FYMM/MMP III   Problem Set 2

Please submit your solutions for grading by Monday 14.9. in Moodle.

1. Show that the group of matrices $M(n, \mathbb{R})$ (with addition as the law of "multiplication") is isomorphic to $\mathbb{R}^{n^2}$.

2. Compute the following multiplications (of the permutation group S_6):

   - $(235)(46) \cdot (14)(265)$
   - $(1635)(24) \cdot (1536)(24)$
   - $(26)(35) \cdot (24536)$

3. Anne, Barbara, Christofer, and David are going to lunch. How many distinct ways there are for them to be seated around a circular table? The room has no features so that all seats are equally good, and thus it only matters who gets to sit next to whom. Use groups to derive your result. (If you find some ambiguity left in the question, pick an interpretation and stay consistent in deriving your answer.)

4. Anne, Barbara and Christofer have seven indistinguishable two euro coins. How many ways are there to distribute the seven coins among them so that each gets at least one coin? How many ways are there to distribute the seven coins so that everyone gets a different number of coins (and at least one coin)?

5. Show that $\mathrm{U}(n)$ is a proper subgroup of $\mathrm{GL}(n, \mathbb{C})$ and $\dim\, \mathrm{U}(n) = n^2$.