Module 7: Sensitivity Analysis – Adding

Reading Material: 7.1 – Introduction

This module is a natural follow-up to Module 6, which illustrated how to use EXCEL to solve LP models. Here, we study the SA reports that can be generated from the EXCEL Solver after a model has been analyzed. The SA reports can help provide additional insightful information about our model scenario beyond just the right answer. This module also represents the last foundational building block that students should have before moving on to bigger, more realistic modeling situations. After completing this module, our strategy throughout the rest of the book will be to build on the basic understanding of mathematical programming models and look at other decision situations in practice in which such modeling approaches can add value to the decision makers.

Reading Material: 7.2 – Sensitivity Analysis and the Pursuit of Insightfulness

7.2.1 Strategy of Approach

As you recall, some of the concepts of SA were introduced back in Module 4 when still solving models using graphical methods. This allowed us to see the impacts or insight that we can determine from the SA printouts. It may be extremely helpful to have this linkage from the EXCEL report back to pictures as we proceed in this module. Sometimes I think SA is a little terminology heavy, and although I'll try not to be excessive, having the previous work done in graphical form has been shown to help understanding (and have no adverse side effects!).

7.2.2 Learning Objectives at a High Level

The goal of understanding SA reports is not about precluding us from exploring our LP models, rerunning them with a variety of parameter changes and "what-if" type analyses with differing sets of assumptions. SA is meant to guide our exploration of the LP models, to provide quick and easy information about the robustness of our optimal solution and measures of which constraints are inhibiting and their marginal value.

Recall from before some of the underlying premises of SA. We are considering the impact of modifying only one parameter at a time – the infamous AOMPC conditions (all other model parameters constant). Our SA report assumes there are no integer restrictions on our decision variables (in fact, it cannot be generated when we add integer constraints!). The concept of SA can be used in any model – studying the impact of one model parameter at a time on the model's objective. But the specific SA that naturally "falls out" from the Simplex method through the underlying algebra goes away with integer requirements.

The SA report addresses two main general model questions: How robust is our optimal solution? How valuable are our resources/constraints?

7.2.3 Learning Objectives – The Tale of Two Tables

The first table in the SA report relates to our decision variables. Specifically, the numerical information provided tells us (AOMPC) how the objective function coefficients can vary and NOT alter the originally determined optimal solution. Thus, the information gives us a feel for the robustness of the optimal solution.

The second table in the SA report provides constraint information. We can determine binding constraints (non-zero shadow prices) and determine the potential impact of a change in a constraint on the objective function value. The shadow price term is this per-unit change, something we've also called the marginal value of the constraint. The range information provided in the second table looks similar to that in the first table, but it has an entirely different interpretation.

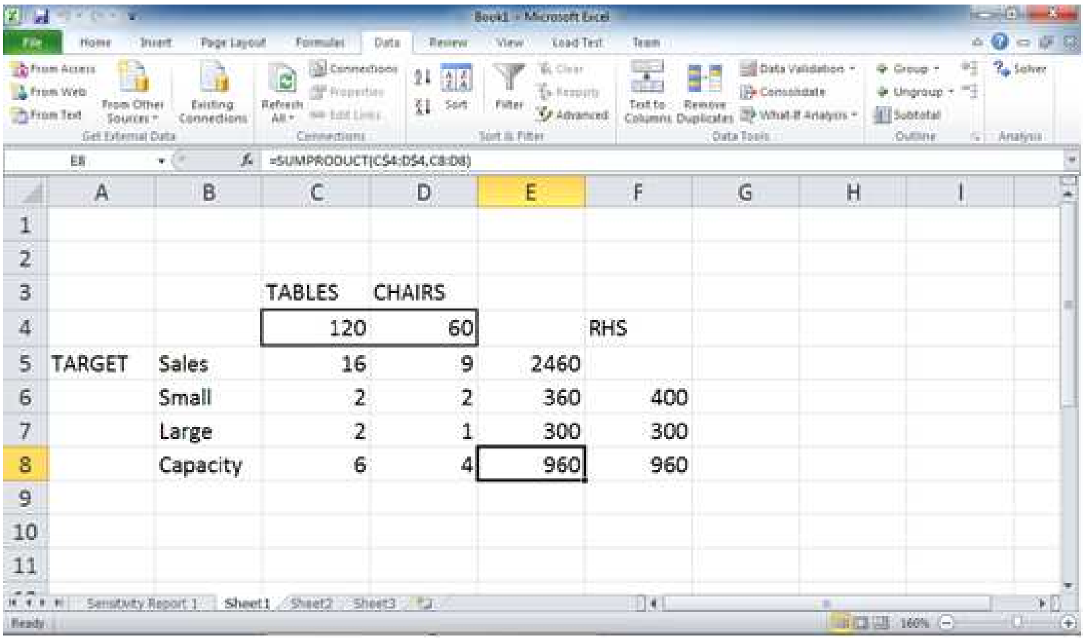
When we say the objective function value changes (this would be, for instance, the total amount of profit, cost, etc. for the optimal solution of the model) because a change in a resource/constraint, because this is the ONLY change to the original model (AOMPC), we also know that THE OPTIMAL SOLUTION OF THE MODEL HAS ALSO CHANGED. By optimal solution change, we mean the values of our decision variables have changed. I have underlined this because this has been the most difficult concept of SA to grasp historically by your colleagues, and I want to change that!

To summarize our introduction, the two SA tables have entirely different interpretations. The examples below further clarify this distinction.

Reading Material: 7.3 – SA Example 1: LEGO Production

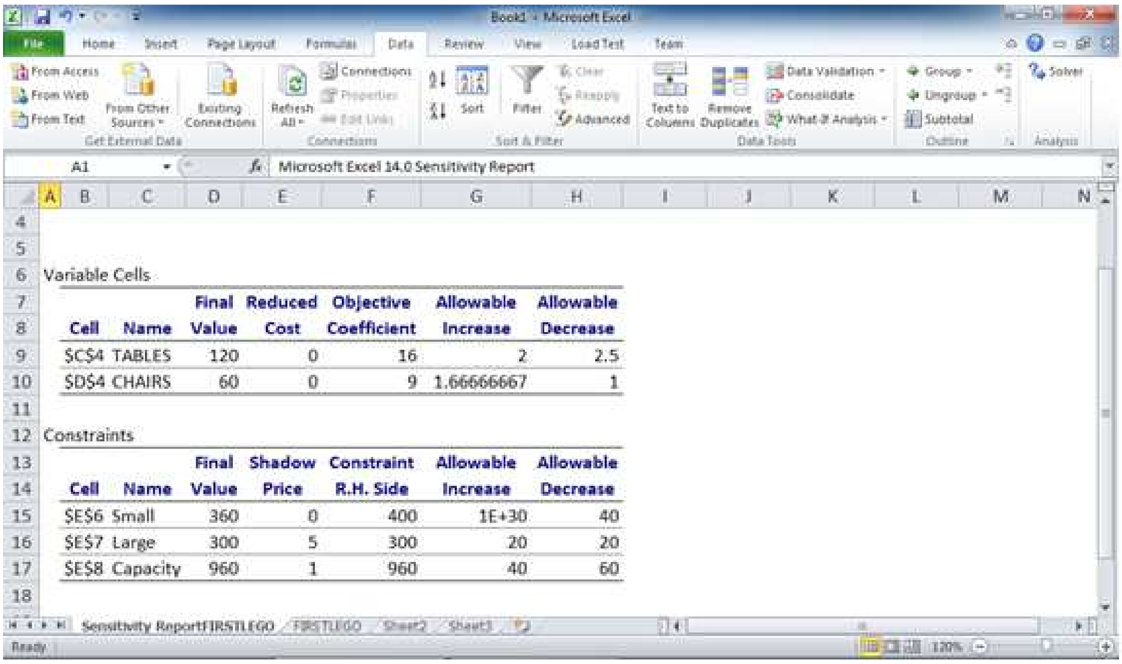
7.3.1. Introduction

Figures 7.1 and 7.2 are the EXCEL/Solver solutions and SA report repeated from the previous module for the basic LEGO production problem.



	A	B	C	D	E	F
1						
2						
3			TABLES	CHAIRS		
4			120	60		RHS
5	TARGET	Sales	16	9	2460	
6		Small	2	2	360	400
7		Large	2	1	300	300
8		Capacity	6	4	960	960

Figure 7.1



Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C4	TABLES	120	0	16	2	2.5
D4	CHAIRS	60	0	9	1.66666667	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E6	Small	360	0	400	1E+30	40
E7	Large	300	5	300	20	20
E8	Capacity	960	1	960	40	60

Figure 7.2

From our results, we can identify the optimal production mix of 120 tables and 60 chairs, providing sales of $2460.

We can also identify our Large LEGO blocks and Capacity as binding constraints. This is known by (at least) two different facts: the left-hand side (LHS) = right-hand side (RHS) in Figure 7.1, the EXCEL spreadsheet. And, in Figure 7.2, the SA report, Large LEGOs and Capacity have non-zero shadow prices. The non-zero shadows prices represent the marginal value of the resource. More about the shadow prices will be discussed below.

We will continue the discussion by following the same path used back in Module 4. There we investigated the following two insightful questions.

a. How can the sales price per table vary and not have an impact on the optimal mix of LEGO furniture? (i.e., how robust is the optimal production mix to variations in the sales price of tables?).
b. What are additional units of large LEGO blocks worth to us? (i.e., what is the marginal value of large LEGO blocks because they are a binding constraint?).

Let's look at each question separately.

7.3.2. The Robustness of the Optimal Solution – The Sales Price of Tables

We are interested in seeing what the SA report tells us about changes in the sales price of tables and how it may (or may not) have an impact on our originally determined optimal production levels of 120 tables and 60 chairs.

This requires examining the first table in the SA report, and specifically the last three columns of information. These columns are "objective coefficient," "allowable increase," and "allowable decrease."

In general, the information here tells us for ONE decision variable, AOMPC, the range that the objective function coefficient can have and NOT have an impact on the originally determined optimal solution. Thus, for any decision variables, there is NO CHANGE in the optimal solution if the objective function coefficient stays between "objective coefficient – allowable decrease" on the lower end and "objective coefficient + allowable increase" on the upper end.

In our case, for tables – the original determined optimal solution is 120 tables and 60 chairs. The original objective function coefficient for tables was 16. The allowable decrease is 2.5, the allowable increase 2. That means that the sales price of tables can vary from \$13.5 (\$16 – \$2.5) to 18 (16+2) and the mix of 120 Tables and 60 Chairs is STILL optimal. Of course, the assumption remains that all other model parameters stay constant (AOMPC).

At a sales price of 13.5 and 18, the exact bounds, there are multiple optimal solutions. The original optimal solution is still optimal, but there is another extreme point that is also optimal (as are all the infinite number of solutions that lie on the line segment connecting the extreme points, visualizing this phenomenon in two dimensions).

For a sales price of tables less than $13.5 and greater than $18, the optimal solution would change. Looking at the SA report only, we would NOT know what the solution would change to without rerunning the LP model. But that of course is fine; the intent of the SA is to give us an idea of how sensitive the optimal solution is to changes in the parameters of the model and nothing more.

So would this range ($13.5 to $18) be considered robust or not robust? It is context dependent, and up to decision maker interpretation whether he or she has developed strong confidence that this solution will be always the best solution if this range is big or if she needs to proceed with caution before implementing the solution or do more model scenario analysis if this range is considered small (making the solution less robust).

One can also calculate this same range for chairs. Of course, this range calculation assumes that the sales price of tables is at its originally stated value of $16 (recall AOMPC – maybe we should play acronym bingo or bet on how many times I'll use that acronym in this module!). So, using the same table, we can determine that the solution of 120 tables and 60 chairs would remain the best production plan for chair sales prices ranging from $8 ($9 – $1) to $10.66 ($9 + $1.66).

So, in summary, for every product (decision variables), we have a quick measure of how sensitive the optimal production mix (solution) is to changes in the objective function coefficients.

An aside: Note that if the profit or sales coefficient of a product changes and our optimal solution includes units of that product, even if the optimal solution values of our decision variables don't change (i.e., the solution stays the same), the objective *function* VALUE will change. Subtle terminology, but trying to help us all keep it sorted out!

7.3.3 – The Marginal Value of Large LEGOs – Shadow Prices and a Different Range

The bottom table of the SA report in Figure 7.2 is relevant for question b) above. Note that the interpretation of the range information here is MUCH DIFFERENT than what was just discussed for the top half of the table.

The shadow price column shows us the specific per-unit impact of changing the RHS of the binding constraint Large LEGO blocks *as long as it is within the allowable decrease/allowable increase ranges stated*. In this case, every one-unit increase in large LEGO blocks will result in an increase of total sales of $5 (increase in the objective function value), up to an increase of 20 units (from the original available resource of 300 up to 320). Beyond that point (320 large LEGO blocks), the report provides no information about what will happen. To find out the impact of having, say, 330 large LEGO blocks available for producing furniture (that would be an increase of 30 units, beyond the allowable increase and, say it with me now – AOMPC!) the model needs to be re-solved with the new RHS data input.

This same analysis is true for every one-unit decrease in the availability of large LEGO blocks – it results in a $5 decrease in the objective function value for up to a decrease in the RHS of 20 units.

In both cases – increases or decreases of the binding resource Large LEGO blocks WITHIN THE ALLOWABLE RANGE, the optimal solution, the numbers of tables and chairs that should be optimally produced, are DIFFERENT. The SA report itself does not tell their new optimal values. But, because we are just considering ONE CHANGE, and that change is the amount of an available resource, and we (correctly) conclude that the objective function value will change, then the only way all of this can happen is if the optimal production levels (the optimal amount of tables and chairs) also change.

This same scenario was examined graphically in Module 4 and is there for "picture back-up." Recall that the specific impacts on the optimal production levels of tables and chairs was found because we looked at the detailed graph. In general, if a specific impact on solution values is desired, there is nothing keeping us from rerunning the model. Our learning objective in this module is different from finding explicit impacts – it concerns gaining additional insight from the SA report with minimal calculations to guide and direct further analysis.

One more comment on the "AOMPC" thread. Consider an explicit increase (10 units) in the availability of large LEGO blocks, AOMPC. From the SA report, we know that this will lead to a $50 increase in total sales (the +10 falls within the allowable increase, thus the shadow price of $5 applies). Question: To achieve this $50 increase in sales, wouldn't we also have to increase capacity, our other binding constraint, to obtain this? Where else would we put the additional tables and chairs?

This type of question came up a number of years ago from one of your colleagues – a very good question. But the answer is "No, we would not also have to increase the capacity. In fact, the assumption is of course that it stays the same!"

The marginal impact of +$5 for every unit increase in large LEGO blocks considers the trade-offs and the relationships in the model concerning the existing constraints (including the 960 limitation on capacity) and other model parameters. To achieve this gain of $50, the optimal number of tables and chairs will change, but the new optimal solution will stay within the 960 capacity limit. The extra 10 large LEGO blocks will be used in the best way possible, but it is likely that we will produce fewer of one product, and more of the other, to achieve this net gain.

In fact, if you solve the model with the new RHS of 310 for large LEGO blocks, the new optimal solution is 140 tables (20 more than before) and 30 chairs (30 less than before). The sales gain is 16 × 20 = 320 and the sales loss 9 × 30 = 270, so a net gain in sales of $50. Capacity is still binding at 960 units as well. But, to begin with, the shadow price assumed Capacity stays the same anyway. There is no information available that gives us a hint on what happens if we change TWO model parameters at the same time . . . we would just rerun the model in a different kind of "what-if" analysis scenario.

For completeness, let's examine the other two constraints in the model: Small LEGO blocks, a non-binding constraint, and capacity, another binding constraint. Note that Small LEGOs have a shadow price of 0. We have extra left over, so adding more small LEGO blocks will just be adding to our surplus and will be worth no addition sales. The 1E+30 (in scientific notation) shown in the Allowable Increase column for small LEGO blocks says that the shadow price of 0 is applicable for up to an INFINITE increase in the amount of small LEGO blocks. That makes sense because we already have leftover small LEGO blocks.

Also, the allowable decrease of small LEGO blocks is 40 units – the exact amount of small LEGO blocks unused in our optimal production mix. If we had a decrease in available small LEGO blocks greater than 40 units, the shadow price of 0 would no longer apply and the model would be re-executed to assess the impact. Beyond the reduction of 40 units, the feasible region would likely change, small LEGO blocks becoming a binding resource, and therefore all the model relationships be altered.

Capacity has a marginal value (shadow price) of 1, which is applicable for changes from 900 units (960 – allowable decrease of 60) to 1000 units (960 + allowable increase of 40). Although a little bit of comparing apples to oranges, large LEGO blocks have a larger marginal impact than units of capacity, and that could be worth noting by the production manager/modeler whose job is to constantly seek ways of increasing profitability.

7.3.4 Summarization of LEGO Production SA Printout

After looking at this first example, here is a summary of the kind of insightful information that the two SA tables can provide.

For the top table: Will solution values of the decision variables change or not change given a specific objective function modification? Put another way – how sensitive is the optimal solution to possible changes, errors, or modifications to objective function coefficients?

For the bottom table: What are the binding restrictions/constraints in our LP model? What are the restrictions/constraints/resources marginal value? For how large of ranges are those marginal values relevant? And, if specific changes are investigated with a binding constraint and we assert that this will have an impact on the objective function value, this implies that the optimal solution values of our decision variables HAVE CHANGED!

The initial production problem for tables and chairs was a maximization problem with only less than or equal to constraint. Our next example, a slight modification and amalgamation of the three LEGO production problems, will illustrate SA reports for minimization problems with mixed constraints. Additionally, the mysterious "Reduced Cost" column in the decision variable table will also be clarified.

Reading Material: 7.4 – Example 2: The Three LEGO Furniture Product Example

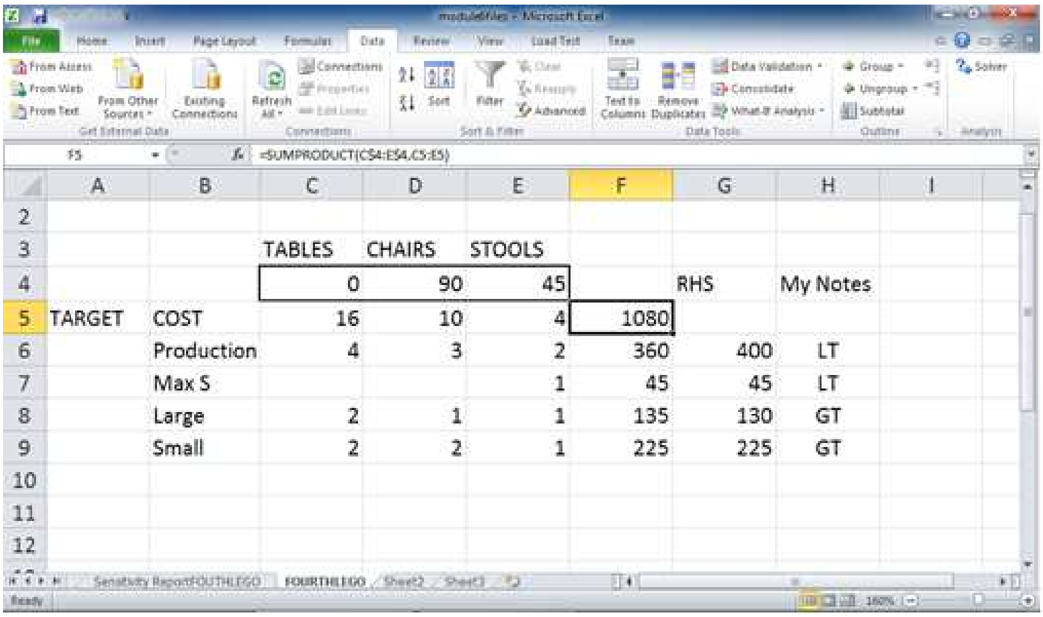
Consider a modified LEGO furniture production problem that includes a third product, stools. Here is a quick description of the problem scenario.

7.4.1. The Problem Statement

Problem: Create a minimum cost production plan for three LEGO furniture products: tables (made of two large LEGO blocks and two small LEGO blocks), chairs (made of one large LEGO and two small LEGO blocks) and stools (made of one large LEGO and one small LEGO blocks). Tables cost $16 each, chairs $10 each, and stools $4 each. No more than 45 stools can be produced because of lack of demand. A capacity constraint that limits the total amount of LEGO blocks to be used in production to no more than 400. Obviously, tables use four LEGO blocks, chairs three LEGO blocks, and stools two LEGO blocks. A minimum number of large LEGO blocks (130) must be used in production, as well as a minimum number of small LEGO blocks (225) as well. Find the optimal production plan in producing the three products at a minimum cost.

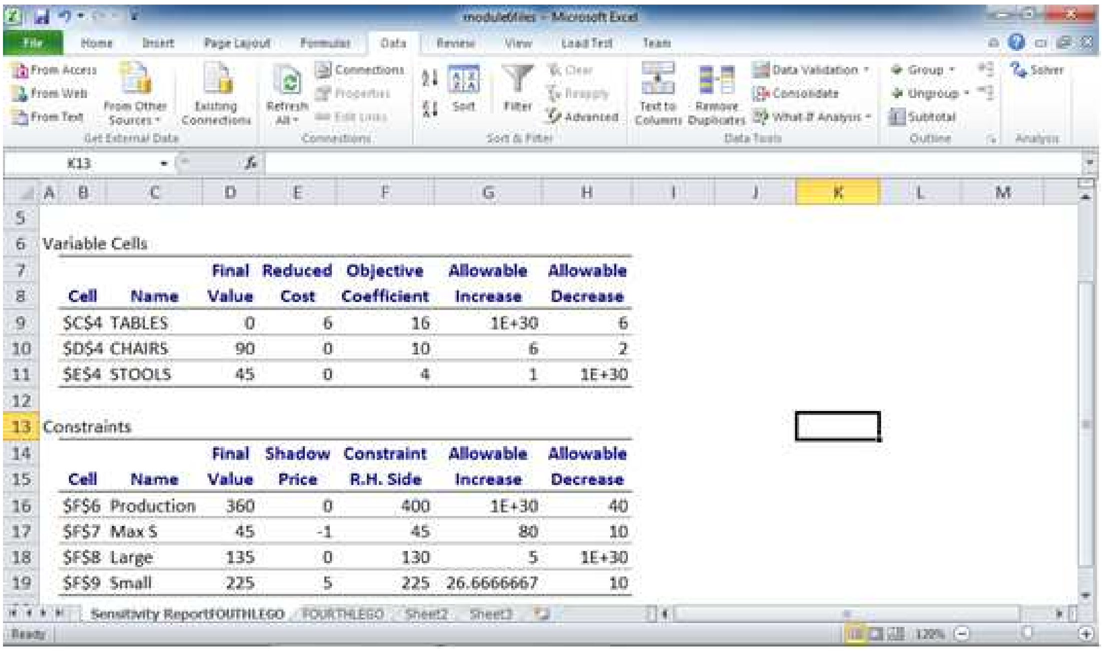
7.4.2 The EXCEL LP Model and SA Report

The EXCEL LP model is shown in Figure 7.3. The SA report that results from solving the model is shown in Figure 7.4.



	A	B	C	D	E	F	G	H
2								
3			TABLES	CHAIRS	STOOLS			
4			0	90	45		RHS	My Notes
5	TARGET	COST	16	10	4	1080		
6		Production	4	3	2	360	400	LT
7		Max S			1	45	45	LT
8		Large	2	1	1	135	130	GT
9		Small	2	2	1	225	225	GT
10								
11								
12								

Figure 7.3



Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C4	TABLES	0	6	16	1E+30	6
D4	CHAIRS	90	0	10	6	2
E4	STOOLS	45	0	4	1	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F6	Production	360	0	400	1E+30	40
F7	Max S	45	-1	45	80	10
F8	Large	135	0	130	5	1E+30
F9	Small	225	5	225	26.6666667	10

Figure 7.4

There are three decision variables (C4:E4) in the model. There are four constraints, Rows 6 through 9. The first two constraints are (<=), the second two (>=). The LHS SUMPRODUCTS are in column F, the RHS values in column G. Thus, the constraints relate column F (LHS) to column G (RHS). The target cell (MIN) is F5.

7.4.3 Basic Solution Findings

The optimal production plan includes producing only 90 chairs, 45 stools, and no tables. The minimum cost of the production plan is $1080. Two of the four constraints are binding: the constraint that allows no more than 45 stools to be produced (Row 7) and the minimum number of small LEGO blocks that must be used in the production plan (Row 9). There are 40 unused units of capacity (360 vs. 400), and our production plan uses five more large LEGO blocks than the minimum required (135 vs. 130).

7.4.4 Unique SA Report Observations

Notice that producing tables was found to be undesirable in our optimal production plan. A further examination of the top table in our SA report gives us a little insight into the degree of the tables unattractiveness. The “Reduced Cost” entry for tables has a value of 6 (as does the allowable decrease column entry – and these are the same not by coincidence!). This indicates that the cost of tables would have to be reduced by at least $6 (made more attractive by $6) for the optimal solution to include at least some tables (i.e., alter the production plan). This is the same thing that the Allowable Decrease is telling us too!

In summary, we see that the Reduced Cost column is only non-zero when we have a decision variable with a “0” optimal solution value. The Reduced Cost value is taken from the Allowable Decrease in a minimization problem (amount by which the objective function coefficient would have to be improved for it to have a non-zero production level), and if taken from the Allowable Increase column in a maximization problem (amount by which the objective function coefficient would have to be improved for it to have a non-zero production level). In the big scheme of things, this is a minor issue, but I wanted to clarify this information that appears in the SA report. The Reduced Cost information is poorly named and redundant, but . . . (fill in the blank from previous module!).

On the second table in the SA report, note the shadow prices of our two binding constraints (MAX Stools and Small LEGOs). The upper bound for stools has a shadow price of (–$1), and the small LEGO blocks minimum requirement has a shadow price of +$5. It is interesting that they differ in signs and are of different sign from the previous model! Further investigation is warranted.

The (–$1) shadow price for the less than or equal to constraint for stools says the following: Within the allowable range (which as you can see is an increase of 80 units), every additional stool that we allow our model to make has a –1 unit impact on the objective function. Thus, it decreases the objective function when we loosen this constraint. Because this is a minimization problem, the shadow price of –$1 tells us we can reduce our cost by one unit for every additional stool we allow (up to the increase of 80). Likewise, if we tighten the constraint (lower the RHS), we would INCREASE (–1*–1) the objective function value, which would make the solution worse (because it is a minimization problem). So, the negative shadow price makes perfectly good sense in this case. C

Now consider Small LEGO blocks, a binding greater than or equal to constraint. If we increase the RHS, we are making that constraint tighter. The +$5 shadow price tells us that for every unit increase (obviously within our allowable range), we increase cost by $5. The reverse is true as well – if we decrease the RHS requirement, we are loosening the constraint and we would see the cost decrease (+5*–1) by $5. This remains consistent with our minimization objective.

As you’re thinking about this, always trust EXCEL’s SA report. The sign of the shadow price is always correct. If you increase the RHS of a binding constraint and that constraint has a positive shadow price, the objective function value will go up. That will be good or bad depending on the model objective. Likewise, if you decrease the RHS of a binding constraint and that constraint has a positive shadow price, the objective function will go down. Again, that is good or bad depending on model objective.

Should I repeat the above paragraph for the reverse (a negative shadow price?). Okay, here it is. If you increase the RHS of a binding constraint and that constraint has a negative shadow price, the objective function value will go down. That will be good' or 'bad' depending upon the model objective. Likewise, if you decrease the RHS of a binding constraint and that constraint has a negative shadow price, the objective function will go up. Again, that is good or bad depending on model objective.

The two examples studied so far have illustrated SA report contents in general terms, not with specific decision maker questions in mind. The third example is meant as an in-class exercise to illustrate how SA can be used to answer specific questions that confront the decision maker in interpreting LP model results.

Reading Material: 7.5 – Example 3: Buffet's Army Revisited

The third example in this module was a practice problem at the end of Module 5. The problem description is repeated here.

7.5.1. Buffet's Army

The Willy Buffett Foundation (WBF) each summer hires local high school (HS) and college (CO) students with local ties to staff its many camp and outreach activities around the upper Midwest. For the upcoming summer, based on cost and demand for student worker help, it can target to hire up to 600 HS students and CO students combined. The Foundation will NOT target to hire more than 600, but could target to hire fewer depending on other factors (described below).

Because the Foundation has been doing this for a number of summers, they have developed a metric that measures the "attractiveness" of the HS and CO hires. This metric combines productivity and cost. Higher levels of this measure (call it a Buffetmeter) is better than lower levels. For this summer, assume that the Buffetmeter for a HS student is 300 and the Buffetmeter for a CO student is 700.

Because of local arrangements with the area HSs, the WBF has agreed that no more than 40% of the total students hired this summer can be college students (COs). Additionally, history has also shown that some students who agree to come invariably never show up. This dropout rate tends to be 5% for the HS students and 10% for the CO students. WBF wishes to target students such that the total expected number of dropouts is no greater than 35.

The WBF wishes to find the optimal number of HS and college students CO to *target to hire* such that the overall Buffetmeter measure is maximized and that the hiring constraints of the situation described above are met.

Note that you are deciding on how many students of the two types to TARGET to hire. Dropout information constrains the decisions of the models, but you are still deciding how many people to TARGET, not how many come.

7.5.2 The LP Algebraic Model

The following LP model represents the above scenario, using HS to represent the number of high school students to target and CO the number of college students to hire.

MAX 300 HS + 700 CO

ST	HS + CO	<= 600	(Overall limit)
	0.05 HS + 0.10 CO	<= 35	(Dropout Limit)
	0.40 HS – 0.60 CO	>= 0	(40% requirement written as >=)
	HS, CO	>= 0	

Note that the third constraint was written as a greater than or equal constraint – it could also have been implemented (still in row/column form) as –0.4 HS + 0.6 CO <= 0.

7.5.3 The EXCEL Spreadsheet/Solution

Figure 7.5 is the EXCEL spreadsheet that implements this LP model. Figure 7.6 is the corresponding Solver model. Figure 7.7 is the SA report derived from this solution.

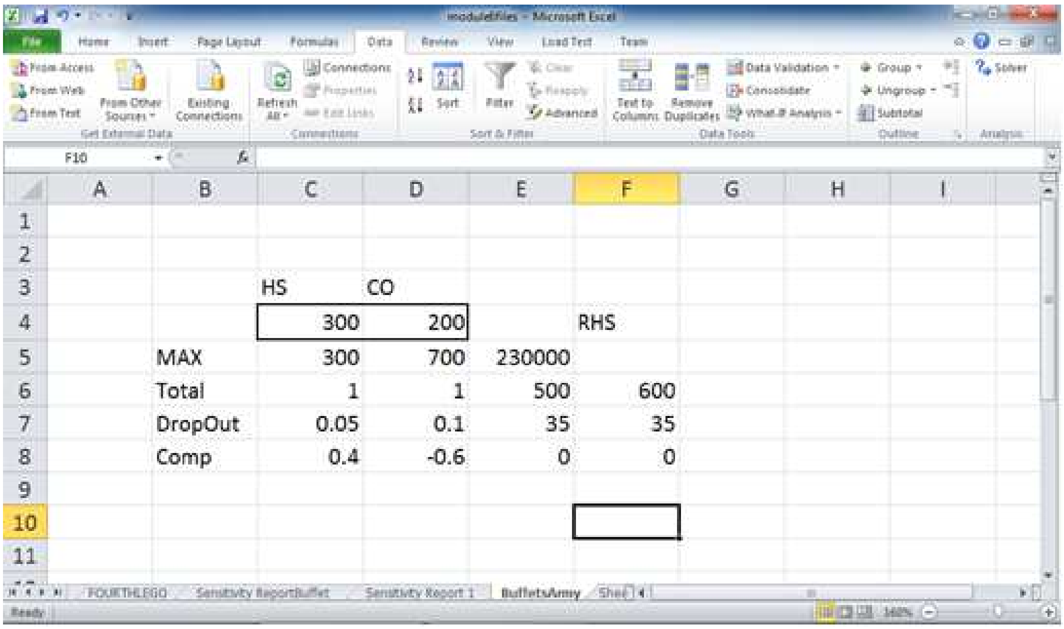


Figure 7.5

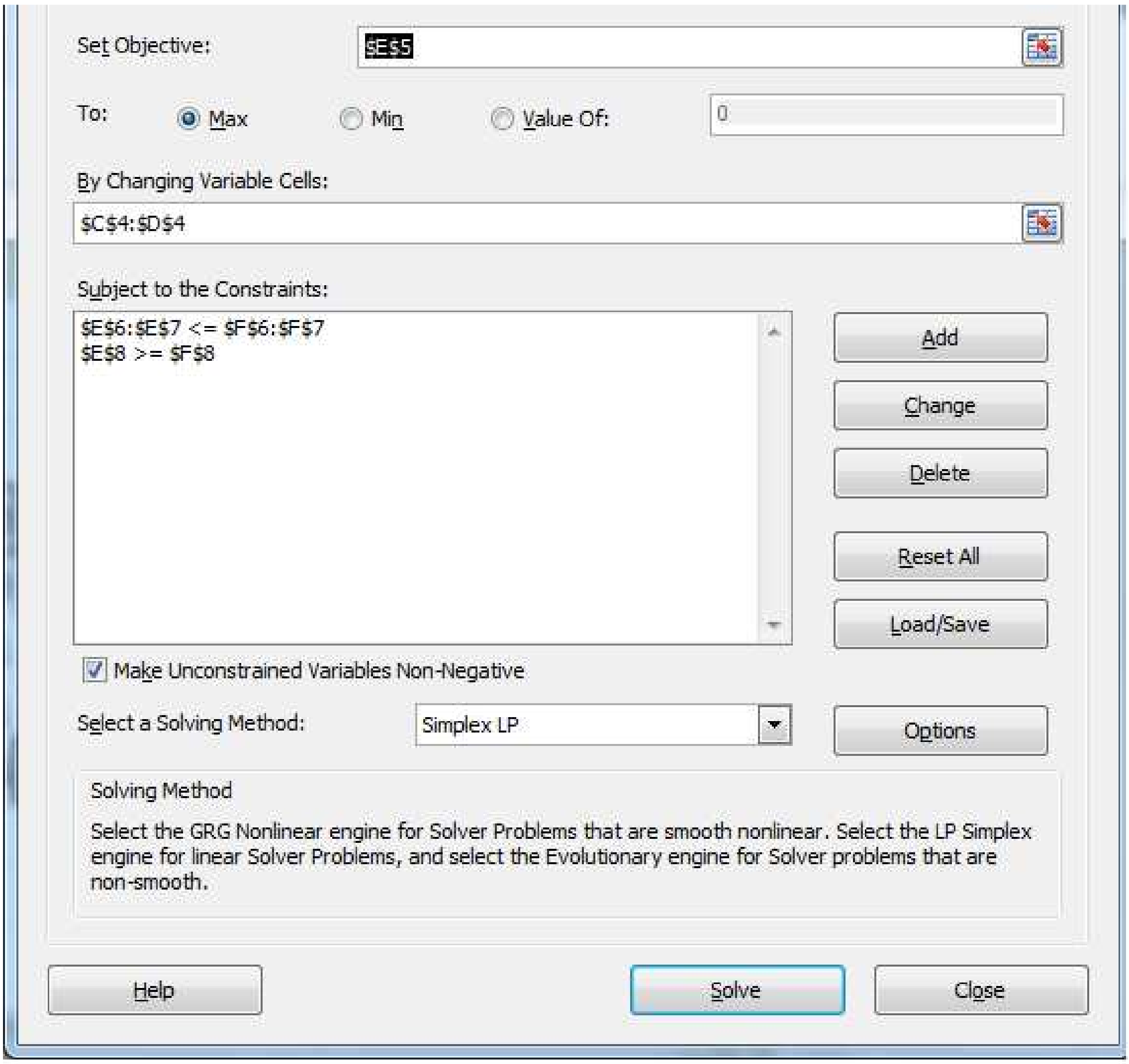
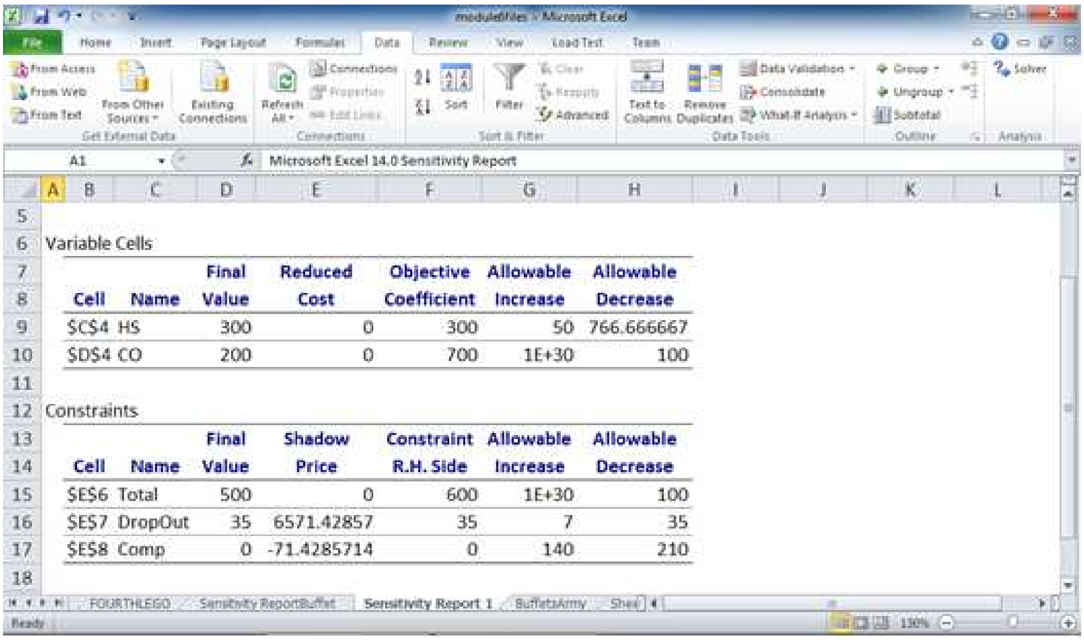


Figure 7.6



Microsoft Excel 14.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C4	HS	300	0	300	50	766.666667
D4	CO	200	0	700	1E+30	100

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E6	Total	500	0	600	1E+30	100
E7	DropOut	35	6571.42857	35	7	35
E8	Comp	0	-71.4285714	0	140	210

Figure 7.7

7.5.4 The "In-Class" Activity

Consider the following questions about the Buffet's Army LP model and solution. Answer all questions utilizing the EXCEL solution and the SA report. Treat each question independently and, where appropriate, with all other model parameters constant (AOMPC).

Question 1

What would the best mix of targeted students be if we assumed that high school students added NO value to the workforce? A little extreme, but let's consider this extreme case. That means that their Buffetmeter value was 0. Explain.

Question 2

Would our optimal mix of targeted students change if the Buffetmeter value for the college students was actually half as large (350 instead of 700)? Explain.

Question 3

Discuss the impact on the optimal measure of Buffetmeters if we lowered the allowable amount of dropouts to 28 (from 35). Would this have an impact on the optimal mix of targeted students? Explain.

Question 4

Does the 40% limitation on college students have any impact on the optimal mix of students? Explain – in general terms.

7.5.5 The "In-Class" Activity – Responses.

Below are suggested responses to the questions asked above.

Question 1

What would the best mix of targeted students be if we assumed that high school students added NO value to the workforce? A little extreme, but let's consider this extreme case. That means that their Buffetmeter value was 0. Explain.

Response: The original solution to the LP model indicated that the optimal targeted mix of students was 300 HS and 200 CO. This provided a weighted Buffetmeter value of 230,000. The Dropout and the Ratio constraint were binding. We could tell this because the constraint LHS = RHS and because they both had non zero shadow prices in the SA report.

Our model used 300 as the Buffetmeter value for HS. This question asks us to consider the impact on the optimal student mix if that value changed to 0. This is a decrease of 300 units.

We consult the first SA table. We see that for the HS variable, the Allowable Decrease value is 766.67. This means that the originally determined optimal mix of students (HS = 300, CO = 200) remains optimal for Buffetmeter values of down to (300 – 766.67) or – 466.67 for the HS folks. This seems crazy! That means that our optimal solution does not change even if the HS kids add no value (or negative value).

Why is this the case? It seems counterintuitive. Consider though that the CO students add a tremendous amount of value in terms of Buffetmeter scores. However, we can only have up to 40% of our students be CO age based on our agreements with the local high schools. So, by the way in which we've assessed the students, we still come out ahead (positive values) by trading off even "negative" HS students against the high-performing CO students. The point where this trade-off is no longer worth it is when the negative Buffetmeter score for HS students goes lower than 466.67 (i.e., -466.67).

To summarize, even if HS had a 0 Buffetmeter score, our optimal solution would remain the same (300 HS, 200 CO).

Question 2

Would our optimal mix of targeted students change if the Buffetmeter value for the college students was actually half as large (350 instead of 700)? Explain.

Response: This requires a similar analysis to Question 1. The top table in SA is relevant. We are looking at a reduction in half of the Buffetmeter score for CO decision variables. This would be lowering it by 350 (from 700 to 350).

We can see that the allowable decrease value for CO is 100. The proposed reduction of 350 exceeds our allowable decrease. Thus, the optimal mix of students WILL CHANGE.

Using SA reports, we do not know what the new solution will look like – we just know it will be different. Likely, we will see fewer college students targeted in the optimal solution, but that is just a guess.

Question 3

Discuss the impact on the optimal measure of Buffetmeters if we lowered the allowable amount of dropouts to 28 (from 35). Would this have an impact on the optimal mix of targeted students? Explain.

Response: The dropout constraint is binding. The shadow price (found in the second table) is 6571.42857 for this constraint, implying that if the RHS of this constraint goes up by one unit, the marginal increase in the objective function (Buffetmeters) would be 6571.42857. Likewise, a one-unit decrease would have the same magnitude of effect, but it would lower the objective function value (Buffetmeters would go down by 6571.42857).

We have been asked to consider a decrease in the allowed dropouts by seven. That falls within the Allowable Decrease range shown in the second table for Dropout (the allowable decrease is 35). So the shadow price is applicable for the entire seven-unit decrease.

The net impact on the objective function value is then (–7) * 6571.42857 = –46,000. So, reducing the allowed number of dropouts by seven would cause a decrease of 46,000 in Buffetmeters in our optimal targeted hiring mix.

Does our optimal mix change? Of course it does. We make only one change to the model – allowed dropouts. And we conclude that this will cause a drop of 46,000 in our objective function. Thus, the optimal mix had to be altered, more than likely targeting more HS students and fewer CO students – because CO students are a hirer dropout risk, and this would also explain the drop in the Buffetmeters.

Question 4

Does the 40% limitation on college students have any impact on the optimal mix of students? Explain – in general terms.

Response: This constraint (the composition/ratio constraint) is a binding constraint (LHS=RHS, shadow price non-zero). Thus, it obviously helps define the feasible region and helps determine the optimal mix of students.

Shadow prices for ratio constraints are not easily interpreted. We will not even try. Suffice it to say that this constraint plays a significant role in the optimal student mix. Relaxing the constraint, you would expect the objective function value to increase and vice versa. This is about all that one can say about the ratio constraint from the SA report.

Reading Material: 7.6 – Summary

This concludes the module on the automatically generated SA reports in the Solver. These reports help the decision maker gain additional insight into the problem being modeled. The reports do NOT preclude the decision maker's inalienable right to manipulate model parameters and run multiple scenarios under different assumptions, etc. SA is just a small piece of the puzzle that helps the LP user be a better decision maker.

The practice problems serve a couple of purposes. Some of them require you to formulate an LP model before answering some SA-related questions. Thus, those problems have a mixture of learning objectives – both LP skills and SA skills. I tried to select fairly straight-forward LP models to use in this case, because obviously the SA questions are dependent on getting the LP model correctly modeled.

I have also included some more strictly SA questions that provide to you an SA report (and perhaps a spreadsheet model) and ask you questions about the report.

Keep in mind the learning objective here is to reach a good, clear understanding of the SA reports. Executing models over and over again is okay to check your conclusions, but the real purpose is to know what the SA reports are telling us and what they can't tell us. The overarching objective is of course gaining modeling insight.

This module also represents the last of what I term the foundational modules of the course. As with home building and life in general, to successfully rise up, one needs a solid foundation. Now that we have it, we will seek to build quickly and reach to the stars in terms of modeling expertise and real-world applicability. Go, Woody!