Two types of degree nominalizations: Degree concepts and qualities[1]

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Abstract. Francez and Koontz-Garboden (2017: FKG) propose that in many languages, gradable predication is fundamentally about not degrees but qualities qua portions of an abstract substance, like wisdom or beauty, that an individual can be said to possess. Both notions—qualities and degrees—are crucial, we propose, to understanding an important distinction between two varieties of degree nominalization in English (e.g. *height* in *Ingo is Bertha's height* vs *tallness* in *Ingo has Bertha's tallness*). Building on Bochnak (2013) and Gobeski (2019), we marshal a range of unnoticed or under-noticed contrasts that we argue show that some nominalizations denote qualities and others denote what we'll call DEGREE CONCEPTS, which are intensionalized degrees.

Keywords: degrees, nominalizations, property concepts, adjectives, type shifting, copular constructions, DP adjuncts, concealed questions

1. Introduction

English and many other well-studied languages generally use adjectives to express gradable properties, as in (1a). But another strategy is available, exemplified in (1b), which involves the use of degree nominalizations (DNs) like *strength*:

(1) a. Floyd is strong. *(adjective)*
b. Floyd has strength. *(nominalization)*

Some languages use DNs as their primary strategy for expressing gradable properties. For example, to describe Floyd in Hausa, one would use the nominalization *ƙarfī* 'strength', as in (2a). The construction also requires the possessive verb *da*̃, which is used to express ownership more generally, as in (2b):

(2) a. Munã **dã** ƙarfī
we.CONT **with** strength
'We **have** strength.' *(Hausa, Newman, 2000)*

b. Yārinyã tana **dã** zōbẽ
girl she.CONT **with** ring
'The girl **has** a ring.'

Returning to English, Francez and Koontz-Garboden (2017) observe that sentences like (3) do not give rise to a contradiction:[2]

[1]We would like to thank Brian Buccola, Eva Csipak, Daiki Matsuoka, Friederike Moltmann, Viola Schmitt, Greg Scontras, Ryan Walter Smith, Todd Snider, Stephanie Solt, Anne-Michelle Tessier, and the audience at Sinn und Bedeutung 29.

[2]In (3) we have actually modified Francez and Koontz-Garboden's example slightly. The original is: *The Taj Mahal has as much beauty as the Stata Center, though their beauties are very different* (p.54). We believe our

(3) The Taj Mahal has the same beauty as the Stata Center, though their beauties are very different.

On an analysis where DNs like *beauty* characterize scales (i.e., sets of degrees), (3) should be contradictory. Francez and Koontz-Garboden argue that DNs instead denote sets of pre-ordered portions of substances, which in turn can be measured. On this view, the first clause in (3) states that the Taj Mahal and Stata Center occupy the same position on the pre-order, while the second clause states that the portions of beauty that they possess are distinct in some other way.

However, we observe that not all English DNs fit in Francez and Koontz-Garboden's frame without contradiction, as shown in (4).

(4) #The Taj Mahal has the same {**height** / **weight** / **size**} as the Stata Center, though their {**heights** / **weights** / **sizes**} are very different.

These DNs can only have the degree/ordering reading, which leads to the feeling that (4) is contradictory. Therefore, there seem to be two types of DN in English, and not all of them can be accounted for under Francez and Koontz-Garboden's portion/substance analysis.

It will be useful to have labels for these two types. We will call them "neutral" and "biased" DNs. Neutral DNs are so named because they do not license entailments to the positive form of the corresponding adjective, while biased DNs *do* entail the positive form:

(5) a. Floyd has a certain height. $\not\Rightarrow$ Floyd is tall. *(neutral)*
b. Floyd has a certain tallness. $\Rightarrow$ Floyd is tall. *(biased)*

(6) a. Floyd has a certain temperature. $\not\Rightarrow$ Floyd is warm. *(neutral)*
b. Floyd has a certain warmth. $\Rightarrow$ Floyd is warm. *(biased)*

Neutral DNs give rise to contradiction in Taj Mahal sentences such as (4), whereas biased DNs do not. While many of these DNs come in neutral-biased pairs (e.g., *height-tallness*), *beauty* is a biased DN that lacks a corresponding neutral DN.[3]

Neutral DNs	**Biased DNs**
height	tallness
weight	heaviness
temperature	warmth
age	youth
size	largeness
	beauty

Many or perhaps even most biased DNs are derivationally related to gradable adjectives, whereas neutral DNs are suppletive relative to their corresponding gradable adjectives.

modification makes the same point, and also sharpens the judgments surrounding the two types of DNs that we introduce below.

[3]The essential distinction between neutral vs biased DNs was already noticed by Moltmann (2009), who referred to these as absolute nominalizations and positive nominalizations, respectively.

In Section 2, we show that neutral and biased DNs differ with respect to the following properties: mode of attribution (using *have* vs *be*); their mass/count behavior; the availability of distributive readings; their compatibility with certain predicates like *write down* and *be permitted*; the ways in which they can be said to change; co-occurrence with factor phrases; and concealed questions. After reviewing previous work on English DNs in Section 3, we present our analysis in Section 4. We analyze biased DNs in terms of QUALITIES, i.e., portions of an abstract substance, in the spirit of Francez and Koontz-Garboden (2017). We analyze neutral DNs in terms of DEGREE CONCEPTS, i.e., intensionalized degrees. Section 5 concludes.

2. Neutral vs. biased DNs

The first difference we highlight between neutral and biased DNs is in whether they use *be* or *have* for attribution. Neutral DNs, but not biased DNs, can occur with the copula in predication. (We find *have* with neutral DNs somewhat less natural, but possible.) Biased DNs, however, must use *have*. This contrast is seen in (7):

(7) a. Ingo {is / ??has} the same {height / age / weight / size} as Bertha. *(neutral)*

b. Ingo {#is / has} the same {tallness / oldness / heaviness / beauty} as Bertha. *(biased)*

This contrast is striking. Francez and Koontz-Garboden especially emphasize the role of possession as part of how degree nominalizations work across languages. In Taj Mahal sentences, biased DNs behave exactly as they would predict, requiring an overtly possessive verb—but neutral DNs clearly pattern differently.

There is also a difference in adjunction possibilities. Neutral DNs can adjoin to NPs, whereas biased DNs cannot.

(8) a. A person that {height / age / weight / size} is bound to make the team. *(neutral)*

b. #A person that {tallness / oldness / heaviness / beauty} is bound to make the team. *(biased)*

If one analyzes these adjuncts as reduced relatives containing an implicit *who is*, it may be possible to explain the facts in (7) and in (8) along the same lines—though as we will see, we will be able to unify these facts without assuming any kind of ellipsis.

The two types of DN also display different behavior with respect to the mass/count distinction. Neutral DNs show features of count nouns, whereas biased DNs definitively behave like mass nouns. For instance, only neutral DNs allow plural morphology:

(9) a. Floyd noticed the participants' {heights / ages / weights / sizes}. *(neutral)*

b. #Floyd noticed the participants' {tallnesses / beauties / wisdoms / patiences}. *(biased)*

While neutral DNs can occur with *many*, biased DNs must occur with *much*:

(10) a. I didn't notice **many** {heights / ages / weights / sizes}. *(neutral)*

b. I didn't notice **much** {beauty / strength / wisdom / patience}. *(biased)*

The two types also differ with respect to intensification with *more*, which is fine with biased DNs but odd with neutral DNs:

(11) a. ?? Floyd has more {height / age / weight / size} than Clyde. *(neutral)*

b. Floyd has more {tallness / oldness / heaviness / beauty} than Clyde. *(biased)*

Another interesting difference is in the availability of distributive and collective readings with group nouns. Singular neutral DNs give rise to collective readings with group nouns. In (12), *age* refers to how long ago the committee was formed, not the ages of the individual members, and *size* refers to the number of committee members, not the sizes of the individual members:

(12) The {age / size} of the committee intimidated Floyd. *(neutral)*

a. *age*: ...#The members were all elderly.

b. *size*: ...#The members were all enormous.

Meanwhile biased DNs allow a distributive interpretation, as in (13). We do also detect an available collective reading here, insofar as the DN represents a property that a committee could plausibly have (i.e., the committee as a whole was formed a long time ago).

(13) The {oldness / tallness} of the committee intimidated Floyd. *(biased)*

a. *age*: The members were all elderly.
b. *size*: The members were all enormous.

Only neutral DNs have a clear interpretation with factor phrases such as *three times* (also called multiplicative or ratio phrases; Sassoon 2010; Gobeski 2019; Coppock 2021):

(14) a. Floyd is three times Ingo's {height / age / weight / size}. *(neutral)*

b. ??Floyd has three times Ingo's {tallness / oldness / heaviness / beauty}. *(biased)*

The two types of DN also differ with respect to how they can be said to change. For a neutral DN, its change is in the degree to which the subject holds the property, as in (15). When a biased DN is said to change, as in (16), its change can be in the *way* in which a property is manifested (Moltmann 2009):

(15) Floyd's **height** has changed. *(neutral)*
a. He's grown a few inches.
b. #He used to be a lanky tall, but now he's bulked up.

(16) Floyd's **tallness** has changed. *(biased)*
a. He used to be a lanky tall, but now he's bulked up.
b. He's grown a few inches.

A degree interpretation is also possible for (16). Possessed neutral DNs occur in positions where concealed questions do, whereas possessed biased DNs do not make suitable concealed questions. Typical positions for a concealed question interpretation of a DP include in the complement of verbs like *guess* and *know* (Grimshaw 1979 among many others), and here the concealed question interpretation available for neutral DNs but not biased DNs, as shown in (17) (cf. Moltmann 2009):

(17) Floyd guessed **Clyde's height**. *(concealed question)*
≈ Floyd guessed **what Clyde's height was**. *(question)*

(18) a. Clyde knew Ingo's $\left\{\begin{matrix}\text{height}\\\text{weight}\end{matrix}\right\}$. *(neutral)*

≈ Clyde knew what Ingo's $\left\{\begin{matrix}\text{height}\\\text{weight}\end{matrix}\right\}$ was.

b. #Clyde knew Ingo's $\left\{\begin{matrix}\text{tallness}\\\text{heaviness}\end{matrix}\right\}$. *(biased)*

$\not\approx$ Clyde knew what Ingo's $\left\{\begin{matrix}\text{tallness}\\\text{heaviness}\end{matrix}\right\}$ was.

Finally, only neutral DNs are compatible with certain predicates, such as *be permitted* and *write down*, whereas biased DNs are odd:

(19) a. This $\left\{\begin{matrix}\text{age/size/height/weight}\\\text{\#tallness/bigness/heaviness/strength}\end{matrix}\right\}$ is permitted.

b. I wrote down my $\left\{\begin{matrix}\text{age/size/height/weight}\\\text{\#tallness/bigness/heaviness/strength}\end{matrix}\right\}$.

The empirical differences between neutral and biased DNs are summarized in Table 1.

	Neutral DNs	**Biased DNs**
Can be the same and different (Taj Mahal)	✓	✗
Simple predication with copula	✓	✗
Can head DP adjuncts	✓	✗
Pattern as count nouns	✓	✗
Distributive readings	✗	✓
Licit with factor phrases	✓	✗
Can change in quality	✗	✓
Concealed questions	✓	✗
Can be written down, permitted	✓	✗

Table 1: Behavior of neutral vs. biased DNs

3. Previous work

3.1. Possessive degree nominalizations

As we noted in (2), Hausa uses nominalizations, not adjectives, to express gradable concepts, and this is not an uncommon strategy. A number of researchers, including Dixon (1982); Moltmann (2009); Portner (1992); Zucchi (1989); Grimm and McNally (2015), among others, have discussed the connection between adjectives and their nominalized forms. We're going to focus in particular on Francez and Koontz-Garboden (2015, 2017) and their discussion of what they refer to as "qualities", which will be crucial to our analysis.

For Francez and Koontz-Garboden, a quality is a term for an abstract mass entity, such as *beauty* or *strength*, which can be divided up into portions of type p. These qualities are ordered by a total preorder $\leq$, which can be thought of as a size ordering. They are also mass entities in the

style of Link (1983), meaning they're partially ordered with the mereological part ordering $\preceq$. The size ordering $\leq$ preserves the mereological part ordering, such that for two given portions p, q and a quality P:

(20) $p, q \in P : p \preceq q \rightarrow p \leq q$

So under their analysis, a degree nominal like *strength* is treated as a portion of the appropriate mass entity with the denotation in (21), where p is some portion of the quality of strength:

(21) $[\![\textit{strength}]\!] = \lambda p \,.\, \textbf{strength}(p)$

Now because portions can be possessed by individuals, the possessive morphology is important. So when the possessive is applied to the nominal, a predicate of individuals is returned. (22) states that x possesses a portion of strength z, where z is restricted to "large enough" in the context C.

(22) $[\![\textit{has strength}]\!]^C = \lambda x \,.\, \iota p \in C[\textbf{strength}(p) \wedge \pi(x, p)]$

Francez and Koontz-Garboden note that because portions aren't antisymmetric, two of them can be the same size without being the same portion. This explains why (23) isn't contradictory:

(23) The Taj Mahal has the same beauty as the Stata Center, though their beauties are very different.

However, this doesn't account for the difference between neutral and biased DNs:

(24) #The Taj Mahal has the same height as the Stata Center, though their heights are very different.

Francez and Koontz-Garboden don't say anything about this, perhaps because their concern is more with cross-linguistic data than these more fine-grained distinctions. But essentially this distinction seems to be present in Moltmann (2009), to which we now turn.

3.2. Neutral vs biased degree nominalizations

What we've been calling neutral and biased DNs seem to have been previously noticed in the literature in slightly different ways, notably by Bierwisch (1989) and Moltmann (2009). For Bierwisch, the distinction reflects the distinction between what he calls dimensional and evaluative adjectives. Moltmann recognizes the distinction between the two kinds of nominalization more directly, and she uses different terminology: what we're calling neutral DNs, she calls absolute nominalizations, and our biased DNs are her positive nominalizations. She notes that neutral DNs can occur with degree predicates, but biased ones can't:

(25) Floyd's $\left\{ \begin{array}{c} \text{height} \\ \#\text{tallness} \end{array} \right\}$ is six feet.

Moltmann (2009) analyzes these nominalizations in a framework of tropes, which are, broadly speaking, particularized properties of individuals. For Moltmann, both types of nominalizations

involve a relationship between a particular trope and its bearer; where they differ is that biased DNs make reference to a standard of comparison (cf. the use of the POS morpheme in degree-based accounts), while neutral DNs don't.[4]

However, despite the differences in specifics, both forms of nominalizations are ultimately tropes for Moltmann, and thus of the same semantic type. But this poses something of a puzzle. This account doesn't adequately explain the grammatical distinctions in (26) for instance:

(26) a. Ingo $\left\{\begin{array}{c}\text{is}\\ \text{??has}\end{array}\right\}$ the same $\left\{\begin{array}{l}\text{height}\\ \text{age}\\ \text{weight}\\ \text{size}\end{array}\right\}$ as Bertha. *(neutral)*

b. Ingo $\left\{\begin{array}{c}\text{\#is}\\ \text{has}\end{array}\right\}$ the same $\left\{\begin{array}{l}\text{tallness}\\ \text{oldness}\\ \text{heaviness}\\ \text{beauty}\end{array}\right\}$ as Bertha. *(biased)*

It's also not clear that it would straightforwardly account for the other facts we raise. So we are going to need something more than just Moltmann's tropes or Francez and Koontz-Garboden's qualities to account for all the facts.

It's worth recognizing where these approaches converge. FKG introduce the idea of qualities, or portions of an abstract substance, and its exact ontological status is not their primary concern. Moltmann's tropes build on a rich tradition of research in philosophy, and it's worth reflecting on whether they could be viewed as one way of spelling out FKG's qualities. We will build on FKG's analysis more directly because it allows us to focus on the crucial linguistic properties. Further reflection about the ontological commitments and trade-offs in this area will be left to future work.

4. Analysis

4.1. Neutral DNs

The first step is to notice that neutral DNs are in some respects similar to measure phrases, which are standardly taken to denote degrees. That might be taken to suggest that neutral DNs denote degrees directly. This first step is reasonable, but even for measure phrases, a simple degree-denoting denotation requires elaboration. Measure phrases can occur with the copula, which suggests that they are—or in any case, can be—predicates:

(27) Ingo is six feet.

Of course, a degree like **6ft** isn't a predicate, so it can't be directly predicated of Ingo. Nor would it suffice to take this to be asserting identity, because of course Ingo is not himself identical to

[4]This is putting it very crudely. What Moltmann actually says is that biased DNs inherit their status of being concrete or abstract entities and their location in space and time only from the bearer and not from the standard. This explains why some biased tropes, such as *tallness*, are concrete despite the standard being abstract. Moltmann refers to these as quasi-relational tropes.

a degree. The way out, we suggest, is to suppose that a type shift is systematically available in English that shifts a degree to a property of individuals with that degree as their measure on the scale associated with that degree. We call this DEG-SHIFT, and it can shift *six feet* as in (28b):

(28) a. $[\![\text{DEG-SHIFT}]\!] = \lambda d \lambda x \,.\, \mu_{\textbf{scale}(d)}(x) \geq d$

b. $[\![\textit{Ingo is DEG-SHIFT six feet}]\!] = [\![\text{DEG-SHIFT}]\!]([\![\textit{six feet}]\!])([\![\textit{Ingo}]\!])$
$= \mu_{\textbf{scale(6ft)}}(\textbf{Ingo}) \geq \textbf{6ft}$
$= \mu_{\textbf{height}}(\textbf{Ingo}) \geq \textbf{6ft}$

The same shift will be crucial in interpreting neutral DNs as well. In copular positions, neutral DNs behave like measure phrases. They appear to be predicated directly of an individual:

(29) Ingo is my height.

Again, Ingo is of course not identical to a height degree. Rather, in this sentence, *my height* is shifted to a predicate of individuals whose measure is the same as the measure of the speaker's height, again using DEG-SHIFT:

(30) a. $[\![\textit{my height}]\!] = \mu_{\textbf{height}}(\textbf{me})$

b. $[\![\text{DEG-SHIFT}]\!] = \lambda d \lambda x \,.\, \mu_{\textbf{scale}(d)}(x) \geq d$

c. $[\![\textit{Ingo is DEG-SHIFT my height}]\!] = [\![\text{DEG-SHIFT}]\!]([\![\textit{my height}]\!])([\![\textit{Ingo}]\!])$
$= \mu_{\textbf{height}}(\textbf{Ingo}) \geq \mu_{\textbf{height}}(\textbf{me})$

If it were only in copular positions that this shift is necessary, one might suspect that this isn't a type shift at all, but rather a built-in part of the meaning of the English copula. But as it turns out, the same effect is needed elsewhere. They can also occur as DP adjuncts:

(31) A player my $\left\{\begin{matrix}\text{height}\\ \text{weight}\\ \text{size}\end{matrix}\right\}$ is bound to make the team.

If we suppose that the neutral DN is shifted using DEG-SHIFT, these structures could straightforwardly be interpreted intersectively:

(32) a. $[\![\text{DEG-SHIFT } \textit{my height}]\!] = \lambda x \,.\, \mu_{\textbf{scale}(d_{that})}(x) \geq \mu_{\textbf{height}}(\textbf{me})$

b. $[\![\textit{player [}\text{DEG-SHIFT } \textit{that height]}]\!] = \lambda x \,.\, \textbf{player}(x) \wedge \mu_{\textbf{scale}(d_{that})}(x) \geq \mu_{\textbf{height}}(\textbf{me})$

This is especially worth noting not only because it provides evidence for treating DEG-SHIFT as a generally available type shift, but also because it sheds some light on the oddly ill-understood area of DP adjuncts (Larson 1985; Whitman 2002; Mills 2005; Haegeman 2002; Bošković 2006 among others). One of the great challenges in this area is that adjunct DPs seem to be possible with only a restricted set of nouns (e.g., *do this every time/*period, do this Sunday/*March*). That neutral DNs are systematically possible here—and that this is because of the availability of this shift—explains some of the crucial data. Inconveniently, it also predicts that *?*a player six feet* should be possible. It seems at best marginal. Perhaps it's morphologically blocked by *a six-foot player*.

In both copular and adjoined positions, neutral DNs are possible with factor phrases:

(33) a. This player is three times my height.
b. a player three times my height

Given the approach we have taken so far, this is expected. If *my height* on its own denotes a degree, it's expected that it would be possible with with factor phrases. That the resulting complex expression is possible both in copular sentences and as DP adjuncts follows from the existence of the type shift:

(34) $[\![\textit{is} \text{ DEG-SHIFT } \textit{three times my height}]\!] = \lambda x \,.\, \mu_{\textbf{height}}(x) \geq [3 \times \mu_{\textbf{height}}(\textbf{me})]$

However, treating neutral DNs as purely degree-denoting only gets us so far, and a more complicated approach will be required. Predicates of change are one of the crucial cases. These systematically allow neutral DNs but not measure phrases:

(35) $\left\{ \begin{array}{l} \text{My height} \\ \text{\#Six feet} \end{array} \right\}$ changed.

Another crucial case is concealed questions. Classic concealed questions involve DPs that seem to be interpreted as though they were indirect questions, as in *Floyd knows the answer*, which means something like 'Floyd knows what the answer is'. Neutral DNs work this way too (cf. Moltmann 2009; Castroviejo Miró and Schwager 2008):

(36) Floyd knows $\left\{ \begin{array}{l} \text{my height} \\ \text{\#six feet} \end{array} \right\}$.

These examples are reminiscent of the temperature paradox (Montague 1973; Lasersohn 2005), which involves invalid inferences like (37a) and could be extended to (37b):

(37) a. The temperature is 25°. The temperature has risen. Therefore, 25° has risen.
b. The temperature is 25°. Floyd knows the temperature. Therefore, Floyd knows 25°.

One way to think about concealed questions is to take them to denote INDIVIDUAL CONCEPTS (Heim 1979; Romero 2005; Aloni and Roelofsen 2011; Bhadra and Mendia 2019), that is, functions from worlds or situations to individuals, type $\langle s, e \rangle$. We will take the situation route here. But because we are dealing with degrees, what we will actually need are DEGREE CONCEPTS, of type $\langle s, d \rangle$, functions from situations to degrees. Thus *Bertha's height* denotes a function from a situation to Bertha's height in that situation:

(38) $[\![\textit{Bertha's height}]\!] = \lambda s \,.\, \mu_{\textbf{height},s}(\textbf{Bertha})$

The neutral DN *height* by itself, therefore, takes an individual argument as well:[5]

(39) $[\![\textit{height}]\!] = \lambda x \lambda s \,.\, \mu_{\textbf{height},s}(x)$

[5]The denotation of *height* in (39) is very close to the one proposed in Bochnak (2013) for degree nominalizations in Luganda (Bantu), modulo reference to situations, and the use here of a measure function type rather than a degree predicate.

Intensionalizing degrees in this way makes possible an account of change predication. Predicates like *changed* are fundamentally intensional. For cases like *The president has changed*, it makes sense to take *change* to take an individual concept as an argument. For degree cases, it makes sense for *change* to take a degree concept argument as an argument. The denotation in (40) requires that evaluating the degree concept δ with respect to earlier situations gives a different result from later ones:

(40) $[\![\textit{changed}]\!] = \lambda\delta_{\langle s,d\rangle}\lambda s\,.\,\exists s'[s' \prec_{\textbf{time}} s \wedge \delta(s') \neq \delta(s)]$

That adds up to the sentence denotation in (41):

(41) $[\![\textit{Bertha's height changed}]\!]$
$= \lambda s\,.\,\exists s'[\,s' \prec_{\textbf{time}} s \wedge \mu_{\textbf{height},s'}(\textbf{Bertha}) \neq \mu_{\textbf{height},s}(\textbf{Bertha})\,]$

In purely extensional contexts, such intensional denotations can actually get in the way, and something will need to be done. The classic strategy is to assume an extensionalizing operator (Montague 1973 among many others), which we will treat as a type shift represented directly in the syntax. Following Montague, we will use the EXTENSIONALIZE symbol:

(42) $[\![^{\vee}]\!]^s = \lambda\alpha\,.\,\alpha(s)$

This shift causes an arbitrary expression α to be interpreted with respect to the current evaluation situation s:

(43) a. $[\![\textit{Bertha's height}]\!]^s = \lambda s\,.\,\mu_{\textbf{height},s}(\textbf{Bertha})$
b. $[\![^{\vee}\ \textit{Bertha's height}]\!]^s = [\![\textit{Bertha's height}]\!]^s(s) = \mu_{\textbf{height},s}(\textbf{Bertha})$

This shift can be freely inserted into the syntax whenever necessary to avoid a type clash. That's what happens in simple copular examples (we've also added a situation argument to DEG-SHIFT):

(44) a. Ingo is Bertha's height.
b. $[\![\text{DEG-SHIFT}]\!]^s = \lambda d\lambda x\,.\,\mu_{\textbf{scale}(d),s}(x) \geq d$
c. $[\![\textit{Ingo is [}\ \text{DEG-SHIFT}\ \textit{[}\ ^{\vee}\ \textit{Bertha's height]]}]\!]^s$
$= [\![\text{DEG-SHIFT}]\!]^s([\![^{\vee}\ \textit{Bertha's height}]\!]^s)([\![\textit{Ingo}]\!]^s)$
$= [\![\text{DEG-SHIFT}]\!]^s(\mu_{\textbf{height},s}(\textbf{Bertha}))(\textbf{Ingo})$
$= [\lambda d\lambda x\,.\,\mu_{\textbf{scale}(d),s}(x) \geq d](\mu_{\textbf{height},s}(\textbf{Bertha}))(\textbf{Ingo})$
$= 1$ iff $\mu_{\textbf{height},s}(\textbf{Ingo}) \geq \mu_{\textbf{height},s}(\textbf{Bertha})$

This correctly predicts that the sentence will be true in exactly the situations in which Ingo's height meets or exceeds Bertha's.

This sort of semantics also explains why neutral DNs can be said written down, as in (45a):

(45) a. Floyd wrote down Bertha's height.
b. #Floyd wrote down Bertha's tallness

Neutral DNs denote a form of degree, so, once extensionalized, they can be written down. As (45b) shows, that's not the case for biased DNs like *tallness*, to which we now turn.

4.2. Biased DNs

Biased DNs, we suggest, are more abstract. Rather than denoting a degree, they instead denote a portion of a quality, in the FKG sense. More precisely, a biased DN definite description like (46) picks out the maximal contextually salient portion p of a quality, in this case tallness, possessed by an individual (π represents the possession relation, following FKG):[6]

(46) $[\![\textit{Esme's tallness}]\!]^{C,s'} = \lambda s\,.\,\iota p \in C\,[\mathbf{tallness}_s(p) \wedge \pi_s(\mathbf{Esme}, p)]$

Hence *Esme's tallness* denotes the maximal salient portion of tallness that Esme has. This has an immediate welcome consequence: that biased DNs denote stuff—masses—and are predicted to require mass determiners and to resist pluralization:

(47) a. I didn't notice much $\left\{\begin{array}{l}\text{tallness}\\ \text{beauty}\\ \text{strength}\end{array}\right\}$.

b. *Floyd has all his friends' $\left\{\begin{array}{l}\text{tallnessses}\\ \text{beauties}\\ \text{strengths}\end{array}\right\}$.

For the sake of emphasizing the similarity to neutral DNs, we have intensionalized the denotation in (46) from the start. It would also have been reasonable to state it in purely extensional terms and use a Montagovian intensionalizing operator $\wedge$ to shift it when necessary.

Either way, biased DNs are predicted to resist multiplication with factor phrases, because multiplication is presumably defined for degrees but not for portions of abstract stuff:

(48) #Floyd is three times Bertha's tallness.

An especially important point of contrast between neutral and biased DNs involves copular sentences. Unlike neutral DNs, biased DNs don't use the copula, but require a verb of possession:

(49) Floyd $\left\{\begin{array}{l}\#\text{is}\\ \text{has}\end{array}\right\}$ Esme's $\left\{\begin{array}{l}\text{tallness}\\ \text{strength}\\ \text{beauty}\end{array}\right\}$.

What rules out the copula is straightforward. Neutral DNs work with the copula because they can be shifted with the DEG-SHIFT type-shift to property denotations. That type shift is only defined for degrees, so it can't combine with an expression that denotes a portion of a quality, however abstract. The same fact correctly predicts that biased DNs are impossible as DP adverbials:

[6] It would perhaps be useful to explicitly invoke a supremum operator here, in the spirit of Link 1983. We use ι for simplicity and to emphasize the connection to singular definite descriptions.

(50) *A player Esme's $\left\{\begin{array}{l}\text{tallness}\\ \text{beauty}\\ \text{strength}\end{array}\right\}$ is bound to make the team.

That all explains a variety of contexts in which biased DNs don't occur. But something more will need to be said about some of the contexts in which they do. In particular, our assumptions about the grammar of possession require some elaboration. A sentence such as (51) seems on its face to assert that Floyd and Esme share a portion of tallness (or strength or beauty) between them:

(51) Floyd has Esme's $\left\{\begin{array}{l}\text{tallness}\\ \text{strength}\\ \text{beauty}\end{array}\right\}$.

It's not clear that this interpretation makes much sense metaphysically. It seems too strong to suppose that Floyd's tallness is literally identical to Esme's. (And indeed, Moltmann's trope-based analysis is designed to capture that fact.) That's the case conceptually, and it's also the case as a matter of truth conditions. For (51) to be true, it's not necessary that Floyd and Esme be exactly the same height, for example. Rather, it seems to mean that Floyd tallness is of the same kind as Esme's. We will take this connection to kinds seriously. Possession in general often involves kind readings:

(52) a. Anton has Eva's nose.
'Anton has the same kind of nose as Eva.'
b. I have Floyd's car.
'I have the same kind of car as Floyd.'

It's certainly true that these sentences have a reading on which there is a single shared nose or car, but this is not the most natural reading. On the natural reading, (52a) claims Anton and Eva's noses realize the same kind of nose. One way to represent the meaning of (52a) is (53):

(53) $\exists k\left[\begin{array}{l}\iota x[\mathbf{nose}_s(x) \wedge \pi_s(\mathbf{Anton}, x)] \text{ realizes } k \wedge\\ \iota y[\mathbf{nose}_s(y) \wedge \pi_s(\mathbf{Eva}, y)] \text{ realizes } k\end{array}\right]$

It would take us too far afield to explore in more detail the semantics of possession and kinds, but the sketch in (53) will be sufficient to shed some light on how possession of qualities works. In the spirit of (53), possessed biased DN can be interpreted as in (54):

(54) a. Floyd has Esme's tallness.
b. $\exists k\left[\begin{array}{l}\iota p \in C\,[\mathbf{tallness}_s(p) \wedge \pi_s(\mathbf{Floyd}, p)] \text{ realizes } k \wedge\\ \iota p' \in C\,[\mathbf{tallness}_s(p') \wedge \pi_s(\mathbf{Esme}, p')] \text{ realizes } k\end{array}\right]$

Importantly, this can be true if Floyd and Esme have the same height. That's a special case of realizing the same kind (Anderson and Morzycki 2015; Scontras 2017). But it can also mean that their heights have in common something other than their linear extent, perhaps both being gracefully tall or hulkingly so.

A similar challenge arises in how biased DNs are interpreted with predicates of change. Because biased and neutral DN denotations vary in type, it will be necessary to shift the type of the

corresponding change predicates slightly. Alongside the version of *change* already proposed and repeated in (55a), we need to introduce the version in (55b):

(55) a. $[\![\textit{changed}]\!] = \lambda \delta_{\langle s,d \rangle} \lambda s \,.\, \exists s' [s' \prec_{\textbf{time}} s \wedge \delta(s') \neq \delta(s)]$
b. $[\![\textit{changed}_2]\!] = \lambda P_{\langle s,p \rangle} \lambda s \,.\, \exists s' [\, s' <_{\textbf{time}} s \wedge P(s) \neq P(s') \,]$

These are identical apart from the type. That's possible because biased DNs already denote the *intension* of a quality portion, like the intensional degree concept denotations of neutral DNs. This predicts sentence denotations like (56):

(56) $[\![\textit{Esme's tallness changed}_2]\!]^C$
$= \lambda s \,.\, \exists s' \left[\begin{array}{c} s' <_{\textbf{time}} s \wedge \iota p \in C[\textbf{tallness}(p) \wedge \pi_s(\textbf{Esme}, p)] \neq \\ \iota p' \in C[\textbf{tallness}(p') \wedge \pi_{s'}(\textbf{Esme}, p')] \end{array} \right]$

This requires that Esme's tallness portion in an earlier situation be non-identical to her tallness portion in a subsequent one. For neutral DNs, change simply required that a particular measure change over time. For biased DNs, change requires non-identical portions of abstract tallness. These can still differ in their measure on the tallness scale, of course. But they can also differ in the other ways envisaged by FKG, including in the abstract ways that separate lanky tallness from bulked-up tallness. More generally, in any comparison of sameness and change, there can only be one way of comparing degree concepts: according to the measure.

4.3. Revisiting the Taj Mahal

We are now in a position to return to the modified FKG example with which we began the paper:

(57) The Taj Mahal and Stata Center have the same $\left\{ \begin{array}{c} \text{height} \\ \text{beauty} \end{array} \right\}$, but their $\left\{ \begin{array}{c} \#\text{heights} \\ \text{beauties} \end{array} \right\}$ are different.

The reason for the contradiction with the neutral DN *height* is that there is no way for heights to vary except in their measure. If two buildings have the same height, their height measure must be identical. But not so with biased DNs. The beauty of one building can vary from that of another along arbitrary abstract dimensions. For us, this is the consequence of the fact that different portions of abstract beauty can differ arbitrarily.

Degrees are just measures. One can intensionalize them, of course, but in the end they are always about measurement. But there is more to a quality than its measure. That makes predication of sameness or difference potentially more complicated.

Of course, to make this fully convincing we would have to provide an explicit semantics for *same* and *different*. We leave that to future work.

5. Taking stock

Qualities were proposed because many languages express gradable predicates in terms of possession rather than simple predication. That suggests a typological distinction, with some languages primarily using degrees and others qualities. English degree nominalizations demon-

strate that both strategies can be used in tandem. They also provide evidence for degree concepts, and they point to the possibility that other languages make a similar distinction among their degree nominalizations.

One topic for future investigation is the distributivity patterns of neutral DNs and biased DNs. As we noted, only biased DNs allow for a distributive reading:

(58) The {age / size} of the committee intimidated Floyd. *(neutral)*

a. *age*: #The members were all elderly.

b. *size*: #The members were all enormous.

(59) The {oldness / tallness} of the committee intimidated Floyd. *(biased)*

a. *age*: The members were all elderly.

b. *size*: The members were all enormous.

Why can a committee's oldness but not age involve the ages of the individual members? It's not obvious to us that this follows from anything we have proposed. The behavior of biased DNs here may be a consequence of their being mass terms. Other mass terms can also be distributed among members of a group:

(60) The committee's {hair / #shirt} was mostly gray.

But what about neutral DNs makes them resist such a reading?

Another curious observation is that neutral DNs seem to be odd with comparative and equative morphology. As biased DNs are possessed masses, it's unsurprising they can be modified by *more* and *much*, but this is not the case for neutral DNs:

(61) a. Floyd has more beauty than Clyde. *(biased)*

b. Floyd has as much beauty than Clyde.

(62) a. #Floyd is more size than Clyde. *(neutral)*

b. #Floyd is as much size as Clyde.

There is then the question of how *ways* in which one holds a property can be represented. We have established that *beauty* can be used in Taj Mahal sentences without contradiction because it involves not to a degree but a quality. Thus the beauties of the two buildings can differ in quality in multiple ways, not just in their amount. But what is a way, concretely? How does it relate to what qualities are? Are ways perhaps Carlsonian *kinds* of qualities? That may provide an avenue for understanding what it means to say that someone's tallness has changed, or that someone's tallness is different from someone else's.

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