Matrix Algebra and Matrix Notation: A Brief Review

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Scalars and Vectors

Scalar: A single value, constant, or observation.

Scalars are denoted by lowercase italics.

$$x = [5]$$

Vector: A row or column of values, responses, or observations.

$$\text{Column vector: } \mathbf{x} = \begin{bmatrix} 5 \\ 2 \\ 4 \\ 8 \end{bmatrix}$$

Vectors are denoted by lowercase bold letters.

$$\text{Row vector: } \mathbf{x}' = \begin{bmatrix} 5 & 2 & 4 & 8 \end{bmatrix}$$

Matrices

Matrix: A rectangular array of values arranged in rows and columns, denoted by its size, $n \times p$ such that n is the number of rows and p is the number of columns. Elements in a matrix are referred to as x_{ij} for the corresponding value of x in the "ith row and jth column"

$$2 \times 3 \text{ Matrix: } \mathbf{X} = \begin{bmatrix} 5 & 2 & 7 \\ 4 & 1 & 3 \end{bmatrix}$$

$$3 \times 2 \text{ Matrix: } \mathbf{X}' = \begin{bmatrix} 5 & 4 \\ 2 & 1 \\ 7 & 3 \end{bmatrix}$$

Matrices are denoted by uppercase bold letters.

Transpose to switch rows and columns

Special Properties of Matrices

Square: $n = p$

$$\mathbf{X}_1 = \begin{bmatrix} 2 & 3 & 6 \\ 5 & 1 & 2 \\ 5 & 8 & 0 \end{bmatrix}$$

Symmetric: Area above the diagonal is a mirror image of the area below the diagonal, or $x_{12} = x_{21}$, $x_{23} = x_{32}$,… $x_{ij} = x_{ji}$.

$$\mathbf{X}_2 = \begin{bmatrix} 1 & 3 & 6 \\ 3 & 1 & 2 \\ 6 & 2 & 1 \end{bmatrix}$$

Transposing Matrices

A matrix **X'** is transposed (**X**) by changing rows to columns and columns to rows:

$$\mathbf{X} = \begin{bmatrix} 5 & 2 & 7 \\ 4 & 1 & 3 \end{bmatrix} \quad \mathbf{X}' = \begin{bmatrix} 5 & 4 \\ 2 & 1 \\ 7 & 3 \end{bmatrix}$$

Transposing matrices can make it possible to multiply any matrix with the transpose of the same matrix.

Commonly Used Matrices

SSCP

$$\mathbf{X'X} = \begin{bmatrix} SS_1 & SP_{12} & SP_{13} \\ SP_{21} & SS_2 & SP_{23} \\ SP_{31} & SP_{32} & SS_3 \end{bmatrix}$$

Variance-covariance

$$\mathbf{S} = \begin{bmatrix} \sigma_1^{\ 2} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^{\ 2} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^{\ 2} \end{bmatrix}$$

Identity

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Correlation

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & r_{13} \\ r_{21} & 1 & r_{23} \\ r_{31} & r_{32} & 1 \end{bmatrix}$$

Matrix Addition and Subtraction

Equal-sized matrices can be added or subtracted by adding or subtracting corresponding elements:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 10 & 8 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 6 & 12 \\ 5 & 4 \end{bmatrix}$$

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 8 & 13 \\ 15 & 12 \end{bmatrix} \qquad \mathbf{A} - \mathbf{B} = \begin{bmatrix} -4 & -11 \\ 5 & 4 \end{bmatrix}$$

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Now for a You Tube Video on Matrix Addition/Subtraction and Multiplying by a Constant (Scalar):

https://www.youtube.com/watch?v=EFApWAI3NJw

Matrix Multiplication

If $p_{\mathbf{A}} = n_{\mathbf{B}}$, the sum of the products of the i^{th} row of **A** and the j^{th} column of **B** make the elements of the resulting matrix.

The resulting matrix will be of size $n_{\mathbf{A}} \times p_{\mathbf{B}}$:

$$\mathbf{AB} = \begin{bmatrix} 5 & -2 \\ 4 & 7 \\ 9 & 3 \end{bmatrix} \begin{bmatrix} -5 & 2 & 4 & 1 \\ 3 & 6 & -2 & 5 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} 5(-5)-2(3) & 5(2)-2(6) & 5(4)-2(-2) & 5(1)-2(5) \\ 4(-5)+7(3) & 4(2)+7(6) & 4(4)+7(-2) & 4(1)+7(5) \\ 9(-5)+3(3) & 9(2)+3(6) & 9(4)+3(-2) & 9(1)+3(5) \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} -31 & -2 & 24 & -5 \\ 1 & 50 & 2 & 39 \\ -36 & 36 & 30 & 24 \end{bmatrix}$$

Now for a You Tube Video on Matrix Multiplication:
https://www.youtube.com/watch?v=kuixY2bCc_0

Matrix Division: Inverse

Inverse: The multivariate equivalent of division.

Denoted $\mathbf{A}^{-1}$

The inverse of a matrix is one that solves the following:
$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$

Trace of a Matrix

Trace: Sum of diagonal elements

$$\mathbf{X}_1 = \begin{bmatrix} 2 & 3 & 6 \\ 5 & 1 & 2 \\ 5 & 8 & 0 \end{bmatrix} \qquad \mathrm{Tr}(\mathbf{X}_1) = 2 + 1 + 0 = 3$$

Determinant of A Matrix: Simplest Case

Determinant: A single value that characterizes a square matrix, the determinant represents the volume of the $n \times p$ space.

The determinant of a 2×2 matrix is simple to calculate:

For a matrix **A**: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|\mathbf{A}| = ad - bc$

It gets complicated to find determinant of a 3×3 or larger dimensional matrices.