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Rational Chebyshev Approximations for the Inverse of the Error Function

By J. M. Blair, C. A. Edwards and J. H. Johnson

Abstract. This report presents near-minimax rational approximations for the inverse of the error function inverf x, for $0 \leqslant x \leqslant 1 - 10^{-10000}$, with relative errors ranging down to 10^{-23}. An asymptotic formula for the region $x \longrightarrow 1$ is also given.

1. **Introduction.** The inverse error function inverf x occurs in the solution of nonlinear heat and diffusion problems [1]. It provides exact solutions when the diffusion coefficient is concentration dependent, and may be used to solve certain moving interface problems. The percentage points of the normal distribution, which are important in statistical calculations, are expressible in terms of inverf x, and a common method of computing normally distributed random numbers [2], [3] requires efficient approximations.

The basic mathematical properties of the related function inverfc x are discussed in [4] and [1], and 10S Chebyshev series expansions are given in [1]. [5] lists 3D rational approximations, and [6] contains 7S rational minimax approximations to inverf x and inverfc x. The most accurate set of approximations is given in [7], which contains Chebyshev series expansions accurate to at least 18S for $0 \leqslant x \leqslant 1 - 10^{-300}$.

This report gives near-minimax rational approximations for inverf x for $0 \leqslant x \leqslant 1 - 10^{-10000}$, with relative errors ranging down to 10^{-23}. An asymptotic series is developed which gives at least twenty-five digits of accuracy over the remaining part of the range $1 - 10^{-10000} \leqslant x < 1$. Tables 1–88 computed by this method are included in the microfiche section of this issue. These tables provide the most efficient representations available, and the low order approximations should be useful in normal random number generators.

2. **Functional Properties.** The error function is defined for all real values of the argument y by

$$x = \operatorname{erf} y = 2\pi^{-1/2}\int_0^y e^{-t^2}\,dt$$

and is an odd function of y. For $y \geqslant 0$, x lies in the range $[0, 1)$. The complementary error function is defined as

$$\operatorname{erfc} y = 1 - \operatorname{erf} y.$$

The inverse error function is defined by

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$$y = \operatorname{inverf} x,$$

and the inverse error function complement by

$$y = \operatorname{inverfc}(1 - x).$$

inverf x exists for x in the range $-1 < x < 1$ and is an odd function of x, with a Maclaurin expansion of the form

$$\operatorname{inverf} x = \sum_{n=1}^{\infty} C_n x^{2n-1}.$$

The first two hundred values of C_n are listed in [7].

By inverting the standard asymptotic series

$$(1) \quad \operatorname{erf} y \sim 1 - \frac{\pi^{-1/2}}{y} e^{-y^2} \left[1 + \sum_{m=1}^{\infty} (-1)^m \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{(2y^2)^m} \right], \qquad y \longrightarrow \infty,$$

we can derive an asymptotic expansion for inverf x of the form

$$(2) \quad \begin{aligned} (\operatorname{inverf} x)^2 \sim{}& \eta - \frac{1}{2}\ln\eta + \eta^{-1}\left(\frac{1}{4}\ln\eta - \frac{1}{2}\right) \\ &+ \eta^{-2}\left(\frac{1}{16}\ln^2\eta - \frac{3}{8}\ln\eta + \frac{7}{8}\right) \\ &+ \eta^{-3}\left(\frac{1}{48}\ln^3\eta - \frac{7}{32}\ln^2\eta + \frac{17}{16}\ln\eta - \frac{107}{48}\right) \\ &+ \eta^{-4}\left(\frac{1}{128}\ln^4\eta - \frac{23}{192}\ln^3\eta + \frac{29}{32}\ln^2\eta - \frac{31}{8}\ln\eta + \frac{1489}{192}\right) \\ &+ \ldots, \qquad x \longrightarrow 1, \end{aligned}$$

where $\eta = -\ln[\pi^{1/2}(1 - x)]$.

3. Generation of Approximations. Rational minimax approximations to inverf x were computed in twenty-nine decimal arithmetic on a CDC 6600 using a version of the second algorithm of Remes due to Ralston [8]. The relative error of the approximations was levelled to three digits.

The approximation forms and intervals are

$$\begin{aligned} \operatorname{inverf} x &\simeq x R_{lm}(x^2), && 0 \leqslant x \leqslant 0.75, \\ &\simeq x R_{lm}(x^2), && 0.75 \leqslant x \leqslant 0.9375, \\ &\simeq \xi^{-1} R_{lm}(\xi), && 0.9375 \leqslant x \leqslant 1 - 10^{-100}, \\ &\simeq \xi^{-1} R_{lm}(\xi), && 1 - 10^{-100} \leqslant x \leqslant 1 - 10^{-10000}, \end{aligned}$$

where $R_{lm}(x)$ is a rational function of degree l in the numerator and m in the denominator, and where $\xi = [-\ln(1 - x)]^{-1/2}$.

The auxiliary variable ξ is necessary in the higher ranges to allow high accuracy approximations with rational functions of reasonable degree. The form of the asymptotic expansion (2) might suggest $\xi^{-1}R_{lm}(\xi^2)$ as a more natural approximating function. This form was checked, in addition to $\xi^{-1}R_{lm}(\xi)$ and $\xi^{-1}R_{lm}(\xi^{1/2})$ for the highest range of x, and the latter found to be the most efficient. However, the improvement in accuracy is not enough to offset the cost of the additional square root evaluation.

For the range $0 \leqslant x \leqslant 0.9975$ the master routine computes inverf x by solving the equation erf $y - x = 0$ by the Newton-Raphson technique. For larger values of x, in the range $0.9975 \leqslant x \leqslant 1 - e^{-625}$, we solve instead the equation erfc $y - (1 - x) = 0$. The computation of erf y and erfc y is based on the algorithm in [9], which was programmed in FORTRAN in 29S arithmetic on a CDC 6600. For $x > 1 - e^{-625}$ underflow occurs in evaluating $1 - x$, and the equation is rewritten as $\xi(-\ln \operatorname{erfc} y)^{1/2} - 1 = 0$, where $\xi = [-\ln(1 - x)]^{-1/2}$. Newton-Raphson iteration is again used, starting with $y = 1/\xi$, and the asymptotic formula (1) is used to compute erfc y. Because of the algorithms used, the computed values of inverf x and inverfc x are expected to be accurate to almost full-working precision.

The master routine was checked by comparing the results against the published formulae of Strecok [7]. The maximum relative differences for the ranges [0, 0.8], [0.8, 0.9975], $[1 - 25 \times 10^{-4}, 1 - 5 \times 10^{-16}]$, and $[1 - 5 \times 10^{-16}, 1 - 10^{-300}]$ are 0.67×10^{-24}, 0.13×10^{-22}, 0.45×10^{-22}, and 0.39×10^{-22}, respectively, which are consistent with the magnitudes of the coefficients of the last terms retained by Strecok in his series expansions.

Additional checks consisted of a comparison of the results on either side of the transition points 0.9975 and $1 - e^{-625}$, a comparison between the master routine and (2) at $x = 1 - 10^{-10000}$, and differencing of the values generated by the master routine. The results indicate that the master routine is accurate to at least twenty-seven digits.

4. **Results.** The details of the approximations are given in Tables 1–88, in a format similar to that used in [10]. Tables 1–4 summarize the best approximations in the L_∞ Walsh arrays of the function, and Tables 5–88 give the coefficients of selected approximations.

The precision is defined as

$$-\log_{10} \max_x \left| \frac{f(x) - R_{lm}(x)}{f(x)} \right|,$$

where $f(x)$ is the function being approximated, and the maximum is taken over the appropriate interval.

For the ranges [0, 0.75] and [0.75, 0.9375] the rational functions are ill-conditioned, both in the power polynomial and Chebyshev polynomial forms and lose up to three significant digits by cancellation. To eliminate the cancellation the numerator and denominator were subsequently converted to minimal Newton form (MNF) [11], and the resulting coefficients rounded off by an algorithm similar to that described in [10]. For each of the approximations in the tables the MNF has a particularly simple form, being a polynomial in $(x - x_R)$, where x_R is the right-hand end of the approximation interval; and hence, the MNF is no more costly to evaluate than the power polynomial form.

The approximations in Tables 5–88 were verified by comparing them with the master routine for 5000 pseudorandom values of the argument in each interval.

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RATIONAL CHEBYSHEV APPROXIMATIONS FOR THE INVERSE OF THE ERROR FUNCTION

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Table 1

$$\operatorname{inverf} x \approx x\, P_{\ell}(x^2)/Q_{m}(x^2)$$

RANGE	PRECISION	ℓ	m	TABLE OF COEFFICIENTS
[0,0.75]	1.00	0	0	5
	2.22	0	1	6
	3.51	1	1	7
	4.78	1	2	8
	6.08	2	2	9
	7.35	2	3	10
	8.64	3	3	11
	9.92	3	4	12
	11.22	4	4	13
	12.50	4	5	14
	13.79	5	5	15
	15.07	5	6	16
	16.37	6	6	17
	17.65	6	7	18
	18.94	7	7	19
	20.23	7	8	20
	21.52	8	8	21
	22.80	8	9	22

Table 2

$$\text{inverf } x \approx x\, P_{\ell}(x^2)/Q_m(x^2)$$

RANGE	PRECISION	ℓ	m	TABLE OF COEFFICIENTS
[.75,.9375]	.89	0	0	23
	1.91	0	1	24
	3.01	1	1	25
	4.09	1	2	26
	5.19	2	2	27
	6.28	2	3	28
	7.38	3	3	29
	8.48	3	4	30
	9.58	4	4	31
	10.67	4	5	32
	11.77	5	5	33
	12.86	5	6	34
	13.97	6	6	35
	15.06	6	7	36
	16.15	7	7	37
	17.25	7	8	38
	18.30	8	8	39
	19.39	8	9	40
	20.42	9	9	41
	21.65	9	10	42
	22.75	10	10	43

Table 3

$$\text{inverf } x \approx \xi^{-1} P_\ell(\xi)/Q_m(\xi) \quad , \quad \xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

RANGE	PRECISION	ℓ	m	TABLE OF COEFFICIENTS
$[.9375, 1-10^{-100}]$	.95	0	0	44
	2.11	1	0	45
	2.88	0	2	46
	4.02	1	2	47
	5.04	3	1	48
	6.33	2	3	49
	7.61	5	2	50
	8.88	2	6	51
	10.14	8	2	52
	11.02	6	5	53
	13.11	6	7	54
	13.69	9	6	55
	15.05	8	8	56
	15.90	8	9	57
	16.51	10	8	58
	17.78	10	10	59
	18.03	11	10	60
	19.37	13	9	61
	20.32	10	13	62
	21.97	12	13	63
	22.85	14	12	64

Table 4

$$\text{inverf } x \approx \xi^{-1}\, P_\ell(\xi)/Q_m(\xi)\ ,\quad \xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

RANGE	PRECISION	ℓ	m	TABLE OF COEFFICIENTS
$[1-10^{-100}, 1-10^{-10000}]$	2.45	0	0	65
	3.26	0	1	66
	4.50	0	2	67
	5.67	2	1	68
	6.53	1	3	69
	7.63	3	2	70
	8.37	2	4	71
	9.69	4	3	72
	10.15	3	5	73
	11.01	4	5	74
	11.88	4	6	75
	12.77	5	6	76
	13.58	5	7	77
	14.55	6	7	78
	15.36	8	6	79
	16.36	7	8	80
	17.29	9	7	81
	18.35	8	9	82
	18.71	9	9	83
	19.67	8	11	84
	20.45	10	10	85
	21.37	9	12	86
	22.19	10	12	87
	23.00	14	9	88

Table 5

$$\text{inverf } x \approx p_0 x \ , \quad |x| \le 0.75$$

P00	(	0)	.9754

Table 6

$$\text{inverf } x \approx x\ p_0 / \sum_{j=0}^{1} q_j (x^2 - .5625)^j \ , \quad |x| \le 0.75$$

P00	(	1)	-.27095 8
Q00	(	1)	-.25134 7
Q01	(	1)	.10000

Table 7

$$\text{inverf } x \approx x \sum_{j=0}^{1} p_j (x^2 - .5625)^j / \sum_{j=0}^{1} q_j (x^2 - .5625)^j \quad |x| \le 0.75$$

P00	(	1)	-.10976 672
P01	(	0)	.53062 1
Q00	(	1)	-.10123 953
Q01	(	1)	.10000 00

Table 8

$$\text{inverf } x \approx x \sum_{j=0}^{1} p_j(x^2-.5625)^j \Big/ \sum_{j=0}^{2} q_j(x^2-.5625)^j \ , \ |x| \le 0.75$$

P00	(1)	.59362 254
P01	(1)	-.58954 591

Q00	(1)	.54734 868
Q01	(1)	-.82675 896
Q02	(1)	.10000 00

Table 9

$$\text{inverf } x \approx x \sum_{j=0}^{2} p_j(x^2-.5625)^j \Big/ \sum_{j=0}^{2} q_j(x^2-.5625)^j \ , \ |x| \le 0.75$$

P00	(1)	.15918 63138
P01	(1)	-.24423 26820
P02	(0)	.37153 461

Q00	(1)	.14677 51692
Q01	(1)	-.30131 36362
Q02	(1)	.10000 0000

Table 10

$$\text{inverf } x \approx x \sum_{j=0}^{2} p_j (x^2-.5625)^j / \sum_{j=0}^{3} q_j (x^2-.5625)^j \ , \quad |x| \le 0.75$$

P00	(	2)	-.13095 99674 22
P01	(	2)	.26785 22576 0
P02	(	1)	-.92890 57365
Q00	(	2)	-.12074 94262 97
Q01	(	2)	.30960 61452 9
Q02	(	2)	-.17149 97799 1
Q03	(	1)	.10000 0000

Table 11

$$\text{inverf } x \approx x \sum_{j=0}^{3} p_j (x^2-.5625)^j / \sum_{j=0}^{3} q_j (x^2-.5625)^j \ , \quad |x| \le 0.75$$

P00	(	1)	-.26252 67819 738
P01	(	1)	.67547 44696 99
P02	(	1)	-.39030 76246 27
P03	(	0)	.28847 75643
Q00	(	1)	-.24205 83755 216
Q01	(	1)	.74837 69729 08
Q02	(	1)	-.59793 36646 15
Q03	(	1)	.10000 00000 0

Table 12

$$\text{inverf } x \approx x \sum_{j=0}^{3} p_j (x^2\ .5625)^j / \sum_{j=0}^{4} q_j (x^2-.5625)^j \ , \quad |x| \le 0.75$$

P00	(	2)	.28883 66896 3037
P01	(	2)	-.89043 76057 796
P02	(	2)	.72122 76821 115
P03	(	2)	-.12813 82901 387

Q00	(	2)	.26631 69799 5602
Q01	(	2)	-.95916 43304 721
Q02	(	2)	.99761 47945 595
Q03	(	2)	-.29116 50931 897
Q04	(	1)	.10000 00000 0

Table 13

$$\text{inverf } x \approx x \sum_{j=0}^{4} p_j (x^2-.5625)^j / \sum_{j=0}^{4} q_j (x^2-.5625)^j \ , \quad |x| \le 0.75$$

P00	(	1)	.46268 02021 25696
P01	(	2)	-.16680 59471 26248
P02	(	2)	.17623 01761 90819
P03	(	1)	-.54309 34290 7266
P04	(	0)	.23693 70191 42

Q00	(	1)	.42660 64476 06664
Q01	(	2)	-.17593 07269 90431
Q02	(	2)	.22733 14645 44494
Q03	(	1)	-.99018 86347 8727
Q04	(	1)	.10000 00000 000

Table 14

$$\text{inverf } x \approx x \sum_{j=0}^{4} p_j (x^2-.5625)^j / \sum_{j=0}^{5} q_j (x^2-.5625)^j \text{ , } |x| \le 0.75$$

P00	(	2)	-.63628 85909 64628 6
P01	(	3)	.26184 31346 81937 6
P02	(	3)	-.34041 22053 75487 0
P03	(	3)	.15187 93313 08045 5
P04	(	2)	-.16434 13225 0140
Q00	(	2)	-.58667 91234 96985 4
Q01	(	3)	.27186 18962 62689 7
Q02	(	3)	-.41856 25576 64677 5
Q03	(	3)	.24376 83358 72050 2
Q04	(	2)	-.44140 10518 4605
Q05	(	1)	.10000 00000 000

Table 15

$$\text{inverf } x \approx x \sum_{j=0}^{5} p_j (x^2-.5625)^j / \sum_{j=0}^{5} q_j (x^2-.5625)^j \text{ , } |x| \le 0.75$$

P00	(	1)	-.84938 46999 07046 41
P01	(	2)	.39367 94338 34839 55
P02	(	2)	-.61087 50573 70550 66
P03	(	2)	.36424 44862 95483 54
P04	(	1)	-.70154 38565 55874 7
P05	(	0)	.20177 10235 1770
Q00	(	1)	-.78316 07832 18833 48
Q01	(	2)	.40361 17723 48169 20
Q02	(	2)	-.72411 39469 39276 69
Q03	(	2)	.53489 35185 41307 87
Q04	(	2)	-.14776 96740 68656 41
Q05	(	1)	.10000 00000 00000

Table 16

$$\text{inverf } x \approx x \sum_{j=0}^{5} p_j(x^2-.5625)^j / \sum_{j=0}^{6} q_j(x^2-.5625)^j \text{ , } |x| \leq 0.75$$

P00	(3)	.14002 16916 16135 32398
P01	(3)	-.72042 75515 68640 660
P02	(4)	.12967 08621 66051 0654
P03	(3)	-.96979 32901 51403 067
P04	(3)	.27624 27049 26942 524
P05	(2)	-.20129 40180 55205 4
Q00	(3)	.12910 46303 11468 48865
Q01	(3)	-.73123 08064 26097 292
Q02	(4)	.14949 70492 91578 8517
Q03	(4)	-.13377 93793 68341 8692
Q04	(3)	.50337 47142 78356 698
Q05	(2)	-.62202 05554 52921 6
Q06	(1)	.10000 00000 00000

Table 17

$$\text{inverf } x \approx x \sum_{j=0}^{6} p_j(x^2-.5625)^j / \sum_{j=0}^{6} q_j(x^2-.5625)^j \text{ , } |x| \leq 0.75$$

P00	(2)	.16030 49558 44066 22931 1
P01	(2)	-.90784 95926 29603 26650
P02	(3)	.18644 91486 16209 87391
P03	(3)	-.16900 14273 46423 82420
P04	(2)	.65454 66284 79448 7048
P05	(1)	-.86421 38115 87247 794
P06	(0)	.17605 87821 39059 0
Q00	(2)	.14780 64707 15138 31611 0
Q01	(2)	-.91374 16702 42603 13936
Q02	(3)	.21015 79048 62053 17714
Q03	(3)	-.22210 25412 18551 32366
Q04	(3)	.10760 45391 60551 23830
Q05	(2)	-.20601 07303 28265 443
Q06	(1)	.10000 00000 00000 00

Table 18

$$\operatorname{inverf} x \approx x \sum_{j=0}^{6} p_j (x^2-.5625)^j / \sum_{j=0}^{7} q_j (x^2-.5625)^j \;, \quad |x| \le 0.75$$

P00	(3)	-.30786	87264	23136	95280 46
P01	(4)	.19007	15359	05281	34753 18
P02	(4)	-.43799	65230	83869	26160 8
P03	(4)	.46631	43353	34343	31286 8
P04	(4)	-.22977	46717	66071	44886 9
P05	(3)	.45560	20427	26891	2817
P06	(2)	-.23886	24010	43087	559
Q00	(3)	-.28386	51472	53666	21129 52
Q01	(4)	.18997	76918	64530	57809 85
Q02	(4)	-.48481	63543	00488	72101 5
Q03	(4)	.59039	34813	48436	65625 5
Q04	(4)	-.35088	97638	38772	64098 4
Q05	(3)	.92741	31916	09353	1880
Q06	(2)	-.83288	32790	19365	700
Q07	(1)	.10000	00000	00000	00

Table 19

$$\operatorname{inverf} x \approx x \sum_{j=0}^{7} p_j (x^2-.5625)^j / \sum_{j=0}^{7} q_j (x^2-.5625)^j \;, \quad |x| \le 0.75$$

P00	(2)	-.30866	88652	77644	97339 502
P01	(3)	.20652	78640	29423	39589 007
P02	(3)	-.52856	77083	50938	23310 87
P03	(3)	.64880	50954	40362	49088 40
P04	(3)	-.39205	50990	19713	91289 20
P05	(3)	.10706	27809	77700	74402 58
P06	(2)	-.10303	48845	54396	78272
P07	(0)	.15641	51082	19238	600
Q00	(2)	-.28460	29017	38823	39383 682
Q01	(3)	.20518	92414	92385	30630 152
Q02	(3)	-.57617	12509	01275	38064 10
Q03	(3)	.79669	38817	05637	70334 37
Q04	(3)	-.56509	30556	40234	24022 86
Q05	(3)	.19450	32048	39540	87700 60
Q06	(2)	-.27371	85230	60026	62877
Q07	(1)	.10000	00000	00000	0000

Table 20

$$\text{inverf } x \approx x \sum_{j=0}^{7} p_j (x^2-.5625)^j \Big/ \sum_{j=0}^{8} q_j (x^2-.5625)^j \; , \quad |x| \le 0.75$$

P00	(	3)	.67645	57952	65851	77112	3606
P01	(	4)	-.48716	23471	88061	34686	6351
P02	(	5)	.13695	96022	55143	14808	5967
P03	(	5)	-.19029	09700	59805	02083	1321
P04	(	5)	.13640	77684	03368	98712	5877
P05	(	4)	-.47913	05081	90746	14860	895
P06	(	3)	.70051	10922	35656	91614	8
P07	(	2)	-.27695	28710	47165	16016	
Q00	(	3)	.62371	46142	27559	42437	6964
Q01	(	4)	-.48153	48706	68920	26495	1208
Q02	(	5)	.14739	77692	34566	19584	5274
Q03	(	5)	-.22793	78583	40551	31263	8455
Q04	(	5)	.18796	28467	69108	65270	7107
Q05	(	4)	-.80145	97236	83333	48784	238
Q06	(	4)	.15731	07732	42888	54698	81
Q07	(	3)	-.10738	78066	58350	86337	5
Q08	(	1)	.10000	00000	00000	0000	

Table 21

$$\operatorname{inverf} x \approx x \sum_{j=0}^{8} p_j (x^2-.5625)^j \Big/ \sum_{j=0}^{8} q_j (x^2-.5625)^j \ , \ |x| \leq 0.75$$

P00	(2)	.60344 38910 01173 87447 91235
P01	(3)	-.46574 48486 78771 21440 29462
P02	(4)	.14283 99779 11513 18404 50665 5
P03	(4)	-.22206 96807 53171 82652 79170
P04	(4)	.18508 59444 60577 30525 36718
P05	(3)	-.80469 85535 48242 52453 4489
P06	(3)	.16364 17674 49627 87245 1786
P07	(2)	-.11994 20276 16122 57349 98
P08	(0)	.14088 94789 13897 90200
Q00	(2)	.55639 52239 27168 70053 53874 1
Q01	(3)	-.45829 50483 21565 58017 38271
Q02	(4)	.15203 10584 86617 42611 44748 2
Q03	(4)	-.26045 08670 10099 65414 27314
Q04	(4)	.24573 13895 58930 43372 91627
Q05	(4)	-.12609 53962 07644 83131 43775
Q06	(3)	.32517 69082 65411 10784 6713
Q07	(2)	-.35087 45112 89498 96062 87
Q08	(1)	.10000 00000 00000 00000 0

Table 22

$$\operatorname{inverf} x \approx x \sum_{j=0}^{8} p_j (x^2-.5625)^j \Big/ \sum_{j=0}^{9} q_j (x^2-.5625)^j , \quad |x| \le 0.75$$

P00	(4)	-.14855 07416 31375 47585 18888 90
P01	(5)	.12224 37770 06384 25027 57234 20
P02	(5)	-.40583 89788 79680 35456 47898 0
P03	(5)	.69757 81844 60969 87204 28582 1
P04	(5)	-.66288 23025 05639 13958 55685 3
P05	(5)	.34463 97685 29636 34922 35886 8
P06	(4)	-.90929 43390 09954 49270 6998
P07	(4)	.10216 63340 31473 91172 4304
P08	(2)	-.31549 39308 61967 72332 97
Q00	(4)	-.13696 86964 88759 97272 79408 64
Q01	(5)	.11981 80299 00809 72031 85263 25
Q02	(5)	-.42786 90319 25295 57109 76717 7
Q03	(5)	.80377 23153 60116 65072 18183 7
Q04	(5)	-.85372 62983 60500 47166 53500 2
Q05	(5)	.51302 86782 64824 34171 83716 8
Q06	(5)	-.16518 01534 34648 83596 03975
Q07	(4)	.25060 38305 29015 98314 1685
Q08	(3)	-.13449 14312 10066 37969 734
Q09	(1)	.10000 00000 00000 00000 0

Table 23

$$\text{inverf } x \approx p_0 x \,,\ 0.75 \le x \le 0.9375$$

```
P00   (  1)   .12240
```

Table 24

$$\text{inverf } x \approx x\ p_0 / \sum_{j=0}^{1} q_j (x^2 - .87890625)^j \,,\ 0.75 \le x \le 0.9375$$

```
P00   (  1) -.14862 5

Q00   (  1) -.10710 4
Q01   (  1)  .10000 0
```

Table 25

$$\text{inverf } x \approx x \sum_{j=0}^{1} p_j (x^2 - .87890625)^j / \sum_{j=0}^{1} q_j (x^2 - .87890625)^j$$

$$0.75 \le x \le 0.9375$$

```
P00   (  0) -.49075 0
P01   (  0)  .73423 2

Q00   (  0) -.34963 8
Q01   (  1)  .10000 00
```

Table 26

$$\text{inverf } x \approx x \sum_{j=0}^{1} p_j(x^2-.87890625)^j / \sum_{j=0}^{2} q_j(x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(1)	.10178 950	
P01	(1)	-.32827 601	
Q00	(0)	.72455 99	
Q01	(1)	-.33871 553	
Q02	(1)	.10000 000	

Table 27

$$\text{inverf } x \approx x \sum_{j=0}^{2} p_j(x^2-.87890625)^j / \sum_{j=0}^{2} q_j(x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	.23286 863
P01	(1)	-.11816 2169
P02	(0)	.53384 223
Q00	(0)	.16574 853
Q01	(1)	-.10835 6783
Q02	(1)	.10000 0000

Table 28

$$\text{inverf } x \approx x \sum_{j=0}^{2} p_j (x^2-.87890625)^j / \sum_{j=0}^{3} q_j (x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	-.70987 5483
P01	(1)	.48420 71339
P02	(1)	-.52017 61073
Q00	(0)	-.50526 3975
Q01	(1)	.41865 85867
Q02	(1)	-.68492 72415
Q03	(1)	.10000 0000

Table 29

$$\text{inverf } x \approx x \sum_{j=0}^{3} p_j (x^2-.87890625)^j / \sum_{j=0}^{3} q_j (x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	-.12402 56522 1
P01	(1)	.10688 05957 4
P02	(1)	-.19594 55607 8
P03	(0)	.42305 81357
Q00	(-1)	-.88276 97997
Q01	(0)	.89007 43359
Q02	(1)	-.21757 03119 6
Q03	(1)	.10000 00000 0

Table 30

$$\text{inverf } x \approx x \sum_{j=0}^{3} p_j (x^2-.87890625)^j / \sum_{j=0}^{4} q_j (x^2-.87890625)^j$$

$$0.75 \le x \le 0.9375$$

P00	(0)	.49633 37483 1
P01	(1)	-.51381 57320 25
P02	(2)	.13411 97417 494
P03	(1)	-.71986 74889 6
Q00	(0)	.35327 24232 3
Q01	(1)	-.41747 50925 62
Q02	(2)	.13568 88557 20
Q03	(2)	-.11436 53583 76
Q04	(1)	.10000 00000 0

Table 31

$$\text{inverf } x \approx x \sum_{j=0}^{4} p_j (x^2-.87890625)^j / \sum_{j=0}^{4} q_j (x^2-.87890625)^j,$$

$$0.75 \le x \le 0.9375$$

P00	(-1)	.70156 83618 57
P01	(0)	-.85081 95568 326
P02	(1)	.29255 14644 677
P03	(1)	-.27905 98322 322
P04	(0)	.35215 10941 60
Q00	(-1)	.49935 10018 90
Q01	(0)	-.67874 50194 062
Q02	(1)	.27807 50537 918
Q03	(1)	-.36178 90396 636
Q04	(1)	.10000 00000 000

Table 32

$$\text{inverf } x \approx x \sum_{j=0}^{4} p_j (x^2-.87890625)^j / \sum_{j=0}^{5} q_j (x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	-.34688 76691 975
P01	(1)	.48077 41226 0565
P02	(2)	-.20478 70119 02191
P03	(2)	.28746 57116 0243
P04	(1)	-.92530 46782 146
Q00	(0)	-.24690 21046 736
Q01	(1)	.37837 25103 8596
Q02	(2)	-.18656 21299 89775
Q03	(2)	.33183 07146 4595
Q04	(2)	-.17137 25392 4242
Q05	(1)	.10000 00000 000

Table 33

$$\text{inverf } x \approx x \sum_{j=0}^{5} p_j (x^2-.87890625)^j / \sum_{j=0}^{5} q_j (x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(-1)	-.41139 81706 7782
P01	(0)	.64372 98540 03468
P02	(1)	-.32890 26740 93993
P03	(1)	.62451 84315 79026
P04	(1)	-.36595 35964 95117
P05	(0)	.30260 11494 3200
Q00	(-1)	-.29324 54062 0124
Q01	(0)	.50114 85005 27886
Q02	(1)	-.29014 46872 99145
Q03	(1)	.66539 30519 63183
Q04	(1)	-.54064 05804 12825
Q05	(1)	.10000 00000 00000

Table 34

$$\operatorname{inverf} x \approx x \sum_{j=0}^{5} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{6} q_j (x^2 - .87890625)^j$$

$$0.75 \le x \le 0.9375$$

P00	(0)	.24224 96886 24227
P01	(1)	-.42046 64715 16659 5
P02	(2)	.25011 19714 04336 44
P03	(2)	-.60090 53042 30262 6
P04	(2)	.52922 02474 16856 0
P05	(2)	-.11352 87386 00315 7
Q00	(0)	.17242 45722 39610
Q01	(1)	-.32453 56272 40065 3
Q02	(2)	.21534 95322 70387 63
Q03	(2)	-.60543 86604 59822 3
Q04	(2)	.68454 59265 64466 8
Q05	(2)	-.23943 36037 07420 7
Q06	(1)	.10000 00000 00000

Table 35

$$\operatorname{inverf} x \approx x \sum_{j=0}^{6} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{6} q_j (x^2 - .87890625)^j$$

$$0.75 \le x \le 0.9375$$

Q00	(-1)	.17668 44629 48063 5
Q01	(0)	-.36363 30704 38225 98
Q02	(1)	.27271 87290 63128 61
Q03	(1)	-.91404 63803 21697 81
Q04	(2)	.13531 59828 30615 354
Q05	(1)	-.75390 55414 34509 52
Q06	(1)	.10000 00000 00000 00
P00	(-1)	.24823 46661 96142 9
P01	(0)	-.47451 99401 35955 27
P02	(1)	.32302 00558 30891 51
P03	(1)	-.94705 89734 85633 36
P04	(2)	.11486 63107 16134 902
P05	(1)	-.45575 02217 01686 04
P06	(0)	.26588 64808 95280 0

Table 36

$$\operatorname{inverf} x \approx x \sum_{j=0}^{6} p_j (x^2-.87890625)^j \Big/ \sum_{j=0}^{7} q_j (x^2-.87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	-.16904 78046 78174 49
P01	(1)	.35243 74318 10022 774
P02	(2)	-.26981 43370 55035 2151
P03	(2)	.93407 83041 01874 299
P04	(3)	-.14553 64428 64673 2178
P05	(2)	.88058 52004 72365 945
P06	(2)	-.13490 18591 23194 732
Q00	(0)	-.12032 21171 31342 87
Q01	(1)	.26848 12231 55663 173
Q02	(2)	-.22424 85268 70486 4906
Q03	(2)	.87234 95028 64349 429
Q04	(3)	-.16043 52408 44431 8626
Q05	(3)	.12591 17982 10152 5057
Q06	(2)	-.31848 61786 24882 403
Q07	(1)	.10000 00000 00000 00

Table 37

$$\text{inverf } x \approx x \sum_{j=0}^{7} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{7} q_j (x^2 - .87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(-1)	-.15238	92634	40726	128
P01	(0)	.34445	56924	13612	5216
P02	(1)	-.29344	39867	25424	78687
P03	(2)	.11763	50570	52178	27302
P04	(2)	-.22655	29282	31011	04193
P05	(2)	.19121	33439	65803	30163
P06	(1)	-.54789	27619	59831	8769
P07	(0)	.23751	66890	24448	000
Q00	(-1)	-.10846	51696	02059	954
Q01	(0)	.26106	28885	84307	8511
Q02	(1)	-.24068	31810	43937	57995
Q03	(2)	.10695	12997	33870	14469
Q04	(2)	-.23716	71552	15965	81025
Q05	(2)	.24640	15894	39172	84883
Q06	(2)	-.10014	37634	97830	70835
Q07	(1)	.10000	00000	00000	0000

<u>Table 38</u>

$$\mathrm{inverf}\ x \approx x \sum_{j=0}^{7} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{8} q_j (x^2 - .87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(0)	.11788 89795 86894 9835
P01	(1)	-.28690 44151 56943 38990
P02	(2)	.26891 12457 63661 39280 1
P03	(3)	-.12244 67479 86750 07204 0
P04	(3)	.28150 48808 08414 72939 1
P05	(3)	-.30866 76924 69725 28252 1
P06	(3)	.13631 28203 54108 03612 4
P07	(2)	-.15659 36326 14413 24707

Q00	(-1)	.83909 11456 88223 996
Q01	(1)	-.21650 21385 97219 93642
Q02	(2)	.21814 84046 40264 70913 5
Q03	(3)	-.10893 83625 72869 91744 5
Q04	(3)	.28314 32465 28204 70264 8
Q05	(3)	-.36907 64811 17669 62886 7
Q06	(3)	.21317 90751 18569 95133 6
Q07	(2)	-.40848 16219 63519 60986
Q08	(1)	.10000 00000 00000 0000

Table 39

$$\operatorname{inverf} x \approx x \sum_{j=0}^{8} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{8} q_j (x^2 - .87890625)^j$$

$$0.75 \le x \le 0.9375$$

P00	(-2)	.94897 36280 86810 8002
P01	(0)	-.24758 24236 28233 55485 8
P02	(1)	.25349 38922 07148 93916 88
P03	(2)	-.12954 19898 06467 71502 07
P04	(2)	.34810 85774 93575 00873 33
P05	(2)	-.47644 36712 97871 81802 84
P06	(2)	.29631 33150 58763 08122 74
P07	(1)	-.64200 07150 72094 48654 9
P08	(0)	.21489 18500 73070 62000
Q00	(-2)	.67544 51277 88509 4594
Q01	(0)	-.18611 65062 73721 78511 4
Q02	(1)	.20369 29504 72163 51160 19
Q03	(2)	-.11315 36062 42380 54876 36
Q04	(2)	.33880 17677 95951 42684 61
Q05	(2)	-.53715 37344 88621 43348 55
Q06	(2)	.41409 99177 84288 88715 67
Q07	(2)	-.12831 38383 39532 26499 44
Q08	(1)	.10000 00000 00000 00000 0

Table 40

$$\operatorname{inverf} x \approx x \sum_{j=0}^{8} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{9} q_j (x^2 - .87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(-1)	-.82168	12190	35724	59683	
P01	(1)	.22861	80685	91703	46309	88
P02	(2)	-.25367	56331	36660	49703	850
P03	(3)	.14367	81666	61149	53140	806
P04	(3)	-.44210	09066	85900	02871	390
P05	(3)	.72855	04644	52946	48562	452
P06	(3)	-.59374	74232	84138	84663	358
P07	(3)	.19987	29891	69772	31980	380
P08	(2)	-.17856	23079	33513	29801	95
Q00	(-1)	-.58484	29920	14349	78481	
Q01	(1)	.17129	09064	02908	92507	23
Q02	(2)	-.20218	71785	32085	94474	378
Q03	(3)	.12358	65726	41248	33025	955
Q04	(3)	-.41892	73760	68941	86680	375
Q05	(3)	.78451	94250	03326	29935	385
Q06	(3)	-.76459	01174	75386	71664	942
Q07	(3)	.33897	18080	99299	65434	408
Q08	(2)	-.50938	07739	23598	69524	61
Q09	(1)	.10000	00000	00000	00000	0

Table 41

$$\text{inverf } x \approx x \sum_{j=0}^{9} p_j (x^2 - .87890625)^j \Big/ \sum_{j=0}^{9} q_j (x^2 - .87890625)^j$$

$$0.75 \leq x \leq 0.9375$$

P00	(-2)	-.59764 86905 88381 75131 8
P01	(0)	.17674 83979 67448 47515 643
P02	(1)	-.21145 24958 94984 66552 6362
P03	(2)	.13166 90005 22637 04798 6855
P04	(2)	-.45790 74446 32397 35517 9198
P05	(2)	.88866 35189 06412 05800 0986
P06	(2)	-.91099 70656 20514 30805 3049
P07	(2)	.43507 10102 00429 18522 7802
P08	(1)	-.73779 42025 51541 85450 674
P09	(0)	.19639 95497 42267 95300 00
Q00	(-2)	-.42538 47359 28801 67288 2
Q01	(0)	.13203 56466 38370 83008 109
Q02	(1)	-.16732 79797 64925 17255 3288
Q03	(2)	.11175 35390 56790 07339 5509
Q04	(2)	-.42412 91295 73455 92290 4538
Q05	(2)	.92124 95357 53296 52561 0751
Q06	(3)	-.10995 52696 68950 14634 81649
Q07	(2)	.65474 13396 56766 42044 5679
Q08	(2)	-.15989 24757 10877 69057 0947
Q09	(1)	.10000 00000 00000 00000 000

Table 42

$$\operatorname{inverf} x \approx x \sum_{j=0}^{9} p_j (x^2-.87890625)^j \Big/ \sum_{j=0}^{10} q_j (x^2-.87890625)^j$$

$$0.75 \le x \le 0.9375$$

P00	(-1)	.57245 10367 06304 13025 736
P01	(1)	-.17922 39918 81163 08457 58333
P02	(2)	.22982 14563 37602 80253 30223
P03	(3)	-.15596 47381 07463 43382 37102 6
P04	(3)	.60488 56566 77555 24751 04163 5
P05	(4)	-.13531 59636 47737 96769 29291 63
P06	(4)	.16816 89509 41079 81541 16292 2
P07	(4)	-.10596 02532 80152 42679 41789 0
P08	(3)	.28095 37086 81439 47177 37933
P09	(2)	-.20077 54828 18109 80130 5957
Q00	(-1)	.40744 99566 65585 29014 277
Q01	(1)	-.13353 49134 41672 33883 49744
Q02	(2)	.18072 83225 79667 13936 04860
Q03	(3)	-.13086 40724 25849 79659 74402 1
Q04	(3)	.54961 76407 56380 40421 50828 7
Q05	(4)	-.13597 68418 15301 54992 87647 42
Q06	(4)	.19289 05692 53913 39287 89331 7
Q07	(4)	-.14607 73580 25996 87012 62205 5
Q08	(3)	.51309 08257 65227 62255 53270
Q09	(2)	-.62115 03816 42844 68933 3398
Q10	(1)	.10000 00000 00000 00000 000

Table 43

$$\operatorname{inverf} x \approx x \sum_{j=0}^{10} p_j (x^2-.87890625)^j \Big/ \sum_{j=0}^{10} q_j (x^2-.87890625)^j$$

$$0.75 \le x \le 0.9375$$

P00	(-2)	.37983 61161 79297 66727 3885
P01	(0)	-.12556 42587 17295 35132 3457
P02	(1)	.17191 66838 39930 98377 33032 2
P03	(2)	-.12641 81923 50003 89155 25868 53
P04	(2)	.54198 57639 60472 07196 29512 88
P05	(3)	-.13784 05371 93031 96737 27419 769
P06	(3)	.20298 24274 37465 85797 76497 21
P07	(3)	-.16186 93620 61499 16906 48635 95
P08	(2)	.61246 55913 29226 67158 38562 1
P09	(1)	-.93506 10553 94993 45138 73439
P10	(0)	.18095 39020 20106 98960 0000
Q00	(-2)	.27035 36183 07234 82129 4499
Q01	(-1)	-.93333 19755 78378 16539 27285
Q02	(1)	.13443 62736 54698 18351 18417 6
Q03	(2)	-.10500 03755 32476 47117 97308 24
Q04	(2)	.48429 10556 29395 17946 31175 19
Q05	(3)	-.13497 31234 71034 81689 34554 936
Q06	(3)	.22315 52950 37569 36487 62242 09
Q07	(3)	-.20810 14743 80799 74854 96484 96
Q08	(2)	.98667 95523 36503 86780 60210 2
Q09	(2)	-.19497 30256 25481 98205 51494 4
Q10	(1)	.10000 00000 00000 00000 00000

Table 44

$$\text{inverf } x \approx \xi^{-1} p_0 \text{ , } 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.881

Table 45

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{1} p_j \xi^j \text{ , } 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(1)	.10256 62
P01	(0)	-.38040

Table 46

$$\text{inverf } x \approx \xi^{-1} p_0 / \sum_{j=0}^{2} q_j \xi^j \text{ , } 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(1)	.22715 88

Q00	(1)	.22517 159
Q01	(0)	.43813 8
Q02	(1)	.10000 00

Table 47

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{1} p_j \xi^j / \sum_{j=0}^{2} q_j \xi^j , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.98650 088
P01	(0)	.92601 777
Q00	(0)	.98424 719
Q01	(1)	.10074 7432
Q02	(1)	.10000 0000

Table 48

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{3} p_j \xi^j / \sum_{j=0}^{1} q_j \xi^j , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.56930 6387
P01	(0)	.97371 2122
P02	(0)	-.77555 5339
P03	(0)	.23380 4987
Q00	(0)	.56881 3561
Q01	(1)	.10000 00000

Table 49

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{2} p_j \xi^j / \sum_{j=0}^{3} q_j \xi^j \ , \ 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.23082 54405 3
P01	(1)	.15148 62509 18
P02	(0)	.76478 59261
Q00	(0)	.23077 97955 8
Q01	(1)	.15194 85204 63
Q02	(1)	.11941 21785 57
Q03	(1)	.10000 00000 0

Table 50

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{5} p_j \xi^j / \sum_{j=0}^{2} q_j \xi^j \ , \ 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.15504 70003 116
P01	(1)	.13827 19649 631
P02	(0)	.69096 93488 87
P03	(1)	-.11280 81391 617
P04	(0)	.68054 42468 25
P05	(0)	-.16444 15679 1
Q00	(0)	.15502 48498 22
Q01	(1)	.13852 28141 995
Q02	(1)	.10000 00000 000

Table 51

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{2} p_j \xi^j / \sum_{j=0}^{6} q_j \xi^j \;,\; 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(1)	.10676 11247 6342
P01	(2)	.19384 61849 9846
P02	(2)	.53288 15015 8236
Q00	(1)	.10675 48625 1484
Q01	(2)	.19395 10752 45869
Q02	(2)	.55724 35357 9446
Q03	(2)	.24416 64030 4774
Q04	(2)	.21251 85280 4419
Q05	(1)	-.70446 97237 727
Q06	(1)	.10000 00000 000

Table 52

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j / \sum_{j=0}^{2} q_j \xi^j \;,\; 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-1)	.11125 21834 95389
P01	(0)	.26833 42048 26738 2
P02	(0)	.97313 42206 25464 2
P03	(0)	-.37745 85822 57143 0
P04	(0)	-.44686 70406 19259
P05	(0)	.71598 15703 61426
P06	(0)	-.46059 71444 09907
P07	(0)	.15164 63463 43608
P08	(-1)	-.20555 09764 649
Q00	(-1)	.11124 79925 39409
Q01	(0)	.26841 98897 32864
Q02	(1)	.10000 00000 00000 0

Table 53

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{6} p_j \xi^j / \sum_{j=0}^{5} q_j \xi^j \text{ , } 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-2)	.47944 77933 46489
P01	(0)	.17121 76690 13084 01
P02	(1)	.12618 65207 19596 240
P03	(1)	.21339 06055 64337 15
P04	(0)	.28387 37490 01951 4
P05	(0)	.23530 35904 89892 9
P06	(-1)	-.40076 38986 44416
Q00	(-2)	.47943 70466 09729
Q01	(0)	.17124 54255 49151 92
Q02	(1)	.12742 21706 54959 402
Q03	(1)	.24161 88379 94885 U9
Q04	(1)	.14957 33748 30505 66
Q05	(1)	.10000 00000 00000 00

Table 54

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{6} p_j \xi^j / \sum_{j=0}^{7} q_j \xi^j \text{ , } 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-2)	.14537 56965 30055 13
P01	(-1)	.75724 22575 02058 771
P02	(0)	.90290 08339 50101 4639
P03	(1)	.30478 36391 60287 47248
P04	(1)	.29528 72269 43320 04428
P05	(1)	.20126 96109 44065 6463
P06	(0)	.87822 13117 50254 978
Q00	(-2)	.14587 37705 96959 530
Q01	(-1)	.75730 50728 17257 843
Q02	(0)	.90690 25318 98225 1188
Q03	(1)	.31885 55138 14325 32505
Q04	(1)	.40562 61832 81058 69543
Q05	(1)	.40734 83581 25808 1838
Q06	(1)	.22017 07144 79728 9713
Q07	(1)	.10000 00000 00000 0000

Table 55

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{9} p_j \xi^j / \sum_{j=0}^{6} q_j \xi^j \;, \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-2)	.14143 58031 57589 907
P01	(-1)	.79276 74143 50942 1103
P02	(1)	.10098 23529 04220 31850 6
P03	(1)	.34235 55244 32719 18780 7
P04	(1)	.20374 08451 13919 36914
P05	(0)	-.10426 94789 24189 6647
P06	(0)	.92342 90226 80556 7008
P07	(0)	-.46722 37712 07606 3860
P08	(0)	.13265 53492 82737 031
P09	(-1)	-.16766 08689 55712 06
Q00	(-2)	.14143 41958 42495 024
Q01	(-1)	.79282 38229 28832 3465
Q02	(1)	.10137 58305 87846 69844 0
Q03	(1)	.35738 19664 98479 43117
Q04	(1)	.32958 99297 13345 45573
Q05	(1)	.20600 52253 44168 39652
Q06	(1)	.10000 00000 00000 00000

Table 56

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j / \sum_{j=0}^{8} q_j \xi^j \ , \ 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-4)	.31008 08562 55295 8465
P01	(-2)	.40974 87603 01193 99241
P02	(0)	.12149 02662 89727 61616 64
P03	(1)	.11091 67694 63902 82026 357
P04	(1)	.32283 79855 66392 39484 387
P05	(1)	.28816 91815 65159 88262 11
P06	(1)	.20479 72087 26299 60497 29
P07	(0)	.85459 22081 97214 83280 6
P08	(-2)	.35510 95884 62238 3139
Q00	(-4)	.31008 09298 56452 2487
Q01	(-2)	.40975 28678 66391 45041
Q02	(0)	.12159 07800 74875 71430 68
Q03	(1)	.11186 27167 63169 64210 428
Q04	(1)	.34323 63984 30529 01805 054
Q05	(1)	.41402 84677 11620 21505 54
Q06	(1)	.41197 97271 27220 41212 85
Q07	(1)	.21629 61962 64143 45609 20
Q08	(1)	.10000 00000 00000 00000 00

Table 57

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j / \sum_{j=0}^{9} q_j \xi^j \,, \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-3)	.10501 31152 37334 38116
P01	(-1)	.10532 61131 42333 38164 25
P02	(0)	.26987 80273 62432 83544 516
P03	(1)	.23268 69578 89196 90806 414
P04	(1)	.71678 54794 91079 96810 001
P05	(1)	.85475 61182 21678 27825 185
P06	(1)	.68738 08807 35438 39802 913
P07	(1)	.36270 02483 09587 08930 02
P08	(0)	.88606 27392 96515 46814 9

Q00	(-3)	.10501 26668 70303 37690
Q01	(-1)	.10532 86230 09333 27531 11
Q02	(0)	.27019 86237 37515 54845 553
Q03	(1)	.23501 43639 79702 53259 123
Q04	(1)	.76078 02878 58012 77064 351
Q05	(2)	.11181 58610 40569 07827 3451
Q06	(2)	.11948 78791 84353 96667 8438
Q07	(1)	.81922 40974 72699 07893 913
Q08	(1)	.40993 87907 63680 15361 45
Q09	(1)	.10000 00000 00000 00000 00

<u>**Table 58**</u>

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{10} p_j \xi^j \Big/ \sum_{j=0}^{8} q_j \xi^j \;, \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-4)	.56451 97770 90644 82298
P01	(-2)	.53504 14748 78930 13765 64
P02	(0)	.12969 55009 97273 52403 0254
P03	(1)	.10426 15854 92982 66122 83637
P04	(1)	.28302 67790 17544 89974 2694
P05	(1)	.26255 67287 94480 72726 6643
P06	(1)	.20789 74263 01749 17228 9354
P07	(0)	.72718 80623 15568 11306 121
P08	(-1)	.66816 80771 18049 89575 0
P09	(-1)	-.17791 00457 51117 59979 1
P10	(-2)	.22419 56322 33463 45828
Q00	(-4)	.56451 69986 27606 51514
Q01	(-2)	.53505 58706 79306 53953 35
Q02	(0)	.12986 61541 69116 46934 5513
Q03	(1)	.10542 93223 26264 91195 2443
Q04	(1)	.30379 33117 35222 06237 2456
Q05	(1)	.37631 16853 64050 28901 0232
Q06	(1)	.38782 85827 70420 11263 5182
Q07	(1)	.20372 43181 74121 77929 8258
Q08	(1)	.10000 00000 00000 00000 000

Table 59

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{10} p_j \xi^j / \sum_{j=0}^{10} q_j \xi^j \ , \ 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-5)	.11738	35250	99916	66680	42	
P01	(-3)	.13858	51811	34967	18807	956	
P02	(-2)	.43836	70987	71260	95665	3116	
P03	(-1)	.53644	86114	71536	48365	69526	
P04	(0)	.34839	28713	96575	22572	31828	0
P05	(1)	.15335	29752	39898	90804	34319	43
P06	(1)	.35780	40256	90859	96758	23779	6
P07	(1)	.31598	51960	11320	90206	35148	4
P08	(1)	.21371	21499	72655	15514	51050	7
P09	(0)	.86030	09752	62802	60580	01956	
P10	(-2)	.27834	01035	37470	01059	96	
Q00	(-5)	.11738	31387	23977	77529	27	
Q01	(-3)	.13858	76216	55322	46059	720	
Q02	(-2)	.43873	42402	27069	35023	4583	
Q03	(-1)	.53963	55030	32008	16743	66763	
Q04	(0)	.35609	08730	59002	65560	07175	1
Q05	(1)	.16068	37770	97190	17608	77943	20
Q06	(1)	.39846	60818	46717	57295	52121	6
Q07	(1)	.46632	96034	87366	35331	33480	6
Q08	(1)	.43859	04525	64495	54653	69550	7
Q09	(1)	.22859	98127	24229	05411	98706	6
Q10	(1)	.10000	00000	00000	00000	00000	

Table 60

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{11} p_j \xi^j \Big/ \sum_{j=0}^{10} q_j \xi^j \ , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-5)	.19953 28896 45372 10824 86
P01	(-3)	.21273 70263 17859 53343 3481
P02	(-2)	.58975 59595 24072 47651 4135
P03	(-1)	.59481 90145 27358 09123 80875
P04	(0)	.31328 28983 09326 67506 43282 3
P05	(1)	.13630 13995 64422 60990 23133 27
P06	(1)	.34152 81520 56529 30673 63048 19
P07	(1)	.30184 18146 89336 06839 71718 9
P08	(1)	.20842 43354 62073 39433 31686 0
P09	(0)	.85545 63502 67434 99993 98754
P10	(-2)	.40273 91840 87128 93132 298
P11	(-3)	-.15196 13911 57447 16810 2

Q00	(-5)	.19953 21037 93742 12953 70
Q01	(-3)	.21274 15696 34040 84598 9564
Q02	(-2)	.59037 06202 37313 54671 0838
Q03	(-1)	.59959 15011 08610 92334 24770
Q04	(0)	.32318 08308 08178 36442 40561 0
Q05	(1)	.14378 33780 47493 44527 82249 33
Q06	(1)	.37644 57150 82579 69664 40074 96
Q07	(1)	.44081 43569 81438 41173 76015 8
Q08	(1)	.42508 71049 71828 04606 34993 8
Q09	(1)	.22127 46942 79697 85343 06411 5
Q10	(1)	.10000 00000 00000 00000 00000 0

Table 61

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{13} p_j \xi^j / \sum_{j=0}^{9} q_j \xi^j \ , \ 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-5)	.17029 70491 07967 08016 142
P01	(-3)	.24570 51233 12105 17459 60803
P02	(-2)	.97783 79899 23616 88749 34800 6
P03	(0)	.14537 65287 12209 34050 49862 15
P04	(0)	.89334 91748 59923 39148 10483 28
P05	(1)	.23889 52874 03955 32949 06263 978
P06	(1)	.30900 23631 60079 67119 52925 016
P07	(1)	.19793 33570 92262 72165 70093 09
P08	(0)	.43854 19046 17763 56012 36815 8
P09	(0)	.48534 12494 85699 74824 04914 3
P10	(0)	-.30475 67857 48365 58342 29150 3
P11	(0)	.12046 31192 43646 27398 14332 3
P12	(-1)	-.28282 24938 04011 78345 9565
P13	(-2)	.30528 44117 38658 20854 745
Q00	(-5)	.17029 66312 38113 98774 963
Q01	(-3)	.24570 81299 62366 99010 02795
Q02	(-2)	.97838 61042 11932 13862 84130
Q03	(0)	.14596 83981 34359 67426 83854 39
Q04	(0)	.91164 65700 46856 86528 08523 17
Q05	(1)	.25957 49106 58118 66488 71838 648
Q06	(1)	.40076 44480 82818 03327 68448 626
Q07	(1)	.36979 40604 47532 54095 38307 84
Q08	(1)	.21138 02548 27004 27901 10478 36
Q09	(1)	.10000 00000 00000 00000 00000 00

Table 62

$$\mathrm{inverf}\ x \approx \xi^{-1} \sum_{j=0}^{10} p_j \xi^j \Big/ \sum_{j=0}^{13} q_j \xi^j \ , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-3)	.39069 61256 82794 49520 567
P01	(-1)	.60163 82750 27446 85594 87812
P02	(1)	.25904 29476 40100 41284 18997 00
P03	(2)	.42502 07027 24353 38918 58417 142
P04	(3)	.29989 63587 93527 13098 49690 1052
P05	(4)	.10301 41294 93188 81936 28784 3003
P06	(4)	.22677 59611 31486 67402 05370 4284
P07	(4)	.35209 52855 51729 17247 48533 8948
P08	(4)	.26815 72676 71807 02438 32495 3694
P09	(4)	.16977 37630 23360 28988 34077 911
P10	(3)	.57120 96628 95364 49955 25523 31

Q00	(-3)	.39069 52579 65350 28249 7741
Q01	(-1)	.60164 48077 83463 38892 21804
Q02	(1)	.25916 98570 69825 96179 93466 60
Q03	(2)	.42649 14050 97571 19936 91622 842
Q04	(3)	.30485 56477 28554 80492 75413 381[illegible]
Q05	(4)	.10929 28031 30911 92428 37105 8821
Q06	(4)	.25995 95937 79846 81611 49506 2678
Q07	(4)	.44114 35295 61956 85242 22359 3112
Q08	(4)	.44443 06154 47785 02985 99991 0432
Q09	(4)	.36874 90162 49684 46389 47049 2427
Q10	(4)	.17525 93828 52523 97097 48231 323
Q11	(3)	.69751 77437 05862 28156 07985 56
Q12	(2)	-.10612 76925 79519 54218 03792 5
Q13	(1)	.10000 00000 00000 00000 00000

Table 63

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{12} p_j \xi^j \Big/ \sum_{j=0}^{13} q_j \xi^j \ , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-6)	.11522	61405	60469	49369	24051	
P01	(-4)	.21819	99668	14587	03482	56875	51
P02	(-2)	.12020	87080	93301	40260	99580	7673
P03	(-1)	.26716	95163	26264	64659	13312	3065
P04	(0)	.27570	68413	75619	89159	20978	9071
P05	(1)	.14646	27875	19201	37024	44941	5715
P06	(1)	.45183	35984	80018	52933	83292	2898
P07	(1)	.88653	55358	42886	66464	44315	7281
P08	(2)	.11130	63973	16379	22481	76029	1236
P09	(2)	.10230	35796	45670	19308	36301	9019
P10	(1)	.67962	24757	84076	19867	16355	1386
P11	(1)	.30104	54413	08139	68517	34479	4706
P12	(0)	.88611	87006	97448	90620	00056	5737
Q00	(-6)	.11522	59575	41306	01406	24614	
Q01	(-4)	.21820	15788	10335	93406	94592	18
Q02	(-2)	.12024	72379	43735	48641	11249	2874
Q03	(-1)	.26772	73698	91010	05853	29925	0042
Q04	(0)	.27817	06139	46817	35880	57652	0533
Q05	(1)	.15085	24376	52335	15563	87854	9036
Q06	(1)	.48769	99209	22942	55757	89186	5613
Q07	(2)	.10377	67812	16985	79281	93858	5565
Q08	(2)	.14958	78195	63311	06554	27518	9149
Q09	(2)	.16322	32437	87484	58073	92297	5853
Q10	(2)	.13167	26933	78105	60031	30634	3686
Q11	(1)	.81793	00909	32540	56092	11112	8117
Q12	(1)	.33952	53241	33505	85425	59088	2338
Q13	(1)	.09999	99999	99999	99999	99999	9999

Table 64

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{14} p_j \xi^j \Big/ \sum_{j=0}^{12} q_j \xi^j , \quad 0.9375 \le x \le 1-10^{-100}$$

$$\xi = [-\ln(1-x)]^{-1/2}$$

P00	(-7)	.55430	47116	50111	60103	79781	
P01	(-4)	.11425	90847	66914	96984	00155	016
P02	(-3)	.68839	21737	78053	02994	29711	5631
P03	(-1)	.16747	34778	29781	53605	80047	8392
P04	(0)	.18722	81387	47262	23861	26543	8230
P05	(1)	.10399	27979	23348	87919	82438	0669
P06	(1)	.31609	87137	84919	25900	92970	6177
P07	(1)	.59873	72841	52602	30257	78610	5671
P08	(1)	.71081	56183	58225	57115	66772	1252
P09	(1)	.49349	55386	73668	83116	16445	1019
P10	(1)	.29386	18828	86453	73079	31788	4758
P11	(0)	.73730	55263	34374	10593	86994	3767
P12	(-1)	.46121	12165	52813	45382	85083	7902
P13	(-2)	-.87363	76813	04722	52465	38710	485
P14	(-3)	.77400	27922	69106	13889	81329	5

Q00	(-7)	.55430	39562	53733	78240	40471	
Q01	(-4)	.11425	98011	15445	85037	78726	516
Q02	(-3)	.68857	97999	25517	97551	38217	4915
Q03	(-1)	.16777	05888	67402	51069	84832	3282
Q04	(0)	.18867	02359	53749	57604	57503	2631
Q05	(1)	.10681	66661	65608	47009	91004	0343
Q06	(1)	.34104	07749	15634	61100	15673	4285
Q07	(1)	.70655	51683	22478	33053	86392	3977
Q08	(1)	.97334	65449	61018	95065	71840	8458
Q09	(1)	.90417	25867	08973	52657	41313	2457
Q10	(1)	.65907	80337	34948	61638	59822	5434
Q11	(1)	.29541	10212	58709	90378	50678	2300
Q12	(1)	.09999	99999	99999	99999	99999	9999

Table 65

$$\mathrm{inverf}\ x \approx \xi^{-1} p_0 \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.99635

Table 66

$$\mathrm{inverf}\ x \approx \xi^{-1} p_0 / \sum_{j=0}^{1} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(1)	.83660 49

Q00	(1)	.83558 56
Q01	(1)	.10000 0

Table 67

$$\operatorname{inverf} x \approx \xi^{-1} p_0 / \sum_{j=0}^{2} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.79038 112
Q00	(0)	.79025 8095
Q01	(-1)	.22877 4
Q02	(1)	.10000 00

Table 68

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{2} p_j \xi^j / \sum_{j=0}^{1} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.20504 66018
P01	(0)	.99789 4857
P02	(0)	-.41635 332
Q00	(0)	.20503 99066
Q01	(1)	.10000 00000

Table 69

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{1} p_j \xi^j / \sum_{j=0}^{3} q_j \xi^j \ , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(0)	.10479 19634 2
P01	(1)	.13707 99584 0
Q00	(0)	.10479 05658 92
Q01	(1)	.13714 19617 2
Q02	(0)	.24919 1738
Q03	(1)	.10000 0000

Table 70

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{3} p_j \xi^j / \sum_{j=0}^{2} q_j \xi^j \ , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-2)	.98045 62029 15
P01	(0)	.36366 78891 71
P02	(0)	.97302 94983 7
P03	(0)	-.53749 47401
Q00	(-2)	.98045 12778 02
Q01	(0)	.36369 99715 44
Q02	(1)	.10000 00000 00

Table 71

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{2} p_j \xi^j \Big/ \sum_{j=0}^{4} q_j \xi^j , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-1/2}$$

P00	(-2)	.61926 76103 55
P01	(0)	.35525 93025 854
P02	(1)	.19887 48035 060
Q00	(-2)	.61926 57895 409
Q01	(0)	.35527 41794 749
Q02	(1)	.20069 01506 325
Q03	(0)	.61930 15752 1
Q04	(1)	.10000 00000 0

Table 72

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{4} p_j \xi^j \Big/ \sum_{j=0}^{3} q_j \xi^j , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-1/2}$$

P00	(-3)	.22899 40827 4324
P01	(-1)	.26383 04418 19694
P02	(0)	.46043 01477 86939
P03	(0)	.94276 21490 4644
P04	(0)	-.57724 78103 254
Q00	(-3)	.22899 37946 4858
Q01	(-1)	.26383 38165 76924
Q02	(0)	.46116 70917 90869
Q03	(1)	.10000 00000 00000

Table 73

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{3} p_j \xi^j / \sum_{j=0}^{5} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$, \ \xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-3)	.25714 37550 27167
P01	(-1)	.35082 77090 74057
P02	(0)	.77626 13473 11802
P03	(1)	.25935 68249 81201

Q00	(-3)	.25714 35002 08623
Q01	(-1)	.35083 10289 17669 6
Q02	(0)	.77710 79252 45628
Q03	(1)	.26730 67273 81966
Q04	(1)	.10876 82207 1345
Q05	(1)	.10000 00000 000

Table 74

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{4} p_j \xi^j / \sum_{j=0}^{5} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-4)	.27519 57458 20285
P01	(-2)	.51858 36044 76728 0
P02	(0)	.17929 40985 73957 58
P03	(1)	.12231 82562 59906 23
P04	(1)	.10011 38992 23338 8

Q00	(-4)	.27519 55709 83484
Q01	(-2)	.51858 63764 09414 1
Q02	(0)	.17938 85590 14912 49
Q03	(1)	.12329 03254 83169 35
Q04	(1)	.13002 90156 06176 5
Q05	(1)	.10000 00000 00000

Table 75

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{4} p_j \xi^j / \sum_{j=0}^{6} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-5)	.92474 31817 50278 8
P01	(-2)	.23140 55182 29532 59
P02	(0)	.11718 59056 07554 639
P03	(1)	.14071 60462 51535 023
P04	(1)	.31711 94588 28176 88
Q00	(-5)	.92474 27882 00114 5
Q01	(-2)	.23140 62642 29138 718
Q02	(0)	.11721 87977 02384 836
Q03	(1)	.14132 11664 35098 655
Q04	(1)	.33931 02957 17151 17
Q05	(1)	.16360 23734 90049 7
Q06	(1)	.10000 00000 00000

Table 76

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{5} p_j \xi^j / \sum_{j=0}^{6} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-6)	.77391 27144 24704 74
P01	(-3)	.25069 18142 76151 471
P02	(-1)	.17675 73152 93550 7734
P03	(0)	.33267 16324 27961 8417
P04	(1)	.14854 99518 98026 6532
P05	(0)	.85249 21034 69995 32
Q00	(-6)	.77391 24889 76299 95
Q01	(-3)	.25059 23213 31364 981
Q02	(-1)	.17678 57332 31619 4143
Q03	(0)	.33336 34697 90176 3300
Q04	(1)	.15223 15596 41231 9796
Q05	(1)	.13260 99445 73021 080
Q06	(1)	.10000 00000 00000 00

Table 77

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{5} p_j \xi^j / \sum_{j=0}^{7} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-6)	.33294 31459 54385 810
P01	(-3)	.13257 91534 02762 0528
P02	(-1)	.12114 20989 92880 95635
P03	(0)	.32187 37361 93217 40052
P04	(1)	.24042 21485 76572 48636
P05	(1)	.37941 93273 44354 1350
Q00	(-6)	.33294 30749 33591 505
Q01	(-3)	.13257 93375 02379 28496
Q02	(-1)	.12115 46347 27806 62088
Q03	(0)	.32225 48145 74589 58227
Q04	(1)	.24312 32642 66548 24962
Q05	(1)	.43170 77761 18013 2816
Q06	(1)	.23385 40499 94681 000
Q07	(1)	.10000 00000 00000 0

Table 78

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{6} p_j \xi^j / \sum_{j=0}^{7} q_j \xi^j \ , \ 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-7)	.18254 68185 28684 4560
P01	(-5)	.91406 68599 59729 64791
P02	(-2)	.11090 52053 11618 46826 4
P03	(-1)	.42414 86064 92510 81711 5
P04	(0)	.51965 87891 46264 29918 5
P05	(1)	.17136 65754 46791 75585 7
P06	(0)	.76015 05889 85647 3869
Q00	(-7)	.18254 67910 67000 1958
Q01	(-5)	.91406 76949 69990 37461
Q02	(-2)	.11091 22677 94056 38421 8
Q03	(-1)	.42442 29108 36817 18937 3
Q04	(0)	.52230 78707 49893 15803 5
Q05	(1)	.17913 57238 85083 71000 7
Q06	(1)	.14103 12873 30222 76749
Q07	(1)	.10000 00000 00000 0000

Table 79

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j \Big/ \sum_{j=0}^{6} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-1/2}$$

P00	(-8)	.32054 05422 06204 9557
P01	(-5)	.18994 79322 63212 78529 1
P02	(-3)	.28142 23189 85853 16084 41
P03	(-1)	.13705 04879 06781 66785 818
P04	(0)	.22681 43542 00597 63291 723
P05	(1)	.10984 21959 89234 01502 27
P06	(0)	.67911 43397 05620 82198
P07	(0)	-.83433 41891 67720 7384
P08	(0)	.34219 51267 24034 321

Q00	(-8)	.32054 05053 28239 7844
Q01	(-5)	.18994 80592 26014 28768 4
Q02	(-3)	.28143 49691 09893 97548 65
Q03	(-1)	.13710 92249 60226 55253 243
Q04	(0)	.22751 72815 17447 32321 08
Q05	(1)	.11253 48514 03695 94014 05
Q06	(1)	.10000 00000 00000 00000 0

Table 80

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{7} p_j \xi^j \Big/ \sum_{j=0}^{8} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-9)	.34654 29858 80863 50177
P01	(-6)	.25084 67928 24075 70520 55
P02	(-4)	.47378 13196 37286 02986 534
P03	(-2)	.31312 60375 97786 96408 3388
P04	(-1)	.77948 76454 41435 36994 854
P05	(0)	.70045 68123 35816 43868 271
P06	(1)	.18710 42034 21679 31668 683
P07	(0)	.71452 54774 31351 45428 3

Q00	(-9)	.34654 29567 31595 11156
Q01	(-6)	.25084 69079 75380 27114 87
Q02	(-4)	.47379 53129 59749 13536 339
Q03	(-2)	.31320 63536 46177 68848 0813
Q04	(-1)	.78073 48906 27648 97214 733
Q05	(0)	.70715 04479 95337 58619 993
Q06	(1)	.19998 51543 49112 15105 214
Q07	(1)	.15072 90269 27316 80008 56
Q08	(1)	.10000 00000 00000 00000 0

Table 81

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{9} p_j \xi^j \Big/ \sum_{j=0}^{7} q_j \xi^j , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-1/2}$$

P00	(-10)	.50069 22457 74829 13579 0
P01	(-7)	.42644 79707 07689 30599 206
P02	(-5)	.97151 77540 82926 26982 995
P03	(-3)	.79888 39561 63239 07020 2216
P04	(-1)	.25754 36302 13164 56967 69417
P05	(0)	.31603 21190 95869 40117 79409
P06	(1)	.12318 55657 87185 87095 32610
P07	(0)	.59095 59395 19430 09590 378
P08	(0)	-.84662 68725 78715 48013 1
P09	(0)	.38309 31372 38705 57727
Q00	(-10)	.50069 22135 48022 43226 8
Q01	(-7)	.42644 81160 13360 72655 398
Q02	(-5)	.97153 83520 90643 26300 944
Q03	(-3)	.79902 42104 44356 79144 0542
Q04	(-1)	.25780 91817 03742 44563 74228
Q05	(0)	.31783 41778 76637 96754 37642
Q06	(1)	.12780 06728 89813 56657 25887
Q07	(1)	.10000 00000 00000 00000 000

Table 82

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j / \sum_{j=0}^{9} q_j \xi^j \;,\; 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-11)	.45344 68956 32093 98449 85
P01	(-8)	.46156 00632 13453 32510 214
P02	(-5)	.12964 48156 06431 97452 43907
P03	(-3)	.13714 32956 96651 28933 38403 9
P04	(-2)	.60537 91473 91621 89689 04216 2
P05	(0)	.11279 04635 36302 80004 88142 45
P06	(0)	.82810 03090 44626 90215 64139 9
P07	(1)	.19507 62028 75805 68829 45716
P08	(0)	.69952 99060 70581 54857 725

Q00	(-11)	.45344 68737 70882 06782 75
Q01	(-8)	.46156 01760 09335 92558 3458
Q02	(-5)	.12964 67185 09449 81712 63671
Q03	(-3)	.13715 89198 83502 05065 39350 4
Q04	(-2)	.60574 83055 00971 40404 30833 6
Q05	(0)	.11311 88933 43557 82064 35024 32
Q06	(0)	.84001 81491 81780 42918 55594 1
Q07	(1)	.21238 24208 74549 93541 81012
Q08	(1)	.15771 92238 66620 40545 9982
Q09	(1)	.10000 00000 00000 00000 000

Table 83

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{9} p_j \xi^j \Big/ \sum_{j=0}^{9} q_j \xi^j , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-1/2}$$

P00	(-11)	.30006 62273 74526 28797 20
P01	(-8)	.31986 69132 73796 11030 9165
P02	(-6)	.94421 97286 45889 91856 01036
P03	(-3)	.10534 57332 40790 18000 26721 71
P04	(-2)	.49218 40580 59009 07732 50148 0
P05	(-1)	.97387 83430 50087 14951 22456 1
P06	(0)	.76186 67260 66484 56564 15906 4
P07	(1)	.19219 51565 14520 53065 38663 7
P08	(0)	.76735 17075 12467 94056 5421
P09	(-1)	-.32127 14293 75016 71268 06

Q00	(-11)	.30006 62140 50252 94571 67
Q01	(-8)	.31986 69847 91603 71893 8965
Q02	(-6)	.94423 23876 89546 96713 73492
Q03	(-3)	.10535 66376 24038 75391 17000 00
Q04	(-2)	.49245 53495 53784 48712 41584 7
Q05	(-1)	.97643 00930 81769 95890 19104 5
Q06	(0)	.77169 91486 19879 15780 48711 3
Q07	(1)	.20743 13944 79558 74487 98422 5
Q08	(1)	.15973 83887 80216 33004 08467
Q09	(1)	.10000 00000 00000 00000 0000

Table 84

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{8} p_j \xi^j / \sum_{j=0}^{11} q_j \xi^j , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-11)	-.13124 90974 79764 24520 781
P01	(-8)	-.16203 38613 00164 28723 49571
P02	(-6)	-.56725 45647 54309 83537 32831 4
P03	(-4)	-.77453 70355 11304 15524 60972 88
P04	(-2)	-.46272 83674 13004 78209 67493 486
P05	(0)	-.12503 32997 07870 34688 03505 488
P06	(1)	-.14896 49339 20549 99557 81110 554
P07	(1)	-.70907 47555 97692 12756 80548 35
P08	(2)	-.10246 72796 27527 56662 48265 46
Q00	(-11)	-.13124 90929 00508 77949 985
Q01	(-8)	-.16203 38888 19818 31999 38604
Q02	(-6)	-.56726 01915 87571 45756 82478 8
Q03	(-4)	-.77459 35507 75199 26589 47788 60
Q04	(-2)	-.46289 65871 01542 83731 72002 547
Q05	(0)	-.12522 94724 58366 06977 72867 7826
Q06	(1)	-.14995 23996 52287 70396 74316 202
Q07	(1)	-.73082 43222 23446 71818 66043 76
Q08	(2)	-.12221 72573 30643 12806 20240 14
Q09	(1)	-.61596 29859 38871 01608 35542
Q10	(1)	-.34057 10449 54562 32403 9575
Q11	(1)	.10000 00000 00000 00000 00

Table 85

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{10} p_j \xi^j / \sum_{j=0}^{10} q_j \xi^j \;,\; 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-\frac{1}{2}}$$

P00	(-13)	.61030 51444 04846 43546 175
P01	(-10)	.83627 01775 54663 16032 40067
P02	(-7)	.32895 40748 21529 40087 14972 96
P03	(-5)	.51207 14652 44177 52859 73927 147
P04	(-3)	.35495 88070 22054 72544 54485 7008
P05	(-1)	.11373 87937 82738 40561 34035 4397
P06	(0)	.16530 28350 24119 50466 80015 9331
P07	(1)	.10005 22126 37720 74438 90021 0815
P08	(1)	.20278 48941 18170 23500 67687 124
P09	(0)	.62007 85767 56069 85548 19286 6
P10	(-1)	.35748 66812 96909 04933 3411
Q00	(-13)	.61030 51265 91503 12030 367
Q01	(-10)	.83627 02940 65339 68793 95595
Q02	(-7)	.32895 67214 26348 14815 49374 81
Q03	(-5)	.51210 10935 46738 53317 60061 077
Q04	(-3)	.35505 83876 39567 11327 31007 8886
Q05	(-1)	.11387 20528 91897 05662 05065 7271
Q06	(0)	.16608 83843 53424 62673 44744 9094
Q07	(1)	.10213 35668 36093 82947 97313 5691
Q08	(1)	.22641 42663 75653 78003 54021 940
Q09	(1)	.16070 39392 03457 26285 89084 08
Q10	(1)	.10000 00000 00000 00000 00000 0

Table 86

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{9} p_j \xi^j \Big/ \sum_{j=0}^{12} q_j \xi^j \;, \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-\frac{1}{2}}$$

P00	(-13)	-.29362 42807 03304 56456 8813
P01	(-10)	-.45569 08392 99777 69467 00986 9
P02	(-7)	-.20661 58807 73254 33313 37468 452
P03	(-5)	-.37915 51991 54100 68941 05205 6780
P04	(-3)	-.31925 24670 13463 99646 34665 9748
P05	(-1)	-.12962 36759 29796 12248 30720 2561
P06	(0)	-.25455 27776 73182 11222 73078 3902
P07	(1)	-.23216 04794 95143 86424 72111 0112
P08	(1)	-.88602 26435 74993 81878 40275 7899
P09	(2)	-.10788 02036 88225 33680 67469 1425

Q00	(-13)	-.29362 42737 48929 90139 0653
Q01	(-10)	-.45569 08894 98461 64006 31663 9
Q02	(-7)	-.20661 71712 53893 11397 80711 678
Q03	(-5)	-.37917 16460 76279 17782 31537 2738
Q04	(-3)	-.31931 65869 67920 51431 33822 0009
Q05	(-1)	-.12972 57169 86378 06521 78702 1667
Q06	(0)	-.25529 32129 20342 34231 33335 7987
Q07	(1)	-.23470 20640 29079 89250 44136 9561
Q08	(1)	-.92668 07918 99399 62247 66735 9296
Q09	(2)	-.13603 46216 78558 15932 90984 2616
Q10	(1)	-.70172 99309 08465 53670 62189 59
Q11	(1)	-.33815 46415 78074 88009 10525 8
Q12	(1)	.10000 00000 00000 00000 0000

Table 87

$$\text{inverf } x \approx \xi^{-1} \sum_{j=0}^{10} p_j \xi^j \Big/ \sum_{j=0}^{12} q_j \xi^j \ , \quad 1-10^{-100} \le x \le 1-10^{-10000}$$

$$\xi = [-\ell n(1-x)]^{-1/2}$$

P00	(-13)	-.11144	11840	89227	21195	24166	5
P01	(-10)	-.19119	39783	48760	64457	87638	96
P02	(-8)	-.96897	42581	21424	53472	26024	603
P03	(-5)	-.20133	93030	04021	83920	09331	4217
P04	(-3)	-.19492	74462	63834	66099	52419	8152
P05	(-2)	-.92692	51667	45455	10243	40721	3849
P06	(0)	-.21798	41478	99542	16188	09754	8642
P07	(1)	-.24458	96454	69975	79364	81984	2773
P08	(2)	-.11876	11205	92536	12747	12825	1973
P09	(2)	-.19546	08206	45925	23676	09201	4877
P10	(1)	-.32100	77829	15892	35002	51937	036

Q00	(-13)	-.11144	11818	69514	24646	54990	3
Q01	(-10)	-.19119	39957	48164	89241	45896	51
Q02	(-8)	-.96897	92090	64174	22653	22195	481
Q03	(-5)	-.20134	63045	02032	96487	10360	9909
Q04	(-3)	-.19495	80857	20015	81837	20558	1439
Q05	(-2)	-.92748	02537	70744	06125	24013	3059
Q06	(0)	-.21845	06536	13219	91038	00982	8534
Q07	(1)	-.24648	46047	73745	58206	25646	3103
Q08	(2)	-.12244	86332	45744	57884	77754	1533
Q09	(2)	-.22767	93429	05551	97989	11476	7526
Q10	(2)	-.13896	04217	65147	58525	64387	1644
Q11	(1)	-.79944	87768	81734	52374	10887	91
Q12	(1)	.10000	00000	00000	00000	00000	

Table 88

$$\operatorname{inverf} x \approx \xi^{-1} \sum_{j=0}^{14} p_j \xi^j \Big/ \sum_{j=0}^{9} q_j \xi^j \; , \quad 1-10^{-100} \leq x \leq 1-10^{-10000}$$

$$\xi = [-\ln(1-x)]^{-1/2}$$

P00	(-15)	.17282 25279 92059 75832 59875 3
P01	(-12)	.32540 28651 66164 78673 62617 888
P02	(-9)	.18265 82587 16819 00382 03640 1575
P03	(-7)	.42476 70049 86077 38551 85422 5216
P04	(-5)	.46568 12807 53131 87330 81084 5795
P05	(-3)	.25404 52862 01718 65179 92289 0421
P06	(-2)	.69476 23100 43492 67072 10122 5981
P07	(-1)	.91641 16260 51727 03216 79292 9794
P08	(0)	.51942 13171 04180 49977 07173 1647
P09	(0)	.87215 81134 38965 94071 98077 4131
P10	(0)	-.49556 07220 18979 93096 62559 0308
P11	(0)	-.24268 94852 28365 49265 16430 345
P12	(0)	.55493 79789 56013 48962 92072 43
P13	(0)	-.36369 94313 49610 35169 96986 3
P14	(-1)	.97875 40375 23532 30765 0497

Q00	(-15)	.17282 25250 66133 22808 +4097 5
Q01	(-12)	.32540 28899 53293 53748 80631 238
Q02	(-9)	.18265 90341 78667 49753 09812 3222
Q03	(-7)	.42477 90796 80458 54200 89419 0714
Q04	(-5)	.46574 00169 14010 99728 27782 8170
Q05	(-3)	.25416 49029 90323 85434 23874 6124
Q06	(-2)	.69590 71551 62581 22692 39712 2897
Q07	(-1)	.92178 72685 85016 04424 95826 8494
Q08	(0)	.53171 13832 16238 32710 92310 6676
Q09	(1)	.09999 99999 99999 99999 99999 9999