Module 8: More LP Models - Part I

Reading Material: 8.1 – Introduction – The Rest of the Text

8.1.1 More LP Applications – Our Path

Now that we have mastered the foundations of creating and solving LP models, it is time to move ahead at a more rapid pace and explore some of the many areas where LP modeling can assist real-world decision making.

The focus will be on applications areas that over time have shown to be useful to our MBA (and other) graduates. It is impossible to cover all possible scenarios in terms of useful applications. But my past students have, in many ways, helped us out. Let me explain.

It seems that every semester, students in this class will come up to me after class (or send e-mails) and tell me how they have already applied an LP model to some work situation or perhaps even an application away from the office (optimizing beer production was one of my favorites). They've told me stories of how they have scheduled employees, created golf teams for a reunion, analyzed work flow, optimized water production systems, modeled distribution and logistics networks, managed baseball concession stands, determined the assignment of workers to specific jobs, and many others (including beer production!). This was WHILE they were enrolled in class.

So, I have used this feedback along with the projects I have been involved with and other examples in the academic literature to create our experience. Although the examples in the text may be small and simplified, they are representative of some very useful LP applications. Most important, they help us practice the important overall philosophy of modeling.

It is always difficult to start this journey. Do we introduce our larger LP models by functional area (such as operations, marketing, finance) or introduce the tools by a traditional "problem" taxonomy (production problems, diet problems, transportation problems, etc.)? Traditional text books typically opt for the latter format, probably a reflection of long ago days when computer power did not exist and we learned specialized solution algorithms for specific problem types – because that was the only way to solve them efficiently (or at all!)

We will follow the latter format too, but not for an archaic reason (at least I don't think so!). By sequencing our applications in the way chosen, it has historically seemed that even as we get to larger and more complex settings, the process of creating LP models for these decision scenarios starts to appear easier and easier!! Let's hope it will work that way for you.

8.1.2 Learning Objectives for Module 8

We have a number of different learning objectives in this module related to gently expanding our LP modeling ability. In the first example, we examine an “allocation” application that illustrates a scenario in which one might want to consider deviating from the row/column form in EXCEL. This example also helps us illustrate the advantageous flexibility of using spreadsheets for creating LP models (compared with being forced to input the algebraic model every time!).

The second example is a marketing application of LP. An LP model is set up to help determine the optimal number of different advertisement media needed to reach people in a public service announcement campaign for ear health (Ear Bud Safety). This case is not only a classic application of LP in a functional area, it is also a great example of what one might call the “think before we enter” learning objective. A natural phenomenon occurs in this class: when a student is confronted by a problem with lots of numbers, a typical (and understandable) reaction is ‘I must find a home for every number!” This example illustrates that when we use patience with our three-step modeling approach (What do we control, what do we want to achieve, and what constrains us), the numbers take care of themselves and an appropriate parsimonious model results from our analysis.

The third example is also a classic type of situation, termed a “diet” problem. To this point, we have seen production problems (e.g., producing LEGO blocks), financial situations (allocating funds in investment models), and design models (Bates Motel problems), among others. Diet problems were one of the first categories of application models (along with gasoline blending models) used in practice back in the 1950s. They were called this because they were used in agricultural-related livestock feeding situations in which different grains and such were blended together to create the optimal diet, subject to nutritional requirements with a goal of minimizing cost. As well as gaining exposure to this important type of modeling situation, the example includes an additional important modeling learning objective: how to linearly model the concept of weighted averages.

These three examples along with the practice problems will increase our modeling skills and comfort in the power of LP.

Reading Material: 8.2 – Example 1: Tulsa SodBusters Farms

8.2.1 Scenario Introduction – Part I

Tulsa SodBusters Farms have a company motto – “There’s more than one way to grow grass”. The company grows fescue sod, and sells it to home builders and other interested parties for use in yards. We will develop an LP model to assist their planning for the next growing year. This model should suggest how many units of two different fescue grasses should be planted (and therefore harvested) at their two different farms.

The two different grasses grown are called Fescue I and Tulsa Fescue. The two farms, both having a maximum capacity of 35 units of sod (you may substitute whatever measure of area you would like), are aptly named Farm 1 and Farm 2.

When harvested, Fescue I sells for $45 per unit and Tulsa Fescue sells for $55 per unit. The grass sales prices are independent of growing location.

The cost to grow units of Fescue I and Tulsa Fescue does depend upon the farm, because of topography and other soil-related conditions. Fescue 1 costs $16 per unit to grow at Farm 1, and $20 per unit to grow at Farm 2. Tulsa Fescue costs $22 per unit to grow at Farm 1 and $32 per unit to grow at Farm 2. Budget-wise, we can plant sod to our hearts' content – as long as it does not exceed the budget of $1414. Thus, not only do costs have an impact on our objective, but it explicitly constrains our sod allocation.

Demand for the sod is always hard to predict season by season. The model should make sure that between 20 and 40 units of Fescue I is grown/allocated and between 10 and 30 units of Tulsa Fescue is grown/allocated, inclusive of the endpoints.

Create an LP model that, when solved, tells us how many units of each grass type should be grown at each farm that maximizes net return subject to the relevant constraints.

8.2.2 The algebraic model for Tulsa SodBusters

The algebraic model for this problem is critical to the learning objective, so it will be presented first in the section.

So let us go through the three-step process: *What do we control*? The units of (Fescue I, Tulsa Fescue) to be planted at (Farm 1, Farm 2).

We have two types of sod and two farms – four different combinations of how grass/sod can be planted. Thus, there are four decision variables. Abbreviate these variables as FIF1, FIF2 for Fescue I at Farm 1 and Farm 2 respectively, and TFF1 and TFF2 for Tulsa Fescue at Farm 1 and Farm 2. Doing a Spock-McCoy mind meld, "remember" the 2×2 combination concept. (Sorry for the old Star Trek II reference).

Now, *what do we want to achieve*? Our goal is to maximize return, which is total sales less total costs. Each farm/fescue has a different combination of sales and costs. For instance, Fescue I at Farm 1 is sold for $45 per unit, but costs $16 per unit. Thus, the net contribution to profit/return is $45 less $16 = $29 per unit. That is its objective function coefficient. The other three decision variables require similar calculations.

What constrains us in our sod allocation decision?

There is a limited budget ($1414). There are farm planting capacities (35 units each farm). There is an upper and a lower bound on the targeted production of both sod types. This equates to seven distinct constraints.

From this description and the problem write-up, the algebraic model for Tulsa SodBusters is listed below.

MAX 29 FIF1 + 25 F1F2 + 33 TFF1 + 23 TFF2
ST

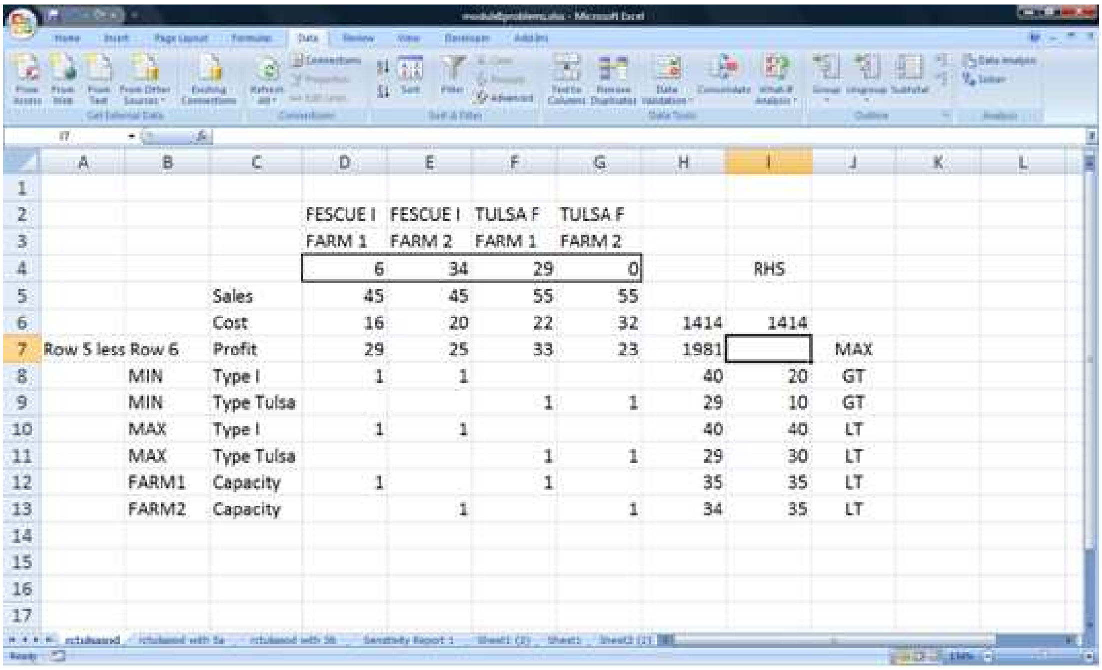
16 FIF1 + 20 F1F2 + 22 TFF1 + 32 TFF2	<= 1414 (Budget)
FIF1 + F1F2	>= 20 (MIN FI)
TFF1 + TFF2	>= 10 (MIN TF)
FIF1 + F1F2	<= 40 (MAX FI)
TFF1 + TFF2	<= 30 (MAX TF)
FIF1 + TFF1	<= 35 (MAX F1)
F1F2 + TFF2	<= 35 (MAX F2)

Non-negativity applies as well.

The next section implements the model in row/column form – which of course simply mimics the algebra. It is always a reasonable approach in EXCEL to solve any LP model.

8.2.3 Row/Column Implementation

Figure 8.1 and 8.2 illustrate the EXCEL implementation of the LP model. By now, we should be fairly comfortable with the mechanics of modeling. Figure 8.2 is the exact Solver input for this spreadsheet.



	A	B	C	D	E	F	G	H	I	J
1										
2				FESCUE I	FESCUE I	TULSA F	TULSA F			
3				FARM 1	FARM 2	FARM 1	FARM 2			
4				6	34	29	0		RHS	
5			Sales	45	45	55	55			
6			Cost	16	20	22	32	1414	1414	
7	Row 5 less Row 6		Profit	29	25	33	23	1981		MAX
8		MIN	Type I	1	1			40	20	GT
9		MIN	Type Tulsa			1	1	29	10	GT
10		MAX	Type I	1	1			40	40	LT
11		MAX	Type Tulsa			1	1	29	30	LT
12		FARM1	Capacity	1		1		35	35	LT
13		FARM2	Capacity		1		1	34	35	LT
14										
15										
16										
17										

Figure 8.1

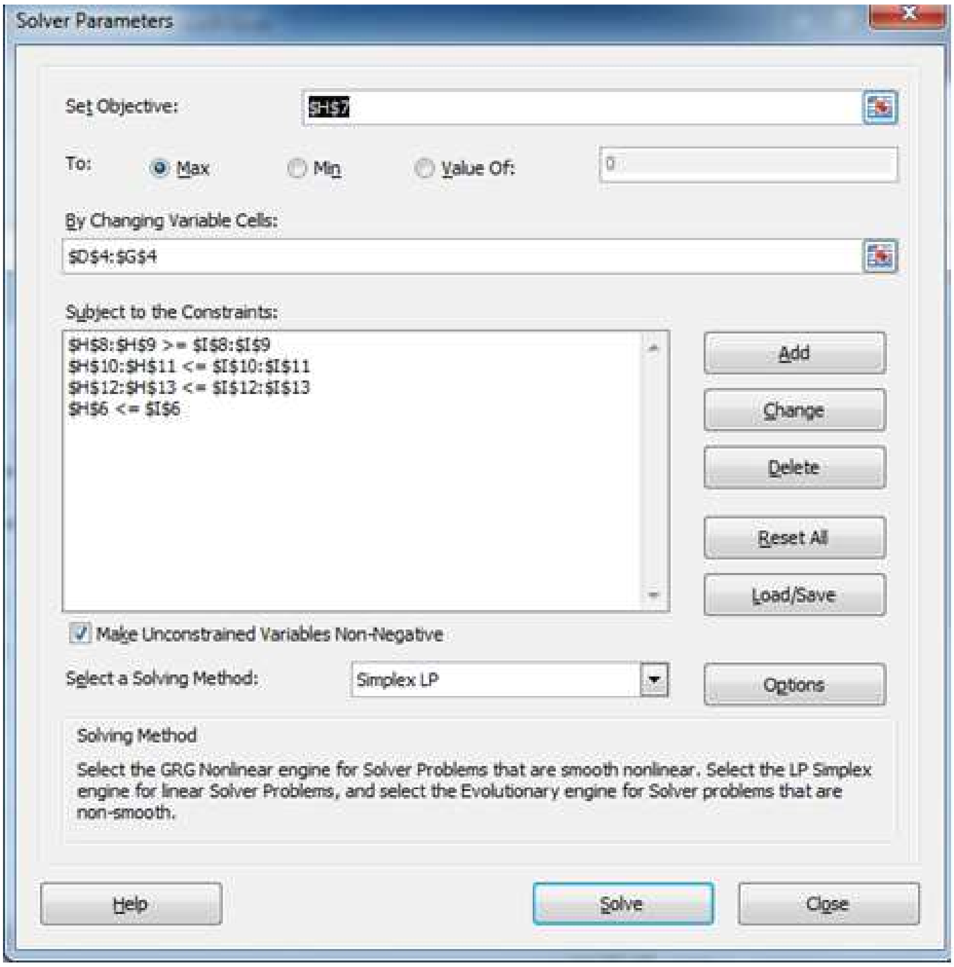


Figure 8.2

Row 4 is the decision variable row. Note that Row 7 calculates the objective function coefficients. Because the objective function coefficients are sales price less cost, Row 7 uses a formula (Row 5 – Row 6) to calculate the net return (an efficient use of EXCEL functionality to help build the LP model).

Column H is our action column, where we calculate the SUMPRODUCT of Row 4 and the corresponding row of interest. H7 is our target cell (Row 7*Row 4 is our profit/return). Note that Row 6 is one of our seven constraints – the cost constraint. Because that information was used in calculating the objective function coefficient, it might seem a little out of place in the spreadsheet (it is fine).

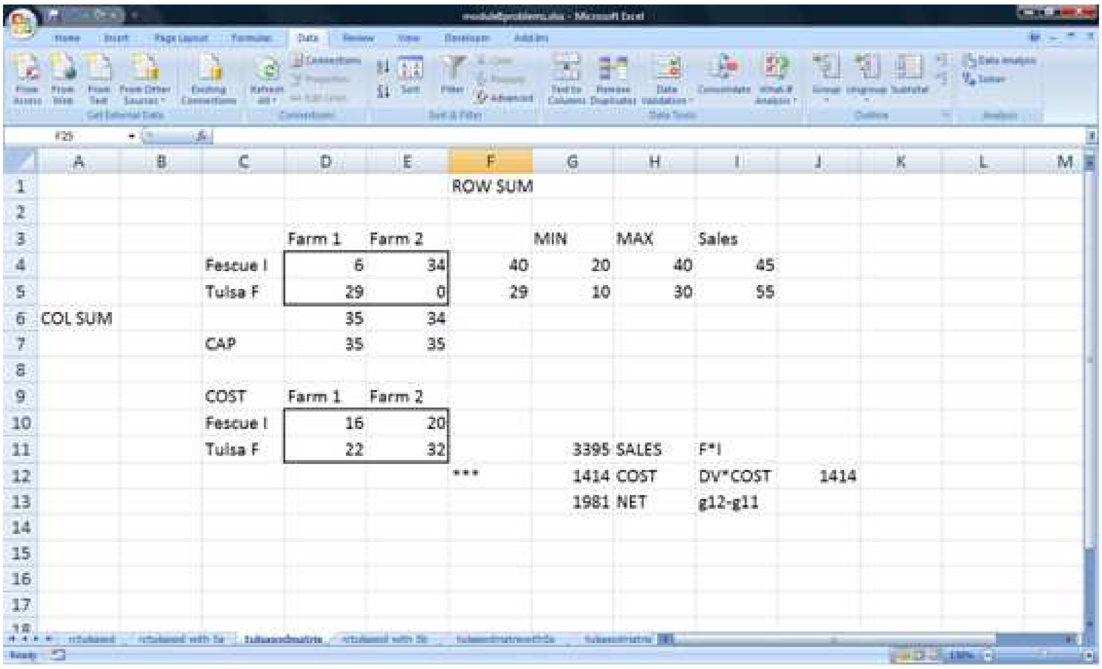
Rows 8 through 13 implement the last six constraints of the LP model. The type of constraint is highlighted in Column J (just a label).

The solution to the model indicates that six units of Fescue I and 29 units of Tulsa Fescue should be planted at Farm 1 and 29 units of Fescue I and NO Tulsa Fescue planted at Farm 2. This maximizes our profit at $1981. Note the binding constraints (LHS=RHS) are cost (Row 6), the maximum allowed of Fescue I production (Row 10), and the capacity of Farm 1 (Row 12).

8.2.4 Alternative Modeling Approach – The "Keenu Reeves" Approach

Some of you perhaps wanted to model this problem differently because of its unique 2×2 structure. It is also okay if you did not get that feeling.

Figures 8.3 and 8.4 show an alternative way in which the EXACT SAME LP MODEL is implemented using what we call a MATRIX approach instead of a row/column approach.



	A	B	C	D	E	F	G	H	I	J
1						ROW SUM				
2										
3				Farm 1	Farm 2		MIN	MAX	Sales	
4			Fescue I	6	34	40	20	40	45	
5			Tulsa F	29	0	29	10	30	55	
6	COL SUM			35	34					
7			CAP	35	35					
8										
9			COST	Farm 1	Farm 2					
10			Fescue I	16	20					
11			Tulsa F	22	32		3395	SALES	F*I	
12						***	1414	COST	DV*COST	1414
13							1981	NET	g12-g11	

Figure 8.3

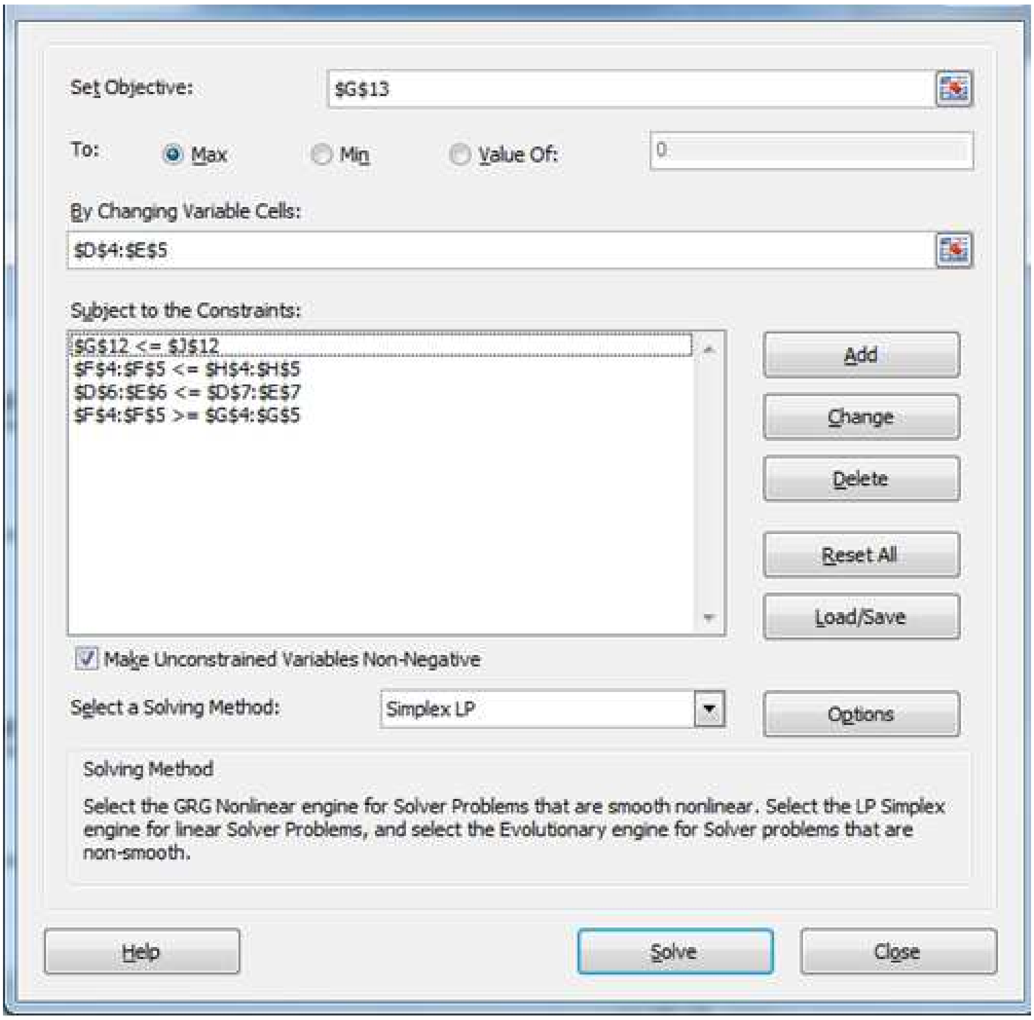


Figure 8.4

What is a matrix approach? The decision variable placeholders in EXCEL exploit the unique combinatorial structure of the decision situation. In this case, there are two products and two locations, thus a natural 2×2 problem. The decision variables can then be modeled in a 2×2 matrix. Column D represents Farm 1, Column E, Farm 2. Fescue I is associated with Row 4, Tulsa Fescue in Row 5. Thus, D4 is the placeholder for the amount of Fescue I grown at Farm 1, D5 the amount of Tulsa Fescue grown at Farm 1, cell E4 is the amount of Fescue I grown at Farm 2, and E5 represents the amount of Tulsa Fescue grown at Farm 2.

Why might this be more natural? With such a combinatorial set of decision variables, we tend to need to "sum the columns" and "sum the rows" in our constraints. Thus, any time the matrix style of decision variables is used, we should automatically SUM each row and each column of our decision variable matrix. Keep that in mind for future modules as well.

In the present situation, Column F and Row 6 sum the rows and columns, respectively. The formula in F4 is =SUM(D4:E4) and in F5, =SUM(D5:E5). Similarly, the formulae in D6 and E6 are column sums (=SUM(D4:D5) and (=SUM(E4:E5), respectively). You might recognize that F4 and F5 represent the total units of our two products, and that D6 and D7 represent the total amount of units grown at each of the two farms. So, by using a matrix, we have quickly created the framework for six of our seven constraints – the two constraints for farm capacity and the four constraints for MIN and MAX production of our two products. We will more explicitly identify these constraints at the end of the section.

Although there may be an opportunity to save effort in creating the corresponding LP model using a matrix approach, there is still work to do. We have to work a little harder in creating our objective function, and our constraints will be spread out more on the spreadsheet. None of this is a problem, just an artifact of this approach. In fact, one could randomly place model components on a spreadsheet – as long as the underlying LP model is accurate, their exact appearance in EXCEL does not matter. It is good modeling practice, though, to have some form of order.

To create the objective function, we will calculate separately the total sales and the total cost, then maximize the difference between the two values (sales less costs). Three different cells will be used. Also note that the corresponding costs to grow products at farms are provided in cells D10:E11, and that the sales prices of the two products are listed in Column I, Rows 4 and 5.

Cell G11 is the placeholder for the solution's total sales. The formula is a SUMPRODUCT of the total amount of each product planted/grown (Column F, because we do not care where it is grown) matched up with the sales price in Column I (= SUMPRODUCT (F4:F5, I4:I5)). Cell G12 calculates the total cost. This SUMPRODUCT takes the 2×2 decision variable matrix and matches them up with the 2×2 cost matrix (=SUMPRODUCT(D4:E5,D10:E11).

Finally, cell G13 is set equal to G11 – G12, the total sales less the total cost. This is the target cell to maximize. As a by-product of calculating the net profit, the left-hand side (LHS) value for the cost constraint has also been calculated in G12 (it is the one unaccounted for constraint in this format of the model). The RHS of this constraint is typed arbitrarily in Cell J12.

Figure 8.4 shows the locations of our target cell, the changing cells, and our seven constraints. The constraints can be summarized as Row 6 <= Row 7 (two capacity constraints), Column F >= Column G (minimum production of the two products), Column F <= Column H (maximum production allowed of the two products), and finally the cost constraint G12 <= J12.

Figures 8.1 and 8.3 therefore represent two totally different EXCEL approaches that create the exact same LP model (and obviously, the exact same optimal solution).

Two important final points: A matrix approach is just as good as row/column (in these circumstances) – but row/column is just as good as a matrix approach as well! When modeling circumstances have combinations (here, it is two types of sod grown at two farms; it could be three places where products are stored in warehouses, with four destinations where the products need to go, etc.), a matrix style of decision variables is worth considering. Again, it is ALWAYS safe to use row/column format for ANY LP problem, past, present, and future.

Second, I hope it is becoming clearer that individual LP models can be implemented differently, perhaps even using our own developing personal EXCEL style, as long as the underlying LP model is the same. This style may include different ways of implementing the same LINEAR constraints, etc. As long as the spreadsheet appropriately maps to the correct LP model, we are okay.

Now, let us by continue this example by considering adding one additional requirement to the core model. This will help illustrate flexibility in specifying Solver constraints.

8.2.5 Tulsa SodBusters Part II – Minimum Production at Each Farm

Consider the following scenario: The owner of SodBusters does not like the originally determined optimal solution for fescue planting due to a perceived lack of diversification, as Farm 2 is only allocated Fescue I (no Tulsa Fescue). (The concern: what happens if a fungus is among us at Farm 1 and kills all of the Tulsa Fescue? Oh, the humanity!). Anyway, let's add one small requirement to the situation –that at each farm, there must be at least five units of each product planted/grown.

Algebraically, this is fairly straight-forward. We would add the following four constraints to our model:

FIF1 >= 5;   FIF2 >=5;   TFF1 >=5;   TFF2 >= 5.

Figure 8.5 shows adding the constraints explicitly to our spreadsheet. Note the solution changed in resolving the model, which should not be unexpected given that we've added more constraints to the model. You can see forcing each farm to plant each Fescue causes the net profit to decrease to 1917.538 (from 1981).

Figure 8.6 shows the additional constraints added in the Solver dialog box.

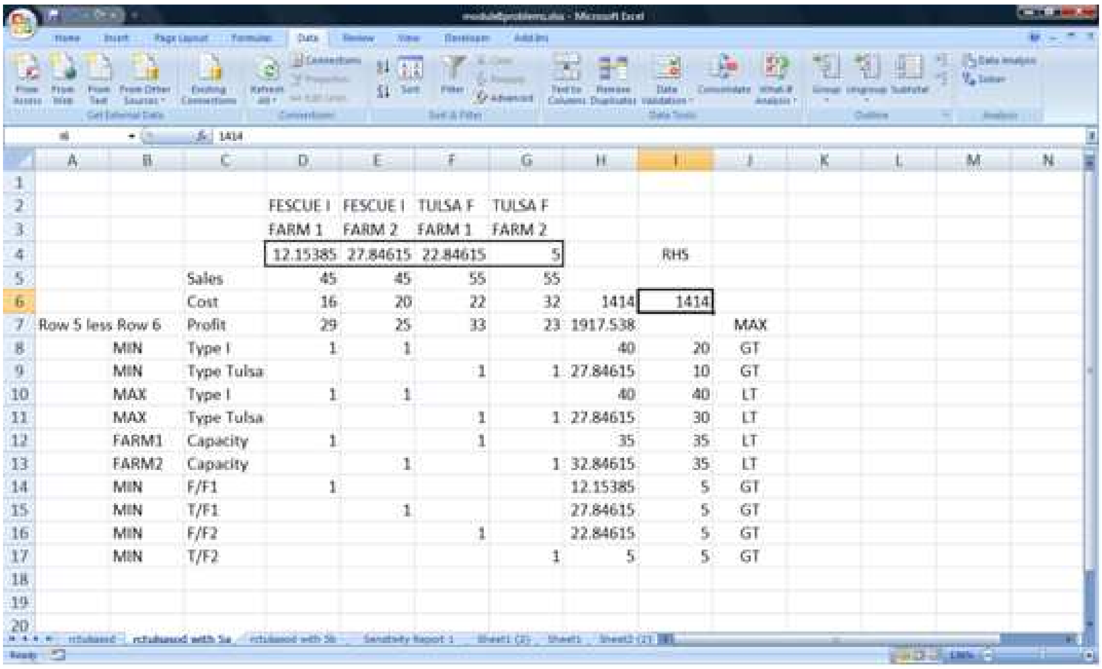


Figure 8.5

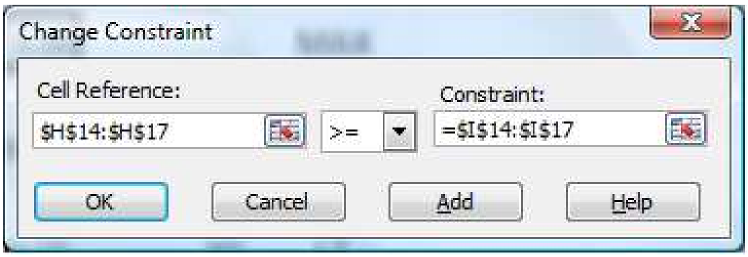


Figure 8.6

Now, for the very subtle learning objective in this additional modified example: We could have added these four constraints WITHOUT changing our spreadsheet! So we'll illustrate a bit more of the EXCEL Solver capability next

Figure 8.7 shows how the constraint could have been added "behind the scenes." Instead of adding the constraints as in Figures 8.5 and 8.6, we would simply highlight the decision variable cells as our LHS (D4:G4), and type the number "5" as our right-hand side (RHS) (and select >=). You can give it a try yourself, and it will result in the same optimal solution.

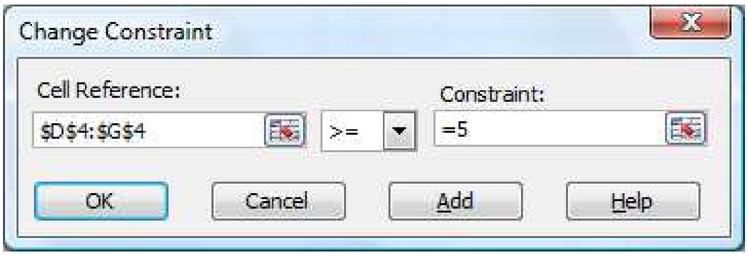


Figure 8.7

Which style is preferred? Explicitly listing all constraints has its advantages in debugging models. But a benefit of EXCEL in creating LP models is its ability to free us (somewhat) from having to always replicate the algebra of the model. This ability gets us to finding solutions and gaining insight quicker. So, there is not a preference per se, just use the approach with which you are most comfortable.

For completeness, Figures 8.8 and 8.9 show the explicit representation of these four constraints using the matrix-style decision variables, and Figure 8.10 is the constraint that would implement the restriction behind the scenes.

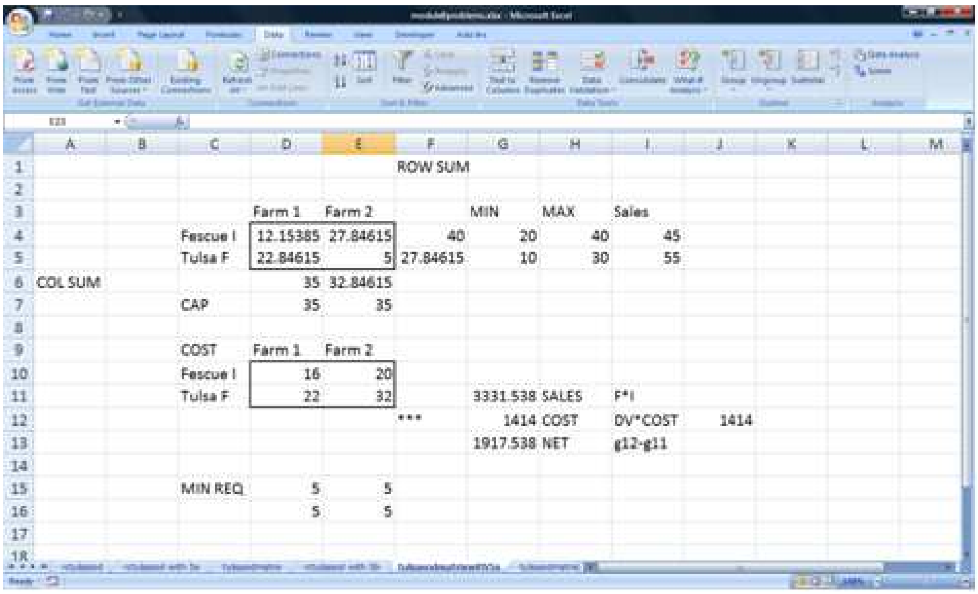


Figure 8.8

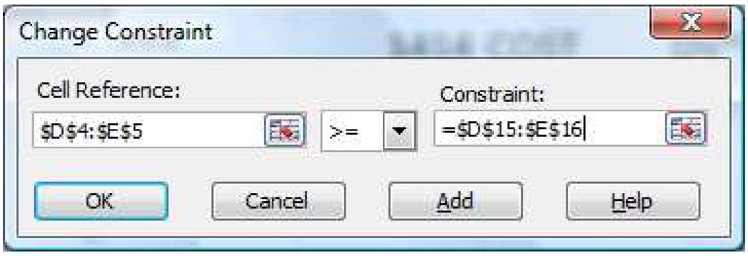


Figure 8.9

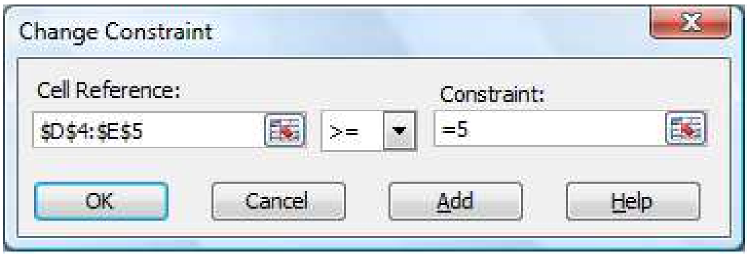


Figure 8.10

A word of caution not related to this example: The Solver Constraint box, as shown here, can be used to input RHS numerical values. It is NOT designed to do calculations/operations. That is EXCEL's job. DO NOT TRY TO DO CALCULATIONS IN THE SOLVER CONSTRAINT BOX. I have seen it done occasionally without harm. Most times, when attempted, it causes the model to barf. There is no reason to tempt fate and NOT use EXCEL for what it is designed to do – help us in the calculations needed to set up the LP framework for the Solver. Disregard this warning about inappropriate Solver use at your own risk. Some have tried; few, if any have succeeded.

This brings an end to this lengthy, multifaceted example. The use of matrix-style variables was shown as an alternative to row/column form for LP models. Matrix-style variables may be useful when a decision situation is characterized by having x products made at y locations, or something similar. We will see this format many times in future Modules. So, to quote Marilyn McCoo and Billy Davis Jr. – "you don't have to be a row and column modeler to be in my show . . . ."

This example also showed the flexibility in constraint specification that is possible, that we can even enter them directly in the Solver constraint dialog box. We are just scratching the surface on modeler flexibility. Whether there are 50 ways to create your model, I don't know – ". . . just give me a Sum, Bum; make a new table, Mable, don't need to be a row Bo, just get yourself solved."

Reading Material: 8.3 – Example 2: The National Ear Bud Safety Campaign

8.3.1 Problem Description

Because increasing numbers of ear injuries have been suffered by the U.S. population over the last few years (primarily caused by iPods and smart phones), the FDA is conducting a small trial campaign in Oklahoma to advertise the dangers of ear buds for aural health.

The Food and Drug Administration (FDA) has a small budget for the trial advertising campaign ($104,000). They want to find the optimal number of advertisements to run on three different media that will maximize the number of people that they reach, *reach* being a standard marketing concept for people who receive the advertised message. The three media chosen for the campaign are satellite radio, internet sites (sites already chosen by the FDA), and TV. Radio ads reach 5000 people for each ad and cost $2,500 each, internet ads reach 12,000 people (per ad) and cost $750 each, and TV ads cost $8,000 and reach 18,000 people each. The FDA has also asked that no more than 10 internet ads be used; there are no restrictions on the other ad types individually, but there is a restriction that the number of satellite radio ads must be greater than or equal to 40% of those on the other two (internet and TV) combined.

The people reached by each ad type can be categorized on many different demographics. Here, the FDA is interested in just two categories – sex and age of the people reached by the advertisements. Sex is broken down into male and female, and the age category uses two groups, younger than 50 years and 50 years and older. The following table breaks down the total people reached for each ad type into the four different category combinations.

		<50	>=50		Total
				SubTotals	
Sat. Radio	Men	500	3000	3500	
	Women	250	1250	1500	
	Subtotals	750	4250		5000
INT	Men	4250	2000	6250	
	Women	4250	1500	5750	
	Subtotals	8500	3500		12000
TV	Men	4000	4000	8000	
	Women	4000	6000	10000	
	Subtotals	8000	10000		18000

For example, the table indicates satellite radio reaches 500 younger men, 250 younger women, 3000 older men, and 1250 older women. At a higher level of differentiation, satellite radio reaches a total of 750 people younger than 50 years old (younger) and 4250 people >= 50 years old (older). Still another slice – these ads reach 3500 men and 1500 women. The overall total reached is, of course, 5000 people.

The FDA has the following two requirements for the composition of advertisements:

1. The number of women reached must be greater than or equal to the number of men.
2. The amount of people reached who are 50 and over must be at least 80% of the number of people less than 50 years old reached.

Create an LP model that will find the optimal way of using the advertising money in determining the optimal number of advertisements such that requirements are met and the total number of people reached is maximized. Do NOT worry about the model forcing the number of ads to be integers (though that obviously makes sense and it will be dealt with in due time).

8.3.2 "We Don't Have to Find a Home for Every Number"

It would be natural to look at the demographic information and think – "Oh, my, what I am going to do with all the data." It is understandable to be a little intimidated by all these numbers early on in our modeling mastery tour. But one of the learning objectives in this example is sort of the Taoist principle of Management Science – somewhat like a *wu wei* – action without action. Or, put another way, contemplating attacking the two demographic composition constraints, resisting the temptation to rush to find a home for every number. (Now I would suggest that by sipping some relaxing green tea or other beverage of choice while modeling, that too will assist, but more as an assist to relaxing.)

Before focusing on finding homes for numbers, some problem basics first – what do we control? The number of the three different ad types to use – call them RAD, INT, and TV. What do we want to achieve? Maximize reach. What constrains us? Cost is one constraint, maximum number of internet ads is another constraint, and the 40% radio requirement a third. Finally, the two FDA requirements are constraints four and five.

Let's map out the easier things first.

Objective function =	5 RAD + 12 INT + 18 TV		(units of 1000)
Cost constraint:	2.5 RAD + 0.75 INT + 8 TV		<= 104 (units of 1000)
Internet MAX:		INT	<= 10

The radio ad requirement will be derived using what has been learned in previous modules.

RAD >= .4 (INT + TV) is the starting point for this constraint. Interestingly, we could implement that expression directly in EXCEL and it would be valid. But let us still stick to row/column form for now. Rearranging the terms, we get the following expression:

RAD – .4 INT – .4 TV >= 0.

This is the constraint format we will implement in EXCEL.

To create the "must reach at least as many women as men" constraint in our advertising mix, we first identify the number of women and men each ad type reaches. This is of course shown in the table. RAD reaches 3500 men and 1500 women, INT ads reach 6250 men and 5750 women, and TV reaches 8000 men and 10,000 women.

So, the amount of women reached by the ad types can be stated as:

1500 RAD + 5750 INT + 10,000 TV or 1.5 RAD + 5.75 INT + 10 TV (using 000s again).

The amount of men reached by the ad types can be stated as (in 000s):

3.5 RAD + 6.25 INT + 8 TV.

Our desired constraint puts those two expressions together: WOMAN >= MEN, which is restated as:

1.5 RAD + 5.75 INT + 10 TV >= 3.5 RAD + 6.25 INT + 8 TV

Rearranging terms to row/column form:

–2 RAD – 0.5 INT + 2 TV >= 0

This is the constraint that represents the first composition requirement stated by the FDA. Note how we took our time, using the appropriate demographic information from the table, and boiled it all down to one very common-looking constraint.

An aside: Note that the Solver is going to favor TVs over the other ad types when it satisfies this constraint. The TV variable has the only positive coefficient in this constraint, and the constraint is a >=. The other two ad types reach more men than women (thus, their negative coefficient), so there will need to be a balance of TV ads in order to meet this requirement. Also, this makes sense if you consider the male/female balance of the ads.

Going back to the LP model, a similar derivation is required for the age demographic requirements.

Going back to the table, the number of people younger than 50 years of age reached by the ads is (again, in 000s):

.75 RAD + 8.5 INT + 8 TV.

The number of >= 50 year olds reached:

4.25 RAD + 3.5 INT + 10 TV.

The expression that relates the two (OLD >= 80% NEW) is:

4.25 RAD + 3.5 INT + 10 TV >= .8 *(.75 RAD + 8.5 INT + 8 TV)

This simplifies to:

4.25 RAD + 3.5 INT + 10 TV >= .6 RAD + 6.8 INT + 6.4 TV

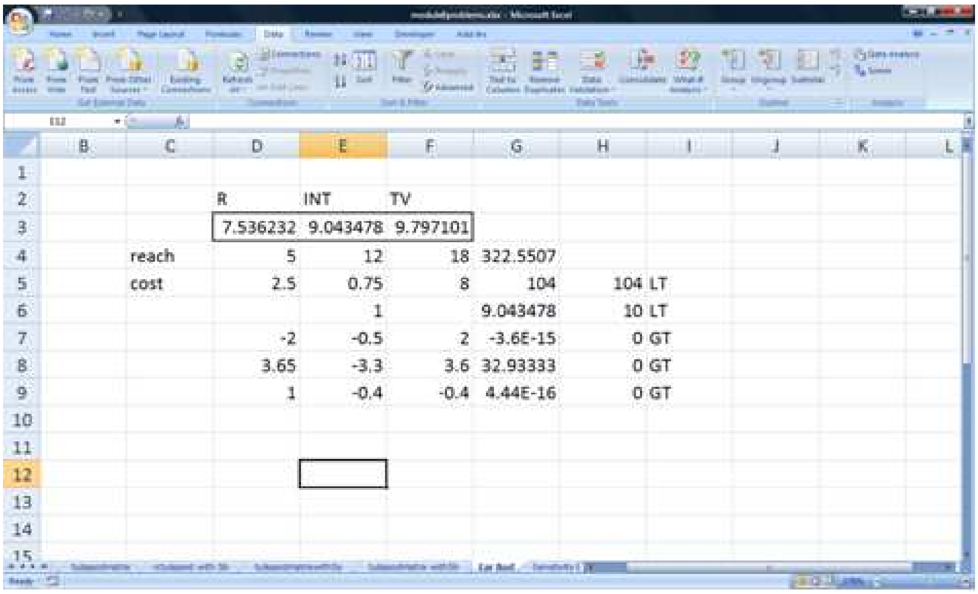
Finally, in row/column form:

3.65 RAD – 3.3 INT + 3.6 TV >= 0.

We are now ready to input our three–decision variable, 5-constraint model into EXCEL.

8.3.3 EXCEL Model for the Ear Bud Safety Campaign

Figure 8.11 shows the row/column implementation and resulting solution. Figure 8.12 is the Solver information. This is a very straight-forward model, so we won't linger on the EXCEL formulas. The order of the constraints: Cost in Row 5, Max Internet ads in Row 6, women/men FDA requirement in Row 7, young/old FDA requirement in Row 8, and the radio 40% requirement in Row 9.



	B	C	D	E	F	G	H	I
1								
2			R	INT	TV			
3			7.536232	9.043478	9.797101			
4		reach	5	12	18	322.5507		
5		cost	2.5	0.75	8	104	104	LT
6				1		9.043478	10	LT
7			-2	-0.5	2	-3.6E-15	0	GT
8			3.65	-3.3	3.6	32.93333	0	GT
9			1	-0.4	-0.4	4.44E-16	0	GT

Figure 8.11

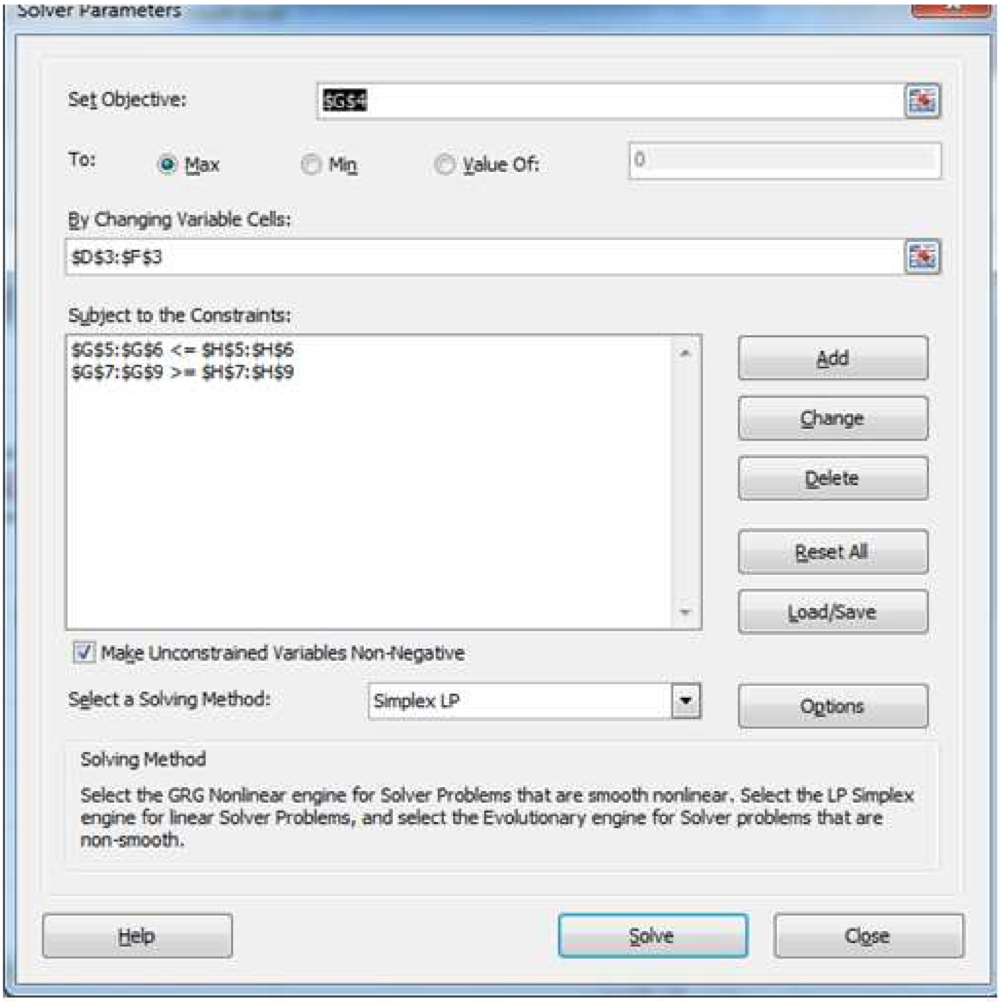


Figure 8.12

Note that there are three binding constraints in the model – cost, the third constraint (the women/men constraint) and the fifth constraint, the radio 40% constraint. The solution values are all "very ugly" in terms of being fractional – we will deal with this in the next module. Needless to say, it is not reasonable to purchase 9.0434 internet ads! There will be much more about integer solutions in Module 9.

To summarize, this model illustrated how to patiently practice *wu wie* in creating LP models. The resulting model for this scenario looks very simple – but effort was expended before any EXCEL input in defining the constraints properly.

So the learning lesson here: Although we can go directly from problem statement to EXCEL model, we must do so without rushing. We don't have to immediately find a home for every number. Practice what the late Coach John Wooden used to teach: “Be quick, but don’t hurry”. Okay, enough preaching and obscure references. I hope you get the point.

Our final example is a classic diet problem with a weighted average constraint, the last modeling-related learning objectives of this module.

Reading Material: 8.4 – Example 3: Brock’s Boar-atorium

8.4.1 Problem Description

Brock’s Boar-atorium is a company that specializes in supporting the fast growing industry of pig racing. Among their many products, they offer a feed supplement specially blended to promote speedy pigs. This supplement consists of corn, oats, flax, and barley blended together.

Each week, the feed manager takes the amount of corn, oats, flax, and barley left over from other uses and blends them together subject to nutritional requirements to make this special supplement. Develop a linear programming model for this month that minimizes costs in creating a mix that satisfies the following requirements. Data are shown below.

1. The amount of fiber in the mix must exceed 520 grams.
2. The “speed” parameter of the mix must average at least 5.
3. Do not exceed the maximum amount of grain available – 125 lb of corn, 150 lb of oats, 200 lb of flax, and 75 lb of barley.
4. The amount of fat in the mix cannot exceed 600 grams.
5. The amount of sodium in the mix cannot exceed 400 grams.
6. Flax cannot compose more than 30% of the total mix.

There is no constraint that suggests a minimum production of mix. The table below shows costs ($), fiber, fat, and sodium (in g/lb) and the speed value (units/lb).

	Corn	**Oats**	**Flax**	**Barley**
Cost/lb	3	2.5	4.2	3.2
Fiber/lb	1.6	1.7	2.2	1.3
Speed/lb	4.5	3.9	7	5.3
Fat/lb	2.1	2.2	2	1.5
Sodium/lb	.9	1.2	1.1	1.4

8.4.2 LP Model Formulation

We will move through the preliminaries quickly and focus on the two more difficult constraints – the weighted average for the "speed" parameter, and the proportion constraint dealing with flax.

How many decision variables? Our model seeks to find the optimal way of mixing four ingredients together to minimize cost. So, the model has four decision variables (CORN, OATS, FLAX, BARLEY), representing the amount of each grain in the mix. The objective function is straight-forward as well:

MIN 3 CORN + 2.5 OATS + 4.2 FLAX + 3.2 BARLEY.

Constraints that implement the requirements 2) and 6) will be postponed to the end of this section. The other requirements are arguably pretty straight-forward.

1) 1.6 CORN + 1.7 OATS + 2.2 FLAX + 1.3 BARLEY >= 520 (FIBER)

3) CORN <= 125

OATS <=150

FLAX <=200

BARLEY <= 75

Four constraints, one for each ingredient.

4) 2.1 CORN + 2.2 OATS + 2.0 FLAX + 1.5 BARLEY <= 600 (Fat)

5) 0.9 CORN + 1.2 OATS + 1.1 FLAX + 1.4 BARLEY <= 400 (Sodium)

Because proportion constraints have appeared numerous times in the past, the flax/30% requirement will be crafted next. We do not know ahead of time (i.e., before solving the model) how much mix our model will make. Algebraically, though, we can specify the total amount of the mix as: CORN + OATS + FLAX + BARLEY. Thus, requirement 5) mathematically is:

FLAX <= .3* (CORN + OATS + FLAX + BARLEY)

This simplifies to (including the rearranging of terms to get to row/column form):

–0.3 CORN – 0.3 OATS + 0.7 FLAX – 0.3 BARLEY <= 0.

After the number of different proportion constraints seen so far, you may have begun to recognize a pattern with the constraint coefficients. If not, that's okay. Just thought I'd mention it if you've discovered a secret code. You may have!

We will next derive the weighted average constraint for the value parameter. First, an average calculation is non-linear. So, from our perspective, it is BAD! (yes, that's a little melodramatic). But we can linearize this easily and implement a useful constraint. There is an average function in EXCEL, but it is a non-linear function that will blow up your model. Please do not go there. It will not work.

Let's take a step back from the LP model for a moment. If we had 10 lb of corn, 5 lb of oats, 10 lb of flax, and 5 lb of barley, what would our average speed value be for that mix? We would calculate it as below:

(4.5 * CORN + 3.9 * OATS + 7 * FLAX + 5.3 * BARLEY)/ (CORN + OATS + FLAX + BARLEY)

The numerator is the "TOTAL SPEED VALUE" for the grain, and the denominator is the "TOTAL GRAIN." TOTAL SPEED VALUE divided by TOTAL GRAIN gives our speed value average.

That would give us an average of:

(4.5 * 10 + 3.9 * 5 + 7 * 10 + 5.3 * 5)/(10+5+10+5) = 161/30 = 5.3666

The average calculation is non-linear because we have decision variables in both the numerator AND the denominator. Multiplying decision variables together and dividing decision variables with each other are definitely non-linear relationships. Adding and subtracting decision variables is a linear operation. So, something like X1*X2 cannot be done in an LP model, just like we cannot divide X1 by X2.

But we can "linearize" requirements related to averages. The requirement (item 2 in our list) asked that the average speed value be at least 5 in our mix. In "words," this would be:

TOTAL SPEED VALUE/TOTAL GRAIN >= 5.

If both sides are multiplied by "TOTAL GRAIN," the expression becomes:

TOTAL SPEED VALUE >= 5*TOTAL GRAIN

This expression is a linear equivalent of the non-linear average requirement, and thus implementable in Solver. Going back to the explicit decision variables, this constraint becomes:

4.5 * CORN + 3.9 * OATS + 7 * FLAX + 5.3 * BARLEY >= 5 * (CORN + OATS + FLAX + BARLEY

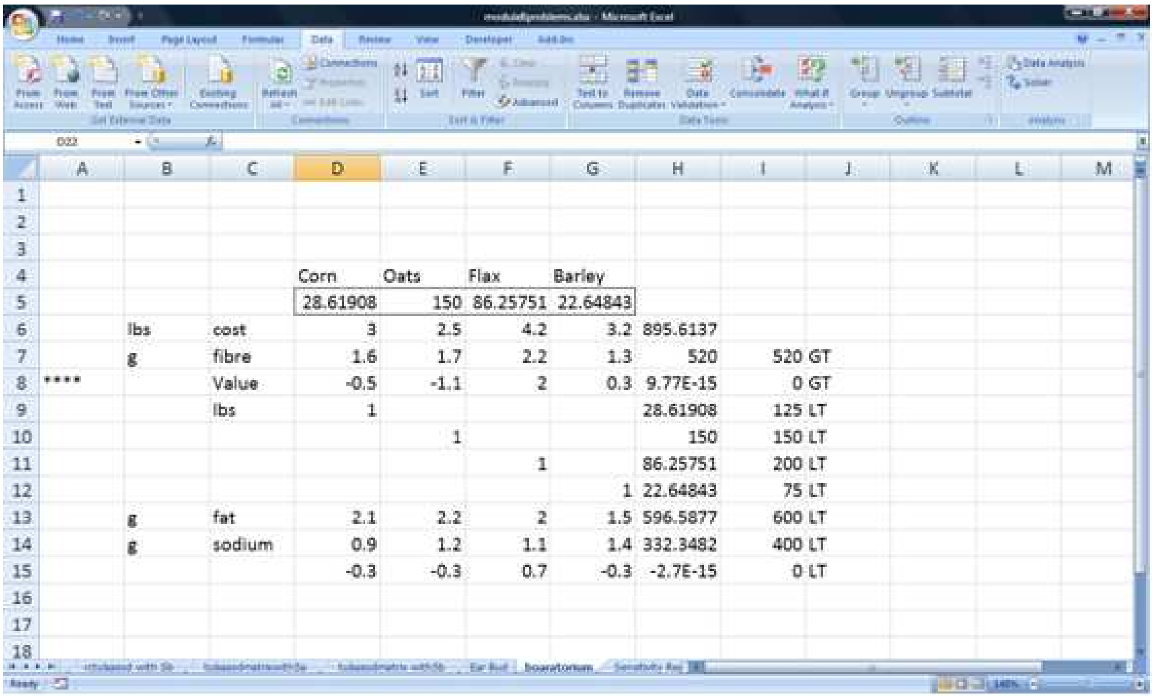
Using our now trustworthy algebra skills, the row/column version becomes:

–0.5 CORN – 1.1 OATS +2 FLAX + 0.3 BARLEY >= 0.

This completes the major learning objective for this ‘diet’ problem – implementing an important non-linear constraint in a linear fashion.

8.4.3 EXCEL Model for Brock’s Boar-atorium

Figure 8.13 is the EXCEL implementation of the LP model. We did not include a Solver dialog box, because this is a pretty standard LP model. The notes in Column J provides the flavor of constraint in case it is not clear. The stars in A8 bring attention to the new type of constraint implemented here in Brock’s problem.



	A	B	C	D	E	F	G	H	I	J
4				Corn	Oats	Flax	Barley			
5				28.61908	150	86.25751	22.64843			
6		lbs	cost	3	2.5	4.2	3.2	895.6137		
7		g	fibre	1.6	1.7	2.2	1.3	520	520	GT
8	****		Value	-0.5	-1.1	2	0.3	9.77E-15	0	GT
9			lbs	1				28.61908	125	LT
10					1			150	150	LT
11						1		86.25751	200	LT
12							1	22.64843	75	LT
13		g	fat	2.1	2.2	2	1.5	596.5877	600	LT
14		g	sodium	0.9	1.2	1.1	1.4	332.3482	400	LT
15				-0.3	-0.3	0.7	-0.3	-2.7E-15	0	LT

Figure 8.13

The solution determines that 28.619 lb of corn, 150 lb of oats, 86.257 lb of flax, and 22.64843 lb of barley should be blended together for the minimum cost of $895.6137. One would expect that fractions are okay with problems such as this, but it may be hard to exactly measure out 28.61908 lb of corn. Again, integers will be discussed soon. Examining the constraints, fiber (yes, I used the English spelling in the spreadsheet – I was going to use it here in the body of the chapter but Word kept underlining it!), the average value requirement of 5, the maximum available amount of oats, and the 30% flax requirement were all binding constraints. Note that for the value and flax ratio constraint, the LHS really does equal the RHS – the small numbers using scientific notation in the spreadsheet are equivalently zero (e.g., 9.77 E-15 in cell H8 is the number 0.00000000000000977). This is just the round-off error one gets when doing decimal arithmetic with a binary computer.

Reading Material: 8.5 – Summary

Three models were presented in this module that expands our modeling ability to include some powerful modeling constructs. We should feel comfortable in varying from row/column format if it appears to be a more efficient and/or effective (and/or likeable!) means of representing a problem in EXCEL. We have seen a case that emphasizes the need to be patient and rather than just fling numbers at EXCEL, execute the human portion of the modeling partnership before model construction. Finally, with the renewed confidence in our algebraic ability, we can take non-linear relationships and make them effectively linear when necessary to implement important modeling requirements.

The practice problems below reinforce the concepts covered in the chapter. Next up will be the first examination of some of the concepts of integer programming.