Events and Probabilities

Session Agenda

- Brief review of
  - Types of events
  - Types of probabilities
  - Types of event probabilities

Introduction to Probability

One of the basic concepts in probability is the …

EVENT --- a state of the world that may or may not occur

For example:

- Head appears when you toss a coin
- OSU Cowboys beat OU Sooners in Football this year
- Google's stock rises in value tomorrow
- A machine breaks down after 1,000 hours of operation
- A person receives an offer for a product and decides to place an order

We often denote an event by a capital letter, e.g.,
A = OSU Cowboys beat OU Sooners in Football this year

Type of Events

Single	state of the world that may or may not occur --- denoted A
Or	event that A occurs, or B occurs, or both --- denoted A+B, same as B+A
Joint	event that both A and B occur --- denoted AB, same as BA
Conditional	event that A occurs given B has occurred --- denoted A|B, different from B|A

Events (cont’d)

Mutually Exclusive (ME)	when events A and B cannot occur at the same time
Collectively Exhaustive (CE)	when events A and B comprise all possible outcomes

Both terms can refer to a collection of two or more events

PROBABILITY
--- the chance that an event may occur

P(A) denotes the probability of event A occurring

P(A) is always a number between 0 and 1, or 0% and 100%

How do you determine P(A)?

- P(A) is *objective* when it can be determined quantitatively (e.g., a flip of a coin, the likelihood of getting the flu)
- P(A) is *subjective* when it is determined by someone's beliefs (e.g., the probability that an employee will do a good job, the chance that a team will win)
- In practice, many probabilities are *hybrid*, i.e., they have both objective and subjective components

Probability Formulas

Session Agenda

- Independent events
- Addition and Multiplication formulas of probabilities

Independent Events

Events A and B are independent if

$P(A|B) = P(A)$

and

$P(B|A) = P(B)$

Intuitively, events A and B are independent if the knowledge that B has happened does NOT change the probability of the other event, A happens

An Example to Clarify Independence

- Event, A : a person living in Stillwater, OK owns a Lexus
- What is P(A)= ?
- Assumption:
  - Stillwater, Oklahoma, population is about 50,000
  - Only one Lexus dealer in Stillwater and according to that dealer there are about 500 Lexus owners in Stillwater
- Then, $P(A) = 500/50{,}000 = 0.01$ or 1%

An Example to Clarify Independence (Contd.)

- Event, B : a person whose annual income is more than \$250,000
- What is $P(A|B)$ or, Probability that a person own a Lexus given his/her annual income is more than \$250,000?
- Do you think that is the same as $P(A)$ or 0.01?
- If $P(A|B) = P(A)$, that means…

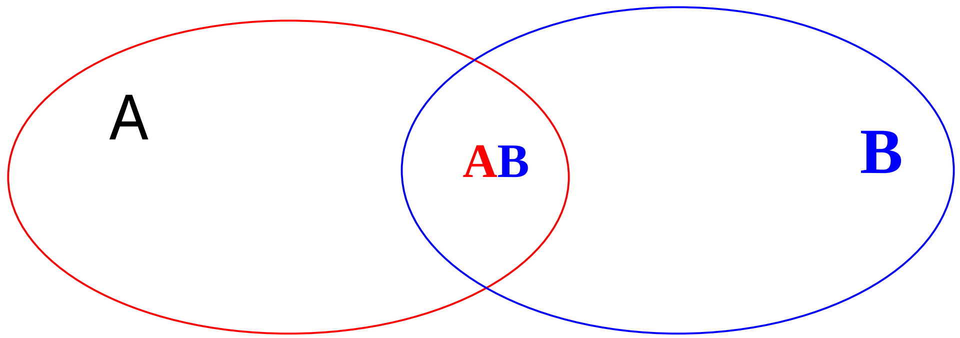
An Example to Clarify Independence (Contd.)

- Assume in Stillwater, there are 200 people whose annual income is $250,000 or more.
  - Reminder Stillwater population is 50,0000
- So, P(B), probability of someone making more than $250,000 is 200/50,000 = 0.004 or. 0.4%
- Assume that of the 200 people whose annual income is $250,000 or more, 50 of them own a Lexus.
- So, P(A|B) or, Probability that a person own a Lexus given his/her annual income is more than $250,000 is
- = P(AB) / P(B)
  - P(AB) = 50/50,000 = 0.001, P(B) = 0.004
- =0.001/0.004 = 0.25 or, 25% - much different than P(A)

Probability Formulas

Addition Formula	$P(A+B) = P(A) + P(B) - P(AB)$

Venn Diagram Representation



$$P(A+B) = P(A) + P(B) - P(AB)$$

Probability Formulas

Multiplication Formula

$$P(AB) = P(A|B)\, P(B)$$

- $P(A|B) = P(AB)/P(B)$
- Cross multiplying, we get, $P(AB) = P(A|B)\, P(B)$

Probability
Formulas (Special Cases)

Addition Formula for **ME** Events

$$P(A+B) = P(A) + P(B)$$

THE MECE RULE

If several events are mutually exclusive *and* collectively exhaustive, then their individual probabilities add to 1

Probability Formulas (Special Cases) Contd.

Multiplication Formula for **Independent** Events	$P(AB) = P(A)\,P(B)$

Independence means, $P(A|B) = P(A)$

So, multiplication formula becomes, $P(AB) = P(A|B)P(B) = P(A)\,P(B)$

Probability Formulas at Work

An Example with Stock Price

Stock Price Example

Today, the price is \$50.00, tomorrow the price could be …

A = \$51.51 ≤ price	P(A) = 0.05
B = \$51.01 ≤ price ≤ \$51.50	P(B) = 0.15
C = \$50.51 ≤ price ≤ \$51.00	P(C) = 0.15
D = \$50.01 ≤ price ≤ \$50.50	P(D) = 0.25
E = price is \$50.00	P(E) = 0.00
F = \$49.50 ≤ price ≤ \$49.99	P(F) = 0.25
G = \$49.00 ≤ price ≤ \$49.49	P(G) = 0.10
H = price ≤ \$48.99	P(H) = 0.05

Mutually exclusive and collectively exhaustive, i.e., no overlap, and sum of individual probabilities equals 1.00

A = \$51.51 ≤ price	P(A) = 0.05
B = \$51.01 ≤ price ≤ \$51.50	P(B) = 0.15
C = \$50.51 ≤ price ≤ \$51.00	P(C) = 0.15
D = \$50.01 ≤ price ≤ \$50.50	P(D) = 0.25
E = price is \$50.00	P(E) = 0.00
F = \$49.50 ≤ price ≤ \$49.99	P(F) = 0.25
G = \$49.00 ≤ price ≤ \$49.49	P(G) = 0.10
H = price ≤ \$48.99	P(H) = 0.05

P(X): Probability that **\$50.01** ≤ price ≤ **\$51.00** tomorrow?

A = \$51.51 ≤ price	P(A) = 0.05
B = \$51.01 ≤ price ≤ \$51.50	P(B) = 0.15
C = \$50.51 ≤ price ≤ \$51.00	P(C) = 0.15
D = \$50.01 ≤ price ≤ \$50.50	P(D) = 0.25
E = price is \$50.00	P(E) = 0.00
F = \$49.50 ≤ price ≤ \$49.99	P(F) = 0.25
G = \$49.00 ≤ price ≤ \$49.49	P(G) = 0.10
H = price ≤ \$48.99	P(H) = 0.05

P(Y): Probability that the price will **go down**?

A = \$51.51 $\leq$ price	P(A) = 0.05
B = \$51.01 $\leq$ price $\leq$ \$51.50	P(B) = 0.15
C = \$50.51 $\leq$ price $\leq$ \$51.00	P(C) = 0.15
D = \$50.01 $\leq$ price $\leq$ \$50.50	P(D) = 0.25
E = price is \$50.00	P(E) = 0.00
F = \$49.50 $\leq$ price $\leq$ \$49.99	P(F) = 0.25
G = \$49.00 $\leq$ price $\leq$ \$49.49	P(G) = 0.10
H = price $\leq$ \$48.99	P(H) = 0.05

P(Z): Assuming the price goes down, probability the price will be **between \$49.00 to \$49.49**?

Z= G given Y, where Y is price going down

P(Z)= P(G|Y) = P(GY)/P(Y) ,
But, recall Y=F+G+H and P(Y)=0.40

GY = G

P(Y) = P(G)/P(Y) = 0.10/0.40= 0.25

A = \$51.51 ≤ price	P(A) = 0.05
B = \$51.01 ≤ price ≤ \$51.50	P(B) = 0.15
C = \$50.51 ≤ price ≤ \$51.00	P(C) = 0.15
D = \$50.01 ≤ price ≤ \$50.50	P(D) = 0.25
E = price is \$50.00	P(E) = 0.00
F = \$49.50 ≤ price ≤ \$49.99	P(F) = 0.25
G = \$49.00 ≤ price ≤ \$49.49	P(G) = 0.10
H = price ≤ \$48.99	P(H) = 0.05

Is the event X **independent** of the event Q?
Where, X is (\$50.51 ≤ price ≤ \$51.50) tomorrow
Q is price goes up tomorrow