Model Essentials for Regressions

Dr. Goutam Chakraborty

Outline

- Predict new cases.
- Select useful inputs.
- Optimize complexity.

Prediction formula

Sequential selection

Best model from sequence

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Model Essentials – Regressions

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Linear Regression Prediction Formula

input measurement

$$\hat{y} = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2$$

prediction estimate

intercept estimate

parameter estimate

Choose intercept and parameter estimates to *minimize*:

squared error function

$$\sum_{\text{training data}} (y_i - \hat{y}_i)^2$$

...

Linear (Multiple) Regression Prediction Formula

input measurement

$$\hat{y} = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2$$

prediction estimate

intercept estimate

parameter estimate

Choose intercept and parameter estimates to *minimize the loss function:*

squared error function

$$\sum_{\text{training data}} (y_i - \hat{y}_i)^2$$

...

Logistic Regression Prediction Formula

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2$$ *logit scores*

...

Logit Link Function

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2 \quad \textit{logit scores}$$

The logit link function transforms probabilities (between 0 and 1) to **logit scores (between −∞ and +∞).**

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Logit Link Function

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2 = \operatorname{logit}(\hat{p})$$

$$\hat{p} = \frac{1}{1 + e^{-\operatorname{logit}(\hat{p})}}$$

To obtain prediction estimates, the logit equation is solved for $\hat{p}$.

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$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2$$

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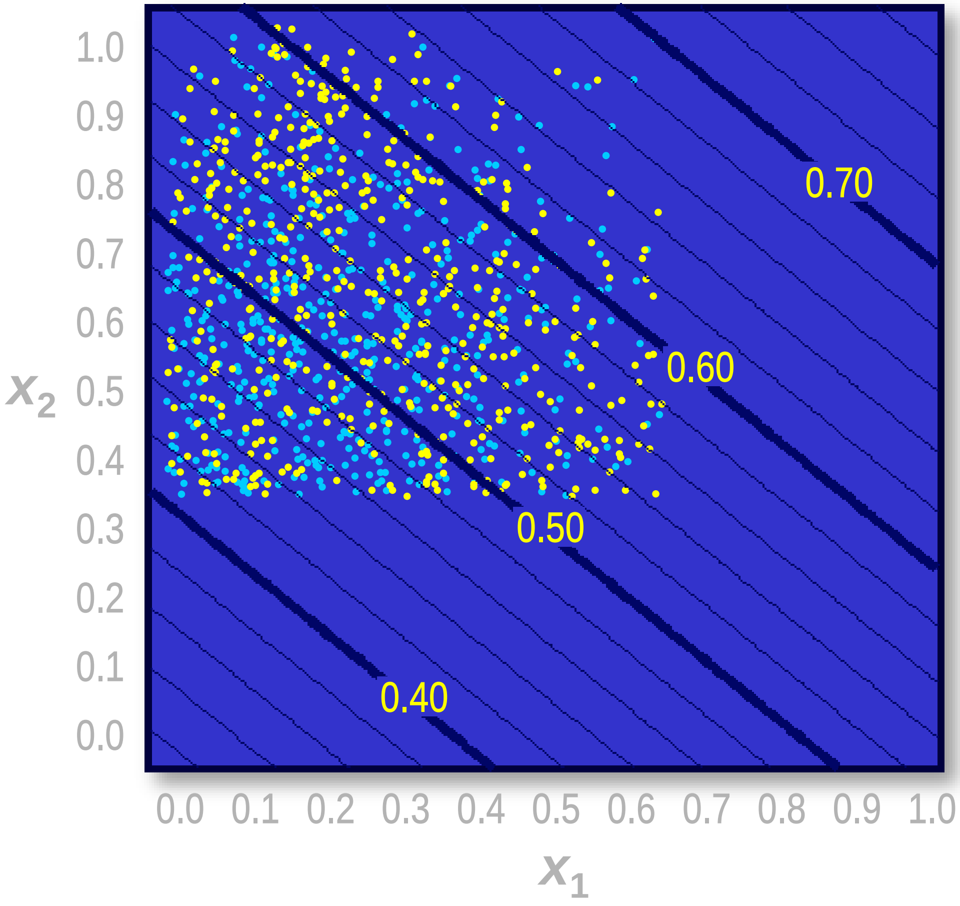
Simple Prediction Illustration: Logistic Regression

$$\text{logit}(\hat{p}) = \boxed{\hat{w}_0} + \boxed{\hat{w}_1} \cdot x_1 + \boxed{\hat{w}_2} \cdot x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

You need intercept and parameter estimates.

Predict dot color for each x_1 and x_2.



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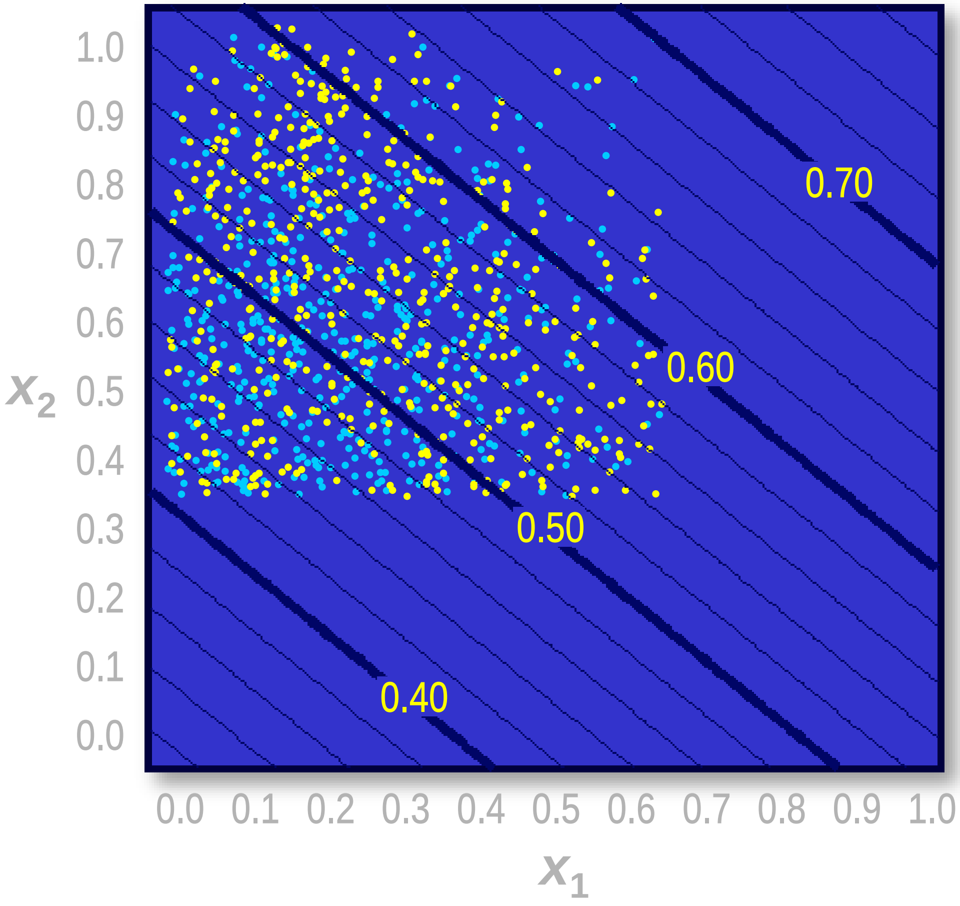

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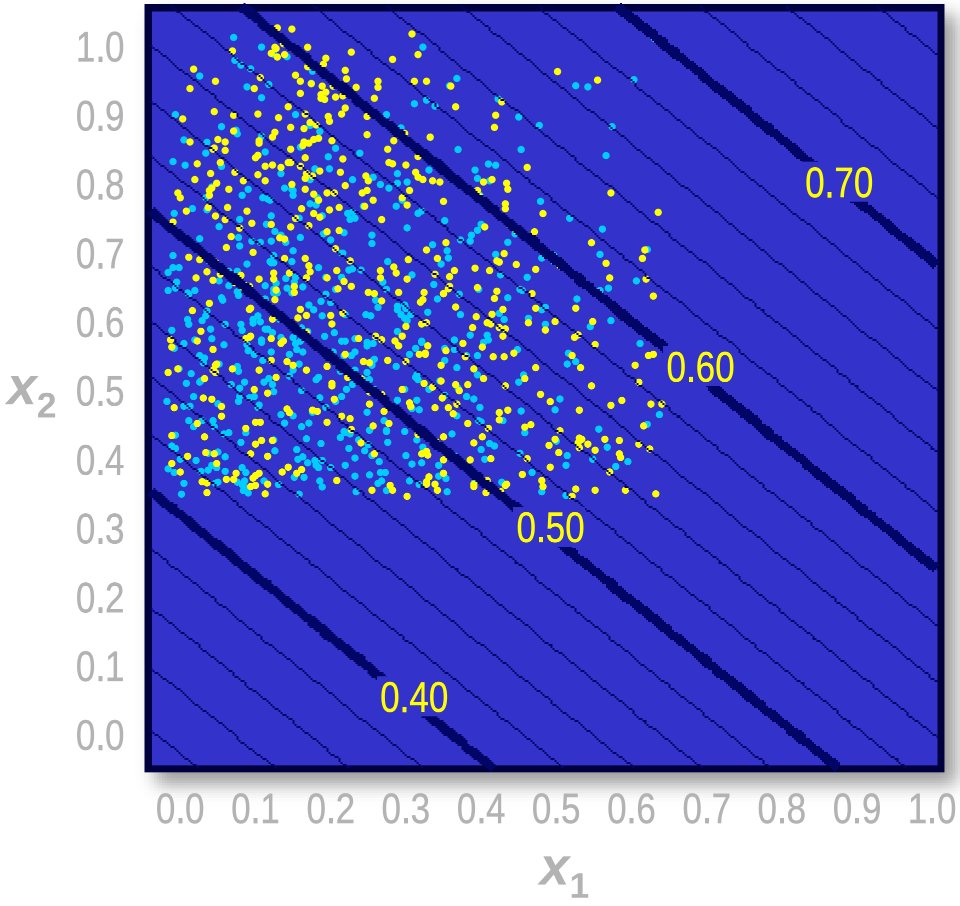
Find parameter estimates by *maximizing*

$$\underset{\substack{\text{primary} \\ \text{outcome} \\ \text{training cases}}}{\sum} \log(\hat{p}_i) + \underset{\substack{\text{secondary} \\ \text{outcome} \\ \text{training cases}}}{\sum} \log(1 - \hat{p}_i)$$

log-likelihood function



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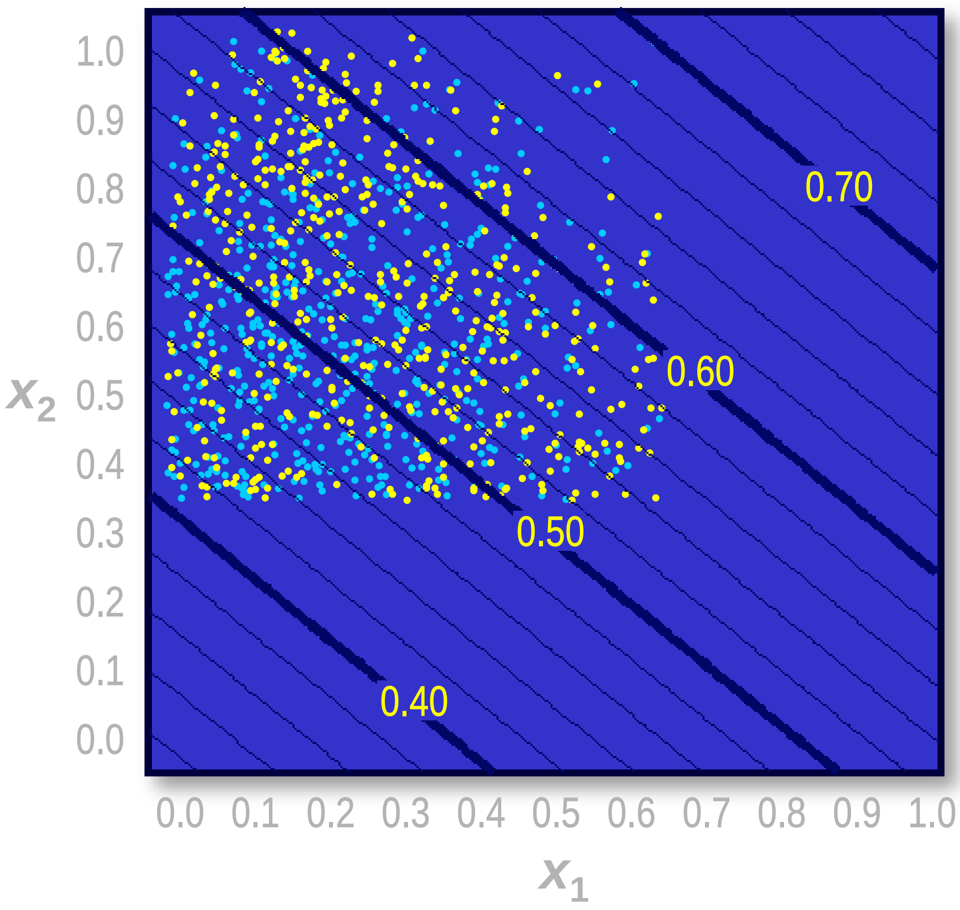

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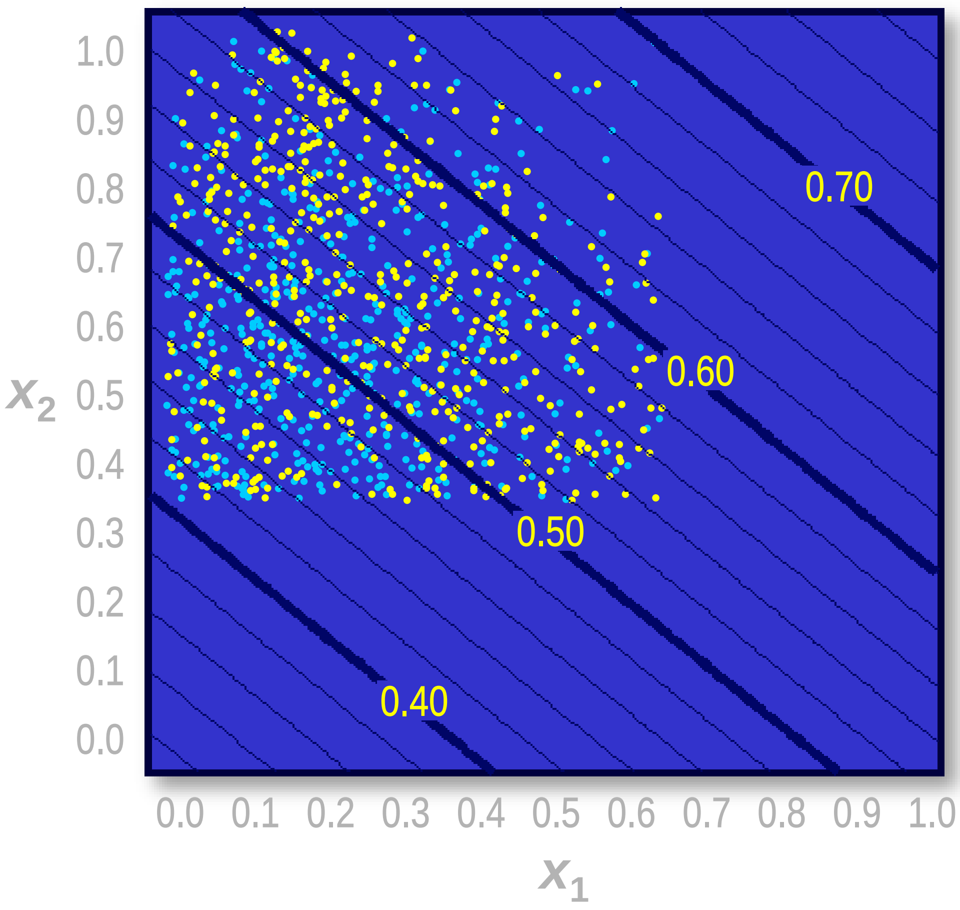
Simple Prediction Illustration: Logistic Regression

$$\text{logit}(\hat{p}) = -0.81 + 0.92\,x_1 + 1.11\,x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

Using the maximum likelihood estimates, the prediction formula assigns a logit score to each x_1 and x_2.



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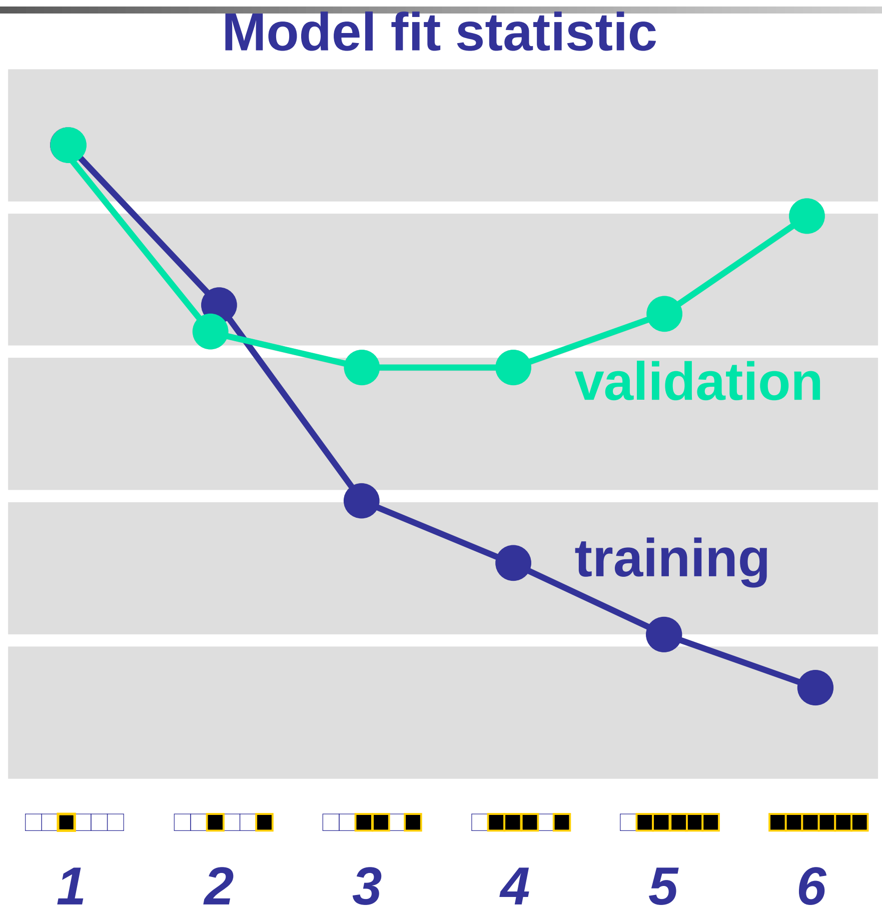
Model Fit versus Complexity



Evaluate each sequence step.

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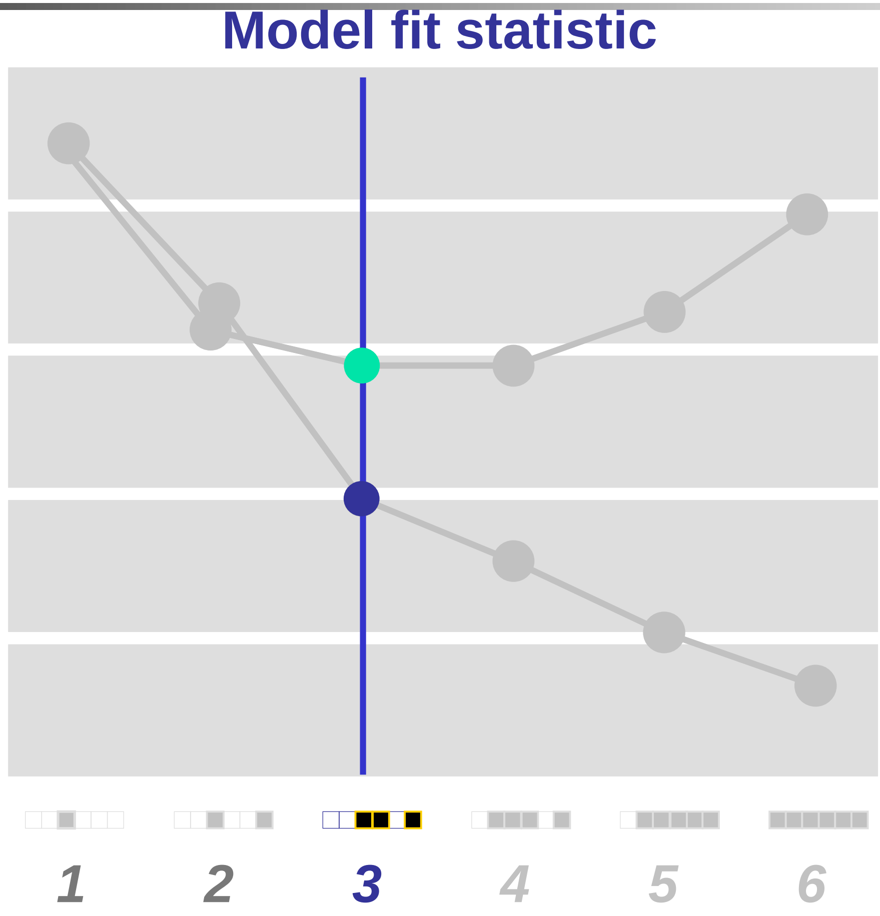
Select Model with Optimal Validation Fit



Evaluate each sequence step.

Choose simplest optimal model.

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