Project_1

November 4, 2024

[1]:
```python
import numpy as np
import pandas as pd
from scipy.sparse import csc_matrix

np.random.seed(412)  # For reproducibility
from scipy.sparse import csc_matrix, csr_matrix
```

[2]:
```python
# Install required packages in Jupyter Notebook
!pip install numpy scipy matplotlib scikit-learn
```

```
Requirement already satisfied: numpy in
/home/anis/jupyter_env/lib/python3.12/site-packages (2.1.3)
Requirement already satisfied: scipy in
/home/anis/jupyter_env/lib/python3.12/site-packages (1.14.1)
Requirement already satisfied: matplotlib in
/home/anis/jupyter_env/lib/python3.12/site-packages (3.9.2)
Requirement already satisfied: scikit-learn in
/home/anis/jupyter_env/lib/python3.12/site-packages (1.5.2)
Requirement already satisfied: contourpy>=1.0.1 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (1.3.0)
Requirement already satisfied: cycler>=0.10 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (4.54.1)
Requirement already satisfied: kiwisolver>=1.3.1 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (1.4.7)
Requirement already satisfied: packaging>=20.0 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (24.1)
Requirement already satisfied: pillow>=8 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (11.0.0)
Requirement already satisfied: pyparsing>=2.3.1 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib) (3.2.0)
Requirement already satisfied: python-dateutil>=2.7 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from matplotlib)
(2.9.0.post0)
Requirement already satisfied: joblib>=1.2.0 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from scikit-learn) (1.4.2)
Requirement already satisfied: threadpoolctl>=3.1.0 in
```

```
/home/anis/jupyter_env/lib/python3.12/site-packages (from scikit-learn) (3.5.0)
Requirement already satisfied: six>=1.5 in
/home/anis/jupyter_env/lib/python3.12/site-packages (from python-
dateutil>=2.7->matplotlib) (1.16.0)
```

0.1 Exercise 1)

[3]:
```python
#Loading data


def load_data(file_path):
    """Loads the data matrix X and target vector y from a CSV file"""

    # load data
    csv_data = pd.read_csv(file_path)
    print(f"Data features: \n {csv_data.columns.values} \n")

    # Separate features (X) and target (y)
    X = csv_data.iloc[:, :-1].values
    y = csv_data.iloc[:, -1].values

    print(y[2])

    num_ones_or_zeros_y=len(y[y == 0])+len(y[y==1])

    print(f"The # of ones and zeros: {num_ones_or_zeros_y} is the same as␣
 ↪length of y {y.shape[0]}")


    # Convert y to ±1 (y is in {0, 1})
    y = np.where(y == 0, -1,1)

    return X, y


# Load the dataset
X, y = load_data('data.csv')
print(X.shape)
```

```
Data features:
 ['android.permission.GET_ACCOUNTS'
 'com.sonyericsson.home.permission.BROADCAST_BADGE'
 'android.permission.READ_PROFILE' 'android.permission.MANAGE_ACCOUNTS'
 'android.permission.WRITE_SYNC_SETTINGS'
 'android.permission.READ_EXTERNAL_STORAGE'
 'android.permission.RECEIVE_SMS'
 'com.android.launcher.permission.READ_SETTINGS'
 'android.permission.WRITE_SETTINGS'
```

'com.google.android.providers.gsf.permission.READ_GSERVICES'
'android.permission.DOWNLOAD_WITHOUT_NOTIFICATION'
'android.permission.GET_TASKS'
'android.permission.WRITE_EXTERNAL_STORAGE'
'android.permission.RECORD_AUDIO'
'com.huawei.android.launcher.permission.CHANGE_BADGE'
'com.oppo.launcher.permission.READ_SETTINGS'
'android.permission.CHANGE_NETWORK_STATE'
'com.android.launcher.permission.INSTALL_SHORTCUT'
'android.permission.android.permission.READ_PHONE_STATE'
'android.permission.CALL_PHONE' 'android.permission.WRITE_CONTACTS'
'android.permission.READ_PHONE_STATE'
'com.samsung.android.providers.context.permission.WRITE_USE_APP_FEATURE_SURVEY'
'android.permission.MODIFY_AUDIO_SETTINGS'
'android.permission.ACCESS_LOCATION_EXTRA_COMMANDS'
'android.permission.INTERNET'
'android.permission.MOUNT_UNMOUNT_FILESYSTEMS'
'com.majeur.launcher.permission.UPDATE_BADGE'
'android.permission.AUTHENTICATE_ACCOUNTS'
'com.htc.launcher.permission.READ_SETTINGS'
'android.permission.ACCESS_WIFI_STATE' 'android.permission.FLASHLIGHT'
'android.permission.READ_APP_BADGE' 'android.permission.USE_CREDENTIALS'
'android.permission.CHANGE_CONFIGURATION'
'android.permission.READ_SYNC_SETTINGS'
'android.permission.BROADCAST_STICKY'
'com.anddoes.launcher.permission.UPDATE_COUNT'
'com.android.alarm.permission.SET_ALARM'
'com.google.android.c2dm.permission.RECEIVE'
'android.permission.KILL_BACKGROUND_PROCESSES'
'com.sonymobile.home.permission.PROVIDER_INSERT_BADGE'
'com.sec.android.provider.badge.permission.READ'
'android.permission.WRITE_CALENDAR' 'android.permission.SEND_SMS'
'com.huawei.android.launcher.permission.WRITE_SETTINGS'
'android.permission.REQUEST_INSTALL_PACKAGES'
'android.permission.SET_WALLPAPER_HINTS'
'android.permission.SET_WALLPAPER'
'com.oppo.launcher.permission.WRITE_SETTINGS'
'android.permission.RESTART_PACKAGES'
'me.everything.badger.permission.BADGE_COUNT_WRITE'
'android.permission.ACCESS_MOCK_LOCATION'
'android.permission.ACCESS_COARSE_LOCATION'
'android.permission.READ_LOGS'
'com.google.android.gms.permission.ACTIVITY_RECOGNITION'
'com.amazon.device.messaging.permission.RECEIVE'
'android.permission.SYSTEM_ALERT_WINDOW'
'android.permission.DISABLE_KEYGUARD'
'android.permission.USE_FINGERPRINT'
'me.everything.badger.permission.BADGE_COUNT_READ'

```
 'android.permission.CHANGE_WIFI_STATE' 'android.permission.READ_CONTACTS'
 'com.android.vending.BILLING' 'android.permission.READ_CALENDAR'
 'android.permission.RECEIVE_BOOT_COMPLETED'
 'android.permission.WAKE_LOCK' 'android.permission.ACCESS_FINE_LOCATION'
 'android.permission.BLUETOOTH' 'android.permission.CAMERA'
 'com.android.vending.CHECK_LICENSE'
 'android.permission.FOREGROUND_SERVICE'
 'android.permission.BLUETOOTH_ADMIN' 'android.permission.VIBRATE'
 'android.permission.NFC' 'android.permission.RECEIVE_USER_PRESENT'
 'android.permission.CLEAR_APP_CACHE'
 'com.android.launcher.permission.UNINSTALL_SHORTCUT'
 'com.sec.android.iap.permission.BILLING'
 'com.htc.launcher.permission.UPDATE_SHORTCUT'
 'com.sec.android.provider.badge.permission.WRITE'
 'android.permission.ACCESS_NETWORK_STATE'
 'com.google.android.finsky.permission.BIND_GET_INSTALL_REFERRER_SERVICE'
 'com.huawei.android.launcher.permission.READ_SETTINGS'
 'android.permission.READ_SMS' 'android.permission.PROCESS_INCOMING_CALLS'
 'Result']
```

```
0
The # of ones and zeros: 29332 is the same as length of y 29332
(29332, 86)
```

[4]:
```
# Display data details
print(f"Loaded dataset with {X.shape[0]} samples and {X.shape[1]} features.")
print(f"Number of malicious data points: {np.sum(y == 1)}")
print(f"Number of non-malicious data points: {np.sum(y == -1)}")

sparsity = len(X[X == 0]) / X.size * 100
print(f"{sparsity:.2f}% of X's entries are 0")
```

```
Loaded dataset with 29332 samples and 86 features.
Number of malicious data points: 14700
Number of non-malicious data points: 14632
89.01% of X's entries are 0
```

We dont need one hot encoding since all the data is binary. the data features and many others, are binary features. Each permission is either granted (1) or not granted (0).

0.2 Exercise 2)

[5]:
```
## Create test and training sets



def split_data(X, y, r=0.5):
    """
```

```
    Splits the data into training and test sets.
    X should be in CSC (Compressed Sparse Column) format.
    y will be returned as a dense vector.
    r: Test size ratio (0 < r < 1).
    """
    # Convert X to sparse CSC matrix (if not already sparse)
    if not isinstance(X, csc_matrix):
        X_sparse = csc_matrix(X)
    else:
        X_sparse = X

    # y is now expected to be a dense array, so no need to convert it to sparse
    y_dense = np.array(y)

    # Shuffle and split data
    indices = np.random.permutation(X_sparse.shape[0])
    split_index = int(X_sparse.shape[0] * (1 - r))

    # Splitting X
    X_train = X_sparse[indices[:split_index], :]
    X_test = X_sparse[indices[split_index:], :]

    # Splitting y (kept as dense)
    y_train = y_dense[indices[:split_index]]
    y_test = y_dense[indices[split_index:]]

    return X_train, X_test, y_train, y_test


# Split the dataset (50/50 split)
X_train, X_test, y_train, y_test = split_data(X, y, r=0.5)
```

0.3 Exercise 3)

[6]:
```
import numpy as np


def classify(X, y, w):
    """Returns the number of correctly classified points using the weight␣
 ↪vector w."""
    # Perform matrix-vector multiplication (X.dot(w)) and predict using the␣
 ↪sign function
    predictions = np.sign(X.dot(w))

    # Convert sparse vector y to dense for comparison
    if isinstance(y, np.ndarray):  # If y is already dense
        y_dense = y
```

```
    else:
        y_dense = y.toarray().flatten()  # Convert sparse matrix to dense and␣
↪flatten it

    # Compare predictions with true labels and count correct classifications
    correct = np.sum(predictions == y_dense)
    accuracy = correct / len(y_dense)

    return correct, accuracy

# Example: Try random weight vector
w_random = np.random.randn(X_train.shape[1])  # Random weight vector of␣
↪appropriate size
correct, accuracy = classify(X_test, y_test, w_random)
print(f"Random classification accuracy: {accuracy * 100:.2f}%")
```

```
Random classification accuracy: 69.68%
```

We can verify that the output makes sense for random weight vectors by calculating the mean over N trials:

[7]:
```
avg_arr=[]
for i in range(0,100):
    w_random = np.random.randn(X_train.shape[1])  # Random weight vector of␣
↪appropriate size
    correct,accuracy=classify(X_test,y_test,w_random)
    avg_arr.append(accuracy)


print(f"average accuracy is: {np.mean(avg_arr)}")
```

```
average accuracy is: 0.5088715396154371
```

As expected, we get a value around 50%

0.4 Exercise 4

Let

$$X \in \mathbb{R}^{n\times d}$$

be a data matrix,

$$\underline{\omega} \in \mathbb{R}^{d\times 1}$$

be the parameter vector, and

$$\underline{y} \in \mathbb{R}^{n\times 1}$$

be the vector of target labels.

Let $\sigma(z)$ denote the **sigmoid function**, defined as:

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

We define the following cost function for logistic regression with L_2 regularization:

$$J(\underline{\omega}) = \sum_{i=1}^{n} L(y_i x_i^T \underline{\omega}) + \frac{\lambda}{2}\|\underline{\omega}\|^2$$

where

$$L(s) = \log(1 + e^{-s})$$

is the logistic loss function.

0.4.1 Gradient of the Logistic Regression Cost Function

Using the chain rule, we find the expression for the gradient of $J(\underline{\omega})$ with respect to ω_j. The gradient of the cost function $J(\underline{\omega})$ with respect to $\underline{\omega}$ is given by:

$$\frac{\partial J(\underline{\omega})}{\partial \omega_j} = \lambda \omega_j - \left(\sum_{i=1}^{n} \frac{\exp(-y_i x_i^T \underline{\omega})}{1 + \exp(-y_i x_i^T \underline{\omega})} y_i x_i^T \right) \underline{e_j}$$

Let z^T represent the row vector in the summation above:

$$z^T = \sum_{i=1}^{n} \frac{\exp(-y_i x_i^T \underline{\omega})}{1 + \exp(-y_i x_i^T \underline{\omega})} y_i x_i^T$$

Then, the gradient can be rewritten as:

$$\nabla J(\underline{\omega}) = \lambda \underline{\omega} - \begin{bmatrix} z^T \underline{e_1} \\ z^T \underline{e_2} \\ \vdots \\ z^T \underline{e_d} \end{bmatrix} = \lambda \underline{\omega} - \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_d \end{bmatrix} = \lambda \underline{\omega} - \underline{z}$$

By transposing z^T to obtain $\underline{z}$, we get:

$$\nabla J(\underline{\omega}) = \lambda \underline{\omega} - \sum_{i=1}^{n} \frac{\exp(-y_i x_i^T \underline{\omega})}{1 + \exp(-y_i x_i^T \underline{\omega})} y_i \underline{x_i}$$

$$\nabla J(\underline{\omega}) = \lambda \underline{\omega} - \sum_{i=1}^{n} \frac{1}{1 + \exp(y_i x_i^T \omega)} y_i \underline{x_i}$$

Since $\frac{1}{1+\exp(y_i x_i^T \underline{\omega})} = \sigma(-y_i x_i^T \underline{\omega})$, this simplifies to:

$$\nabla J(\underline{\omega}) = \lambda \underline{\omega} - \sum_{i=1}^{n} \sigma(-y_i x_i^T \underline{\omega}) y_i \underline{x_i}$$

Let $u_i = \sigma(-y_i x_i^T \underline{\omega}) y_i$, which essentially rescales $\underline{y}$ by the sigmoid function. Defining a new vector $\underline{u}$ with entries u_i, we can rewrite the gradient as:

$$\nabla J(\underline{\omega}) = \lambda \underline{\omega} - \sum_{i=1}^{n} u_i \underline{x_i}$$

Since x_i^T represents a row of the matrix X, the vector $\underline{x_i}$ is, consequently, a column of the transposed matrix X^T. With this observation, we can see that the sum above simplifies to a matrix-vector product:

$$\nabla J(\underline{\omega}) = \lambda\underline{\omega} - X^T\underline{u}$$

where the components of $\underline{u}$ are given by $u_i = \sigma(-y_i x_i^T \underline{\omega}) y_i$.

0.5 Exercise 5

[8]:

```
import numpy as np
from scipy.special import expit  # Efficient sigmoid for sparse

def sigmoid(z):
    """Using Scipy's own Sigmoid function optimized for sparse matrices."""
    return expit(z)

def logistic_regression(X, y, alpha=0.001, reg_lambda=10, K=100):
    """
    Logistic regression using gradient descent with L2 regularization.
    Arguments:
    X -- sparse data matrix (CSC format)
    y -- labels
    alpha -- learning rate
    reg_lambda -- regularization constant
    K -- number of gradient descent steps

    Returns:
    w -- weight vector
    """
    # Initialize weights
    w = np.zeros(X.shape[1])

    # Gradient descent
    for step in range(K):
        # Prediction and gradient calculation
        scale_vector = sigmoid(-X.dot(w) * y)
        y_rescaled = scale_vector * y
        grad = -X.T.dot(y_rescaled) + reg_lambda * w
        # Update weights
        w -= alpha * grad

    return w


# Hyperparameters
alpha = 0.001
reg_lambda = 10
K = 100
```

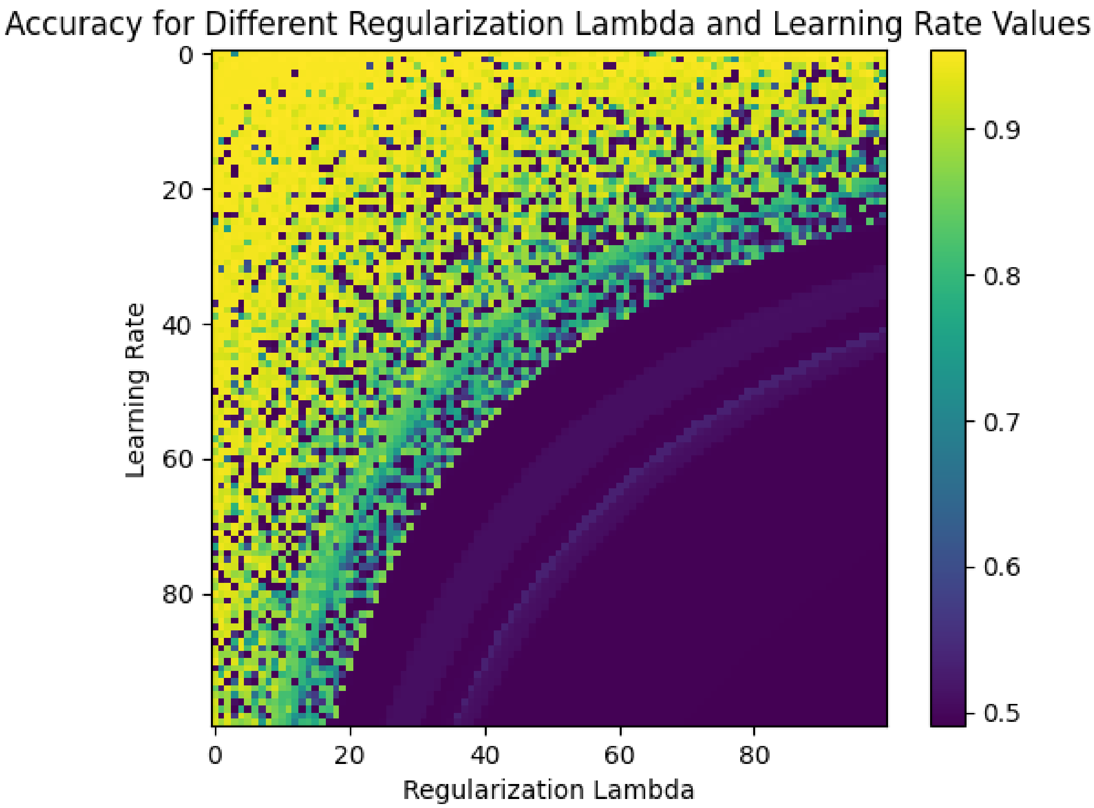
```python
# Train model
w = logistic_regression(X_train, y_train, alpha, reg_lambda, K)

# Evaluate on test set
correct, accuracy = classify(X_test, y_test, w)
print(f"Test set classification accuracy: {accuracy * 100:.2f}%")

# Calculate cost on training set
weights_vector = sigmoid(-X_train.dot(w) * y_train)
cost = -np.sum(np.log(weights_vector)) + 0.5 * reg_lambda * w.dot(w)
print(f"Cost: {cost / len(y_train):.4f}")
```

```
Test set classification accuracy: 94.86%
Cost: 5.1566
```

[9]:
```python
# Gathering Data to plot the accuracy of logistic gradient descent
# for different values of reg_lambda and alpha
num_lambdas = 100
num_alphas = 100
reg_lambdas = np.linspace(1, 100, num_lambdas)
alphas = np.linspace(0.001, 0.01, num_alphas)

accuracies = np.zeros((num_lambdas, num_alphas))
for p1, reg_lambda in enumerate(reg_lambdas):
    for p2, alpha in enumerate(alphas):
        w = logistic_regression(X_train, y_train, alpha, reg_lambda, K)
        _, accuracies[p1, p2] = classify(X_test, y_test, w)
```

[10]:
```python
# Heatmap plot of accuracy
import matplotlib.pyplot as plt
fig, ax = plt.subplots()
plt.xlabel("Regularization Lambda")
plt.ylabel("Learning Rate")
plt.imshow(accuracies)
plt.colorbar()
plt.title("Accuracy for Different Regularization Lambda and Learning Rate␣
↪Values")
```

[10]:
```
Text(0.5, 1.0, 'Accuracy for Different Regularization Lambda and Learning Rate
Values')
```



0.5.1 Exercise 6)

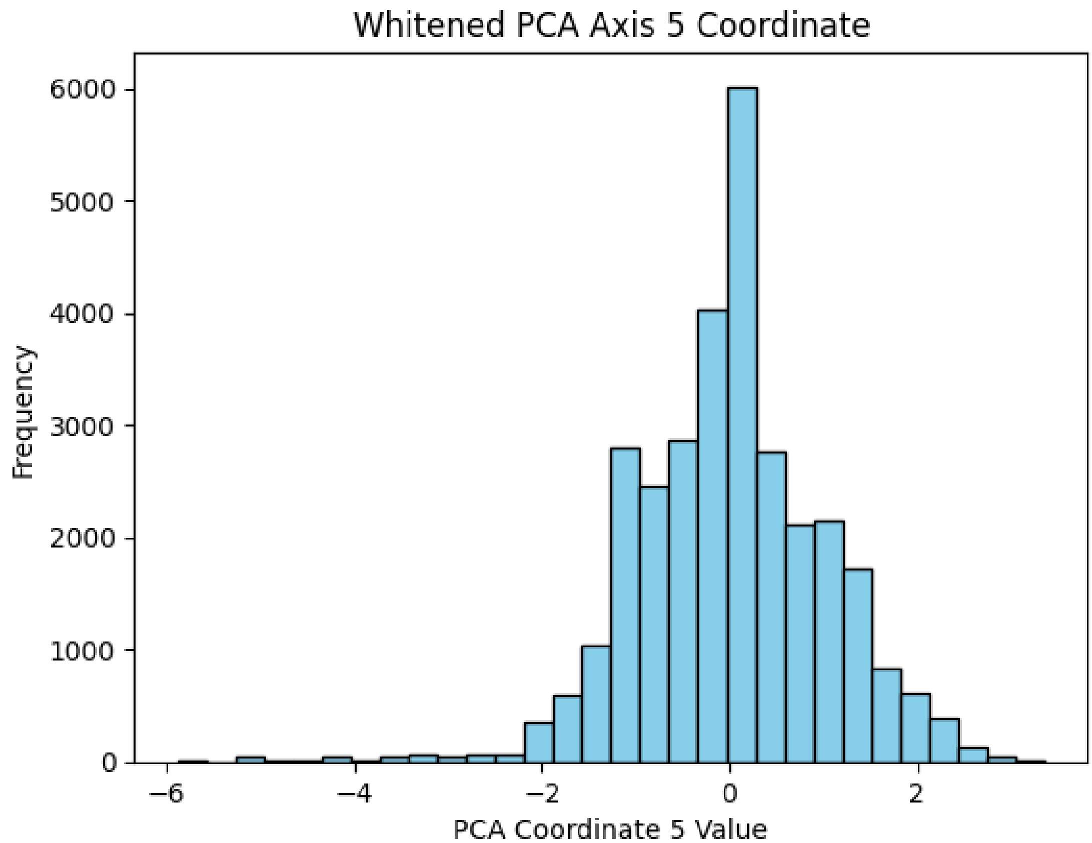
[11]:
```
from sklearn.decomposition import PCA
import numpy as np
import pandas as pd

# Load the dataset with fake points.
csv_data = pd.read_csv("data2.csv")
X = csv_data.iloc[:, :-1].values

# Use PCA with the number of PCA axes the same as our number of features.
# Additionaly whiten the data, meaning dividing each column by its standard␣
↪deviation.
pca = PCA(whiten=True)
pca.fit(X)
X_pca = pca.transform(X)

# Affirming that each column has a standard deviation of 1.
epsilon = 0.01
assert np.all(np.abs(np.std(X_pca, axis=0) - 1) <= epsilon)
```

```
# Calculate the norm of points in the whitened PCA basis.
# Points with a large norm can be identified as potential outliers (refer to
↪Lecture 8).
norm = np.linalg.norm(X_pca, axis=1)

# Assuming we know that 2000 fake datapoints were added,
# find the norm threshold that is less than exactly 2000 datapoints
sorted_norm = np.sort(norm)
threshold = sorted_norm[-2000]
print(threshold)
outliers = norm > threshold
# print(len(outliers[outliers==1]))
```

```
18.703055600592254
```

[12]:

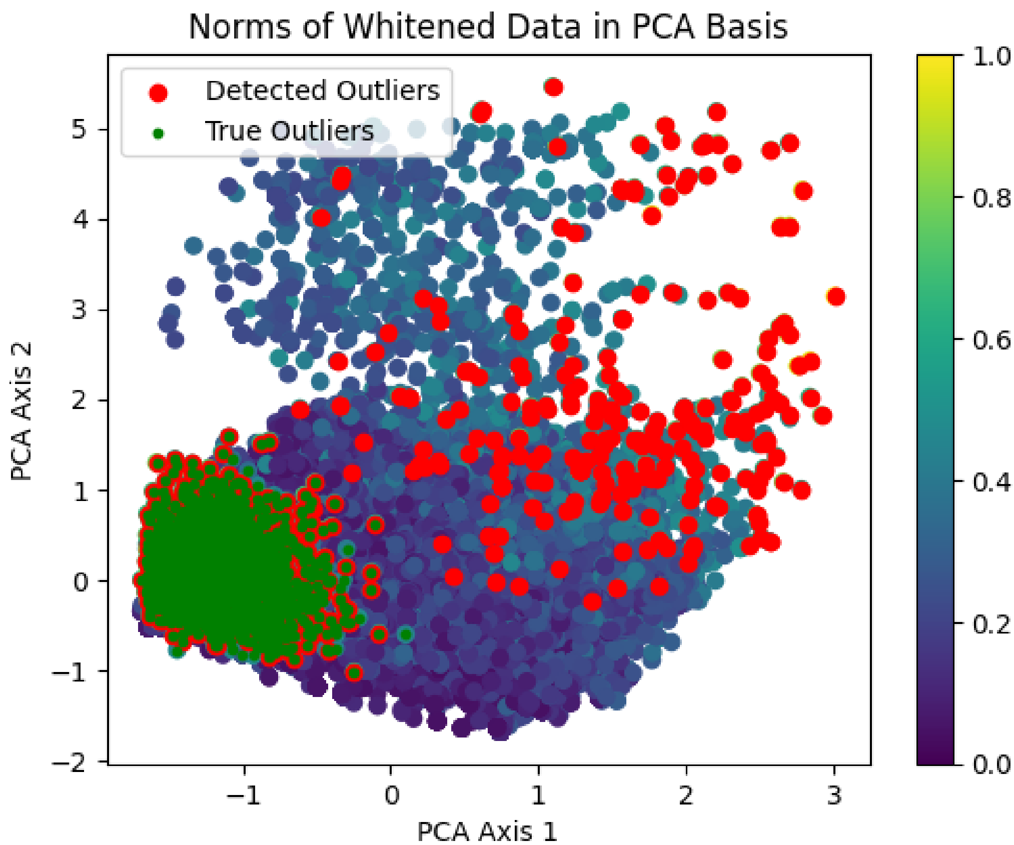
```
import matplotlib.pyplot as plt

# One assumption we made is that the PCA coordinates
# should be normally distributed and in fact we can see this is the case.
plt.hist(X_pca[:, 5], bins=30, color='skyblue', edgecolor='black')
plt.title('Whitened PCA Axis 5 Coordinate')
plt.xlabel('PCA Coordinate 5 Value')
plt.ylabel('Frequency')
plt.show()
```



[13]:
```python
# This visualization shows the outlier points detected by thresholding the
 ↪norms
# of the whitened data, compared to the actual fake points that were added.
# Most of the true fake points are clustered around (-1, 0) in the
# PCA-1 vs. PCA-2 coordinate space. We can also see that most of fake points
 ↪were identified.
# Note: We found that the true outliers were the last 2000 points of X in data2.
 ↪csv.

ax1 = 0
ax2 = 1
plt.scatter(X_pca[:,ax1], X_pca[:,ax2], c=norm)
plt.scatter(X_pca[outliers,ax1], X_pca[outliers,ax2], c="red", label="Detected
 ↪Outliers")
plt.scatter(X_pca[-2000:,ax1], X_pca[-2000:,ax2],10, c="green", label="True
 ↪Outliers")
plt.xlabel("PCA Axis 1")
plt.ylabel("PCA Axis 2")
plt.title("Norms of Whitened Data in PCA Basis")
```

```
plt.legend()
plt.colorbar()
```

[13]: <matplotlib.colorbar.Colorbar at 0x73e3ba9b3890>



[14]:
```
# Fake points detected vs false positive statistics.

outlier_indices = np.array(np.where(outliers)).T
number_fake_detected = np.sum(outlier_indices > X.shape[0]-2000)
print(f"Number of fake points detected: {number_fake_detected}")
print(f"Number of real points falsely detected as fake: {2000 -␣
↪number_fake_detected}")
```

```
Number of fake points detected: 1741
Number of real points falsely detected as fake: 259
```