UNIVERSITÀ DEGLI STUDI DI GENOVA

Master Degree in Robotics Engineering
Research Track 2

Assignment report on statistics

Author: Luca Mosetti
Academic year: 2021-2022

---

1. Introduction

This report reviews the statistical analysis that has been carried out in order to compare two different approaches for solving a robotics-related problem. In particular, the problem concerns driving a mobile robot inside a simulated circuit to grasp targets in it, while avoiding collisions with obstacles.
The two implementations to be compared are the following:

- *Algorithm 1*: is a solution to the problem that has been developed for an assignment given during Research Track 1 course;
- *Algorithm 2*: is an algorithm provided by the professor;

2. Formal hypothesis

As performance evaluator it has been decided to choose the average time required to make the robot completing a lap around the circuit. Hence, the null hypothesis H_0 to be tested through the statistical analysis is the following:

H_0: *"Algorithms 1 and 2, on average, make the robot completing a lap in about the same time* $(\mu_1 = \mu_2)$*"*.

While the alternative hypothesis H_a results to be:

H_a: *"Algorithms 1 and 2, on average, do not make the robot completing a lap in about the same time* $(\mu_1 \neq \mu_2)$*"*.

In order to check whether H_0 should be rejected or not, a two-tailed test with 5% level of significance has been adopted.

3. Choice of the statistical test

In practise, to apply any statistical test, it is required to perform some experiments to collect empirical data that can be used to compute some statistics. As we want to compare two different approaches to the same problem, we had to draw a sample of each.
Specifically, both the algorithms have been tested on same randomly-generated scenarios, so that the observations in the samples can be paired. Therefore, the most appropriate statistical test to use in this context is the *paired t-Test*. This kind of test is parametric, in fact it is based on the assumption that data come from a normally distributed population. Hence, before applying it, we must be sure that this condition is verified.
In order to do so we have two options: the first one consists in using quite large samples $(n \geq 30)$ so that the Central Limit Theorem holds, while the second one is drawing smaller samples and use the Lilliefors test to prove that they belongs to a normal distribution.

4. Lilliefors test

In our case it has been decided to adopted the second solution mentioned in *section 3*, so that it has been sufficient to test the algorithms only on 15 randomly-generated scenarios.
The two samples, along with the Lilliefors test performed on each, are shown in *figure 1* and *figure 2*.
To clarify the content of the figures, it's worth to point out which are the steps followed by the Lilliefors test to prove that a sample is drawn from a normal distribution (null hypothesis):

1. Estimate mean and standard deviation of the population using the data;
2. Compute the empirical cumulative distribution function ($eCDF$) of the normalized data;
3. Evaluate the maximum discrepancy D between $eCDF$ and the normal cumulative distribution function ($nCDF$);
4. Check, using the appropriate table, whether D is large enough to be statistically significant to reject the null hypothesis;

As it can be seen from *figure 1* and *figure 2*, the test confirm the null hypotesis with a 5% level of significance in both cases. Therefore, it is possible to conclude that the samples belongs to a normal distribution.

5. Paired t-Test

Finally, after having shown that the samples are drawn from a normal distribution, we can apply the *paired t-Test* to check whether the null hypothesis stated in *section 2* has to be rejected or not.
The procedure, reported also in *figure 3*, is the following:

1. Compute the difference $d = t_1 - t_2$ between each pair of corresponding observations in the two samples;
2. Compute mean μ_d and standard deviation σ_d of the differences obtained at the previous step;
3. Compute the standard error of the mean difference, defined as $SE(\mu_d) = \sigma_d/\sqrt{n}$;
4. Compute the index $T = \mu_d/SE(\mu_d)$;
5. Use the t-distribution tables to compare T with the values for a t-distribution with $n-1$ DoF;

Since we have chosen a 5% level of significance and our samples have 15 DoF, the T index has to be compared with the value associated to a t-distribution with 14 DoF and 0.05 significance. Looking at the tables we find out that $T < T_{14/0.05}$.
Hence, it is possible to conclude that the statistic is not significant enough to reject the null hypotesis. In other terms, on average, the two algorithms make the robot complete a lap in about the same time.

	t_1 [s]	μ_{t1}	σ_{t1}	norm. t_1	sorted data	eCDF	nCDF	nCDF - eCDF	D	D 15/0.05	Lilliefors result
1	206	181	23,72	1,05	-2,11	0,07	0,02	0,05	0,111	0,220	normal
2	197			0,67	-1,44	0,13	0,08	0,06			
3	201			0,84	-1,14	0,20	0,13	0,07			
4	131			-2,11	-0,51	0,27	0,31	0,04			
5	154			-1,14	-0,34	0,33	0,37	0,03			
6	173			-0,34	-0,30	0,40	0,38	0,02			
7	181			0,00	-0,13	0,47	0,45	0,02			
8	178			-0,13	0,00	0,53	0,50	0,03			
9	169			-0,51	0,29	0,60	0,61	0,01			
10	174			-0,30	0,67	0,67	0,75	0,08			
11	188			0,29	0,76	0,73	0,78	0,04			
12	199			0,76	0,84	0,80	0,80	0,00			
13	210			1,22	1,05	0,87	0,85	0,01			
14	208			1,14	1,14	0,93	0,87	0,06			
15	147			-1,44	1,22	1,00	0,89	0,11			

Figure 1: Lilliefors test on the first sample

	t_2 [s]	μ_{t2}	σ_{t2}	norm. t_2	sorted data	eCDF	nCDF	nCDF - eCDF	D	D 15/0.05	Lilliefors result
1	200	185	12,56	1,17	-2,09	0,07	0,02	0,05	0,091	0,220	normal
2	196			0,85	-1,38	0,13	0,08	0,05			
3	202			1,33	-1,14	0,20	0,13	0,07			
4	159			-2,09	-0,58	0,27	0,28	0,01			
5	171			-1,14	-0,50	0,33	0,31	0,02			
6	187			0,14	-0,42	0,40	0,34	0,06			
7	180			-0,42	0,14	0,47	0,55	0,09			
8	188			0,22	0,22	0,53	0,59	0,05			
9	179			-0,50	0,22	0,60	0,59	0,01			
10	178			-0,58	0,46	0,67	0,68	0,01			
11	191			0,46	0,78	0,73	0,78	0,05			
12	188			0,22	0,85	0,80	0,80	0,00			
13	197			0,93	0,93	0,87	0,82	0,04			
14	195			0,78	1,17	0,93	0,88	0,05			
15	168			-1,38	1,33	1,00	0,91	0,09			

Figure 2: Lilliefors test on the second sample

	t_1 [s]	t_2 [s]	d	μ_d	σ_d	SE(μ_d)	T	T 14/0.05	t-Test result
1	206	200	6	-4	12,45	3,21	1,307	2,145	H_0 confirmed
2	197	196	1						
3	201	202	-1						
4	131	159	-28						
5	154	171	-17						
6	173	187	-14						
7	181	180	1						
8	178	188	-10						
9	169	179	-10						
10	173	179	-4						
11	188	191	-3						
12	199	188	11						
13	210	197	13						
14	208	195	13						
15	147	168	-21						

Figure 3: t-Test results