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Regularized four-sided cavity flows: A spectral bifurcation benchmark implemented in Julia

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Abstract

Driven cavity flows are commonly used as benchmarks to validate Navier-Stokes solvers. These problems can test spatial discretization methodologies such as finite elements, finite differences, and spectral methods. They also assess a variety of boundary condition implementations and time-stepping schemes. The simple lid-driven cavity flow has received considerable attention. This flow is steady for high Reynolds numbers, with the first instability occurring within a large uncertainty interval (7500, 8100) [Kuhlmann (2018)].

More recent variants, such as the four-sided version of the cavity flow, have been proposed [Wahba (2009)]. In this problem, lids move with the same velocity profile and parallel lids move in opposite directions. Later works tested different numerical techniques and studied the bifurcations with linear stability analysis, arc-length continuation, and time-stepping [Perumal (2011), Cadou (2012), Chen (2013)]. This cavity has the computational advantage of exhibiting a variety of bifurcations at low or moderate Reynolds numbers.

Still, the problem suffers from corner singularities due to the discontinuous boundary conditions and affects the exponential convergence of spectral methods. This work presents a regularized version of the four-sided cavity flow to address the issue. A spectral Chebyshev discretization of the flow problem is implemented in Julia, an open-source, high-performance language for scientific computing. A developed Julia module provides a reproducible example of the proposed cavity.

The regularized four-sided lid-driven cavity shows most of the primary bifurcation scenarios. The flow undergoes instabilities, such as pitchfork, saddle-node (fold), and Hopf. Predicting the precise location of the bifurcations could present an amenable Navier-Stokes bifurcation benchmark when testing and comparing different schemes and implementations.

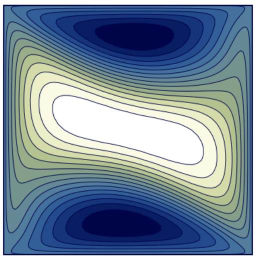

Figure 1: Asymmetric solution on a 64×64 grid at Reynolds 100

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References

Wahba E. M. (2009) “Multiplicity of states for two-sided and four-sided lid driven cavity flows“, *Comput. Fluids*, **38**, 247–253

Arumuga Perumal D., Dass A. K. (2011) “Multiplicity of steady solutions in two-dimensional lid-driven cavity flows by Lattice Boltzmann Method“, *Comput. Math. with Appl.*, **61**, 3711–3721

Cadou J. M., Guevel Y. and Girault G. (2012) “Numerical tools for the stability analysis of 2D flows: application to the two- and four-sided lid- driven cavity“, *Fluid Dyn. Res.*, **44** (3), 031403

Chen, K. T., Tsai C. C. and Luo W. J. (2013) “Multiplicity Flow Solutions in A Four-sided Lid-driven Cavity“, *Appl. Mech. Mater.*, **368-370**, 838–843

Kuhlmann H., Romanò F. (2018) “The Lid-Driven Cavity“, *Comput. Methods Appl. Sci.*