In this article you’ll learn about Permutation and Combination problems: Definition, formulas, solved examples and a quiz with practice questions.

**Permutations**

**Definition**

Permutations are the different ways in which a collection of items can be arranged.

For example:

The different ways in which the alphabets A, B and C can be grouped together, taken all at a time, are ABC, ACB, BCA, CBA, CAB, BAC.

Note that ABC and CBA are not same as the order of arrangement is different. The same rule applies while solving any problem in Permutations.

The number of ways in which n things can be arranged, taken all at a time, nPn = n!, called ‘n factorial.’

**Factorial Formula**

Factorial of a number n is defined as the product of all the numbers from n to 1.

For example, the factorial of 5, 5! = 5\*4\*3\*2\*1 = 120.

Therefore, the number of ways in which the 3 letters can be arranged, taken all a time, is 3! = 3\*2\*1 = 6 ways.

Number of permutations of n things, taken r at a time, denoted by:

nPr = n! / (n-r)!

For example:

The different ways in which the 3 letters, taken 2 at a time, can be arranged is 3!/(3-2)! = 3!/1! = 6 ways.

**Important Permutation Formulas**

1! = 1

0! = 1  
  
Let us take a look at some examples:

**Problem 1:** Find the number of words, with or without meaning, that can be formed with the letters of the word ‘CHAIR’.

Solution:

‘CHAIR’ contains 5 letters.

Therefore, the number of words that can be formed with these 5 letters = 5! = 5\*4\*3\*2\*1 = 120.

**Problem 2:** Find the number of words, with or without meaning, that can be formed with the letters of the word ‘INDIA’.

Solution:

The word ‘INDIA’ contains 5 letters and ‘I’ comes twice.

When a letter occurs more than once in a word, we divide the factorial of the number of all letters in the word by the number of occurrences of each letter.

Therefore, the number of words formed by ‘INDIA’ = 5!/2! = 60.

**Problem 3:** Find the number of words, with or without meaning, that can be formed with the letters of the word ‘SWIMMING?

Solution:

The word ‘SWIMMING contains 8 letters. Of which, I occurs twice and M occurs twice.

Therefore, the number of words formed by this word = 8! / (2!\*2!) = 10080.

**Problem 4:** How many different words can be formed with the letters of the word ‘SUPER’ such that the vowels always come together?

Solution:

The word ‘SUPER’ contains 5 letters.

In order to find the number of permutations that can be formed where the two vowels U and E come together.

In these cases, we group the letters that should come together and consider that group as one letter.

So, the letters are S,P,R, (UE). Now the number of words are 4.

Therefore, the number of ways in which 4 letters can be arranged is 4!

In U and E, the number of ways in which U and E can be arranged is 2!

Hence, the total number of ways in which the letters of the ‘SUPER’ can be arranged such that vowels are always together are 4! \* 2! = 48 ways.

**Problem 5:** Find the number of different words that can be formed with the letters of the word ‘BUTTER’ so that the vowels are always together.

Solution:

The word ‘BUTTER’ contains 6 letters.

The letters U and E should always come together. So the letters are B, T, T, R, (UE).

Number of ways in which the letters above can be arranged = 5!/2! = 60 (since the letter ‘T’ is repeated twice).

Number of ways in which U and E can be arranged = 2! = 2 ways

Therefore, total number of permutations possible = 60\*2 = 120 ways.  
  
**Problem 6:** Find the number of permutations of the letters of the word ‘REMAINS’ such that the vowels always occur in odd places.

Solution:

The word ‘REMAINS’ has 7 letters.

There are 4 consonants and 3 vowels in it.

Writing in the following way makes it easier to solve these type of questions.

(1) (2) (3) (4) (5) (6) (7)

No. of ways 3 vowels can occur in 4 different places = 4P3 = 24 ways.

After 3 vowels take 3 places, no. of ways 4 consonants can take 4 places = 4P4 = 4! = 24 ways.

Therefore, total number of permutations possible = 24\*24 = 576 ways.

**Combinations**

**Definition**

The different selections possible from a collection of items are called combinations.

For example:

The different selections possible from the alphabets A, B, C, taken 2 at a time, are AB, BC and CA.

It does not matter whether we select A after B or B after A. The order of selection is not important in combinations.

To find the number of combinations possible from a given group of items n, taken r at a time, the formula, denoted by nCr is

nCr = n! / [r! \* (n-r)!]

For example, verifying the above example, the different selections possible from the alphabets A, B, C, taken two at a time are

3C2 = 3! / (2! \* (3-2)!) = 3 possible selections (i.e., AB, BC, CA)

**Important Combination formulas**

nCn = 1

nC0 = 1

nC1 = n

nCr = nC(n-r)

The number of selections possible with A, B, C, taken all at a time is 3C3 = 1 (i.e. ABC)

**Solved examples of Combination**

Let us take a look at some examples to understand how Combinations work:

**Problem 1:** In how many ways can a committee of 1 man and 3 women can be formed from a group of 3 men and 4 women?

Solution:

No. of ways 1 man can be selected from a group of 3 men = 3C1 = 3! / 1!\*(3-1)! = 3 ways.

No. of ways 3 women can be selected from a group of 4 women = 4C3 = 4! / (3!\*1!) = 4 ways.

**Problem 2:** Among a set of 5 black balls and 3 red balls, how many selections of 5 balls can be made such that at least 3 of them are black balls.

Solution:

Selecting at least 3 black balls from a set of 5 black balls in a total selection of 5 balls can be

3 B and 2 R

4 B and 1 R and

5 B and 0 R balls.

Therefore, our solution expression looks like this.  
5C3 \* 3C2 + 5C4 \* 3C1 + 5C5 \* 3C0 = 46 ways .

**Problem 3:** How many 4 digit numbers that are divisible by 10 can be formed from the numbers 3, 5, 7, 8, 9, 0 such that no number repeats?

Solution:

If a number is divisible by 10, its units place should contain a 0.  
\_ \_ \_ 0

After 0 is placed in the units place, the tens place can be filled with any of the other 5 digits.

Selecting one digit out of 5 digits can be done in 5C1 = 5 ways.

After filling the tens place, we are left with 4 digits. Selecting 1 digit out of 4 digits can be done in 4C1 = 4 ways.

After filling the hundreds place, the thousands place can be filled in 3C1 = 3 ways.

Therefore, the total combinations possible = 5\*4\*3 = 60.

**Permutations and Combinations Quiz**

Try these practice problems.

Problem 1

Solve the following.

i) 30P2   
ii) 30C2

A. 870, 435  
B. 435, 870  
C. 870, 470  
D. 435, 835

Answer 1

A

**Explanation:**

30P2 = 30! / 28! = 30\*29\*28! / 28! = 30\*29 = 870.

30C2 = 30! / (2!\*28!) = 435.

Problem 2

How many different possible permutations can be made from the word ‘BULLET’ such that the vowels are never together?

A. 360  
B. 120  
C. 480  
D. 240

Answer 2

D.

**Explanation**:

The word ‘BULLET’ contains 6 letters of which 1 letter occurs twice = 6! / 2! = 360

No. of permutations possible with vowels always together = 5! \* 2! / 2! = 120

No. of permutations possible with vowels never together = 360-120 = 240.

Problem 3

In how many ways can a selection of 3 men and 2 women can be made from a group of 5 men and 5 women ?

A. 10  
B. 20  
C. 30  
D. 100

Answer 3

D.

**Explanation**:

5C3 \* 5C2 = 100