0.0.1 Rotation about an arbitrary axis

`http://tom.cs.byu.edu/~455/hw/rotation.pdf`

A vector $\vec{v} = (x, y, z)$ is rotated by an angle θ about an axis given by the unit vector $\hat{n} = (a, b, c)$. It can be shown that the 3×3 rotation matrix has the form

$$\begin{bmatrix} a^2 + (1 - a^2)\cos\theta & ab(1 - \cos\theta) - c\sin\theta & ac(1 - \cos\theta) + b\sin\theta \\ ab(1 - \cos\theta) + \sin\theta & b^2 + (1 - b^2)\cos\theta & bc(1 - \cos\theta) - a\sin\theta \\ ac(1 - \cos\theta) - b\sin\theta & bc(1 - \cos\theta) + a\sin\theta & c^2 + (1 - c^2)\cos\theta \end{bmatrix} \tag{1}$$

Alternative

A more geometrically intuitive[1] approach starts with noting that the vector $\vec{v}$ can be decomposed into parts parallel and perpendicular to the rotation axis $\hat{n}$

$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp} \tag{2}$$

where

$$\vec{v}_{\parallel} = \hat{n}(\hat{n} \cdot \vec{v}) \tag{3a}$$
$$\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} \tag{3b}$$

The process of rotation has no effect on the parallel component, with all changes in the plane perpendicular to the rotation axis. The rotated perpendicular component $= \vec{v}'_{\perp}$ will be in a plane is spanned by $\vec{v}_{\perp}$ and a second vector defined as

$$\vec{u} = \hat{n} \times \vec{v}_{\perp} = \hat{n} \times \vec{v} \qquad \textbf{because } \hat{n} \times \vec{v}_{\parallel} = 0 \tag{4}$$

with the the standard right-handed coordinate system so that $\hat{v}_{\perp} \times \hat{u} = \hat{n}$ and a positive angle θ rotates $\vec{v}_{\perp}$ towards $\vec{u}$. The rotated perpendicular component can be written in terms of two orthogonal basis vectors and the rotation angle where $|\vec{u}| = |\vec{v}_{\perp}| = |\vec{v}'_{\perp}|$, so

$$\vec{v}'_{\perp} = \vec{v}_{\perp}\cos\theta + \vec{u}\sin\theta \tag{5}$$

The final rotated vector can be written in several different ways

$$\vec{v}' = \vec{v}_{\parallel} + \vec{v}'_{\perp} \tag{6a}$$
$$= \vec{v}_{\parallel} + \vec{v}_{\perp}\cos\theta + \vec{u}\sin\theta \tag{6b}$$
$$= \vec{v} + \vec{v}_{\perp}(\cos\theta - 1) + \vec{u}\sin\theta \tag{6c}$$

I suspect that the computational requirements are sub-optimal...

1. Calculate $\vec{v}_{\parallel}$: dot product (3 mult, 2 add) followed by vector-scalar multiplication (3 mult) = 6 multiply, 2 addition
2. Calculate $\vec{v}_{\perp}$: vector subtraction = 3 addition
3. Calculate $\vec{u}$: cross product = 6 multiply, 3 addition
4. Combine terms = 6 multiply + 6 addition

...certainly less elegant than a matrix multiplication

[1] but computationally inferior?

0.1 IDL code

Would it be worth introducing a "skymap_aim" object that is an overloaded vector with za/az options? Or just overload "skymap_vector" spherical conversions?

There are 4 combinations: 1) theta/phi co-latitude/longitude , right ascension/declination 2) /phi latitude/longitude 3) theta/ zenith angle/azimuth (clockwise from north) 4) / elevation/azimith 5) some wacky camera convention?

Try keyword SPHERICAL=[1—2—4—6] where bit 0 is basic conversion, bit 1 is co-latitude, bit 2 is co-azimuth.

For Cartesian x, y, z the azimuth is the angle in the x-y plane relative to the x-axis and positive towards +y

$$\phi = (y, x)$$

but for geographic East,North,Up azimuth is clockwise from north so we get

$$\phi = (x, y)$$

with the alternative being North,East Down

$$\phi = (y, x)$$

or left-handed coordinates