

# حل امتحان كالكولس 1-البولتكناك

للأستاذ أحمد عرفه

0786543665



شرح المادة بتطبيق اسمه بروفيسور للأستاذ أحمد عرفه

الدورة ستكلفك فقط ١٥ دينار ستحصل خلالها على

شرح لكل تفاصيل المادة بجودة تصوير عالية جداً

متابعة خلال الفصل حتى تنهي الفايمل بالإجابة عن جميع أسئلتك من خلال مجموعة الدورة على الفيس

ملخص المادة و دوسيات لأسئلة السنوات



Name:

Lecture Time:

Instructor:

Date: 17/11/2019.

Write the correct answers

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20

Q1) If  $f(x) = \sqrt{4-x^2}$ , then domain  $f$  and Range  $f$  are

- a)  $D_f: (-\infty, -2] \cup [2, \infty)$   $R_f: [0, 2]$  b)  $D_f: [-2, 2]$   $R_f: [0, 2]$  c)  $D_f: [-2, 2]$   $R_f: [0, \infty)$  d)  $D_f: [0, 2]$   $R_f: [0, \infty)$

Q2) If  $f(x) = \frac{3x+1}{x-2}$ , then  $f^{-1}(x) =$

- a)  $\frac{2x+1}{x+3}$  b)  $\frac{x-2}{3x+1}$  c)  $\frac{x+1}{x-3}$  d)  $\frac{2x+1}{x-3}$

Q3) If  $f(x) = \begin{cases} [x-c^2] & , x \leq 3 \\ 2x-4 & , x > 3 \end{cases}$

Where  $[..]$  in  $f$  is the greatest integer function is continuous at  $x=2$ , then the values of  $c$

- a)  $[-1, 0]$  b)  $[0, 1]$  c)  $[-1, 1]$  d)  $[-1, 1] \setminus \{0\}$

Q4) One of the following statements is false

- a) The domain of  $f - g$  is the intersection of the domains of  $f$  and  $g$ .  
 b) The domain of  $g \circ f$  consists of all values in domain  $f$  and values of  $x$  where  $f(x)$  is in domain  $g$ .  
 c) The graph of  $f(-x)$  obtained by reflect graph  $f$  about  $x$ -axis.  
 d) The graph of even functions is symmetric about  $y$ -axis.

Q5) If  $g(x) = \frac{f(x) + f(-x)}{2}$ ,  $h(x) = \frac{f(x) - f(-x)}{2}$ , where  $f$  has domain of all real numbers, then

- a)  $g$  is odd and  $h$  is even function  
 b)  $g$  is even and  $h$  is odd function  
 c)  $g$  and  $h$  are odd functions  
 d)  $g$  and  $h$  are even functions

Q6) The vertical and horizontal asymptotes for  $f(x) = \frac{2x^2 - 8}{x^2 - 3x + 2}$  respectively are

- a)  $x=1, 2$   $y=2$   
 b)  $x=-1, y=2$   
 c)  $x=2, y=1$   
 d)  $x=1, y=2$

Q7) If  $\lim_{x \rightarrow \infty} \csc\left(\frac{\pi x^2}{2 - 3x^2}\right)$

- a)  $\frac{-1}{\sqrt{3}}$   
 b)  $-2$   
 c)  $-\frac{2}{\sqrt{3}}$   
 d)  $\frac{-1}{3}$

Q8) If the graph of the function  $f$  is reflected about  $x$ -axis, stretched vertically by 2 and then shifted left by 3 units. The new function will be =

- a)  $-2f(x+3)$   
 b)  $2f(-x+3)$   
 c)  $-\frac{1}{2}f(x+3)$   
 d)  $2f(-x-3)$

Q9) The discontinuity points of  $f(x) = \frac{|x-2|}{\csc x - 2}$

- a)  $x = \frac{\pi}{6} \pm 2n\pi, \frac{5\pi}{6} \pm 2n\pi$   
 b)  $x = \frac{\pi}{3} \pm 2n\pi, \frac{5\pi}{3} \pm 2n\pi$   
 c)  $x = \frac{\pi}{3} \pm 2n\pi, \frac{2\pi}{3} \pm 2n\pi$   
 d) otherwise where  $n=0, 1, 2, \dots$

Q10) one of the following functions has inverse on its natural domain

- a)  $f(x) = x^2 - 3$   
 b)  $f(x) = x^3 + 4$   
 c)  $f(x) = |x - 5|$   
 d)  $f(x) = \cos x$

Q11)  $\lim_{x \rightarrow \infty} \frac{(3x-1)^2(2x^2+1)}{5x(3x+3)^3} =$

- a) 1  
 b)  $\frac{2}{3}$   
 c)  $\frac{2}{15}$   
 d)  $\frac{3}{5}$

Q12) The interval of continuity for  $f(x) = \sqrt{|x-4|+3x}$  is

- a)  $[4, \infty)$   
 b)  $[-2, \infty)$   
 c)  $[2, 4]$   
 d)  $[0, 4]$

Q13) The graph of the curve  $y^3 = 2x \cos x$  is symmetric about

- a) origin  
 b)  $x$ -axis  
 c)  $y$ -axis  
 d) No symmetry

Q14) If  $f(x) = \csc^2 \sqrt{x}$ , then

a)  $D_f: x \geq 0$  and  $x \neq n^2 \pi^2, R_f: (0,1]$

b)  $D_f: x \geq 0$  and  $x \neq n^2 \pi^2, R_f: [0,1]$

c)  $D_f: x > 0$  and  $x \neq 2n^2 \pi^2, R_f: [1, \infty)$

d)  $D_f: x > 0$  and  $x \neq n^2 \pi^2, R_f: [1, \infty)$

Q15)  $\lim_{x \rightarrow \infty} x \left( \sin \frac{2}{x} + \frac{1}{x^2} \cos 3x \right) =$

a)  $+\infty$

b) 5

c) 3

d) 2

Q16)  $\lim_{x \rightarrow 0} \frac{\tan^2 ax - 2x^2}{4x^2 - \sin ax^2} = 1$ , then  $a =$

a) -3, 2

b) 2

c) 3, -2

d) 1, -3

Q17)  $f(x) = \begin{cases} |x-5| & , x \leq 2 \\ x^2 - 9 & , 2 < x \leq 3 \\ 2x - 6 & , x > 3 \end{cases}$ ,  $g(x) = \begin{cases} \cos x + 2 & , x \leq 0 \\ x^2 - 2x + 3 & , 0 < x < 1 \\ |x-2| + 1 & , x \geq 1 \end{cases}$

$f \circ g$  is continuous at

a)  $x=0, 1$

b)  $x=1$

c)  $x=1, 3$

d)  $x=0$

Q18)  $f(x) = x + \frac{1}{x}$ ,  $g(x) = \frac{x-1}{x+2}$ , the domain of  $f \circ g$  is

a)  $x \neq 0, 1$

b)  $x \neq 1, -2$

c)  $x \neq 0, 1, -2$

d)  $x \neq 0, -2$

Q19)  $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 6x + 9}}{x^2 - 6x + 9}$

a)  $-\infty$

b) 0

c) -1

d)  $+\infty$

Q20)  $\lim_{x \rightarrow -\infty} x + \sqrt{x^2 + 4x}$

a) -1

b) -2

c)  $-\infty$

d)  $+\infty$

Best wishes

17/11/2019

1<sup>st</sup> semester

Solved by Teacher

Ahmad Arafeh

Q1)

Doman  $\sqrt{4 - x^2}$  is  $4 - x^2 \geq 0$

$$x^2 \leq 4 \rightarrow |x| \leq 2 \rightarrow -2 \leq x \leq 2$$

$$D\sqrt{4 - x^2} = [-2, 2]$$

Range  $x^2 \geq 0$

Range  $-x^2 \leq 0$

Range  $4 - x^2 \leq 4$

Range  $\sqrt{4 - x^2} \leq \sqrt{4}$

الآن أي جذر زوجي مداه (مخرجاته) أكبر  
أو يساوي صفر .

$$0 \leq \sqrt{4 - x^2} \leq 2$$

$$[0, 2] \quad \textcircled{B}$$

$$\text{Q2)} \quad y = \frac{3x+1}{x-2}$$

$$x = \frac{3y+1}{y-2}$$

نبدأ بالضرب التبادلي

$$xy - 2x = 3y + 1$$

$$xy - 3y = 1 + 2x$$

$$y(x - 3) = 1 + 2x$$

$$y = \frac{1 + 2x}{x - 3} = f^{-1}(x)$$

$\textcircled{D}$

Q3) Cts at  $x=3$  خطأ مطبعي بالسؤال

$\lim_{x \rightarrow 3} f(x)$  exist

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^-} f(x)$$

$$\lim_{x \rightarrow 3^+} 2x - 4 = \lim_{x \rightarrow 3^-} [x - c^2]$$

$$2 = [3 - c^2]$$

$$2 < 3 - c^2 < 3$$

$$-1 < -c^2 < 0$$

$$0 < c^2 < 1$$

$$-1 < c < 1$$

$$[-1, 1] \quad \textcircled{C}$$

Q4)

False statement

C) The graph of  $f(-x)$  obtained  
by reflect graph  $f$  about  $x$  - axis

الصواب

The graph of  $f(-x)$  obtained by  
reflect graph  $f$  about  $-axis$

Q5)

$$g(-x) = \frac{f(-x) + f(x)}{2}$$

$$= \frac{f(x) + f(-x)}{2}$$

$$= g(x)$$

*g is even function*

$$h(-x) = \frac{f(-x) - f(x)}{2}$$

$$-h(x) = -\frac{f(x) - f(-x)}{2}$$

$$= \frac{f(-x) - f(x)}{2}$$

$$h(-x) = -h(x)$$

*h is odd function*

(B)

Q6)

V.A

$$f(x) = \frac{2(x-2)(x+2)}{(x-2)(x-1)}$$

V.A is  $x = 1$  Only

H.A

$$\lim_{x \rightarrow \pm\infty} \frac{2x^2 - 8}{x^2 - 3x + 2} = \lim_{x \rightarrow \pm\infty} \frac{2x^2}{x^2} = 2$$

H.A is  $y = 2$

(D)

Q7)

$$\lim_{x \rightarrow \infty} \csc\left(\frac{\pi x^2}{2 - 3x^2}\right) =$$

$$\csc\left(\lim_{x \rightarrow \infty} \frac{\pi x^2}{2 - 3x^2}\right)$$

$$= \csc\left(\lim_{x \rightarrow \infty} \frac{\pi x^2}{-3x^2}\right)$$

$$= \csc\left(\frac{-\pi}{3}\right)$$

$$= -\csc\left(\frac{\pi}{3}\right) = -\frac{2}{\sqrt{3}}$$

(C)

Q8) Reflected about x-axis  $-f(x)$

Stretched vertically by 2  $-2f(x)$

Shifted left 3 units  $-2f(x + 3)$

(A)

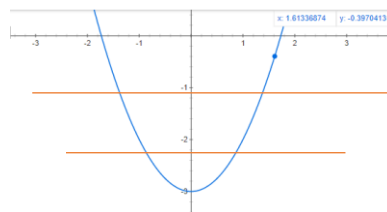
Q9) أصفار المقام

$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$

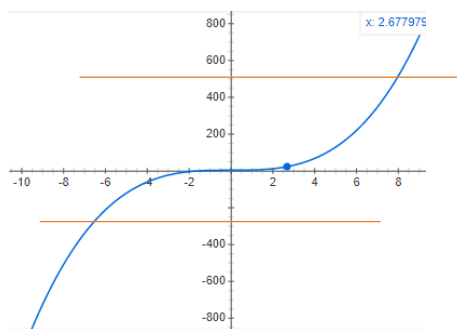
$$x = \frac{\pi}{6} \pm 2n\pi, \frac{5\pi}{6} \pm 2n\pi \quad (A)$$

Q10) a)  $x^2 - 3$  Not (1-1)



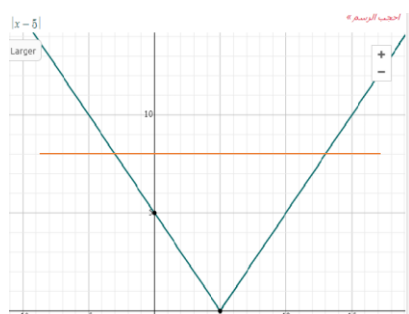
No inverse

$$x^3 + 4 \quad (1-1)$$



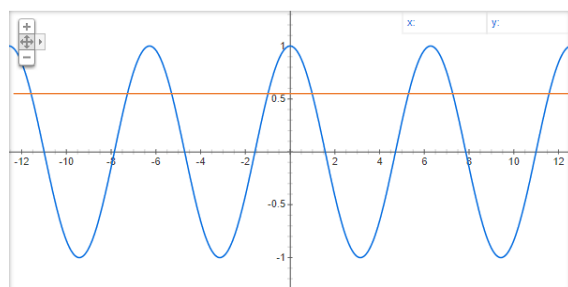
Has inverse

$$c) |x - 5| \quad \text{Not (1-1)}$$



Has no inverse

$$d) \cos x \quad \text{Not (1-1)}$$



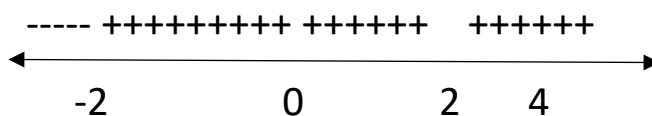
Has no inverse

(B)

$$Q11) \lim_{x \rightarrow \infty} \frac{(3x-1)^2(2x^2+1)}{5x(3x+3)^3} =$$

$$\lim_{x \rightarrow \infty} \frac{(9x^2)(2x^2)}{5x(27x^3)} = \lim_{x \rightarrow \infty} \frac{2x^4}{15x^4} = \frac{2}{15} \quad (C)$$

$$Q12) |x - 4| + 3x \geq 0$$



الأرقام وضعناهم من خيارات السؤال

$$[-2, \infty)$$

(B)

$$Q13) y^3 = 2x \cos x$$

$$\text{About y-axis} \quad y^3 = 2(-x) \cos(-x)$$

$$(x \rightarrow -x) \quad y^3 = 2(-x) \cos x$$

Not Symm. About y

$$\text{About x-axis} \quad (-y)^3 = 2x \cos x$$

$$(y \rightarrow -y) \quad -y^3 = 2x \cos x$$

Not Symm. About x

About the origin

$$(x \rightarrow -x) \quad (-y)^3 = 2(-x) \cos(-x)$$

$$(y \rightarrow -y) \quad -y^3 = 2(-x) \cos x$$

$$-y^3 = -2x \cos x$$

$$y^3 = 2x \cos x$$

Symmetric about the origin

(A)

**Ahmad Arafah**

0786543665

$$Q14) f(x) = \frac{1}{\sin^2(\sqrt{x})}$$

D f(x):

$\sqrt{x}$  is  $x > 0$  (تم استثناء الصفر (أصفار مقام)

$\sqrt{x} = \pm n\pi \rightarrow x = n^2\pi^2$  أصفار المقام

Df:  $x > 0$  and  $x \neq n^2\pi^2$

R f(x):

$$0 \leq \sin^2(\sqrt{x}) \leq 1$$

$$1 \leq \frac{1}{\sin^2(\sqrt{x})} < \infty$$

$[1, \infty)$

(D)

$$Q15) \lim_{x \rightarrow \infty} x \sin\left(\frac{2}{x}\right) + \frac{1}{x} \cos(3x)$$

$$\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{x}} = \lim_{y \rightarrow 0} \frac{\sin 2y}{y} = 2$$

$$\text{Let } y = \frac{1}{x}, x \rightarrow \infty, y \rightarrow 0$$

And the second limit

$$-1 \leq \cos(3x) \leq 1$$

$$\begin{array}{ccc} -1 & \leq \frac{1}{x} \cos(3x) \leq & \frac{1}{x} \\ \swarrow & & \searrow \\ \lim_{x \rightarrow \infty} \frac{-1}{x} = 0 & & \lim_{x \rightarrow \infty} \frac{1}{x} = 0 \end{array}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} \cos(3x) = 0$$

Then

$$\begin{aligned} \lim_{x \rightarrow \infty} x \sin\left(\frac{2}{x}\right) + \frac{1}{x} \cos(3x) \\ = 2 + 0 \\ = 2 \end{aligned}$$

(D)

Q16) بقسمة كل حد بالبسط و المقام على  $x^2$

$$\lim_{x \rightarrow 0} \frac{\frac{\tan^2 ax}{x^2} - \frac{2x^2}{x^2}}{\frac{4x^2}{x^2} - \frac{\sin ax^2}{x^2}} = 1$$

$$\frac{a^2 - 2}{4 - a} = 1$$

$$a^2 - 2 = 4 - a$$

$$a^2 + a - 6 = 0$$

$$(a + 3)(a - 2) = 0$$

$$a = -3, 2$$

القيمتان تحققان النهاية عند التعويض

(A)

أحمد عرفه

0786543665



Q17) i)  $g$  is cts at  $x = 0$

$$g(0) = 3$$

Now, study if  $f$  cts at  $x = 3$

$$f(3) = 9 - 9 = 0$$

$$\lim_{x \rightarrow 3} f(x) = 0$$

$f$  cts at  $x = 3$

$f \circ g$  cts at  $x = 0$

ii)  $g$  is cts at  $x = 1$

$$g(1) = 2$$

Now, study if  $f$  cts at  $x = 2$

$$f(2) = |2 - 5| = 3$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2 - 9 = -5$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} |x - 5| = 3$$

$$\lim_{x \rightarrow 2} f(x) = D.N.E$$

$f$  not cts at  $x = 2$

$f \circ g$  not cts at  $x = 1$

iii)  $g$  is cts at  $x = 3$

$$g(3) = 2$$

$f$  not cts at  $x = 2$

$f \circ g$  not cts at  $x = 3$

(D)

Q18)  $D g(x) = \mathbb{R} - \{-2\}$

$$f \circ g(x) = \frac{x-1}{x+2} + \frac{x+2}{x-1}$$

$$D = \mathbb{R} - \{-2, 1\}$$

$$D f \circ g = (\mathbb{R} - \{-2, 1\}) \cap \mathbb{R} - \{-2\} \\ = \mathbb{R} - \{-2, 1\}$$

$$x \neq -2, 1$$

(B)

Q19)

$$\lim_{x \rightarrow 3^-} \frac{\sqrt{(x-3)(x-3)}}{x^2 - 6x + 9} = \lim_{x \rightarrow 3^-} \frac{\sqrt{(x-3)^2}}{x^2 - 6x + 9}$$

$$= \lim_{x \rightarrow 3^-} \frac{|x-3|}{x^2 - 6x + 9}$$

$$= \lim_{x \rightarrow 3^-} \frac{3-x}{x^2 - 6x + 9}$$

$$= \lim_{x \rightarrow 3^-} \frac{3-x}{(x-3)(x-3)}$$

$$= \lim_{x \rightarrow 3^-} \frac{-1}{(x-3)} = \frac{-1}{0^-} = \infty \quad (D)$$

$$\text{Q20) } \lim_{x \rightarrow -\infty} x + \sqrt{x^2 + 4x} \cdot \frac{x - \sqrt{x^2 + 4x}}{x - \sqrt{x^2 + 4x}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 - x^2 - 4x}{x - \sqrt{x^2 + 4x}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-4x}{x - \sqrt{x^2}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-4x}{x - |x|} = \lim_{x \rightarrow -\infty} \frac{-4x}{2x} = -2$$

(B)