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-Al-Balqa' Applied University, Faculty of Engineering Technology

Mid Exam, Linear Algebra

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Lecture Time: Date.

Part One: Choose the correct answer **(10 marks)**

1	2	3	4	5
A	B	C	D	B

4

1- One of the following matrices is in R.R.E form

A) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

2- The corresponding linear system for the augmented matrix $\begin{bmatrix} 2 & 1 & 3 & -5 \\ 1 & 2 & 4 & 0 \\ -1 & 6 & 2 & 1 \end{bmatrix}$ is:

A) $2X_1 + X_2 + 3X_3 + 5 = 0$
$X_1 + 2X_2 + 4X_3 = 0$
$-X_1 + 6X_2 + 2X_3 - 1 = 0$

B) $2X_1 + X_2 + 3X_3 = 5$
$X_1 + 2X_2 + 4X_3 = 0$
$-X_1 + 6X_2 + 2X_3 = 1$

C) $2X_1 + X_2 + 3X_3 + 5 = 0$
$X_2 + 2X_1 + 4X_3 = 0$
$-X_1 + 6X_2 + 2X_3 - 1 = 0$

D) $2X_1 + X_2 + 3X_3 + 5 = 0$
$X_1 + 2X_2 + 4X_3 = 0$
$X_1 - 6X_2 - 2X_3 - 1 = 0$

3- The solution of the linear system with the corresponding augmented matrix $\begin{bmatrix} 2 & 5 & 8 \\ 4 & 10 & 16 \end{bmatrix}$ is:

A) The system has no solution

B) $X_1 = 4$ and $X_2 = 0$

C) $X_1 = 4 - \frac{5t}{2}$ and $X_2 = t;\ t \in \mathbb{R}$

D) $X_1 = 8 - 5t$ and $X_2 = t;\ t \in \mathbb{R}$

4- One of the following statements is false for any symmetric matrix A

A) $tr(2A) = tr(A) + tr(A^T)$.

B) AA^T is symmetric.

C) A is a square matrix.

D) The equation $AX = B$ has only one solution.

5- Let A be a matrix with $det(A) = 7$, and B is the matrix result by multiplying the third row in A by the scalar $\alpha = 3$ and the second column in A by the scalar $\beta = -2$ then $det(B) =$

A) $\frac{-7}{6}$ B) -42
C) 21 D) -14

②

Part two: Fill the blank. (10 marks)

1- Let $A = \begin{bmatrix} 2 & 7 \\ -3 & 5 \end{bmatrix}$ then $A^{-1} =$ $\frac{1}{31}\begin{bmatrix} 5 & -7 \\ 3 & 2 \end{bmatrix}$

2- Let $B = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 7 \\ 0 & 0 & b \end{bmatrix}$ with $det(B) = 10$ then $b =$ $\frac{10}{3} = 3.333$

3- If $A^{-1} = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 3 & -2 \\ 4 & 0 & 1 \end{bmatrix}$ then the solution of the equation $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is .... $\begin{bmatrix} 1 \\ 6/3 \\ 1 \end{bmatrix}$

4- If A is a square matrix satisfies $A^2 - 3A = I$, then $A^{-1} =$ ~~$A^2 - 3A$~~ I

5- Let A *and* B be two symmetric matrix with the same size, then AB is symmetric matrix iff $AB =$ ~~I~~ I I

Part Three: Essay Questions

1- Find the inverse matrix for $A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 1 & 4 \\ 3 & 5 & 0 \end{bmatrix}$ (4 marks)

~~$c_{11} = -20$~~
~~$c_{12} = 12$~~
~~$c_{13} = -13$~~
~~$c_{21} = 10$~~
~~$c_{22} = -6$~~
~~$c_{23} = 4$~~
~~$c_{31} = 10$~~
~~$c_{32} = -8$~~
~~$c_{33} = 7$~~

$c_{11} = -20$
$c_{12} = -12$
$c_{13} = -13$
$c_{22} = 10$
$c_{22} = -6$
$c_{23} = 4$
$c_{31} = 10$
$c_{32} = -8$
$c_{33} = 20$

$A^{-1} =$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ -2 & 1 & 4 & 0 & 1 & 0 \\ 3 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$

$$R_2+2R_1,\ R_3-3R_1 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 7 & 8 & 2 & 1 & 0 \\ 0 & -4 & -6 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-1 * R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 7 & 8 & 2 & 1 & 0 \\ 0 & 4 & 6 & 3 & 0 & -1 \end{array}\right]$$

$$\frac{R_2}{7} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 4 & 6 & 3 & 0 & -1 \end{array}\right] \xrightarrow{R_1-3R_2,\ R_3-4R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

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$$\Rightarrow A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -20 & 22 & -13 \\ 10 & -6 & 4 \\ 10 & -8 & 7 \end{bmatrix} = \frac{1}{-20} \begin{bmatrix} -20 & 22 & -13 \\ 10 & -6 & 4 \\ 10 & -8 & 7 \end{bmatrix}$$

2- Use Cramer's Rule to solve the linear system **(4 marks)**

$$x + 2y - z = 5$$
$$2x + y + z = 0$$
$$-x + y + \cancel{z}\ 2z = 1$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

$$|A| = 2 - 1 = \boxed{1}$$

$$A_1 = \begin{bmatrix} 5 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \Rightarrow |A_1| = 1$$

$$A_2 = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix} = |A_2| = -1$$

2

$$A_3 = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = |A_3| = 1$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{1}{1} = \boxed{1},\quad y = \frac{|A_2|}{|A|} = \frac{-1}{1} = \boxed{-1},\quad z = \frac{|A_3|}{|A|} = \boxed{1}$$

$$\Rightarrow \quad X = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

3- Use Gauss-Jordan Method to solve the system **(2 marks)**

$$x + 3y = 5$$
$$2x + 4y = 8$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 4 & 8 \end{array}\right]$$

2

$$R_2 - 2R_1 \begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & -2 \end{bmatrix}$$

$$\frac{R_2}{-2} \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_1 - 3R_2$$
$$\cancel{R_1 - 3R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \underline{\underline{\text{sol}}}\ (x, y) = (2, 1)$$