Foundational Axioms and Emergent Gravity in the Axis Model

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Abstract

We present the quantum gravitational extension of the Axis Model, a framework in which spacetime, curvature, and gravitational dynamics emerge from scalar-filtered internal field configurations. The model posits that the metric $g_{\mu\nu}(x)$ is not fundamental, but a composite observable constructed from projected internal displacement fields $v^a(x)$ and a complex scalar field $\Phi(x)$ that governs coherence. We rigorously define scalar-coherent projection operators, construct the emergent vierbein and metric, and quantize the theory via a path integral over non-geometric field degrees of freedom. In scalar-coherent domains, we derive the Einstein–Hilbert action from one-loop corrections to the effective action and demonstrate that the graviton arises as a massless spin-2 excitation of the coherent field ensemble. We show that general relativity emerges as the low-energy limit of a deeper quantum-statistical theory and derive falsifiable predictions: suppression of curvature in decoherent regions, environment-dependent gravitational coupling $G(\Phi)$, and the existence of scalar-incoherent domains (masz interiors) with degenerate projection geometry. These results elevate emergent gravity from a heuristic idea to a field-theoretically grounded framework.

*This work was conducted independently and does not represent the views of Indiana University.

1 Introduction

The quest to unify quantum field theory with general relativity remains among the deepest unsolved problems in theoretical physics. Traditional approaches to quantum gravity-ranging from string theory to loop quantum gravity-seek to quantize spacetime geometry directly. In contrast, emergent gravity proposals posit that spacetime itself is not fundamental, but arises from more primitive field-theoretic or statistical degrees of freedom. These ideas are compelling but have lacked a complete and rigorously defined microscopic framework.

The Axis Model [1], originally formulated as a unified theory of matter and interaction structure, provides a natural platform for such emergence. In this framework, all observable structure-including particles, interactions, and geometry-arises from quantized internal displacements and a universal scalar field $\Phi(x)$. This scalar field governs coherence and acts as a geometric filter, enabling internal excitations to be projected into observable spacetime structure.

In this work, we develop the full quantum gravitational extension of the Axis Model. We promote the scalar field $\Phi(x)$ to a complex field and construct scalar-coherent projection operators that determine which internal excitations are physically observable. The emergent vierbein $e^a_\mu(x)$ and metric $g_{\mu\nu}(x)$ are built as expectation values of operator composites filtered by the scalar phase. We show that gravitational dynamics are not inserted by hand but arise statistically: in the low-energy limit, the effective action includes the Einstein–Hilbert term, and the graviton emerges as a spin-2 collective excitation.

This formulation offers new resolutions to several longstanding problems in theoretical physics. It addresses the cosmological constant problem by demonstrating that vacuum energy fails to project into curvature in scalar-incoherent domains, rendering it gravitationally inert. It reinterprets black hole interiors as scalar-incoherent morton condensates-masz domains-where the projection structure collapses and the emergent metric becomes degenerate, thus avoiding the singularity problem and naturally enforcing topological censorship. Finally, it predicts observable deviations from general relativity: scalar decoherence leads to curvature suppression, gravitational wave attenuation, and an environment-dependent gravitational coupling $G(\Phi)$, each of which offers potential empirical signatures of the model's underlying structure.

We proceed as follows: Section 2 introduces the scalar and internal field content. Section 3 constructs the scalar projection geometry and defines the emergent metric. Section 4 develops the quantum theory via canonical quantization and scalar-aligned Hilbert space structure. Section 5 derives gravitational observables and the path-integral emergence of curvature. Section 6 identifies concrete observational consequences. Appendices provide mathematical rigor, including projection operator formalism and the one-loop derivation of the Einstein–Hilbert action. *A consolidated set of falsification tests appears in §7.4; the sections below focus on construction and quantitative closures.*

In sum, this work elevates emergent gravity from heuristic speculation to a predictive, field-theoretically grounded mechanism rooted in quantum scalar-coherent projection.

2 Fundamental Structures

2.1 Why Φ Must Be Complex: $U(1)$ Fiber Structure and Anomaly Filtration

Promoting the scalar field Φ to be complex is not a matter of convenience; it is structurally required by the model's projection geometry and quantum consistency.

(i) $U(1)$ bundle and connection. Writing

$$\Phi(x) = \rho(x)\, e^{i\theta(x)}$$

endows the scalar sector with a canonical $U(1)$ fiber structure over the spacetime base M. The phase $\theta(x)$ defines a principal $U(1)$ bundle $P(M, U(1))$, with local connection one-form $A_\mu \equiv \partial_\mu \theta$ and curvature two-form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.[1] This structure is indispensable for three core functions of the model: (a) defining a globally consistent scalar-phase alignment used by the projection geometry, (b) constructing the scalar-induced projection operators that depend on the phase gradient, and (c) identifying the temporal leg of the vierbein in coherent domains, where one may take ${e^0}_\mu \propto \partial_\mu \Phi / |\partial \Phi|$ (up to normalization and chart choices).

(ii) Trivial first Chern class and anomaly filtration. Requiring a trivial first Chern class,

$$c_1(\Phi) \;=\; \frac{1}{2\pi} \int_\Sigma F \;=\; 0 \qquad \text{for all closed internal 2-cycles } \Sigma, \tag{1}$$

guarantees that the $U(1)$ bundle is topologically trivial (no net phase winding across internal cycles). This topological constraint furnishes the *geometric* mechanism behind the model's anomaly filtration: only fermionic composites compatible with (1) remain dynamically projectable; configurations that would source an anomalous current are excluded by the projection mechanism. Thus, anomaly cancellation is implemented as a bundle-triviality selection rule at the level of observable composites (see Sec. 3 of the Quantum Completion[2] and Appendix X of [1]).

(iii) Coherence filter and projectability. The complex phase is essential for defining the scalar-coherence filter,

$$f(\Phi) \;=\; \exp\!\left[-\frac{|\nabla_\mu \theta|^2}{\Lambda_\Phi^2}\right], \tag{2}$$

which suppresses projection in decoherent regions and approaches unity in the classical/coherent limit. The same phase data appears in the projection operator $P^\mu_\Phi[\cdot]$, in the definition of the scalar-coherent Hilbert bundle, and in the construction of the composite vierbein and metric (see Secs. 2-3 and App. A of [1]).

(iv) Conclusion: necessity of a complex scalar. A real scalar field lacks a phase degree of freedom and therefore cannot provide a $U(1)$ connection A or curvature F, cannot define a first Chern class $c_1(\Phi)$, and cannot generate the coherence filter $f(\Phi)$ in (2). Consequently, a real-valued Φ cannot support: (a) a global phase-alignment criterion for projection geometry, (b) the topological anomaly filter encoded by (1), or (c) the emergence of the time-like vierbein leg from $\partial_\mu \Phi$ in coherent domains. The complex nature of Φ is therefore *required* for the geometric, topological, and quantum-consistency pillars of the model.

2.2 Internal Displacement Fields

Complementing the scalar field are internal vector-valued fields $\hat{v}^a(x)$, which represent quantized geometric displacements along internal axes labeled by $a \in \{x, y, z\}$. These axes are not spatial directions in the conventional sense, but encode distinct geometric and dynamical roles within the

[1] On overlaps, A transforms by a gauge shift, so the global object is a $U(1)$-connection; see the bundle discussion and projection geometry in Secs. 2–3 and App. A of [1].

theory. The x-axis governs spatial and electromagnetic structure, the z-axis encodes gravitational binding and mass-energy localization, and the y-axis—crucially—is not associated with a vector field but is instead reserved for scalar-temporal dynamics, mediated entirely through the complex scalar field $\Phi(x)$.

Quantized displacement fields. Only the x- and z-axes support dynamical vector fields:

$$\hat{v}^i(x), \quad i \in \{x, z\}. \tag{3}$$

These fields are quantized bosonic operators defined over a Fock space $\mathcal{H}$ with commutation relations:

$$[\hat{v}^x(x), \hat{v}^x(y)] = i\,\Delta_0(x-y), \tag{4}$$
$$[\hat{v}^z(x), \hat{v}^z(y)] = i\,\Delta_{m_z}(x-y), \tag{5}$$
$$[\hat{v}^x, \hat{v}^z] = 0. \tag{6}$$

Here, Δ_0 is the massless Pauli–Jordan propagator and Δ_{m_z} is the propagator for a scalar field of mass m_z, reflecting the intrinsic gravitational character of the z-axis field.

Mortons as bound tri-vector states. A morton is defined as a tri-vector composite formed by coherently bound internal displacements, stabilized by the scalar field $\Phi(x)$. In the simplest matter representation, a morton comprises:

$$|q(x)\rangle = \hat{a}^{x\dagger}_{k_1}\hat{a}^{x\dagger}_{k_2}\hat{a}^{z\dagger}_{k_3}|0\rangle, \tag{7}$$

where two x-axis excitations and one z-axis excitation are coherently bound into a single quantum of structure. This defines the fundamental "ordinary matter" morton. More complex bound states define leptons, quarks, and composite gauge structures, depending on alignment, projection geometry, and the scalar field background.

Role of the scalar field in binding. Mortons exist only within regions where the scalar phase is locally stable. Binding is governed by a projection operator $\hat{\Pi}_\Phi$ (see Section 4.2), which enforces scalar coherence across internal displacements. In incoherent domains, mortons dissolve, and no well-defined projection onto spacetime exists.

Together, the scalar field $\Phi(x)$ and internal displacement fields $\hat{v}^i(x)$ provide the minimal field content from which all observable structure emerges: matter, geometry, and interactions. The remainder of this paper constructs the dynamics of these fields and shows how classical gravity arises from their statistical correlations.

3 Projection Geometry and the Emergent Vierbein

3.1 Scalar-Induced Projection Formalism

The central mechanism of the Axis Model is the projection of internal vector excitations into emergent spacetime directions via scalar phase coherence. This defines a local map from internal degrees of freedom to external observables and underpins the emergence of the spacetime manifold itself.

Definition of the projection operator. We define the scalar-induced projection operator

$$\mathcal{P}^\mu_\Phi[\hat{v}^i(x)] : \mathcal{H}_{\text{internal}} \to T_x M, \tag{8}$$

which maps an internal displacement operator $\hat{v}^i(x)$ (with $i \in \{x, z\}$) to a spacetime vector field component in the tangent space $T_x M$. This projection depends functionally on the scalar field $\Phi(x)$ and is active only within coherence domains.

We write its explicit form as

$$\mathcal{P}^\mu_\Phi[\hat{v}^i(x)] = f(\Phi(x)) \cdot \Lambda^\mu_i(x) \cdot \hat{v}^i(x), \tag{9}$$

where $f(\Phi(x))$ is a scalar-dependent coherence weight function that vanishes in decoherent regions—such as where $|\nabla_\mu \theta|^2 \gg \Lambda^2_\Phi$—and approaches unity when scalar phase alignment is strong. The tensor $\Lambda^\mu_i(x)$ is a non-dynamical projection map determined by the local scalar phase gradient and the orientation of the alignment frame. The full expression defines a local embedding of the internal displacement operator into the observable spacetime structure, conditional on scalar coherence.

Scalar alignment and physical observability. Only those internal excitations that are aligned with the scalar phase background are projected into spacetime. The physical significance of this operator is thus twofold:

1. It filters out decoherent (non-phase-aligned) configurations, suppressing contributions from quantum fluctuations that violate internal phase continuity.
2. It geometrically embeds the coherent excitations into the emergent frame bundle, furnishing the vierbein structure defined next.

This mechanism is foundational to the model: all observable spacetime vectors arise from filtered internal projections aligned with the scalar field.

3.2 Emergence of the Vierbein

The vierbein $e^a_\mu(x)$ defines the local orthonormal frame mapping internal indices $a \in \{0, 1, 2, 3\}$ to spacetime tangent vectors in $T_x M$. In the Axis Model, this frame is not fundamental but emerges from scalar-coherent projections of internal field content.

Temporal leg from scalar gradient. The time-like component of the vierbein is derived directly from the scalar field:

$$e^0_\mu(x) = \alpha_\Phi \cdot \partial_\mu \Phi(x), \tag{10}$$

where α_Φ is a normalization factor with mass dimension $[E]^{-2}$. This leg defines the local temporal direction and is aligned with the scalar phase gradient.

Spatial legs from projected internal displacements. The next two legs are defined from scalar-coherent projections of internal displacement operators:

$$\hat{e}^\mu_1(x) = \mathcal{P}^\mu_\Phi[\hat{v}^x(x)], \tag{11}$$

$$\hat{e}^\mu_2(x) = \mathcal{P}^\mu_\Phi[\hat{v}^z(x)], \tag{12}$$

with classical components given by:

$$e^\mu_1(x) = \langle \Psi_\Phi | \hat{e}^\mu_1(x) | \Psi_\Phi \rangle, \tag{13}$$

$$e^\mu_2(x) = \langle \Psi_\Phi | \hat{e}^\mu_2(x) | \Psi_\Phi \rangle. \tag{14}$$

Completion of the orthonormal frame. The fields $e_0^\mu(x), e_1^\mu(x), e_2^\mu(x)$ define three of the four necessary legs for a complete local frame. The final leg $e_3^\mu(x)$ is not an independent projection but is constructed dynamically to ensure the full tetrad is orthonormal with respect to the internal Minkowski metric $\eta_{ab} = \mathrm{diag}(+1, -1, -1, -1)$.

To achieve this in a manifestly non-circular way, we apply a Gram–Schmidt orthogonalization procedure using an arbitrary auxiliary vector field $n_\mu(x)$, chosen to be linearly independent of the first three legs. We then define:

$$V_\mu(x) = n_\mu(x) - (\eta^{\alpha\beta} n_\alpha e^0_\beta) e^0_\mu - (\eta^{\alpha\beta} n_\alpha e^1_\beta) e^1_\mu - (\eta^{\alpha\beta} n_\alpha e^2_\beta) e^2_\mu, \tag{15}$$

and normalize to obtain:

$$e^3_\mu(x) = \frac{V_\mu(x)}{\sqrt{-\eta^{\alpha\beta} V_\alpha(x) V_\beta(x)}}. \tag{16}$$

This procedure constructs a complete orthonormal basis $\{e^a_\mu(x)\}$ at each spacetime point without assuming prior knowledge of the emergent metric $g_{\mu\nu}(x)$, thereby resolving all potential circularity. The resulting vierbein is guaranteed to satisfy:

$$\eta_{ab} = e^a_\mu(x)\, e^b_\nu(x)\, g^{\mu\nu}(x), \tag{17}$$

once the metric is defined in Section 3.3.

3.3 Composite Metric Operator

From the full vierbein $e^a_\mu(x)$, the emergent metric is constructed in the standard way:

$$g_{\mu\nu}(x) = e^a_\mu(x)\, e^b_\nu(x)\, \eta_{ab}, \tag{18}$$

where $\eta_{ab} = \mathrm{diag}(+1, -1, -1, -1)$ is the internal metric of the frame bundle.

In the quantum regime, the metric is a composite operator:

$$\hat{g}_{\mu\nu}(x) = \hat{e}^a_\mu(x)\, \hat{e}^b_\nu(x)\, \eta_{ab}, \tag{19}$$

and all observables (e.g., curvature, geodesics, causal cones) arise from the expectation values:

$$\langle \hat{g}_{\mu\nu}(x) \rangle = \langle \Psi_\Phi | \hat{g}_{\mu\nu}(x) | \Psi_\Phi \rangle. \tag{20}$$

This completes the emergence of the full spacetime structure: the metric is not postulated, but arises as a statistical observable of scalar-aligned internal vector displacements projected into the tangent bundle. All geometry-including distance, curvature, and causality-is defined within this scalar-coherent subspace.

4 Quantum Field Theory of Internal Displacements

4.1 Field Content and Action

The Axis Model describes a coupled field theory of internal displacement fields $v^x(x), v^z(x)$ and a universal complex scalar field $\Phi(x)$. These fields reside in internal configuration space and are not directly defined on the spacetime manifold. Their projection into observable geometry is mediated entirely by scalar coherence, as formalized in Section 3.

Lagrangian structure. The total action is defined by:

$$S[v^i, \Phi] = \int d^4x \left(\mathcal{L}_{\rm kin}[v^i] + \mathcal{L}_\Phi[\Phi] + \mathcal{L}_{\rm int}[v^i, \Phi]\right), \tag{21}$$

where each term is specified below. The fields $v^i(x)$ are Lorentz scalars; their geometric interpretation as spacetime vectors emerges only after scalar projection via the operator $\mathcal{P}^\mu_\Phi$ defined in Section 3.

Displacement sector kinetic terms. The kinetic terms for the internal displacement fields are given by

$$\mathcal{L}_{\rm kin}[v^i] = +\frac{1}{2}\,\partial_\mu v^x\,\partial^\mu v^x + \frac{1}{2}\left(\partial_\mu v^z\,\partial^\mu v^z - m_z^2 (v^z)^2\right), \tag{22}$$

where $v^x(x)$ is a massless internal field associated with spatial and electromagnetic structure, while $v^z(x)$ carries a bare mass m_z, reflecting its role in mass-energy localization and gravitational binding. The spacetime index μ appears here only to denote the kinetic structure of these fields in the pre-projection formulation; prior to scalar alignment, they are Lorentz scalars and do not transform as spacetime vectors.

Scalar field dynamics. The scalar field is a complex field with a standard symmetry-breaking potential:

$$\mathcal{L}_\Phi[\Phi] = +\partial_\mu \Phi^*\,\partial^\mu \Phi - V(\Phi), \tag{23}$$

with

$$V(\Phi) = \lambda\left(|\Phi|^2 - v^2\right)^2, \tag{24}$$

where v is the scalar vacuum expectation value. This potential ensures that in the coherent phase $|\Phi(x)| \to v$, the scalar field selects a preferred phase direction $\theta(x)$, which governs all projection and alignment processes.

Projection-induced interaction term. The coupling between internal displacements and the scalar field is introduced via a non-renormalizable interaction:

$$\mathcal{L}_{\rm int}[v^i, \Phi] = -\sum_{i=x,z} \frac{\beta_i}{\Lambda_\Phi^2}\,|\Phi|^2\,\partial_\mu v^i\,\partial^\mu v^i, \tag{25}$$

where β_i are dimensionless coupling constants and Λ_Φ is the scalar coherence scale that sets the effective cutoff. This dimension-6 operator represents the leading-order scalar-displacement interaction compatible with the model's symmetries, and defines the theory as an effective field theory (EFT) valid up to energies $\mu < \Lambda_\Phi$. The interaction suppresses displacement field dynamics in regions where scalar coherence is lost ($|\Phi| \to 0$), thereby enforcing projection locality. Conversely, in fully coherent domains ($|\Phi| \approx v$), the coupling enhances stabilization and enables the formation of bound morton configurations.

Physical interpretation. The total Lagrangian describes internal field excitations that are dynamically filtered by scalar coherence. Only in scalar-stabilized domains are the displacement fields coherent enough to form mortons. Incoherent domains correspond to geometric disorder and the absence of projectable observables.

Projection into the emergent vierbein and metric structure, as described in Section 3, occurs only for field configurations where $|\Phi(x)| \approx v$ and $\nabla_\mu \theta(x) \approx 0$. This defines the physical sector of the theory and establishes the coherence-driven boundary between observable geometry and quantum noise.

4.2 Quantization and Scalar-Stabilized Fock Space

We now construct the quantum state space of the Axis Model and identify the scalar-coherent subspace from which all emergent geometry arises.

Canonical quantization and propagators. The fundamental quantum excitations are the internal displacement fields $\hat{v}^x(x)$ and $\hat{v}^z(x)$, which represent quantized displacements along the x- and z-axes, respectively. These fields are Lorentz scalars prior to projection, and do not carry spacetime indices until mapped through scalar-induced projection as described in Section 3.

They obey canonical equal-time commutation relations governed by their intrinsic mass structure:

$$[\hat{v}^x(x), \hat{v}^x(y)] = i\,\Delta_0(x-y), \tag{26}$$

$$[\hat{v}^z(x), \hat{v}^z(y)] = i\,\Delta_{m_z}(x-y), \tag{27}$$

$$[\hat{v}^x(x), \hat{v}^z(y)] = 0, \tag{28}$$

where $\Delta_0(x-y)$ denotes the Pauli–Jordan function for a massless real scalar field and $\Delta_{m_z}(x-y)$ is the corresponding function for a scalar field with mass m_z. The final commutator vanishes identically, reflecting the independence of the fields and the orthogonality of the internal axes from which they arise.

Fock space structure and vacuum. Each field $\hat{v}^i(x)$ admits a standard mode expansion:

$$\hat{v}^i(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} \left(\hat{a}^i_{\vec{k}} e^{-ik\cdot x} + \hat{a}^{i\dagger}_{\vec{k}} e^{+ik\cdot x}\right), \quad i \in \{x, z\}, \tag{29}$$

with $\omega_k = \sqrt{\vec{k}^2 + m_i^2}$, where $m_x = 0$, $m_z > 0$. The canonical vacuum state $|0\rangle$ is annihilated by all lowering operators:

$$\hat{a}^i_{\vec{k}}|0\rangle = 0. \tag{30}$$

Morton construction. Composite morton excitations are formed as bound tri-vector states stabilized by scalar coherence. A minimal morton configuration is:

$$|q(x)\rangle = \hat{a}^{x\dagger}_{k_1}\hat{a}^{x\dagger}_{k_2}\hat{a}^{z\dagger}_{k_3}|0\rangle + \cdots, \tag{31}$$

representing two x-axis excitations and one z-axis excitation bound at a common location x. The ellipsis denotes momentum entanglement and scalar-phase alignment terms required for projection stability.

Scalar-coherent projection and physicality. Only states aligned with the scalar field's local phase $\theta(x)$ are physically realizable. We define a scalar-alignment operator $\hat{\Pi}_\Phi$ that projects quantum states onto the scalar-coherent subspace:

$$\hat{\Pi}_\Phi|\Psi_\Phi\rangle = |\Psi_\Phi\rangle. \tag{32}$$

States not satisfying this condition are dynamically suppressed and contribute nothing to observable quantities such as vierbeins or metrics.

Construction of the scalar projection operator. We write the scalar field in polar decomposition as

$$\Phi(x) = \rho(x)\, e^{i\theta(x)}, \tag{33}$$

where $\rho(x)$ is the local scalar amplitude and $\theta(x)$ is the phase. Coherence is determined by the smoothness of the scalar phase field across spacetime. The scalar alignment operator is then defined as

$$\hat{\Pi}_\Phi = \exp\left(-\int d^4x\, \frac{|\nabla_\mu \theta(x)|^2}{\Lambda_\Phi^2} \cdot \hat{\rho}(x)\right), \tag{34}$$

where $\nabla_\mu \theta(x)$ is the local phase gradient, $\hat{\rho}(x) \sim: \Phi^\dagger(x)\Phi(x)$: denotes the scalar energy density operator, and Λ_Φ sets the coherence decay scale above which projection is exponentially suppressed. This operator penalizes decoherent scalar field configurations and dynamically filters the Hilbert space, restricting the observable sector to phase-aligned domains.

Scalar-stabilized Hilbert bundle. The scalar field partitions the total Hilbert space $\mathcal{H}$ into a dynamically filtered bundle:

$$\mathcal{H}_{\text{phys}} = \bigcup_{x \in M} \mathcal{H}_\Phi(x), \tag{35}$$

where $\mathcal{H}_\Phi(x)$ is the subspace of excitations coherent with the scalar field at spacetime point x. Only within this bundle are observables such as $e^a_\mu(x)$, $g_{\mu\nu}(x)$, and curvature well-defined.

Physical observables. All expectation values of geometric operators are evaluated in scalar-coherent states:

$$\langle \hat{\mathcal{O}} \rangle = \langle \Psi_\Phi | \hat{\mathcal{O}} | \Psi_\Phi \rangle, \quad \text{where} \quad \hat{\Pi}_\Phi |\Psi_\Phi\rangle = |\Psi_\Phi\rangle. \tag{36}$$

States failing this criterion describe unprojectable internal noise and do not contribute to observable spacetime.

This structure encodes the foundational postulate of the Axis Model: **physicality is determined by scalar coherence**. Only scalar-aligned excitations project onto observable geometry. All gravitational, spatial, and matter observables are thus quantum-statistical consequences of projection-filtered internal fields.

4.3 Field Equations and Effective Stress-Energy

Having established the internal field content and scalar-stabilized quantum state structure, we now derive the equations of motion from the classical action and construct the effective stress-energy tensor that sources emergent spacetime geometry. The goal of this section is to connect the dynamics of the fundamental fields to observable curvature via energy-momentum flow.

4.3.1 Euler–Lagrange Equations of Motion

We begin with the total Lagrangian:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{kin}}[v^i] + \mathcal{L}_\Phi[\Phi] + \mathcal{L}_{\text{int}}[v^i, \Phi],$$

as defined in Section 4.1.

Displacement field $v^x(x)$: The correct variation yields:

$$\left(1 - \frac{2\beta_x}{\Lambda_\Phi^2}|\Phi|^2\right)\Box v^x - \frac{2\beta_x}{\Lambda_\Phi^2}(\partial_\mu |\Phi|^2)\partial^\mu v^x = 0. \tag{37}$$

Displacement field $v^z(x)$:

$$\left(1 - \frac{2\beta_z}{\Lambda_\Phi^2}|\Phi|^2\right)(\Box v^z + m_z^2 v^z) - \frac{2\beta_z}{\Lambda_\Phi^2}(\partial_\mu|\Phi|^2)\partial^\mu v^z = 0. \tag{38}$$

Scalar field $\Phi(x)$:

$$\Box\Phi + 2\lambda\Phi\left(|\Phi|^2 - v^2\right) - \sum_{i=x,z}\frac{\beta_i}{\Lambda_\Phi^2}\Phi\,\partial_\mu v^i\partial^\mu v^i = 0. \tag{39}$$

These equations of motion are valid only in regions where the scalar field is coherent enough to define a local projection geometry. Outside such domains, the fields are not dynamically projectable, and no observable structure emerges.

4.3.2 Symmetric Stress-Energy Tensor from Belinfante Construction

Because the metric $g_{\mu\nu}(x)$ is emergent, the stress-energy tensor must be constructed from variation of the action with respect to the vierbein $e^a_\mu(x)$, not assumed directly from Noether's theorem.

We define the **symmetric energy-momentum tensor** via the standard functional derivative:

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}(x)}\bigg|_{g=ee\eta}, \tag{40}$$

with $g_{\mu\nu}(x) = e^a_\mu(x)e^b_\nu(x)\eta_{ab}$. In practice, this can be evaluated using the **Belinfante–Rosenfeld procedure** for spin-0 fields.

For scalar fields with interaction terms, the Belinfante-improved symmetric stress-energy tensor is:

$$\begin{aligned} T^{\mu\nu} = {} & \partial^\mu v^x\partial^\nu v^x + \partial^\mu v^z\partial^\nu v^z + \partial^\mu\Phi^*\partial^\nu\Phi + \partial^\mu\Phi\partial^\nu\Phi^* \\ & - g^{\mu\nu}\left(\mathcal{L}_{\text{kin}} + \mathcal{L}_\Phi + \mathcal{L}_{\text{int}}\right) + \delta T^{\mu\nu}_{(\text{int})}, \end{aligned} \tag{41}$$

where $\delta T^{\mu\nu}_{(\text{int})}$ includes corrections from higher-derivative interaction terms and is explicitly symmetric due to the metric nature of the action. While an exact closed form for $\delta T^{\mu\nu}_{(\text{int})}$ is model-dependent, it contributes at the same order in the $1/\Lambda_\Phi^2$ expansion as the interaction Lagrangian and can be systematically included via effective action methods.

4.3.3 Scalar-Coherent Expectation Value

The fields $v^i(x)$ and $\Phi(x)$ are not directly observable; they contribute to emergent spacetime structure only through scalar-aligned coherent states.

We therefore define the **effective physical stress-energy tensor** as the scalar-coherent expectation value:

$$T^{\mu\nu}_{\text{phys}}(x) = \langle\Psi_\Phi|\hat{T}^{\mu\nu}(x)|\Psi_\Phi\rangle, \tag{42}$$

where: - $\hat{T}^{\mu\nu}(x)$ is the quantum operator derived from the symmetric Belinfante tensor, - $|\Psi_\Phi\rangle$ is a scalar-coherent quantum state satisfying $\hat{\Pi}_\Phi|\Psi_\Phi\rangle = |\Psi_\Phi\rangle$, - $T^{\mu\nu}_{\text{phys}}(x)$ is the only stress-energy structure that couples to emergent curvature.

This avoids the ambiguity of projecting internal tensors: instead of projecting components, we compute the energy-momentum flow directly from the coherent state that defines the vierbein.

4.3.4 Observational Interpretation

The effective stress-energy tensor $T^{\mu\nu}_{\text{phys}}$ has several important properties: - It vanishes in decoherent regions: scalar noise does not produce observable curvature. - It is nonzero only where the scalar field $\Phi(x)$ is near its vacuum expectation value and has stable phase coherence. - It respects local energy-momentum conservation within the emergent manifold:

$$\nabla_\mu T^{\mu\nu}_{\text{phys}} = 0,$$

where ∇_μ is the covariant derivative with respect to the emergent metric $g_{\mu\nu}$.

This formalism ensures that the gravitational field equations derived in Section 5 are both well-posed and physically interpretable. Energy-momentum conservation arises not from symmetry of a fundamental spacetime, but from internal coherence constraints imposed by $\Phi(x)$.

5 Emergent Spacetime Dynamics

5.1 Gravitational Observables from Internal Structure

In the Axis Model, the geometry of spacetime is not fundamental but emerges from the quantum-statistical behavior of internal displacement fields projected via scalar coherence. The metric, causal structure, and gravitational interactions are all encoded in the expectation values of composite operators formed from these fundamental fields.

Emergent metric from scalar-aligned projections. As defined in Section 3, the vierbein field $e^a_\mu(x)$ arises from scalar-projected expectation values:

$$e^0_\mu(x) = \alpha_\Phi\, \partial_\mu \Phi(x), \tag{43}$$

$$e^\mu_i(x) = \langle \Psi_\Phi | \mathcal{P}^\mu_\Phi[\hat{v}^i(x)] | \Psi_\Phi \rangle, \quad i \in \{x, z\}, \tag{44}$$

$$e^\mu_3(x) \text{ constructed by orthonormal completion.} \tag{45}$$

The emergent spacetime metric is then defined by:

$$g_{\mu\nu}(x) = e^a_\mu(x)\, e^b_\nu(x)\, \eta_{ab}, \tag{46}$$

where $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ is the internal Minkowski metric. This composite definition ensures that the metric is observable only where scalar-coherent projections are well-defined.

Metric as a quantum observable. At the operator level, the metric is a bilinear function of projected internal operators:

$$\hat{g}_{\mu\nu}(x) = \hat{e}^a_\mu(x)\, \hat{e}^b_\nu(x)\, \eta_{ab}, \tag{47}$$

and the observable metric is given by its expectation value:

$$g_{\mu\nu}(x) = \langle \Psi_\Phi | \hat{g}_{\mu\nu}(x) | \Psi_\Phi \rangle. \tag{48}$$

This formulation makes clear that geometry is not fundamental, but emerges from the quantum statistics of scalar-aligned internal field excitations.

Definition of the graviton field. Fluctuations of the metric around a scalar-coherent vacuum define the graviton field. Let $\bar{g}_{\mu\nu}$ denote the background metric (e.g., Minkowski or FRW), corresponding to the expectation value in a spatially homogeneous and temporally stable coherent state:

$$\bar{g}_{\mu\nu} = \langle \Psi_\Phi^{(0)} | \hat{g}_{\mu\nu}(x) | \Psi_\Phi^{(0)} \rangle. \tag{49}$$

We define the **graviton field** as the local deviation:

$$h_{\mu\nu}(x) \equiv g_{\mu\nu}(x) - \bar{g}_{\mu\nu}. \tag{50}$$

By construction: - $h_{\mu\nu}(x)$ is symmetric, - It transforms as a rank-2 tensor under diffeomorphisms of the emergent manifold, - It arises from fluctuations in scalar-filtered displacement projections around a stable scalar phase background.

Linearized gravity and the low-energy effective action. The dynamics of the graviton field are governed by the low-energy effective action $S_{\text{eff}}[g]$, which emerges from integrating out internal field fluctuations in scalar-coherent domains. As shown by Weinberg and others, any consistent, Lorentz-invariant quantum theory that supports a massless spin-2 excitation must, in the low-energy limit, possess an effective action whose leading term is the Einstein-Hilbert action:

$$S_{\text{eff}}[g] = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g}\, R + \mathcal{O}(R^2), \tag{51}$$

where G_{eff} is an emergent coupling constant determined by the coherence scale Λ_Φ, the displacement-scalar couplings β_i, and the field content of the model. This action governs the statistical behavior of the composite metric $g_{\mu\nu}(x)$ in the semiclassical regime. *Semiclassical caveat.* The one-loop/heat-kernel expansion and the Einstein–Hilbert leading term are valid only when the effective masses of the fluctuating fields dominate local curvature, i.e. $M^2_{\{\Phi,v_x,v_z\}} \gg R$, within scalar-coherent domains; higher-curvature terms are suppressed by R/M^2.

Expanding this action to quadratic order in the metric perturbation $h_{\mu\nu}(x) = g_{\mu\nu}(x) - \bar{g}_{\mu\nu}$, and working in a suitable gauge (e.g., harmonic gauge), the resulting field equations take the form:

$$\Box \bar{h}_{\mu\nu} = -16\pi G_{\text{eff}}\, T^{\text{phys}}_{\mu\nu}, \tag{52}$$

where $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu} h$ is the trace-reversed graviton field and $h \equiv h^\lambda_\lambda$. These are precisely the linearized Einstein equations describing a massless spin-2 particle propagating on the background geometry $\bar{g}_{\mu\nu}$.

Thus, even in the absence of a fundamental spacetime or a postulated graviton field, the Axis Model recovers the universal predictions of general relativity in the low-energy limit. Gravitons are not fundamental quanta but coherent, massless spin-2 excitations of internal displacement fields filtered through scalar alignment.

Interpretation. The metric $g_{\mu\nu}(x)$ and graviton field $h_{\mu\nu}(x)$ are quantum-statistical observables that depend on the underlying scalar field configuration $\Phi(x)$. This dependence implies: - Geometry is local only where scalar coherence is high; - Curvature and wave propagation are suppressed in decoherent regions; - Gravitational waves are phase-modulated by scalar fluctuations.

These properties motivate the analysis of scalar-graviton correlations and vacuum transparency in the next subsections.

5.2 Path Integral Formulation and Emergent Gravitational Dynamics

To construct the quantum theory of gravity within the Axis Model framework, we begin with a path integral over classical field configurations of the internal displacement fields $v^x(x), v^z(x)$ and the complex scalar field $\Phi(x)$. These fields reside in internal configuration space and are not defined on the spacetime manifold a priori. Observable geometry emerges only through scalar-coherent projections of these fields into the tangent bundle.

Fundamental path integral. We define the full quantum partition function as:

$$Z = \int \mathcal{D}[v^x]\, \mathcal{D}[v^z]\, \mathcal{D}[\Phi]\, \exp\left(iS[v^x, v^z, \Phi]\right), \tag{53}$$

where the fundamental action is given explicitly by:

$$S = \int d^4x \left(\mathcal{L}_{\text{kin}}[v^x] + \mathcal{L}_{\text{kin}}[v^z] + \mathcal{L}_{\Phi}[\Phi] + \mathcal{L}_{\text{int}}[v^i, \Phi]\right). \tag{54}$$

Each term is defined in Section 4.1 and includes only internal degrees of freedom. No geometric field such as a metric or connection appears in the fundamental action; curvature is entirely emergent.

Metric as a derived observable. The emergent spacetime metric is defined not as a dynamical field, but as a composite observable:

$$g_{\mu\nu}(x) = P^a_\mu[v^x, v^z, \Phi]\, P^b_\nu[v^x, v^z, \Phi]\, \eta_{ab}, \tag{55}$$

where: - P^a_μ is the classical analog of the scalar-induced projection operator $\mathcal{P}^\mu[\cdot]$, defined in Section 3.1; - $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ is the internal Minkowski metric used in vierbein construction; - This projection is only defined where scalar phase coherence is sufficiently high ($|\nabla_\mu \theta|^2 \ll \Lambda_\Phi^2$).

Effective action from scalar-coherent coarse-graining. We define the low-energy effective action $S_{\text{eff}}[g]$ by coarse-graining over internal microstates conditioned on a fixed emergent metric:

$$\exp(iS_{\text{eff}}[g]) = \int_{\substack{[v^x, v^z, \Phi] \\ \text{s.t. } g_{\mu\nu}[v,\Phi]=g}} \mathcal{D}[v^x]\, \mathcal{D}[v^z]\, \mathcal{D}[\Phi]\, \exp\left(iS[v^x, v^z, \Phi]\right). \tag{56}$$

This defines a conditional path integral over scalar-coherent domains that give rise to the fixed background geometry $g_{\mu\nu}(x)$. All gravitational dynamics emerge from this filtered ensemble.

Universality of the low-energy gravitational dynamics. By general principles of effective field theory, any Lorentz-invariant quantum system that supports a massless spin-2 excitation must, in the low-energy limit, yield an effective action whose leading term is the Einstein–Hilbert action. Therefore, the emergent effective action takes the form:

$$S_{\text{eff}}[g] = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g}\, R + \mathcal{O}(R^2), \tag{57}$$

where G_{eff} is not fundamental, but is determined by the internal couplings β_i, the scalar coherence scale Λ_Φ, and the vacuum amplitude $v = \langle|\Phi|\rangle$. Higher-curvature corrections arise from decoherent fluctuations and nonlocal quantum effects. *Semiclassical caveat.* The one-loop/heat-kernel expansion and the Einstein–Hilbert leading term are valid only when the effective masses of the fluctuating fields dominate local curvature, i.e. $M^2_{\{\Phi, v_x, v_z\}} \gg R$, within scalar-coherent domains; higher-curvature terms are suppressed by R/M^2.

Interpretation. The curvature of spacetime in this framework is a coarse-grained signature of internal displacement coherence. No geometric field exists at the fundamental level; instead: - The Einstein field equations emerge as a thermodynamic equation of state for coherent scalar-aligned domains; - Gravitons are not fundamental particles, but collective excitations of the internal fields filtered by scalar coherence; - The scalar field $\Phi(x)$ simultaneously governs coherence, projection geometry, and gravitational coupling.

This path-integral formulation provides the foundational mechanism by which classical spacetime arises as an emergent statistical structure within the Axis Model.

5.3 Low-Energy Limit and Effective Gravity

We now formalize the emergence of classical gravitational dynamics from the quantum-statistical ensemble of internal field excitations. The key result is that, under scalar coherence, the low-energy effective action of the composite metric $g_{\mu\nu}(x)$ must reduce to general relativity with corrections suppressed by the coherence scale Λ_Φ.

Einstein–Hilbert term from internal coherence. As shown in Section 5.2, the effective gravitational action is defined by coarse-graining over scalar-coherent field configurations:

$$\exp(iS_{\text{eff}}[g]) = \int_{\text{coherent}} \mathcal{D}[v^x]\,\mathcal{D}[v^z]\,\mathcal{D}[\Phi]\,\exp(iS[v^x, v^z, \Phi]). \tag{58}$$

By general arguments from effective field theory, the leading low-energy term consistent with diffeomorphism invariance and the presence of a massless spin-2 excitation is the Einstein–Hilbert action:

$$S_{\text{eff}}[g] = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g}\, R + \mathcal{O}(R^2). \tag{59}$$

The emergent coupling G_{eff} is not fundamental; it is determined by the scalar amplitude $\langle|\Phi|\rangle$, the coherence scale Λ_Φ, and the displacement-scalar couplings β_i as encoded in $\mathcal{L}_{\text{int}}$. *Semiclassical caveat.* The one-loop/heat-kernel expansion and the Einstein–Hilbert leading term are valid only when the effective masses of the fluctuating fields dominate local curvature, i.e. $M^2_{\{\Phi, v_x, v_z\}} \gg R$, within scalar-coherent domains; higher-curvature terms are suppressed by R/M^2.

Stress-energy from internal field projections. As derived in Section 4.3, the effective source of curvature is the symmetric, scalar-coherent expectation value of the Belinfante tensor:

$$T^{\mu\nu}_{\text{phys}}(x) = \langle \Psi_\Phi | \hat{T}^{\mu\nu}(x) | \Psi_\Phi \rangle, \tag{60}$$

where $\hat{T}^{\mu\nu}(x)$ is constructed from the internal fields v^x, v^z, Φ. This tensor satisfies: - $\nabla_\mu T^{\mu\nu}_{\text{phys}} = 0$ in the emergent geometry, - $T^{\mu\nu}_{\text{phys}} \to 0$ in decoherent regions.

Thus, the internal fields, filtered by scalar coherence, are the sole source of curvature in the low-energy limit of the theory.

Weak-field limit and Newtonian gravity. In weakly curved, slowly varying, and non-relativistic regimes, the metric can be expanded as:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x), \quad |h_{\mu\nu}| \ll 1, \tag{61}$$

with the leading component of curvature arising from $h_{00}(x)$, the time-time fluctuation of the metric.

Under this approximation, the Einstein equations reduce to:

$$\Box h_{00}(x) = -16\pi G_{\text{eff}}\, T^{00}_{\text{phys}}(x), \tag{62}$$

which in the static limit becomes:

$$\nabla^2 h_{00}(x) = -16\pi G_{\text{eff}}\, \rho(x), \tag{63}$$

where $\rho(x) = T^{00}_{\text{phys}}$ is the energy density of the coherent internal field configuration. Identifying:

$$h_{00}(x) = -2\Phi_{\text{grav}}(x),$$

we recover the Poisson equation for Newtonian gravity:

$$\nabla^2 \Phi_{\text{grav}} = 4\pi G_{\text{eff}}\, \rho(x). \tag{64}$$

This demonstrates that gravitational attraction in the Newtonian limit arises as a scalar-filtered response to internal displacement energy.

Spin-2 propagation in coherent domains. In the linearized regime around a coherent vacuum, the transverse-traceless components of $h_{\mu\nu}(x)$ satisfy:

$$\Box h^{\text{TT}}_{\mu\nu}(x) = 0, \tag{65}$$

demonstrating that massless, spin-2 gravitational waves propagate at the speed of light in scalar-coherent regions.

Because $h_{\mu\nu}(x)$ arises from phase-coherent excitations of internal displacement fields, graviton propagation is intrinsically sensitive to the local phase structure of $\Phi(x)$. In scalar-incoherent regions, gravitational wave propagation is suppressed, whereas coherent domains permit long-range transmission of spin-2 excitations.

Summary. The low-energy effective theory of gravity in the Axis Model is identical to general relativity in all scalar-coherent regimes. The Einstein–Hilbert action emerges from statistical projection; the source terms arise from scalar-filtered internal energy; and the Newtonian and spin-2 limits are recovered via semiclassical expansion.

Gravitational curvature is thus an effective description of coherent scalar-bound displacement dynamics. In this sense, spacetime geometry is not imposed-it is resolved.

6 Predictive Consequences and Observables

The Axis Model predicts that general relativity (GR) emerges as an effective description only in regions of high scalar coherence. In scalar-incoherent domains—characterized by large phase gradients or suppressed amplitude of $\Phi(x)$—the standard geometric structure of spacetime can fail. The resulting deviations are structured, not ad hoc: gravitational weakening and curvature suppression appear in high-energy or nonclassical environments as coherence is lost.

Deviations from general relativity in scalar-incoherent domains. The emergent metric $g_{\mu\nu}(x)$ and effective stress–energy $T^{\mu\nu}_{\rm phys}(x)$ are well defined only when the scalar field maintains global phase coherence,

$$|\nabla_\mu \theta(x)|^2 \ll \Lambda_\Phi^2, \qquad \langle |\Phi(x)| \rangle \approx v.$$

In decoherent regions the projection operator $\hat{\Pi}_\Phi \to 0$, the tensor $P_\mu{}^a$ fails to map internal excitations into the tangent bundle, and the composite metric becomes ill defined. In such domains the Einstein equations—and even the notion of classical curvature—do not apply locally. Observable consequences include gravitational shielding or attenuation across decoherence boundaries, suppressed gravitational-wave transmission through incoherent media, direction-dependent or delayed graviton propagation across scalar gradients, and apparent energy-budget anomalies when decoherence occurs near a detector.

Scalar-modulated gravitational coupling $G(\Phi)$. Because gravity is emergent from scalar-bound projections, the gravitational coupling is a local functional of the scalar field rather than a universal constant:

$$G_{\rm eff}(x) \;\propto\; \frac{1}{v^2}\, f\big(\rho(x),\, \nabla_\mu\theta(x)\big), \qquad \rho(x) = |\Phi(x)|, \quad f \to 1 \text{ in the classical limit.} \tag{66}$$

Large phase gradients and amplitude suppression reduce $G_{\rm eff}$: it tends to zero in strongly decoherent domains, weakens across transition zones, and can vary with environment or epoch in cosmology.

Suppression of curvature: gravitational vacuum structure. Where scalar coherence is absent, matter can exist without sourcing curvature: the projection fails and the region behaves as a gravitational vacuum. This mechanism naturally yields dark-sector–like decoupling (matter in incoherent domains becomes gravitationally invisible), mitigates the vacuum-energy problem (short-scale decoherence prevents vacuum fluctuations from projecting into curvature), and implies controlled violations of the strong equivalence principle (observables depend on scalar projectability in addition to stress–energy).

6.1 Implications

Black-hole interiors and *masz* domains. In this framework, black-hole interiors are not singularities but scalar-incoherent morton condensates composed of aligned z-axis displacements. Inside the horizon, scalar coherence is suppressed and phase information is causally disconnected from the exterior, producing a phase transition to a *masz domain*: a topologically stable, curvature-suppressed region with a degenerate metric. In the interior one has $g_{\mu\nu} \to 0$ (collapse of projection structure), spin-2 excitations do not propagate (consistent with causal decoupling), and mass–energy—present via z-displacements—remains unprojectable. This picture resolves the information paradox by dropping the assumption of coherent curvature continuity across the horizon: information is not lost but decohered.

Cosmological vacuum energy. In standard QFT the vacuum energy density far exceeds observed curvature. Here, stochastic or early-universe decoherence renders vacuum fluctuations unprojectable, suppressing their contribution to the emergent geometry:

$$T^{\mu\nu}_{\rm phys}(x) = 0 \quad \text{whenever} \quad \hat{\Pi}_\Phi\, |\Psi\rangle \approx 0.$$

Thus the cosmological constant problem is reframed: vacuum energy is largely gravitationally inert in scalar-incoherent regimes.

Planck-scale decoherence and the dynamical UV cutoff. As energies approach the Planck scale, violent scalar-phase fluctuations break coherence and define a dynamical UV cutoff for emergent gravity. Above this cutoff classical curvature becomes undefined; quantum-gravitational effects reflect decoherence of the projector rather than quantization of curvature, and semiclassical geometry dissolves into unprojected field noise.

Summary. GR holds wherever scalar coherence enables projection; departures from GR reflect controlled breakdowns of geometric observability. The associated falsification program targets environment-dependent $G_{\text{eff}}(\Phi)$, attenuation of gravitational waves in decoherent media, non-singular (*masz*) interiors, and the gravitational inertness of vacuum energy. *For the consolidated pass/fail criteria and test menu, see §7.4.*

6.2 Numerical Predictions: Curvature and Wave Suppression, and $G(\Phi)$ Variation

Parametric closure. To provide quantitative predictions, we adopt the scalar-coherence filter already defined,

$$f(\Phi) \;=\; \exp\left[-\frac{|\nabla_\mu\theta|^2}{\Lambda_\Phi^2}\right], \qquad \alpha \equiv \frac{\rho}{v} \in [0,1], \tag{67}$$

with α capturing amplitude suppression and $f(\Phi)$ capturing phase decoherence. We then impose the minimal and maximally predictive ansatz that *all emergent gravitational phenomena share the same suppression factor*:

$$\boxed{\frac{G_{\text{eff}}}{G_0} = \alpha^2 f(\Phi), \qquad \mathcal{S}_R \equiv \frac{R_{\text{obs}}}{R_{\text{coh}}} = \alpha^2 f(\Phi), \qquad \mathcal{S}_{\text{GW}} \equiv \frac{A_{\text{out}}}{A_{\text{in}}} = \alpha^2 f(\Phi)} \tag{68}$$

where G_0 is the coherent-domain limit ($G_0 \simeq G_N$).[2] This ansatz reduces to general relativity when scalar coherence is maximal ($\alpha \to 1,\ f(\Phi) \to 1$).

Tabulated predictions. Defining the dimensionless phase-gradient parameter $\zeta \equiv |\nabla\theta|^2/\Lambda_\Phi^2$, Tables 1–3 display the predicted suppression factors for representative (α, ζ). The identical form across G_{eff}, $\mathcal{S}_R$, and $\mathcal{S}_{\text{GW}}$ visually emphasizes the unifying power of the ansatz.

Table 1: Predicted suppression of the effective gravitational constant $G_{\text{eff}}/G_0 = \alpha^2 e^{-\zeta}$.

$\alpha\backslash\zeta$	0	0.25	1	2
1.0	1.000	0.779	0.368	0.135
0.9	0.810	0.631	0.298	0.109
0.7	0.490	0.382	0.181	0.067
0.5	0.250	0.195	0.092	0.034

[2]These closures reflect that: (i) the source of curvature is the coherent, scalar-filtered stress-energy; (ii) the Einstein–Hilbert term emerges with an effective coupling G_{eff}; and (iii) spin-2 amplitudes are collective excitations of the scalar-coherent sector and are attenuated when propagating through decoherent domains. The quadratic dependence α^2 arises because the emergent metric $g_{\mu\nu}$ is bilinear in the vierbein, each leg scaling linearly with ρ. See Secs. 5-6 and App. A for the underlying operators and the coherent-domain limit.

Table 2: Predicted curvature suppression $\mathcal{S}_R = \alpha^2 e^{-\zeta}$.

$\alpha\backslash\zeta$	0	0.25	1	2
1.0	1.000	0.779	0.368	0.135
0.9	0.810	0.631	0.298	0.109
0.7	0.490	0.382	0.181	0.067
0.5	0.250	0.195	0.092	0.034

Table 3: Predicted gravitational-wave amplitude transmission $\mathcal{S}_{\rm GW} = \alpha^2 e^{-\zeta}$.

$\alpha\backslash\zeta$	0	0.25	1	2
1.0	1.000	0.779	0.368	0.135
0.9	0.810	0.631	0.298	0.109
0.7	0.490	0.382	0.181	0.067
0.5	0.250	0.195	0.092	0.034

MC lensing workflow with uncertainty bands. To demonstrate computational testability, Algorithm 1 outlines a Monte Carlo workflow for applying this framework to lensing. We model scalar-decoherent slabs or patches along a line of sight. For each realization: (1) draw (α, ζ) from priors, (2) compute $G_{\rm eff}(x)$ from Eq. (68), (3) propagate null rays and accumulate the lensing potential, and (4) record observables. Repeating yields posterior distributions and credible bands.

Algorithm 1 Monte Carlo lensing under scalar coherence

1: Specify priors for $(\alpha(x), \zeta(x))$ and coherence-domain geometry.
2: **for** N realizations **do**
3: Sample (α, ζ); set $G_{\rm eff}(x) = G_0 \alpha(x)^2 e^{-\zeta(x)}$.
4: Solve ray tracing with $G_{\rm eff}(x)$; compute lensing potential ψ.
5: Output $(\hat{\alpha}, \mu, \Delta t)$ and summary statistics.
6: **end for**
7: Report medians and 68%/95% credible bands.

Notes. This closure is deliberately conservative: it ties all three observables to the same filter and amplitude, ensuring that the model's predictions are both minimal and falsifiable. Alternative closures (e.g., $\mathcal{S}_{\rm GW} \propto e^{-\tau}$ with $\tau = \int \zeta \, d\ell$) can be explored, but the central claim remains: *effective gravity, observable curvature, and GW transmission are strongly correlated and governed by the same suppression law.*

6.2.1 Thin-lens mapping and uncertainty propagation

Let

$$s \equiv \alpha^2 e^{-\zeta} \quad \Rightarrow \quad G_{\rm eff} = s\, G_0.$$

In the thin-lens approximation with surface density $\Sigma(\boldsymbol{\theta})$, the GR critical surface density is

$$\Sigma_{\rm crit}^{\rm GR} = \frac{c^2}{4\pi G_0} \frac{D_s}{D_l D_{ls}}.$$

Replacing $G_0 \to G_{\rm eff}$ gives

$$\Sigma_{\rm crit}^{\rm eff} = \frac{\Sigma_{\rm crit}^{\rm GR}}{s}, \qquad \kappa_{\rm eff}(\boldsymbol{\theta}) = \frac{\Sigma}{\Sigma_{\rm crit}^{\rm eff}} = s\,\kappa_{\rm GR}(\boldsymbol{\theta}), \qquad \gamma_{\rm eff}(\boldsymbol{\theta}) = s\,\gamma_{\rm GR}(\boldsymbol{\theta}),$$

and hence

$$\boldsymbol{\alpha}_{\rm eff}(\boldsymbol{\theta}) = s\,\boldsymbol{\alpha}_{\rm GR}(\boldsymbol{\theta}), \qquad \mu_{\rm eff}^{-1} = \big[1 - s\,\kappa_{\rm GR}\big]^2 - s^2|\gamma_{\rm GR}|^2.$$

For a point mass lens, the Einstein angle rescales as

$$\theta_{E,\rm eff} = \sqrt{s}\,\theta_{E,\rm GR},$$

and the geometric/Shapiro time delay scales as

$$\Delta t_{\rm eff} = s\,\Delta t_{\rm GR}.$$

Therefore all standard lensing observables are fixed by the single suppression factor s.

Uncertainty propagation. If (α, ζ) are treated as random fields with means $(\bar{\alpha}, \bar{\zeta})$ and small fluctuations, then

$$\bar{s} = \mathbb{E}[s] = \mathbb{E}[\alpha^2]\,\mathbb{E}[e^{-\zeta}], \qquad \frac{\delta\theta_E}{\theta_E} = \tfrac{1}{2}\,\frac{\delta s}{s}, \qquad \frac{\delta\Delta t}{\Delta t} = \frac{\delta s}{s},$$

and, to first order,

$$\delta\mu^{-1} = -2\big(1 - s\,\kappa_{\rm GR}\big)\kappa_{\rm GR}\,\delta s - 2s|\gamma_{\rm GR}|^2\,\delta s.$$

Thus credible bands for $(\theta_E, \mu, \Delta t)$ follow directly from those of s (equivalently from (α, ζ)), without running a full simulation.

Degeneracy note. Because $\kappa \to s\,\kappa_{\rm GR}$ and $\gamma \to s\,\gamma_{\rm GR}$, the effect is mass-sheet–like; single-source imaging alone cannot break it. Time delays and multi-plane/multi-source configurations do, which is why $\Delta t_{\rm eff} = s\,\Delta t_{\rm GR}$ is especially diagnostic in this framework.

6.2.2 Numerical Predictions from the Suppression Factor

We adopt the suppression factor

$$s(\alpha, \zeta) \;=\; \alpha^2\,e^{-\zeta}, \tag{69}$$

with $0 < \alpha \leq 1$ encoding coherence amplitude and $\zeta \geq 0$ the decoherence depth. In the weak–field regimes derived in this work, the principal observables scale as

$$\frac{\theta_E}{\theta_{E,\rm GR}} \;=\; \sqrt{s}, \qquad \frac{\Delta t}{\Delta t_{\rm GR}} \;=\; s, \qquad \frac{\{\kappa, \gamma\}}{\{\kappa, \gamma\}_{\rm GR}} \;=\; s, \qquad \frac{h}{h_{\rm GR}} \;=\; s, \qquad \frac{G_{\rm eff}}{G_0} \;=\; s. \tag{70}$$

These relations permit immediate numerical predictions once (α, ζ) is specified for a given environment.

Baseline Grid: $s(\alpha,\zeta)$ and Derived Scalings

Table 4: Baseline suppression grid and induced scalings (rounded to three decimals). Entries are $s = \alpha^2 e^{-\zeta}$, the Einstein angle ratio $\theta_E/\theta_{E,\mathrm{GR}} = \sqrt{s}$, the time–delay ratio $\Delta t/\Delta t_{\mathrm{GR}} = s$, the weak–lensing shear/convergence ratio $\{\gamma,\kappa\}/\{\gamma,\kappa\}_{\mathrm{GR}} = s$, the gravitational–wave amplitude ratio $h/h_{\mathrm{GR}} = s$, and the effective coupling ratio $G_{\mathrm{eff}}/G_0 = s$. *All entries are direct evaluations of Eqs. (69)–(70) for the listed (α,ζ) values; e.g. with $(\alpha,\zeta) = (1.0, 0.25)$, $s = e^{-0.25} = 0.779$ and $\theta_E/\theta_{E,\mathrm{GR}} = \sqrt{s} = 0.884$. No numerical fitting or simulation is involved.*

α	ζ	s	$\theta_E/\theta_{E,\mathrm{GR}}$	$\Delta t/\Delta t_{\mathrm{GR}}$	$\{\gamma,\kappa\}/\{\gamma,\kappa\}_{\mathrm{GR}}$	$h/h_{\mathrm{GR}} = G_{\mathrm{eff}}/G_0$
1.00	0.00	1.000	1.000	1.000	1.000	1.000
1.00	0.25	0.779	0.884	0.779	0.779	0.779
1.00	1.00	0.368	0.607	0.368	0.368	0.368
1.00	2.00	0.135	0.367	0.135	0.135	0.135
0.90	0.25	0.632	0.795	0.632	0.632	0.632
0.70	1.00	0.180	0.425	0.180	0.180	0.180
0.50	2.00	0.034	0.184	0.034	0.034	0.034

Illustrative Environment Mapping

The table below translates representative environments into plausible (α,ζ) inputs and shows the induced scalings via Eq. (70). These entries are illustrative and intended to guide observational cross-checks.

Table 5: Illustrative environment–to–observable mapping using $s(\alpha,\zeta) = \alpha^2 e^{-\zeta}$. Numbers are order–of–magnitude inputs to demonstrate how environment choices propagate to observables.

Environment (illustrative)	(α,ζ)	s	$\theta_E/\theta_{E,\mathrm{GR}}$	$\Delta t/\Delta t_{\mathrm{GR}}$	h/h_{GR}
Galactic disk (coherent)	(1.0, 0.0)	1.000	1.000	1.000	1.000
Cluster core (mild decoherence)	(0.9, 0.25)	0.632	0.795	0.632	0.632
Halo outskirts	(0.7, 1.0)	0.180	0.425	0.180	0.180
Cosmic void	(0.5, 2.0)	0.034	0.184	0.034	0.034

Worked Example (SIS lens; scaling only)

Consider a singular isothermal sphere (SIS) lens with GR Einstein angle $\theta_{E,\mathrm{GR}} = 1.20''$ and baseline time delay $\Delta t_{\mathrm{GR}} = 20.0$ days for a given image pair. For $(\alpha,\zeta) = (1.0, 1.0)$ we have $s = 0.368$ and therefore

$$\theta_E = \sqrt{s}\,\theta_{E,\mathrm{GR}} = 0.607 \times 1.20'' = 0.728'', \qquad \Delta t = s\,\Delta t_{\mathrm{GR}} = 0.368 \times 20.0 \text{ days} = 7.36 \text{ days},$$

with weak–lensing shear/convergence and GW strain each reduced by the same factor s. All other lens parameters and cosmological distances remain as in the GR baseline; only the observables listed in Eq. (70) rescale.

6.3 Compatibility With Precision Tests in Coherent Domains

All precision tests relevant to the solar system, binary pulsars, and multimessenger GW events probe regimes that, in this framework, are scalar-coherent: $\alpha \to 1$ and $|\nabla_\mu\theta|^2/\Lambda_\Phi^2 \to 0$. In this limit the effective theory reduces to GR.

(i) Solar-system tests (PPN). In coherent domains the effective action reduces to the Einstein–Hilbert form and the source is the coherent, conserved stress-energy. Hence the PPN parameters coincide with GR at leading order. No anomalous fifth-force terms appear, and the Newtonian limit is recovered from $\nabla^2\Phi_{\rm grav} = 4\pi G_{\rm eff}\rho$ with $G_{\rm eff} \to G_0$.

(ii) Gravitational-wave speed. Linearized propagation in coherent domains yields massless spin-2 waves satisfying $\Box h^{\rm TT}_{\mu\nu} = 0$, so the tensor speed equals the luminal speed, $c_T = 1$. No dispersion or birefringence is induced by the scalar sector in this limit.

(iii) Equivalence principle. Matter couples through the emergent metric constructed from scalar-filtered projections; in coherent domains this is indistinguishable from minimal coupling to $g_{\mu\nu}$. Thus both the universality of free fall and local Lorentz invariance are preserved at leading order.

(iv) Scope of deviations. Deviations from GR—variable $G(\Phi)$, curvature suppression, and wave attenuation—are confined to scalar-decoherent regions where the projection filter suppresses observables. These regimes are explicitly outside the domain of precision tests and are the target of the phenomenology developed in Sec. 6.2.

7 Limitations, Scope, and Falsification

7.1 Assumptions

Emergent geometry. The metric $g_{\mu\nu}$ is a composite observable from projected internal displacements and a *complex* scalar Φ; propagation and curvature exist only in scalar-coherent domains.[3]

Scalar-coherent projection. Physical observables live in the scalar-coherent subspace $\mathcal{H}_\Phi$; projection suppresses geometry in decoherent regions and regulates UV behavior.[4]

7.2 Known limits

Semiclassical/low-curvature derivation. The Einstein–Hilbert term arises from one-loop corrections to a scalar-filtered effective action; the heat-kernel/loop expansion requires curvature to be small relative to effective masses.

Coherence dependence. Predictions apply where scalar phase is coherent; in decoherent regions, $G_{\rm eff}$ and curvature are suppressed and conventional GR does not obtain.

7.3 Open problems

Closed one-loop expression for $G_{\rm eff}$. A fully calculable expression awaits the explicit evaluation of scalar-filtered one-loop corrections.

Spinor geodesics. Geodesic motion derived from spinor propagation in emergent backgrounds remains to be completed.

Possible holographic dual. A boundary description of the scalar–displacement theory is left for future work.

[3] Abstract and summary emphasize projection-filtered emergent geometry and coherence.

[4] Appendix A formalizes the projection operator Π_Φ and local map $P^\mu_\Phi[\cdot]$.

7.4 Falsification tests

Environment-dependent gravity. The effective coupling $G_{\text{eff}}(\Phi)$ varies with scalar coherence; coherent domains recover GR while decoherent media weaken gravity. This predicts measurable lensing/time-delay scalings and environment-dependent inferences of G.

Gravitational-wave attenuation. Spin-2 propagation is suppressed across scalar-incoherent regions, implying transmission deficits or direction-dependent transparency in media with large phase gradients (cf. the $(\alpha^2 e^{-\zeta})$ closures).

Non-singular interiors (masz domains). Black-hole interiors behave as scalar-incoherent condensates with degenerate projection geometry; curvature is suppressed and the interior is causally decoupled. Observables include systematic compactness/shadow deviations consistent with curvature suppression.

Vacuum-energy inertness in decoherent domains. Vacuum energy fails to project into curvature when scalar coherence is lost; any observation requiring vacuum energy to gravitate *in decoherent regimes* would directly challenge the projection mechanism.

Vacuum energy inertness in decoherent domains. If early-universe or astrophysical observations require vacuum energy to gravitate in decoherent regions, this would directly challenge the projection mechanism.

8 Conclusion

This work has established the formal quantum field-theoretic foundation for emergent gravity within the Axis Model. We have shown that a consistent spacetime metric and its dynamics arise not from a fundamental geometric substrate, but from the statistical projection of internal displacement fields and a complex scalar field that governs coherence. By constructing the theory at the level of canonical fields, path integrals, and scalar-filtered expectation values, we have demonstrated that general relativity emerges as a low-energy effective limit in domains of sustained scalar coherence.

Summary of key results. We defined a scalar-projected Hilbert space $\mathcal{H}_\Phi$, in which all geometric observables are constructed as expectation values of composite operators formed from internal field content. The emergent vierbein and metric were shown to arise dynamically, with curvature appearing only in regions where the scalar phase is coherent. We derived the semiclassical Einstein equations from the low-energy limit of the scalar-filtered effective action $S_{\text{eff}}[g]$, and demonstrated that stress-energy arises solely from scalar-projectable field configurations.

Central physical implications. The most profound consequence of this framework is that general relativity is not universally valid-it is the statistical thermodynamic behavior of a deeper scalar-bound field theory. Regions of scalar decoherence do not curve spacetime and may support energy distributions that are gravitationally inert. This leads to concrete and falsifiable predictions: - Environment-dependent gravitational coupling $G(\Phi)$, - Curvature suppression in black hole interiors (masz domains), - Natural resolution of the cosmological constant problem via projection filtering, - Gravitational shielding and wave suppression in scalar-incoherent regions.

The role of scalar coherence. Scalar coherence plays a central role at every level of the theory. It: - Filters quantum field fluctuations to define a physical (projectable) state space, - Determines where and how a spacetime manifold exists, - Acts as a dynamical UV cutoff that protects the emergent metric from short-scale quantum instability, - Enforces locality, observability, and energy-momentum conservation in the emergent regime.

In this sense, coherence is not merely a condition for observability-it is the precondition for geometry itself.

The nature of the graviton. In contrast to conventional approaches that quantize the metric as a fundamental field, the graviton in the Axis Model is not a primitive degree of freedom. Rather, it emerges as a collective, massless, spin-2 excitation of the scalar-coherent internal displacement fields. Its propagation is confined to regions where scalar coherence is maintained; in the presence of phase decoherence, graviton modes are suppressed and the notion of curvature becomes ill-defined. This interpretation provides a physically grounded origin for the graviton's tensor structure and links gravitational wave phenomena directly to deeper field coherence mechanisms.

Future directions. This paper lays the groundwork for several avenues of future investigation. One priority is the explicit derivation of one-loop corrections to the scalar-filtered path integral, which would yield a fully calculable expression for the emergent gravitational constant G_{eff} and test the universality of the Einstein–Hilbert term under projection-based coarse-graining. Another important extension involves incorporating Standard Model fermion fields into the projection formalism, enabling the derivation of geodesic motion as an emergent effect of spinor propagation in scalar-coherent backgrounds. Finally, the internal scalar–displacement theory may admit a holographic interpretation, offering a dual boundary description of emergent bulk gravitational dynamics. These directions collectively extend both the geometric framework and the predictive scope of the Axis Model, and will be explored in future work.

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References

[1] Andrew Morton. The axis model: A unified framework for emergent particle structure, cosmology, and gravitational phenomena, 2025.

[2] Andrew Morton. Quantum completion of the axis model: Gauge structure, brst invariance, and renormalization stability, 2025.

A Scalar-Coherent Projection Operator Formalism

This appendix formalizes the mathematical structure underlying scalar-coherent projection in the Axis Model. The key ingredients are two projection constructs:

1. The scalar alignment operator $\hat{\Pi}_\Phi$, which filters quantum states in the internal Hilbert space based on the coherence properties of the scalar field $\Phi(x)$,
2. The local projection map $\mathcal{P}^\mu_\Phi[\cdot]$, which defines the embedding of internal field excitations into spacetime vectors only in domains of phase coherence.

These objects are foundational to the construction of emergent geometry and energy-momentum observables.

A.1 Scalar Alignment Operator $\hat{\Pi}_\Phi$

Let $\Phi(x) = \rho(x)\, e^{i\theta(x)} \in \mathbb{C}$ be the complex scalar field defined over the manifold M. We define coherence via the local smoothness of the phase $\theta(x)$. A region is scalar-coherent if:

$$|\nabla_\mu \theta(x)|^2 \ll \Lambda_\Phi^2,$$

where Λ_Φ is the scalar coherence scale.

We define the scalar alignment operator $\hat{\Pi}_\Phi$ acting on the quantum state space $\mathcal{H}$ by:

$$\hat{\Pi}_\Phi \equiv \exp\left(-\int d^4x\, \frac{|\nabla_\mu \theta(x)|^2}{\Lambda_\Phi^2} \cdot \hat{\rho}(x)\right), \tag{71}$$

where $\hat{\rho}(x) \equiv\, :\Phi^\dagger(x)\Phi(x):$ is the scalar energy density operator (normal ordered). This operator acts as a **functional filter** on quantum states, exponentially suppressing those with large scalar phase gradients.

We define the scalar-coherent subspace:

$$\mathcal{H}_\Phi = \left\{ |\Psi\rangle \in \mathcal{H} \;\middle|\; \hat{\Pi}_\Phi |\Psi\rangle = |\Psi\rangle \right\}, \tag{72}$$

and interpret this as the physical Hilbert space over which geometric observables are defined.

A.2 Local Projection Map $\mathcal{P}^\mu_\Phi[\cdot]$

Let $v^a(x)$ denote an internal displacement field along axis $a \in \{x, z\}$. These fields are Lorentz scalars with respect to the external spacetime manifold prior to projection. Their interpretation as spacetime vectors depends on scalar-induced coherence.

We define the local projection operator $\mathcal{P}^\mu_\Phi[\cdot]$ as a scalar-modulated functional acting on internal fields:

$$\mathcal{P}^\mu_\Phi[v^a](x) = f(\Phi(x)) \cdot \lambda^{\mu a}(x) \cdot v^a(x), \tag{73}$$

where the function $f(\Phi(x))$ approaches unity in the limit of vanishing scalar phase gradient ($\nabla_\mu \theta \to 0$) and vanishes in decoherent regions, thereby controlling the local projectability of internal displacements. The tensor $\lambda^{\mu a}(x)$ is a scalar-dependent projection map that embeds internal displacements into the spacetime tangent bundle. In scalar-coherent domains, $\lambda^{\mu a}(x)$ becomes approximately constant and defines the emergent vierbein legs $e^\mu_a(x)$.

The projection function $f(\Phi)$ may be taken to have the form:

$$f(\Phi(x)) = \exp\left(-\frac{|\nabla_\mu \theta(x)|^2}{\Lambda_\Phi^2}\right), \tag{74}$$

ensuring a smooth suppression of projection away from coherent domains.

A.3 Composite Vierbein and Metric Definitions

The projected displacement fields define the emergent vierbein as:

$$e_a^\mu(x) = \langle \Psi_\Phi | \mathcal{P}_\Phi^\mu[\hat{v}^a(x)] | \Psi_\Phi \rangle, \quad a \in \{1,2\}, \tag{75}$$

with the third spatial leg constructed via orthonormal completion and the temporal leg given by:

$$e_\mu^0(x) = \alpha_\Phi \, \partial_\mu \Phi(x), \tag{76}$$

as defined in Section 3.2.

The emergent metric is then:

$$g_{\mu\nu}(x) = e_\mu^a(x) \, e_\nu^b(x) \, \eta_{ab}, \quad \eta_{ab} = \mathrm{diag}(+1, -1, -1, -1), \tag{77}$$

which is defined only in scalar-coherent domains. Outside these regions, projection fails and $g_{\mu\nu}(x)$ becomes ill-defined.

Together, the operators $\hat{\Pi}_\Phi$ and $\mathcal{P}_\Phi^\mu[\cdot]$ define the scalar-modulated filtering structure that underlies all geometric emergence in the Axis Model. The alignment operator $\hat{\Pi}_\Phi$ selects the physical Hilbert subspace in which observables are well-defined, while the projection operator $\mathcal{P}_\Phi^\mu[\cdot]$ determines which internal field configurations are mapped into spacetime geometry. In scalar-incoherent regions, these operators suppress curvature, block projection, and enforce gravitational silence. Energy, momentum, and geometry are thus not universally defined—they emerge only where scalar coherence permits projection. These operators are not auxiliary—they are the mechanism by which geometry, gravity, and physicality arise.

B Composite Field Propagators and Commutators

In this appendix, we analyze the propagators and commutation relations of the emergent gravitational fields in the Axis Model. Since the metric and graviton are defined as composite operators built from internal displacement fields, their quantum behavior must be derived from the canonical structure of those underlying fields. We show that causal structure is inherited from the Pauli–Jordan propagators of the fundamental fields; that both the metric and graviton are local composite operators; and that the graviton satisfies the correct commutation structure for a massless spin-2 excitation in scalar-coherent domains.

B.1 Canonical Commutators for Displacement and Scalar Fields

The fundamental displacement fields $\hat{v}^i(x)$, where $i \in \{x, z\}$, obey canonical equal-time commutation relations:

$$[\hat{v}^x(x), \hat{v}^x(y)] = i \, \Delta_0(x-y), \tag{78}$$

$$[\hat{v}^z(x), \hat{v}^z(y)] = i \, \Delta_{m_z}(x-y), \tag{79}$$

$$[\hat{v}^x(x), \hat{v}^z(y)] = 0, \tag{80}$$

where $\Delta_m(x-y)$ is the Pauli–Jordan commutator function for a real scalar field of mass m. These vanish for spacelike separation:

$$\Delta_m(x-y) = 0 \quad \text{for} \quad (x-y)^2 < 0,$$

ensuring microcausality.

The scalar field $\hat{\Phi}(x) \in \mathbb{C}$ also satisfies canonical equal-time commutation relations:

$$[\hat{\Phi}(x), \hat{\Phi}^\dagger(y)] = i\,\Delta_{m_\Phi}(x-y), \tag{81}$$

where m_Φ is the scalar mass scale determined by the symmetry-breaking potential $V(\Phi)$. As with the displacement fields, this vanishes at spacelike separation, preserving causality throughout the full internal field theory.

B.2 Composite Metric Operator

The emergent metric is defined via coherent projections of the internal displacement fields:

$$\hat{g}_{\mu\nu}(x) = \hat{e}^a_\mu(x)\,\hat{e}^b_\nu(x)\,\eta_{ab}, \tag{82}$$

with the spatial legs given by:

$$\hat{e}^\mu_i(x) = \mathcal{P}^\mu_\Phi[\hat{v}^i(x)], \quad i \in \{x, z\}, \tag{83}$$

and the temporal leg constructed from the scalar gradient:

$$\hat{e}^0_\mu(x) = \alpha_\Phi\,\partial_\mu\hat{\Phi}(x). \tag{84}$$

Because the metric is quadratic in local field operators, it is a **local composite operator**, well-defined via point-splitting or normal-ordering techniques.

B.3 Commutation Relations of the Metric Operator

We now evaluate the commutator:

$$[\hat{g}_{\mu\nu}(x), \hat{g}_{\rho\sigma}(y)].$$

This can be expanded using bilinearity and the fact that $\hat{g}_{\mu\nu}(x)$ is built from products of projected $\hat{v}^i(x)$ and $\partial_\mu\Phi(x)$. Using Wick's theorem and the canonical commutators, we obtain:

$$\begin{aligned}[\hat{g}_{\mu\nu}(x), \hat{g}_{\rho\sigma}(y)] = \sum_{i,j}\eta_{ab}\eta_{cd}\Big(&[\hat{e}^a_\mu(x), \hat{e}^c_\rho(y)]\,\hat{e}^b_\nu(x)\hat{e}^d_\sigma(y) \\ &+ \hat{e}^a_\mu(x)\hat{e}^c_\rho(y)\,[\hat{e}^b_\nu(x), \hat{e}^d_\sigma(y)]\Big) + \cdots\end{aligned} \tag{85}$$

Each term in the commutator is built from the fundamental Pauli–Jordan functions $\Delta_m(x-y)$, and thus vanishes for spacelike separation. Therefore:

$$[\hat{g}_{\mu\nu}(x), \hat{g}_{\rho\sigma}(y)] = 0 \quad \text{for} \quad (x-y)^2 < 0. \tag{86}$$

This confirms that the emergent metric operator is causal.

B.4 Graviton Commutators in the Linearized Regime

Define the graviton operator as:

$$\hat{h}_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) - \bar{g}_{\mu\nu}, \tag{87}$$

where $\bar{g}_{\mu\nu}$ is the coherent vacuum metric. In the linearized regime (i.e., small fluctuations in scalar-coherent backgrounds), the graviton inherits its commutation relations from the underlying bilinear operators:

$$[\hat{h}_{\mu\nu}(x), \hat{h}_{\rho\sigma}(y)] \propto i\,\Gamma_{\mu\nu\rho\sigma}(x-y), \tag{88}$$

where $\Gamma_{\mu\nu\rho\sigma}$ is the spin-2 commutator kernel derived from bilinear combinations of $\Delta_0(x-y)$. This shows that: - $\hat{h}_{\mu\nu}(x)$ propagates causal disturbances, - It obeys the correct symmetric, massless tensor structure in coherent domains, - Its spin-2 character arises from the symmetric product of two spin-0 field excitations.

The metric and graviton operators in the Axis Model are local, causal, and composite, with their commutation structure inherited entirely from the fundamental displacement and scalar field operators. As a result, the emergent geometry respects locality and microcausality, graviton propagation is causal and well-defined, and no hidden degrees of freedom are introduced at the emergent level. These results confirm that the projection-based emergence of spacetime geometry is consistent with the principles of quantum field theory and supports a physically well-behaved framework for gravitational dynamics.

C Path Integral Derivation of Effective Curvature Terms

This appendix outlines the derivation of the emergent gravitational action in the Axis Model from first principles, using a semiclassical (one-loop) expansion around scalar-coherent backgrounds. The goal is to show that the effective action $S_{\text{eff}}[g]$ contains the Einstein–Hilbert term in its infrared limit. This elevates the claim of emergent general relativity from an EFT argument to a derivable result.

C.1 Framework and Expansion Strategy

We begin with the scalar-displacement path integral:

$$Z = \int \mathcal{D}[v^x]\mathcal{D}[v^z]\mathcal{D}[\Phi]\, \exp\left(iS[v^x, v^z, \Phi]\right), \tag{89}$$

where the action is:

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu v^x \partial^\mu v^x + \frac{1}{2}\partial_\mu v^z \partial^\mu v^z - \frac{1}{2}m_z^2 (v^z)^2 + \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi) + \mathcal{L}_{\text{int}}[v^i, \Phi] \right). \tag{90}$$

We restrict to scalar-coherent domains in which the emergent metric is defined:

$$g_{\mu\nu}(x) = \langle \Psi_\Phi | \hat{g}_{\mu\nu}(x) | \Psi_\Phi \rangle.$$

To extract the effective action $S_{\text{eff}}[g]$, we follow the background field method: - Decompose fields into background + fluctuation: $\Phi = \bar{\Phi} + \delta\Phi$, $v^i = \bar{v}^i + \delta v^i$, - Fix a background projection geometry $\bar{g}_{\mu\nu}$ defined by $\bar{\Phi}, \bar{v}^i$, - Integrate out Gaussian fluctuations around this background to compute $\Delta S_{\text{eff}}[g]$.

C.2 One-Loop Determinants and Heat Kernel Method

To leading order, the effective action is:

$$S_{\text{eff}}[g] = S_{\text{cl}}[\bar{\Phi}, \bar{v}^i] + \frac{i}{2} \operatorname{Tr} \log\left(-\Box + M_\Phi^2\right) + \frac{i}{2} \sum_{i=x,z} \operatorname{Tr} \log\left(-\Box + M_{v^i}^2\right) + \cdots, \tag{91}$$

where: - $M_\Phi^2, M_{v^i}^2$ are local mass operators derived from the second functional derivatives of the action, - The trace-log terms represent the one-loop determinants.

We evaluate these contributions using the standard heat kernel expansion on curved backgrounds:

$$\mathrm{Tr}\log\bigl(-\Box + M^2\bigr) = -\int d^4x\sqrt{-g}\left(\frac{M^4}{32\pi^2}\log\frac{M^2}{\mu^2} + \frac{M^2 R}{96\pi^2} + \cdots\right), \tag{92}$$

valid when $M^2 \gg R$. The crucial observation is that the **Ricci scalar R** appears as the first curvature-sensitive correction.

C.3 Emergence of the Einstein–Hilbert Term

Applying the heat kernel result to all fluctuating fields, we obtain:

$$S_{\mathrm{eff}}[g] \supset \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_{\mathrm{eff}}}R + \Lambda_{\mathrm{eff}} + \cdots\right), \tag{93}$$

where the emergent coupling is:

$$\frac{1}{16\pi G_{\mathrm{eff}}} = \frac{1}{96\pi^2}\left(M_\Phi^2 + M_{v^x}^2 + M_{v^z}^2\right), \tag{94}$$

and Λ_{eff} includes quartic divergences and vacuum energy contributions. In the Axis Model, decoherence suppresses contributions to Λ_{eff}, explaining the small observed curvature in vacuum.

This result explicitly demonstrates that integrating out scalar and displacement fluctuations in scalar-coherent domains generates the Einstein–Hilbert term. The gravitational constant is not inserted by hand-it arises from field fluctuations and projection geometry.

C.4 Validity and Limitations

This derivation assumes a scalar-coherent background that supports a smooth projection geometry, along with suppression of higher-derivative curvature terms—such as R^2 and $R_{\mu\nu}R^{\mu\nu}$—consistent with a low-energy or small-curvature expansion. It also relies on the use of a regularization scheme, such as dimensional or zeta-function regularization, that preserves general covariance. Regions in which scalar coherence breaks down ($\hat{\Pi}_\Phi \to 0$) lie outside the domain of this expansion and do not contribute to curvature. As such, the emergent gravitational action derived here is valid only within scalar-coherent domains.

This derivation rests on three core assumptions: the existence of a scalar-coherent background that defines a smooth projection geometry; the suppression of higher-derivative curvature terms such as R^2 and $R_{\mu\nu}R^{\mu\nu}$, consistent with low-energy or weak-curvature limits; and the use of a regularization scheme—such as dimensional or zeta-function regularization—that preserves general covariance. Regions where scalar coherence fails ($\hat{\Pi}_\Phi \to 0$) lie outside the domain of this expansion and do not generate curvature contributions. Within coherent domains, however, we have shown that integrating out scalar and displacement field fluctuations yields an effective action that contains the Einstein–Hilbert term,

$$S_{\mathrm{eff}}[g] \supset \frac{1}{16\pi G_{\mathrm{eff}}}\int \sqrt{-g}\,R + \cdots,$$

with a calculable gravitational coupling determined by the underlying field content and coherence structure. This confirms that general relativity emerges not as a postulate, but as the universal low-energy limit of scalar-filtered quantum field theory.