Matrix Inversion, Singularity and Special Matrices (SSCP)

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Matrix Division: Inverse

Inverse: The multivariate equivalent of division.

Denoted $\mathbf{A}^{-1}$

Inverse: Matrix Division

The inverse of a matrix is one that solves the following:

$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$

To find the inverse of a 2 × 2 matrix by hand, first create a pattern matrix with alternating + and – signs across each row:

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

Next, find the determinant of the original matrix, **A**:

$$|\mathbf{A}| = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = (2*4) - (1*3) = 5$$

Matrix Algebra Review

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Inverse: Matrix Division

Attach the signs from the pattern matrix to the original matrix elements and swap the elements on the positive diagonal:

$$\begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$$

Finally, divide each element by the determinant of the matrix:

$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{4}{5} & \frac{-1}{5} \\ \frac{-3}{5} & \frac{2}{5} \end{bmatrix}$$

Matrix Algebra Review

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Singular Matrices: A Real Problem

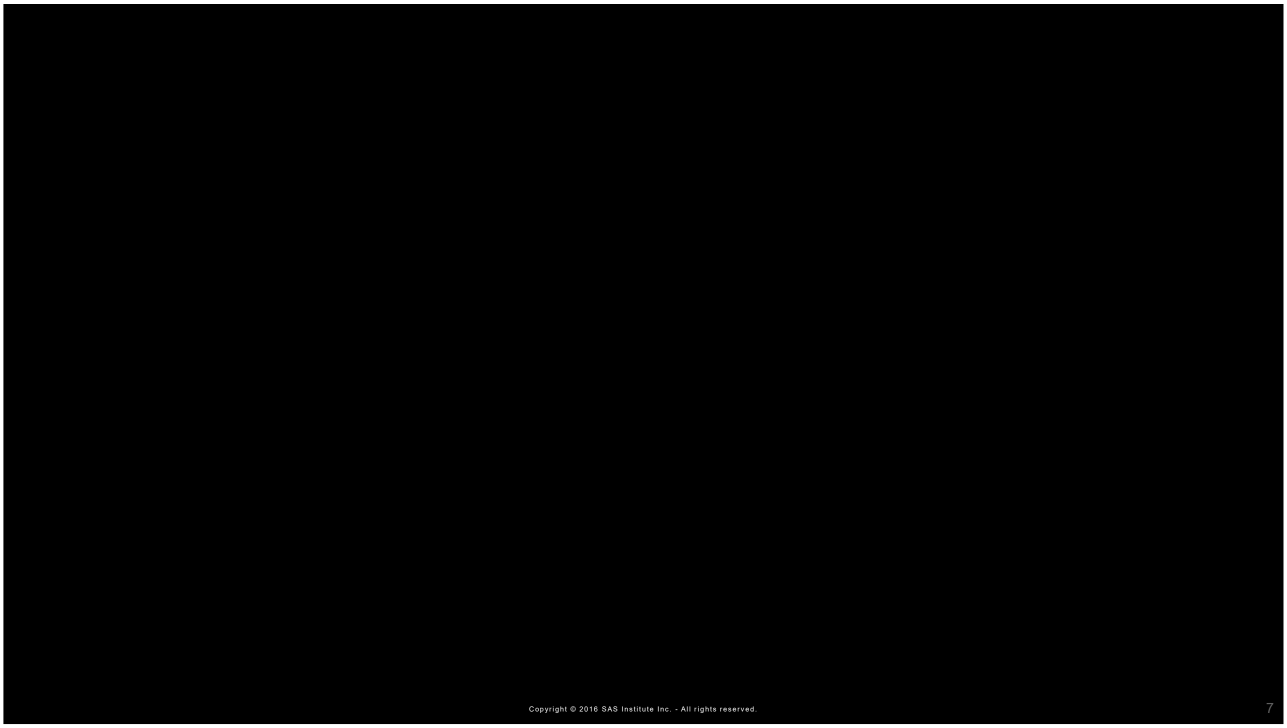
•Notice that if the determinant of a matrix=0, the inverse cannot be calculated.

- Matrices whose determinant=0 are known as *singular* matrices.

• Collinear (perfectly linearly related) variables in a matrix (**X**) can cause singularity in the **SSCP** (**X'X**) matrix.

- Matrix inversion is used extensively in multivariate statistics, and therefore collinear variables (and singular matrices) pose a real problem for analysis.
- When two variables in data *are perfectly correlated*, the SSCP becomes singular.

Examples of Different Singular SSCP Matrices



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