Regularized four-sided cavity flows: A spectral bifurcation benchmark implemented in Julia

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Abstract

Driven cavity flows are commonly used as benchmarks to validate Navier-Stokes solvers. These problems can test spatial discretization methodologies such as finite elements, finite differences, and spectral methods. They also assess a variety of boundary condition implementations and time-stepping schemes. The simple lid-driven cavity flow has received considerable attention. This flow is steady for high Reynolds numbers, with the first instability occurring within a large uncertainty interval (7500, 8100) [Kuhlmann (2018)].

More recent variants, such as the four-sided version of the cavity flow, have been proposed [Wahba (2009)]. In this problem, lids move with the same velocity profile and parallel lids move in opposite directions. Later works tested different numerical techniques and studied the bifurcations with linear stability analysis, arc-length continuation, and time-stepping [Perumal (2011), Cadou (2012), Chen (2013)]. This cavity has the computational advantage of exhibiting a variety of bifurcations at low or moderate Reynolds numbers.

Still, the problem suffers from corner singularities due to the discontinuous boundary conditions and affects the exponential convergence of spectral methods. This work presents a regularized version of the four-sided cavity flow to address the issue. A spectral Chebyshev discretization of the flow problem is implemented in Julia, an open-source, high-performance language for scientific computing. A developed Julia module provides a reproducible example of the proposed cavity.

The regularized four-sided lid-driven cavity shows most of the primary bifurcation scenarios. The flow undergoes instabilities, such as pitchfork, saddle-node (fold), and Hopf. Predicting the precise location of the bifurcations could present an amenable Navier-Stokes bifurcation benchmark when testing and comparing different schemes and implementations.

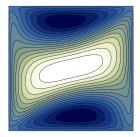


Figure 1: Asymmetric solution on a 64×64 grid at Reynolds 100

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