Regular Model Checking

for Formal Verification of Infinite-State Systems

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Sources of Infinity

- Unbounded communication queues (channels), unbounded waiting queues.
- Unbounded push-down stacks: recursion.
- Unbounded counters, unbounded capacity of places in Petri nets.
- Unbounded string variables.
- Continuous variables: time, temperature, ...
- Unbounded dynamic spawning of threads, dynamic memory allocation:
  - dynamic linked (circular/shared/nested/...) lists, trees, skip-lists, ...
- Parameterisation:
  - parametric bounds of queues, counters, ...,
  - parametric networks of processes.

Model Checking Infinite-State Systems

❖ Cut-offs: safe, finite bounds on the sources of infinity such that when a system is verified up to these bounds, the results may be generalised.

❖ Abstraction:

- predicate abstraction: $x \in \{5, 6, 7, ...\} \leadsto x \geq 5$,
- abstractions for parameterised networks of processes: 0-1-∞ abstraction, ...

❖ Symbolic methods: finite representation of infinite sets of states using

- logics,
- grammars,
- automata, ...

❖ Automated induction, ...

Decidability Issues

- Formal verification of infinite state systems is usually **undecidable**.
- There exist (sub)classes of systems for which various problems are decidable:
  - push-down systems—model checking LTL is even polynomial for a fixed formula,
  - lossy channel systems—reachability, safety, inevitability, and (fair) termination are decidable (though non-primitive recursive),
  - various parameterised systems for which finite cut-offs exist,
  - ...
- Otherwise, semi-algorithmic solutions can be used:
  - termination is not guaranteed,
  - an indefinite answer may be returned, or
  - a help from the user is needed: invariants, predicates, ...

Regular Model Checking

The Basic Idea

Regular Model Checking

[Pnueli et al. 97], [Wolper, Boigelot 98], [Bouajjani, Nilsson, Jonsson, Touili 00]

- A **generic** framework for verification of infinite-state systems:
  - a configuration $\rightsquigarrow$ a word w over a suitable alphabet Σ,
  - a set of configurations $\rightsquigarrow$ a regular language:
    - usually described by a finite-state automaton A,
    - two distinguished sets of configurations:
      - initial configurations $Init$ and
      - bad configurations Bad,
  - an action (transition) $\rightsquigarrow$ a rational relation τ:
    - usually described by a finite-state transducer T,
    - sometimes, more general, regularity-preserving relations are used.
      - Implemented, e.g., as specialised operations on automata.
- Safety verification $\rightsquigarrow$ check that $\tau^*(Init) \cap Bad = \emptyset$,
  - implies a need to compute $\tau^*(Init)$ or its sufficiently precise approximation.

Regular Model Checking: Applications

- Communication protocols.
  - Lossy/non-lossy FIFO channels systems / cyclic rewrite systems.
- Sequential programs with recursive procedure calls.
  - Push-down systems / prefix rewrite systems.
- Counter systems, Petri nets.
  - Various systems may be (automatically) translated to counter systems.
- String manipulating programs. [Yu, Alkhalaf, Bultan, Ibarra et al 08–17]
- Programs with (unbounded) dynamic linked data structures:
  - lists, cyclic lists, shared lists. [Bouajjani, Habermehl, V., Moro 05]
- Parameterized networks of processes:
  - mutual exclusion and cache coherence protocols, ..., [many of the mentioned works]

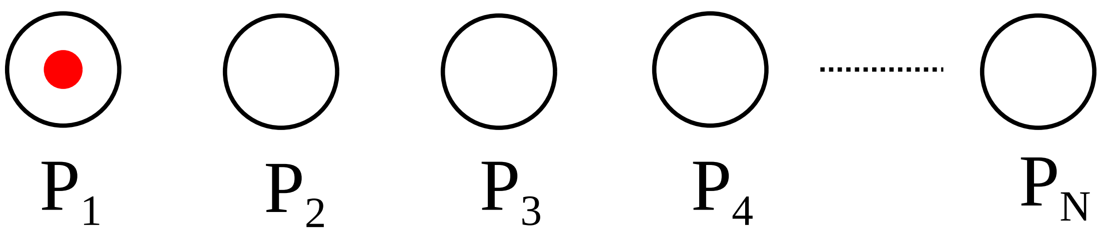
$$q_1 q_2 \cdots q_{i-1} q_i q_{i+1} \cdots q_j \cdots q_n \mapsto q_1 q_2 \cdots q_{i-1} q_i' q_{i+1} \cdots q_j' \cdots q_n$$

  - pipelined microprocessors. [Charvát, Smrčka, V. 14–19]

Example: A Simple Token Passing

- A simple protocol in a linear process network:
  - a parametric number of processes,
  - a process does or does not have a token,
  - a process that has a token passes it to the right.

- Initially, a token is in the left-most process.



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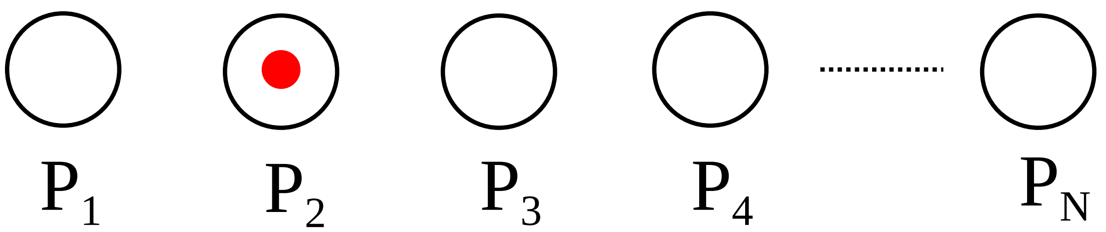

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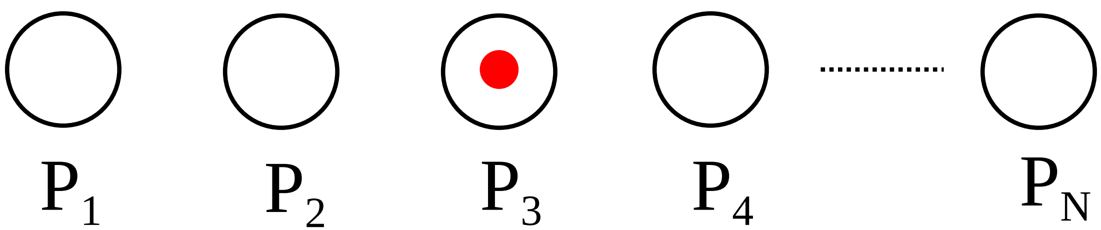

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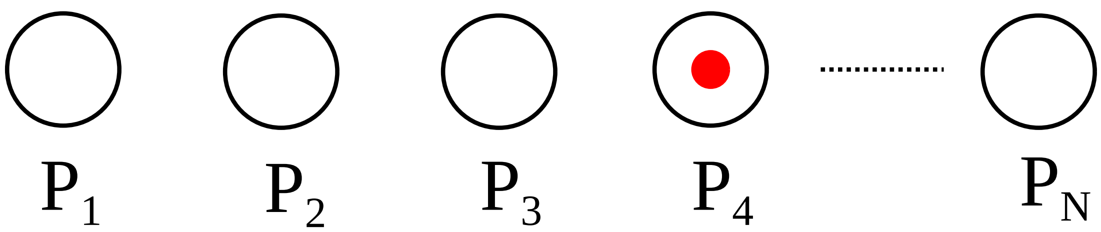

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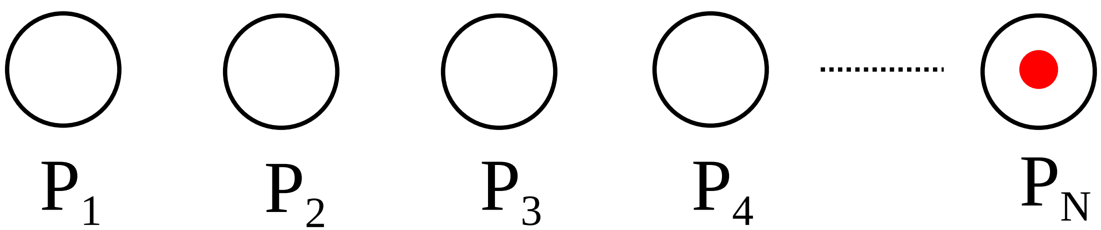

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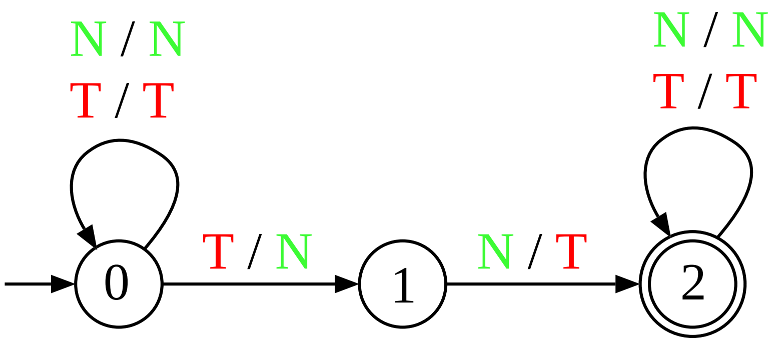
Example: A Simple Token Passing

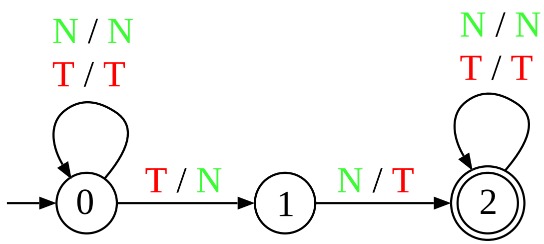
- An **encoding** of the simple token passing protocol for the needs of RMC:
  - the alphabet: $\Sigma = \{T, N\}$,
  - configurations: words from Σ^*, e.g., $N\ N\ T\ N$,
  - initial configurations: $T\ N^*$ (a regular language),
  - bad configurations: $N^* + (T + N)^*\ T\ N^* T\ (T + N)^*$ (a regular language),
  - transitions—in the form of a finite-state transducer:



Example: A Simple Token Passing

❖ An application of the transducer on a single configuration:

$T\ N\ N\ N \xrightarrow{\tau} N\ T\ N\ N \xrightarrow{\tau} N\ N\ T\ N \xrightarrow{\tau} N\ N\ N\ T$



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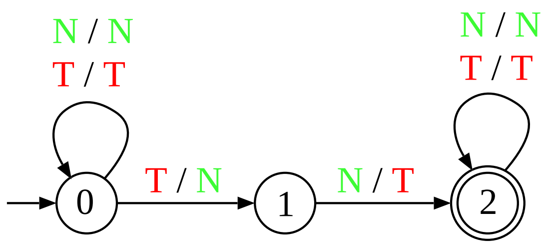
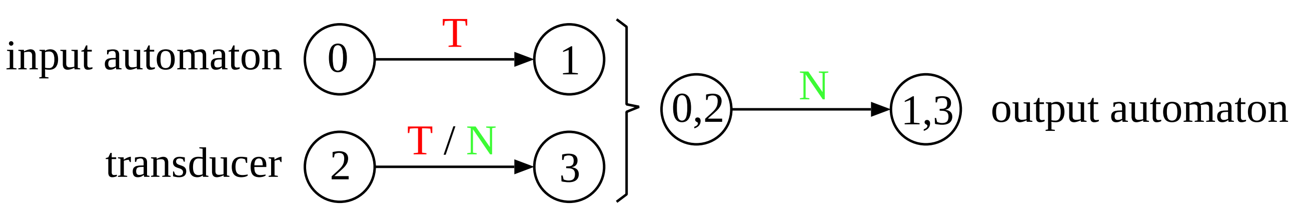


- An application of the transducer on all initial configurations:

$$T\ N^* \xrightarrow{\tau} N\ T\ N^* \xrightarrow{\tau} N\ N\ T\ N^* \xrightarrow{\tau} N\ N\ N\ T\ N^* \xrightarrow{\tau} \ldots$$



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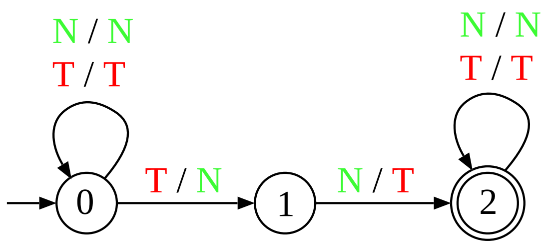

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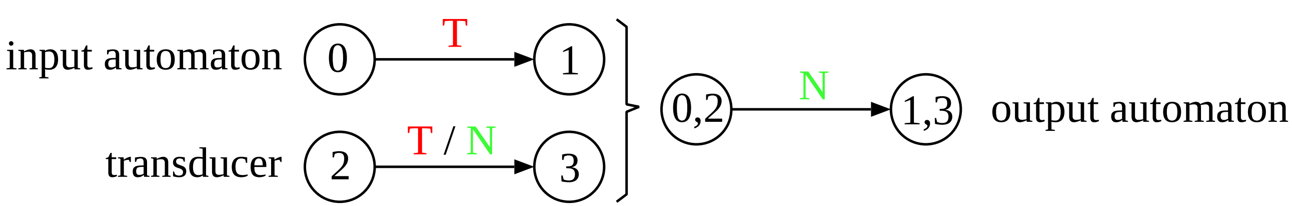

❖ A simple iterative computation of all reachable configurations will **never converge** to the desired set $N^*\ T\ N^*$.

- Need special (accelerated) ways for computing/over-approximating $\tau^*(Init)$.

Regular Model Checking

Computing Closures

RMC: Computing Closures

The task: compute/over-approximate $\tau^*(Init)$.

- Problems to face:
  - Non-regularity / non-constructibility of $\tau^*(Init)$.
  - Termination of the constructions.
  - State explosion in the automata / transducers.

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❖ Solutions:

- Specialised constructions: LCS, PDS, classes of arithmetical relations, lists, ...
- General-purpose constructions:
  - widening by extrapolating repeated patterns, [Bouajjani, Touili], [Wolper, Boigelot, Legay]
  - merging states wrt the history of their creation, [Abdulla, Nilsson, Jonsson, d'Orso]
  - widening by merging states wrt their fw/bw languages, [Yu, Alkhalaf, Bultan, Ibarra]
  - refinable abstraction by state merging, [Bouajjani, Habermehl, V.]
  - automata learning, [Habermehl, V.], [Vardhan, Sen, Viswanathan, Agha], [Chen, Hong, Lin, Rümmer]
  - ...

Abstract Regular Model Checking

- Given a relation τ, and two automata I (initial states) and B (bad states), check:

$$\tau^*(I) \cap B = \emptyset$$

1. Define a finite-range abstraction α on automata s.t. $L(A) \subseteq L(\alpha(A))$.
2. Compute iteratively $(\alpha \circ \tau)^*(I)$.
3. If $(\alpha \circ \tau)^*(I) \cap B = \emptyset$, then answer YES.

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4. Otherwise, let θ be the computed symbolic path from I to B.
5. Check if θ includes a concrete counterexample.
   - If yes, then answer NO.
   - Otherwise, refine α s.t. it excludes θ and goto (2).

Abstract Regular Model Checking

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$\Longrightarrow$ *Counter-Example Guided Abstraction Refinement (CEGAR) loop*

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Abstractions Based on State Collapsing

❖ We abstract automata by collapsing their states that are equal wrt some criterion s.t.:

$$L(A) \subseteq L(\alpha(A)).$$

❖ Various equivalences on automata states can be used, e.g.:

- Equivalence wrt languages of words of a bounded length k:

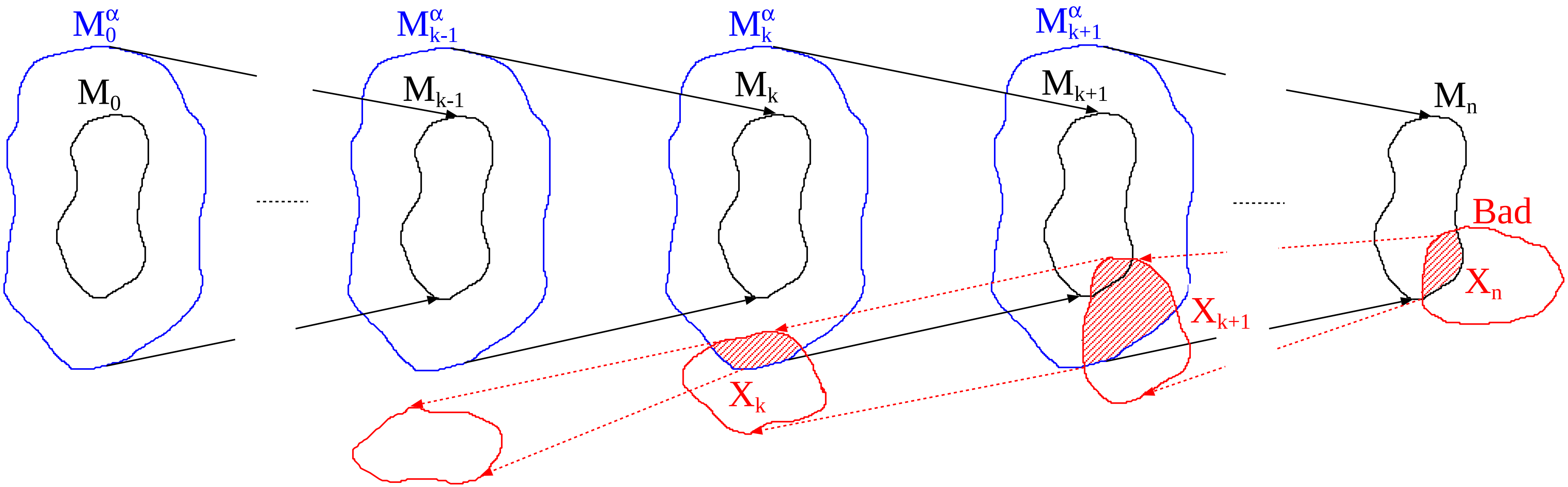
$$q_1 \simeq_k q_2 \ \text{ iff } \ L(A, q_1)^{\leq k} = L(A, q_2)^{\leq k}$$

  $L(A, q)^{\leq k}$: the set of words of length at most k accepted in A from q.

- Equivalence wrt a set of predicate languages $\mathcal{P} = \{P_1, ..., P_n\}$:

$$q_1 \simeq_{\mathcal{P}} q_2 \ \text{ iff } \ \forall 1 \leq i \leq n : L(A, q_1) \cap P_i \neq \emptyset \Leftrightarrow L(A, q_2) \cap P_i \neq \emptyset$$

Counterexample-Guided Refinement



❖ For abstraction based on bounded length languages: increment the bound.

❖ For predicate automata abstraction: take as predicates languages of all states of the last non-empty intersection of the forward and backward run:

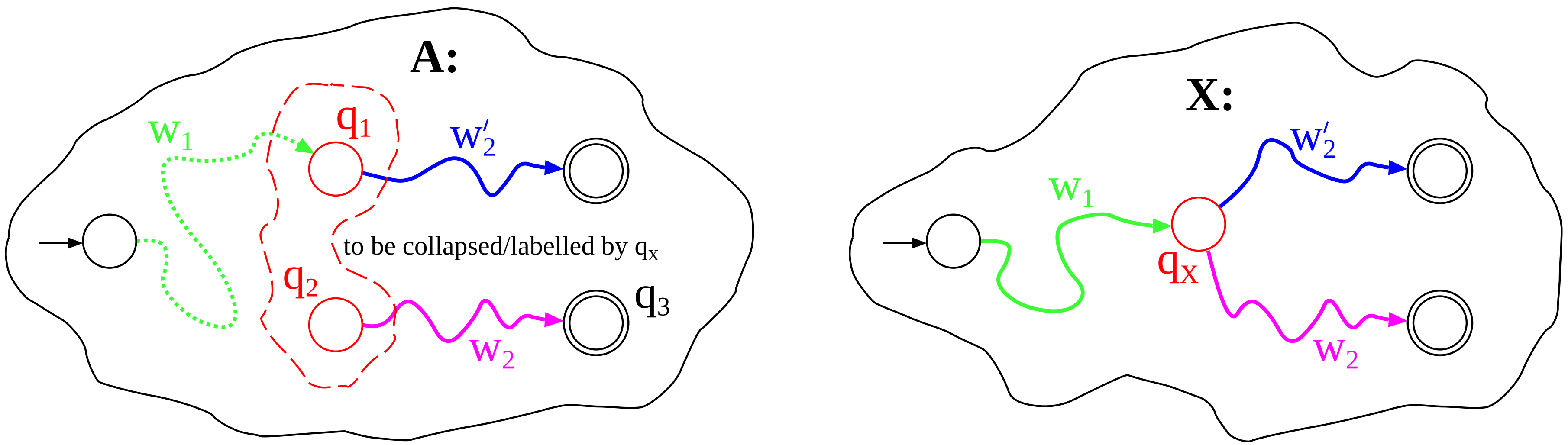
$$\mathcal{P}' = \mathcal{P} \cup \{L(X_k, q) \mid q \text{ is a state in } X_k\}.$$

Predicate Automata Abstraction: Refinement

Theorem:

Let A and X be two finite automata, and let $\mathcal{P}$ be a finite set of predicate languages such that $\forall q \in Q_X .\ L(X, q) \in \mathcal{P}$.
Then, if $L(A) \cap L(X) = \emptyset$, we have $L\big(\alpha_{\mathcal{P}}(A)\big) \cap L(X) = \emptyset$ too.

❖ Proof sketch: Assume $w \notin L(A) \wedge w \in L\big(\alpha_{\mathcal{P}}(A)\big) \cap L(X)$ with a minimum number of *jumps* needed to accept it in A – the last jump being $q_1 \rightsquigarrow q_2$ from where w_2 is accepted.



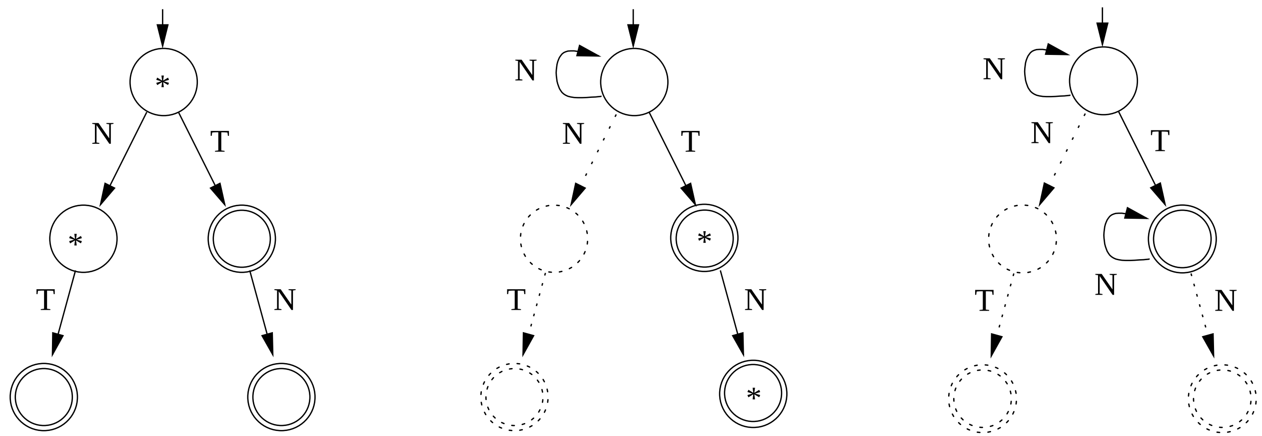
For $w_1 w_2'$, an even smaller number of jumps is needed which is a contradiction.

RMC by Automata Learning

❖ Use algorithms for learning automata from positive/negative samples of their languages to obtain an approximation of $\tau^*(I)$.

❖ Trakhtenbrot-Barzdin:

- based on having n-complete sets of positive and negative samples,
- can be obtained for length-preserving systems as $\tau^*(I^{\leq n})$,
- if $\tau^*(I^{\leq n}) \cap B \neq \emptyset$, error found,
- generalise the sample represented as a loop-free automaton by folding transitions back to compatible states, obtain an automaton A,



- if $\tau(L(A)) \subseteq L(A) \ \wedge \ I \subseteq L(A) \ \wedge \ L(A) \cap B = \emptyset$, verified; otherwise, increase n.

RMC by Automata Learning

- Angluin L* and variants:
  - membership query for a configuration w: check $w \in \tau^*(I^{=|w|})$.
  - equivalence query – replaced by $\tau(L(A)) \subseteq L(A) \ \wedge \ I \subseteq L(A) \ \wedge \ L(A) \cap B = \emptyset$.
- Guaranteed to terminate with the correct answer for length-preserving systems.

Regular Model Checking

String Analysis

RMC and String Analysis

[Yu, Alkhalaf, Bultan, Ibarra et al 08–17]

❖ Deterministic finite automata from MONA with transitions encoded using MTBDDs used for representing sets of strings that may appear in string variables of a program.

❖ Program statements (concatenation, replacement, ...) implemented as specialised automata operations.

❖ Non-refinable widening – collapsing states considered equal:

- states having the same language,
- states having a common access string (for non-sink states),
- closed under transitivity.

❖ Implemented in the STRANGER tool.

❖ Applied for finding XSS vulnerabilities in php-based web applications.

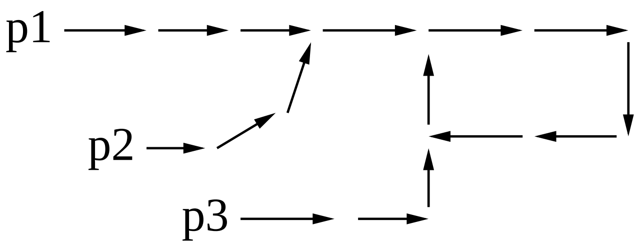
Regular Model Checking

Tricks

RMC and Programs with 1-Selector-Linked Structures

[Bouajjani, Habermehl, Moro, V. 05]

- Heap structures (even with 1 selector) are complex:



- Use pairs of from-to markers m_f/m_t:

$$p1 \to\to\to n_t \to m_t \to\to\to\to\to h_t \to m_f \mid p2 \to\to\to n_f \mid p3 \to\to\to h_f$$

- Pointer operations expressible by transducers up to marker elimination:
  - $\mid y\ m_t \to\to \cdots \to \perp \mid x \to\to \cdots \to m_f \mid$ is changed to $\mid x \to\to \cdots \to y \to\to \cdots \to \perp \mid$,
  - Not rational!
  - Move letter-by-letter and use widening to converge: overapproximation.
  - Use automata surgery instead of transducers.

RMC and Programs with 1-Selector-Linked Structures

❖ RMC can overapproximate sets of reachable configurations at any line, including loop invariants.

❖ Basic memory safety checked directly by the transducers of the program statements:

- no garbage is created,
- no null pointer dereferences,
- no undefined pointer dereferences.

❖ More complex properties can be checked from the invariants.

❖ Generating and checking code (test harness) can be added to the original code to check more complex properties: transforming the checks to error-line reachability.

❖ Sometimes, one may use special markers injected into random positions:

- e.g., when checking a function for reversing lists,
- one may check whether $bgn\ l \rightarrow^* fst \rightarrow snd \rightarrow^* end \rightarrow \perp$ gets transformed to $end\ l \rightarrow^* snd \rightarrow fst \rightarrow^* bgn \rightarrow \perp$.

RMC and Checking Liveness

- For length-preserving systems, liveness can be reduced to checking reachability:
  - Choose any configuration $a_1 a_2 \ldots a_n$ as a candidate for the beginning of a loop.
  - Double every symbol: $a_1 a_1 a_2 a_2 \ldots a_n a_n$.
    - In order to avoid words of the form $w.w$.
  - Go on execution on the "red" symbols: $a_1 \tau'(a_1) a_2 \tau'(a_2) \ldots a_n \tau'(a_n)$.
  - Check whether the system can get back to $a_1 a_1 a_2 a_2 \ldots a_n a_n$.

- Monitoring via some property automaton can be done within the transducer implementing the transition relation.

- More general approaches have been proposed, covering even the non-length preserving case. [Bouajjani, Legay, Wolper 05], [Vardhan, Sen, Viswanathan, Agha 05]

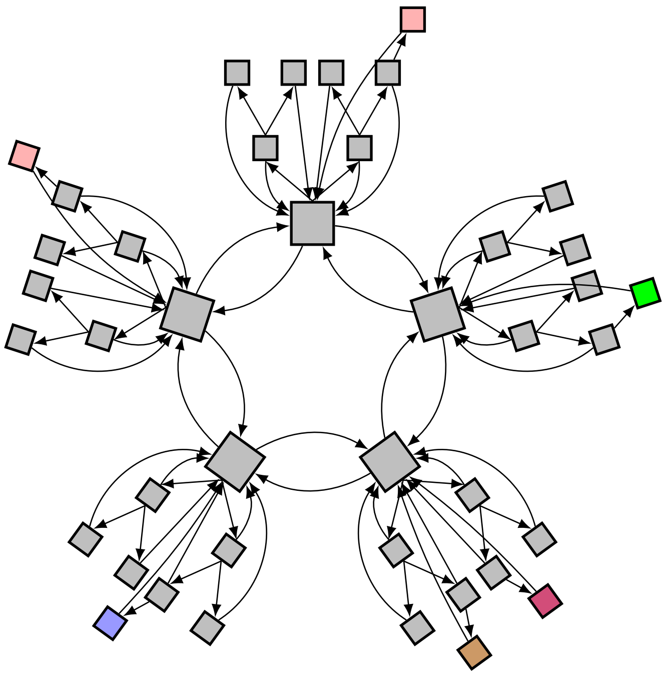
Regular Model Checking

Extensions, Improvements

RMC: Extensions, Improvements

- *Omega* regular model checking:
  - based on variants of Büchi automata,
  - systems with real-valued variables, liveness checking.

    [Boigelot, Bouajjani, Legay, Wolper]



- Regular *tree* model checking:
  - based on variants of tree automata (TAs),
  - parametric protocols with tree topology,
  - programs with complex dynamic data structures:
    - forest automata (FAs): tuples of (nested) TAs,
    - graphs decomposed to tuples of trees whose leaves can refer to roots,
    - symbols can be FAs describing repeated substructures.

[Holik, Hruska, Lengal, Rogalewicz, Simacek, V.]

- R(T)MC based on non-deterministic automata:
  - simulation-based minimisation,
  - antichain-based inclusion checking.