Module 2: Introduction to "Solving the Solver" – Linear Programming and Then Some!

Reading Material: 2.1 – Introduction

The previous module was meant to be a class warm-up. A lot of students enter this class with great trepidation about the level of math sophistication necessary to be successful. I hope to have reduced this by showing that basic algebra will serve us fine.

We've also established that the course will be "story problem" driven – basically, we will be creating mathematical models to help better analyze and understand real-world business settings. In a classroom, these real-world business settings are often captured as a story problem.

Therefore, our goal in the present module is to provide more background on expectations for the remainder of the course/book, building on the somewhat unusual opening module. We will also talk a little about the philosophy of this book and the MBA class is it meant to serve.

Reading Material: 2.2 – Algebra on Steroids: Linear Programming, Management Science, and Operations Research

The major subject of the course is "algebra on steroids" – linear programming (LP). The term linear programming is a much more scientific term than it needs to be, thus, my using the (hopefully) humorous term algebra on steroids – implying that the majority of what we will be learning is just a small extension beyond algebra. But this is not "math for the sake of math" – we will be talking about (and using) approachable, useful, and practical models that can help real, practicing managers do their jobs better, make their organizations more money, and save the world. At the very end of the chapter, we will give a specific example on what we mean by "algebra on steroids" by comparing two versions of an example warm-up problem from Module 1.

These mathematical, quantitative models are a subset of the field called Management Science (MS) or alternatively Operations Research (OR). One can do a lot of reading on the origins of the field – which trace back only as far as the 1930s and 1940s when scientists were doing "research on operations" in attempt to help the Allies win the Second World War. In some ways, this makes MS/OR a very young field, one that continues to grow with the continued emergence and ubiquity of the digital computer.

History is always important to a field, but regurgitating what others have written is not particularly interesting to me, nor likely to you. What I hope to do in this module is to give you enough of a background of WHAT it is we are going to be doing, WHY we are going to

be doing it, perhaps point to a few resources that will HELP us do it, and in general give you a flavor of the class execution.

Our approach will be to use Excel's Solver to implement linear programming and other similar decision tools. Excel has become the de facto communication and calculation tool of business and industry, and it makes sense to use it to learn the concepts of linear programming and management science. Other tools are available and, in some cases, might be superior to Excel. I would be more than happy to point you to the relevant literature and other sophisticated optimization tools.

Reading Material: 2.3 – A View from the Syllabus – Course Objectives

At the risk of sounding pedantic, borrowing from my usual syllabus for the course, here is an overview of what we'd like to accomplish.

> The primary objective of this course is to develop skills in quantitative modeling of business problems and opportunities. The main focus of the course exposes students to the readily available optimization analysis tools (such as linear programming) that are standard in today's spreadsheets. Emphasis will be placed on understanding how such modeling techniques can be used to **assist** the decision-maker, when they are applicable, and an identification of technique limitations . . .

Also from my syllabus:

> Decision making in organizations is a partnership between humans, models and data. This course focuses on primarily the partnership between humans and mathematical models, and will provide the student with additional 'tools in their tool-belt' to facilitate more effective decisions…

The theme in these two quotes: this class is about a partnership between decision maker and modeler and involves real, useful, and accessible to all, mathematical models.

As with any quantitative course, we will work very hard on crafting models in Excel that get "the right answer" to homework problems, examination questions, etc. This is the limitation of any university course, but this is still the best way to learn about useful quantitative tools. It is important for you to know that I recognize (and that WE will recognize) that our models are approximations of reality and their true value to us is not necessarily the "right answer" but the insight we gain in studying a particular situation with our quantitative models.

I could bore you with details about example situations, but I'll just bore you with one anecdote and then move on. A Ph.D. student in Civil Engineering (who was also the city manager for Enid, OK) and I developed a partnership in creating an LP model to manage the city's water system. After much study, we realized that although water flow is a non-linear phenomenon, if valid assumptions about the operation of the water system were made, a strictly linear model would handle the operation of the system 98% to 99% of the time. Starting there, we created a model not much more sophisticated then models we will be

developing later in the book (though it was bigger and we had to program the solution algorithm from scratch!), and found we could save municipalities about 30% or more on electricity costs in operating their water systems. We implemented this system in a number of locations in California as well.

The punch line for this system – we found "insight" that cities could save a lot of energy consumption by treating their water like inventory. When electric rates were low (middle of the night), demand for water was low, so the best strategy was to build up water inventory (in water towers, other containers, etc.) in low usage times, then stop running water pumps (the sources of water) during the day when electric rates were higher.

You might say, well, you didn't need a model to tell us that, but at the time the operators (and the modelers!) did. This is an example of the insight that can be gained through a partnership of decision maker, model, data, and simplifying assumptions. The philosophy of the water system is implemented in many locations in the old Pacific Gas and Electric territory in California.

This is the ultimate use of the tools we'll talk about here in this book – a partnership, in which it is not just about the solution of the model but also about simplifying reality to a point at which we can experiment with a model, generate some solutions, try different parameters and assumptions, and gain some insight about the scenario so we can go and make better decisions, earn more money, and of course, save the world.

This philosophy is perhaps depicted in Figure 2.1. We have reality, we simplify it as an LP model, we solve it, we gain insight into the problem under study through sensitivity analysis or other means, and then we refine our model and continue to improve our understanding of the situation.

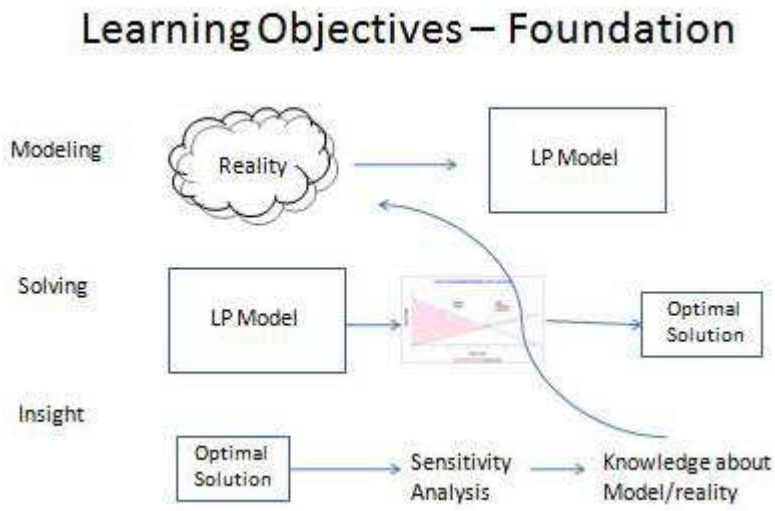

Figure 2.1

Figure 2.1 is also a good picture of what this book hopes to accomplish in the early, foundational stages of the text. Basically, we have reality, which is fairly ill-defined. We create an LP model that approximates reality (the straight lines representing "linearizing" or simplifying the complete complexity of the system under analysis, the cloud-like squiggly lines), and then, through algebra, we find the optimal solution of the model. (Note that we will learn just a little about the algebra of solutions in the foundational material, moving forward quickly to Excel doing the heavy lifting).

Upon finding the optimal solution, one also considers what we call sensitivity analysis, which are automatically generated reports from the algebraic solution to the model, to gain further "insight" about the situation being modeled. We might identify factors such as "this resource constrains our production" or "the solution is very sensitive to what we assume the profit of our product will be," and so forth. This is also the stage when we need to make sure the model solution passes the 'snift test' – does the solution make practical sense, or did we accidently omit a key constraint, variables, etc.?

Given this analysis by the decision maker with her/his knowledge of the solution and the situation being analyzed, one might make changes to the original model, rerun, and start the whole process again. Again, the iterative, model-decision maker partnership looking to gain insight about the situation under study. This partnership is sometimes difficult to implement in a classroom. Nonetheless, our students often see immediate application of the tools and techniques (and situations) we talk about in class and begin to use these models in their organizations while they are still in class.

Reading Material: 2.4 – Outside Resources: Briefly

Most introductory chapters in textbooks for the Management Science class might spend a lot of time talking about large-scale implementation success stories in practice that saved companies millions upon millions of dollars. This can be a very useful addition. I'd prefer to reserve a section on the eBook website for someday posting brief stories about what I refer to as "killer applications."

OR/MS Today and the journal *Interfaces* are two good outside resources that focus on applications of MS and OR. We will take most of our "killer applications" examples from these publication. These are very readable publications that are designed to talk about the practice of using the tools and techniques we discuss in this book. I encourage you to track them down and graze through them, and certainly I make it a habit in my classes to discuss them from time to time as well.

The main academic organization related to this field is the Institute for Operations Research and the Management Sciences (INFORMS; check it out!)

Reading Material: 2.5 – Approach of the Book and Author's comments

With each passing revision to this book (which no doubt will require many!), I will update my thoughts here. Many years ago, I wrote a much less extensive set of notes for a class in Knowledge Management Tools and Techniques when I could not find a book that matched my learning objectives and style and wanted to provide a cheaper alternative to the students than some overpriced book that I really didn't use. As a take-off on CliffsNotes, those publications that got me through my English classes, I called them Rick's Notes. They were effective, and students just bought them at Kinko's for the 5 years I used them. I even had a colleague of mine use them 3 or 4 years later as a faculty member at the University of Evansville.

I've been teaching versions of this class (Management Science Methods) for almost 25 years now. I've not only used the tools in practice but have also gained insight into how to make the useable tools accessible to my (primarily) MBA students. Each semester, in the spirit of continuous improvement, I change in-class examples, class formats, examination approaches, etc. I have found effective ways to approach the class – always looking for improved examples and improving experiences. I recently reached a point at which it bothered me that I was making students buy a $250 textbook for really just doing practice homework problems; the rest of the course I felt I had found improved ways to engage the students so they could feel firsthand how our topics were practical, useful, and scalable to problems faced by their organizations.

As luck would have it, I found an outlet for my guilt: a publisher willing to help me make my validated and successful philosophical approach to the class available to the students and at less than 50% of the previous cost. I had thought about it for years, waiting for the perfect time. Well, I'll be retired before the perfect time hits, so I said yes, got $0 for agreeing to do it (not very optimal signing bonus!), and jumped in.

It will be a work in progress for years because I want to create the perfect class environment … and I know the odds of me doing that is also 0%. I missed nearly every deadline the publishers gave me for the first installment of the book. They still talk to me, by the way. So look for future improvements to be discussed here.

The book is meant for MBA students, but it can be used by undergraduates as well. I tried to avoid a lot of algebra and Greek symbols. Pure academics will likely hate it for that very reason. I will ALWAYS be proud to hold up my students and the preparation that they get through our partnership in class as examples and to compare their abilities with those of any MBA student around the country being taught in a more traditional (and perhaps, higher "pedigree") program. At the end of the semester, we have been rigorous, with value added in a practical, understandable way. After all, that is our jobs – make learning fun and accessible – not painful and obtuse (had to use that term after watching *The Shawshank Redemption* for the twenty-third time last night).

I didn't write this book to make money or to be adopted by anyone else. I wrote this to supplement the way my class has evolved over 25 years (without trying to brag too much, almost everyone finds it "one of the most useful classes in my MBA program") and to save students some money. I do not claim this to be a literary masterpiece, nor do I want it to be. I want it to help you learn and master the material, how to apply some very useful

quantitative techniques to decision making that you can implement right now on your desktop to be a better decision maker and to save the world.

Basically, each module mimics a major course learning objective. Typically, each module begins with one or more problems meant as in-class exercises. When they are then solved, the "A-ha" moment occurs for the readers. Practice problems are provided at the back of the modules. Good old-fashioned hard work rather than some newly discovered "approach" to education (a la "the new math"). Now, unless hard work, lots of effort, and clear explanations are a 'new' approach to quantitative classes.

Finally, let me steal from my syllabus a final philosophical comment. By the end of the course, it is my intent that you, the reader/student, will be able to:

- Create sophisticated spreadsheet models using Excel Solver that address so-called desktop applications in scheduling, resource allocation, distribution, etc.
- Analyze decision-making situations and understand limitations, drawbacks, and advantages of using spreadsheet models and their output to assist in making decisions.
- Understand the limitless potential of larger "killer applications" that are possible even with just a semester's worth of exposure.
- Employ the concepts of management science modeling in practice even WITHOUT sophisticated models, specifically related to identifying objectives, decisions under the control of the organization, the constraints faced in the situation, and the usefulness of sensitivity analysis to derive alternative solutions.

The fourth point is important. There is actually an underlying philosophy in this class – the three-step decision philosophy of attacking a challenge or opportunity. The decision maker needs to identify what it is we control, what we wish to accomplish, and what holds us back. This is a good decision process to adopt in all forms of life, not just in a business sense and/or in this class. We will see how these three questions are turned into decision variables, objectives, and constraints in the subsequent modules. But in general, this is a useful decision strategy in all walks of life.

Reading Material: 2.6 – Conclusion

As we dive into this thing we've called "algebra on steroids" or linear programming (LP) in the next module, and in the context of the three key aspects of decisions, let us look at this problem from Module 1.

Example

A friend brought small bags of cookies to sell at a fairly large BCS Championship Game Watch Party (there were no TCU fans present, however). Three kinds of cookies were sold: Stars (sold for $1 per bag), Circles (sold for $0.75 per bag), and Stars and Stripes (sold for $1.50 per bag).

He brought the cookies to the Watch Party in three large boxes. By volume, it is a known fact that one of the large boxes can hold 100 bags of Stars, 120 bags of Circles, or 80 bags of Stars and Stripes (or a corresponding mix of cookies). HINT: Don't concern yourself with what each box held; view this as an aggregate limit of numbers of cookies.

All cookies brought were sold, a total of 300 bags. The total amount of money raised was $312. All bags of cookies brought in the boxes were sold.

Can you determine how many of each of the three cookie types were brought/sold? HINT: Yes, you can!

Now, if we recast the situation a little bit and turned it into more about making decisions, it might look like this.

Example 2

A friend was bringing small bags of cookies to sell at a fairly large BCS Championship Game Watch Party (there were no TCU fans present, however). Three kinds of cookies were sold: Stars (sold for $1 per bag), Circles (sold for $0.75 per bag), and Stars and Stripes (sold for $1.50 per bag).

He was to bring the cookies to the Watch Party in three large boxes. (The boxes did not have to be full, but he could not bring more than three large boxes of cookies). By volume, it is a known fact that one of the large boxes can hold 100 bags of Stars, 120 bags of Circles, or 80 bags of Stars and Stripes (or a corresponding mix of cookies). HINT: Don't concern yourself with what each box held; view this as an aggregate limit in the numbers of cookies.

Previous parties had given him some hints on the demand for cookies – he knew that for the sake of variety, he needed to make at least 45 bags of each type of cookie.

As he was planning his cookie composition, he also realized he was constrained by time in putting together the cookie bags. Circle cookies and Stars cookies took 1 minute per bag to finish. Because Stars and Stripes had more icing, it took 2 minutes to finish each bag. He allocated 420 minutes (7 hours) to put the bags together.

Can you determine how many of each of the three cookie types your friend should make to maximize sales (a surrogate for profit)? To quote POTUS: "Yes, you can!" (Eventually.)

The second version is in fact a word problem that we can solve using LP. You'll be asked to solve this problem in the future, but it's a little too big for us initially to attack. But as you can see, we have the numbers of the three types of cookies to decide (or that we control), we want to maximize sales (our objective), and we have a variety of things that constrain us (number of cookies to bring in boxes, demand, and time to make the cookie bags).

So, no more talk – let's get to our first example so we can tackle the BCS problem as soon as possible (at least in terms of cookie making – I will not speak here about football playoffs!).