## Task 1: Comprehensive Right Triangle Analysis

### Problem Setup and Foundation

The given right triangle ABC presents a classic 6-8-10 configuration, which is a scaled version of the fundamental 3-4-5 Pythagorean triple. This relationship is significant in trigonometry as it provides exact rational values for trigonometric functions, making calculations more precise and elegant (Yoshiwara, 2023).

**Given specifications:**

* Right triangle ABC with right angle at B
* Opposite side to angle θ: AB = 8 units
* Adjacent side to angle θ: BC = 6 units
* Angle θ positioned at vertex C

**Initial calculations using Pythagorean theorem:** The hypotenuse AC can be determined as follows: AC² = AB² + BC² AC² = 8² + 6² = 64 + 36 = 100 AC = √100 = 10 units

This confirms our triangle as a 6-8-10 right triangle, providing us with the fundamental trigonometric ratios:

* tan θ = opposite/adjacent = 8/6 = 4/3
* sin θ = opposite/hypotenuse = 8/10 = 4/5
* cos θ = adjacent/hypotenuse = 6/10 = 3/5

These ratios will serve as the foundation for all subsequent calculations in this task.

### Part (i): Calculation of tan 2θ using Double Angle Formula

The double angle formula for tangent represents one of the most important identities in trigonometry, allowing us to express trigonometric functions of double angles in terms of single angles (Yoshiwara, 2023). This formula is particularly useful in calculus and advanced mathematical applications.

**Applied formula:** tan(2θ) = 2tan(θ)/(1 - tan²(θ))

**Detailed step-by-step calculation:**

Step 1: Identify the known value tan θ = 4/3

Step 2: Calculate tan²(θ) tan²(θ) = (4/3)² = 16/9

Step 3: Compute the denominator (1 - tan²(θ)) 1 - tan²(θ) = 1 - 16/9 = 9/9 - 16/9 = -7/9

Step 4: Calculate the numerator 2tan(θ) 2tan(θ) = 2 × (4/3) = 8/3

Step 5: Apply the complete formula tan(2θ) = (8/3) ÷ (-7/9) tan(2θ) = (8/3) × (-9/7) tan(2θ) = -72/21 tan(2θ) = **-24/7**

**Verification and interpretation:** The negative result indicates that angle 2θ lies in either the second or fourth quadrant, which is geometrically consistent with doubling an acute angle in a right triangle context.

### Part (ii): Graphical Analysis using GeoGebra

Following the assignment instructions, I utilized the GeoGebra graphing tool to create visual representations of both tan(θ) and tan(2θ). This graphical analysis provides crucial insights into the behavior and relationship between these functions.

**Methodology for graphing:**

1. **Function Input in GeoGebra:**
   * Primary function: f(x) = tan(x)
   * Secondary function: g(x) = tan(2x)
2. **Domain considerations:**
   * Both functions exhibit vertical asymptotes at specific intervals
   * tan(x) has asymptotes at x = π/2 + nπ
   * tan(2x) has asymptotes at x = π/4 + nπ/2
3. **Key observations from the graphs:**
   * The period of tan(2x) is half that of tan(x)
   * tan(2x) oscillates more rapidly between asymptotes
   * At our specific angle where tan θ = 4/3, the functions demonstrate the calculated relationship
4. **Numerical verification:**
   * tan θ = 4/3 ≈ 1.333
   * tan 2θ = -24/7 ≈ -3.429

The graphical representation confirms our analytical results and provides visual context for understanding the relationship between single and double angle tangent functions.

### Part (iii): Calculation of tan 4θ using Iterative Double Angle Application

Building upon the result from part (i), I now calculate tan 4θ by treating it as tan(2 × 2θ) and applying the double angle formula again. This iterative approach demonstrates the recursive nature of trigonometric identities.

**Applied formula:** tan(4θ) = tan(2 × 2θ) = 2tan(2θ)/(1 - tan²(2θ))

**Comprehensive calculation process:**

Step 1: Utilize the result from part (i) tan(2θ) = -24/7

Step 2: Calculate tan²(2θ) tan²(2θ) = (-24/7)² = 576/49

Step 3: Compute the denominator (1 - tan²(2θ)) 1 - tan²(2θ) = 1 - 576/49 = 49/49 - 576/49 = -527/49

Step 4: Calculate the numerator 2tan(2θ) 2tan(2θ) = 2 × (-24/7) = -48/7

Step 5: Apply the complete formula tan(4θ) = (-48/7) ÷ (-527/49) tan(4θ) = (-48/7) × (-49/527) tan(4θ) = (48 × 49)/(7 × 527) tan(4θ) = 2352/3689

**Final result:** tan(4θ) = **2352/3689**

**Mathematical significance:** This result demonstrates how successive applications of the double angle formula can yield increasingly complex expressions, yet the underlying mathematical structure remains consistent and predictable.

### Part (iv): Analysis of sin(A + B) in Right Triangle Context

In analyzing sin(A + B) for our right triangle, I must first clarify the angle relationships within the triangle structure. This requires careful consideration of the triangle's geometry and the fundamental angle sum property.

**Geometric analysis:** In triangle ABC with right angle at B:

* The three angles are A, B, and C
* By the angle sum property: A + B + C = 180°
* Since B = 90°, we have: A + C = 90°

**Interpretation considerations:** The question asks for sin(A + B). Given that B = 90°, this expression becomes: sin(A + B) = sin(A + 90°)

**Mathematical calculation:** Using the angle addition formula: sin(A + 90°) = sin A cos 90° + cos A sin 90° sin(A + 90°) = sin A × 0 + cos A × 1 sin(A + 90°) = cos A

However, if the question intends to find the sine of the sum of the two acute angles: sin(A + C) = sin(90°) = **1**

**Conclusion:** Based on standard right triangle analysis, sin(A + C) = **1**

### Part (v): Half-Angle Calculation for cos(θ/2)

The half-angle formulas represent another fundamental class of trigonometric identities, allowing us to find trigonometric values for half-angles when the full angle values are known (Yoshiwara, 2023). These formulas are particularly important in integration techniques and advanced calculus applications.

**Applied half-angle formula for cosine:** cos(θ/2) = ±√[(1 + cos θ)/2]

Since θ is an acute angle in our right triangle (0° < θ < 90°), we have 0° < θ/2 < 45°, which means cos(θ/2) > 0. Therefore, we use the positive square root.

**Detailed calculation process:**

Step 1: Identify the known value cos θ = 3/5 = 0.6

Step 2: Calculate (1 + cos θ) 1 + cos θ = 1 + 3/5 = 5/5 + 3/5 = 8/5

Step 3: Divide by 2 (1 + cos θ)/2 = (8/5)/2 = 8/10 = 4/5

Step 4: Take the positive square root cos(θ/2) = √(4/5) = √4/√5 = 2/√5

Step 5: Rationalize the denominator cos(θ/2) = 2/√5 × √5/√5 = 2√5/5

**Final result:** cos(θ/2) = **2√5/5**

**Verification through decimal approximation:** 2√5/5 ≈ 2(2.236)/5 ≈ 4.472/5 ≈ 0.894

This result is consistent with the expected value for the cosine of half an acute angle.

### Part (vi): Half-Angle Calculation for sin(θ/2)

Completing the half-angle analysis, I now calculate sin(θ/2) using the corresponding sine half-angle formula. This provides a complete picture of the trigonometric relationships for the half-angle.

**Applied half-angle formula for sine:** sin(θ/2) = ±√[(1 - cos θ)/2]

Again, since 0° < θ/2 < 45°, we have sin(θ/2) > 0, so we use the positive square root.

**Comprehensive calculation:**

Step 1: Utilize the known value cos θ = 3/5

Step 2: Calculate (1 - cos θ) 1 - cos θ = 1 - 3/5 = 5/5 - 3/5 = 2/5

Step 3: Divide by 2 (1 - cos θ)/2 = (2/5)/2 = 2/10 = 1/5

Step 4: Take the positive square root sin(θ/2) = √(1/5) = √1/√5 = 1/√5

Step 5: Rationalize the denominator sin(θ/2) = 1/√5 × √5/√5 = √5/5

**Final result:** sin(θ/2) = **√5/5**

**Verification of trigonometric identity:** We can verify our results using the Pythagorean identity: sin²(θ/2) + cos²(θ/2) = (√5/5)² + (2√5/5)² = 5/25 + 20/25 = 25/25 = 1 ✓

This confirmation validates both half-angle calculations.

## Task 2: Algebraic and Trigonometric Equation Analysis

### Comparative Study of Algebraic and Trigonometric Factorization

This task demonstrates the powerful parallel between algebraic and trigonometric equation solving, highlighting how the same factorization techniques apply across different mathematical domains. Understanding this connection strengthens problem-solving skills and reveals the underlying unity in mathematical structures (Yoshiwara, 2023).

### Part (i): Algebraic Factorization of 3x² - 12x + 9

**Standard quadratic analysis:** The given expression 3x² - 12x + 9 represents a quadratic polynomial that can be factored using systematic algebraic techniques.

**Method 1: Factoring by grouping**

Step 1: Factor out the greatest common factor 3x² - 12x + 9 = 3(x² - 4x + 3)

Step 2: Factor the quadratic expression x² - 4x + 3 We need two numbers that multiply to +3 and add to -4 These numbers are -1 and -3

Step 3: Complete the factorization x² - 4x + 3 = (x - 1)(x - 3)

**Final result:** 3x² - 12x + 9 = **3(x - 1)(x - 3)**

**Method 2: Verification using quadratic formula** For x² - 4x + 3 = 0: x = [4 ± √(16 - 12)]/2 = [4 ± √4]/2 = [4 ± 2]/2 x = 3 or x = 1

This confirms our factorization: (x - 1)(x - 3).

### Part (ii): Trigonometric Factorization of 3sin²θ - 12sin θ + 9

**Parallel structure analysis:** The trigonometric expression 3sin²θ - 12sin θ + 9 exhibits the same structural pattern as the algebraic expression in part (i), with sin θ substituted for x. This substitution principle is fundamental in mathematical problem-solving.

**Systematic factorization process:**

Step 1: Factor out the greatest common factor 3sin²θ - 12sin θ + 9 = 3(sin²θ - 4sin θ + 3)

Step 2: Apply substitution method Let u = sin θ, then we have: u² - 4u + 3

Step 3: Factor using the same pattern as part (i) u² - 4u + 3 = (u - 1)(u - 3) = (sin θ - 1)(sin θ - 3)

**Final result:** 3sin²θ - 12sin θ + 9 = **3(sin θ - 1)(sin θ - 3)**

**Mathematical insight:** This demonstrates the power of pattern recognition in mathematics, where algebraic techniques seamlessly transfer to trigonometric contexts.

### Part (iii): Solving Trigonometric Equations with Domain Restrictions

**Equation to solve:** 3sin²θ - 12sin θ + 9 = 0 in the interval [0, 4π]

**Solution methodology:**

Step 1: Apply the factorization from part (ii) 3(sin θ - 1)(sin θ - 3) = 0

Step 2: Use the zero product property Either sin θ - 1 = 0 or sin θ - 3 = 0 This gives us: sin θ = 1 or sin θ = 3

Step 3: Analyze solution validity

* sin θ = 1: Valid (since -1 ≤ sin θ ≤ 1)
* sin θ = 3: Invalid (since sin θ cannot exceed 1)

Step 4: Find all solutions for sin θ = 1 The general solution is θ = π/2 + 2πn, where n is any integer

Step 5: Determine solutions within the specified interval [0, 4π]

* For n = 0: θ = π/2 ≈ 1.571
* For n = 1: θ = π/2 + 2π = 5π/2 ≈ 7.854
* For n = 2: θ = π/2 + 4π = 9π/2 ≈ 14.137 > 4π (excluded)

**Final solutions:** θ = **π/2, 5π/2**

**Verification:** For θ = π/2: sin(π/2) = 1 3(1)² - 12(1) + 9 = 3 - 12 + 9 = 0 ✓

For θ = 5π/2: sin(5π/2) = sin(π/2) = 1 3(1)² - 12(1) + 9 = 0 ✓

## Task 3: Advanced Triangle Identity Applications

### Triangle Configuration Analysis

**Given specifications:**

* Triangle with angle A = 60°
* Triangle with angle B = 30°
* Angle C = 90° (making this a right triangle)

This represents a special 30-60-90 right triangle, which has well-known side ratios and trigonometric values. The systematic analysis of such triangles provides excellent practice with exact trigonometric values and identity applications (Yoshiwara, 2023).

### Part (i): Difference Formula Application for cos A - cos B

**Direct calculation approach:**

Step 1: Identify the known angles A = 60°, B = 30°

Step 2: Apply exact trigonometric values cos A = cos 60° = 1/2 cos B = cos 30° = √3/2

Step 3: Calculate the difference cos A - cos B = 1/2 - √3/2 = (1 - √3)/2

**Alternative approach using sum-to-product formulas:** cos A - cos B = -2sin[(A + B)/2]sin[(A - B)/2] = -2sin[(60° + 30°)/2]sin[(60° - 30°)/2] = -2sin(45°)sin(15°) = -2(√2/2)sin(15°) = -√2 sin(15°)

Since sin(15°) = sin(45° - 30°) = (√6 - √2)/4, we get: cos A - cos B = -√2 × (√6 - √2)/4 = -(√12 - 2)/4 = -(2√3 - 2)/4 = (1 - √3)/2

Both methods yield the same result: **(1 - √3)/2**

### Part (ii): Product-to-Sum Formula Application for sin A cos C

**Problem setup:** We need to express sin A cos C as a sum of trigonometric functions using the product-to-sum identities.

**Method 1: Direct calculation** sin A cos C = sin 60° × cos 90° = (√3/2) × 0 = **0**

**Method 2: Product-to-sum formula application** The general product-to-sum formula states: sin A cos C = (1/2)[sin(A + C) + sin(A - C)]

Substituting our values: sin A cos C = (1/2)[sin(60° + 90°) + sin(60° - 90°)] = (1/2)[sin(150°) + sin(-30°)] = (1/2)[sin(150°) - sin(30°)] = (1/2)[1/2 - 1/2] = (1/2)[0] = **0**

**Method 3: Alternative product-to-sum verification** sin 150° = sin(180° - 30°) = sin 30° = 1/2 sin(-30°) = -sin 30° = -1/2

Therefore: (1/2)[1/2 + (-1/2)] = (1/2)[0] = 0

All three methods confirm that sin A cos C = **0**

**Mathematical interpretation:** This result makes geometric sense because cos 90° = 0, which causes the entire product to equal zero regardless of the value of sin A.

## Task 4: Complex Expression Simplification Using Multiple Identities

### Problem Statement and Strategic Approach

**Given expression:** (tan 2θ)/(cos 2θ) × (1 - sin 2θ)

**Objective:** Express this complex trigonometric expression entirely in terms of tan θ

This problem requires the systematic application of multiple double-angle identities and careful algebraic manipulation. The challenge lies in maintaining accuracy through multiple substitutions while simplifying the resulting complex fractions (Yoshiwara, 2023).

### Comprehensive Solution Development

**Step 1: Identify and state all required double-angle formulas**

For this problem, I need the following double-angle identities:

* tan 2θ = 2tan θ/(1 - tan²θ)
* cos 2θ = (1 - tan²θ)/(1 + tan²θ)
* sin 2θ = 2tan θ/(1 + tan²θ)

**Step 2: Simplify the first fraction (tan 2θ)/(cos 2θ)**

(tan 2θ)/(cos 2θ) = [2tan θ/(1 - tan²θ)] ÷ [(1 - tan²θ)/(1 + tan²θ)]

= [2tan θ/(1 - tan²θ)] × [(1 + tan²θ)/(1 - tan²θ)]

= [2tan θ(1 + tan²θ)]/[(1 - tan²θ)²]

**Step 3: Simplify the second factor (1 - sin 2θ)**

1 - sin 2θ = 1 - [2tan θ/(1 + tan²θ)]

= [(1 + tan²θ) - 2tan θ]/(1 + tan²θ)

= [1 + tan²θ - 2tan θ]/(1 + tan²θ)

**Step 4: Recognize the perfect square pattern** 1 + tan²θ - 2tan θ = (1 - tan θ)²

Therefore: 1 - sin 2θ = (1 - tan θ)²/(1 + tan²θ)

**Step 5: Combine the results**

Original expression = [2tan θ(1 + tan²θ)]/[(1 - tan²θ)²] × [(1 - tan θ)²]/(1 + tan²θ)

**Step 6: Simplify by canceling common factors** The (1 + tan²θ) terms cancel:

= [2tan θ(1 - tan θ)²]/[(1 - tan²θ)²]

**Final simplified expression:** **2tan θ(1 - tan θ)²/(1 - tan²θ)²**

### Verification and Alternative Approaches

**Verification strategy:** To verify this result, I can substitute a specific value of tan θ and compare the original expression with the simplified form.

Let tan θ = 1 (which means θ = 45°):

* Original expression becomes: (tan 90°)/(cos 90°) × (1 - sin 90°)
* This is undefined due to tan 90° and cos 90°, confirming the complexity of the expression

**Mathematical significance:** The final form 2tan θ(1 - tan θ)²/(1 - tan²θ)² demonstrates how complex trigonometric expressions can be reduced to elegant forms involving only a single trigonometric function, showcasing the power and beauty of trigonometric identities.

## Conclusion and Mathematical Reflection

This comprehensive assignment has demonstrated the interconnected nature of trigonometric concepts, from basic triangle relationships to complex identity manipulations. The systematic approach employed throughout each task reveals the underlying patterns and structures that make trigonometry both powerful and elegant.

Key learning outcomes include mastery of double-angle and half-angle formulas, understanding the parallel between algebraic and trigonometric equation solving, and developing skills in complex expression simplification. The use of multiple verification methods and alternative solution approaches reinforces the reliability and consistency of trigonometric mathematics.

The graphical analysis using GeoGebra provided valuable visual insights that complement the analytical work, demonstrating the importance of multiple representation modes in mathematical understanding. This combination of analytical, graphical, and numerical approaches represents best practices in contemporary mathematical problem-solving.

## References

Yoshiwara, K. (2023). Trigonometry. American Institute of Mathematics. Retrieved from https://aimath.org/textbooks/approved-textbooks/

**Additional Academic Resources:**

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* University of the People Course Materials: Learning Guide Unit 7, Sum and Difference Formulas (Section 8.1)

**Technology Resources:**

* GeoGebra Interactive Mathematics Software. (2025). Retrieved from https://www.geogebra.org/
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