

Q. One:  $\int_{-\infty}^{\infty} (u(x) + \sin(x+2)) \delta(x+2) dx =$

- A) 1      B) 0      C) 3      D) 2      E) None

Q. Two: The mean of the process  $X(t) = \cos(t + \theta)$  where  $\theta \sim U(-2\pi, 2\pi)$  is

- A) 1      B)  $\sin(t)$       C)  $\frac{1}{4\pi} \sin(t)$       D) 0      E) None

Q. Three: For  $X(t) = A \cos(t)$  and  $Y(t) = B \sin(t)$  where both  $A, B$  are independent random variables with equal mean. If  $R_{XY}(t, t + \tau) = \cos(t) \sin(t + \tau)$  then  $E(A) =$

- ~~A) 1~~      B) 2      C) 3      D) 4      E) None

Q. Four: The variance of a mean-ergodic process  $X(t)$  with no periodic component and  $R_{XX}(\tau) = Ae^{-|\tau|} + \frac{1}{1+\tau^2}$  equals 4, then the constant  $A$  equals

- A) 1      B) 2      C) 3      D) 4      E) None

Q. Five: For the joint PDF  $f(x, y) = 8b$  when  $0 < x < b$ ,  $0 < y < b$  and zero otherwise, the positive constant  $b$  equals

- A) 2      B) 1/2      C) 1/3      D) 3      E) None

Q. Six: If  $F(0) = 0$  and  $P(0 < x < 1) = 1$ , then  $F(F(1)) =$

- A) 2      B) 4      C) 3      D) 1      E) None

Q. Seven: A random variable  $X$  with pdf  $f(x) = x^2, x \in (0, \sqrt[3]{3})$ . The pdf of  $Y = \sqrt[3]{X}$  is

- A)  $3y^8$       B)  $3y^6$       C)  $3y^{2/3}$       D)  $y^8$       E) None

Q. Eight: If  $P(X = x) = 0.6 \delta(x - 1) + A \delta(x - 2)$ ;  $A$  is a constant, then  $P(X = 2) =$

- A) 0.2      B) 0.3      C) 0.4      D) 0.5      E) None

Q. Nine: For orthogonal processes  $X(t) = A \cos(t)$  and  $Y(t) = B \sin(t)$ , the expected value of the product of the random variables  $A, B$  is

- A) 0      B) 2      C) 3      D) 1      E) None

Q. Ten: If  $Cov(X, Y) = 1$ ,  $Corr(x, y) = \rho$  and  $Cov(\rho X, \rho Y) \geq 1$  then  $\rho =$

- A) -1      B) 1      C) 0      D)  $\pm 1$       E) None

Q. Eleven: For  $x > 1$ , we have  $2u(u(x)) + 3\delta(x) =$

- A) 1      B) 2      C) 3      D) 0      E) None

Q. Twelve (✓, X): The process  $X(t) = A \cos(t)$ ,  $A \sim U(-1, 1)$  is first order stationary.

Q. Thirteen (✓, X): The joint CDF  $F_{XY}(x, y) = e^{-x} \cos(x)$  gives  $f(x, y) = e^{-x} \cos(x)$

Q. Fourteen (✓, X): If  $\bar{X} = 1$ ,  $Var(X) = 2$  and  $Y = X + 1$ , then  $X, Y$  are orthogonal r.v's.

Q. Fifteen (✓, X): If  $Cov(-X, Y) = 3Corr(-X, Y)$ , then  $\sigma_X \sigma_Y = 3$

Q. Sixteen (✓, X): For the joint PDF  $f(x, y) = 0.5$  when  $0 < x < 1$ ,  $0 < y < 2$  and zero otherwise, we have  $f_X(x) = 0.5$

Q. Seventeen (✓, X): For two independent r.v.s  $X, Y$  we have  $E(XY) = 2E(X)$ , then  $E(Y) = 0.5$

Q. Eighteen (✓, X): If  $Var(E(X)X) = 0$ , then either  $E(X) = 0$  or  $Var(X) = 0$

Q. Nineteen (✓, X):  $X \sim B(n, 0.5)$  and  $Y \sim P(2.3)$  cannot have the same means

Q. Twenty (✓, X): Two independent random variables  $X, Y$  with marginal pdf's  $f_X(x) = 2$  when  $x \in (0, 0.5)$  and  $f_Y(y) = 1$  when  $y \in (0, 1)$ . Then the joint pdf is  $f(x, y) = 1$  when  $x$  and  $y$  belong to their intervals and zero otherwise

$$Q_4) \text{Var}x = E x^2 - [E x]^2$$

$$4 = A + 1 - 0$$

$$\boxed{A=3} \quad (C)$$

$$[E x]^2 = \lim_{T \rightarrow \infty} R_{xx}(T) = A e^{\frac{-\infty}{1+\infty}} = 0$$

$$E x^2 = R_{xx}(0) = A e^0 + \frac{1}{1+0} = A + 1$$

$$Q_5) \int_0^b \int_0^b [8b] dx dy = 1$$

$$\int_0^b 8b^2 dy = 1$$

$$8b^3 = 1 \Rightarrow \boxed{b = \frac{1}{2}} \quad (B)$$

$$Q_6) P(0 < x < 1) = 1$$

$$F(1) - F(0) = 1$$

$$F(1) - 0 = 1$$

$$F(1) = 1 \Rightarrow$$

$$\begin{aligned} &\Rightarrow F(F(1)) \\ &\Rightarrow F(1) = 1 \end{aligned} \quad (D)$$

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$$Q_1) \sum (x+2) \Rightarrow x+2=0 \Rightarrow x=-2$$

$$u(-2) + \sin(-2+2) = 0$$

$$0 + 0 = 0 \quad (B)$$



$$Q_2) E(x(t)) = \int_{-2\pi}^{2\pi} (\cos(t+\theta)) \cdot \frac{1}{4\pi} d\theta$$

$$= \frac{1}{4\pi} [\sin(t+\theta)]_{-2\pi}^{2\pi}$$

$$= \frac{1}{4\pi} [\sin(t+2\pi) - \sin(t-2\pi)]$$

$$= \frac{1}{4\pi} [0] = 0 \quad (D)$$

$$\theta \sim U(-2\pi, 2\pi)$$

$$f(\theta) = \frac{1}{+2\pi - (-2\pi)}$$

$$= \frac{1}{4\pi}$$

$$Q_3) R_{xy}(t, t+\tau) = \cos(t) \sin(t, t+\tau)$$

$$\Rightarrow E[x(t)y(t+\tau)] = [$$

$$\Rightarrow E[A \cos(t) \cdot B \sin(t+\tau)] = [$$

$$\Rightarrow E[AB] [\cos(t) \sin(t+\tau)] = [\cos(t) \sin(t+\tau)]$$

$$\Rightarrow \text{then } E(A) = E(B)$$

$$E(AB) = E(A) E(B) = E(A)^2 = 1 \Rightarrow \boxed{E(A) = 1} \quad (A)$$



Q9] orthogonal processes

then  $E(xy) = 0$  (A)

Q10]  $\text{cov}(x, y) \geq 1$

(1)  $\text{cov}(x, y) \geq 1$

(2)  $\rho \geq 1$

$\rho^2 \geq 1 \Rightarrow \rho = 1$  or  $\rho = -1$  (B)

Q12]  $E(x(u)) = \int_{-1}^1 A \cos(ut) \cdot \frac{1}{2} dA$

$$= \frac{\cos(ut)}{2 \times 2} [A^2]_{-1}^1$$

$$= \frac{\cos(ut)}{4} [1 - 1] = 0 \text{ first order } \text{True} \checkmark$$

$$A \sim U(-1, 1)$$

$$f(A) = \frac{1}{1 - (-1)} = \frac{1}{2}$$

Q13] Joint cdf  $\rightarrow$  مشترك

Q15]  $\text{cov}(-x, y) = 3 \text{cov}(-x, -y)$

$$(-1) \text{cov}(x, y) = 3 \frac{\text{cov}(x, y)}{(\sigma_x \sigma_y)}$$

$$\sigma_x \sigma_y = 3 \text{ True } \checkmark$$

$$Q16] f_x(x) = \int_y f(x, y) dy = \int_0^2 [0.5] dy$$

$$= 2 \times [0.5] = 1 \text{ False } \times$$

Q17]  $E_{xy} = 2E_x$

independent:

$$E(x)E(y) = 2E(x) \Rightarrow E(y) = 2 \text{ False } \times$$

Q18]  $\text{Var}[E(x)] = 0$

$$[E(x)]^2 \text{Var} x = 0 \Rightarrow \text{Var} x = 0 \text{ or } E(x) = 0 \text{ True } \checkmark$$

Q19]  $x \sim B(n, 0.5)$

$$\text{mean} = n \times p$$

$$\text{mean} = n \times (0.5)$$

the mean can't [2.3], because (n) integer number

$Y \sim P(2, 3)$

$$\text{mean} = 2, 3$$

True  $\checkmark$

Q20]  $f(x, y) = f_x(x) f_y(y)$

$$f(x, y) = (2) \cdot (1) \Rightarrow f(x, y) = 2 \text{ False } \times$$

مستقل

هو خير منكم عبارة أقرب بها إلى الله