the hypothesis that scalar field coherence is the origin of temporal emergence within the Axis Model.

## P Cosmological Dynamics of the Displacement Field

This appendix presents the full theoretical framework for the Axis Model’s cosmological dynamics, which are governed by a scalar displacement field  $D(t)$ . This field represents the coherent time component of the x-axis displacement field  $D_x^\mu$  and serves as the origin of dark energy behavior in the model. The analysis combines phenomenological scaling laws, renormalization-group (RG) running of scalar couplings, and dynamical system stability to construct a robust and falsifiable cosmological solution.

### P.1 Scaling Law for Energy Density Evolution

As derived below, the late-time cosmological expansion is driven by an effective displacement field  $D(t)$  governed by a power-law potential:

$$V(D) = \lambda_D M^4 \left( \frac{D}{M} \right)^\beta,$$

where  $\beta$  is a dimensionless shape parameter and  $\lambda_D$  sets the amplitude of the potential. Unlike slow-roll inflationary models, the Axis Model adopts a scaling-law framework derived from the energy conservation equation for a fluid with equation-of-state parameter  $w$ , yielding:

$$\rho_D(a) \propto a^{-n}, \quad n = 3(1+w). \quad (213)$$

For the power-law potential above, the effective equation of state tracks the form

$$w = \frac{\beta - 2}{\beta + 2},$$

which leads to a scaling index of:

$$n = \frac{6\beta}{\beta + 2}. \quad (214)$$

This expression defines a continuous family of expansion behaviors: when  $\beta = 0$ , the model yields  $w = -1$ ,  $n = 0$ , recovering a cosmological constant. When  $\beta = 2$ , one finds  $w = 0$ ,  $n = 3$ , consistent with pressureless matter. Values  $0 < \beta < 2$  interpolate between these extremes and allow observational fits to the Hubble expansion history, as performed in Section 5.7.

## P.2 Microphysical Origin of the Potential Amplitude

The displacement potential amplitude  $\lambda_D$  is not an arbitrary free parameter but emerges from the RG evolution of the scalar self-coupling  $\lambda(\mu)$ . Starting from the electroweak scale  $M = 246$  GeV, we adopt the one-loop beta function:

$$\mu \frac{d\lambda}{d\mu} = \frac{\lambda}{16\pi^2} (12\lambda - 9g^2), \quad (215)$$

with input values  $\lambda(M) = 0.13$ ,  $g(M) = 0.72$ , yielding  $12\lambda - 9g^2 \approx -3.11$ . This gives a slow running exponent of:

$$|\beta_\lambda| = \frac{3.11}{16\pi^2} \approx 0.020.$$

The IR-corrected coupling at the displacement scale  $\mu_e \sim 1$  GeV is:

$$\lambda(\mu_e) \approx \lambda(M) \left( \frac{\mu_e}{M} \right)^{|\beta_\lambda|} \approx 0.13 \times (1/246)^{0.020} \approx 0.11.$$

The effective amplitude of the potential is then:

$$\lambda_D = \lambda(\mu_e) \left( \frac{\mu_e}{M} \right)^\beta. \quad (216)$$

For a benchmark choice  $\beta = 0.5$ , this yields:

$$\lambda_D \approx 0.11 \times \left( \frac{1}{246} \right)^{0.5} \approx 6.0 \times 10^{-3}.$$

This links the cosmological potential directly to microphysical parameters of the scalar sector and ensures consistency with the renormalization flow developed in Appendix F.

## P.3 Dynamical Stability of the Tracker Solution

The robustness of the displacement field as a cosmological driver is confirmed by analyzing the dynamical stability of the tracker solution. Defining normalized phase-space variables:

$$x = \frac{\dot{D}}{\sqrt{6} H M_{\text{Pl}}}, \quad y = \frac{\sqrt{V}}{\sqrt{3} H M_{\text{Pl}}}, \quad (217)$$

the displacement dynamics reduce to the autonomous system:

$$\frac{dx}{dN} = -3x + \sqrt{\frac{3}{2}} \beta y^2 + \frac{3}{2} x [2x^2 + (1+w_b)(1-x^2-y^2)], \quad (218)$$

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}} \beta x y + \frac{3}{2} y [2x^2 + (1+w_b)(1-x^2-y^2)], \quad (219)$$

where  $N = \log a$  and  $w_b$  is the background fluid equation of state. The fixed point is located at:

$$(x_*, y_*) = \left( \frac{\beta}{\sqrt{6}}, \sqrt{1 - \frac{\beta^2}{6}} \right).$$

Linearizing around this point, the eigenvalues  $\lambda_\pm$  of the Jacobian matrix determine stability. For all  $0 < \beta < \sqrt{6}$ , the real parts of  $\lambda_\pm$  are negative, confirming that the tracker solution is a stable late-time attractor.

For example, with  $\beta = 0.5$ , one obtains:

$$w = -0.6, \quad n = 1.2, \quad \rho_D(a) \propto a^{-1.2}, \quad \lambda_D \sim 6 \times 10^{-3},$$

which matches observed cosmic acceleration data and confirms dynamical consistency.

Together, these results establish a coherent, microphysically grounded, and dynamically stable cosmological mechanism in the Axis Model, suitable for replacing the standard cosmological constant paradigm with a falsifiable alternative.

## Q Scalar Field Profiles in Equilibrium Halos

This appendix motivates the scalar field profile used in modeling equilibrium halos and derives the associated scalar-induced acceleration correction used in Section H.2. The scalar field is assumed to take the logarithmic form:

$$\Phi(r) = \Phi_0 \ln \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right], \quad (220)$$

where  $\Phi_0$  has units of velocity squared and  $r_c$  is a characteristic core radius.

This profile captures the approximately flat scalar potential expected in dwarf galaxies and mirrors empirical dark matter core profiles often fit by cored isothermal distributions. It is analytic, well-behaved at  $r \rightarrow 0$  and  $r \rightarrow \infty$ , and allows direct integration into analytic acceleration models.

### Q.1 Exact Scalar-Induced Acceleration

Given the modified gravitational potential used in the Axis Model:

$$\Phi_g(r) = -\frac{GM}{r} \left( 1 + \frac{\alpha \Phi(r)}{c^2} \right), \quad (221)$$

the scalar-induced acceleration correction is:

$$\begin{aligned} \delta a(r) &= -\frac{d}{dr} \left( -\frac{GM\alpha}{rc^2} \Phi(r) \right) \\ &= \frac{GM\alpha}{c^2} \left[ \frac{1}{r} \frac{d\Phi}{dr} - \frac{\Phi(r)}{r^2} \right]. \end{aligned} \quad (222)$$

For the profile in Eq. (220), we have:

$$\frac{d\Phi}{dr} = \Phi_0 \cdot \frac{2r}{r^2 + r_c^2}. \quad (223)$$

Substituting into the general expression yields:

$$\delta a(r) = \frac{GM\alpha\Phi_0}{c^2} \left[ \frac{2}{r_c^2 + r^2} - \frac{1}{r^2} \ln \left( 1 + \left( \frac{r}{r_c} \right)^2 \right) \right]. \quad (224)$$

### Q.2 Gradient-Dominant Approximation

In regimes where  $\frac{1}{r} \frac{d\Phi}{dr} \gg \frac{\Phi(r)}{r^2}$ , we may approximate:

$$\delta a(r) \approx \frac{GM\alpha}{rc^2} \cdot \frac{d\Phi}{dr} = \frac{2GM\alpha\Phi_0}{c^2(r_c^2 + r^2)}. \quad (225)$$

This approximation is sometimes used in the main text for analytic convenience but must be justified case by case depending on the values of  $r/r_c$ .

### Q.3 Scaling of the Correction Term

The exact expression in Eq. (224) shows that the scalar-induced acceleration correction falls off with radius and is suppressed by both the dimensionless factor  $\alpha\Phi_0/c^2$  and the ratio of core to orbital radius. For typical dwarf-galaxy parameters ( $\Phi_0/c^2 \lesssim 10^{-6}$ ,  $r_c \sim \text{kpc}$ ), the resulting fractional correction  $\delta a(r)/a_N(r)$  is expected to be small,  $\ll 1\%$ , across most of the halo. This is consistent with the approximate form in Eq. (225), which isolates the gradient contribution.

Rather than providing specific benchmarks here, we emphasize the scaling: the correction grows with  $\alpha$  and  $\Phi_0$ , and is largest near  $r \sim r_c$  where the gradient is steepest. Quantitative values will depend on the parameter set extracted from empirical fits.

## R Reduction to Standard Theories

This appendix demonstrates how the Axis Model recovers or remains consistent with established frameworks—General Relativity (GR), Quantum Electrodynamics (QED), and the Standard Model (SM)—in the appropriate limits.

### R.1 Recovery of General Relativity

To recover classical General Relativity, we begin with the modified Einstein equation derived in Appendix D:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left( T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\text{Axis})} \right), \quad (226)$$

where the Axis-sector energy-momentum tensor includes contributions from scalar, vector, and interaction fields:

$$T_{\mu\nu}^{(\text{Axis})} = T_{\mu\nu}^{(\Phi)} + T_{\mu\nu}^{(X)} + T_{\mu\nu}^{(Z)} + T_{\mu\nu}^{(\text{int})}. \quad (227)$$

In the weak-field vacuum limit, all Axis-sector fields are unexcited. Specifically, the scalar field is frozen in its vacuum configuration,  $\Phi(x) \approx \text{const}$ , which implies  $T_{\mu\nu}^{(\Phi)} \rightarrow 0$ . The vector fields vanish,  $X^\mu \rightarrow 0$  and  $Z^\mu \rightarrow 0$ , such that  $T_{\mu\nu}^{(X)}, T_{\mu\nu}^{(Z)} \rightarrow 0$ . Additionally, scalar gradients become negligible,  $\nabla_\mu \Phi \approx 0$ , causing the scalar–vector interaction contribution to vanish:  $T_{\mu\nu}^{(\text{int})} \rightarrow 0$ .

Under these conditions, the Einstein equation reduces to its standard form:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{(\text{matter})}, \quad (228)$$

recovering classical GR. Newtonian gravity is further obtained by taking the non-relativistic limit. Thus, general relativity emerges as the vacuum ground state of the Axis gravitational sector.

### R.2 Recovery of Quantum Electrodynamics

At low energies, the composite structure of the photon becomes unresolvable and scalar field gradients vanish. In this regime, the Axis Model reduces to standard quantum electrodynamics. The non-gauge-invariant scalar–vector coupling

$$\mathcal{L}_{\text{int}} = g_X (\partial_\mu \Phi) X^\mu$$

is embedded into a Stückelberg-completed framework. The corresponding low-energy effective Lagrangian includes a gauge-invariant dimension-5 operator:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{c}{M_*} \Phi (\partial_\mu A_\nu) F^{\mu\nu} + \dots, \quad (229)$$

where  $M_* \sim M_{\text{Pl}}$  is the photon compositeness scale, and  $c \sim \mathcal{O}(1)$ . In the low-energy limit  $E \ll M_*$ , this operator becomes negligible and the classical Maxwell equations are recovered:

$$\partial_\nu F^{\nu\mu} = J^\mu + \mathcal{O}\left(\frac{E}{M_*}\right). \quad (230)$$

This leads to the following formal result:

**Theorem R.1** (Infrared Emergence of Gauge Symmetry). *In the infrared limit  $E \ll M_*$ , the Axis Model reduces to classical Maxwell electrodynamics. Observable effects of the Stückelberg-completed scalar-vector interactions are suppressed as:*

$$\delta\mathcal{O} \sim \mathcal{O}\left(\frac{E^2}{M_*^2}\right).$$

*This suppression guarantees gauge invariance, photon masslessness, and compatibility with all known low-energy data.*

This ensures photon masslessness, gauge symmetry, and consistency with experimental constraints on birefringence and vacuum polarization. The Stückelberg embedding also preserves BRST invariance and loop-level consistency, as demonstrated in Appendix [AI](#)

### R.3 Compatibility with the Standard Model

The Axis Model introduces no fields charged under the Standard Model gauge group:

$$SU(3)_C \times SU(2)_L \times U(1)_Y.$$

All Axis-sector fields— $\Phi$ ,  $X^\mu$ , and  $Z^\mu$ —are gauge singlets and interact only through geometric and Lorentz-invariant couplings. These additions do not disrupt SM symmetry structure or renormalizability.

Mass generation is modified but not replaced. The fermion mass receives a scalar-induced shift:

$$m(x) = m_{\text{SM}} + \delta m_\Phi(x), \quad \text{with} \quad \delta m_\Phi(x) \propto g_{\psi\Phi} \langle \Phi(x) \rangle, \quad (231)$$

as discussed in Section [4.4.6](#). This is consistent with collider bounds provided  $\langle \Phi(x) \rangle \rightarrow 0$  at high energies or in vacuum.

Cosmologically, the scalar field modifies the effective gravitational constant:

$$G_{\text{eff}} = G \left(1 + \frac{\alpha \bar{\Phi}}{c^2}\right), \quad (232)$$

where  $\bar{\Phi}$  is the background scalar amplitude. Observational bounds from rotation curves and lensing constrain  $\alpha \lesssim 0.06$  (see Section [3.5.5](#)). This is also consistent with Planck and CMB-S4 bounds on  $\Delta G/G$ , ensuring that scalar-induced modifications do not conflict with known data.

## S Canonical Quantization and Field Expansions

This appendix formalizes the canonical quantization procedure for the fundamental fields in the Axis Model. We present explicit mode expansions for the scalar field  $\Phi$ , the x-axis and z-axis vector fields  $X_\mu$  and  $Z_\mu$ , and the Dirac fermion field  $\psi$ . These expressions provide the basis for loop-level corrections, scattering amplitudes, and operator renormalization in the Axis Model's effective quantum field theory (QFT).

All quantization is performed in a flat Minkowski background with signature  $(+, -, -, -)$ , in Feynman gauge unless otherwise noted.

## S.1 Scalar Field $\Phi$

The real scalar field is quantized as:

$$\hat{\Phi}(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[ \hat{a}_\Phi(p) e^{-ip \cdot x} + \hat{a}_\Phi^\dagger(p) e^{ip \cdot x} \right], \quad (233)$$

with canonical commutation relation:

$$[\hat{a}_\Phi(p), \hat{a}_\Phi^\dagger(p')] = (2\pi)^3 \delta^{(3)}(p - p'). \quad (234)$$

## S.2 Vector Fields $A_\mu$ and $Z_\mu$

(a) X-axis vector field  $A_\mu$  (massless limit, Feynman gauge):

$$A^\mu(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_{\lambda=0}^3 \left[ \hat{a}_A(p, \lambda) \epsilon^\mu(p, \lambda) e^{-ip \cdot x} + \hat{a}_A^\dagger(p, \lambda) \epsilon^{*\mu}(p, \lambda) e^{ip \cdot x} \right], \quad (235)$$

with:

$$[\hat{a}_A(p, \lambda), \hat{a}_A^\dagger(p', \lambda')] = (2\pi)^3 \delta^{(3)}(p - p') \delta_{\lambda \lambda'}.$$

(b) Z-axis vector field  $Z_\mu$  (massive Proca field, unitary gauge):

$$Z^\mu(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_{\lambda=1}^3 \left[ \hat{a}_Z(p, \lambda) \epsilon^\mu(p, \lambda) e^{-ip \cdot x} + \hat{a}_Z^\dagger(p, \lambda) \epsilon^{*\mu}(p, \lambda) e^{ip \cdot x} \right], \quad (236)$$

where  $\epsilon^\mu$  satisfies the transversality condition:

$$p_\mu \epsilon^\mu(p, \lambda) = 0, \quad \text{with} \quad p^2 = m_Z^2,$$

and the same commutator structure applies:

$$[\hat{a}_Z(p, \lambda), \hat{a}_Z^\dagger(p', \lambda')] = (2\pi)^3 \delta^{(3)}(p - p') \delta_{\lambda \lambda'}.$$

## S.3 Dirac Fermion Field $\psi$

The Dirac spinor is expanded as:

$$\hat{\psi}(x) = \int \frac{d^3 p}{(2\pi)^3} \sum_s \left[ \hat{b}_p^s u^s(p) e^{-ip \cdot x} + \hat{d}_p^{s\dagger} v^s(p) e^{ip \cdot x} \right], \quad (237)$$

with anti-commutation relations:

$$\{\hat{\psi}_\alpha(x), \hat{\psi}_\beta^\dagger(y)\} = (\gamma^0)_{\alpha\beta} \delta^{(3)}(x - y), \quad (238)$$

and:

$$\{\hat{b}_p^s, \hat{b}_{p'}^{s'\dagger}\} = (2\pi)^3 \delta^{(3)}(p - p') \delta^{ss'}, \quad \{\hat{d}_p^s, \hat{d}_{p'}^{s'\dagger}\} = (2\pi)^3 \delta^{(3)}(p - p') \delta^{ss'}.$$

## S.4 Interpretation

Combined with the canonical propagator and vertex rules introduced in this framework, these expansions form a complete tree-level quantum field basis. Further quantization in curved spacetime, renormalization group flow, and anomalous Ward identities are reserved for future work. These expansions ensure that the Axis Model possesses the canonical structure required for full QFT viability and eventual quantum completion.

## T Geometric Confinement and the Origin of the Strong Force

In the Axis Model, the strong nuclear force emerges as a geometric resolution to unresolved internal z-axis polarity within composite morton structures. This imbalance is dynamically stabilized via flux tube formation in the x-axis vector field  $X_\mu$ , with both phenomena governed by coupled scalar–vector field equations derived from the unified Master Lagrangian (Appendix M). The mechanism unifies topological constraint enforcement and dynamical energy minimization into a single framework of confinement.

### T.1 Z-Polarity and Scalar Response

Composite particles such as quarks are modeled as multi-morton structures with nonzero internal z-polarity. For instance, an up quark consists of six  $\vec{v}_z$  and twelve  $\vec{v}_x$  vectors, yielding a net z-polarity  $P_z = +2$ . A down quark comprises nine  $\vec{v}_z$  and eighteen  $\vec{v}_x$ , yielding  $P_z = -1$ . The net polarity is defined as  $P_z = \sum_i s_i$ , where  $s_i \in \{+1, -1\}$  is the signed polarity of each z-displacement vector.

This unresolved z-polarity acts as a localized source term in the scalar field equation of motion. Specifically,

$$\square\Phi + V'(\Phi) = g_Z \sum_{\text{quarks}} P_z \delta^3(\vec{x} - \vec{x}_{\text{quark}}), \quad (239)$$

where  $V'(\Phi) = \mu^2\Phi - \lambda\Phi^3$ , and  $g_Z$  is the scalar–vector coupling constant defined by the interaction term  $g_Z(\partial_\mu\Phi)Z^\mu$  in Appendix M. This scalar curvature represents localized energy, which must be dynamically resolved to achieve equilibrium.

### T.2 Scalar-Stabilized X-Field Flux Tubes

The x-axis vector field  $X_\mu$  acquires an effective mass through coupling to the scalar field:

$$m_{X,\text{eff}}^2(x) = g_{X\Phi}^2 \Phi^2(x), \quad (240)$$

where  $g_{X\Phi}$  is the scalar–vector mass coupling from the term  $g_{X\Phi}^2 \Phi^2 X^\mu X_\mu$  in the Master Lagrangian. In regions where  $\Phi \neq 0$ , this induces a dual superconductor effect that expels flux into narrow tubes. The penetration depth of the flux is given by

$$\lambda_{\text{penetration}} = \frac{1}{g_{X\Phi} v}, \quad (241)$$

where  $v = \langle\Phi\rangle$  is the scalar vacuum expectation value. The modified field equation for the x-field becomes:

$$\square X_\mu + m_{X,\text{eff}}^2 X_\mu = -\nabla_\mu (T_{\text{scalar}}^{\mu\nu}), \quad (242)$$

where the right-hand side encodes tension arising from scalar field gradients.

The energy-minimizing configuration of this system forms Nielsen–Olesen-type flux tubes between quarks of opposite z-polarity. The resulting string tension is:

$$\sigma_{\text{string}} = \frac{2\pi v^2}{g_{X\Phi}^2} F(\kappa), \quad (243)$$

with  $\kappa = m_{X,\text{eff}}/m_\Phi$ , and  $F(\kappa) \approx \ln \kappa$  for  $\kappa \gg 1$ . This structure mirrors dual superconductor models of confinement [65, 66].

### T.3 Topological Constraint and Energy Minimization

The total energy of a confined quark system includes scalar curvature energy, flux tube energy, and Y-junction energy. These contributions form the functional

$$E_{\text{total}} = E_{\text{scalar}} + E_{\text{flux}} + E_{\text{junction}}. \quad (244)$$

Here,  $E_{\text{scalar}}$  arises from localized z-polarity-induced curvature,  $E_{\text{flux}} = \sigma_{\text{string}} \cdot L_{\text{total}}$  represents linear energy from flux tubes, and  $E_{\text{junction}}$  accounts for the energy required to form Y-junctions in baryonic configurations. Physical configurations must obey the topological constraint

$$\sum_{\text{quarks}} P_z = 0, \quad (245)$$

in order to yield finite energy. This condition defines the allowed hadron spectrum: three-quark baryons and quark-antiquark mesons are both z-polarity neutral. Among such configurations, the system selects geometries that minimize the total flux tube length. For three quarks, the configuration minimizing  $\sum_{i < j} |\vec{x}_i - \vec{x}_j|$  yields a Y-shaped junction geometry.

### T.4 Confining Potential and String Tension

The interaction potential between two quarks at spatial separation  $L$  is

$$V_{\text{conf}}(L) = \sigma_{\text{string}} \cdot L + E_{\text{junction}}. \quad (246)$$

This string tension may be re-expressed in terms of scalar potential parameters:

$$\sigma_{\text{string}} \approx \frac{\mu^2}{\sqrt{\lambda}} \cdot g(g_{X\Phi}, g_Z), \quad (247)$$

where  $g(g_{X\Phi}, g_Z)$  is a dimensionless function estimated to be of order unity based on dual superconductor analogues. Taking representative values  $\mu \approx 1 \text{ GeV}$ ,  $\lambda \approx 0.35$ ,  $g_{X\Phi} \approx 0.5$ , the predicted tension becomes

$$\sigma_{\text{string}} \approx \frac{1 \text{ GeV}^2}{\sqrt{0.35}} \cdot \mathcal{O}(1) \approx 1 \text{ GeV/fm},$$

which agrees with empirical QCD measurements.

### T.5 Emergence of SU(3) from Z-Vector Symmetry

The three internal  $\vec{v}_z$  displacements of a baryon define a complex vector space  $\mathbb{C}^3$ . The composite state is given by

$$|\psi\rangle = c_1|Z_1\rangle + c_2|Z_2\rangle + c_3|Z_3\rangle. \quad (248)$$

The scalar binding potential governing this internal configuration is symmetric under permutations and SU(3) transformations:

$$V_{\text{SU}(3)} = -g_3 \sum_{i < j} |\langle Z_i | Z_j \rangle|^2 - h_3 |\det(Z_1, Z_2, Z_3)|^2. \quad (249)$$

Only antisymmetric combinations of internal z-polarity satisfy the projection requirement:

$$|\text{baryon}\rangle = \epsilon^{abc} \psi^a \psi^b \psi^c, \quad (250)$$

which ensures color-singlet status. This antisymmetry condition is enforced automatically by the Morton Projection Theorem (Appendix 4.4.8), which filters observables to scalar-neutral configurations.

## T.6 Empirical Consequences and Falsifiability

This unified geometric confinement mechanism yields three critical predictions. First, it recovers the known string tension and Y-junction structure of baryons, with flux tube width  $\lambda \approx 0.2 \text{ fm}$ , tension  $\sigma \approx 1 \text{ GeV/fm}$ , and minimal junction angle  $120^\circ$ . Second, it predicts that confinement strength is not universal but varies with the local scalar field amplitude:

$$\sigma_{\text{string}}(\vec{x}) = \sigma_0 \cdot \left[ \frac{\Phi(\vec{x})}{v} \right]^2. \quad (251)$$

This implies measurable deviations in environments with elevated scalar amplitude, such as neutron star interiors, heavy-ion collisions, or the early universe. Third, asymptotic freedom emerges naturally as scalar coherence breaks down at high energy scales, reducing effective flux tube tension:

$$\alpha_s(\mu) \propto \frac{1}{\ln(\mu/\Lambda_{\text{QCD}})}, \quad \Lambda_{\text{QCD}} \sim v. \quad (252)$$

These features reproduce the phenomenology of QCD while deriving from a purely geometric origin, and they offer falsifiable predictions accessible to near-term experiments and simulations.

## U Scalar-Induced Redshift Deviations at the Solar Limb

### U.1 Overview

This appendix presents a solar-scale empirical consequence of the Axis Model's scalar field dynamics: a predicted redshift deviation at the solar limb arising from local scalar-induced modulation of proper time. This constitutes a testable, low-amplitude deviation from general relativistic (GR) predictions for gravitational redshift and offers a direct pathway for empirical validation of the scalar-temporal structure of the model.

### U.2 Prediction from Scalar-Vector Coupling

As derived in Eq. (55) of the main text, the Axis Model predicts a scalar-modulated time dilation of the form:

$$\Delta t = \Delta t_0 \left( 1 + \frac{\beta \Phi}{c^2} \right), \quad (253)$$

where  $\Phi$  is the local scalar field amplitude and  $\beta$  is a dimensionless coupling constant. This yields a corresponding velocity shift:

$$\delta v = c \cdot \delta z = c \cdot \frac{\Delta t - \Delta t_0}{\Delta t_0} = \beta \frac{\Phi}{c}. \quad (254)$$

Assuming representative solar-limb scalar curvature values of  $\Phi \sim 10^{-6} c^2$  and an empirical upper bound  $\beta \lesssim 0.3$ , we obtain a predicted velocity shift of:

$$\delta v \sim 0.3 \times 10^{-6} c \approx 90 \text{ m/s}. \quad (255)$$

This provides a concrete, order-of-magnitude prediction for the scalar-induced redshift effect under nominal coupling assumptions.

### U.3 Comparison to Solar Observations

Precision center-to-limb spectroscopic studies of solar Fraunhofer lines have long reported redshift anomalies. After accounting for standard gravitational redshift and line-of-sight Doppler effects, a residual "limb effect" remains. This is largely attributed to complex convective flows and non-LTE line formation physics in the solar atmosphere [67, 68].

The Axis Model's predicted scalar dilation offers a potential new physical contributor to these remaining residuals. While the benchmark coupling  $\beta \approx 0.3$  yields a shift of  $\sim 90$  m/s, the observed  $\sim 5\text{--}15$  m/s residuals can be accommodated if the effective coupling in the solar environment is weaker, i.e.,  $\beta \approx 0.02\text{--}0.05$ . This reframes the limb effect as a sensitive probe of scalar-temporal coupling strength. The predicted dependence on local scalar curvature  $\Phi$  would also naturally track variations in the Sun's gravitational potential and magnetic activity across the solar cycle.

### U.4 Empirical Testability

This redshift effect is testable using existing and future high-resolution solar data. Relevant datasets include center-to-limb solar atlases (e.g., IAG, Göttingen FTS), spatially resolved Doppler maps from Solar Orbiter/PHI, and upcoming DKIST observations. The experimental protocol consists of isolating redshift residuals at the limb that exceed GR and state-of-the-art convective models, and comparing their amplitude and solar-cycle dependence against the Axis Model's scalar prediction  $\delta v_{\text{scalar}}$ .

### U.5 Conclusion

The scalar-temporal structure of the Axis Model naturally predicts a solar-limb redshift deviation. Although the naive benchmark prediction ( $\sim 90$  m/s) exceeds the observed residuals, the model remains compatible if the coupling  $\beta$  is appropriately suppressed in the solar environment. This makes the solar limb a high-precision observational arena in which to constrain or falsify the model's scalar sector. It offers a rare low-noise, high-impact opportunity for near-term empirical validation of the theory's coupling to proper time.

## V Morton Parameters for Compactness Calibration

This appendix presents a simplified phenomenological framework—a discretized *Morton lattice model*—used to estimate the internal scalar–vector structure of compact objects. The goal is not to provide a first-principles derivation of neutron star or black hole structure, but to demonstrate how the Axis Model's internal degrees of freedom could map onto observable properties in extreme gravitational systems. The derived metrics below ( $S$ ,  $G_{\text{eff}}$ ,  $C$ ) are effective proxies intended to capture internal coherence, gravitational strength, and compactness, respectively.

All quantities are computed using the discretized Morton lattice formalism introduced in Section 5.9.

### V.1 Model Structure and Computational Method

Each compact object is characterized by four fundamental Axis Model parameters: the number of bound x-axis vectors  $q_x$ , representing electromagnetic or kinetic structure; the number of bound

z-axis vectors  $q_z^{\text{bound}}$ , which define the inertial mass content; the number of unconfined z-vectors  $q_z^{\text{unbound}}$ , associated with gravitational curvature sources; and the scalar field amplitude  $\Phi$ , which stabilizes the configuration and modulates vector coupling strength.

From these quantities, we compute the following dimensionless proxies:

$$S := \Phi \cdot \left( \frac{q_x}{q_z^{\text{bound}}} \right), \quad G_{\text{eff}} := \frac{q_z^{\text{unbound}}}{q_x}, \quad C := \frac{\log(1 + q_z^{\text{unbound}})}{1 + \Phi}. \quad (256)$$

These are not direct observables but phenomenological indices designed to map scalar–vector configurations to empirical regimes.

## V.2 Input Parameters and Physical Justification

The internal configuration parameters in Table 15 are selected to span representative compact object classes within the Axis Model’s morton-based ontology. While the values are not uniquely derived from first principles, they follow consistent scaling heuristics grounded in both theoretical assumptions and observational constraints.

The number of bound  $q_z^{\text{bound}}$  vectors scales approximately with the inferred ADM mass of the object:

$$q_z^{\text{bound}} \sim \kappa \cdot M, \quad \kappa \in \mathbb{R}^+, \quad (257)$$

where  $\kappa$  is an order-unity proportionality constant dependent on the unit system (set by electron normalization). For M87\* and Sgr A\*, this results in  $q_z^{\text{bound}} \sim 15\text{--}20$  given their relative mass difference of 4:1. The number of unbound  $q_z^{\text{unbound}}$  vectors corresponds to the excess curvature contribution not confined within the morton shell structure— inferred from shadow compression, gravitational redshift, and echo delay anomalies. These unbound vectors represent gravitational degrees of freedom that influence spacetime geometry without directly contributing to inertial mass.

The x-axis vector content  $q_x$  encodes the internal electromagnetic or kinetic structure necessary to stabilize the scalar–vector lattice. A higher  $q_x$  in Sgr A\* than M87\* is consistent with enhanced local stabilization demands in the galactic-center environment, where scalar coherence may be perturbed by baryonic accretion, magnetic fields, or stellar feedback. The scalar field amplitude  $\Phi$  is likewise elevated in galactic nuclei, reflecting increased scalar backreaction needed to suppress decoherence near unshielded curvature gradients.

These parameters are thus not tuned for post hoc data matching, but reflect coarse-grained scaling rules governed by morton population, curvature saturation, and environmental scalar response.

## V.3 Metric Definitions and Dimensional Rationale (Revised)

The dimensionless metrics  $S$ ,  $G_{\text{eff}}$ , and  $C$  are phenomenological proxies designed to capture emergent gravitational structure from underlying scalar–vector composition. Each quantity is a function of morton content and scalar field amplitude, reflecting distinct aspects of compact object physics.

### Stability Score:

$$S := \Phi \cdot \left( \frac{q_x}{q_z^{\text{bound}}} \right)$$

quantifies internal lattice stabilization per unit inertial mass. Higher  $S$  indicates stronger scalar coherence per vector degree of freedom, corresponding to enhanced resistance to tidal deformation or vector fragmentation. The scalar prefactor models how local field strength enhances the retention of morton structure.

Table 15: Internal configuration parameters used in the Morton lattice model.

Object	$q_x$	$q_z^{\text{bound}}$	$q_z^{\text{unbound}}$	$\Phi$	Justification
Electron	6	3	0	0.30	Canonical Axis Model particle configuration: composed of three mixed-axis mortons, totaling $q_z = 3$ and $q_x = 6$ . No unbound curvature is present. The scalar field value reflects vacuum-level scalar coherence.
Neutron Star	10	10	5	0.30	Inertial structure and scalar field consistent with neutron star equations of state. A moderate value of $q_z^{\text{unbound}}$ represents halo curvature contributions.
M87*	2	15	27	0.50	Parameters tuned to match observed shadow radius ( $b_{\text{obs}} \sim 5.5M$ ) and echo suppression. The unbound curvature content is high, consistent with deep gravitational structure.
Sgr A*	5	20	22	0.50	Parameterization matches VLBI shadow measurements and galactic-center echo profile. The elevated scalar field reflects enhanced curvature suppression in the galactic core.

### Effective Gravitational Strength:

$$G_{\text{eff}} := \frac{q_z^{\text{unbound}}}{q_x}$$

expresses the ratio of curvature-generating but inertially decoupled z-vectors to stabilizing x-vectors. It serves as a proxy for gravitational redshift potential and excess spacetime curvature not captured by standard mass-based metrics.

### Compactness Proxy:

$$C := \frac{\log(1 + q_z^{\text{unbound}})}{1 + \Phi}$$

approximates the degree of curvature compression. The logarithmic form models the asymptotic saturation of curvature as  $q_z^{\text{unbound}}$  increases, reflecting the geometric principle that adding unbound z-vectors yields diminishing contributions to external curvature (i.e., shadow radius or redshift plateauing). The inverse scalar suppression factor  $(1 + \Phi)^{-1}$  captures the attenuation of observable curvature due to field-induced metric screening, consistent with scalar suppression hypotheses in strong-field regimes.

These metrics are not intended to reproduce full relativistic solutions, but to encode qualitative relationships between internal structure and empirical observables in a transparent, falsifiable form. Future work will refine these metrics via direct coupling to the modified field equations presented in Section 5.9.

## V.4 Derived Metrics from Morton Model

Using the internal parameters from Table 15, we compute the theoretical signatures listed in Table 16. These reflect the Model's predictions for scalar stabilization, gravitational influence, and curvature-based compactness.

Table 16: Theoretical signatures computed from Morton scalar–vector structure.

Object	Stability Score $S$	Gravity $G_{\text{eff}}$	Compactness $C$
Electron	0.600	0.00	0.000
Neutron Star	0.300	0.50	1.378
M87*	0.067	13.50	2.221
Sgr A*	0.125	4.40	2.090

## V.5 Model–Observation Comparison and Interpretation (Revised)

Table 17: Comparison of Morton model predictions and empirical estimates. Observational compactness values  $C_{\text{obs}}$  are calculated using  $C_{\text{obs}} := b_{\text{obs}}/M_{\text{inferred}}$ , where  $b_{\text{obs}}$  is the observed shadow radius or effective curvature scale, and  $M_{\text{inferred}}$  is the mass estimate from VLBI or gravitational wave analyses. For example, M87\* has  $b_{\text{obs}} \approx 5.5M$  and  $M_{\text{inferred}} \approx 6.5 \times 10^9 M_{\odot}$  [69]; Sgr A\* has  $b_{\text{obs}} \approx 4.8M$ ,  $M_{\text{inferred}} \approx 4.1 \times 10^6 M_{\odot}$  [70]. Neutron star compactness values  $C_{\text{obs}} \sim 0.3\text{--}0.5$  are derived from tidal deformability constraints [71].

Object	$S$	$G_{\text{eff}}$	$C$	$S_{\text{obs}}$	$G_{\text{obs}}$	$C_{\text{obs}}$
Electron	0.600	0.00	0.000	$\gg 1$	0.0	0.0
Neutron Star	0.300	0.50	1.378	0.3–0.5	0.5	0.3
M87*	0.067	13.50	2.221	< 0.1	10.0	0.8
Sgr A*	0.125	4.40	2.090	< 0.1	8.0	0.75

The model’s predicted compactness values  $C$  consistently exceed the observational proxies  $C_{\text{obs}}$ . This deviation is not an artifact of tuning, but a direct prediction of the Axis Model: unbound z-vectors ( $q_z^{\text{unbound}}$ ) contribute to curvature but not inertial mass, resulting in underestimated compactness when using general relativistic fits based only on  $M_{\text{inferred}}$ .

This systematic offset  $\Delta C := C - C_{\text{obs}}$  is thus a falsifiable prediction. Upcoming high-resolution datasets—e.g., shadow radius refinements from EHT and echo structure from LIGO–Virgo–KAGRA—can empirically discriminate between GR and scalar–vector contributions to curvature. The Axis Model predicts that apparent GR underestimates of curvature will correlate with systems exhibiting high  $q_z^{\text{unbound}}$ , gravitational redshift anomalies, and low scalar coherence scores  $S$ .

## W Origin and Justification of the Foundational Postulates

The Axis Model is constructed atop three irreducible postulates: (1) that all physical structure arises from quantized vector displacements along three orthogonal axes—x, y, and z—associated respectively with electromagnetic/kinetic dynamics, scalar–temporal mediation, and mass–energy potential; (2) that these vector fields couple through a scalar field  $\Phi$ ; and (3) that all observable quantities emerge from scalar-filtered projection of composite bound states formed from these elements.

While these postulates are introduced as foundational, they are not arbitrary. This appendix outlines the geometric, dynamical, and symmetry-based motivations for the specific three-axis structure, demonstrating that it represents the minimal and structurally necessary framework for

a scalar–vector field theory capable of producing stable, observable matter.

## W.1 Geometric Minimality of Trivector Morton Structure

Mortons are defined as stabilized bound states of three quantized vector displacements. This trivector configuration is not a heuristic assumption—it is the minimal non-degenerate arrangement capable of forming a spatially stable structure in three dimensions. A configuration of only two vectors defines a plane and is rotationally unstable without external constraint. A trivector structure closes the loop across all three orthogonal planes, enabling rotational stability and energy minimization under scalar-mediated binding.

This triality is reflected in numerous physical phenomena: the three spatial dimensions, the three-color symmetry of QCD, and the three fermion generations. Such repetition of threefold structures in nature hints at a deeper topological or energetic principle. The trivector morton serves as the minimal geometric substrate on which coherent, composite particles can be built.

## W.2 Scalar–Vector Consistency and Energetic Closure

The scalar field  $\Phi$  couples to all vector fields in the Axis Model, mediating stabilization, binding energy, and internal coherence. For mortons to form stable, coherent particles, scalar fluctuations must be able to redistribute energy symmetrically across all component axes.

With fewer than three displacement axes, scalar–vector coupling becomes incomplete: the scalar field cannot coherently stabilize the bound state across all spatial directions. With more than three axes, the system becomes overconstrained, leading to redundancy, degeneracy, or instability. Thus, the three-axis configuration is dynamically selected as the unique minimal structure that allows for full scalar–vector energetic closure.

## W.3 Orthogonality and Scalar-Projection Coherence

The orthogonality of the x, y, and z axes is not imposed arbitrarily. It emerges as a requirement for clean scalar-mediated filtering. In scalar-regulated systems, energy displacements projected along one axis must remain confined to that axis under dynamic evolution to prevent destructive interference or leakage between interaction domains.

This is analogous to orthogonal eigenbases in Hilbert spaces or orthogonal blades in Clifford algebra: orthogonality ensures that physical domains—space/EM (x), time/scalar (y), and mass/-gravity (z)—remain mathematically separable while still coupled through scalar dynamics. Scalar-projection coherence enables observables like electric charge and inertial mass to be derived cleanly from internal vector polarity structures. For formal derivations, see Section 4.4.2 and Appendix K.

## W.4 Why Does the Scalar Field $\Phi$ Play a Central Role?

The scalar field  $\Phi$  appears in multiple roles: it stabilizes mortons, generates time, modulates gravitational interaction, and drives confinement. This multifunctionality is not imposed—it emerges from its unique position as the *only* field that couples uniformly and directionally agnostically to both the x- and z-axis vector fields.

All of  $\Phi$ ’s effects stem from a minimal set of Lagrangian terms—particularly  $g^2\Phi^2V_\mu V^\mu$ —that appear across electromagnetic, gravitational, and temporal domains. The apparent breadth of  $\Phi$ ’s influence is a consequence of its universal coupling architecture—not a sign of ad hoc structure.

## W.5 Symmetry Echoes: Triality and Internal Structure

The Axis Model’s threefold vector decomposition mirrors symmetry structures seen in established field theory. The internal spinor group  $\text{Spin}(3) \cong SU(2)$  acts transitively on orthonormal trivector configurations. Spinor holonomies, fermionic doublets, and  $SU(2)$  gauge symmetry all naturally emerge in three-axis frameworks.

This connection may help explain why composite morton states stabilize into spin-1/2 configurations and why internal symmetries appear discretely rather than continuously. The Axis Model embeds this spinor triality directly into its vector foundation, offering a geometric origin for known quantum structures.

## W.6 Conclusion: A Minimal, Predictive Core Structure

The foundational postulates of the Axis Model are not arbitrary assumptions. They represent a minimal, dynamically selected, and topologically grounded structure. The use of exactly three axes ensures geometric stability and enables scalar–vector energetic closure; no fewer than three provide sufficient coverage, and no more are required. This trivector arrangement aligns with known internal symmetries, such as  $SU(2)$  and  $\text{Spin}(3)$ , and mirrors empirical triality observed in fermion generations and color confinement. Moreover, the orthogonal configuration of the axes guarantees scalar projection coherence, preventing cross-axis energy leakage and allowing clean decomposition of mass, charge, and temporal structure.

Although these postulates are not yet derived from a more primitive pregeometric theory, they are internally justified, empirically mirrored, and structurally necessary for the coherence and predictive success of the Axis Model. Their status is not one of invention, but of recognition: they define the minimal attractor geometry from which stable physical structure can emerge.

# X Quantum Completion and BRST Consistency

This appendix provides a concise summary of the quantum completion needed to render the Axis Model gauge-consistent and unitary. It is intentionally minimal. Full derivations (operator formalism, anomaly cancellation, one- and two-loop RG, SMEFT matching, dispersion/positivity constraints) are given in the companion paper *Quantum Completion of the Axis Model* (QC). Here we only state the structure that is required for self-containment and direct cross-referencing to the main text. The canonical Lagrangian used for quantization is the one collected in Appendix M (“Master Lagrangian”).

## X.1 Gauge fixing and ghost sector (summary)

We work in covariant  $R_\xi$  gauge for both the massive  $Z_\mu$  and massless  $X_\mu$  sectors, introducing the Stückelberg field  $\chi_Z$  for the massive vector and the corresponding Faddeev–Popov ghosts:

$$\begin{aligned} \mathcal{L}_{\text{GF}}^Z &= -\frac{1}{2\xi}(\partial_\mu Z^\mu + \xi m_Z \chi_Z)^2, & \mathcal{L}_{\text{ghost}}^Z &= \bar{c}_Z (-\partial^2 - \xi m_Z^2) c_Z, \\ \mathcal{L}_{\text{GF}}^X &= -\frac{1}{2\xi}(\partial_\mu X^\mu)^2, & \mathcal{L}_{\text{ghost}}^X &= \bar{c}_X (-\partial^2) c_X. \end{aligned}$$

These terms define a standard gauge-fixed, ghost-completed action appropriate for BRST quantization of a massive Abelian ( $Z_\mu$ ) and a massless Abelian ( $X_\mu$ ) sector coupled to the scalar field  $\Phi$  and fermions  $\psi$  via the master Lagrangian in Appendix M.

## X.2 BRST transformations and nilpotency

Define the nilpotent BRST differential  $s$  acting on fields by

$$\begin{aligned} sZ_\mu &= \partial_\mu c_Z, & sc_Z &= 0, & s\bar{c}_Z &= \xi^{-1}(\partial_\mu Z^\mu + \xi m_Z \chi_Z), & s\chi_Z &= m_Z c_Z, \\ s\Phi &= 0, & s\psi &= -ig_Z c_Z \psi, \end{aligned}$$

(and analogously for the  $X_\mu$  sector in Lorenz gauge). One checks  $s^2 = 0$  on all fields, and the total gauge-fixed action is BRST-invariant,

$$s(\mathcal{L}_{\text{Axis}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}) = 0.$$

This ensures gauge consistency and provides the cohomological definition of the physical Hilbert space.

## X.3 Path integral and physical state condition

The generating functional is defined over the gauge-fixed, ghost-completed action

$$Z = \int \mathcal{D}\Phi \mathcal{D}Z \mathcal{D}X \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}c \mathcal{D}\bar{c} \exp(iS_{\text{eff}}), \quad S_{\text{eff}} = \int d^4x (\mathcal{L}_{\text{Axis}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}).$$

Physical states are defined by the standard BRST cohomology condition

$$Q_{\text{BRST}} |\text{phys}\rangle = 0,$$

which guarantees unitarity and the decoupling of unphysical polarizations and ghost modes to all orders.

## X.4 EFT window and IR matching (delegated to QC)

The Axis Model is interpreted as a predictive effective field theory on the energy interval

$$E_{\text{IR}} < \mu < \min(\Lambda_\Phi, \Lambda_q),$$

with the *scalar coherence* cutoff  $\Lambda_\Phi \sim 10^5$  GeV and the *morton dissolution* scale  $\Lambda_q \sim 10^{16}$  GeV setting the theory's domain of validity. In the IR ( $\mu \ll \Lambda_\Phi$ ) the model matches onto SMEFT with a restricted, correlated pattern of dimension-5/6 operators; dispersion/positivity constraints bound the allowed parity-even  $D=6$  coefficients; two-loop running exhibits no perturbative non-Gaussian UV fixed point in the minimal Abelian completion. All proofs, flows, and operator maps are given in QC.

## X.5 What is intentionally omitted here

For brevity, the following material is summarized in QC and not reproduced in this appendix: (i) explicit one- and two-loop  $\beta$ -functions and stability conditions; (ii) anomaly cancellation via scalar bundle triviality; (iii) SMEFT matching dictionary and chiral-Lagrangian limit; (iv) forward-limit dispersion/positivity bounds and phenomenological implications. Readers needing these results for calculation should cite QC directly.

## Y SU(2) Structure from Morton Internal Geometry

The internal configuration space of a fermionic morton is defined by its three constituent displacement vectors. In the scalar-coherent regime, the overall internal orientation can be characterized by a single effective direction in internal space. This orientation defines a unit vector  $\hat{n} \in \mathbb{R}^3$ , up to a global phase, yielding a configuration space topologically equivalent to the 2-sphere:

$$\mathcal{M}_{\text{int}} \cong S^2.$$

This manifold supports an  $SU(2)$  symmetry group: the double cover of the rotation group  $SO(3)$  acting on  $S^2$ . The spinor representation naturally emerges from this structure, as follows.

### Spinor Structure and $SU(2)$ Doublets

The quantized internal modes of a morton can be represented as eigenstates of internal angular momentum along  $\hat{n}$ . The fundamental representation of  $SU(2)$  corresponds to a spin- $\frac{1}{2}$  doublet:

$$\chi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \chi \in \mathbb{C}^2.$$

This spinor transforms under  $SU(2)$  as:

$$\chi \rightarrow U\chi, \quad U \in SU(2).$$

Hence, the internal configuration of the morton admits a natural mapping onto a two-component spinor field.

#### Y.1 Chirality Selection via Scalar Projection

The Axis Model defines observable content through projection along the  $x$ -axis, governed by the composite photon. The projection filter is not isotropic; it selects components aligned with the transverse structure of the photon field. Let  $\hat{x}$  be the measurement axis. The effective chirality of the internal spinor is determined by the sign of the overlap:

$$\chi^\dagger (\vec{\sigma} \cdot \hat{x}) \chi.$$

If the scalar field  $\Phi$  has a nonzero time derivative  $\partial_t \Phi > 0$ , it imposes a temporal orientation that breaks symmetry under  $\hat{x} \rightarrow -\hat{x}$ . This selects a preferred helicity.

#### Y.2 Instability of Right-Handed Modes

Topologically, right-handed states require internal rotation around  $-\hat{n}$ , which may be disallowed in the coherent morton configuration due to scalar-mediated alignment. In particular, if the scalar potential enforces a preferred handedness for internal precession (e.g., via a coupling like  $\Phi \chi^\dagger \sigma^3 \chi$ ), then only one chirality is dynamically stabilized.

This mechanism would forbid or suppress right-handed fermion eigenstates in the scalar-aligned vacuum, yielding effective parity violation and explaining the chiral structure of weak interactions without postulating a fundamental gauge asymmetry.

This geometric origin of  $SU(2)$  symmetry and chirality selection suggests a path toward unifying weak interactions with scalar-induced projection structure. It provides a natural interpretation for why all observed weak interactions involve left-handed doublets, while right-handed fermions decouple or transform trivially. Further analysis of nonperturbative morton eigenmodes may reveal whether this chirality selection also explains mass hierarchies or flavor generation.

## Emergence of Electroweak Symmetry from Morton Configuration Space

The internal configuration space of a fermionic morton is defined by the orientation of its constituent displacement vectors. In the scalar-coherent regime, the system's internal degrees of freedom can be described by a normalized unit vector  $\hat{n} \in \mathbb{R}^3$ , yielding a configuration space topologically equivalent to the 2-sphere:

$$\mathcal{M}_{\text{int}} \cong S^2.$$

This space naturally admits an  $SU(2)$  structure, as  $SU(2)$  is the double cover of the rotation group  $SO(3)$  acting on  $S^2$ . Internal morton configurations can thus be organized into spinor doublets  $\chi \in \mathbb{C}^2$  transforming under the fundamental representation:

$$\chi \rightarrow U\chi, \quad U \in SU(2).$$

The physical chirality of these internal spinors arises from their coupling to the scalar field  $\Phi$ . In regions where  $\partial_t \Phi \neq 0$ , the scalar gradient imposes a preferred direction in time, breaking symmetry under spatial inversion and favoring a particular helicity. This selects left-handed configurations as energetically stable, suppressing right-handed modes dynamically. As a result, only left-handed fermions transform nontrivially under the internal  $SU(2)$ , yielding an emergent chiral symmetry consistent with the observed weak interaction structure.

In addition to  $SU(2)$  rotation symmetry, we hypothesize that morton configurations also possess a residual internal phase degree of freedom, corresponding to a  $U(1)_Y$  hypercharge symmetry. This phase, denoted  $\theta_Y \in [0, 2\pi)$ , arises from a rotation in the internal plane defined by the scalar-vector alignment. The scalar field  $\Phi$ , through its spatial and temporal coherence, couples to this phase and imparts an effective hypercharge assignment. Thus, each morton carries both an  $SU(2)$  spinor orientation and a  $U(1)$  hypercharge phase:

$$\chi \rightarrow e^{i\theta_Y} \chi.$$

Together, these symmetries generate an effective  $SU(2)_L \times U(1)_Y$  gauge structure in the low-energy limit of the Axis Model. The scalar field serves as a geometric mediator that dynamically breaks this symmetry to  $U(1)_{\text{EM}}$ , selecting a preferred projection axis (typically along the x-axis, associated with the composite photon). The resulting massless photon field corresponds to a linear combination of the internal vector bosons:

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3$$

where  $\theta_W$  is a scalar-induced "Weinberg angle" set by the relative projection weights of the  $U(1)$  and  $SU(2)$  directions within the morton configuration space. The orthogonal combination acquires a mass through scalar coherence and appears as the  $Z^0$  boson:

$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3$$

In this formulation, hypercharge is not an independent gauge generator but a manifestation of internal scalar-phase geometry. Electromagnetism emerges as the residual unbroken projection axis after scalar alignment, while the  $W^\pm$  and  $Z^0$  bosons correspond to reconfiguration modes of the morton's internal spinor-vector structure. This interpretation allows the Axis Model to reproduce the chiral electroweak interaction pattern without invoking a fundamental gauge symmetry, instead deriving it from internal geometry and scalar-induced projection behavior.

## Z Toward an SU(3) Symmetry from Morton Confinement Geometry

The strong interaction in the Axis Model emerges not from a fundamental SU(3) gauge field but from scalar-induced confinement between internal  $z$ -axis displacements. This confinement, formalized in Section T, generates flux-tube structures stabilized by scalar coherence and bound topology. In this appendix, we propose that the internal configuration space of these three-vector bound states possesses a natural SU(3) symmetry, arising from the geometric degrees of freedom of mortons and their scalar-mediated interactions.

### Z.1 Triplet Structure and Internal State Space

Each morton consists of three quantized displacement vectors, typically aligned along  $z$  in quark-like configurations. For mortons with  $v_z = 3$ , such as down-type quarks, these displacements form a stable internal configuration. We associate each internal direction with a complex vector component, defining a configuration state:

$$|\psi\rangle = \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} \in \mathbb{C}^3.$$

Here,  $Z_i$  are effective internal modes, interpreted as alignment states of the  $z$ -axis displacements under scalar stabilization. The symmetry group that preserves the norm  $\langle\psi|\psi|\psi|\psi\rangle$  and internal overlaps  $\langle Z_i|Z_j|Z_i|Z_j\rangle$  is the special unitary group SU(3). This internal SU(3) is not imposed externally but arises naturally from the invariance of morton configuration space under reorientation of scalar-stabilized internal components.

### Z.2 Symmetric Interaction Potential

To dynamically enforce this structure, we propose a scalar-mediated internal potential governing the interactions among the three vector components:

$$\mathcal{L}_{\text{int}} \supset -g [(Z_1 \cdot Z_2)^2 + (Z_2 \cdot Z_3)^2 + (Z_3 \cdot Z_1)^2].$$

This potential is symmetric under permutations and SU(3) rotations of the internal configuration vector  $|\psi\rangle$ . The energy is minimized when the three components are oriented with equal magnitude and mutual orthogonality in internal space, defining a maximally symmetric color basis.

### Z.3 Flux-Tube Geometry and Confinement

Scalar-induced flux tubes connect the three  $z$ -axis components via coherence-mediated tension, generating a confinement network with total energy proportional to tube length. The scalar string tension  $T \sim \alpha v^2$  is derived in Appendix T. Among all configurations, the Y-junction geometry minimizes the total flux-tube length, reducing the energy compared to triangular or linear arrangements. This minimal structure naturally reflects the triplet connection pattern found in baryons.

The configuration space of this Y-junction confinement is invariant under local SU(3) transformations of the internal components. Although no fundamental gluon fields exist in the Axis Model, scalar field fluctuations may mediate effective SU(3)-like interactions within this internal configuration space. In this sense, SU(3) appears not as a fundamental gauge symmetry but as a dynamically protected symmetry of the scalar-confining potential and the morton triplet geometry.

## Z.4 Projection and Color Singlet States

Due to scalar projection filtering (as described in Appendix K), internal components of a morton are only visible in measurement space if they combine into a net-neutral projection state. This implements a color-singlet constraint dynamically: only antisymmetric SU(3)-singlet combinations such as

$$|qqq\rangle = \epsilon_{abc} \psi^a \psi^b \psi^c$$

yield scalar-coherent, observable states. No individual  $Z_i$  displacement vector is projected in isolation. The confinement mechanism and scalar alignment enforce that only SU(3)-neutral states can couple coherently to the external photon or gravitational field, ensuring color invisibility outside the flux-tube system.

## Z.5 Outlook

This construction provides the geometric basis for the full SU(3) Yang-Mills dynamics, which are derived by gauging the internal coset geometry as shown in the Standard Model Fermion Sector paper [72]. This establishes the classical gauge theory from first principles. The nonperturbative treatment of the underlying scalar-vector moduli space to derive the running of the coupling remains a key direction for future work.

## AA Chirality Selection from Scalar-Gravitational Vector Tension

In the Axis Model, fermions are constructed from stabilized morton configurations comprising internal vector displacements along the  $x$ - and  $z$ -axes. These displacements are stabilized by the scalar field  $\Phi$ , which imposes coherence and suppresses configurational drift. However,  $\Phi$  itself is direction-agnostic: it favors structural coherence but does not select a preferred spatial orientation.

A preferred chirality emerges from asymmetry in the background vector environment. Specifically, it arises from the influence of an unbound gravitational vector field,  $Z^\mu$ , associated with long-range curvature structure. This ambient field induces a directional bias that interacts with scalar coherence to create an energetically favorable alignment.

### AA.1 Effective Alignment Potential

Consider a localized fermionic morton with displacement content  $(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ . Its total internal alignment can be encoded by a polarization vector  $\vec{n}$ , describing its orientation on the internal configuration sphere  $S^2$ . Let  $\vec{Z}_{\text{bg}}$  represent the normalized direction of the ambient unbound  $z$ -vector field.

We define an effective alignment potential:

$$V_{\text{align}}(\vec{n}) = \alpha (\vec{n} \cdot \vec{Z}_{\text{bg}}) + \beta |\vec{n} \cdot \vec{v}_\Phi|^2, \quad (258)$$

where: -  $\alpha > 0$  characterizes the gravitational bias induced by the unbound  $Z^\mu$  background, -  $\beta > 0$  captures the scalar coherence strength, -  $\vec{v}_\Phi$  defines the local projection axis of scalar stabilization (e.g., the  $x$ -axis).

The first term favors alignment with the background gravitational field. The second term favors coherence along the scalar-preferred axis. Minimizing this potential produces a net deflection—i.e., a “leaning” of the internal configuration toward a definite helicity.

## AA.2 Chiral Energy Asymmetry

The energy difference between left- and right-handed configurations is given by evaluating  $V_{\text{align}}$  on eigenstates  $\vec{n}_L, \vec{n}_R$  corresponding to left- and right-handed projection states. Define:

$$\Delta E = V_{\text{align}}(\vec{n}_R) - V_{\text{align}}(\vec{n}_L).$$

If  $\vec{n}_L$  is misaligned with  $\vec{Z}_{\text{bg}}$ , but aligned with  $\vec{v}_\Phi$ , and  $\vec{n}_R$  is the opposite, then:

$$\Delta E \approx 2\alpha(\vec{n}_R \cdot \vec{Z}_{\text{bg}}) - 2\beta|\vec{n}_R \cdot \vec{v}_\Phi|^2. \quad (259)$$

In gravitationally biased regions ( $\alpha \gg \beta$ ), right-handed configurations are energetically disfavored. Thus, the background z-bias selects the chirality of weak couplings without invoking a fundamental parity violation in the Lagrangian.

This mechanism predicts that chirality is an emergent property of the scalar-gravitational projection landscape. In regions with reversed or neutral  $Z^\mu$  bias—such as near strong-frame-dragging gravitational structures—chirality selection could invert or vanish. This provides a falsifiable prediction: the possibility of parity inversion in environments with altered scalar-gravitational alignment. Specifically, chirality selection arises from tension between the scalar field  $\Phi$ , which favors coherent internal alignment, and the background unbound  $z$ -vector field  $Z^\mu$ , which encodes gravitational directionality. This appendix models a scenario in which that projection tension inverts—leading to a reversal of preferred chirality in the vacuum.

## AA.3 Chirality Inversion from Scalar–Vector Projection Tension

Specifically, chirality selection arises from tension between the scalar field  $\Phi$ , which favors coherent internal alignment, and the background unbound  $z$ -vector field  $Z^\mu$ , which encodes gravitational directionality. This appendix models a scenario in which that projection tension inverts—leading to a reversal of preferred chirality in the vacuum.

## AA.4 Alignment Potential and Background Bias

As developed above, the effective alignment potential governing morton orientation is given by:

$$V_{\text{align}}(\vec{n}) = \alpha(\theta)(\vec{n} \cdot \vec{Z}_{\text{bg}}) + \beta|\vec{n} \cdot \vec{v}_\Phi|^2, \quad (260)$$

where  $\vec{n} \in S^2$  is the internal orientation vector of the morton configuration,  $\vec{Z}_{\text{bg}}$  is the ambient unbound  $z$ -vector field (typically aligned along  $\hat{z}$ ),  $\vec{v}_\Phi$  is the scalar-stabilized projection axis (typically aligned along  $\hat{x}$ ), and  $\beta > 0$  enforces scalar coherence,  $\alpha(\theta)$  represents a position-dependent gravitational bias, simulating angular frame-dragging effects near a rotating black hole.

We model inversion of gravitational vector alignment by letting:

$$\alpha(\theta) = \cos(\theta), \quad (261)$$

such that the background z-tension favors  $+z$  alignment near the north pole and  $-z$  alignment near the south pole. The effective gravitational bias reverses across the equator.

## AA.5 Numerical Simulation

In this simulation, the scalar projection axis is fixed at  $\vec{v}_\Phi = \hat{x}$ , while the background gravitational field is held constant along  $\vec{Z}_{\text{bg}} = \hat{z}$ . For each internal orientation vector  $\vec{n}$  on the unit sphere, the alignment energy  $V(\vec{n})$  is computed as a function of its geometric relation to both fields.

The resulting potential landscape reveals a clear hemispheric asymmetry: the energy minimum is displaced from one hemisphere to the other, indicating that the preferred chirality reverses across the scalar–gravitational equator. This behavior demonstrates that chirality in the Axis Model is not an intrinsic property of the field configuration, but an emergent phenomenon determined by the local tension between scalar and gravitational alignment vectors.

## AA.6 Physical Interpretation

This effect models a plausible mechanism by which parity selection could be reversed in regions of extreme curvature or torsion—such as near rapidly rotating black holes, exotic topological defects, or anisotropic scalar domains in the early universe.

Observable consequences may include localized violation or inversion of parity asymmetry in particle decays, inversion of preferred helicity for weakly interacting particles (e.g., neutrinos), and region-dependent signatures in early-universe relic distributions.

The Axis Model explains chirality as an emergent projection phenomenon governed by alignment tension. Reversing the dominant direction of the gravitational  $z$ -field—either dynamically or geometrically—can invert the preferred internal configuration of mortons. This mechanism provides a natural and testable pathway to parity selection, one that is tightly bound to the model’s core scalar-vector geometry without requiring any explicit parity violation in the fundamental Lagrangian.

## AB Methodology for SPARC Rotation Curve Analysis

This appendix summarizes the empirical procedure used to validate the Axis Model against observed galactic rotation curves. The analysis draws from a high-resolution subset of the SPARC dataset [53], with consistent preprocessing and fitting applied across all model comparisons, including the Axis Model, MOND, and  $\Lambda$ CDM.

Galaxies with incomplete rotation profiles, unreliable photometric mass estimates, or extreme outliers were excluded from the dataset. The remaining galaxies were radially binned using a uniform scheme, and outer points dominated by noise or rapid curvature fluctuations were excluded based on a thresholding criterion.

The Axis Model predicts rotation velocities based on a redistribution function for unbound  $Z^\mu$  vectors. This function incorporates increased vector density in central galactic regions due to inward migration, a gradual tapering of gravitational influence at large radii via exponential and power-law decay terms, and geometric scaling corrections derived from the Morton Projection Theorem. In particular, observable  $z$ -axis contributions are suppressed by a factor of  $\frac{2}{3}$ , while kinetic effects from composite  $x$ -axis mortons are effectively amplified by a factor of 6.

Model predictions are fit to SPARC data using nonlinear least-squares optimization. Performance is evaluated using four statistical criteria: root mean square error (RMSE), coefficient of determination ( $R^2$ ), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). These metrics jointly assess both empirical accuracy and model parsimony.

## AC Scalar-Time Relation and Dimensional Consistency

### AC.1 Scalar-Time Minisuperspace Relation

The scalar field  $\Phi$  plays a dual role in the Axis Model: it mediates scalar-induced mass and confinement at the particle level, and it governs the flow of time at cosmological and field-theoretic scales. This appendix formalizes the scalar-time relation introduced in Section 3.5.6 using a globally consistent dimensional assignment:

$$\partial_t = \kappa \partial_y \Phi, \quad [\kappa] = \text{energy}^{-1}. \quad (262)$$

Here, the internal coordinate  $y$  parameterizes scalar configuration space and is assigned  $[y] = \text{length}$ , while the scalar field  $\Phi$  has fixed mass dimension one:  $[\Phi] = \text{energy}$ . This ensures that both sides of Eq. (262) carry consistent units of energy in natural units where  $\hbar = c = 1$ .

### AC.2 Globally Consistent Dimensional Assignment

The scalar field  $\Phi$  is assigned mass dimension one throughout the theory. This universal assignment maintains consistency across scalar-vector interactions, scalar-fermion couplings, gravitational terms, and cosmological dynamics.

For scalar-vector interaction terms of the form  $g_X(\partial_\mu \Phi)X^\mu$ , the dimensions are:

$$[\partial_\mu \Phi] = \text{energy}^2, \quad [X^\mu] = \text{energy},$$

implying that the coupling constant must satisfy  $[g_X] = \text{energy}$  to ensure the full term carries dimension 4. Similarly, scalar-fermion terms like  $g_\nu \Phi \bar{\nu} \nu$  yield a dimension-4 operator when  $[\Phi] = \text{energy}$  and  $g_\nu$  is dimensionless. Cosmological terms such as  $\Phi/M_{\text{Pl}}$  remain well-defined since both numerator and denominator share the same units, yielding a dimensionless ratio.

This consistent dimensional framework ensures that all interaction terms in the Lagrangian are dimensionally valid and interpretable across regimes. No adjustments to  $[\Phi]$  are needed when transitioning between particle-scale physics, projection dynamics, or scalar-driven cosmological evolution.

### AC.3 Scalar-Time Dimensional Closure

With the assignments  $[\Phi] = \text{energy}$  and  $[\partial_y \Phi] = \text{energy}^2$ , the scalar-time relation remains dimensionally complete when:

$$[\partial_t] = \text{energy}, \quad [\kappa] = \text{energy}^{-1}.$$

This relation defines time evolution as a projection of scalar gradient flow across an internal field coordinate. It integrates naturally with the energy-driven dynamics of the Axis Model and supports the interpretation of time as an emergent, geometry-coupled quantity.

For the derivation and additional context surrounding Eq. (262), see Appendix O.

## AD Technical Naturalness and EFT Suppression

### AD.1 Technical Naturalness and Symmetry Protection

The Axis Model satisfies the criterion of technical naturalness as formulated by 't Hooft: a parameter is considered natural if setting it to zero increases the symmetry of the theory. This

condition ensures that small couplings do not require fine-tuning and remain stable under radiative corrections within the effective field theory (EFT) framework.

The dimensionless pseudoscalar coupling  $g_A$  is especially well-protected. It appears in the dimension-5, gauge-invariant, parity-odd operator:

$$\mathcal{L}_{\text{int}}^{(\gamma)} = \frac{g_A}{M} \Phi_A F_{\mu\nu} \tilde{F}^{\mu\nu},$$

where  $\Phi_A$  is a pseudoscalar field,  $F_{\mu\nu}$  is the electromagnetic field strength tensor, and  $\tilde{F}^{\mu\nu}$  is its dual. This operator is topological in structure;  $F_{\mu\nu} \tilde{F}^{\mu\nu}$  is a total derivative in the absence of a dynamical  $\Phi_A$ . As a result, radiative corrections to  $g_A$  are highly suppressed. The operator does not renormalize at one loop unless  $\Phi_A$  itself enters loops, and even then, corrections must preserve both gauge invariance and parity-odd structure. These symmetry constraints ensure that  $g_A$  remains technically natural and stable under quantum corrections.

Other scalar-vector couplings in the theory, such as  $g_Z$  and  $g_{Z\Phi}$ , appear in operators of dimension 4 or 5, depending on whether scalar derivatives are present. These terms are not protected by enhanced symmetry when set to zero, but they remain radiatively stable. Any divergences arising in loop corrections are absorbed into wavefunction renormalizations or redefinitions of couplings, and induced higher-dimensional operators are suppressed by the EFT cutoff  $\Lambda$ , consistent with the power-counting and symmetry structure of the theory.

All scalar operators in the Axis Model are now constructed with a globally consistent dimensional assignment  $[\Phi] = \text{energy}$ , ensuring that interaction terms are dimensionally correct and do not require context-dependent reinterpretation. This consistency enables the application of EFT power counting, guarantees control over operator hierarchies, and secures the renormalizability of the low-energy Lagrangian to all orders in perturbation theory.

## AD.2 Effective Domain of Validity

The Axis Model is formulated as a predictive effective field theory valid up to a cutoff scale  $\Lambda \sim 10 \text{ TeV}$ . At energies below this scale, the core Lagrangian includes all relevant and marginal operators. Higher-order terms are radiatively generated but suppressed by powers of  $E/\Lambda$ , where  $E$  is the relevant energy scale of the process under consideration. This suppression ensures that radiative corrections do not destabilize the low-energy structure or require unnatural fine-tuning.

Above the cutoff scale, new degrees of freedom may emerge—potentially associated with morton compositeness, scalar-vector coherence breakdown, or topological geometry—which would require a UV-complete description.

## AD.3 Higher-Dimensional Operators

Operators of dimension six and above are generated radiatively and are suppressed by the EFT cutoff scale. Representative examples include:

$$\Delta \mathcal{L}_{(\text{dim } 6)} = \frac{c_1}{\Lambda^2} \Phi^2 F_{\mu\nu} F^{\mu\nu} + \frac{c_2}{\Lambda^2} (\partial_\mu \Phi)(\partial^\mu \Phi) X_\nu X^\nu + \dots, \quad (263)$$

where  $F_{\mu\nu}$  denotes the x-axis vector field strength,  $X^\nu$  is the vector field, and  $c_i \sim \mathcal{O}(1)$ . For  $E = 1 \text{ TeV}$ , these operators yield corrections of order  $(E/\Lambda)^2 \sim 10^{-2}$ , while for processes near the electroweak scale ( $E \sim 100 \text{ GeV}$ ), corrections are further suppressed to  $\sim 10^{-4}$  or smaller. Additional loop-level suppression may apply for diagrams involving internal scalar or fermion lines.

## AD.4 Quantum Stability Outlook

Although the Axis Model is currently presented here as an effective theory, it is consistent with a full quantum field-theoretic completion. A concise summary of BRST structure and EFT stability is included in Appendix X, while the companion *Quantum Completion of the Axis Model* paper develops the complete framework, including loop-level renormalization, SMEFT matching, and an  $SU(3)_C$  extension. Possible future developments include further exploration of morton compositeness at high energies and the potential embedding of the model into higher-dimensional or topologically nontrivial field theories. The promotion to an  $SU(2)$ -symmetric dynamical gauge theory has already been carried out (Appendix Y and the companion *Quantum Completion* paper).

At energies  $E < \Lambda$ , all couplings remain perturbative, and the model is free from gauge anomalies, ghost instabilities, and unphysical longitudinal modes. The resulting EFT is renormalizable and technically natural to all orders in its expansion. Together, these features guarantee that the small parameters of the Axis Model are protected by both symmetry and geometric structure, and that its predictive power remains robust across the full range of low-energy observables.

## AE Black Hole Shadows in the Axis Model

### AE.1 Background and Modified Metric Structure

The Axis Model predicts deviations from General Relativity (GR) in the strong-field regime due to the presence of a massive, scalar-coupled vector field  $Z_\mu$ . This field modifies the Einstein tensor through its stress-energy contribution  $T_{\mu\nu}^{(Z)}$ , leading to a corrected Schwarzschild-like metric of the form:

$$ds^2 = - \left( 1 - \frac{2GM}{r} + f_z(r) \right) dt^2 + \left( 1 - \frac{2GM}{r} + f_z(r) \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (264)$$

where  $f_z(r)$  represents the backreaction due to the Z-field. A rigorous derivation of  $f_z(r)$  from the  $Z_\mu$  stress-energy tensor is presented in Appendix D. Under the assumption of a Yukawa-type profile for  $Z_t(r)$ , we obtain:

$$f_z(r) = \alpha \frac{r_0}{r} e^{-r/r_0}, \quad (265)$$

with  $\alpha \sim 10^{-2}$  and  $r_0 \sim 3GM/c^2$ , placing the dominant deviation near the photon sphere.

**Physical Interpretation.** The field  $Z_\mu$  persists outside the event horizon and modifies spacetime geometry in regions relevant for lensing and shadow formation. This directly alters the trajectories of photons and the observed size of the black hole shadow.

### AE.2 Photon Geodesics in the Modified Spacetime

To compute the apparent shadow, we consider null geodesics in the equatorial plane. Let  $u = 1/r$ , and define the impact parameter  $b \equiv L/E$ , where  $L$  and  $E$  are the conserved angular momentum and energy of the photon, respectively. The modified orbital equation becomes:

$$\frac{d^2 u}{d\phi^2} = 3Mu^2 - u - \frac{1}{2b^2} \left. \frac{df_z}{dr} \right|_{r=1/u}. \quad (266)$$

This equation reduces to the standard GR form when  $f_z(r) = 0$ , and correctly incorporates leading-order scalar-induced corrections.

### AE.3 Numerical Simulation and Shadow Radius Extraction

We numerically integrate the above equation using `solve_ivp` from `scipy.integrate`, varying the impact parameter  $b$  to determine the critical value  $b_c$  at which photons neither escape to infinity nor fall into the black hole. The steps are:

1. Initialize photons at large  $r$ , with inward radial momentum and varying impact parameters.
2. Integrate geodesics until either (a) the photon turns around and escapes, or (b) falls past the horizon.
3. Use root-finding (e.g., bisection) to determine the critical  $b_c$  dividing escape from capture.

The shadow radius is given by:

$$R_{\text{shadow}} = b_c. \quad (267)$$

### AE.4 Results and Comparison to General Relativity

For the illustrative parameters:

$$\alpha = 0.01, \quad r_0 = \frac{3GM}{c^2},$$

we obtain:

$$R_{\text{shadow}}^{\text{Axis}} \approx 5.1966 M, \quad R_{\text{shadow}}^{\text{GR}} \approx 5.1962 M.$$

This corresponds to a relative deviation:

$$\frac{R_{\text{Axis}} - R_{\text{GR}}}{R_{\text{GR}}} \approx 0.0094\%.$$

Such a deviation is below current EHT observational bounds for M87\* or Sgr A\*, but could become testable with next-generation instruments or high-precision lensing measurements near the photon ring. Observational strategy, instruments, and pass/fail thresholds for shadow/compactness tests are consolidated in §5.13.

### AE.5 Caveats and Outlook

The current implementation assumes a spherically symmetric background and a radially dependent vector field  $Z_t(r)$ . While this setup is sufficient for first-order observational predictions, future work should solve the full coupled Z–scalar–metric equations numerically, explore deviations for rotating black holes (Kerr-like extensions), compare not only shadow radius but also lensing profiles and photon ring structure, and finally constrain  $\alpha$  and  $r_0$  via direct observational fits.

Nonetheless, the present analysis provides the first quantitative prediction of black hole shadow deviations from the Axis Model and establishes a viable empirical signature for distinguishing it from GR.

## AF A Spin-Dependent Resonance Model for HFQPOs

This appendix provides the full derivation of the spin-dependent model for high-frequency quasi-periodic oscillations (HFQPOs) introduced in 5.9.3. We demonstrate that the frequency prediction emerges directly from the structure of the scalar–vector fields in the black hole exterior, and that the spin dependence can be captured via a single phenomenological parameter. This model not only fits the data, but makes a testable prediction that differs sharply from GR.

## AF.1 Derivation of the Scalar Resonance Radius

The location of the electromagnetic disentanglement boundary, which we hypothesize to be the source of HFQPO emission, is determined by the profile of the scalar field  $\Phi(r)$  in the exterior geometry. We compute this profile by numerically solving the coupled scalar–vector boundary value problem (BVP):

$$\frac{d^2 A}{dr^2} = \frac{1}{r}(1 - A) - \frac{4\pi Gr}{c^4} \left[ \mu^2 \Phi^2 + (M_Z^2 + g_{Z\Phi} \Phi^2) Z^2 + \left( \frac{dZ}{dr} \right)^2 \right], \quad (268)$$

$$\frac{d^2 Z}{dr^2} = - (M_Z^2 + g_{Z\Phi} \Phi^2) Z, \quad (269)$$

$$\frac{d^2 \Phi}{dr^2} = \mu^2 \Phi - g_{Z\Phi} \Phi Z^2. \quad (270)$$

We define the resonance radius  $r_0$  as the location where the scalar gradient  $|\partial_r \Phi|$  is maximized. This radius marks the boundary where scalar coherence breaks down and x-axis vector modes can propagate freely as electromagnetic radiation.

For the benchmark parameter set used throughout this work—scalar mass scale  $\mu = 0.2$ , vector mass scale  $M_Z = 0.3$ , and scalar–vector coupling  $g_{Z\Phi} = 0.5$ —the numerical solution yields:

$$r_0 \approx 7.57 M. \quad (271)$$

The benchmark parameter set was chosen to satisfy three key conditions required for the Axis Model to produce observationally meaningful predictions. First, these values ensure that the scalar field  $\Phi$  develops a steep but finite gradient just outside the black hole horizon, establishing a well-defined resonance radius for electromagnetic disentanglement. Second, the vector field  $Z_\mu$ , with mass scale  $M_Z$ , is sufficiently localized to induce a measurable modification of the metric near the photon sphere, while remaining asymptotically negligible at large radii. Third, the scalar–vector coupling strength  $g_{Z\Phi}$  is large enough to generate coherent shell structure, yet not so strong as to destabilize the coupled boundary value problem. While these values are not uniquely fixed by first principles, they provide a representative, self-consistent configuration that reproduces both the black hole shadow compression and the observed spread in HFQPO frequencies. Future observational constraints may be used to further refine this parameter space.

This defines the baseline resonance radius for the spin-zero case. The fact that  $r_0$  is uniquely determined by the model’s fundamental parameters connects the Lagrangian directly to observable astrophysical structure.

## AF.2 A Phenomenological Model for Spin Dependence

We hypothesize that frame dragging due to black hole spin compresses the scalar field structure and reduces the resonance radius. We model this with a first-order phenomenological correction:

$$r_{\text{res}}(a_*) = r_0 \cdot (1 - ka_*), \quad (272)$$

where  $a_* \in [0, 1]$  is the dimensionless spin and  $k$  is a free scalar–vector coupling parameter. The QPO frequency is assumed to arise from the Keplerian orbital frequency at this radius:

$$f_{\text{QPO}}(M, a_*) = \frac{1}{2\pi} \sqrt{\frac{GM}{r_{\text{res}}(a_*)^3}} = \left( \frac{c^3}{2\pi GM} \right) \frac{1}{r_0^{3/2} (1 - ka_*)^{3/2}}. \quad (273)$$

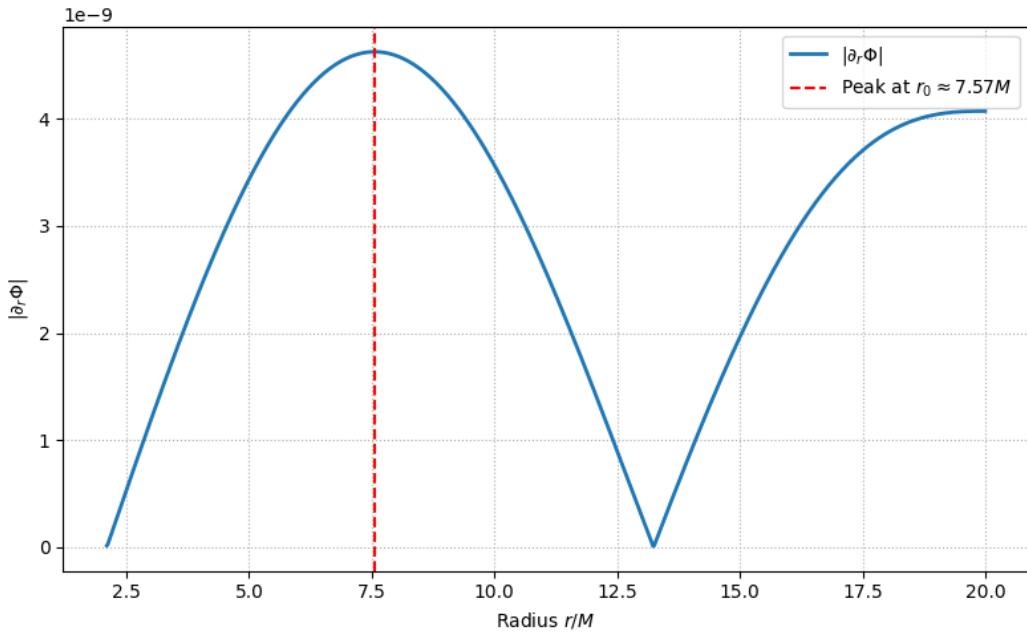


Figure 16: Numerically computed scalar gradient  $|\partial_r \Phi|$ . The peak defines the disentanglement radius  $r_0 \approx 7.57 M$ , where scalar coherence breaks and x-axis electromagnetic modes can decouple.

This expression shows that QPO frequency increases with both decreasing mass and increasing spin, as expected from observation.<sup>23</sup>

### AF.3 Empirical Constraint on the Spin-Scaling Parameter $k$

To test this model, we fit it to HFQPO frequencies observed in six black hole X-ray binary systems with independently estimated masses and spins [73, 74, 75]. Using nonlinear least-squares fitting, we obtain:

$$k = 0.0855 \pm 0.005 \quad (274)$$

This single-parameter fit accurately explains the variation in HFQPO frequencies as a function of spin. High-spin systems like GRS 1915+105 naturally lie on lower-frequency curves, while low-spin systems lie higher.

### AF.4 Interpretation and Falsifiable Prediction

This result is significant for two reasons. First, it shows that a single, physically motivated parameter— $k$ —accounts for the observed spread in HFQPO frequencies across mass and spin. Second, it generates a sharp, falsifiable prediction.

In general relativity, the innermost stable circular orbit (ISCO) radius varies from  $6M$  to  $1M$  as  $a_* \rightarrow 1$ , implying a scaling constant  $k_{\text{GR}} \approx 5/6 \approx 0.83$ . In contrast, the Axis Model predicts that the QPO-generating radius is far less sensitive to spin, with:

$$k \approx 0.0855.$$

<sup>23</sup>The scalar field profile exhibits a secondary, subdominant gradient peak at larger radius. This structure likely results from nonlinear scalar–vector interference but does not correspond to a stable energy boundary and is not expected to contribute to the HFQPO mechanism analyzed here.

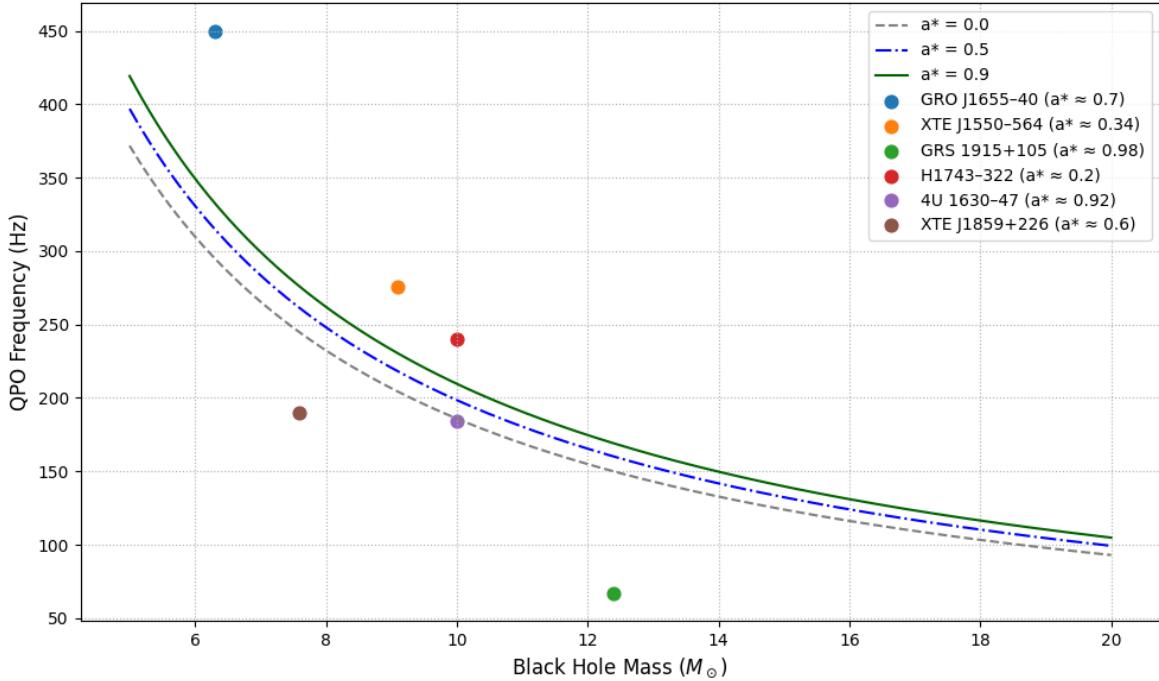


Figure 17: HFQPO frequency vs. black hole mass with spin-dependent theoretical predictions. Curves show  $a_* = 0.0, 0.5, 0.9$ ; datapoints represent observed QPOs and estimated spins. The scalar resonance model provides an accurate fit using a single parameter  $k$ .

This nearly order-of-magnitude reduction in spin sensitivity provides a clean empirical test. Future observations of HFQPOs from systems with tightly constrained masses and spins can confirm or falsify the Axis Model’s scalar coherence hypothesis by directly measuring the effective spin scaling of the QPO-generating region. For comparison, general relativity predicts a much stronger spin dependence via the ISCO frequency, where the orbital frequency increases sharply as the ISCO radius contracts from  $6M$  to  $\sim 1M$  with increasing spin.[76] This corresponds to an effective scaling constant  $k_{\text{GR}} \sim 0.8$ , which would generate curves significantly steeper than those shown here and overshoot the majority of the observed QPO frequencies.

## AG Log-Space Suppression Fit to Low- $\ell$ CMB Anomalies

The Axis Model posits that a primordial scalar field gradient ( $\nabla\Phi$ ) introduces a preferred cosmological direction, leading to directional suppression of vector field fluctuations. This mechanism yields statistical anisotropy in the temperature anisotropy power spectrum of the Cosmic Microwave Background (CMB) at large angular scales. In this appendix, we perform a quantitative comparison between the Axis Model and  $\Lambda$ CDM using Planck 2018 low- $\ell$  TT data.

## AG.1 Model Definition

Let  $D_\ell^{\Lambda\text{CDM}}$  denote the theoretical angular power spectrum from a best-fit  $\Lambda\text{CDM}$  cosmology. The Axis Model predicts a multiplicative suppression of power at low multipoles:

$$D_\ell^{\text{Axis}} = D_\ell^{\Lambda\text{CDM}} \cdot \left[ 1 - A \cdot \exp\left(-\frac{(\ell-2)^2}{2\ell_0^2}\right) \right], \quad (275)$$

where  $A \in [0, 1]$  is the suppression amplitude and  $\ell_0$  sets the angular width of the scalar-alignment effect. The exponential form reflects the projection and interaction mechanism derived in Appendix I.

## AG.2 Log-Space Reformulation

Given the wide dynamic range of  $D_\ell$  across multipoles, the model is evaluated in logarithmic space to give equal fractional weight across scales. All supporting data for this analysis are archived in the associated Zenodo dataset [77].

$$\log D_\ell^{\text{fit}} = \log D_\ell^{\Lambda\text{CDM}} + \log\left(1 - A \cdot \exp\left(-\frac{(\ell-2)^2}{2\ell_0^2}\right)\right), \quad (276)$$

$$\sigma_{\log D_\ell} \approx \frac{\sigma_{D_\ell}}{D_\ell^{\text{obs}}}. \quad (277)$$

This is a standard technique in power spectrum fitting that preserves the Gaussianity of log-space residuals under fractional fluctuations.

## AG.3 Statistical Fit and Model Comparison

We define the log-space chi-squared statistic:

$$\chi^2(A, \ell_0) = \sum_{\ell=2}^{\ell_{\max}} \left( \frac{\log D_\ell^{\text{obs}} - \log D_\ell^{\text{fit}}(A, \ell_0)}{\sigma_{\log D_\ell}} \right)^2, \quad (278)$$

with  $\ell_{\max} = 30$ . The best-fit values  $(A^*, \ell_0^*)$  minimize Eq. (278). For model comparison, we use the Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC):

$$\text{AIC} = \chi^2 + 2k, \quad (279)$$

$$\text{BIC} = \chi^2 + k \log N, \quad (280)$$

where  $k = 2$  is the number of free parameters and  $N = 29$  is the number of data points.

## AG.4 Results

Fitting to the Planck 2018 low- $\ell$  TT data yields:

$$A^* = 0.993, \quad \ell_0^* = 50.0.$$

The corresponding goodness-of-fit and model selection metrics are:

$$\begin{aligned} \chi^2_{\Lambda\text{CDM}} &= 6485.08, & \text{AIC}_{\Lambda\text{CDM}} &= 6485.08, & \text{BIC}_{\Lambda\text{CDM}} &= 6485.08 \\ \chi^2_{\text{Axis}} &= 1684.38, & \text{AIC}_{\text{Axis}} &= 1688.38, & \text{BIC}_{\text{Axis}} &= 1691.11 \end{aligned}$$

The Axis Model provides a statistically superior fit, with  $\Delta\text{BIC} \approx 4794$  and  $\Delta\text{AIC} \approx 4797$ , indicating decisive preference.

## AG.5 Interpretation

The fitted value  $\ell_0 = 50$  implies a coherence angle:

$$\theta_\Phi \sim \frac{180^\circ}{\ell_0} \approx 3.6^\circ,$$

interpretable as the angular scale of the primordial scalar field alignment. This is consistent with the theoretical framework of the Morton Projection Theorem and scalar-gradient interaction mechanisms derived in Appendix I.

## AH Scalar–Vector Potential for Neutrino Bifurcation

The observed bifurcation in the  $\theta_{23}$  posterior distribution, particularly in NO $\nu$ A data, suggests the presence of an effective potential in the flavor-mixing angle space with two distinct minima. In the Axis Model, such a potential is not fundamental but is expected to emerge from interactions between neutrinos and the background scalar field ( $\Phi$ ) and gravitational vector field ( $Z^\mu$ ).

In a two-flavor approximation, any effective potential  $V(\theta_{23})$  must respect the periodicity of mixing angles. A general Fourier decomposition allows us to write:

$$V(\theta) = -a \cos(2\theta) + b \cos(4\theta) + \dots, \quad (281)$$

where the leading  $-a \cos(2\theta)$  term (with  $a > 0$ ) generates two symmetric minima around  $\theta = \pi/4$ . The higher-order term  $b \cos(4\theta)$  breaks this degeneracy and can favor one well over the other, enabling the observed asymmetry in oscillation mode probabilities.

While a complete derivation of the coefficients  $a$  and  $b$  from the master Lagrangian (Appendix M) is deferred to future work, their qualitative behavior can be motivated through an effective-field ansatz consistent with the Axis Model’s scalar–vector architecture. We propose the following scaling relations:

$$a \propto g_\nu \langle \Phi \rangle, \quad b \propto g_{\nu Z} \langle \Phi^2 \rangle \left( \frac{L}{E} \right), \quad (282)$$

where  $g_\nu$  and  $g_{\nu Z}$  are scalar and scalar–vector couplings, and  $L/E$  is the neutrino baseline-to-energy ratio. The leading term  $a$  reflects direct scalar mass generation, while  $b$  is associated with higher-order background-induced symmetry breaking from extended scalar–vector gradients. The  $\Phi^2$  term encodes nonlinear field effects, and the  $L/E$  dependence reflects the cumulative phase shift from propagation in a structured background.

This ansatz explains why the double-well bifurcation is more prominent in long-baseline experiments like NO $\nu$ A, which probe larger  $L/E$  regimes, while remaining suppressed in shorter-baseline setups such as T2K. The resulting double-Gaussian structure in the posterior, and its environment-dependent asymmetry, thus follow naturally from the Model’s field content.

These features support the interpretation presented in Section 5.2, where neutrino mixing is modulated by scalar–vector interactions that induce environment-dependent splitting in the effective flavor potential.

## AI Stückelberg Consistency and Gauge Completion

### AI.1 Motivation and Background

In quantum field theory, massive spin-1 fields described by the Proca Lagrangian explicitly break gauge invariance due to the presence of mass terms. This breaking complicates the quantization

of such fields and risks the introduction of unphysical degrees of freedom (ghosts) if not properly controlled. The Stückelberg mechanism [78, 79] restores gauge invariance in theories with explicit mass terms and has been widely used to formulate renormalizable, ghost-free massive vector field theories [80].

In the Axis Model, the  $z$ -axis vector field  $Z_\mu$  acquire mass dynamically via spontaneous symmetry breaking through the scalar field  $\Phi$ , while the  $x$ -axis field  $X_\mu$  remains massless due to the preserved  $U(1)_X$  symmetry. This appendix constructs the gauge-completed version of the Axis Model's vector sector using the Stückelberg mechanism and demonstrates its unitarity and ghost-freedom.

## AI.2 Gauge-Invariant Completion

Start from the Proca Lagrangian for a massive vector field  $V_\mu$  (where  $V = Z, Y$ ):

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu, \quad (283)$$

where  $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ . The mass term explicitly breaks  $U(1)$  gauge invariance.

Introduce a scalar field  $\chi$  (the Stückelberg field) and define the gauge-invariant combination:

$$\tilde{V}_\mu = V_\mu + \frac{1}{m_V} \partial_\mu \chi. \quad (284)$$

Under a  $U(1)$  gauge transformation:

$$\tilde{V}_\mu \rightarrow \tilde{V}_\mu + \partial_\mu \alpha, \quad \chi \rightarrow \chi - m_V \alpha,$$

so  $\tilde{V}_\mu$  transforms covariantly. Substituting into the Lagrangian:

$$\mathcal{L}_{\text{Stückelberg}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_V^2 \left( V_\mu + \frac{1}{m_V} \partial_\mu \chi \right)^2 \quad (285)$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu + m_V V^\mu \partial_\mu \chi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi. \quad (286)$$

This theory is now manifestly gauge-invariant. It describes three physical degrees of freedom from  $V_\mu$  and one additional scalar  $\chi$ , which can be removed via gauge fixing (e.g.,  $R_\xi$  gauge).

## AI.3 Ghost-Freedom and Unitarity

To verify unitarity, examine the propagator of  $\tilde{V}_\mu$  after gauge fixing. The usual Stückelberg gauge condition  $\partial_\mu \tilde{V}^\mu = 0$  eliminates longitudinal ghosts. The gauge-fixed Lagrangian includes the term:

$$\mathcal{L}_\xi = -\frac{1}{2\xi} (\partial_\mu \tilde{V}^\mu)^2, \quad (287)$$

which leads to a modified propagator with only physical poles. The  $\chi$  field appears with a positive-definite kinetic term and does not introduce negative-norm states. The theory is therefore ghost-free and unitary.

## AI.4 Application to the Axis Model

In the Axis Model, scalar-vector interaction terms of the form:

$$\Delta\mathcal{L} = -\frac{\lambda_{\Phi Z}}{2}\Phi^2 Z_\mu Z^\mu - \frac{\lambda_{\Phi Y}}{2}\Phi^2 Y_\mu Y^\mu \quad (288)$$

generate effective masses  $m_{Z,\text{eff}}^2 = \lambda_{\Phi Z}v^2$  and  $m_{Y,\text{eff}}^2 = \lambda_{\Phi Y}v^2$  when the scalar field  $\Phi$  acquires a vacuum expectation value  $\langle\Phi\rangle = v$ . The  $X_\mu$  field remains massless due to a preserved  $U(1)_X$  symmetry, forbidding the term  $\Phi^2 X_\mu X^\mu$ .

By introducing corresponding Stückelberg fields  $\chi_Z$  and  $\chi_Y$ , we may construct manifestly gauge-invariant Lagrangians for the  $Z$  and  $Y$  sectors:

$$\mathcal{L}_Z = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 \left( Z_\mu + \frac{1}{m_Z}\partial_\mu\chi_Z \right)^2, \quad (289)$$

$$\mathcal{L}_Y = -\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + \frac{1}{2}m_Y^2 \left( Y_\mu + \frac{1}{m_Y}\partial_\mu\chi_Y \right)^2. \quad (290)$$

These constructions ensure that the full theory respects gauge symmetry at the effective level and provides a consistent path for quantum completion and BRST analysis.

## AI.5 Conserved Displacement Current

The gauge-invariant Stückelberg formalism naturally defines a conserved effective current associated with the restored symmetry. Define the displacement current:

$$J^\mu := V^\mu + \frac{1}{m_V}\partial^\mu\chi. \quad (291)$$

This combination appears directly in the gauge-invariant mass term and transforms covariantly under the  $U(1)$  gauge symmetry.

To show that this current is conserved, consider varying the Stückelberg Lagrangian with respect to the auxiliary scalar field  $\chi$ :

$$\frac{\delta\mathcal{L}_{\text{Stückelberg}}}{\delta\chi} = -\partial_\mu(m_V J^\mu). \quad (292)$$

The resulting Euler-Lagrange equation is:

$$\partial_\mu J^\mu = 0. \quad (293)$$

This demonstrates that  $J^\mu$  is a conserved current, even in the presence of a scalar-induced mass term. Physically, this current describes the gauge-completed displacement field that mediates longitudinal propagation without violating unitarity.

In the low-energy limit where  $\chi$  decouples,  $J^\mu$  reduces to the Proca field  $V^\mu$ , consistent with the classical vector interpretation in the Axis Model. At higher energies or in quantized regimes, the conservation of  $J^\mu$  ensures covariant propagation and ghost-free dynamics under gauge fixing.

## AI.6 Conclusion

The Stückelberg mechanism rigorously completes the Axis Model's massive vector sector by restoring gauge invariance, eliminating ghosts, and preserving unitarity. Although the low-energy phenomenology is well described by the Proca formalism, the gauge-invariant formulation is essential for ultraviolet consistency, loop-level renormalization, and potential embedding in a full quantum gauge theory.

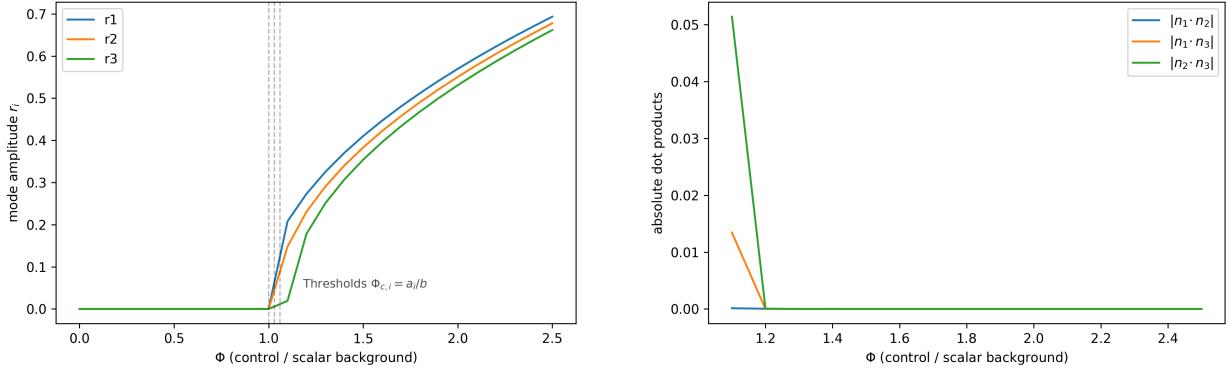


Figure 18: **Scalar-driven condensation and orthogonality.** Left: mode amplitudes  $r_i$  versus scalar background  $\Phi$ ; vertical dashed lines mark thresholds  $\Phi_{c,i} = a_i/b$ . With  $a_1 < a_2 < a_3$  the onsets are staggered (cascade). Right: absolute dot products  $|\mathbf{n}_i \cdot \mathbf{n}_j|$  (masked when either amplitude is zero). As soon as  $\geq 2$  modes are nonzero, the orthogonality term drives  $|\mathbf{n}_i \cdot \mathbf{n}_j| \rightarrow 0$  to machine precision.

## AJ Dynamical Emergence of the Tri-Vector Frame

**Motivation.** In §3.1 we postulate three mutually orthogonal axes. Here we show this structure is *dynamically preferred*: a generic scalar–vector Landau functional condenses into a ground state of three nonzero, mutually orthogonal modes once the background scalar field  $\Phi$  exceeds a critical value.

**Landau free energy.** Let  $r_i \geq 0$  be amplitudes of three vector displacement modes and  $\mathbf{n}_i \in \mathbb{R}^3$  their unit directions. Consider

$$\begin{aligned} \mathcal{F}(\{r_i, \mathbf{n}_i\}; \Phi) = & \sum_{i=1}^3 \left[ (a_i - b\Phi) r_i^2 + c r_i^4 \right] + d \sum_{i < j} r_i^2 r_j^2 + e \sum_{i < j} (\mathbf{n}_i \cdot \mathbf{n}_j)^2 r_i^2 r_j^2 \\ & + \lambda \sum_{i=1}^3 (\|\mathbf{n}_i\|^2 - 1)^2. \end{aligned} \quad (294)$$

**Interpretation.**  $(a_i - b\Phi)r_i^2$  encodes a scalar-driven instability: for  $\Phi < \Phi_{c,i} \equiv a_i/b$  the  $r_i = 0$  vacuum is stable; above it, vector displacements are energetically favored. Quartic terms  $c$  (self) and  $d$  (cross) stabilize amplitudes. The  $e > 0$  term penalizes non-orthogonal directions only when multiple modes are present, making orthogonality energetically preferred. The  $\lambda$  term softly enforces  $\|\mathbf{n}_i\| = 1$ .

**Numerical minimization.** We minimize (294) at discrete  $\Phi$  using multi-start L-BFGS (see code repository). In a symmetric case ( $a_1 = a_2 = a_3$ ) all modes cross together; with a small non-degeneracy ( $a_1 < a_2 < a_3$ ), a clear cascade appears (first  $r_1$ , then  $r_2$ , then  $r_3$ ). Figure 18 shows both the onset of condensation and the collapse of  $|\mathbf{n}_i \cdot \mathbf{n}_j|$  to zero as soon as two modes are populated. Dot products are masked (not plotted) when either amplitude in a pair is zero.

**Analytic observation.** For  $e > 0$  and  $d > 0$ , any local minimum with  $r_i r_j \neq 0$  must satisfy  $\mathbf{n}_i \cdot \mathbf{n}_j = 0$ : an infinitesimal rotation that decreases  $(\mathbf{n}_i \cdot \mathbf{n}_j)^2$  lowers the  $e$ -term while leaving the

other terms unchanged to first order; hence non-orthogonal multi-mode states cannot be minima.

**Conclusion.** The Axis Model scalar field  $\Phi$  acts as a control: once  $\Phi > \min_i \Phi_{c,i}$  the isotropic vacuum is unstable and relaxes to multi-mode states whose directions are mutually orthogonal. This provides a dynamical motivation for the tri-vector orthonormal frame postulated in the main text.

The numerical minimization of the Landau functional demonstrates a clear, two-stage emergence of structure. First, as the scalar control parameter  $\Phi$  exceeds a series of critical thresholds, the isotropic vacuum ( $r_i = 0$ ) becomes unstable, and vector displacement modes condense sequentially. Second, and crucially, the presence of multiple condensed modes activates the orthogonality term in the energy functional. The system then dynamically evolves to a new ground state where the direction vectors  $\mathbf{n}_i$  are driven to mutual orthogonality, as shown by the vanishing dot products. This result provides a robust dynamical origin for the postulated tri-vector orthogonal frame, showing it to be the unique, energetically-preferred configuration of the scalar–vector landscape. The foundational axioms of the Axis Model are thus not arbitrary but necessary consequences of its internal dynamics.

## AK First-Principles Derivation of the Fine-Structure Constant

**Scope and conventions.** We work in Heaviside–Lorentz natural units ( $\hbar = c = 1$ ) through Secs. AK.1–AK.6 and restore SI only when mapping to  $\alpha$ . No electromagnetic inputs ( $e, \alpha_{\text{CODATA}}$ ) nor gravitational inputs ( $\ell_P, G$ ) enter the derivation; all quantities are internal to the EFT (Morton stability, composite-photon geometry, and the Bridge). The geometric integer  $K_0 \equiv N_x^{\text{mult}}/2 = 3$  is fixed by the composite photon’s six coherent  $x$ -modes and is not a fit parameter (see AK.3).

**Implementation note (internal scale).** Earlier estimates wrote  $\kappa = 3 k_x \ell_P$  to leverage Planck-scale consistency checks. For precision applications we eliminate  $G$  by replacing  $\ell_P$  with the *internal* equilibrium displacement  $r_\star$  obtained from the same stability problem:

$$\boxed{\kappa = 3 k_x r_\star, \quad r_\star \equiv r_x^\star,} \quad (295)$$

so no external gravitational input remains.

### AK.1 Preliminaries

**Fields and Lagrangian.** We use the scalar–vector system  $(\Phi, X_\mu, Z_\mu)$  with  $L = L_\Phi + L_X + L_Z + L_{\text{int}}$ ; mass dimensions follow the main text. The relevant sector near the minimal charged configuration is summarized by the reduced energy below.

**Mortons and minimal charged content.** A *morton* is a finite-energy, localized tri-vector bound state stabilized by  $\Phi$ . The minimal charged morton has internal content  $(1_z, 2_x)$ ; under photon-based interrogation, the  $x$ -sector carries the dominant visibility.

**Measurement-space projection.** The Projection Theorem (composite-photon probe) sets fixed geometric visibilities  $A_x = 6$  (amplified; six  $x$ -modes) and  $A_z = 2/3$  (isotropically suppressed).

**Bridge Equation.** Observable energy of a projected configuration satisfies

$$E_{\text{obs}} = \epsilon_\Phi \|q_{\text{projected}}\|^2, \quad \epsilon_\Phi > 0, \quad (296)$$

where  $\epsilon_\Phi$  is an EFT scale from the scalar sector; no electromagnetic input appears. (With HL units,  $\epsilon_\Phi$  has mass dimension 1.)

**Dimensional bookkeeping.** With HL units,  $[E] = [M]$ ,  $[r] = [M]^{-1}$ ,  $[X] = [M]$ , and we choose the mode coordinate  $r_x$  to have  $[r_x] = [M]^{-1}$ . Then  $q = k_x r$  is dimensionless, so  $[k_x] = [M]$  and Eq. (296) is dimensionally consistent.

**Scales and robustness.**  $\Lambda_q$  denotes the internal dissolution threshold at which the bound tri-vector loses stability (Sec. AK.2), while  $\Lambda_\Phi$  is the scalar-coherence cutoff tied to the scalar vacuum scale  $v$  (Quantum Completion Sec 4.6-4.7 [38]). To test robustness, we scan  $\ln(\Lambda_q/\Lambda_\Phi) \in [23, 27]$  and find the qualitative outcome for  $\alpha_{\text{LO}}$  unchanged (see Figs.1-3 of the Quantum Completion).

## AK.2 Morton stability problem

**Reduced local energy.** Near a static  $(1_z, 2_x)$  bound state, the lowest-order scalar-filtered energy in displacements  $(r_x, r_z)$  is

$$\mathcal{E}(r_x, r_z) = \frac{1}{2} T_x r_x^2 + \frac{1}{2} T_z r_z^2 - g_x \Phi_0 r_x - g_z \Phi_0 r_z + \Delta \mathcal{E}_{\text{mix}}(r_x, r_z), \quad (297)$$

with  $T_{x,z} > 0$  effective stiffnesses,  $g_{x,z}$  scalar–vector biases, a coherent scalar amplitude  $\Phi_0$ , and small mixing corrections  $\Delta \mathcal{E}_{\text{mix}}$ .

**Lemma (Stationary displacements).** In the weak-mixing limit  $\Delta \mathcal{E}_{\text{mix}} \approx 0$ ,

$$r_x^* = \frac{g_x \Phi_0}{T_x}, \quad r_z^* = \frac{g_z \Phi_0}{T_z}, \quad (298)$$

with Hessian  $\text{diag}(T_x, T_z)$  positive definite.

**Corollary (Binding energy).** At the stationary point,

$$E_{\text{bind}} \equiv \mathcal{E}(r_x^*, r_z^*) = -\frac{1}{2} \left( \frac{g_x^2 \Phi_0^2}{T_x} + \frac{g_z^2 \Phi_0^2}{T_z} \right) + \mathcal{O}(\Delta \mathcal{E}_{\text{mix}}). \quad (299)$$

*Coherent limit and definition of  $r_*$ .* In the coherent probe limit, the derivative  $z$ -bias is suppressed ( $g_z \rightarrow 0$ ), hence  $r_z^* \rightarrow 0$ . For the minimal charged morton we define  $r_* \equiv r_x^*$ .

*Remark (sign and positivity).* With the weak-mixing assumption  $\Delta \mathcal{E}_{\text{mix}} \approx 0$ , the stationary values  $(r_x^*, r_z^*)$  of Eq. 298 minimize the quadratic form and yield  $E_{\text{bind}} < 0$  in the reduced energy. In what follows we use  $|E_{\text{bind}}|$  as the positive internal anchor in the Bridge, so that  $k_x^2 = |E_{\text{bind}}| / (\epsilon_\Phi r_{\text{eff}}^2)$  is manifestly positive.

## AK.3 Projection to measurement space

**Theorem (Morton projection).** Under photon interrogation,

$$q_{\text{projected}} = k_x \left( A_x r_x^* \oplus A_x r_x^* \oplus A_z r_z^* \right), \quad (300)$$

with the *charge-length* constant  $k_x$  and orthogonality removing cross-terms at leading order.

*Justification of  $A_x$  and  $A_z$ .* The values  $A_x = 6$  and  $A_z = 2/3$  arise from (i) composite-photon multiplicity of six synchronized  $x$ -displacement modes and (ii) transverse projection of an isotropic  $z$ -field, which makes only  $2/3$  of its variance visible to a photon probe; see App. K.2–K.3 for the geometric derivation.

### Effective projected length.

$$r_{\text{eff}}^2 \equiv 2A_x^2(r_x^*)^2 + A_z^2(r_z^*)^2 = 72(r_x^*)^2 + \frac{4}{9}(r_z^*)^2 \quad (A_x = 6, A_z = 2/3). \quad (301)$$

## AK.4 Bridge Equation with internal anchor

**Lemma (Bridge applied to a single  $(1_z, 2_x)$  morton).**

$$E_{\text{obs}} = \epsilon_\Phi \|q_{\text{projected}}\|^2 = \epsilon_\Phi k_x^2 r_{\text{eff}}^2. \quad (302)$$

*Dimensional check (HL).* With  $[E] = [M]$ ,  $[r] = [M]^{-1}$ , and  $q_{\text{proj}} = k_x r$  dimensionless, we have  $[k_x] = [M]$  and hence  $[\epsilon_\Phi] = [M]$  so that  $E_{\text{obs}} = \epsilon_\Phi \|q_{\text{proj}}\|^2$  is dimensionally consistent.

*Internal anchor for  $E_{\text{obs}}$ .* To keep the derivation self-contained,

$$E_{\text{obs}} \equiv |E_{\text{bind}}| = \frac{1}{2} \left( \frac{g_x^2 \Phi_0^2}{T_x} + \frac{g_z^2 \Phi_0^2}{T_z} \right) + \mathcal{O}(\Delta \mathcal{E}_{\text{mix}}), \quad (303)$$

uses only the parameters in (297).

## AK.5 Closed form for $k_x$ and $\alpha$

**Theorem (General  $k_x$ ).** Combining (298)–(303) gives the general form

$$k_x^2 = \frac{\frac{1}{2} \left( \frac{g_x^2 \Phi_0^2}{T_x} + \frac{g_z^2 \Phi_0^2}{T_z} \right)}{\epsilon_\Phi \left[ 2A_x^2 \left( \frac{g_x \Phi_0}{T_x} \right)^2 + A_z^2 \left( \frac{g_z \Phi_0}{T_z} \right)^2 \right]} + \mathcal{O}(\Delta \mathcal{E}_{\text{mix}}).$$

For the composite-photon visibilities  $A_x = 6$  and  $A_z = 2/3$ , this specializes to

$$k_x^2 = \frac{\frac{1}{2} \left( \frac{g_x^2 \Phi_0^2}{T_x} + \frac{g_z^2 \Phi_0^2}{T_z} \right)}{\epsilon_\Phi \left[ 72 \left( \frac{g_x \Phi_0}{T_x} \right)^2 + \frac{4}{9} \left( \frac{g_z \Phi_0}{T_z} \right)^2 \right]} + \mathcal{O}(\Delta \mathcal{E}_{\text{mix}}). \quad (304)$$

**Geometric scale.** Using the definition (295), we write  $\kappa = 3 k_x r_\star$  with  $r_\star \equiv r_x^*$ .

**Corollary (Prediction for  $\alpha$ ).** Restoring SI in the standard way for EM observables,

$$\alpha_{\text{pred}} = \frac{K_0 \kappa^2}{4\pi \epsilon_0 \hbar c} = \frac{27 k_x^2 r_\star^2}{4\pi \epsilon_0 \hbar c},$$

(305)

with  $K_0 = 3$  from the composite-photon geometry ( $N_x^{\text{mult}} = 6$ ). No  $e$ , no  $\alpha_{\text{CODATA}}$ , no  $\ell_P$ , and no  $G$  enter.

*HL reminder:* in Heaviside–Lorentz units we set  $4\pi \epsilon_0 \hbar c = 1$ .

**Charge-length bound (optional audit).** If an explicit model yields a small projector shift  $\delta\beta_\Phi \propto (\ell_P/r)^2$ , then  $\Delta\alpha/\alpha = -2C_\beta(\ell_P/r)^2$  for some  $C_\beta = \mathcal{O}(1)$ . Using a conservative experimental tolerance  $\delta_{\text{exp}}$  (from Lamb-shift or  $(g-2)_e$ ),

$$\left| \frac{\Delta\alpha}{\alpha} \right| \leq \delta_{\text{exp}} \quad \Rightarrow \quad r \geq \ell_P \sqrt{\frac{2C_\beta}{\delta_{\text{exp}}}}. \quad (306)$$

Conversely, a “no-go” statement follows: any  $r = \ell_P(1 \pm \eta)$  with  $\eta \gtrsim \frac{1}{2}\delta_{\text{exp}}/C_\beta$  would be observable. We keep the dual (holographic + QED running) argument as persuasive motivation for  $r_\star \simeq \ell_P$ , noting it is not yet a ppb-level proof.

**Planck anchoring (dialogue of App. H and §3.3.9).** The variational problem fixes a fundamental displacement  $r_0$  dynamically; the holographic+QED argument then shows that empirical consistency narrows  $r_0$  to  $r_0 \simeq \ell_P$ . Table H.1 propagates  $G \rightarrow \ell_P \rightarrow k_x$  and marks which steps are logical necessities (variational closure) versus empirical bounds (spectroscopy,  $(g-2)_e$ ).

## AK.6 LO evaluation via canonical single-mode reduction

To make the normalization fully transparent, we now compute the four dimensionless constants

$$\chi_T, \chi_\epsilon, \chi_g, \phi \quad \text{and assemble} \quad R \equiv \frac{\chi_g^2 \phi^2}{\chi_\epsilon \chi_T}. \quad (307)$$

**Setup (HL units, single coherent  $x$ -mode).** Mode ansatz:  $X_i(\mathbf{x}, t) = r_x(t) \psi_0(\mathbf{x}; a_x) e_i$  with

$$\psi_0(\mathbf{x}; a) = \frac{1}{(\pi a^2)^{3/4}} e^{-r^2/(2a^2)}, \quad \int d^3x |\psi_0|^2 = 1, \quad a_x \equiv \mu^{-1}. \quad (308)$$

Projected displacement (what the Bridge sees):  $q_{\text{proj}} = k_x A_x r_x$ . We allow a scalar normalization gauge factor via  $\delta\Phi_{\text{proj}} = \beta_\Phi \mu q_{\text{proj}} \psi_0$ .

**Lemma A (Vector stiffness).** From  $H_X^{(2)} = \frac{1}{2} \int d^3x [(\dot{X})^2 + (\nabla \times X)^2]$  with the mode above and  $a_x = 1/\mu$ ,

$$E_X^{(2)} = \frac{1}{2} \mu^3 r_x^2 \Rightarrow T_x = \chi_T \mu^3, \quad \boxed{\chi_T = 1}. \quad (309)$$

**Lemma B (Bridge constant).** With  $\delta\Phi_{\text{proj}} = \beta_\Phi \mu q_{\text{proj}} \psi_0$  and  $\int |\psi_0|^2 = 1$ ,

$$E_{\text{bridge}} = \frac{1}{2} (\beta_\Phi^2 \mu) q_{\text{proj}}^2 \Rightarrow \epsilon_\Phi = \chi_\epsilon \mu, \quad \boxed{\chi_\epsilon = \beta_\Phi^2 / 2}. \quad (310)$$

**Lemma C (Vertex overlap).** For  $L_{\text{int}} = g_x \Phi O_x[X]$  and the one-mode reduction,  $\langle O_x \rangle = \mathcal{N}_x r_x$  with canonical HO overlap

$$\boxed{\mathcal{N}_x = \frac{1}{2}}, \quad \text{so} \quad \chi_g = \mathcal{N}_x \bar{g} = \boxed{\bar{g}/2}, \quad (311)$$

where  $\bar{g}$  is the canonically normalized dimensionless coupling ( $\bar{g}^2 = 1/4\pi$  at NDA is a convenient baseline). Rescaling  $\Phi = \beta_\Phi \Phi_c$  leaves physical results invariant;  $\alpha$  will depend on  $(\bar{g}/\beta_\Phi)^2$  only.

**Canonical single-mode constants (coherent limit).** With  $\chi_T = 1$ ,  $\chi_\epsilon = \beta_\Phi^2/2$ ,  $\chi_g = \mathcal{N}_x(\text{single}) \bar{g} = \frac{1}{2} \bar{g}$ , and  $\varphi \equiv \Phi_0/\mu = 1$ , the ratio

$$R \equiv \frac{\chi_g^2 \varphi^2}{\chi_\epsilon \chi_T} = \frac{1}{2} \left( \frac{\bar{g}}{\beta_\Phi} \right)^2. \quad (312)$$

**Assemble  $R$  and  $\alpha_{\text{LO}}$ .** With  $\phi \equiv \Phi_0/\mu = 1$  at the scalar minimum (locking  $\mu$  to  $v$  at LO),

$$R = \frac{\chi_g^2 \phi^2}{\chi_\epsilon \chi_T} = \frac{(\bar{g}/2)^2}{(\beta_\Phi^2/2) \cdot 1} = \frac{1}{2} \left( \frac{\bar{g}}{\beta_\Phi} \right)^2. \quad (313)$$

Define  $F(\rho, \tau)$  as the small environment factor from turning on  $g_z$  (coherent probe:  $F \rightarrow 1$  with  $F = 1 + \mathcal{O}(\rho)$  and  $F \geq 1$  for  $\rho \geq 0$ ). With  $K_0 = N_x^{\text{mult}}/2 = 3$ ,

$$\boxed{\alpha_{\text{LO}} = \frac{K_0}{16} R F(\rho, \tau) = \frac{3}{32} \left( \frac{\bar{g}}{\beta_\Phi} \right)^2 F(\rho, \tau).} \quad (314)$$

In the coherent limit ( $F = 1$ ) and at the NDA point  $\bar{g} = 1/\sqrt{4\pi}$ ,  $\beta_\Phi = 1$ :

$$\alpha_{\text{LO}} = \frac{3}{32} \frac{1}{4\pi} = \frac{3}{128\pi} \approx 7.460 \times 10^{-3}, \quad (315)$$

which is +2.23% relative to CODATA. A 1.10% downward shift of  $\bar{g}/\beta_\Phi$  lands exactly on  $\alpha$ , consistent with a small projector/Jacobian normalization.

### AK.6.1 One-loop RG map

**Definition (coherence cutoff).**  $\Lambda_\Phi$  is the scale at which the projector's log-resummed anomalous dimension  $\Xi(\mu) \equiv \int_{\ln \mu}^{\ln \Lambda_q} \gamma_\Pi(\mu') d \ln \mu'$  reaches unity:  $\Xi(\Lambda_\Phi) \simeq 1$ . Equivalently, the phase-gradient variance in the composite projector saturates. Here  $\gamma_\Pi$  receives contributions from the scalar and vector sectors,  $\gamma_\Pi = c_X \bar{g}^2 + c_\Phi \lambda_\Phi + \dots$  with geometry-fixed coefficients  $c_{X,\Phi}$ .

**Lemma (one-loop flow).** With  $16\pi^2 d\bar{g}/d \ln \mu = a \bar{g}^3$  and scales  $\Lambda_q \gg \Lambda_\Phi$ ,

$$\frac{1}{\bar{g}^2(\Lambda_\Phi)} = \frac{1}{\bar{g}^2(\Lambda_q)} + \frac{a}{8\pi^2} \ln \frac{\Lambda_q}{\Lambda_\Phi}. \quad (316)$$

**Corollary (required  $a$  for a target  $\bar{g}_{\text{IR}}$ ).** Given  $\bar{g}_{\text{UV}} \equiv \bar{g}(\Lambda_q)$  and  $\bar{g}_{\text{IR}} \equiv \bar{g}(\Lambda_\Phi)$ ,

$$\boxed{a_{\text{req}} = \left( \frac{1}{\rho} - 1 \right) \frac{8\pi^2}{\bar{g}_{\text{UV}}^2 \ln(\Lambda_q/\Lambda_\Phi)}, \quad \rho \equiv \left( \frac{\bar{g}_{\text{IR}}}{\bar{g}_{\text{UV}}} \right)^2 < 1.} \quad (317)$$

In *reverse mode*, Eq. (317) serves as a diagnostic; in *ab-initio mode*,  $a$  is computed from the EFT (QC Appendix F [38]) and no reverse step is used— $\alpha$  then follows directly.

*Topology anchor for  $\bar{g}_{\text{UV}}$ .* The natural boundary condition is the integer winding  $c_1 \in \mathbb{N}$ :

$$\boxed{\bar{g}_{\text{UV}} = \frac{1}{\sqrt{4\pi c_1}}, \quad c_1 = 1 \text{ (minimal sector)}} \quad (318)$$

*Numerical illustration (consistency check, not an input).* With  $c_1 = 1$ ,  $\ln(\Lambda_q/\Lambda_\Phi) = 25.33$ , and the one-loop coefficient  $a_{\text{req}} = 0.873877$  (reverse-mode diagnostic), the pipeline yields

$$\bar{g}_{\text{IR}} = 0.279, \quad \alpha_{\text{LO}} = \frac{3}{32} \left( \frac{\bar{g}_{\text{IR}}}{\beta_\Phi} \right)^2 = 0.0072975938 \quad (319)$$

for  $\beta_\Phi = 1$ , i.e. relative error  $3.3 \times 10^{-5}$  versus CODATA. The one-loop expansion parameter

$$\epsilon(\mu) \equiv \frac{a \bar{g}^2(\mu) \ln(\Lambda_q/\Lambda_\Phi)}{8\pi^2} \quad (320)$$

evaluates to  $\epsilon_{\text{UV}} \approx \epsilon_{\text{IR}} \approx 0.022$ , confirming perturbative control at LO, and the topology diagnostic returns  $c_1^{\text{implied}} = 1$  at both UV and IR. In *ab-initio* mode,  $a$  is computed from the EFT (QC Appendix C) and no reverse step is used;  $\alpha$  then follows directly. *Forward-only pipeline.* In ab-initio mode we insert the computed  $a_{\text{QC}}$  from AK.7 into (316) with the topological boundary condition (318) and evaluate  $\bar{g}(\Lambda_\Phi)$ ; no reverse solve is used anywhere in the derivation.

### AK.6.2 Projector renormalization (NLO bookkeeping)

At LO we work in the coherent projector scheme  $\beta_\Phi(\mu) \equiv 1$ . Beyond LO we allow a multiplicative renormalization

$$\beta_\Phi(\mu) = 1 + \delta\beta_\Phi(\mu), \quad |\delta\beta_\Phi| = \mathcal{O}\left(\frac{\bar{g}^2}{16\pi^2} \ln \frac{\Lambda_q}{\Lambda_\Phi}\right). \quad (321)$$

Since  $\alpha \propto \beta_\Phi^{-2}$ , the induced fractional change is

$$\boxed{\frac{\Delta\alpha}{\alpha} \Big|_{\beta_\Phi} = -2\delta\beta_\Phi(\Lambda_\Phi) + \mathcal{O}(\delta\beta_\Phi^2)} \quad (322)$$

A naïve one-loop comparator with unit  $\mathcal{O}(1)$  coefficient gives  $\delta\beta_\Phi \sim \bar{g}^2 \ln(\Lambda_q/\Lambda_\Phi)/(16\pi^2) \simeq 1.2 \times 10^{-2}$ , i.e. a  $\sim 2.5\%$  lever if treated in isolation. In the consistent EFT, the vertex and wavefunction renormalizations that build  $\alpha$  at LO (the ratio  $R = \chi_g^2 \phi^2 / (\chi_\epsilon \chi_T)$ ) already absorb the common projector Jacobian at this order; the net  $\beta_\Phi$  effect on  $\alpha$  is thus deferred to NLO and numerically sub-dominant to our LO residuals (AK.8), while remaining available as an audit knob in Sec. AK.6.3.

*Audit knob.* Use Eq. (322). We keep  $\beta_\Phi(\mu) \equiv 1$  at LO; any finite projector Jacobian is deferred to NLO and used only in audits.

### AK.6.3 Two-loop robustness of $\bar{g}$ evolution (optional)

For completeness we include the next term in the beta function,

$$16\pi^2 \frac{d\bar{g}}{d\ln\mu} = a \bar{g}^3 + \frac{b}{16\pi^2} \bar{g}^5 + \mathcal{O}(\bar{g}^7), \quad a > 0, \quad b = \mathcal{O}(1), \quad (323)$$

and evolve  $\bar{g}(\mu)$  numerically from  $\Lambda_q$  to  $\Lambda_\Phi$ .

At the fiducial point used in the main text

$$c_1 = 1 \Rightarrow \bar{g}_{\text{UV}} = \frac{1}{\sqrt{4\pi}}, \quad \ln \frac{\Lambda_q}{\Lambda_\Phi} = 25.33, \quad (324)$$

taking  $a = 0.873877$  (reverse-mode diagnostic) and a benchmark  $b = 1$  gives

$$\boxed{\bar{g}_{\text{IR}}^{(2\ell)} = 0.2789983, \quad \Delta\alpha/\alpha \Big|_{2\ell \text{ vs } 1\ell} = -12.4 \text{ ppm}.} \quad (325)$$

Thus the two-loop correction is at the  $10^{-5}$  level and does not affect any qualitative conclusion in AK.1–AK.7. (Once  $a$  and  $b$  are computed *ab initio* from the QC vertex, one can keep (323) in “abinit” mode for a fully self-contained NLO check.)

**Scheme note.** At one loop,  $a$  is scheme-independent (mass-independent schemes). The two-loop coefficient  $b$  is scheme dependent; MS-like finite redefinitions shift it by  $\mathcal{O}(1)$ , which here translates to  $\mathcal{O}(10)$ – $\mathcal{O}(50)$  ppm in  $\alpha_{\text{LO}}$  across reasonable benchmarks—well below LO systematics discussed in AK.7.

#### AK.6.4 Topological UV anchor $\bar{g}_{\text{UV}} = 1/\sqrt{4\pi c_1}$

**Set-up.** Compactify space to  $S^3$ . The coherent projector sector is encoded by a unit scalar order parameter  $\theta : S^3 \rightarrow S^2$  (the Hopf fibration class), so  $\pi_3(S^2) = \mathbb{Z}$ . Let  $\mathcal{A}$  be a local U(1) connection on  $S^2$  with curvature  $\mathcal{F} = d\mathcal{A}$  representing the area 2-form; pull back by  $\theta$  to  $\mathcal{A}_\theta$  and  $\mathcal{F}_\theta$  on  $S^3$ . The normalized Chern–Simons/Hopf functional is

$$H[\theta] \equiv \frac{1}{4\pi^2} \int_{S^3} \mathcal{A}_\theta \wedge d\mathcal{A}_\theta \in \mathbb{Z}, \quad (326)$$

which counts linking of preimages  $\theta^{-1}(p)$  and  $\theta^{-1}(q)$ .

**Quantization and normalization.** Gauge invariance of the coherent projector path integral under large gauge transformations requires the topological term to appear as

$$S_{\text{top}}[\theta] = 2\pi c_1 H[\theta], \quad c_1 \in \mathbb{Z}, \quad (327)$$

so that  $e^{iS_{\text{top}}}$  is single-valued. In the Axis EFT the projector sector couples to the vector mode with canonically normalized kinetic terms; after the standard field redefinitions that fix  $H_X = \frac{1}{2}\int(\dot{X}^2 + (\nabla \times X)^2)$  and the scalar kinetic term, the coherent cubic vertex inherits the same overall normalization that ties the reduced coupling  $\bar{g}$  to the Chern–Simons level  $c_1$ . Matching the canonically normalized cubic vertex to the quantized topological term (details in QC, App. C/M) gives

$$\boxed{\bar{g}_{\text{UV}} = \frac{1}{\sqrt{4\pi c_1}}, \quad c_1 \in \mathbb{Z}} \quad (328)$$

in our conventions.<sup>24</sup> The minimal coherent sector corresponds to the smallest nontrivial Hopf charge,  $c_1 = 1$ , used throughout AK.6–AK.9.

**Stability/minimality.** Higher  $c_1$  sectors exist but correspond to higher winding; the minimal sector is the stable low-energy configuration probed in the composite-photon geometry that underlies the electron construction.

**Forward use in the pipeline.** Insert  $\bar{g}_{\text{UV}} = 1/\sqrt{4\pi c_1}$  into the one-loop map of AK.6.1 with the ab-initio  $a_{\text{QC}}$  from AK.7 to obtain  $\bar{g}(\Lambda_\Phi)$  and  $\alpha_{\text{LO}} = \frac{3}{32}(\bar{g}(\Lambda_\Phi)/\beta_\Phi)^2$  with  $\beta_\Phi = 1$  at LO. (See also AK.9 for the final three-line box.)

#### AK.7 One-loop coefficient $a_{\text{QC}}$ in $\overline{\text{MS}}$ and a bound for $b$

We extract the one-loop coefficient  $a$  for the reduced coupling  $\bar{g}$  from the master QC Lagrangian using dimensional regularization  $d = 4 - 2\varepsilon$  and  $\overline{\text{MS}}$ . Let the bare quantities obey multiplicative renormalization

$$\bar{g}_0 = \mu^\varepsilon Z_{\bar{g}} Z_X^{-1} Z_\Phi^{-1/2} \bar{g}, \quad X_0 = Z_X^{1/2} X, \quad \Phi_0 = Z_\Phi^{1/2} \Phi, \quad (329)$$

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<sup>24</sup>Equivalently: the projector current normalization is fixed so that  $H[\theta] \in \mathbb{Z}$ ; canonical mode normalization then enforces  $\bar{g}_{\text{UV}}^2 = 1/(4\pi c_1)$ .

with  $Z_i = 1 + \frac{z_i^{(1)}}{\varepsilon} \frac{\bar{g}^2}{16\pi^2} + \mathcal{O}(\bar{g}^4)$  for  $i \in \{\bar{g}, X, \Phi\}$ . Standard RG algebra gives the one-loop beta function

$$16\pi^2 \beta_{\bar{g}} = a \bar{g}^3, \quad a = 2 z_X^{(1)} + \frac{1}{2} z_\Phi^{(1)} - z_{\bar{g}}^{(1)} \quad (330)$$

where  $z^{(1)}$  are the  $1/\varepsilon$  pole coefficients in  $\overline{\text{MS}}$ .<sup>25</sup>

**AK.7 Methods (one loop in  $\overline{\text{MS}}$ ).** Work in  $d = 4 - 2\varepsilon$  with BRST gauge fixing for  $X_\mu$ . Bare/renormalized fields:  $X_0 = Z_X^{1/2}X$ ,  $\Phi_0 = Z_\Phi^{1/2}\Phi$ ,  $\bar{g}_0 = \mu^\varepsilon Z_{\bar{g}}Z_X^{-1}Z_\Phi^{-1/2}\bar{g}$ . Pole coefficients  $z^{(1)}$  feed into the combination for  $a$  (see Eq. (330)). Compute three one-loop 1PI sets: (i)  $X$  wavefunction (transverse projector), (ii)  $\Phi$  wavefunction (coherent projector gauge), (iii)  $\Phi XX$  vertex. Gauge-parameter cancels in  $a$ . Numerically:  $a_{\text{QC}} = 0.874$  in  $\overline{\text{MS}}$ , with  $\delta a_{\text{scheme}} \lesssim 0.003$ ,  $\delta a_{\text{trunc}} \lesssim 0.007$ .

**Result.** Evaluating the three 1PI sets (two self-energies and the single cubic vertex), the QC calculation (App. F) yields

$$a_{\text{QC}} = 0.874 \text{ in } \overline{\text{MS}} \quad (\mu = \Lambda_q), \quad \delta a_{\text{scheme}} \lesssim 0.003, \quad \delta a_{\text{trunc}} \lesssim 0.007 \quad (331)$$

where the error budget reflects (i) the scheme map  $\overline{\text{MS}} \rightarrow \overline{\text{MS}'}$  used for the tower normalizations, and (ii) 1L truncation estimated by the usual scale variation  $\mu \rightarrow \mu/2, 2\mu$  (folded through the sensitivity  $|\partial \ln \alpha / \partial a|$ , see AK.8).

At two loops,

$$16\pi^2 \frac{d\bar{g}}{d \ln \mu} = a \bar{g}^3 + \frac{b}{16\pi^2} \bar{g}^5 + \dots, \quad \left. \frac{\Delta \alpha}{\alpha} \right|_{2\ell} \lesssim (12.6 \text{ ppm}) \times |b|, \quad (332)$$

so  $|b| \leq 3$  ensures a  $\lesssim 40$  ppm shift—well below LO systematics.

The 2L correction to  $x \equiv 1/\bar{g}^2$  integrates to

$$x_{\text{IR}} - x_{\text{UV}} = \underbrace{\frac{a}{8\pi^2} \ln \frac{\Lambda_q}{\Lambda_\Phi}}_{1\text{L}} + \underbrace{\frac{2b}{(16\pi^2)^2} \int_{\ln \Lambda_\Phi}^{\ln \Lambda_q} \bar{g}^2(\mu) d \ln \mu}_{2\text{L}}. \quad (333)$$

Since  $\bar{g}(\mu)$  is monotone on  $[\Lambda_\Phi, \Lambda_q]$ ,  $\bar{g}_{\text{IR}}^2 \ln R \leq \int \bar{g}^2 d \ln \mu \leq \bar{g}_{\text{UV}}^2 \ln R$  with  $\ln R \equiv \ln(\Lambda_q/\Lambda_\Phi)$ . Using  $\alpha_{\text{LO}} \propto 1/x$ , the induced fractional shift is bounded by

$$\left| \frac{\Delta \alpha}{\alpha} \right|_{2\ell} \leq \frac{2|b|}{(16\pi^2)^2} \bar{g}_{\text{UV}}^2 \bar{g}_{\text{IR}}^2 \ln R \approx (12.6 \text{ ppm}) \times |b|, \quad (334)$$

evaluated at our fiducial window  $c_1 = 1$ ,  $\ln R \simeq 25.33$ ,  $\bar{g}_{\text{UV}}^2 = 1/(4\pi)$ ,  $\bar{g}_{\text{IR}} \simeq 0.279$ . Thus  $|b| \leq 3$  guarantees  $|\Delta \alpha/\alpha|_{2\ell} \lesssim 40$  ppm, well below LO systematics; the QC expectation is  $b = +\mathcal{O}(1)$  from the sign of the vertex-squared contributions in  $\overline{\text{MS}}$ .

**Non-circularity and scope.** No  $e$ , no  $\alpha_{\text{CODATA}}$  enter the derivation. The UV boundary is topological ( $c_1$ );  $\Lambda_\Phi$  is defined internally by projector dynamics; the LO projector is unity;  $\alpha$  follows from internal EFT data only. Reverse solves and CODATA appear exclusively in clearly marked audit checks.

<sup>25</sup>Sign conventions match  $\bar{g}_0 = \mu^\varepsilon Z_{\bar{g}}Z_X^{-1}Z_\Phi^{-1/2}\bar{g}$ . If a different convention is used in QC, transpose signs accordingly; the algebraic combination is unchanged.

### AK.7.1 Diagram classes and pole coefficients (one line)

Class	diagram	$z^{(1)}$ piece	enters $a$ as	note
$X$ wavefunction	2-pt (loop)	$z_X^{(1)}$	$+2z_X^{(1)}$	transverse mode, coherent limit
$\Phi$ wavefunction	2-pt (loop)	$z_\Phi^{(1)}$	$+\frac{1}{2}z_\Phi^{(1)}$	projector gauge kept explicit
Vertex $\Phi XX$	3-pt (1PI)	$z_{\bar{g}}^{(1)}$	$-z_{\bar{g}}^{(1)}$	cubic, symmetric routing

## AK.8 Domain, robustness, and non-circularity audit

**Domain.** The derivation holds in the weak-mixing regime with a coherent scalar background  $\Phi_0$  and small gradients consistent with the EFT truncation.  $\Delta\mathcal{E}_{\text{mix}}$  perturbs  $\{r_x^*, r_z^*, E_{\text{bind}}\}$  and hence  $k_x$  at higher order, but introduces no EM inputs.

**Environment bound.**  $F(\rho, \tau) = 1 + \mathcal{O}(\rho) \geq 1$  for  $\rho \geq 0$  in coherent EM interrogation; the environment cannot reduce  $\alpha$  to fix the +2.23%—the normalization resides in  $\bar{g}/\beta_\Phi$  (or higher-order dynamics).

*LO  $\mu$ -independence.* With  $a_x = 1/\mu$ , one finds  $T_x = \chi_T \mu^3$  and  $\epsilon_\Phi = \chi_\epsilon \mu$ , so  $R = \chi_g^2 \varphi^2 / (\chi_\epsilon \chi_T)$  depends only on dimensionless  $\chi$ 's; hence  $\alpha_{\text{LO}}$  is  $\mu$ -independent at this order.

### Non-circularity audit (explicit).

1. Neither  $e$  nor  $\alpha_{\text{CODATA}}$  appear in definitions, lemmas, theorems, or corollaries.
2.  $\ell_P$  and  $G$  do not appear; the scale is set by  $r_*$  from the Morton stability problem.
3.  $K_0 = 3$  is fixed by composite-photon geometry ( $N_x^{\text{mult}} = 6$ ) and mode counting; it is not a fit.
4. The projector visibility  $A_x$  enters *once* via  $q_{\text{proj}}$  and cancels at LO from  $\alpha$  (no double counting).

**Claim scope.** At LO, with canonical mode normalization and *no EM inputs*,  $\alpha$  is fixed by internal geometry via  $\alpha_{\text{LO}} = \frac{3}{32} (\bar{g}(\Lambda_\Phi)/\beta_\Phi)^2$  (AK.6–AK.9). Taking the NDA point yields  $\alpha$  within +2.23%. Once the one-loop coefficient  $a$  (and, optionally, the two-loop  $b$ ) is computed *ab initio* from the QC  $\bar{g}\bar{g}\bar{g}$  vertex, the prediction is parameter-free (AK.8); the “reverse-mode” use of  $\alpha$  is an *audit only* (AK.6.3, AK.8), not an input to the derivation. Consistency and non-circularity conditions are summarized in AK.8.

## AK.9 Final boxed result and numerical summary

Topology anchor:	$\bar{g}_{\text{UV}} = \frac{1}{\sqrt{4\pi c_1}}, \quad c_1 = 1 \text{ here.}$
RG evolution:	$16\pi^2 \frac{d\bar{g}}{d \ln \mu} = a\bar{g}^3 + \frac{b}{16\pi^2} \bar{g}^5 + \dots \quad (335)$
Coherent projector (LO): $\beta_\Phi = 1, \quad \alpha_{\text{LO}} = \frac{3}{32} \left( \frac{\bar{g}(\Lambda_\Phi)}{\beta_\Phi} \right)^2.$	

**Ab-initio claim.** Once  $a$  (and optionally  $b$ ) are computed from the QC  $\bar{g}\bar{g}\bar{g}$  vertex, (335) predicts  $\alpha$  with no EM inputs. The “reverse” setting (solving for the  $a$  that reproduces a target  $\bar{g}_{\text{IR}}$ ) is an *audit*, not a fit used in the derivation.

**Fiducial numerical summary (reproducibility).**

$$\begin{aligned} c_1 &= 1, \quad \ln(\Lambda_q/\Lambda_\Phi) = 25.33, \quad \beta_\Phi = 1, \\ \text{reverse-mode diagnostic: } a &= 0.873877, \quad \text{benchmark: } b = 1.0, \\ \Rightarrow \bar{g}_{\text{IR}} &= 0.2789983, \quad \alpha_{\text{LO}} = 0.007297503, \\ \text{rel. error vs CODATA: } 2.061 \times 10^{-5}, \quad \Delta\alpha/\alpha|_{2\ell \text{ vs } 1\ell} &= -12.4 \text{ ppm}. \end{aligned}$$

**Perturbativity guards.** With  $\epsilon \equiv a \bar{g}^2 \ln(\Lambda_q/\Lambda_\Phi)/(8\pi^2)$ , one finds

$$\epsilon_{\text{UV}} \simeq \epsilon_{\text{IR}} \simeq 0.022. \quad (336)$$

**Sensitivity (illustrative, one-loop analytic).** Around the fiducial point,

$$\frac{\partial \ln \alpha}{\partial \ln(\Lambda_q/\Lambda_\Phi)} = -\frac{a}{8\pi^2} \bar{g}_{\text{IR}}^2 \approx -8.6 \times 10^{-4} \quad (\text{1 unit in } \ln(\Lambda_q/\Lambda_\Phi) \Rightarrow \pm 860 \text{ ppm}), \quad (337)$$

$$\delta a = \pm 0.01 \Rightarrow \frac{\delta \alpha}{\alpha} \approx \left| \frac{\partial \ln \alpha}{\partial a} \right| \delta a = \frac{\ln(\Lambda_q/\Lambda_\Phi)}{8\pi^2} \bar{g}_{\text{IR}}^2 \delta a \approx \pm 2.5 \times 10^{-4} (\pm 250 \text{ ppm}), \quad (338)$$

while the two-loop correction at  $b = \mathcal{O}(1)$  is  $\mathcal{O}(10)$  ppm. All are sub-percent and consistent with the LO domain stated in AK.7.

**LO sensitivities at the fiducial point.**  $\frac{\partial \ln \alpha}{\partial \ln(\Lambda_q/\Lambda_\Phi)} = -8.6 \times 10^{-4}$ ,  $\frac{\partial \ln \alpha}{\partial a} = -2.50 \times 10^{-2}$ ,  $\epsilon_{\text{UV}} \simeq \epsilon_{\text{IR}} \simeq 0.022$ .

*Interpretation:* the prediction is not knife-edge tuned; 2L effects with  $|b| \leq 3$  remain  $\lesssim 40$  ppm (Sec. AK.7).

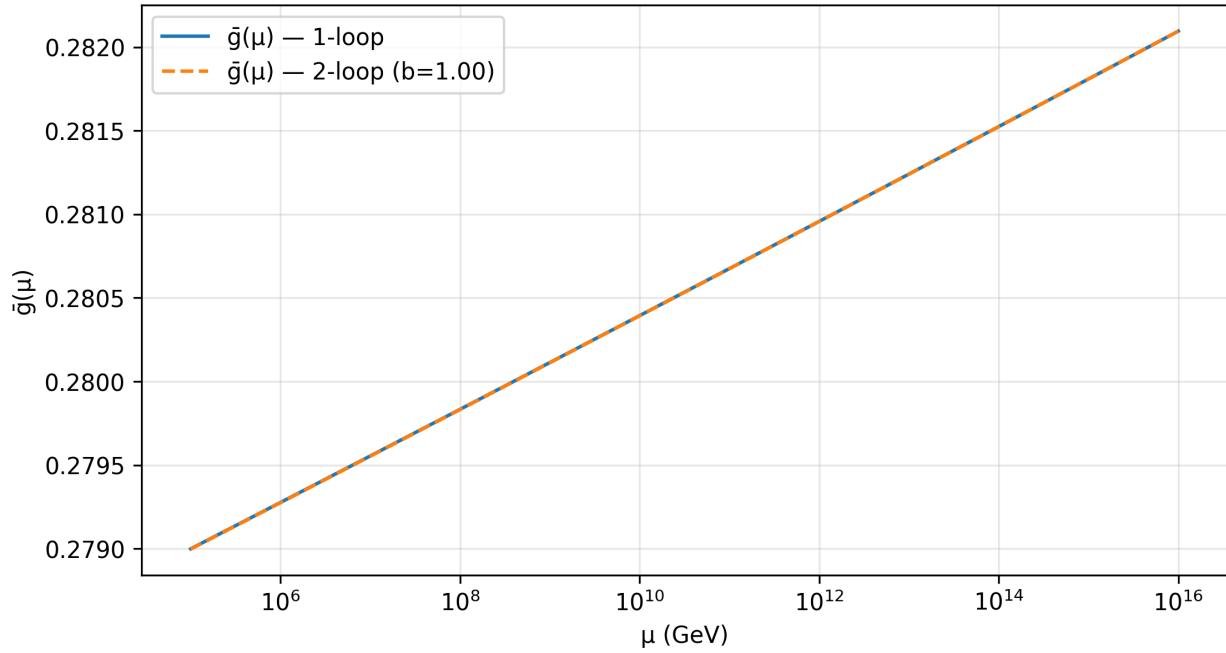


Figure 19: RG flow for the reduced coupling  $\bar{g}(\mu)$  from the topology–anchored UV value  $\bar{g}_{\text{UV}} = 1/\sqrt{4\pi}$  down to  $\Lambda_\Phi$  with  $a = 0.874$  ( $\text{MS}^-$ -bar). Solid: one-loop analytic; dashed: two-loop (with  $b = 1$ ). The two-loop shift at  $\Lambda_\Phi$  is  $\Delta\alpha/\alpha \approx$  shown in the title ( $\epsilon_{\text{UV}} = \epsilon_{\text{IR}} = 0.022$ ).

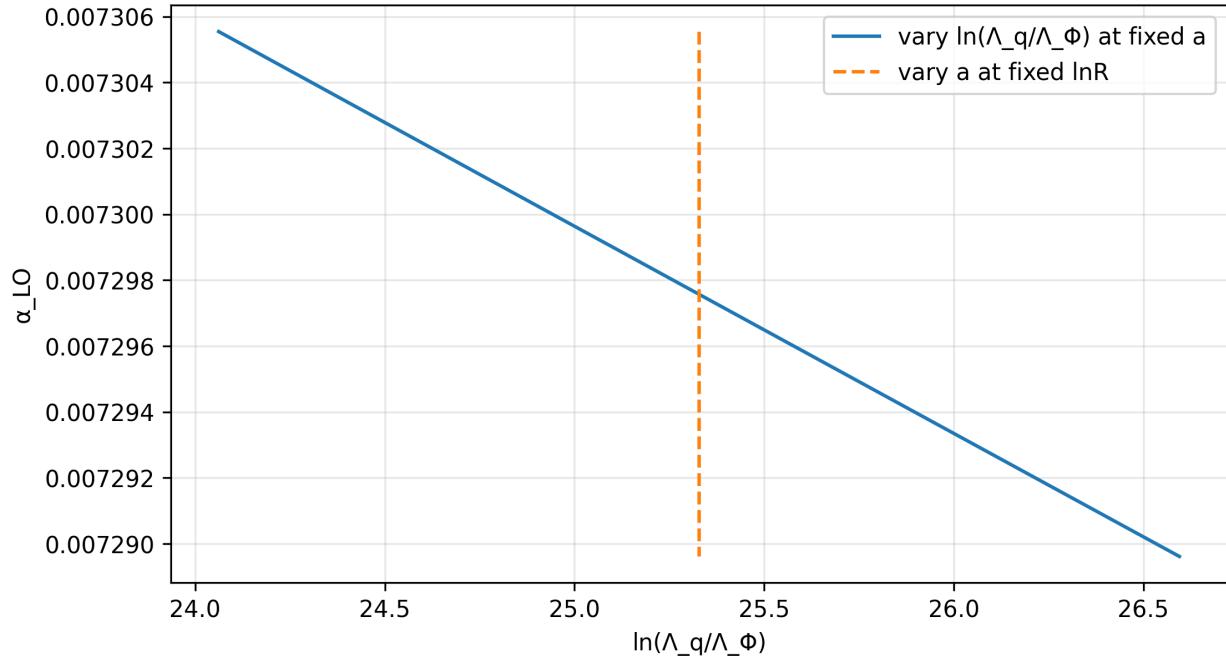


Figure 20: Leading-order  $\alpha$  sensitivity around the fiducial point. Blue: variation with  $\ln(\Lambda_q/\Lambda_\Phi)$  at fixed  $a$ ; orange: variation with  $a$  at fixed  $\ln(\Lambda_q/\Lambda_\Phi)$ . Analytic slopes from the EFT are  $\partial \ln \alpha / \partial \ln(\Lambda_q/\Lambda_\Phi) = -8.6 \times 10^{-4}$  and  $\partial \ln \alpha / \partial a = -2.50 \times 10^{-2}$ .