خدمتكم عبادة نتقرب بها الى الله

2021

أسئلة سنوات
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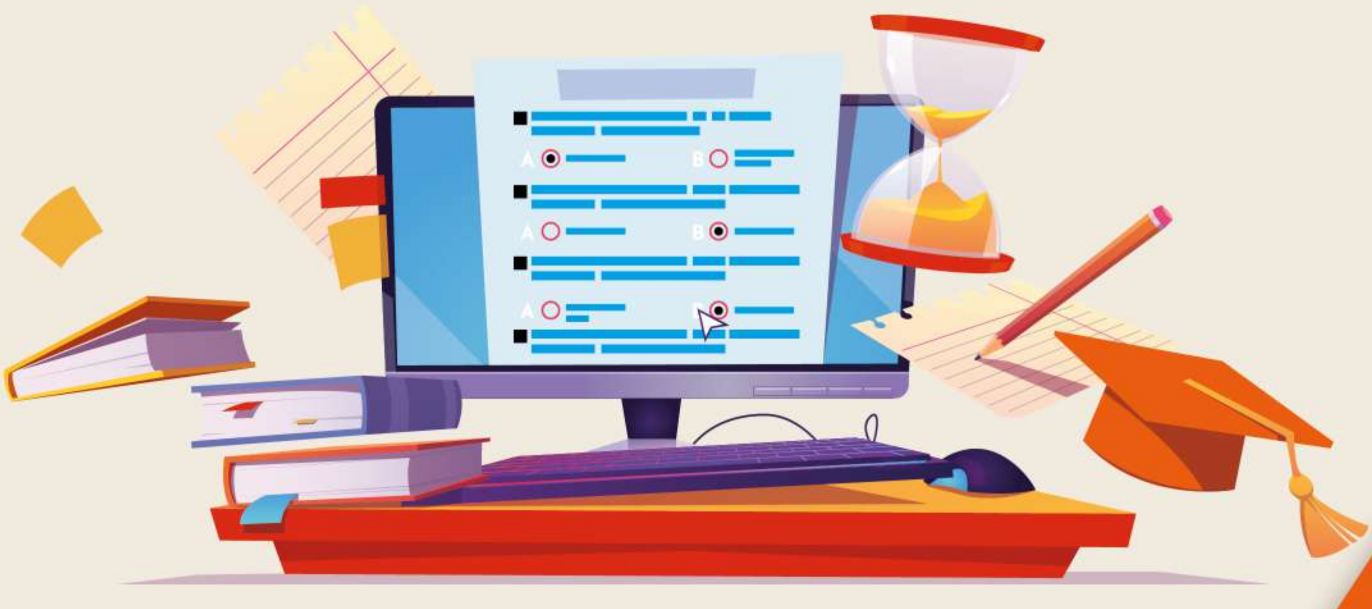
تفاضل وتكامل 2



لجنة السنافر

Question 1 Choose the correct answer for the questions (1-18) and complete the table

Q(1)	Q(2)	Q(3)	Q(4)	Q(5)	Q(6)	Q(7)	Q(8)	Q(9)	Q(10)
Q(11)	**Q(12)**	**Q(13)**	**Q(14)**	**Q(15)**	**Q(16)**	**Q(17)**	**Q(18)**		

Q(1) By converting from cartesian to polar, the cartesian point $(x, y) = (-3, 0)$ becomes

a)$(13, \pi)$ b)$(3, \pi \pm 2n\pi)$ c)$(-3, \pi \pm 2n\pi)$ d)$(16, \pi \pm 2n\pi)$

Q(2) Converting from polar to cartesian, the polar point $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$ becomes

a)$(3, 3)$ b)$(-3, 3\sqrt{3})$ c)$(-3, 3)$ d)$((-\sqrt{3}, 3\sqrt{3}))$

Q(3) In Polar system, $4\cos\theta = 3$ defines a

a)Horizontal line b) Vertical Line c) Circle d)Square

Q(4) The sequence $\left\{\frac{n^2}{2n^2+3}\right\}$ converges to

a) $\frac{3}{2}$ b)$-\frac{1}{2}$ c)$\frac{5}{2}$ d)$\frac{1}{2}$

Q(5) By telescoping method we get $\sum_{1}^{\infty} \frac{2}{k(k+1)} =$

a) $\frac{1}{4}$ b)$\frac{1}{3}$ c)$\frac{1}{2}$ d)2

Q(6) The series $\sum_{1}^{\infty} \frac{1}{\sqrt{2k}}$

2 1 3 2

a) converges conditionally b) converges absolutely c) diverges d) None

Q(7) The series $\sum_{1}^{\infty} \frac{k^3+2k+22}{2k^8+1}$

2 2

a) converges b)diverges c)absolutely diverges d) None

Q(8) The Maclaurin series for $\sin x^3$ is

Q(9) $\sum_{1}^{\infty} (\ln(2k+1))^{k}$

a) converges b) converges conditionally c) diverges d) diverges absolutely

Q(10) The value of the improper integral $\int_{1}^{2} \frac{3}{1-x}\,dx$ is

a) 1 b) 2 c) 6 d) diverges

Q(11) The value of the derivative $\frac{d}{dx} x^{x}$ at 2 is

a) 4 b) $1 + \ln 4$ c) $4 + \ln 4$ d) $4 + 4\ln 4$

Q(12) $\lim_{x \to 0} (1 + 2x)^{\frac{3}{x}} =$

a) e^{6} b) 6 c) -6 d) e^{-6}

Q(13) $\lim_{x \to \infty} \frac{x^{11}}{e^{2x}} =$

a) 1 b) 0 c) 4 d) 3

Q(14) $\lim_{x \to \infty} \left(2 (\ln x)^{\frac{1}{x}}\right) =$

a) 1 b) 2 c) $\ln 2$ d) $\ln 3$

Q(15) $\lim_{x \to 0} (e^{2x} + x)^{\frac{1}{x}} =$

a) 3 b) 4 c) e d) e^{3}

Q(16) $\int_{1}^{\sqrt{2}} \frac{4dx}{x^{2}\sqrt{4 - x^{2}}}dx$

a) $\sqrt{3} - 2$ b) $2\sqrt{3} - 1$ c) $\sqrt{3} - 1$ d) $\sqrt{3} - 3$

Q(17) $\int_{0}^{\pi} 40 \sin 7x \cos 3x\,dx =$

a) -17 b) -7 c) 0 d) -71

Q(18) $\int_{0}^{\ln 3} x e^{x}\,dx =$

a) $3\ln 3 - 2$ b) $3\ln 3$ c) $\ln 3 - 2$ d) $5\ln 3 - 2$

Question 2 Find the value of the integral $\int_{1}^{\infty} \frac{dx}{x^{12}}$ =

Question 3 Find the value of the integral $\int_{-1}^{0} \frac{dx}{x^{2} + x - 2}$ = $\frac{1}{2}$

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1

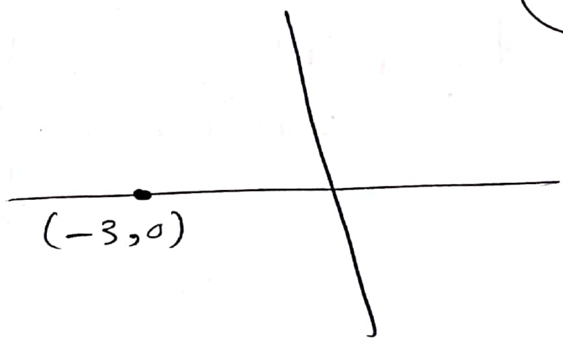
$Q_1)$ $r^2 = x^2 + y^2$

$r^2 = (-3)^2 + 0^2$

$r^2 = 9$

$r = 3$

$\theta = \pi \pm 2n\pi$



$(3, \pi \pm 2n\pi)$. b

$Q_2)$ $x = r\cos\theta = 6\cos\frac{2\pi}{3} = 6.\left(\frac{-1}{2}\right) = -3$

$y = r\sin\theta = 6\sin\frac{2\pi}{3} = 6 \cdot \frac{1}{2} = 3$

$(-3, 3)$ c

②

Q4) $\lim\limits_{n\to\infty} \frac{n^2}{2n^2+3} = \lim\limits_{n\to\infty} \frac{n^2}{2n^2}$

$= \frac{1}{2}$ ⓓ

Q5) $\frac{A}{k} + \frac{B}{(k+1)} = \frac{2}{k(k+1)}$

$2 = A(k+1) + Bk$

If $k=0 \longrightarrow A=2$

$k=-1 \longrightarrow 2=-B$

$B=-2$

$S_n = \sum_{i=1}^{n} \left(\frac{2}{i} - \frac{2}{i+1}\right) = 2\sum_{i=1} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$S_n = 2\left[\left(1-\cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) \cancel{+} \cdots \cancel{+} \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)\right]$

$S_n = 2\left(1 - \frac{1}{n+1}\right)$

$\lim\limits_{n\to\infty} S_n = 2\left(1 - \frac{1}{\infty}\right)$

$= 2.1$

$= 2$ ⓓ

③

6) $\sum_{1}^{\infty} \frac{1}{\sqrt{2}\sqrt{k}} = \sum \frac{1}{\sqrt{2}\, k^{1/2}}$ $p = \frac{1}{2} < 1$

Div.

(c)

7) $b_k = \frac{\cancel{5} k^3}{k^8} = \frac{1}{k^5}$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^3+2k+22}{2k^8+1} \cdot \frac{k^5}{1}$$

$$= \lim_{k\to\infty} \frac{k^8+2k^6+22k^5}{2k^8}$$

$$= \lim_{k\to\infty} \frac{\cancel{k^8}}{2\cancel{k^8}}$$

$$= \frac{1}{2}$$

$0 < \frac{1}{2} < 1$

$\Rightarrow \sum b_k = \sum \frac{1}{k^5}$ $p = 5 > 1$

Conv.

(a)

Q8) $$\sin x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k (x^3)^{2k+1}}{(2k+1)!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+3}}{(2k+1)!}$$

Q9) $\sum_{1}^{\infty} \frac{1}{(\operatorname{Ln}(2k+1))^k}$ Root. (A)

$\lim_{k \to \infty} \left(\frac{1}{\operatorname{Ln}(2k+1)^k} \right)^{1/k}$

$= \lim_{k \to \infty} \frac{1}{\operatorname{Ln}(2k+1)} = \frac{1}{\infty} = 0 < 1$

Conv. (a)

Q10) $\lim_{m \to 1^+} \int_m^2 \frac{3}{1-x} dx = \lim_{m \to 1^+} 3 \operatorname{Ln} |1-x| \Big|_m^2$

$= \lim_{m \to 1^+} \left[3 \operatorname{Ln}|1-2| - 3 \operatorname{Ln}|1-m| \right]$

$= 0 - 3(-\infty)$

$= \infty$ (d)

Div.

Q11) $y = x^x$

$\operatorname{Ln} y = \operatorname{Ln} x^x$

$\operatorname{Ln} y = x \operatorname{Ln} x$

نشتق

$\frac{y'}{y} = x \cdot \frac{1}{x} + \operatorname{Ln} x \cdot 1$

$y' = y(1 + \operatorname{Ln} x)$

$y' = x^x (1 + \operatorname{Ln} x)$

$y'(2) = 2^2 (1 + \operatorname{Ln} 2)$

$= 4(1 + \operatorname{Ln} 2) = 4 + 4 \operatorname{Ln} 2$ غير موجودة

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Q_{12}) $\lim_{x\to 0} (1+2x)^{\frac{-3}{x}}$

let $x = \frac{1}{y}$

$x \to 0$
$y \to \infty$

$\lim_{y\to\infty} \left(1+\frac{2}{y}\right)^{-3y} = e^{2\cdot(-3)} = e^{-6}$ (d)

Q_{13}) $\lim_{x\to\infty} \frac{x^{11}}{e^{2x}} =$ لوبيتال

Finally $\lim_{x\to\infty} \frac{11!}{2^{11} e^{2x}} = \frac{11!}{\infty} = 0$ (b)

Q_{14}) $\lim_{x\to\infty} \left(2(\ln x)^{\frac{1}{x}}\right)$

$2 \lim_{x\to\infty} (\ln x)^{1/x}$

$= 2 \cdot 1$

$= 2$ (b)

$y = (\ln x)^{1/x}$

$\ln y = \ln (\ln x)^{1/x}$

$\ln y = \frac{\ln(\ln x)}{x}$

$\ln y = \lim_{x\to\infty} \frac{\ln(\ln x)}{x} \quad \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{\frac{1}{x}}{\ln x}}{1}$

$\ln y = \lim_{x\to\infty} \frac{1}{x \ln x}$

$\ln y = 0$

$y = e^{0}$

$y = 1$

Q15) $\lim_{x \to 0} (e^{2x} + x)^{\frac{1}{x}} =$

$= e^{2}$

6

$y = (e^{2x} + x)^{\frac{1}{x}}$

$\ln y = \ln(e^{2x} + x)^{\frac{1}{x}}$

$\ln y = \dfrac{\ln(e^{2x} + x)}{x}$ $\quad \frac{\infty}{\infty}$

$\ln y = \lim_{x \to \infty} \dfrac{\ln(e^{2x} + x)}{x}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \dfrac{\frac{2e^{2x}+1}{e^{2x}+x}}{1}$ $\quad \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \dfrac{4e^{2x}}{2e^{2x}+1}$ $\quad \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \dfrac{8\cancel{e^{2x}}}{4\cancel{e^{2x}}}$

$\ln y = 2$

$y = e^{2}$

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Q16) $x = 2\sin\theta$

$x^2 = 4\sin^2\theta$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\cdot\cos\theta$

$dx = 2\cos\theta \, d\theta$

If $x=1 \longrightarrow 1 = 2\sin\theta \rightarrow \frac{1}{2} = \sin\theta \longrightarrow \theta = \frac{\pi}{6}$

$x = \sqrt{2} \longrightarrow \sqrt{2} = 2\sin\theta \longrightarrow \frac{1}{\sqrt{2}} = \sin\theta \longrightarrow \theta = \frac{\pi}{4}$

$$\int_{\pi/6}^{\pi/4} \frac{\cancel{4}\cdot \cancel{2\cos\theta}\, d\theta}{\cancel{4}\sin^2\theta \cdot \cancel{2\cos\theta}} = \int_{\pi/6}^{\pi/4} \csc^2\theta \, d\theta$$

$$= -\cot\theta \Big|_{\pi/6}^{\pi/4}$$

$$= -1 + \sqrt{3}$$

$$= \sqrt{3} - 1 \quad \text{(C)}$$

Q17) $\int_0^{\pi} 40 \sin 7x \cos 3x \, dx$

$$40 * \frac{1}{2} \int_0^{\pi} \sin(4x) + \sin(10x) \, dx$$

$$20 \left(-\frac{\cos 4x}{4} - \frac{\cos 10x}{10} \right) \Big|_0^{\pi}$$

$$= 20 \left[\left(\frac{-1}{4} - \frac{1}{10} \right) - \left(\frac{-1}{4} - \frac{1}{10} \right) \right]$$

$$= 20\left(\cancel{\frac{-1}{4}} - \cancel{\frac{1}{10}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{10}} \right) = 0 \quad \text{(C)}$$

8

Q18) $\int_0^{Ln 3} x e^x \, dx$

$$\begin{array}{ccc} x & & e^x \\ & + & \\ 1 & & e^x \\ & - & \\ 0 & & e^x \end{array}$$

$$= xe^x - e^x \Big|_0^{Ln 3}$$

$$= (Ln 3 \cdot e^{Ln 3} - e^{Ln 3}) - (0 - e^0)$$

$$= 3 Ln 3 - 3 + 1$$

$$= 3 Ln 3 - 2 \quad a$$

Q2) $\lim\limits_{m \to \infty} \int_1^m x^{-12} \, dx = \lim\limits_{m \to \infty} \frac{x^{-11}}{-11} \Big|_1^m$

$$= \lim_{m \to \infty} \frac{-1}{11 x^{11}} \Big|_1^m$$

$$= \lim_{m \to \infty} \frac{-1}{11 m^{11}} + \frac{1}{11}$$

$$= \frac{-1}{\infty} + \frac{1}{11}$$

$$= \frac{1}{11}$$

Q3) $\int_{-1}^{0} \frac{dx}{x^2+x-2}$

9

$$\frac{1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$1 = A(x-1) + B(x+2)$$

if $x=1 \longrightarrow 1 = 3B \longrightarrow B = \frac{1}{3}$

$x = -2 \longrightarrow 1 = -3A \longrightarrow A = -\frac{1}{3}$

$$\int_{-1}^{0} \frac{-1/3}{x+2} + \frac{1/3}{x-1}\, dx$$

$$-\frac{1}{3} \int_{-1}^{0} \frac{1}{x+2} - \frac{1}{x-1}\, dx$$

$$-\frac{1}{3} \left(\ln(x+2) - \ln(x-1) \right) \Big|_{-1}^{0}$$

$$-\frac{1}{3} \ln \left| \frac{x+2}{x-1} \right| \Big|_{-1}^{0}$$

$$= -\frac{1}{3} \ln|2| + \frac{1}{3} \ln\left|\frac{1}{2}\right|$$