



Decision Tree Basics

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Model Essentials – Decision Trees

- ▶ **Predict new cases.**
- ▶ **Select useful inputs.**
- ▶ **Optimize complexity.**

Prediction rules

Split search

Pruning



Model Essentials – Decision Trees

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- ▶ **Select useful inputs.**
- ▶ **Optimize complexity.**

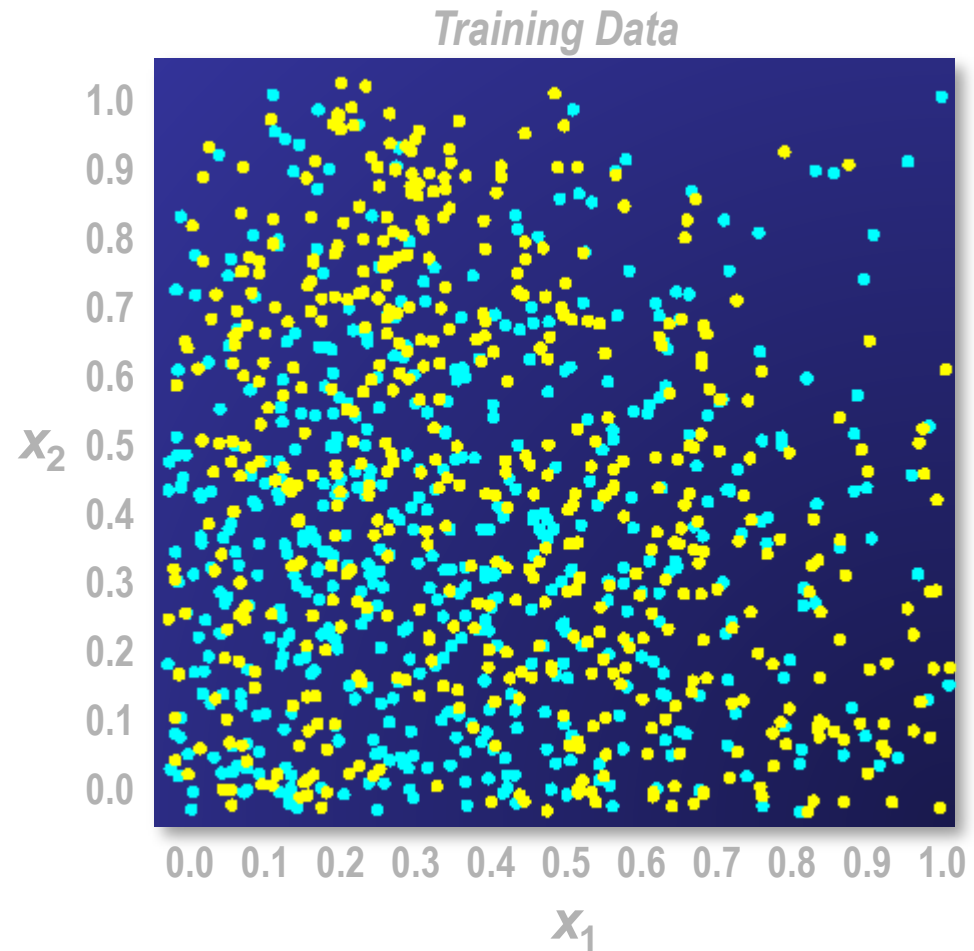
Prediction rules

Split search

Pruning

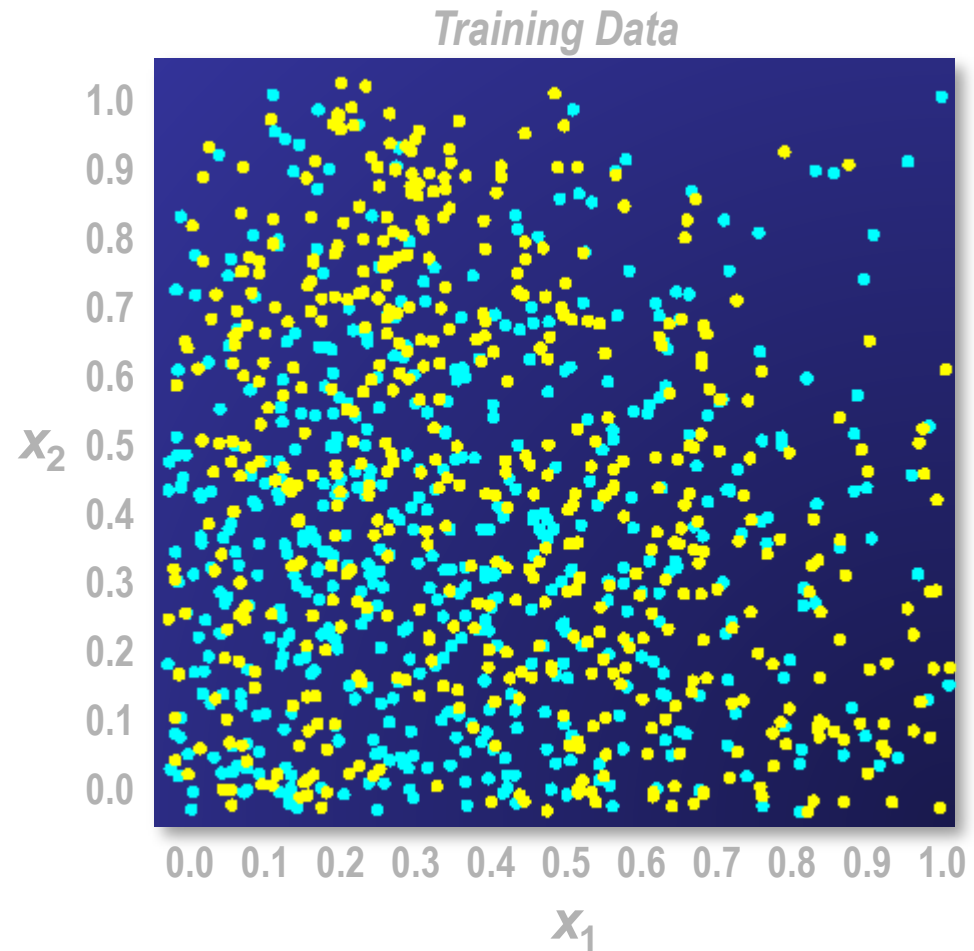
Simple Prediction Illustration

**Predict dot color
for each x_1 and x_2 .**

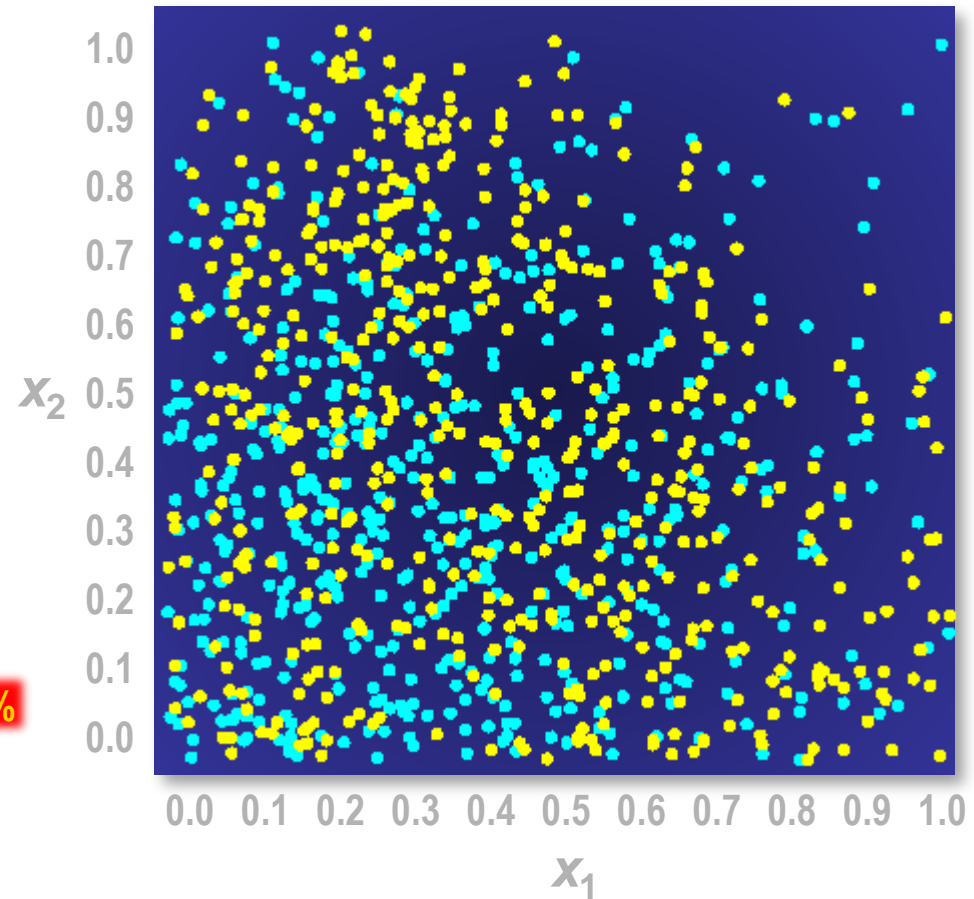
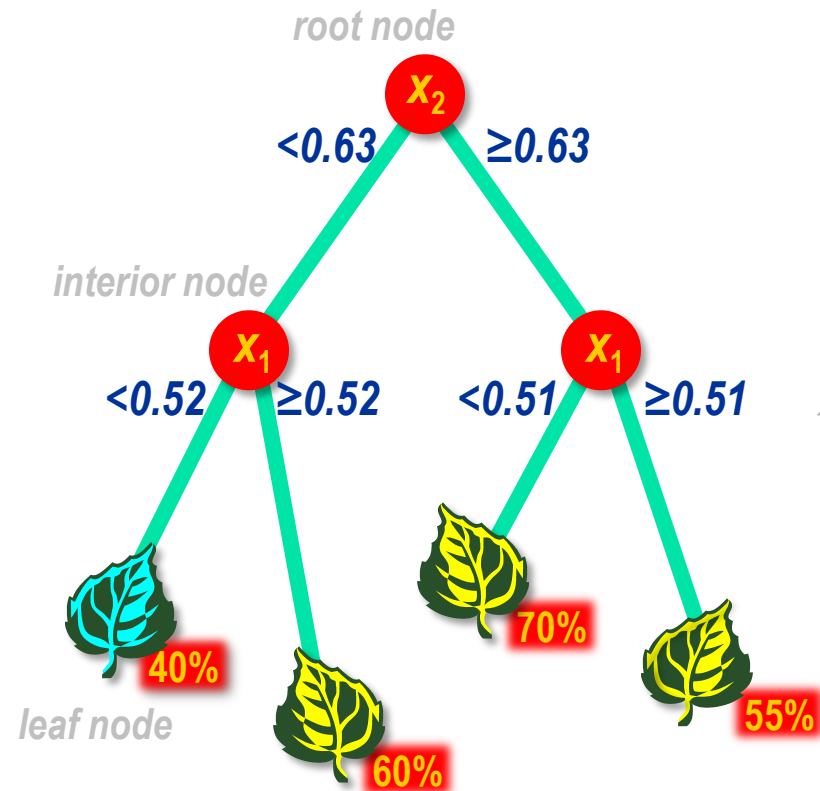


Simple Prediction Illustration

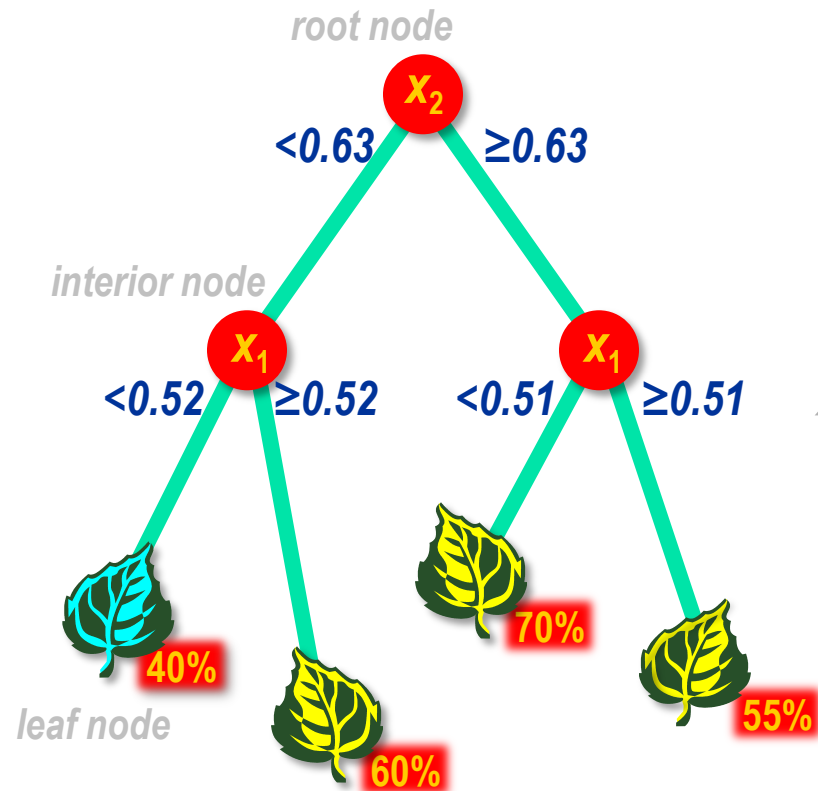
**Predict dot color
for each x_1 and x_2 .**



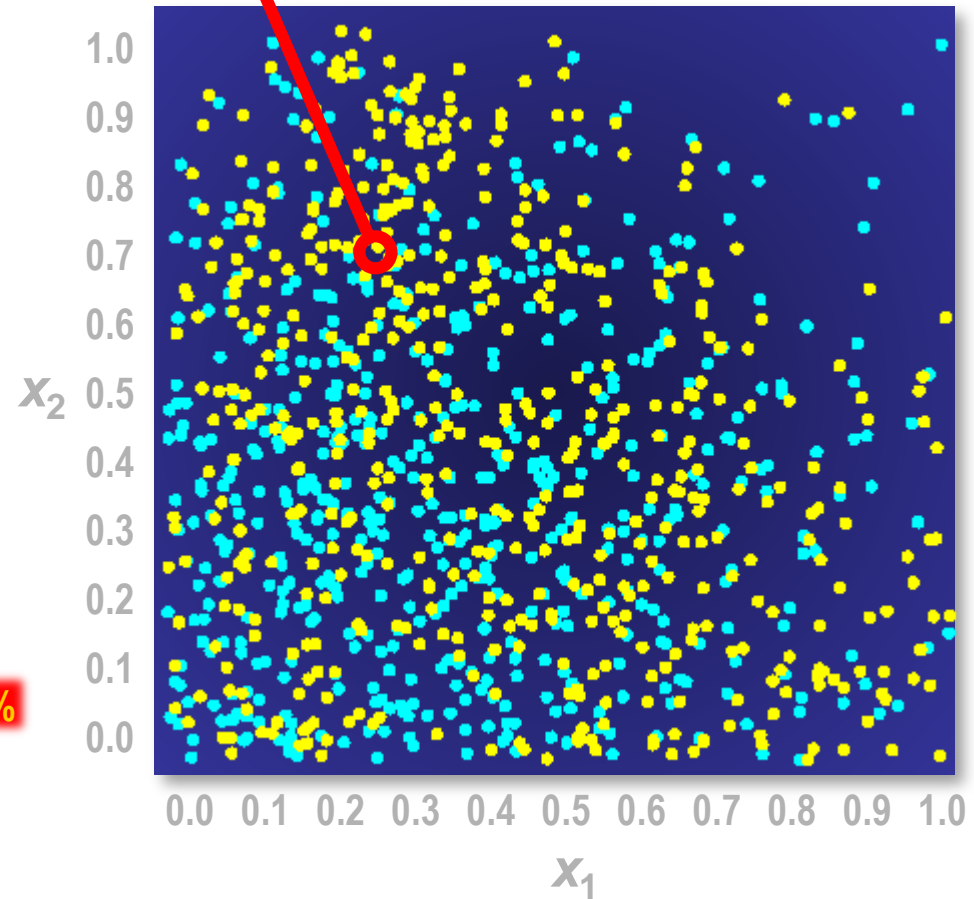
Decision Tree Prediction Rules



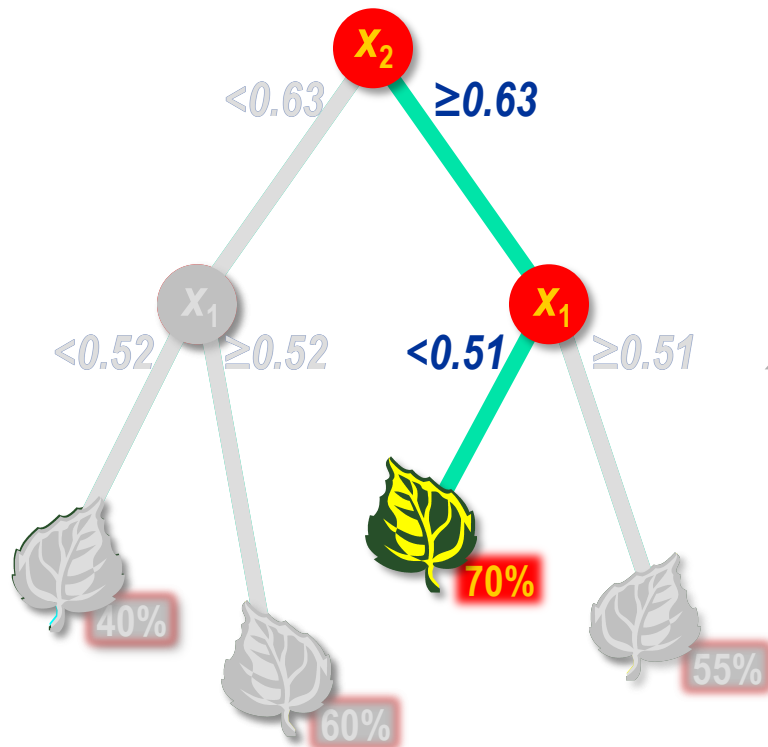
Decision Tree Prediction Rules



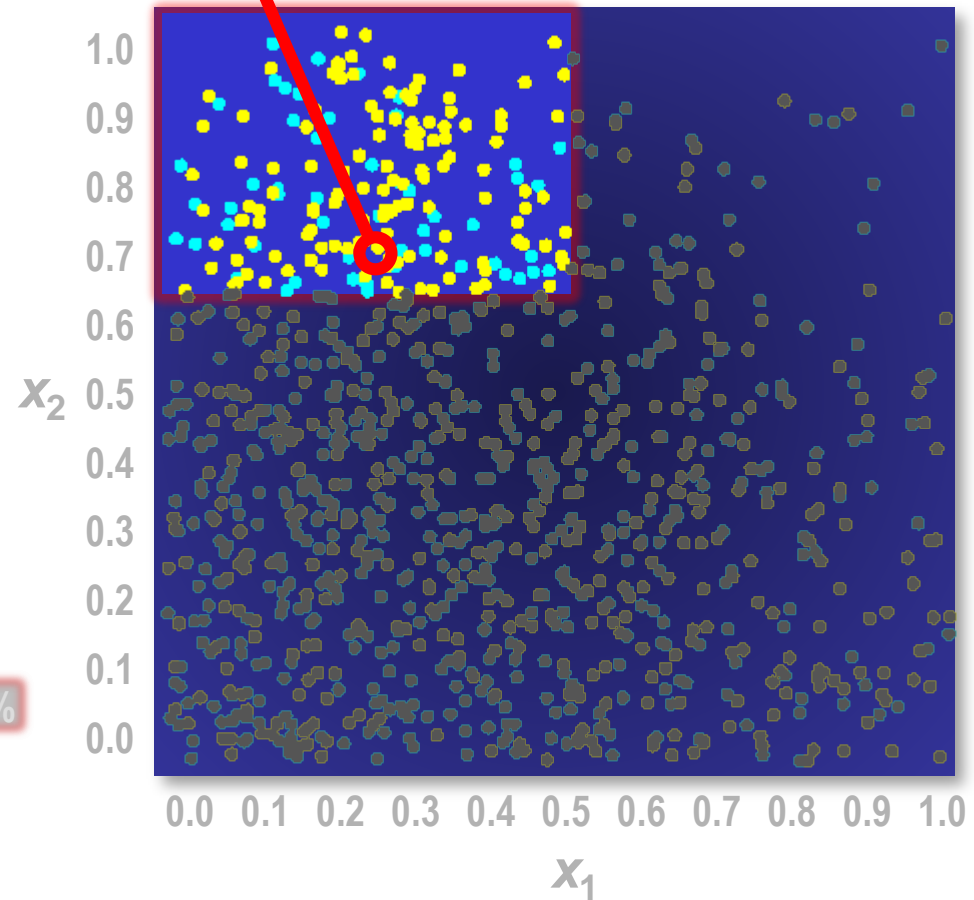
Predict:



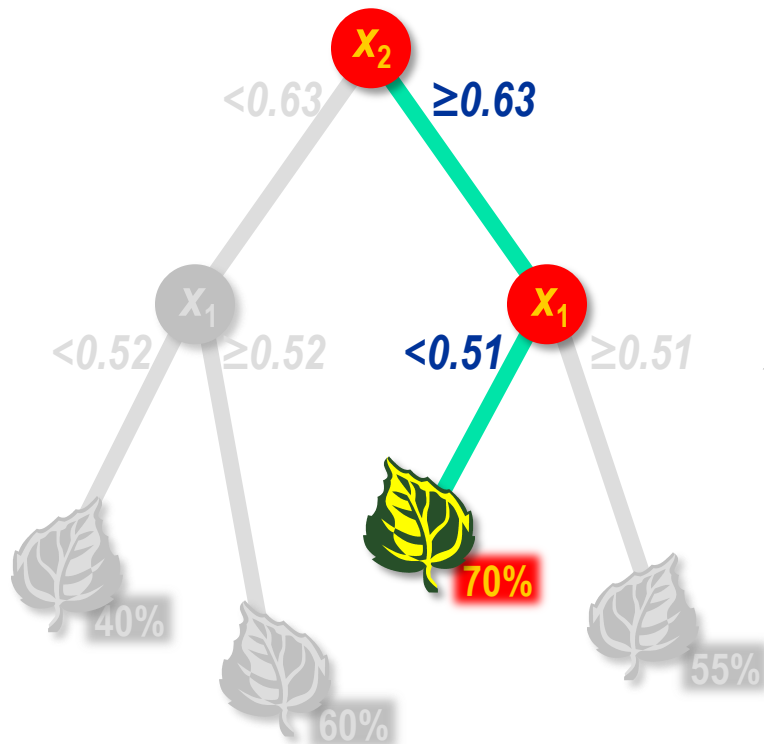
Decision Tree Prediction Rules



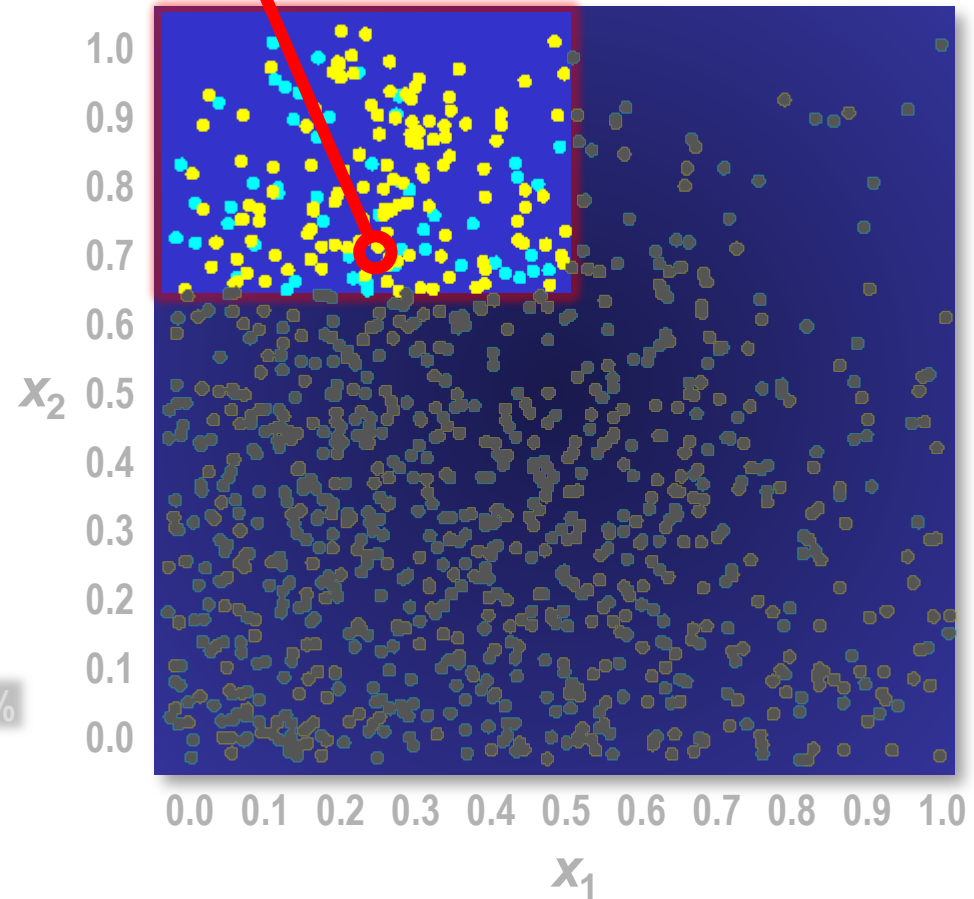
Predict: Decision = ●
Estimate = 0.70



Decision Tree Prediction Rules



Predict: Decision = ●
Estimate = 0.70



Model Essentials – Decision Trees

▶ **Predict new cases.**

Prediction rules

▶ Select useful inputs.

Split search

▶ Optimize complexity.

Pruning

Model Essentials – Decision Trees

- ▶ **Predict new cases.**
- ▶ **Select useful inputs.**
- ▶ **Optimize complexity.**

Prediction rules

Split search

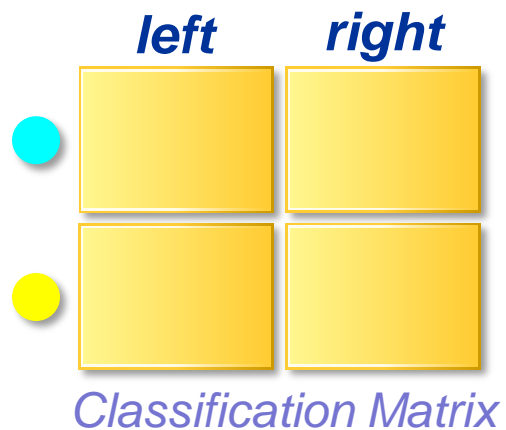
Pruning



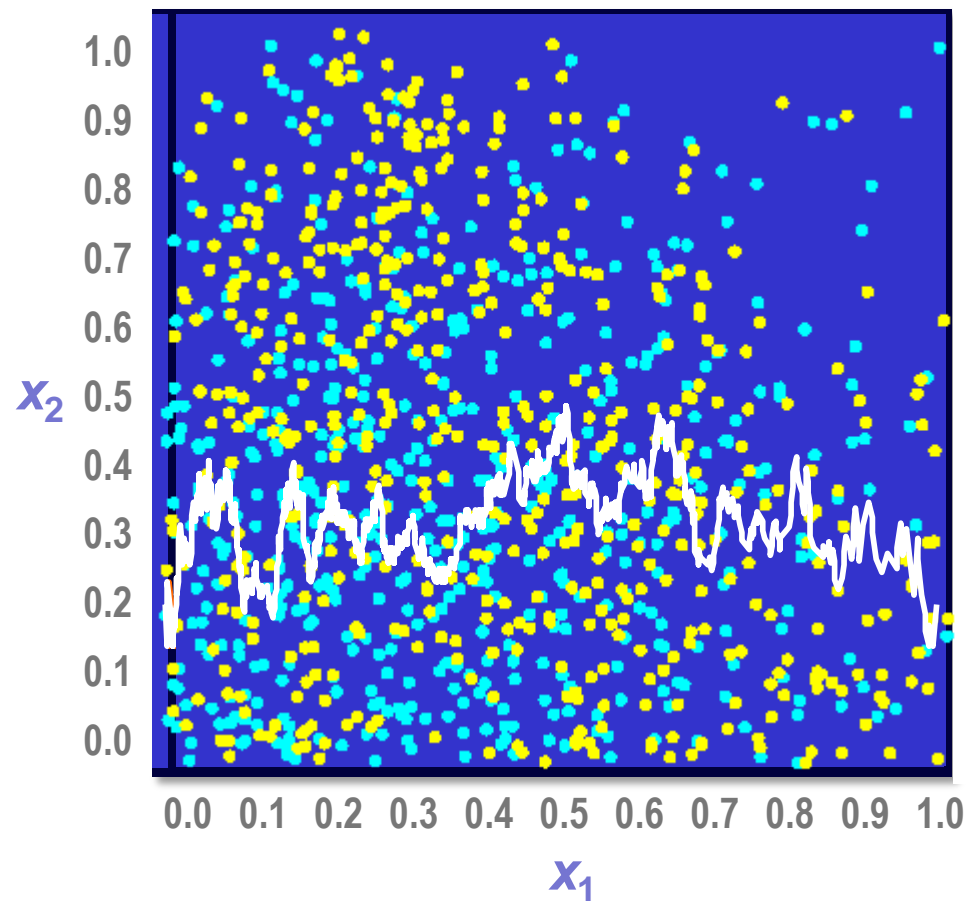
Model Essentials – Decision Trees

- ▶ Predict new cases. Prediction rules
- ▶ **Select useful inputs.** **Split search**
- ▶ Optimize complexity. Pruning



Decision Tree Split Search



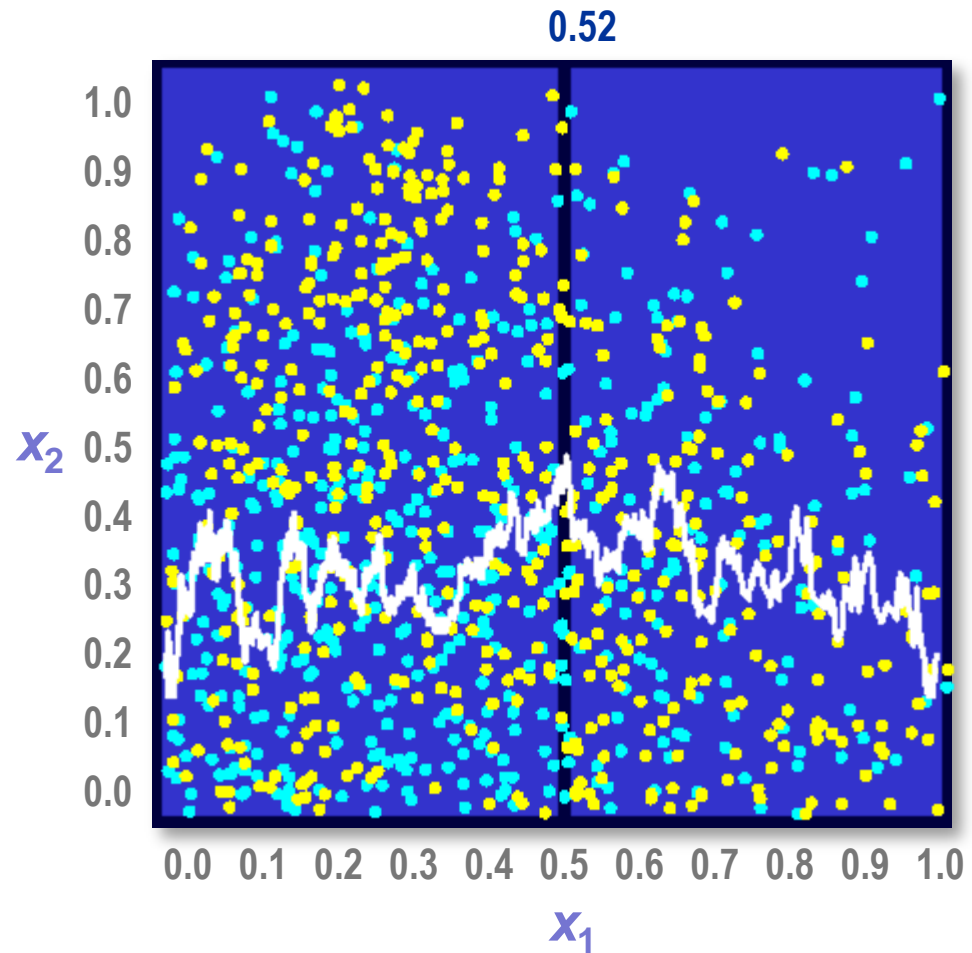
Calculate the ***logworth*** of every partition on input x_1 .



Decision Tree Split Search

	<i>left</i>	<i>right</i>	
	53%	42%	max logworth(x_1) 0.95
	47%	58%	

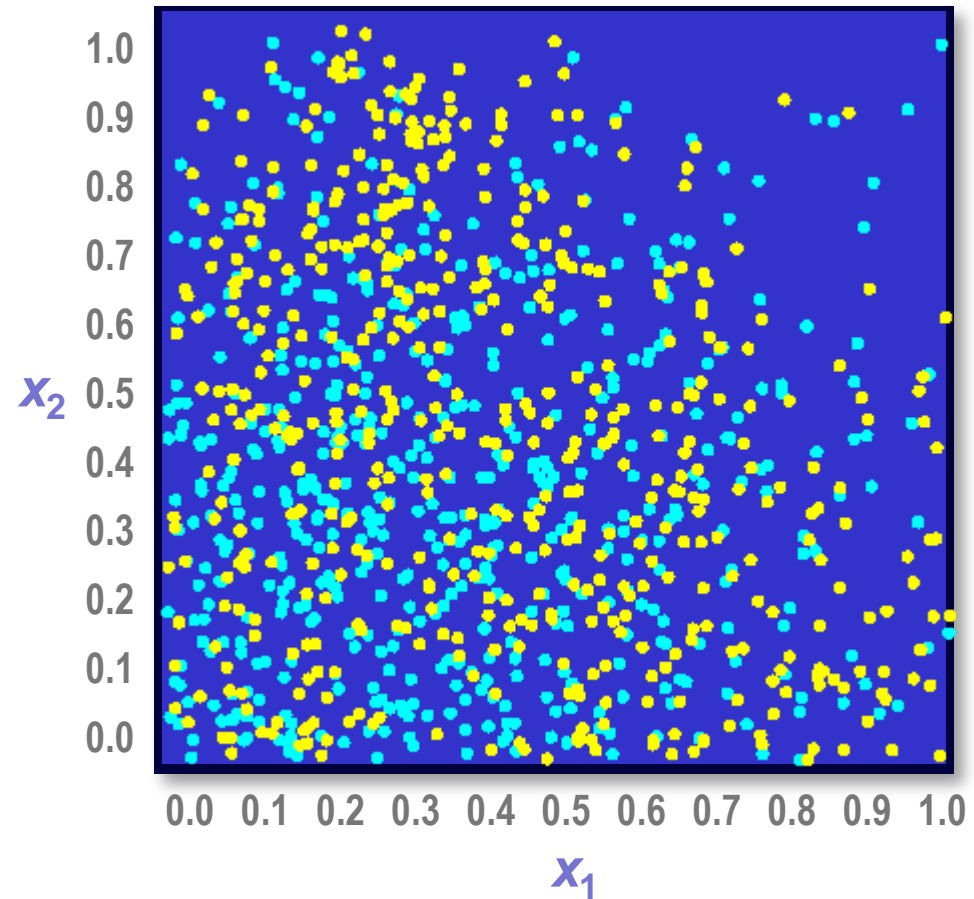
Select the partition with
the maximum *logworth*.



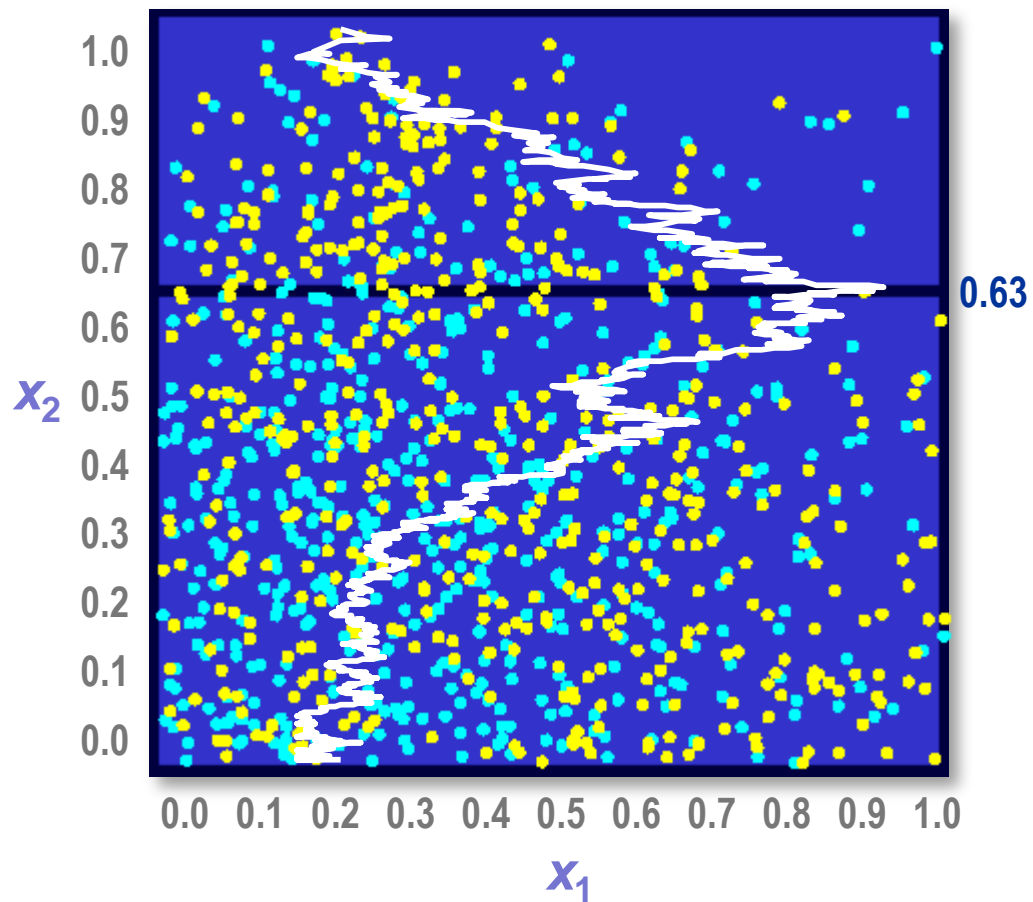
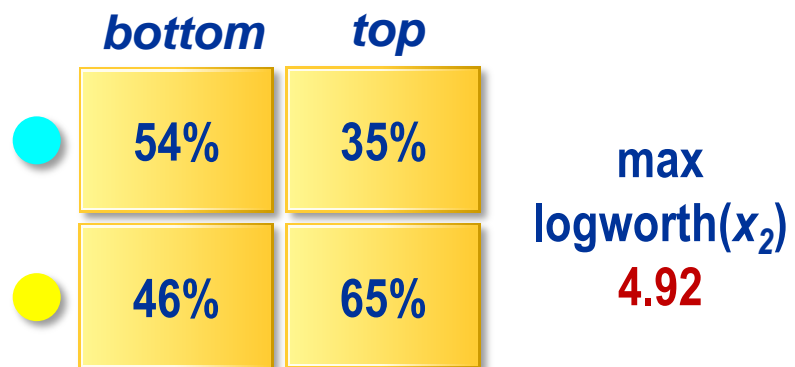
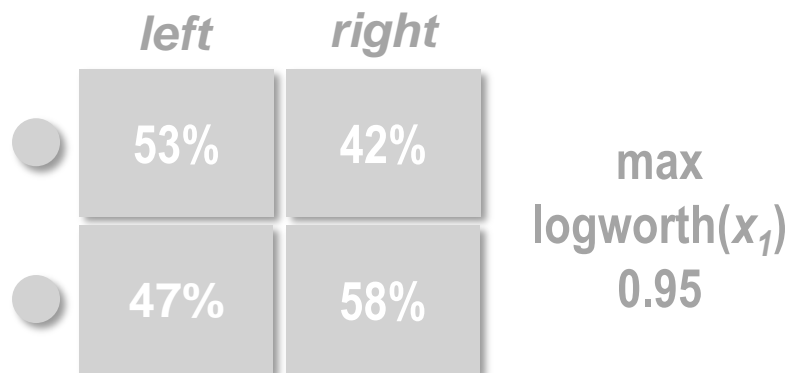
Decision Tree Split Search

	<i>left</i>	<i>right</i>	
●	53%	42%	max logworth(x_1) 0.95
●	47%	58%	

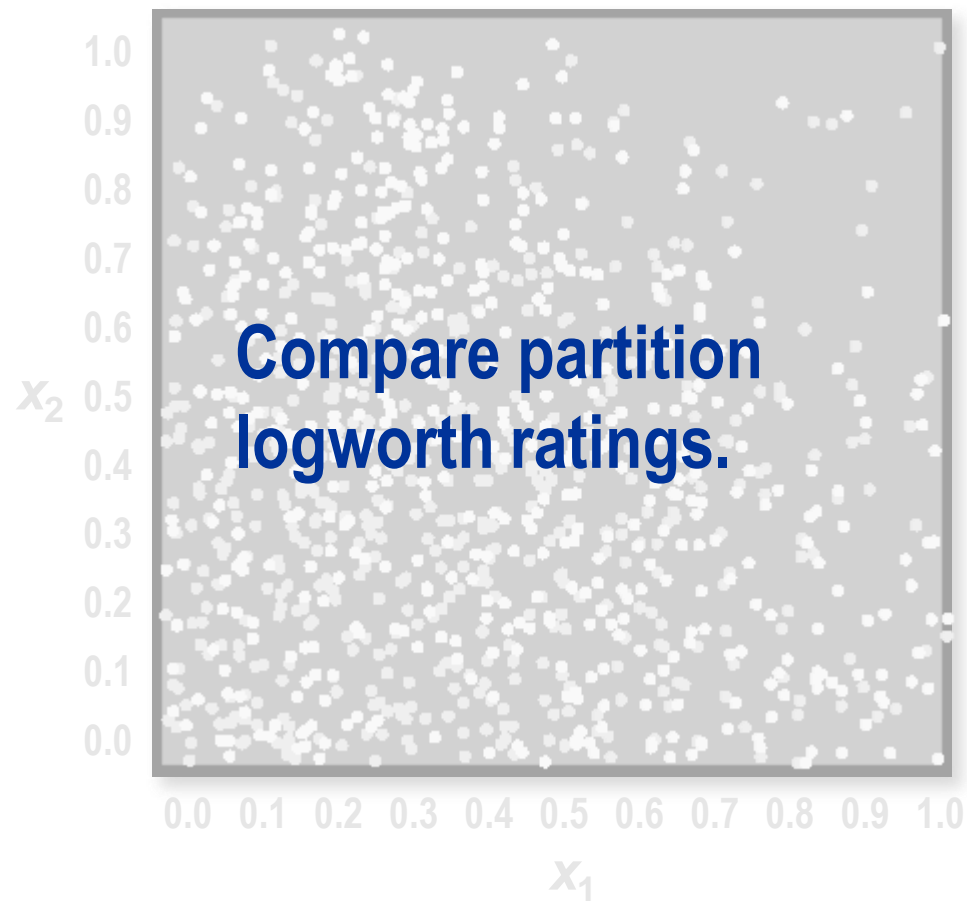
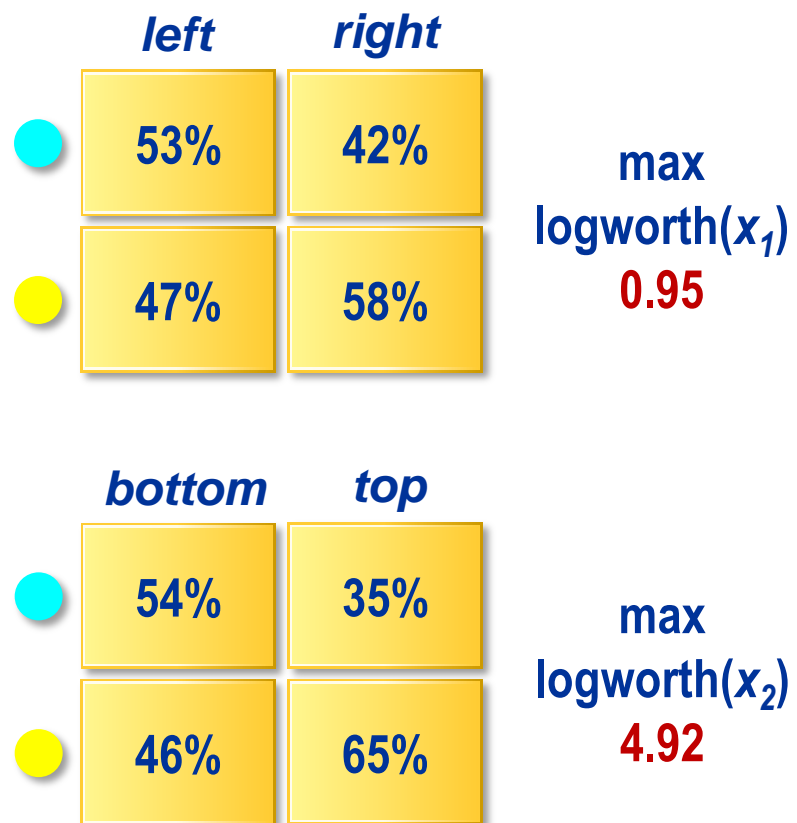
Repeat for input x_2 .



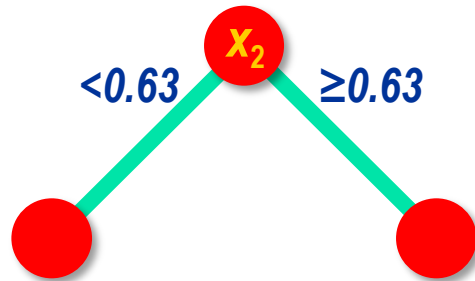
Decision Tree Split Search



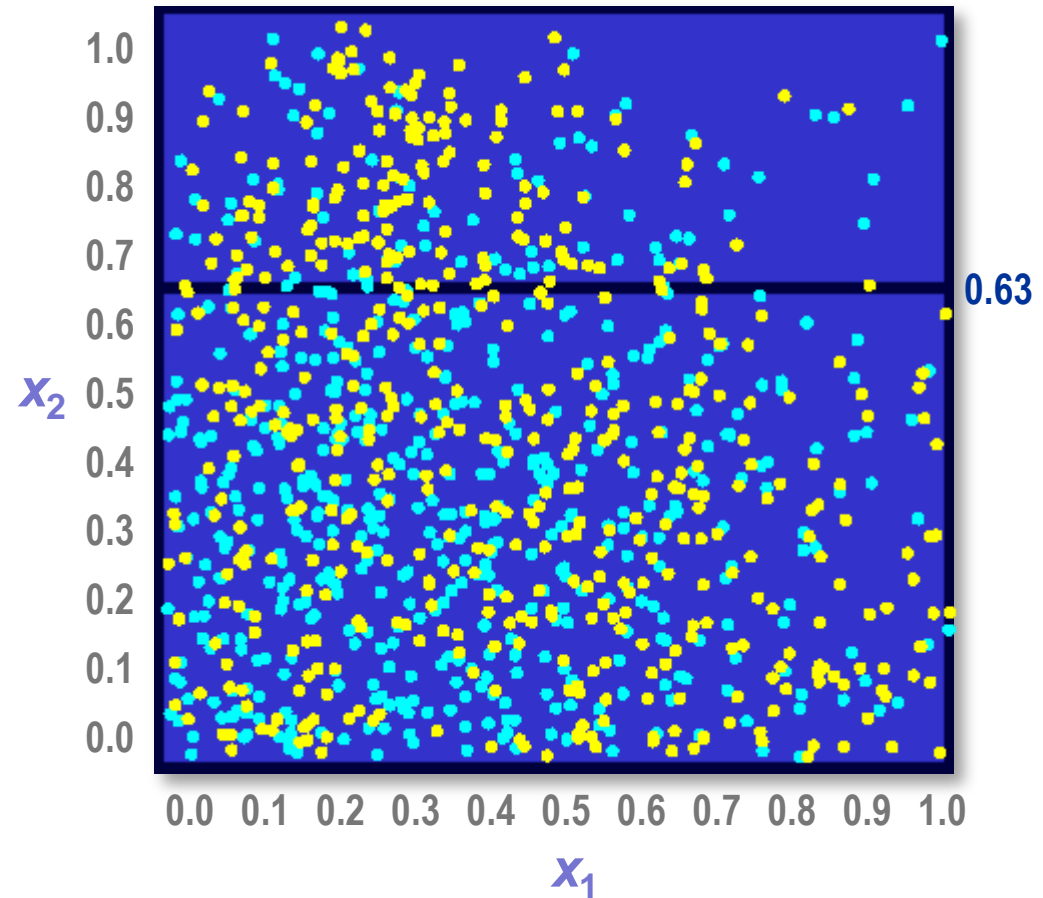
Decision Tree Split Search



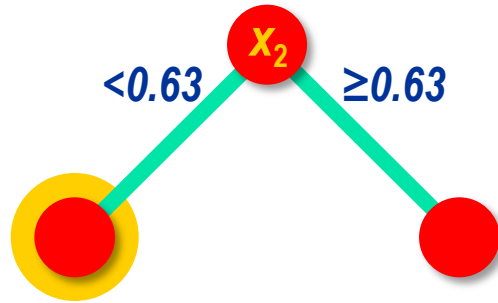
Decision Tree Split Search



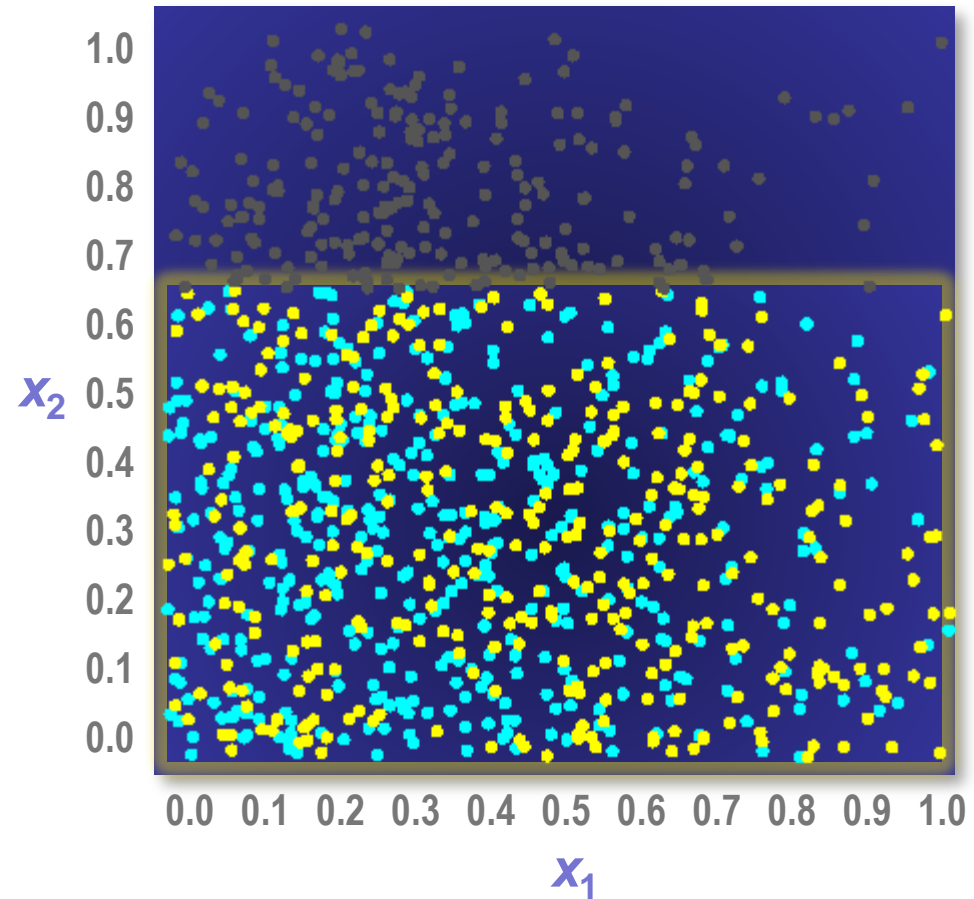
Create a partition rule
from the best partition
across all inputs.



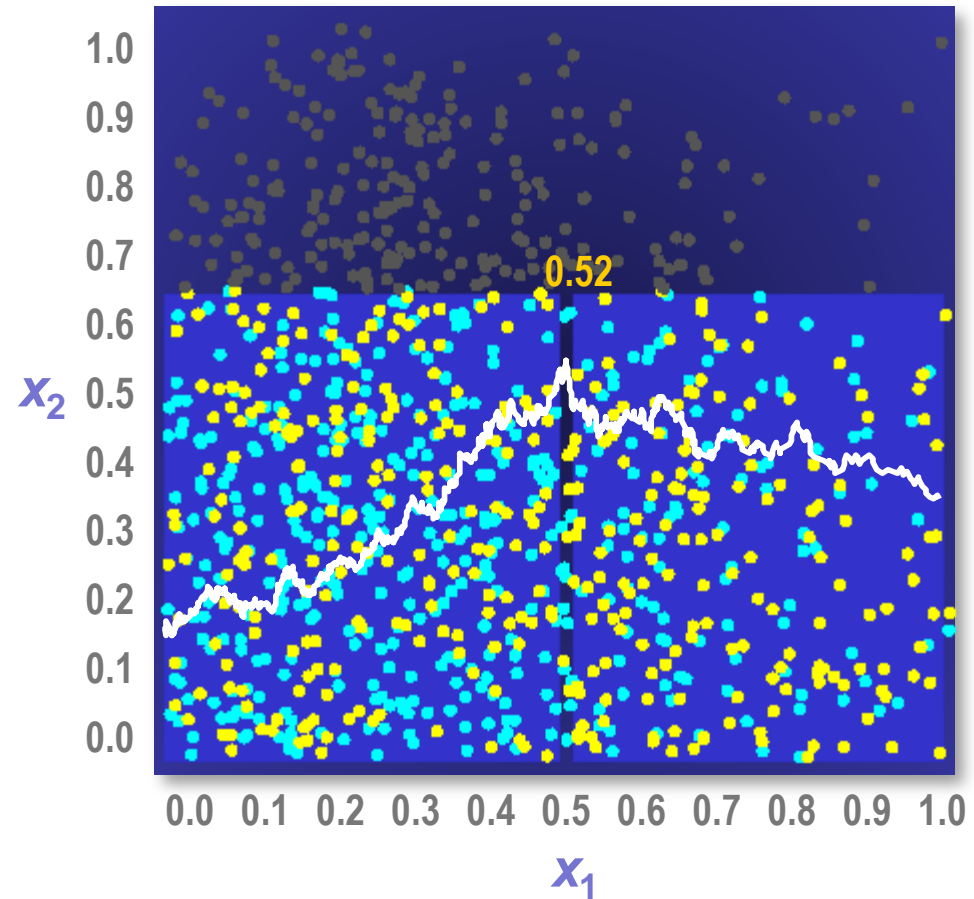
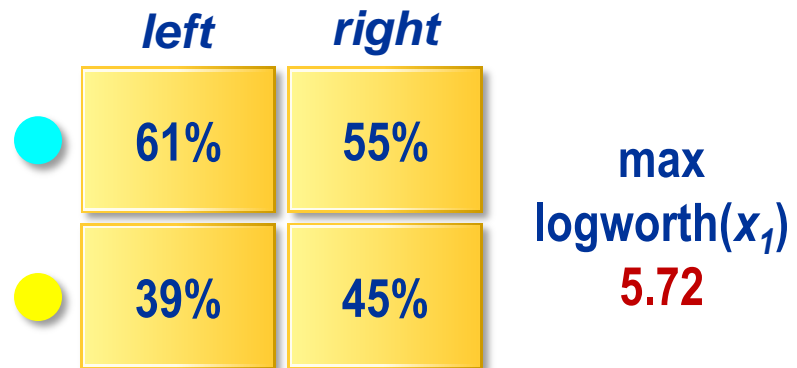
Decision Tree Split Search



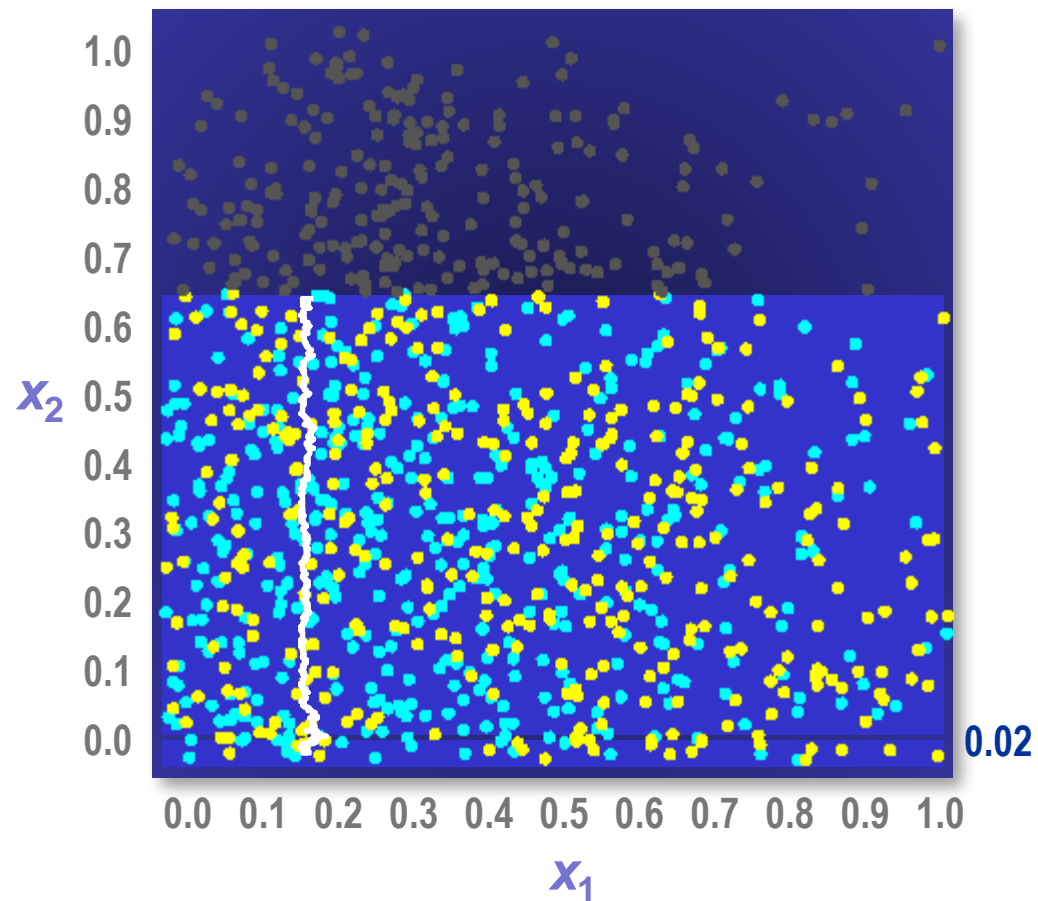
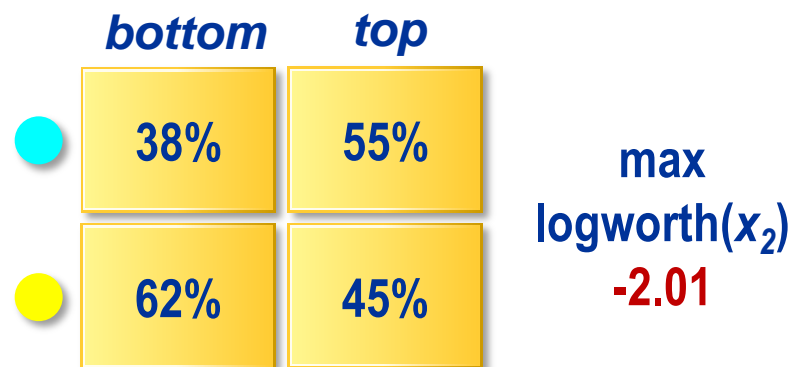
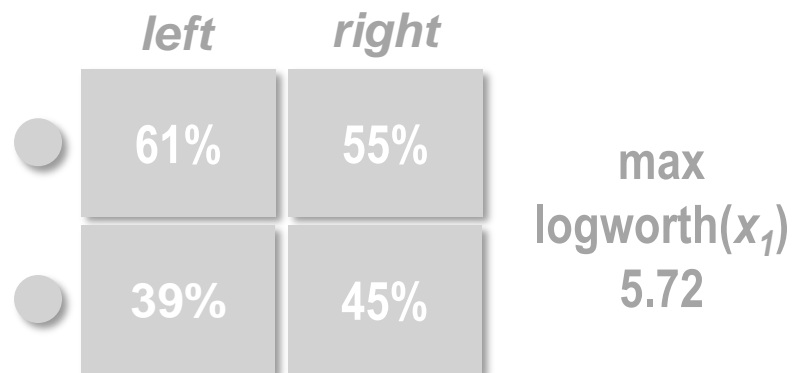
Repeat the process
in each subset.



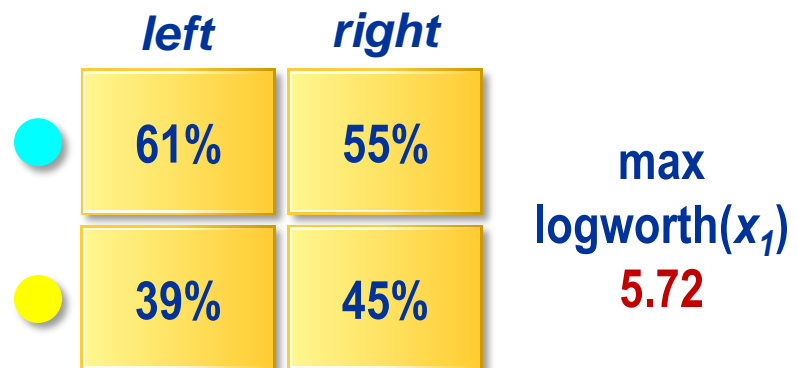
Decision Tree Split Search



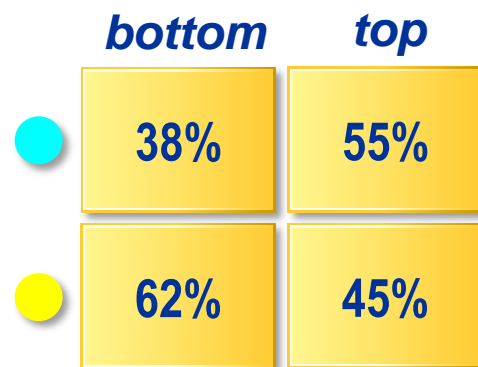
Decision Tree Split Search



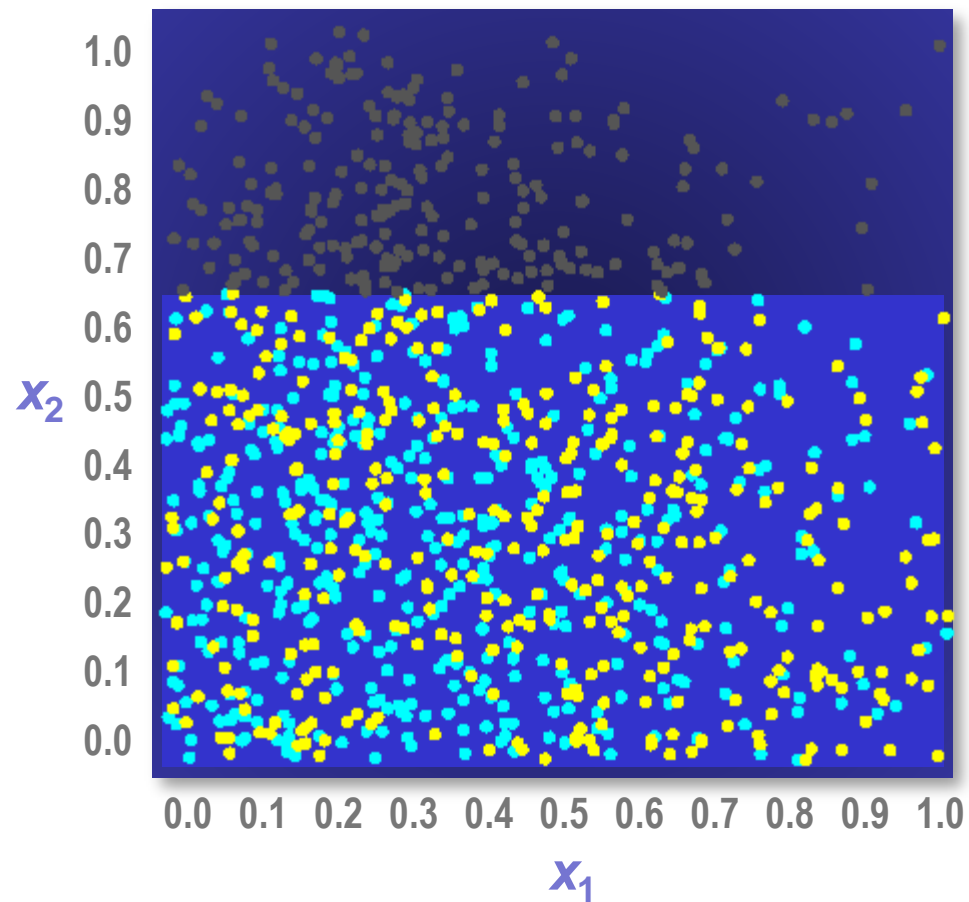
Decision Tree Split Search



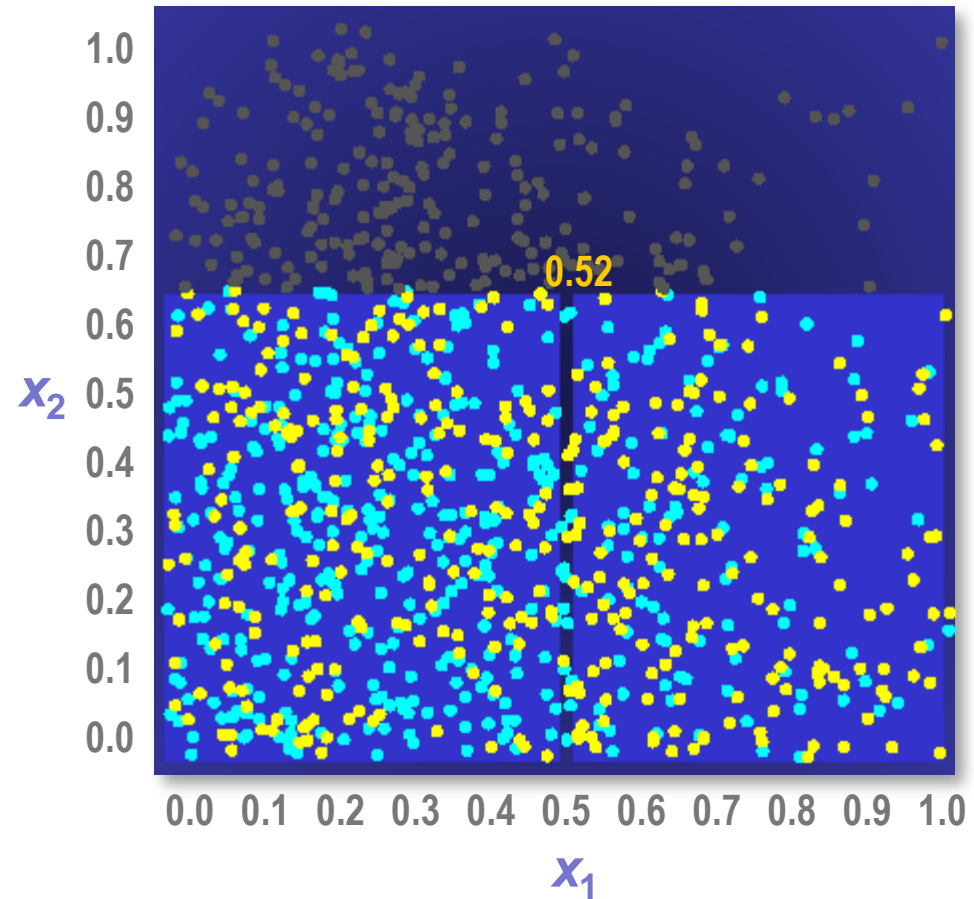
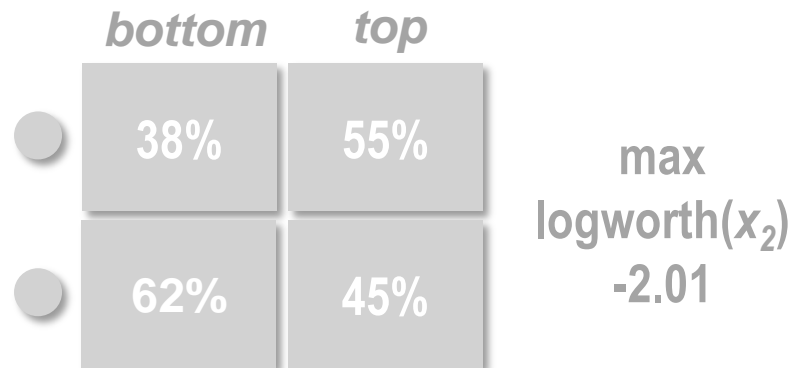
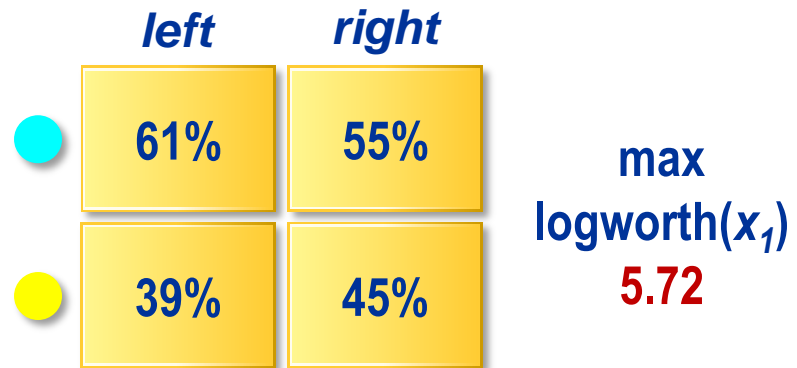
max
logworth(x_1)
5.72



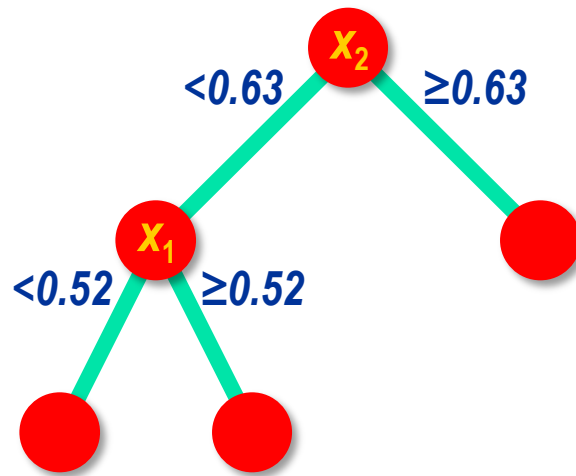
max
logworth(x_2)
-2.01



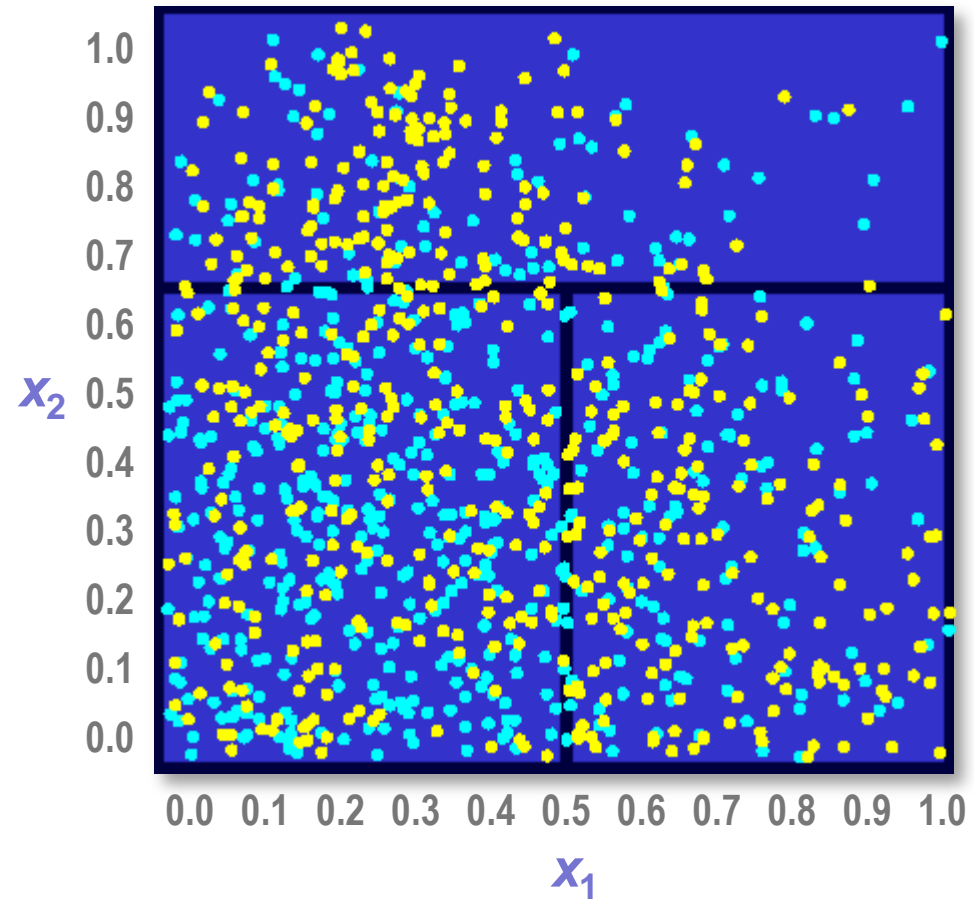
Decision Tree Split Search



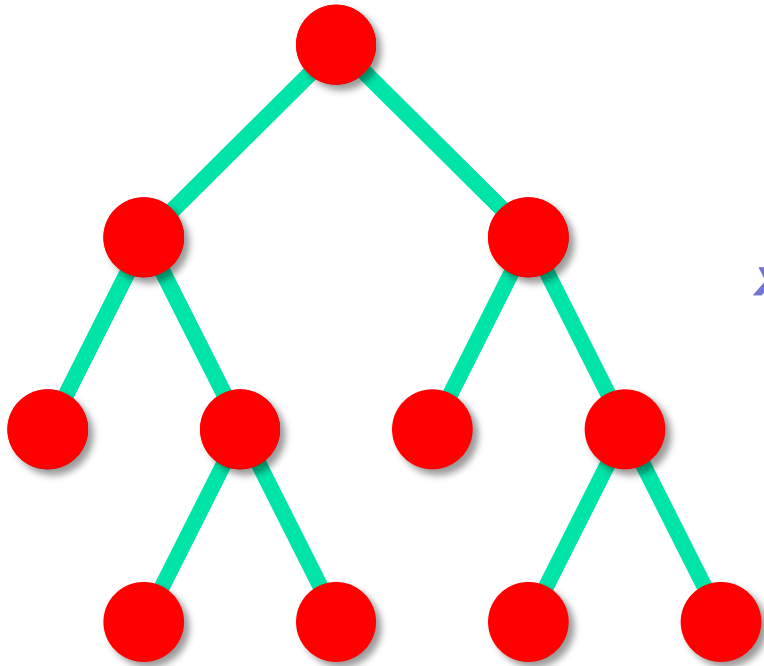
Decision Tree Split Search



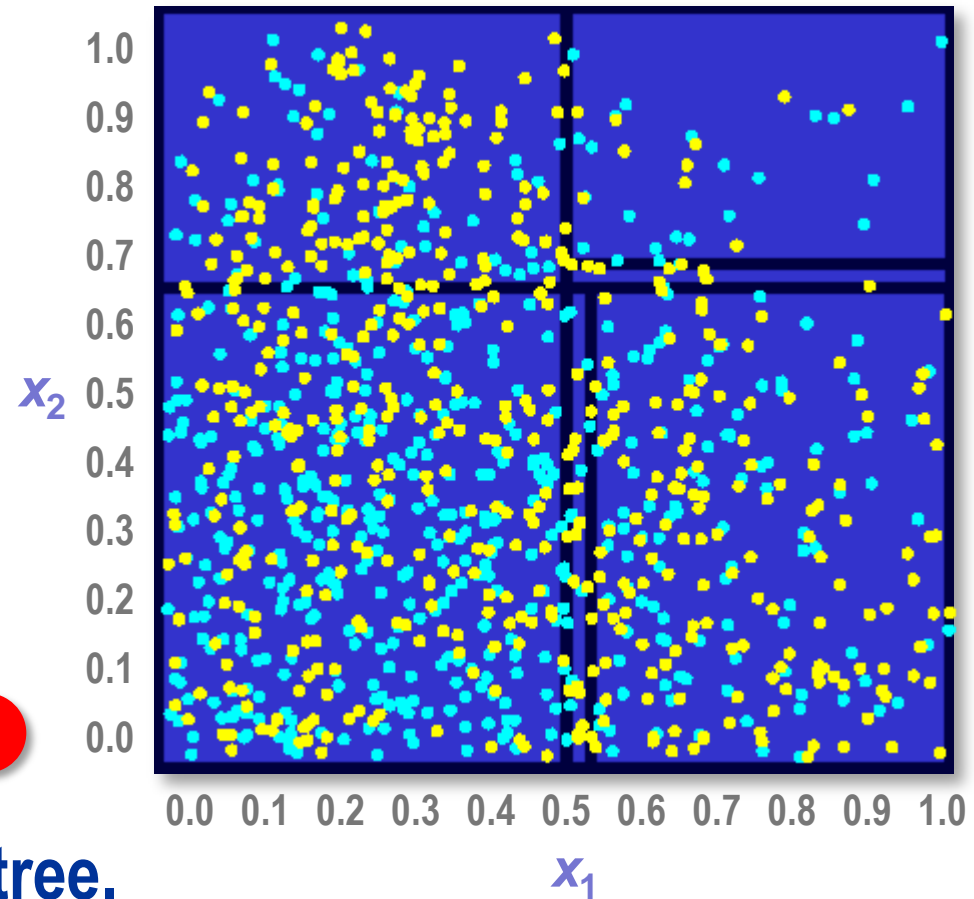
**Create a second
partition rule.**



Decision Tree Split Search



Repeat to form a maximal tree.





Model Essentials – Decision Trees

▶ Predict new cases.

Prediction rules

▶ **Select useful inputs.**

Split search

▶ Optimize complexity.

Pruning



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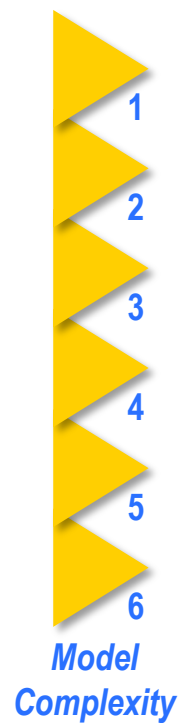
Predictive Model Sequence

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Create a sequence of models with increasing complexity.

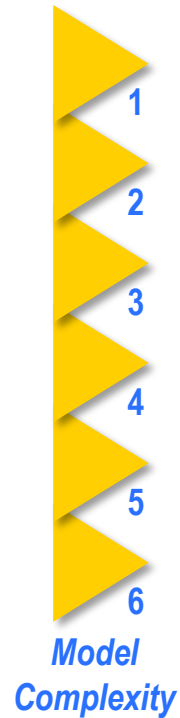
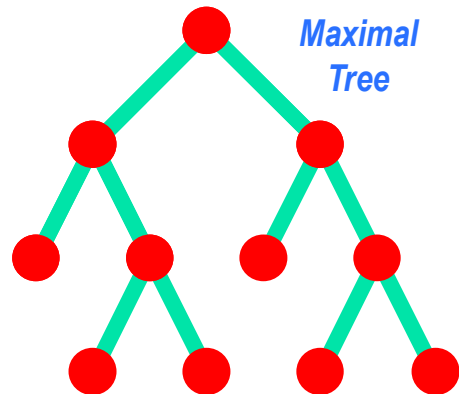
Maximal Tree

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Create a sequence of models with increasing complexity.

A maximal tree is the most complex model in the sequence.

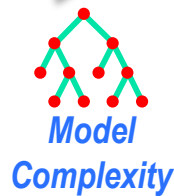
Maximal Tree

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



A maximal tree is the most complex model in the sequence.

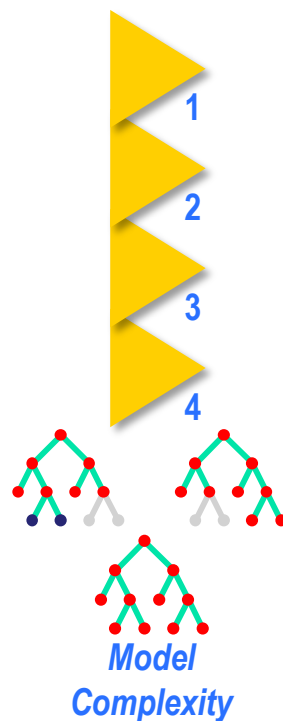
Pruning One Split

Training Data

	inputs			target

Validation Data

	inputs			target



The next model in the sequence is formed by *pruning* one split from the maximal tree.

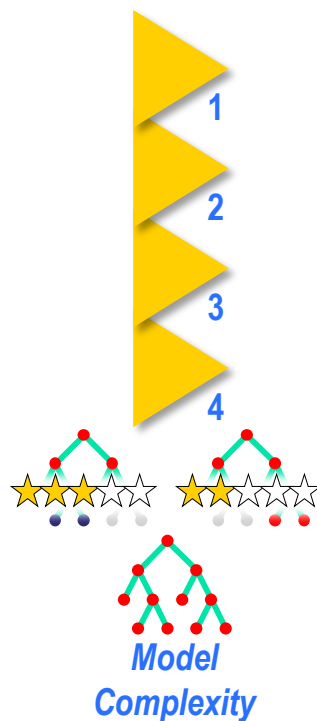
Pruning One Split

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Each subtree's predictive performance is rated on validation data.

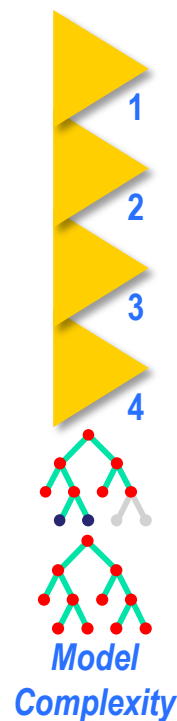
Pruning One Split

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



The subtree with the highest validation assessment is selected.

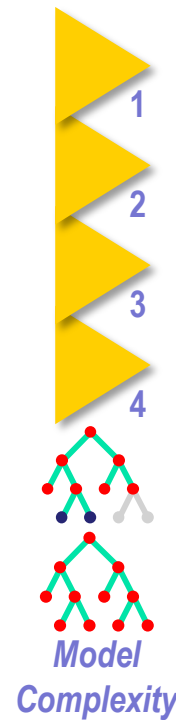
Pruning Two Splits

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Similarly, this is done for subsequent models.

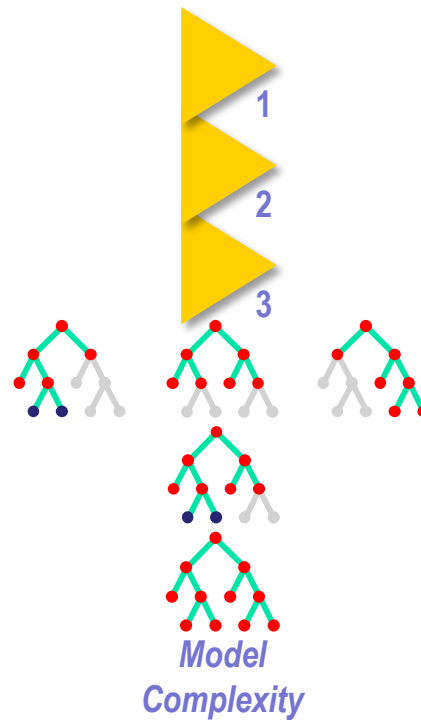
Pruning Two Splits

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Prune two splits from the maximal tree,...

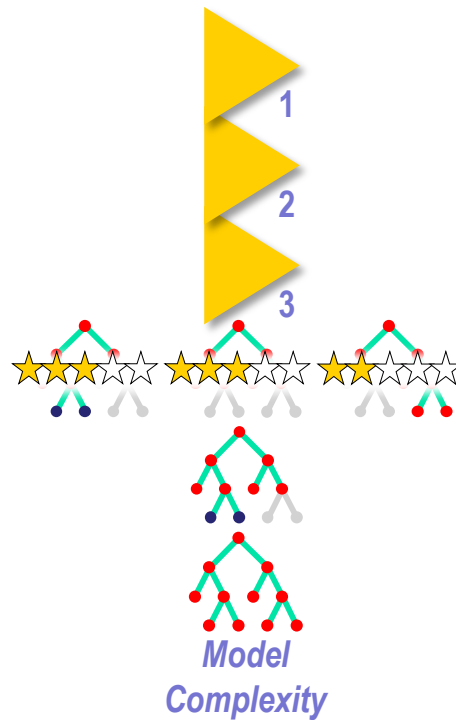
Pruning Two Splits

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



...rate each subtree using validation assessment, and...

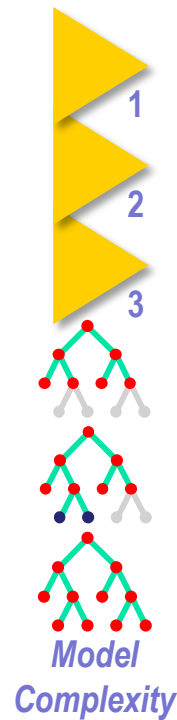
Pruning Two Splits

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



...select the subtree with the best assessment rating.

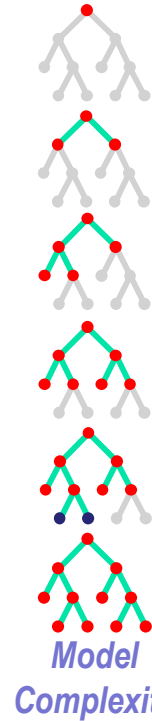
Subsequent Pruning

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Continue pruning until all subtrees are considered.

Selecting the Best Tree

Training Data

	inputs			target

Validation Data

	inputs			target



Compare validation assessment
between tree complexities.

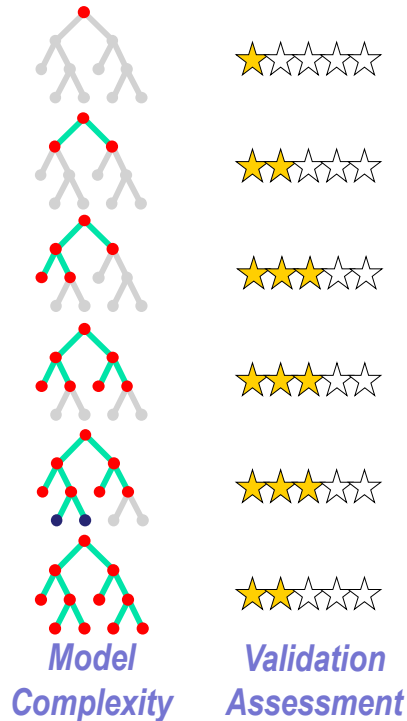
Validation Assessment

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



Choose the simplest model with the highest validation assessment.

Validation Assessment

Training Data

	<i>inputs</i>			<i>target</i>

Validation Data

	<i>inputs</i>			<i>target</i>



What are appropriate validation
assessment
ratings?

Assessment Statistics

Validation Data

	<i>inputs</i>			<i>target</i>

Ratings depend on...



target measurement (binary, continuous, and so on)



prediction type (decisions, rankings, estimates)



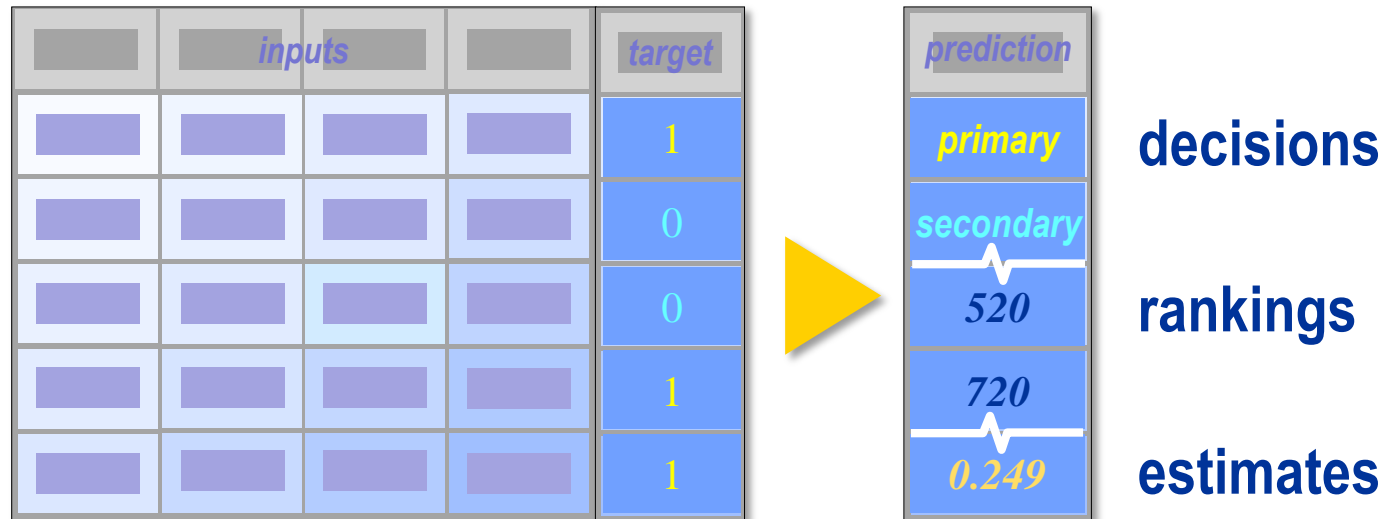
Binary Targets

	<i>inputs</i>			<i>target</i>
				1
				0
				0
				1
				1

primary outcome

secondary outcome

Binary Target Predictions



Decision Optimization

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	<i>primary</i>
				0	<i>secondary</i>
				0	520
				1	720
				1	0.249

decisions

Decision Optimization: Accuracy

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	<i>primary</i>
				0	<i>secondary</i>
				0	520
				1	720
				1	0.249

true positive

true negative

Maximize *accuracy*: agreement
between outcome and prediction

Decision Optimization: Misclassification

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	secondary
				0	primary
				0	520
				1	720
				1	0.249

false negative

false positive

Minimize *misclassification*:
disagreement between outcome and
prediction

Ranking Optimization

	<i>inputs</i>			<i>target</i>	<i>prediction</i>	
				1	<i>secondary</i>	decisions
				0	<i>primary</i>	
				0	520	rankings
				1	720	
				1	0.249	estimates

Ranking Optimization: Concordance

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	<i>secondary</i>
				0	<i>primary</i>
				0	520
				1	720
				1	0.249

target=0 → low score
target=1 → high score

Maximize *concordance*: proper
ordering of primary and
secondary outcomes

Ranking Optimization: Discordance

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	secondary
				0	primary
				0	720
				1	520
				1	0.249

target=0 → high score

target=1 → low score

Minimize *discordance*: improper ordering of primary and secondary outcomes

Estimate Optimization

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	secondary
				0	primary
				0	720
				1	520
				1	0.249

decisions

rankings

estimates

Estimate Optimization: Squared Error

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	secondary
				0	primary
				0	720
				1	520
				1	0.249

$(\text{target} - \text{estimate})^2$

Minimize *squared error*:
squared difference between target
and prediction

Complexity Optimization: Summary

	<i>inputs</i>			<i>target</i>	<i>prediction</i>
				1	secondary
				0	primary
				0	720
				1	520
				1	0.249

decisions

accuracy / misclassification

rankings

concordance / discordance

estimates

squared error