SSCP Matrix from Raw Data:

Raw Data

ID	x_1	x_2
1	1	1
2	3	4
3	2	7

$\bar{x}_1 = 2 \quad \bar{x}_2 = 4$

$$\underset{(3\times2)}{\underset{\sim}{X}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 7 \end{bmatrix}$$

$$\underset{(3\times2)}{\underset{\sim}{X}_d} = \begin{bmatrix} 1-2 & 2-4 \\ 3-2 & 4-4 \\ 2-2 & 7-4 \end{bmatrix}$$

$$\underset{\sim}{X}_d \triangleq \begin{bmatrix} -1 & -3 \\ 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\underset{(2\times3)}{X_d'} = \begin{bmatrix} -1 & 1 & 0 \\ -3 & 0 & 3 \end{bmatrix}$$

$$\underset{(2\times2)}{SSCP} = \underset{(2\times3)}{\underset{\sim}{X}_d'} \cdot \underset{(3\times2)}{X_d} = \begin{bmatrix} -1 & 1 & 0 \\ -3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 \\ 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1.-1)+(1.1)+(0.0) & (-1.-3)+(1.0)+(0.3) \\ (-3.-1)+(0.1)+(3.0) & (-3.-3)+(0.0)+(3.3) \end{bmatrix}$$

$$\underset{(2\times2)}{SSCP} = \begin{bmatrix} 2 & 3 \\ 3 & 18 \end{bmatrix}$$

$$\underset{(2\times2)}{\text{Var-Covar matrix}} = \frac{SSCP}{n-1} = \frac{SSCP}{2} = \begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix}$$

Summarizing Var-Covar Matrices

$$\underset{(2\times 2)}{S_1} = \begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix}$$

$$\text{Det } S_1 = |S_1| = 1\times 9 - 1.5\times 1.5 = \underline{\underline{6.75}}$$

$$\text{Tr } S_1 = 1+9 = \underline{\underline{10}}$$

$$\underset{(2\times 2)}{S_2} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\text{Det } S_2 = |S_2| = 1\times 9 - 0 = \underline{\underline{9}}$$

$$\text{Tr. } S_2 = 1+9 = \underline{\underline{10}}$$

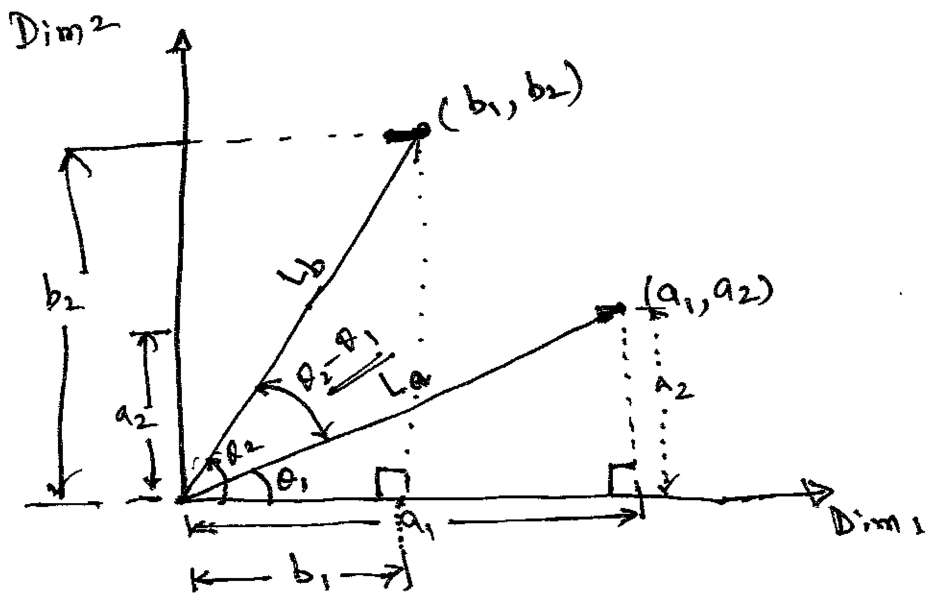
Length, Angle of Vectors

$$\underset{(2\times1)}{\underset{\sim}{a}} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad \underset{(2\times1)}{\underset{\sim}{b}} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Inner product of vector, $\underset{(n\times1)}{\underset{\sim}{a}} = \underset{(1\times n)}{\underset{\sim}{a}'} \cdot \underset{(n\times1)}{\underset{\sim}{a}} = \underset{(1\times2)}{[a_1 \; a_2]} \cdot \underset{(2\times1)}{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}} = \underset{(1\times1)}{a_1^2 + a_2^2}$

(1×1)

Inner product of vector, $\underset{\sim}{b} = \underset{\sim}{b}' \cdot \underset{\sim}{b} = [b_1 \; b_2]\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = b_1^2 + b_2^2$



L_a = Length of vector $\underset{\sim}{a} = \sqrt{\underset{\sim}{a}' \cdot \underset{\sim}{a}}$

θ_1 = Angle of vector $\underset{\sim}{a}$

$L_a^2 = \underset{\sim}{a}' \cdot \underset{\sim}{a} = a_1^2 + a_2^2$

$L_b^2 = \underset{\sim}{b}' \cdot \underset{\sim}{b} = b_1^2 + b_2^2$

Angle between two vectors

$= \theta_2 - \theta_1$

$$\theta = \theta_2 - \theta_1$$

$$\begin{aligned}\cos(\theta) &= \cos(\theta_2 - \theta_1)\\ &= \cos\theta_2 . \cos\theta_1 + \sin\theta_2 . \sin\theta_1\\ &= \frac{b_1}{L_b} . \frac{a_1}{L_a} + \frac{b_2}{L_b} . \frac{a_2}{L_a}\\ &= \frac{a_1 b_1 + a_2 b_2}{L_a . L_b}\end{aligned}$$

$$\boxed{\cos(\theta) = \frac{\underset{\sim}{a}' . \underset{\sim}{b}}{\sqrt{\underset{\sim}{a}' . \underset{\sim}{a}}\ \sqrt{\underset{\sim}{b}' . \underset{\sim}{b}}}}$$

Inner product of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ $= \underset{\sim}{a}' . \underset{\sim}{b}$

$$= [a_1\ a_2]\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$= a_1 b + a_2 b_2$$

$$\cos(\theta) = 0$$

$$\underline{\underline{\theta = 90^0}}$$

$\underline{\underline{\text{0 Correlation}}}$ = Orthogonal vectors (variables)

$$\cos(\theta) = 1$$

$$\underline{\underline{\theta = 0^0}}$$

Corr^{n} is 1 = perfectly correlated