ARMAX Lecture

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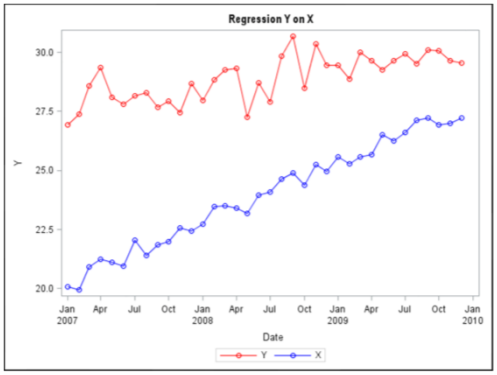
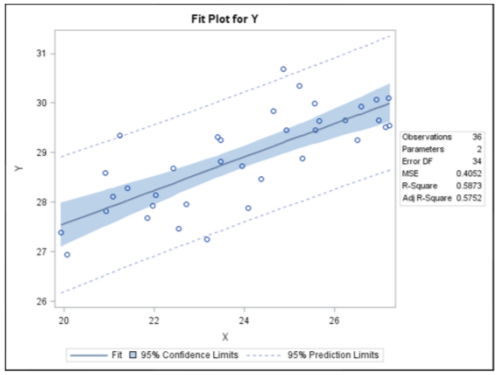
Regression of Y on X

- Linear Regression Model: $Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$ *
- * X_t is an external or *exogenous* predictor of Y_t.





Multiple Linear Regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

- Assumptions include:
  - The predictor variables are known and measured *without* error.
  - The functional relationship between inputs and target is linear.
  - The error term represents a set of random variables that are independent and identically distributed with a Gaussian normal distribution having a mean of 0 and a variance of σ^2.

Time Series Regression Terminology

- *Ordinary Regressor*
  - an input variable that has only a concurrent influence on the target variable
    - X at time *t* is correlated with Y at time *t*.
      X at times before *t* is uncorrelated with Y at time *t*.
- *Dynamic Regressor*
  - an input variable that influences the target variable at current and past values
    - X at times *t*, *t*-1, *t*-2, …, influences Y at time *t*.
- *Transfer Function*
  - a function that provides the mathematical relationship between a dynamic regressor and the target variable

Types of Regressors: Measurement Scale

- Binary (dummy) variables
  - take the value of zero or one
  - can be used to quantify nominal data
- Categorical variables
  - nominal scaled ⇨ nonquantitative categories
  - Ordinal scaled variables can be treated as categorical.
  - They must be coded into a quantitative input. Usually they use a form of dummy coding for each level (less one if a constant term is used in the model).
- Quantitative variables
  - interval or ratio scaled
  - can be transformed

Types of Regressors: Randomness

- *Deterministic*
  - controlled by experimenter
  - alternatively, can be perfectly predicted without error
- *Stochastic*
  - governed by unknown probability distributions
  - cannot be perfectly predicted

Types of Regressors

- Deterministic examples
  - dummy coding for …..
  - settings on a machine (for example, electric current, temperature, and pressure on production equipment)
  - intervention weights (for example, saturation for legislation that is phased in uniformly by month over a year: 1/12, 2/12, 3/12, …,12/12)
  - advertising expenditures by your company when these are…

Types of Regressors

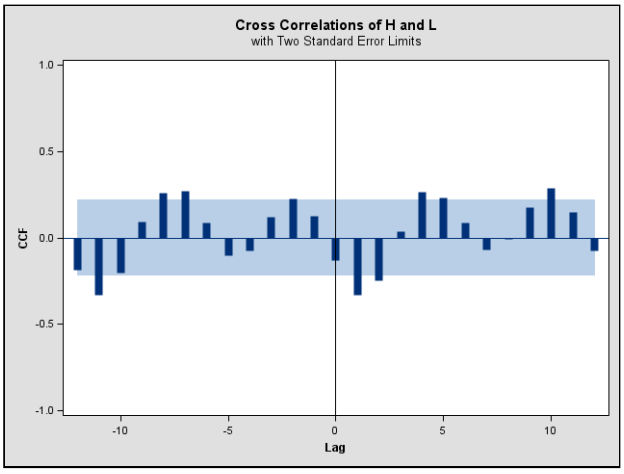
- Stochastic examples
  - ambient, outside, air temperature
  - competitor sales
  - interest rates
  - consumer price index
  - unemployment rate
  - rate per 1000 households of television viewership
  - stock market indices

continued...

The Cross-Correlation Function (CCF)

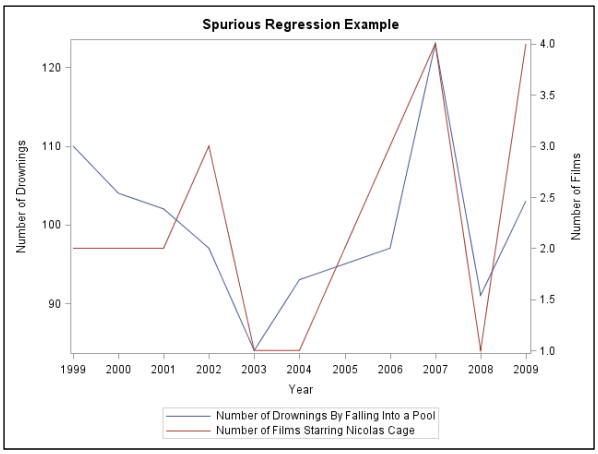
- CCF(k) is the cross-correlation of target Y with input X at lag k.
  - A significant value at lag k implies that Y_t and X_{t-k} are correlated.
  - Spikes and decay patterns in the cross-correlation function can help determine the form of the transfer function.
  - The sample CCF estimates an unknown population CCF.

The Cross-Correlation Function (CCF)



Spurious Correlation

"Why Do We Sometimes Get Nonsense Correlations Between Time Series?"
- George Yule, 1926



Correlation	Number of Drownings
Number of Films	0.680233