Different Statistical Tests

An Overview

Module Agenda (Contd.)

- Part 1 (overview of different tests)
- Part 2A (Lecture)
  - Two sample t-test
    - Analysis variable: **continuous (interval)**
    - Grouping variable: **binary**
- Part 2B (Demo using software) of two sample t-test
- Part 3A (Lecture)
  - Paired sample t-test
    - Analysis variable: **continuous (interval)**, two measures on each subject (person)
- Part 3B (Demo using software) of paired sample t-test

Module Agenda

- Part 4A (Lecture)
  - Cross-tab, Chi-square tests and measures of association between two **categorical** variables
- Part 4B (Demo using software) of cross-tab and Chi-square
- Part 5A (Lecture)
  - Correlation between two **continuous** variables
- Part 5B (Demo using software) of correlation and tests of significance

Overview of Different Statistical Tests

- So far, we looked at one variable at a time (CI or hypothesis test about means, proportions etc.)
- Life gets more interesting when we explore relationships between two variables (Bivariate Analysis)
  - Why should we learn this?
- How would we approach these tests in this class?
  - Understand what types of business questions we want to answer
  - Understand measurement levels of the variables involved in answering those business questions
  - Choose and conduct appropriate tests based on above
    - More emphasis on **application** rather than mechanics of tests
  - Interpret results from test and tie-back to business questions

Data Analysis Tools and Techniques

- Univariate analysis : Using one-variable at a time (e.g., descriptive statistics, CI for any variable in a data set)
- Bivariate analysis : Analysis involving two variables simultaneously (e.g., cross-tabs, two sample t-test, paired-sample t-test, correlation and simple regression)
- Multivariate analysis : Analysis involving three or more variables (e.g., multiple regression, logistic regression, analysis of variance, ….)

Words of Caution Before We Begin

- Before proceeding with any of the analysis mentioned here, please ensure following:
  - Check your data (each variable) to make sure there are no obvious errors (e.g., a quick look at the minimum and maximum value for each variable will indicate if there is any obvious errors)
  - Think of how each variable was defined and how the data were coded (such as male = 1 and female = 0) and whether these make sense for the analysis you are attempting
  - Think of how and why the data were collected

Two Sample T-Tests

Lecture

Two-Sample t-Test (Business Questions)

- This test can be used to answer business questions such as:
  - Is the average salary of male employees equal to the **average** salary of female employees in a firm?
  - Is the **average** $ charged on credit cards by male customers equal to the average $ charged on credit cards by female customers?
  - Is the **click-through rate** equal for version A and version B of an ad displayed in Google search?
- Why do we need a test? Why can't we just calculate the averages/response rates separately for each group and answer the business questions?

Two-Sample t-Test Mechanics

- Uses a grouping (dummy or binary) variable (with **non-overlapping** categories) to group data and another variable for which averages are calculated and compared for each groups.
- The difference between the average of groups is tested against 0!
  - Think what does that imply?
- The distribution of the variable (difference between sample means divided by its standard error) follows a *t-distribution*
- The standard error (and hence the p-value) for the two sample t-test **is calculated in two ways**:
  - **Assuming** equal variance between groups
  - **Assuming** unequal variance between groups
- Another test (*F-test*) and its p-value is needed for you to check the assumption of **equal variance**.

Procedure for Doing Two-Sample t-Test

- Data must be separable by groups and there should be no overlap between groups – that is, same observation can not be in both groups (if that's not true we need a different test)
- Write hypotheses about differences in averages between groups in the population
- Choose level of significance *(usually 5%)*
- **First**, use *F-test* to test if the variances are equal between groups
  - Choose **Equal** variance *t-test* if *F-test* is **not rejected**
  - Choose **Unequal** variance *t-test* if *F-test* is **rejected**
- **Second**, make decisions about differences in averages based on the p-value from the t-test chosen in the first step
  - Be careful with whether your hypotheses about differences in averages was two-sided or one-sided (ignore for this lecture)

Two Sample T-Tests

Demonstration by JMP

Business Questions and Hypotheses

- Question: Is the average overall satisfaction same or different between those who use bill pay vs. those who do not?
- Hypotheses for testing means:
  - Null: No difference in average satisfaction in between those who use bill pay vs. those who do not (H_o: $\mu_1 - \mu_2 = 0$ or, H_o: $\mu_1 = \mu_2$)
  - Alternative: Null is not true (H_o is not true)

Assuming equal variances, the test statistic is calculated as:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Assuming unequal variances, the test statistic is calculated as:

$$d = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

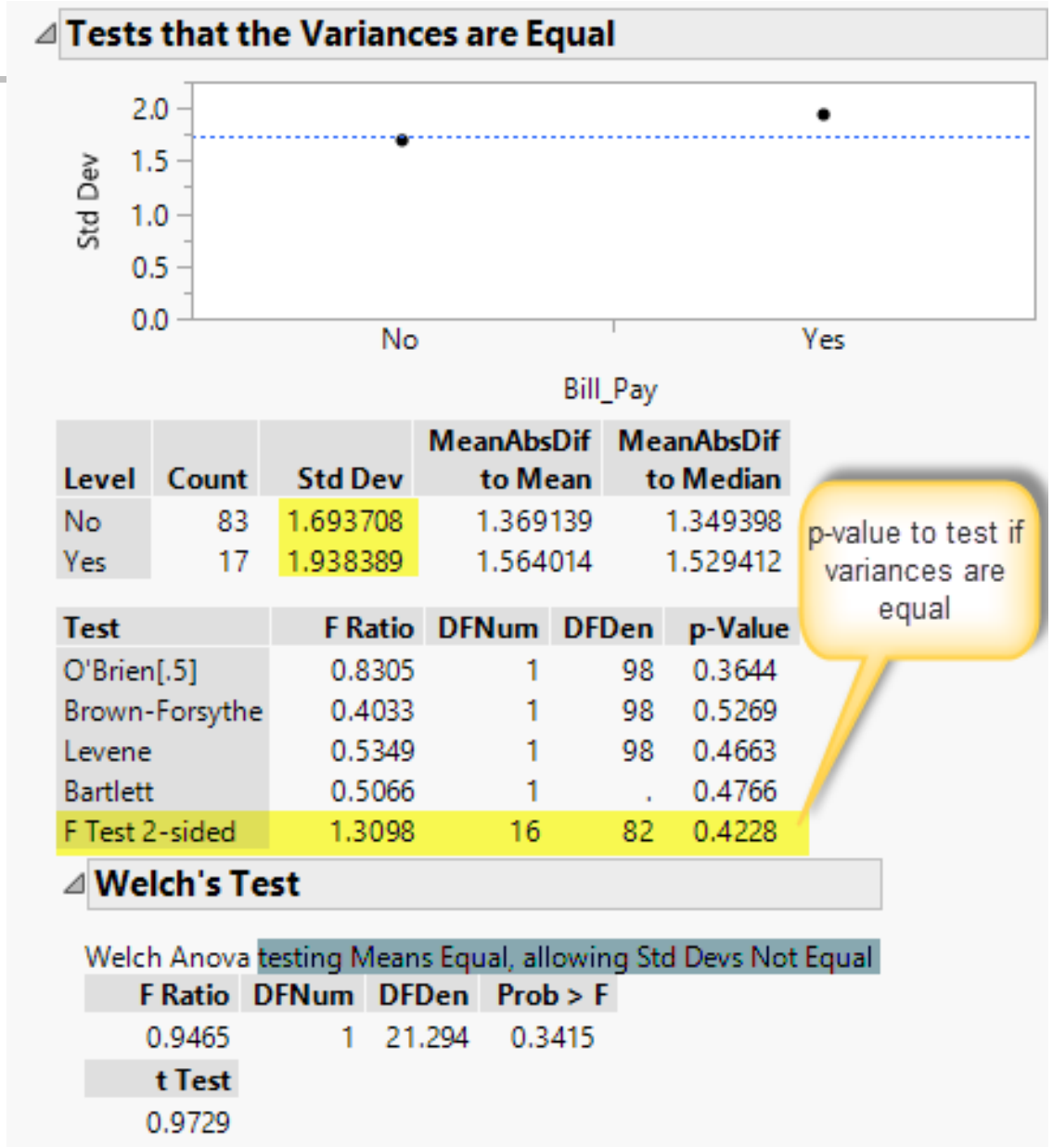
So, we first need to test if the variances of overall satisfaction are equal between the two groups

Data: Customer Survey

- JMP> Analyze >Fit Y by X> Overall as Y, Response > Bill_Pay as X, Factor>Run
- Red triangle next to Oneway Analysis of Overall by Bill_Pay > Unequal Variances

Selected Results

- Is the variances of overall satisfaction equal between the groups (those who use bill pay vs. not)?
  - H_o : No difference in variance of satisfaction in between those who use bill pay vs. those who do not (H_o: $\sigma_1^2 = \sigma_2^2$).
  - H_1 : H_o is not true



Selected Results

- Is the mean of overall satisfaction equal between the groups (those who use bill pay vs. not)?
  - Null: No difference in average satisfaction in between those who use bill pay vs. those who do not (H_o: $\mu_1 - \mu_2 = 0$ or, H_o: $\mu_1 = \mu_2$)
  - Alternative: Null is not true (H_o is not true)
- Red triangle next to Oneway Analysis of Overall by Bill_Pay > Means/Anova/Pooled t

Oneway Anova

Summary of Fit

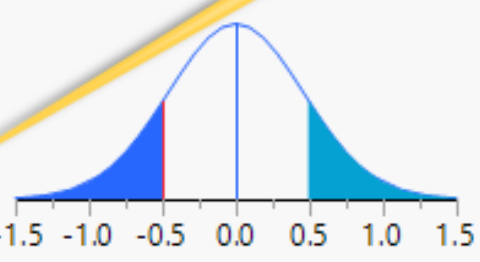
Rsquare	0.011425
Adj Rsquare	0.001338
Root Mean Square Error	1.736013
Mean of Response	4.82
Observations (or Sum Wgts)	100

Pooled t Test

Yes-No
Assuming equal variances

Difference	-0.4918	t Ratio	-1.06425
Std Err Dif	0.4622	DF	98
Upper CL Dif	0.4253	Prob > |t|	0.2898
Lower CL Dif	-1.4090	Prob > t	0.8551
Confidence	0.95	Prob < t	0.1449

p-value to test if means are equal assuming equal variance



Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
No	83	4.90361	0.19055	4.5255	5.2818
Yes	17	4.41176	0.42104	3.5762	5.2473

Std Error uses a pooled estimate of error variance

Business Questions and Hypotheses

- Question: Is the rate of ATM use same or different between those who use bill pay vs. those who do not?
- Hypotheses for testing means:
  - Null: No difference in average ATM usage rate between those who use bill pay vs. those who do not (H_o: $\mu_1 - \mu_2 = 0$ or, H_o: $\mu_1 = \mu_2$)
  - Alternative: Null is not true (H_o is not true)

As before, we first need to test if the variances of ATM are equal between the two groups

Data: Customer Survey (Change ATM from Nominal to Continuous Variable)

- JMP> Click red icon in front of ATM in columns menu> Select Continuous
- JMP >Analyze >Fit Y by X> ATM as Y, Response > Bill_Pay as X, Factor>Run
- Red triangle next to Oneway Analysis of Overall by Bill_Pay > Unequal Variances
- Red triangle next to Oneway Analysis of ATM by Bill_Pay > Means/Anova/Pooled t

Bill_Pay

Level	Count	Std Dev	MeanAbsDif to Mean	MeanAbsDif to Median
No	83	0.4923750	0.4790245	0.3975904
Yes	17	0.5072997	0.4844291	0.4117647

Test	F Ratio	DFNum	DFDen	p-Value
O'Brien[.5]	0.3019	1	98	0.5839
Brown-Forsythe	0.0116	1	98	0.9145
Levene	0.0420	1	98	0.8381
Bartlett	0.0237	1	.	0.8777
F Test 2-sided	1.0615	16	82	0.8091

Welch's Test

Welch Anova testing Means Equal, allowing Std Devs Not Equal

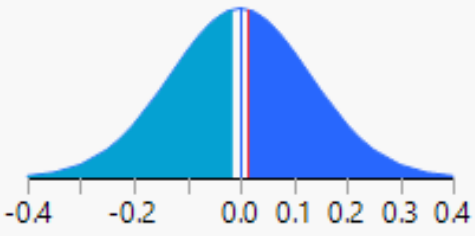
F Ratio	DFNum	DFDen	Prob > F
0.0111	1	22.606	0.9169

t Test
0.1055

Pooled t Test

Yes-No
Assuming equal variances

Difference	0.01417	t Ratio	0.107597
Std Err Dif	0.13174	DF	98
Upper CL Dif	0.27560	Prob > |t|	0.9145
Lower CL Dif	-0.24725	Prob > t	0.4573
Confidence	0.95	Prob < t	0.5427



Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
No	83	0.397590	0.05432	0.28980	0.50538
Yes	17	0.411765	0.12002	0.17360	0.64993

Std Error uses a pooled estimate of error variance

Paired Sample (or, Matched Pair) t-Tests

Lecture

Paired Sample T-test (Business Questions)

- Customers are asked to rate importance of price and reliability on a 9-point scale
  - Marketer wants to know if the average importance of “price” is **same** as the average importance of “reliability”
- Employees of a firm are asked to rate manager’s leadership skills on a 7-point scale last year and this year
  - Manager wants to know if the average perception of his leadership skills is **same** between last year and this year.

Paired Sample t-Test Mechanics

- It computes a difference score (difference between two variables) for each subject.
- Then, it calculates average of this difference score
  - If the average of the difference score is 0 that means….
- It then uses *t-statistic* to test if the average of this difference score is significantly different from 0!
- The p-value for the t-test is reported for you to make decisions.
  - No need to worry about equal or unequal variance!

Procedure for Paired Sample t-Test

- Two variables must be available for each subject or observation.
- Write hypotheses about “differences in averages” of the two variables
- Choose level of significance (usually 5%)
- Use *Paired sample t-test* to test if the “differences in averages between variables” is equal to 0
- Make decisions about “differences in averages” of the two variables based on the p-value from the test
  - Be careful with whether your hypotheses about differences in averages of the two variables was two-sided or one-sided (**ignored for this class**)

A Few Additional Comments About t-Tests

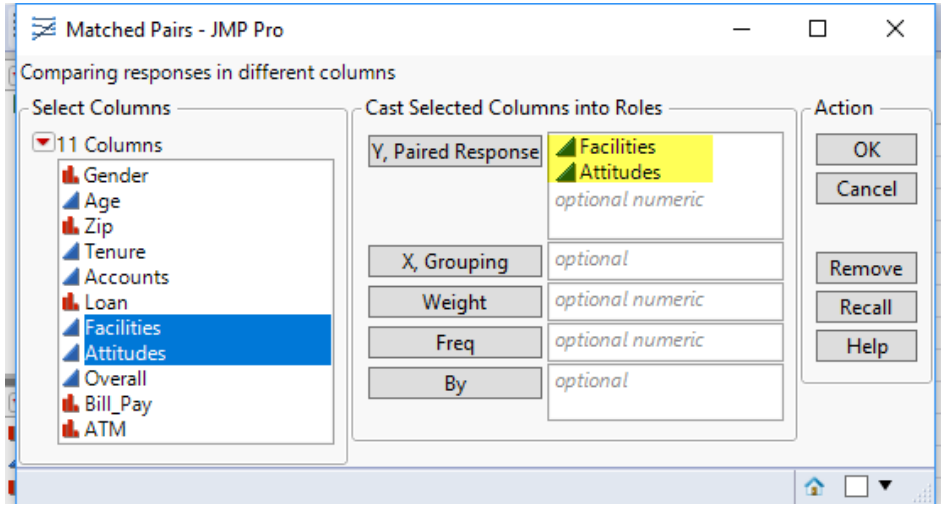
- Use appropriate visualization to get a feel for data.
  - For two-sample t-tests, you can use the Box plot or Histogram plot by grouping variable
  - For paired-sample t-tests, use summary statistics and histograms on the paired variables.

Paired Sample (or, Matched Pair) t-Tests

Demonstration using JMP

Data Set : Customer Survey

- Business Question: Is the average rating for facilities is same or different from the average rating for attitudes?
- Hypotheses for testing means:
  - Null: No difference in average ratings of facilities and attitudes (H_o: $\mu_1 - \mu_2 = 0$ or, H_o: $\mu_1 = \mu_2$) – here 1 and 2 refers to the two variables, facilities and attitude
  - Alternative: H_o is not true
- JMP> Analyze > Specialized Modeling > Matched Pairs> Select Facilities and Attitudes for Y, Paired Responses > OK



Results of Paired Test

Attitudes	4.94	t-Ratio	0.335959
Facilities	4.86	DF	99
Mean Difference	0.08	Prob > |t|	0.7376
Std Error	0.23812	Prob > t	0.3688
Upper 95%	0.55249	Prob < t	0.6312
Lower 95%	-0.3925		
N	100		
Correlation	0.06168		