Eigen Values & Eigen Vectors

- Valid for any square symmetric matrix, $\underline{M}$

$$\underset{(2\times2)}{M} = \begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix}$$

λ = Eigen value

I = Identity matrix of same size as M

$$= \underset{(2\times2)}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$\left| \underset{(2\times2)}{M} - \lambda . \underset{(2\times2)}{I} \right| = 0$$

$$M - \lambda . I = \begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 1.5 \\ 1.5 & 9-\lambda \end{bmatrix}$$

$$|M - \lambda I| = \begin{vmatrix} 1-\lambda & 1.5 \\ 1.5 & 9-\lambda \end{vmatrix} = (1-\lambda)(9-\lambda) - 1.5 * 1.5$$

$$= 9 - 9\lambda - \lambda + \lambda^2 - 2.25$$

$$= \lambda^2 - 10\lambda + 6.75$$

$$\boxed{\lambda^2 - 10\lambda + 6.75 = 0}$$

$a = 1, \; b = -10, \; c = 6.75$

$$\lambda = \frac{-(-10) \pm \sqrt{(-10)^2 - 4.1.6.75}}{2.1}$$

$$= \frac{10 \pm \sqrt{100 - 27}}{2} = \frac{10 \pm \sqrt{73}}{2}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2.a}$$

$$\lambda_1 = \frac{10+\sqrt{73}}{2}, \quad \lambda_2 = \frac{10-\sqrt{73}}{2}$$

$$\lambda_1 = 9.27 \qquad \lambda_2 = 0.73$$

Note:

$$\left\{\begin{array}{l} \lambda_1 + \lambda_2 = 9.27 + 0.73 = 10 = \text{Trace of Matrix, } M \\ \lambda_1 \cdot \lambda_2 = 9.27 * 0.73 = 6.77 = \text{Det. of } '' \text{, } M \end{array}\right\}$$

$\Rightarrow$ M is 100 X100 matrix, 100 eigen value

Eigen Vectore ?

$$\underset{(n\times n)}{M} \cdot \underset{(n\times 1)}{\underset{\sim}{c}} = \lambda_1 \cdot \underset{(n\times 1)}{\underset{\sim}{c}}$$

$$\underset{(2\times 2)}{\begin{bmatrix} 1 & 1.5 \\ 1.5 & 9 \end{bmatrix}} \cdot \underset{(2\times 1)}{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}} = 9.27 \underset{(2\times 1)}{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}$$

$$\begin{bmatrix} c_1 + 1.5c_2 \\ 1.5c_1 + 9c_2 \end{bmatrix} = \begin{bmatrix} 9.27c_1 \\ 9.27c_2 \end{bmatrix}$$

$$\left\{\begin{array}{l} c_1 + 1.5c_2 = 9.27c_1 \Rightarrow 1.5c_2 = 8.27c_1 \Rightarrow c_1 = \frac{1.5}{8.27}c_2 \\ \qquad \underline{c_1 = 0.18c_2} \\ 1.5c_1 + 9c_2 = 9.27c_2 \Rightarrow 1.5c_1 = 0.27c_2 \Rightarrow c_1 = \frac{0.27}{1.5}c_2 \\ \qquad \underline{c_1 = 0.18c_2} \end{array}\right.$$

$$L_c^2 = \underset{\sim}{c}' \cdot \underset{\sim}{c} = c_1^2 + c_2^2$$

<u>**Assumption**</u> : $L_c = 1$

$$c_1^2 + c_2^2 = 1$$

$$\underline{\underline{c_1 = 0.18 C_2}}$$

or, $(0.18 C_2)^2 + C_2^2 = 1$

or, $0.0324 C_2^2 + C_2^2 = 1$

or, $1.0324 \, C_2^2 = 1$

or, $C_2^2 = \dfrac{1}{1.0324}$

or, $C_2 = \sqrt{\dfrac{1}{1.0324}} = 0.984$

$C_1 = 0.18 . C_2 = \underline{\underline{0.178}}$

$$\boxed{\begin{array}{l}\lambda_1 = 9.27 \\ \underset{\sim}{c} = \begin{bmatrix} 0.178 \\ 0.984 \end{bmatrix}\end{array}}$$

$$\boxed{\begin{array}{l}\lambda_2 = 0.73 \\ \underset{\sim}{c} = \begin{bmatrix} -0.984 \\ 0.178 \end{bmatrix}\end{array}}$$

$$\underset{(2\times2)}{P} = \begin{bmatrix} 0.178 & -0.984 \\ 0.984 & 0.178 \end{bmatrix}$$

$$\underset{(2\times2)}{P'} = \begin{bmatrix} 0.178 & 0.984 \\ -0.984 & 0.178 \end{bmatrix}$$

$$\underset{(2\times2)}{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 9.27 & 0 \\ 0 & 0.73 \end{bmatrix}$$

1. $P \cdot P' = I \Rightarrow P' = P^{-1}$

2. $P \cdot \underline{\Lambda} \cdot P' = M$