

### **Additional Practice Problems:**

#### **1) Halsten Lake Golf Club Clubs!**

Halsten Lake Golf Club provides box lunches for \$5 to its members from the snack bar. It makes the sandwiches the previous night and/or early in the morning before golfers arrive. They make four kinds of sandwiches – Club, Turkey and Cheese (T&C), Ham and Cheese (H&C), and Vegetarian.

Although each sandwich generates \$5 in sales, for accounting purposes, each sandwich has a distinct cost to the Golf Club. Club costs \$2.45 each, T&C \$2.60 each, H&C \$3.00 each, and Vegetarian \$1.75. The time it takes to make each sandwich is also different – each Club requires 2 minutes, each T&C 2.5 minutes, each H&C 1.75 minutes and each Vegetarian 1.25 minutes.

The Golf Club can only store 300 total sandwiches. Consider that an upper bound on overall production. Also consider that there is a total amount of sandwich assembly time available of 598 minutes.

Historic demand requirements should also be placed on your sandwich production model. For variety sakes, at least 40 Club sandwiches, 40 T&C sandwiches, 40 H&C sandwiches, and 20 Vegetarian sandwiches should be made. Also, no more than 40 Vegetarian sandwiches should be made. Additionally, the number of T&C sandwiches should be equal to or exceed the number of Club AND H&C COMBINED. (T&C are the highest demand sandwich).

Create an LP model that will determine the optimal number of the four sandwiches to make that MAXIMIZES PROFIT (Sales less costs). Upon creating your model, analyze the optimal solution with the SA questions below – USING THE SA REPORT.

You can obviously check your work by rerunning the model, but that is not the learning objective of this module, so don't cut corners! You will only hurt yourself.

- a) State the optimal mix of sandwiches and the total profit and identify the binding constraints.
- b) If the cost of the H&C is overstated by 25 cents (i.e., profit understated by 25 cents), would that make a difference in the optimal mix of sandwiches? What if that error was 50 cents? Explain.
- c) Which of the four sandwiches' optimal production level seems to be the most sensitive with regard to its calculated profit? Explain your reasoning and discuss implications.
- d) In the production of sandwiches, which seems to be more restraining – time to make the sandwiches or the capacity to store them? Explain.

e) What would be the impact on the amount of profit anticipated from the sandwiches if you lost an hour (60 minutes) of assembly/production time? Explain. Would this change have an impact on the optimal number of sandwiches to be made?

f) Based on the minimum production constraints, which product seems the “least attractive” from a profit standpoint? If you could remove this sandwich from the mix, can the SA tell you the impact on overall model profit? Can the SA tell you what the new optimal solution (i.e., optimal number of sandwiches) would be? Explain very thoroughly for all aspects of this question.

g) Say that you hear that this golf group has a higher proportion of vegetarians playing than usual. What is the impact on the profit if you raise the upper limit on Vegetarian sandwiches to 60? Will the optimal number of sandwiches change? Explain thoroughly.

## 2) Mattress Investment Company

This is a two-part problem – for the first part, create an LP model for the Mattress Investment Company that will help them make investment allocation decisions for individual investors for the next 12 month period.

After completing this model and identifying the optimal investment strategies, answer the SA-related questions using ONLY the SA reports.

### Part a) – Model Building

This model will consider investing dollars in five growth mutual funds. Model parameters of course are estimates. These five fund options are: Amount of \$\$\$ suggested to be invested/allocated in Vanguard (van1), Piper Jaffrey (pj2), T. Rowe Price (trowe3), Trickens Hedge Fund (mt4), and Oppenheimer (Opp5).

Objective: Maximize return – coefficients are estimated percent of annual returns in a BULL market. The objective is obviously measured in dollars (estimated dollar return). The coefficients for the five funds (in order stated above) are 15%, 12%, 24%, 17%, and 10%.

### Constraints

Return 2 – Coefficients are estimated percent of annual return in a BEAR market. Units are dollars. The coefficient for the five funds (in order) are –3%, 4%, 1%, –2%, and 5%. Represents minimum estimated return of investments if it is a BEAR market. Model should invest funds such that this return is at least \$200.

Betapoints – Weighted dollars times beta (risk) – This is an approximate measure of beta (it could be translated to a weighted average for those of you finance folks who think there is some conspiracy against you here – this is a reasonable approximation of riskiness). Individual betas for the five funds (in order) are 2.1, 1.7, 3.1, 2, and 1.3. The total weighted beta (investment

amount times beta's summed for all funds) should not exceed 15,000. Again, this represents maximum risk points in investment allocation.

MIN (one for all funds but trowe3) – Each fund (except for trowe3) should have a minimum of \$1250 invested in it. Trowe3 has no minimum investment amount.

MAX – The maximum the model has available to invest is \$10,000. Any leftover will go “in the mattress.” So, let the model decide whether or not to invest the entire \$10,000.

Ratio constraint – The total amount invested in the first two funds combined (van1, pj2) should be greater than or equal to the amount invested in the last three funds combined (trowe3, mt4, opp5). A policy or philosophy.

Create an appropriate LP model that determines how to allocate the money among the five funds that maximizes the return in a BULL market and meets all the constraints outlined above.

Part b) After solving this model, turn your attention to the SA report.

Use the SA information to answer the following questions. NOTE: CORRECT RESPONSES WILL SOLELY USE THE SENSITIVITY ANALYSIS PRINTOUT INFORMATION IN YOUR EXPLANATIONS – you can probably reverse engineer the LP model. If you want to, that's great and a good double check. But the learning objective for this exercise is to understand the SENSITIVITY ANALYSIS PRINTOUT. Use ALL appropriate information FROM SENSITIVITY ANALYSIS REPORT in your answers. When the question asks for IMPACTS, this means specific numerical impacts on returns, etc. if appropriate. DETAILS! Clearly justify your conclusions. *Assume each question is independent of others and assumes that all other model parameters are constant.*

- 1) Identify the investment amounts that are suggested by the solution of the model and which constraints are binding in the model.
- 2) For any funds that the model does NOT suggest investment (in 1): How much would the return of these funds need to increase for the model to suggest investing dollars in said fund? Again, treat independently.
- 3) Moon Trickens says, “I can't believe you're not investing more in my hedge fund (mt4) – it always returns 30% to 40%; you should invest more and revise the solution of your model by using an estimated return of .30 for my fund.” If you alter the model according to Moon's suggestion, is he right? (i.e., would you feel compelled to alter the optimal investment amounts?). Clearly explain.
- 4) Looking through the history of fund performance forecasting, we find that the estimated yearly returns estimated at the start of the year for the Oppenheimer funds can be off by as much as +1.5% (.015) to –1.5% (–.015). So, if we assume a worst-case variance

scenario (in both directions!), how confident would you be in the optimal investments suggested by the solution of the model? Explain thoroughly.

- 5) The Return2 constraint is in the model for the “Bears” – the market pessimists, not the Super Bowl Shuffle people – those who anticipate a poor 2011 investment season. If we lowered the dollar return requirements of this constraint by 100 units (dollars), what would the impact be on the return of our optimal solution? Would our optimal allocation of money to funds be the same or different? Explain.
- 6) If you increased the upper bound for beta points to, say, 16,000, describe the impact on both the return and the optimal allocation of money to the funds, if any.
- 7) What can you say about the potential impact of changing the minimum investment requirement of the Vanguard fund to \$1500? Anything? Be specific, thorough, and explicit.

3) SA SA! – As part of a consulting project, you have been asked to create an LP model to help a South African Agricultural firm create a crop allocation model for some land it manages. Your objective is to maximize anticipated return (measured in U.S. dollars) in deciding how many hectares (1 ha = 2.471 acres) should be planted in one of five crops – maize, wheat, sugar cane, sunflowers, and vegetables. Also, the government sponsors a crop rotation plan that pays farm cooperatives a small amount if they leave land fallow (let the land rest for a season).

The return calculation is an approximation based on anticipated yield, futures in the crop export market, and other items. Constraints for the model include total available land (TOTAL, in ha.), fertilizer requirements (in kilograms), water requirements (in cubic meters), labor requirements (In U.S. dollars), the maximum the government allows cooperatives to be compensated for fallow land (MAX-F), minimum requirements for the five crops (Rows 11 to 15, in ha), and a ratio constraint of some kind.

Use the SA to answer the following questions. NOTE: CORRECT RESPONSES WILL SOLELY USE THE SENSITIVITY ANALYSIS PRINTOUT INFORMATION IN YOUR EXPLANATIONS – you can probably reverse engineer the LP model. If you want to, that’s great and a good double check. But the learning objective for this exercise is to understand the SENSITIVITY ANALYSIS PRINTOUT. Use ALL appropriate information FROM SENSITIVITY ANALYSIS REPORT in your answers. When the question asks for IMPACTS, this means specific numerical impacts on returns, etc., if appropriate. DETAILS! Clearly justify your conclusions. *Assume each question is independent of others and assumes that all other model parameters are constant.*

- 1) Identify the crop allocation amounts that are suggested by the solution of the model and which constraints are binding in the model.

- 2) Describe whether the optimal allocations would change if the government reduces what it pays for fallow land – assume that a change would cause a 15% decrease in the present return coefficient used in the LP model. Explain.
- 3) Describe whether the optimal crop allocations would change if the return for maize was only 90 units rather than 100. Explain.
- 4) Because of canceling of a highway construction project, it is possible that we really will have 15 more hectares than originally included in the model as available land. If that is the case, what would the impact be on the return of our optimal solution? To obtain this impact, would we also have to increase the amount of water available? And/or also the amount of labor? Explain thoroughly.
- 5) Instead of modifying what it pays for fallow land, discuss the impact on return if the amount of land that this firm could keep fallow is reduced to 60 ha.? Would our optimal allocation of land to crops be the same or different? Explain.
- 6) Suppose we alter the constraint that forces the minimum amount of land planted in wheat to a minimum amount of 70? What would the impact be on the return of our optimal solution? Would our optimal allocation of land to crops be the same or different? Explain.
- 7) Finally, describe (in words) what the ratio constraint attempts to implement.

## Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$4	MAIZE	187.9878049	0	100	0.918367346	0.544235924
\$D\$4	WHEAT	100	0	75	0.548780487	1E+30
\$E\$4	SuCANE	222.804878	0	84	0.474299065	0.381355932
\$F\$4	SuFLOW	304.2073171	0	91	7.518518512	1.481751826
\$G\$4	VEGS	25	0	54	2.475609758	1E+30
\$H\$4	FALLOW	70	0	16	1E+30	2.475609754

## Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$I\$6	TOTAL	910	10.82926829	910	18.88743455	22.47767857
\$I\$7	FERT	718.3628049	0	800	1E+30	81.63719512
\$I\$8	WATER	42000	0.719512195	42000	5035	1443
\$I\$9	LABOR	13000	2.695121951	13000	503.5	494.0405904
\$I\$10	MAX-F	70	2.475609754	70	23.52803738	19.34316354
\$I\$11	MIN-M	187.9878049	0	100	87.98780488	1E+30
\$I\$12	MIN-W	100	-0.548780487	100	85.33898305	69.33454169
\$I\$13	MIN-SC	222.804878	0	100	122.804878	1E+30
\$I\$14	MIN-SF	304.2073171	0	60	244.2073171	1E+30
\$I\$15	MIN-VEG	25	-2.475609758	25	59	25
\$I\$16	RATIO	-59	0	0	1E+30	59

	MAIZE	WHEAT	SuCANE	SuFLOW	VEGS	FALLOW
Profit	187.9878	100	222.8049	304.2073	25	70
TOTAL	1	1	1	1	1	1
FERT	0.75	0.5	1.4	0.7	0.1	0
WATER	64	45	53	44	11	0
LABOR	16	12	13	18	14	1
MAX-F						1
MIN-M	1					
MIN-W		1				
MIN-SC			1			
MIN-SF				1		
MIN-VEG					1	
RATIO	-0.1	-0.1	-0.1	-0.1	0.9	-59

4) EToyBoxes, Inc. – The spreadsheet above describes a production model for EToyboxes, an internet-based company that produces and sells five different wooden toy boxes. The toy box types are Deluxe, Super Deluxe, Custom Deluxe, Economy, and Super Economy. (They are somewhat abbreviated on the printout). Toy boxes are made from two different lumber types (Lumber1 and Lumber2) and are assembled (Assembly). There is a limited number of toy boxes that can be made each month because of storage (Storage). The model also forces at least 10 toy boxes of each type (except for Economy) to be produced each month for the sake of variety (Atleast10-x). Finally, there are two ratio constraints that influence production based on forecasted demand, one for the Super Deluxe (Ratio-SD) and one for the Economy toybox (Ratio-Ec). The model has been coded to maximize profit (Sales price less costs) in deciding how many toy boxes of each kind to make. We are not worried about integer number of toy boxes.

Use the above printouts to answer the following questions. Treat each question independently.

- Identify which constraints are binding for the model.
- The Accounting Department made an error – the per-unit cost of the Super Economy toy box was really only \$35. Would the best production mix of toy boxes stay the same or change under this altered circumstance? Why? Explain.
- Often in an attempt to increase sales late in a given month, sales prices are cut as much as 25% on selected models. Would the optimal solution of toy boxes above remain optimal if the sales price of:
  - Deluxe was reduced by 25%? Clearly explain.

- b. Custom Deluxe was reduced by 25%? Clearly Explain.  
Treat a. and b. as separate events/scenarios.
- d) Describe what you know would happen to our optimal toy box mix and profit level (if anything) if we could totally remove the constraint that requires a minimum of 10 Deluxe toy boxes. Be very clear in your explanation.
- e) If you had a choice of between acquiring 10 more units of Lumber1, 20 more units of Lumber2, and 3 more units of assembly time, which might you want to do and why? Be very clear.
- f) If you added 10 more units of Lumber1, to achieve any change in profit, wouldn't you also have to increase the amount of assembly time accordingly? Clearly explain.
- g) Pick a ratio constraint that is binding (if any fit this category) and explain the impact that this constraint has on the optimal production of toy boxes. In other words, explain in words the limitation that this particular constraint imposes on the solution.



Microsoft Excel 11.0 Sensitivity Report  
Worksheet: [2008ex1sa.xls]Sheet1  
Report Created: 2/8/2008 6:03:10 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$4	Deluxe	10	0	40	6.318901795	1E+30
\$D\$4	SuperDlx	11.37275607	0	60	215	27.07566462
\$E\$4	CustomDlx	25.38542767	0	48	7.953846154	15.91489362
\$F\$4	Economy	22.74551214	0	30	3.513789581	129.25
\$G\$4	Super Ec	44.22386484	0	43	39.77966102	2.725832013

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$8	Lumber1	900	1.09186906	900	30.125	205.1265823
\$H\$9	Lumber2	318.7328405	0	500	1E+30	181.2671595
\$H\$10	Assembly	200	19.82682154	200	3.347222222	20.12430939
\$H\$11	Ratio-SD	-1.77636E-15	27.96198522	0	3.887096774	1.329243354
\$H\$12	Ratio-Ec	-3.55271E-15	-3.632523759	0	7.53125	22.0020429
\$H\$13	Storage	113.7275607	0	115	1E+30	1.272439282
\$H\$14	Atleast10-A	10	-6.318901795	10	38.75	10
\$H\$15	Atleast10-B	11.37275607	0	10	1.372756072	1E+30
\$H\$16	Atleast10-C	25.38542767	0	10	15.38542767	1E+30
\$H\$17	Atleast10-D	44.22386484	0	10	34.22386484	1E+30

	A	B	C	D	E	F	G	H	I
1									
2									
3			Deluxe	SuperDlx	CustomDlx	Economy	Super Ec		
4			10	11.37276	25.38543	22.74551	44.22386		
5		Sales	100	125	110	70	85		
6		Cost	80	65	62	40	42		
7		Net	40	60	48	30	43	4884.857445	RHS
8		Lumber1	8	4	5	10	9.5	900	900 <
9		Lumber2	4	8	6.5	1	0	318.7328405	500 <
10		Assembly	2	1.5	2.25	1.25	1.75	200	200 <
11		Ratio-SD	-0.1	0.9	-0.1	-0.1	-0.1	-1.77636E-15	0 <
12		Ratio-Ec	-0.2	-0.2	-0.2	0.8	-0.2	-3.55271E-15	0 >
13		Storage	1	1	1	1	1	113.7275607	115 <
14		Atleast10-A	1					10	10 >
15		Atleast10-B		1				11.37275607	10 >
16		Atleast10-C			1			25.38542767	10 >
17		Atleast10-D					1	44.22386484	10 >
18									
19									
20									
21									
22									
23									

5) The Obama Bank Bailout

The following spreadsheet model and SA results come from the following problem situation.

Six banks, three in Texas and three in Oklahoma, qualify for bailout money from the federal government. A total of 1,000(000) dollars must be allocated among the six banks (Row 7 in model). Riskiness of the banks is scored on a 100-point scale, with 100 being a bank that is most likely to fail and 1, least likely. Banks are scored on a 1-100 point scale, and the objective of allocating money to banks is such that the product of risk\*dollars allocated is minimized across all six banks. (In other words, we want to provide bailout money to the best banks). This information is found in Row 4 of the spreadsheet model.

Rows 5 and 6 of the model indicate the location of the banks. At least \$500(000) must be allocated to Texas banks, and \$200(000) to Oklahoma banks. Rows 8 to 13 show the maximum amount of bailout money that can be assigned to each individual bank. Each bank provided that information to the federal government. Row 14 is some composition constraint, and Row 15 limits the amount of overhead overall to 37(500) – overhead being the amount of money used for non-lending purposes, a measure of the leanness of the administrative structure of the banks.

Use the model solution and SA to answer the following questions INDEPENDENTLY. Clearly show all work and be very clear in your explanations and justifications.

- a) Which banks are getting the maximum amount of bailout money that they requested?
- b) Individually, if each banks' risk assessment (Row 4) increased by 20%, would any such change for any particular bank change the bailout allocation? Be very specific for all the banks.
- c) Eddie Sutton, a Senator from Oklahoma, is trying to implement a change in the allocating model that will decrease the minimum required bailout money allocated to Texas banks to \$450(000). What is the impact (if anything) in the overall measure of Risk times Dollars (the objective function value)?
- d) If Bank (B5) from Oklahoma decided they wanted to request \$275(000) from the federal government bailout money (instead of \$250(000)), how would that change in the model impact the overall measure of Risk times Dollars (the objective function value)?
- e) Does the optimal allocation of dollars to banks change in either part c or part d? Explain thoroughly.
- f) The federal government has asked you to analyze the model under a decrease in the allowed overhead expense to \$35(000). Discuss the impact in terms of the overall measure of Risk times Dollars (the objective function value).
- g) In words, what explicit ratio/proportion constraint does Row 14 implement? Be very explicit.

## Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$3	B1	175	0	75	5	11
\$D\$3	B2	200	0	32	28.75	1E+30
\$E\$3	B3	125	0	56	19	5
\$F\$3	B4	200	0	44	12.5	1E+30
\$G\$3	B5	250	0	48	3.75	1E+30
\$H\$3	B6	50	0	66	13.75	3.75

## Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$I\$5	State	500	13.75	500	50	60
\$I\$6	State	500	0	200	300	1E+30
\$I\$7	Total	1000	70.75	1000	100	50
\$I\$8	Max-B1	175	0	275	1E+30	100
\$I\$9	Max-B2	200	-28.75	200	166.6666667	20
\$I\$10	Max-B3	125	0	140	1E+30	15
\$I\$11	Max-B4	200	-12.5	200	50	30
\$I\$12	Max-B5	250	-3.75	250	50	20
\$I\$13	Max-B6	50	0	150	1E+30	100
\$I\$14	Ratio	200	0	0	200	1E+30
\$I\$15	Overhead	37.5	-475	37.5	0.6	4

[illegible]