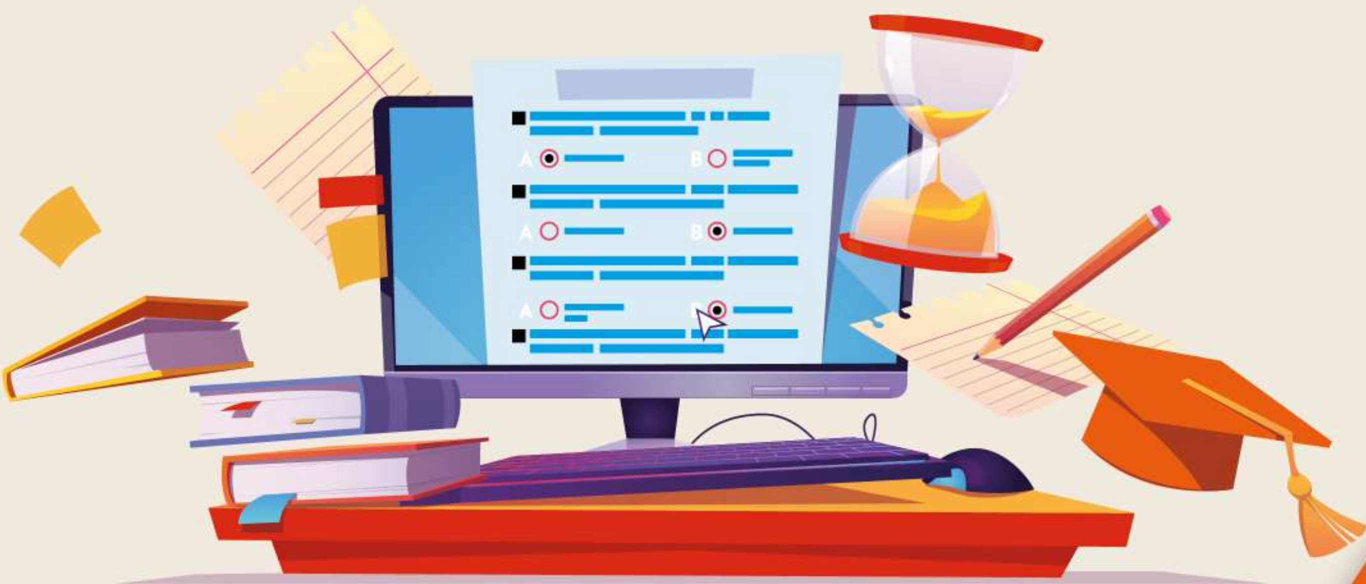
خدمتكم طريق خضناه لرضى الله

2021

# أسئلة سنوات
# فاينال
# تفاضل وتكامل 2



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**Question 1** Choose the correct answer for the questions (1-18) and complete the table

| Q(1) | Q(2) | Q(3) | Q(4) | Q(5) | Q(6) | Q(7) | Q(8) | Q(9) | Q(10) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| Q(11) | Q(12) | Q(13) | Q(14) | Q(15) | Q(16) | Q(17) | Q(18) | | |
| | | | | | | | | | |

Q(1) By converting from cartesian to polar, the cartesian point $(x, y) = (-3, 0)$ becomes

a)$(13, \pi)$ b)$(3, \pi \pm 2n\pi)$ c)$(-3, \pi \pm 2n\pi)$ d)$(16, \pi \pm 2n\pi)$

Q(2) Converting from polar to cartesian, the polar point $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$ becomes

a)$(3, 3)$ b)$(-3, 3\sqrt{3})$ c)$(-3, 3)$ d)$((-\sqrt{3}, 3\sqrt{3}))$

Q(3) In Polar system, $4\cos\theta = 3$ defines a

a)Horizontal line b) Vertical Line c) Circle d)Square

Q(4) The sequence $\left\{\frac{n^2}{2n^2+3}\right\}$ converges to

a) $\frac{3}{2}$ b)$-\frac{1}{2}$ c)$\frac{5}{2}$ d)$\frac{1}{2}$

Q(5) By telescoping method we get $\sum_1^{\infty} \frac{2}{k(k+1)} =$

a) $\frac{1}{4}$ b)$\frac{1}{3}$ c)$\frac{1}{2}$ d)2

Q(6) The series $\sum_1^{\infty} \frac{1}{\sqrt{2k}}$

2 1 3 2

a) converges conditionally b) converges absolutely c) diverges d) None

Q(7) The series $\sum_1^{\infty} \frac{k^3+2k+22}{2k^8+1}$

2 2

a) converges b)diverges c)absolutely diverges d) None

Q(8) The Maclaurin series for $\sin x^3$ is

Q(9) $\sum_{1}^{\infty} (\ln (2k+1))^{k}$

a) converges b) converges conditionally c) diverges d) diverges absolutely

Q(10) The value of the improper integral $\int_1^2 \frac{3}{1-x}dx$ is

a) 1 b)2 c)6 d) diverges

Q(11) The value of the derivative $\frac{d}{dx}x^x$ at 2 is

a)4 b) $1+\ln 4$ c) $4+\ln 4$ d)$4+4\ln 4$

Q(12) $\lim_{x\to 0}(1+2x)^{\frac{3}{x}} =$

a)$e^6$ b) 6 c) -6 d) $e^{-6}$

Q(13) $\lim_{x\to\infty}\frac{x^{11}}{e^{2x}} =$

a) 1 b)0 c)4 d)3

Q(14) $\lim_{x\to\infty}\left(2(\ln x)^{\frac{1}{x}}\right) =$

a)1 b) 2 c)$\ln 2$ d) ln3

Q(15) $\lim_{x\to 0}(e^{2x}+x)^{\frac{1}{x}} =$

a)3 b)4 c)$e$ d)$e^3$

Q(16) $\int_1^{\sqrt{2}} \frac{4dx}{x^2\sqrt{4-x^2}}dx$

a) $\sqrt{3}-2$ b) $2\sqrt{3}-1$ c) $\sqrt{3}-1$ d) $\sqrt{3}-3$

Q(17) $\int_0^{\pi} 40\sin 7x\cos 3x dx =$

a) $-17$ b)$-7$ c)0 d)$-71$

Q(18) $\int_0^{\ln 3} xe^x dx =$

a) $3\ln 3-2$ b) $3\ln 3$ c) $\ln 3-2$ d) $5\ln 3-2$

**Question 2** Find the value of the integral $\int_1^{\infty}\frac{dx}{x^{12}}$ =

**Question 3** Find the value of the integral $\int_{-1}^{0}\frac{dx}{x^2+x-2}$ = $\frac{1}{2}$

1

Q1) $r^2 = x^2 + y^2$

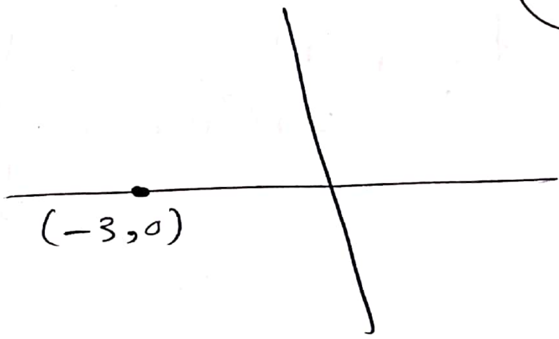
$r^2 = (-3)^2 + 0^2$

$r^2 = 9$

$r = 3$

$\theta = \pi \pm 2n\pi$

$(3, \pi \pm 2n\pi)$. b

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Q2) $x = r\cos\theta = 6\cos\frac{2\pi}{3} = 6 \cdot \left(\frac{-1}{2}\right) = -3$

$y = r\sin\theta = 6\sin\frac{2\pi}{3} = 6 \cdot \frac{1}{2} = 3$

$(-3, 3)$ c

2

Q4) $\lim_{n\to\infty} \frac{n^2}{2n^2+3} = \lim_{n\to\infty} \frac{n^2}{2n^2}$

$= \frac{1}{2}$ d

Q5) $\frac{A}{k} + \frac{B}{(k+1)} = \frac{2}{k(k+1)}$

$2 = A(k+1) + Bk$

If $k=0 \longrightarrow A=2$

$k=-1 \longrightarrow 2=-B$

$B=-2$

$S_n = \sum_{i=1}^{n} \left(\frac{2}{i} - \frac{2}{i+1}\right) = 2\sum_{i=1} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$S_n = 2\left[\left(1-\cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots + \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)\right]$

$S_n = 2\left(1 - \frac{1}{n+1}\right)$

$\lim_{n\to\infty} S_n = 2\left(1 - \frac{1}{\infty}\right)$

$= 2.1$

$= 2$ d

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$$6) \quad \sum_{1}^{\infty} \frac{1}{\sqrt{2}\sqrt{k}} = \sum \frac{1}{\sqrt{2}\,k^{1/2}} \qquad p = \tfrac{1}{2} < 1$$

Div.

(c)

3

$$7) \quad b_k = \frac{\cancel{5}\,k^3}{k^8} = \frac{1}{k^5}$$

$$\lim_{k \to \infty} \frac{a_k}{b_k} = \lim_{k \to \infty} \frac{k^3 + 2k + 22}{2k^8 + 1} \cdot \frac{k^5}{1}$$

$$= \lim_{k \to \infty} \frac{k^8 + 2k^6 + 22k^5}{2k^8}$$

$$= \lim_{k \to \infty} \frac{\cancel{k^8}}{2\cancel{k^8}}$$

$$= \frac{1}{2}$$

$$0 < \tfrac{1}{2} < 1$$

$$\Rightarrow \sum b_k = \sum \frac{1}{k^5} \qquad p = 5 > 1$$

Conv.

(a)

$$Q8) \quad \sin x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k (x^3)^{2k+1}}{(2k+1)!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+3}}{(2k+1)!}$$

Q9) $\sum_{1}^{\infty} \frac{1}{(\ln(2k+1))^k}$ Root. (A)

$$\lim_{k\to\infty} \left(\frac{1}{\ln(2k+1)^k}\right)^{1/k}$$

$$= \lim_{k\to\infty} \frac{1}{\ln(2k+1)} = \frac{1}{\infty} = 0 < 1$$

Conv. (a)

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Q10) $\lim_{m\to 1^+} \int_m^2 \frac{3}{1-x}\,dx = \lim_{m\to 1^+} 3\ln|1-x| \Big|_m^2$

$$= \lim_{m\to 1^+} \left[3\ln|1-\cancel{2}| - 3\ln|1-m|\right]$$

$$= 0 - 3(-\infty)$$

$$= \infty \quad \text{(d)}$$

Div.

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Q11) $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

نشتق

$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$

$y' = y(1 + \ln x)$

$y' = x^x (1 + \ln x)$

$y'(2) = 2^2 (1 + \ln 2)$

$= 4(1 + \ln 2) = 4 + 4\ln 2$ غير موجودة

Q12) $\lim_{x\to 0}(1+2x)^{\frac{-3}{x}}$

let $x=\frac{1}{y}$

$x\to 0$
$y\to\infty$

$\lim_{y\to\infty}\left(1+\frac{2}{y}\right)^{-3y} = e^{2.(-3)} = e^{-6}$ d

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Q13) $\lim_{x\to\infty}\frac{x^{11}}{e^{2x}}$ = لوبيتال

Finally $\lim_{x\to\infty}\frac{11!}{2^{11}e^{2x}} = \frac{11!}{\infty} = 0$ b

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Q14) $\lim_{x\to\infty}\left(2(\ln x)^{\frac{1}{x}}\right)$

$2\lim_{x\to\infty}(\ln x)^{\frac{1}{x}}$

$=2.1$

$=2$ b

$y=(\ln x)^{1/x}$

$\ln y = \ln(\ln x)^{1/x}$

$\ln y = \frac{\ln(\ln x)}{x}$

$\ln y = \lim_{x\to\infty}\frac{\ln(\ln x)}{x}\ \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{\frac{1}{x}}{\ln x}}{1}$

$\ln y = \lim_{x\to\infty}\frac{1}{x\ln x}$

$\ln y = 0$

$y=e^{0}$

$y=1$

Q15) $\lim_{x \to 0} (e^{2x} + x)^{\frac{1}{x}} =$

$= e^{2}$

(6)

$y = (e^{2x} + x)^{\frac{1}{x}}$

$\ln y = \ln(e^{2x} + x)^{\frac{1}{x}}$

$\ln y = \frac{\ln(e^{2x} + x)}{x}$ $\quad \frac{\infty}{\infty}$

$\ln y = \lim_{x \to \infty} \frac{\ln(e^{2x} + x)}{x}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{2e^{2x} + 1}{e^{2x} + x}}{1}$ $\quad \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \frac{4e^{2x}}{2e^{2x} + 1}$ $\quad \frac{\infty}{\infty}$

$\ln y \overset{L'H}{=} \lim_{x \to \infty} \frac{8\cancel{e^{2x}}}{4\cancel{e^{2x}}}$

$\ln y = 2$

$y = e^{2}$

Q16) $x = 2\sin\theta$

$x^2 = 4\sin^2\theta$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\cdot\cos\theta$

$dx = 2\cos\theta \, d\theta$

If $x=1 \longrightarrow 1 = 2\sin\theta \rightarrow \frac{1}{2} = \sin\theta \longrightarrow \theta = \frac{\pi}{6}$

$x = \sqrt{2} \longrightarrow \sqrt{2} = 2\sin\theta \longrightarrow \frac{1}{\sqrt{2}} = \sin\theta \longrightarrow \theta = \frac{\pi}{4}$

$$\int_{\pi/6}^{\pi/4} \frac{\cancel{4}\cdot \cancel{2\cos\theta}\, d\theta}{\cancel{4}\sin^2\theta \cdot \cancel{2\cos\theta}} = \int_{\pi/6}^{\pi/4} \csc^2\theta \, d\theta$$

$$= -\cot\theta \Big|_{\pi/6}^{\pi/4}$$

$$= -1 + \sqrt{3}$$

$$= \sqrt{3} - 1 \quad C$$

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Q17) $\int_0^{\pi} 40 \sin 7x \cos 3x \, dx$

$$40 * \frac{1}{2} \int_0^{\pi} \sin(4x) + \sin(10x) \, dx$$

$$20 \left(-\frac{\cos 4x}{4} - \frac{\cos 10x}{10}\right)\Big|_0^{\pi}$$

$$= 20\left[\left(\frac{-1}{4} - \frac{1}{10}\right) - \left(\frac{-1}{4} - \frac{1}{10}\right)\right]$$

$$= 20\left(\cancel{\frac{-1}{4}} - \cancel{\frac{1}{10}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{10}}\right) = 0 \quad C$$

Q18) $\int_0^{\ln 3} x e^x \, dx$

| | | |
|---|---|---|
| $x$ | | $e^x$ |
| | $+$ | |
| $1$ | | $e^x$ |
| | $-$ | |
| $0$ | | $e^x$ |

$$= xe^x - e^x \Big|_0^{\ln 3}$$

$$= (\ln 3 \cdot e^{\ln 3} - e^{\ln 3}) - (0 - e^0)$$

$$= 3\ln 3 - 3 + 1$$

$$= 3\ln 3 - 2 \quad a$$

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Q2) $\lim_{m\to\infty} \int_1^m x^{-12} \, dx = \lim_{m\to\infty} \frac{x^{-11}}{-11} \Big|_1^m$

$$= \lim_{m\to\infty} \frac{-1}{11x^{11}} \Big|_1^m$$

$$= \lim_{m\to\infty} \frac{-1}{11m^{11}} + \frac{1}{11}$$

$$= \frac{-1}{\infty} + \frac{1}{11}$$

$$= \frac{1}{11}$$

Q3) $\int_{-1}^{0} \frac{dx}{x^2+x-2}$



$$\frac{1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$1 = A(x-1) + B(x+2)$$

if $x=1 \longrightarrow 1 = 3B \longrightarrow B = \frac{1}{3}$

$x=-2 \longrightarrow 1 = -3A \longrightarrow A = -\frac{1}{3}$

$$\int_{-1}^{0} \frac{-1/3}{x+2} + \frac{1/3}{x-1}\, dx$$

$$-\frac{1}{3} \int_{-1}^{0} \frac{1}{x+2} - \frac{1}{x-1}\, dx$$

$$-\frac{1}{3} \left( Ln(x+2) - Ln(x-1) \right) \Big|_{-1}^{0}$$

$$-\frac{1}{3} Ln \left| \frac{x+2}{x-1} \right| \Big|_{-1}^{0}$$

$$= -\frac{1}{3} Ln|2| + \frac{1}{3} Ln \left| \frac{1}{2} \right|$$