Calculus 1 Final 12/1/2020 Solved by Teacher Ahmad Arafeh 0786543665

Q1) Find the natural Domain $f(x) = 3 + \sqrt{x-2}$

Solution $x - 2 \geq 0 \to x \geq 2 \to [2, \infty)$

Q2) $\lim\limits_{x \to -\infty} \dfrac{2x^3 - 4x + 1}{5x + 3x^3}$

Solution $\lim\limits_{x \to -\infty} \dfrac{2x^3}{3x^3} = \dfrac{2}{3}$

Q3) The slope of the tangent line of the graph $y = \cot\sqrt{x}$ at $x = \frac{\pi^2}{4}$

Solution "Slope $= \frac{dy}{dx}$ at $x = \frac{\pi^2}{4}$ "

$\frac{dy}{dx} = -\csc^2\sqrt{x}.\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx}\Big|_{x=\frac{\pi^2}{4}} = -\csc^2\sqrt{\frac{\pi^2}{4}}.\frac{1}{2\sqrt{\frac{\pi^2}{4}}} = -\csc^2\frac{\pi}{2}.\frac{1}{2\frac{\pi}{2}} = \frac{-1}{\pi}$

Q4) $f(x) = \tan^4 3x$, Find$f'(x) =$

Solution $f(x) = (\tan 3x)^4$

$f'(x) = 4(\tan(3x))^3.\sec^2(3x).3 = 12\tan^3(3x).\sec^2(3x)$

Q5) If $x^2 - y^2 = 3x^2y^2$ then $\frac{dy}{dx} =$

Solution "differentiate both sides"

$2x - 2y\frac{dy}{dx} = 3x^2.2y\frac{dy}{dx} + y^2.6x$

$-2y\frac{dy}{dx} - 6x^2y\frac{dy}{dx} = y^2.6x - 2x$

$\frac{dy}{dx}(-2y - 6x^2y) = y^2.6x - 2x$

$\frac{dy}{dx} = \frac{y^2.6x-2x}{-2y-6x^2y}$

Q6)If $3x + \sin y = 4$ then $\frac{d^2y}{dx^2} =$

Solution "differentiate both sides"

$3 + \cos y\frac{dy}{dx} = 0 \to \frac{dy}{dx} = \frac{-3}{\cos y} = -3\sec y$

$\frac{d^2y}{dx^2} = -3.\sec y.\tan y\frac{dy}{dx} = -3\sec y.\tan y.(-3\sec y) = 9\sec^2 y.\tan y$

Q7) $\int_0^1 (2x-1)^6 dx$

Solution $\int_0^1 (2x-1)^6 dx = \frac{(2x-1)^7}{7.2}\Big|_0^1 = \frac{1}{14} + \frac{1}{14} = \frac{1}{7}$

Q8) The equation of the tangent line of the graph $y = x\cos 3x$ at $x = \pi$.

Solution $at\ x = \pi \rightarrow y = \pi\cos 3\pi = -\pi \rightarrow (\pi, -\pi)$

Slope $\frac{dy}{dx} = -x\sin(3x).3 + \cos(3x).1$

$m = -\pi\sin(3\pi).3 + \cos(3\pi) = -1$

Tangent line equation $y + \pi = -1(x - \pi) \rightarrow y = -x$

Q9) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{3}{\sin^2 x} dx$

Solution $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 3\csc^2 x dx = -3\cot x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 3\sqrt{3}$

Q10) $\frac{d}{dx}(\int_{3x}^{2} \frac{dt}{1+t^2}) = \frac{1}{1+2^2}.0 - \frac{1}{1+(3x)^2}.3 = \frac{-3}{1+9x^2}$

Q11) $\int_0^{\frac{pi}{4}} \sqrt{\tan x}\sec^2 x dx$

Solution

$y = \tan x$

$dy = \sec^2 x dx \rightarrow dx = \frac{dy}{\sec^2 x}$

$If\ x = 0 \rightarrow y = 0, \quad x = \frac{\pi}{4} \rightarrow y = 1$

$\int_0^1 \sqrt{y} dy = \frac{2}{3} y^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}$

Q12) $\int_0^2 x f(x^2) dx = 5$, then $\int_0^1 f(4x) dx =$

Solution

$\int_0^1 f(4x) dx \rightarrow y = 4x, \quad dy = 4dx \rightarrow dx = \frac{dy}{4}$

$x = 0 \rightarrow y = 0, x = 1 \rightarrow y = 4$

$\int_0^1 f(4x)dx = \frac{1}{4}\int_0^4 f(y)dy$

Now,

$$\int_0^2 xf(x^2)dx = 5$$

let $y = x^2 \rightarrow dy = 2xdx \rightarrow dx = \frac{dy}{2x}$

$x = 0 \rightarrow y = 0, x = 2 \rightarrow y = 4$

$$\frac{1}{2}\int_0^4 f(y)dy = 5$$

$$\int_0^4 f(y)dy = 10$$

$4\int_0^1 f(4x)dx = 10 \rightarrow \int_0^1 f(4x)dx = \frac{10}{4} = \frac{5}{2}$

Q13) $\int \csc^3 3x.\cot 3x dx$

Solution

$$\int \frac{1}{\sin^3 3x}.\frac{\cos 3x}{\sin 3x}dx = \int \frac{\cos 3x}{\sin^4 3x}dx$$

let $y = \sin 3x \rightarrow dy = \cos 3x.3dx \rightarrow dx = \frac{dy}{3\cos 3x}$

$= \frac{1}{3}\int \frac{dy}{y^4} = \frac{1}{3}.\frac{y^{-3}}{-3} + c = \frac{(\sin 3x)^{-3}}{-9} + c$

Q14) The Absolute Min for $f(x) = 2x - x^2$ on $[0, 4]$

Solution

$f'(x) = 2 - 2x$

$f'(x) = 0 \rightarrow 2 - 2x = 0 \rightarrow 2 = 2x \rightarrow x = 1$

$f(0) = 0 \qquad f(1) = 1 \qquad f(4) = -8$

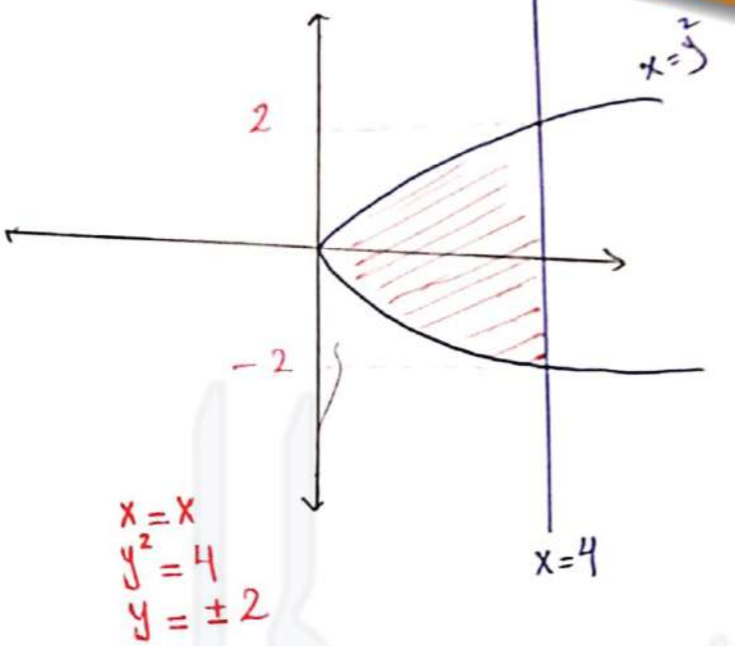
Abs Min at $x = 4$

Q15) Range $\sin x = [-1, 1]$

Range $2\sin x = 2 * [-1, 1] = [-2, 2]$

Q16) The Volume of the solid that result when the region enclosed by the curves $x = y^2$ and $x = 4$ is revolved about y-axis is

Solution



$$V = \pi\int_{-2}^{2}(4^2-(y^2)^2)dy$$
$$= \pi\int_{-2}^{2}(16-y^4)dy$$
$$= \pi(16y-\tfrac{y^5}{5})\Big|_{-2}^{2}$$
$$= \pi[(32-\tfrac{32}{5})-(-32+\tfrac{32}{5})]$$
$$= \pi(64-\tfrac{64}{5}) = \tfrac{256\pi}{5}$$

Q17) $f(x) = 3x^{\frac{2}{3}} - 2x$ Find : a)increasing and deccreasing , concavity .

b) critical point and the csup .

Solution

a) $f'(x) = 3.\frac{2}{3}x^{\frac{-1}{3}} - 2 = 2x^{\frac{-1}{3}} - 2 = \frac{2}{x^{\frac{1}{3}}} - 2$

$f'(x) = \frac{2-2\sqrt[3]{x}}{x^{\frac{1}{3}}}$

$f' = 0 \to 2 = 2\sqrt[3]{x} \to 1 = \sqrt[3]{x} \to x = 1$

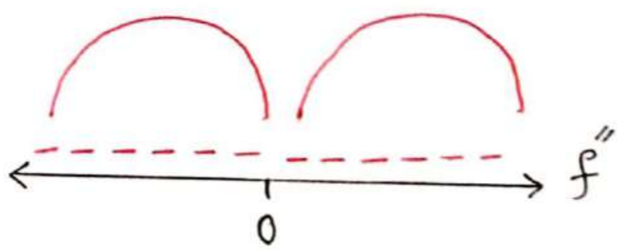
$f'\ d.n.e \to x = 0$

f increasing on $[0,1]$ and f decreasing on $(-\infty,0],[1,\infty)$

Concavity $f''(x) = \frac{-2}{3}x^{\frac{-4}{3}}$

$f''(x) = \frac{-2}{3x^{\frac{4}{3}}}$

$f''(x) d.n.e \quad \to x = 0$



Concave down $(-\infty, \infty)$

b) **Criticals and Cusp**

at $x = 0,\ x = 1$ critical numbers

$(0, f(0)),\ (1, f(1)) \to (0,0),\ \ (1,1)$ critical points

Cusp: $f'(x)$ d.n.e at $x = 0$

$$\lim_{x\to 0^+} \frac{2 - 2\sqrt[3]{x}}{x^{\frac{1}{3}}} = \frac{2}{0^+} = \infty$$

$$\lim_{x\to 0^-} \frac{2 - 2\sqrt[3]{x}}{x^{\frac{1}{3}}} = \frac{2}{0^-} = -\infty$$

Q18) **Solution**

$$A = \int_0^{\frac{\pi}{4}} 2\cos 2x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos 2x\,dx$$

$$= 2\frac{\sin 2x}{2}\Big|_0^{\frac{\pi}{4}} - 2\frac{\sin 2x}{2}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

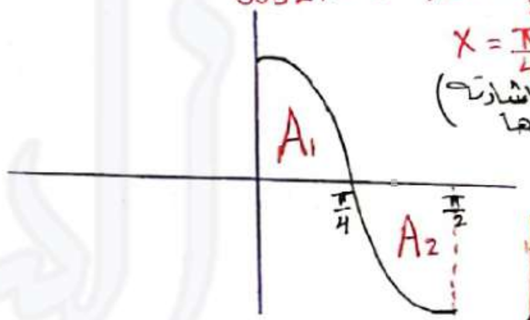
$$= 1 - (0 - 1) = 1 + 1 = 2$$

$y = 0 \to 2\cos 2x = 0$

$\cos 2x = 0 \quad , 2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

(يُغني اشارته عنها)



Q19) $V = 2\pi \int_1^3 x(2x - 1 - (3 - 2x))dx$

$$= 2\pi \int_1^3 x(4x - 4)dx$$

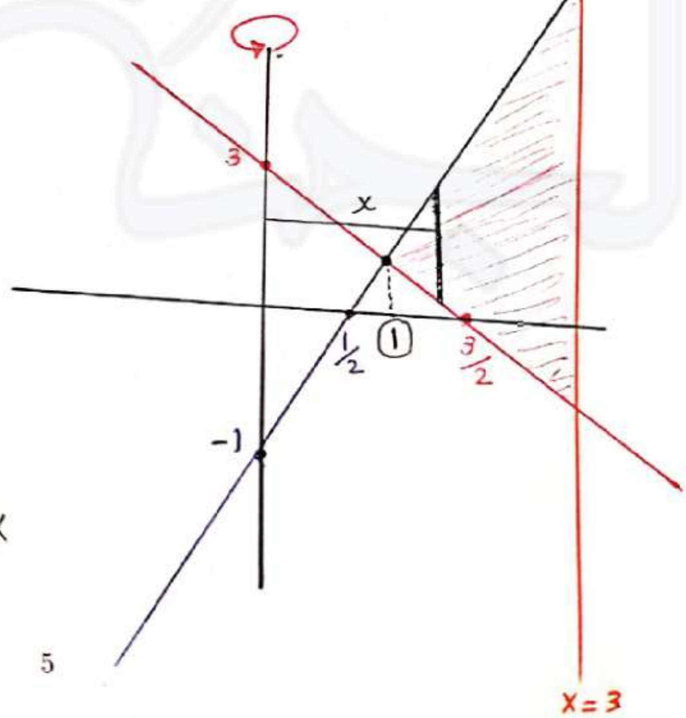
$$= 2\pi \int_1^3 4x^2 - 4x\,dx$$

$$= 2\pi\left(\frac{4x^3}{3} - 2x^2\right)\Big|_1^3$$

$$= 2\pi[(36 - 18) - (\tfrac{4}{3} - 2)]$$

$$= 2\pi(18 + \tfrac{2}{3})$$

$$= \frac{112\pi}{3}$$



$y = y$

$2x - 1 = 3 - 2x$

$4x = 4$

$\boxed{x = 1}$