

Module 1: An Escape from the Library

Reading Material: 1.1 – Square One

The first chapter in a book is usually a mundane, broad, wordy attempt at an overview of the intended material that will be covered in a course – in other words, a lot of sentences strung together just like this one! Normally, a lot of meaningful diagrams showing the important relationships of everything under the sun to the topic being discussed are also shown.

Yes, that was sarcasm.

So I'm going to try to be different – in this chapter, and throughout the book. Ultimately, you will be the judge.

This book mimics the process that my past students and I have used *together* to successfully master a practical, useful set of quantitative tools called Management Science to help solve real business “problems.” And, through this process, my past students have become more valuable employees and better leaders and decision makers, and they are on target to save the world.

Most of my past students have been in MBA programs. A fairly common reaction when the terms *quantitative*, *tools*, and *science* are mentioned in the same paragraph or sentence – a return to a not so pleasant place in their memories, perhaps where a high school football coach masquerading as a math teacher made algebra look like quantum physics or where the graduate student teaching your college math class was more interested in the quantum physics she was studying instead of helping you learn how to apply quantitative principles to your major.

This book is for all of you.

And if you don't have an adverse reaction when you hear those terms, are you going to be left out? Absolutely not. We are about to embark on a semester-long journey learning about some of the most useful quantitative techniques that we can apply in business. So what if they just require algebra and a little bit more? (I like to refer to it as “algebra on steroids”). If you understand the concept of optimization (the main premise of calculus), that means you're just a little ahead of the game.

So, this book is for all of you as well.

My mantra for the class has always been “Let no MBA be left behind.” The material in this book is meant to be useful, practical, understandable, and valuable to everyone who enters its hallowed pages. I am confident by the time we reach the end together, you will feel like you have added a lot to your ‘tool belt’ and that “Yes, I can!” master a quantitative course.

Let's move to our first exercise, which has a couple of different learning objectives. First, it helps create an initial understanding of what kinds (or the format) of "business problems" we will be focusing our efforts on in this class. Second, it gives you an idea of the quantitative sophistication required for you to be successful.

Reading Material: 1.2 – Gary Larsen's contribution to understanding Management Science

In the 1980s, my favorite cartoonist was Gary Larson, who drew "The Far Side." Sometimes the humor was so subtle that I had to ask my intellectual friends what the cartoon meant. But he really had some side-splitters. His cartoons live on in compilation books galore. Others have tried to copy him, but there was only one "Far Side." **For legal reasons, we can't include the cartoon here, so see the author for a description.**

The cartoon captures a very vivid part of my youth. I have revisited it often as my offspring have gone through their own math classes. Maybe you experienced the following as well?

A disclaimer: I enjoyed my math and quantitative classes all throughout my studies (OK, so maybe not geometry that much). I liked to do the abstract problems that were meant to build up our fundamental skills. But I always dreaded the end-of-the-chapter *word problems*, the dastardly story problems. You know, the ones that tried to get you to read a little and apply the abstract concepts you were learning to something real? Thus, my affection for the cartoon.

Well, my friends, our class book, strictly from an "execution" standpoint, belongs in the library depicted in the cartoon. (I believe that a book with the title "Solving the Solver" is hidden on the shelves by the gentleman in the picture facing the guy with horns). Yes, to master the material in this book, you will need to face many, many story problems.

A quick comment about this – these story problems that appear here are meant to represent (in a simplified, controlled learning environment) reality in which the tools and techniques we will learn are useful in helping managers make better decisions and gain insight about their business, help the organizations run more effectively, and making you a more valuable human capital asset. Yes, these story problems will simplify reality, but we will develop skills and insights that will be applicable and scalable to large-scale and realistic real-world settings. We will revisit this from time to time throughout the book.

So, a theme in the first chapter is "quantitative" and "story problems." I may not have successfully lowered blood pressure in some of you as you read through this. To help clarify what we mean by story problems and the level of mathematical sophistication that is needed for success in this class, let's open one of the books on the top shelf of the Library and pull out an example problem or two to clarify our expectations. We will use the second book from the left on the top shelf "Big Book of Story Problems 3" for the following example.

Reading Material: 1.3 – The PTA Example

Example 1.1: On a fourth-grade class field trip, the PTA purchases box lunches for the adults (parents/teachers) and for the kids from McAlister's Deli. The adult lunches cost \$6 each and those for the kids \$4 each. You are auditing the purchase. You have been told that 34 people were on the field trip, and lunch cost \$160. Based on this, determine how many adults and kids attended the field trip.

So, have those past memories came rushing back to give you warm fuzzies? Let us attack this story problem and brush up on our algebra and modeling skills. Specifically, we are going to need to decide what we want an abstract mathematical model to do for us. First question:

How many different “values” are we going to need to determine (i.e., “solve for”) to satisfy the requirements of this story problem?

Yes, there are two different values that we are asked to determine: The number of adults and the number of kids that attended the field trip. We are going to use VARIABLES as the term to represent the number of adults (A) and the number of kids (K). In this setting, variables are what we want to determine to “solve” the story problem.

So, next question: Perhaps you recall that to solve for a number of unknown variables in an algebraic context, you need to somehow relate these variables together through the use of EQUATIONS. (We might also keep in the back of our mind the term CONSTRAINTS for this as well). How many equations do you think are relevant and can relate the number of adults A with the number of kids K?

Again, Correct – two equations. So, maybe you recall an algebraic mantra from the past – “To solve a system of linear equations, you must have the same number of unknowns as equations defining those unknowns.” This is mainly true.

Anyway, the two equations we can construct relating A and K together are cost (\$160 spent) and the number of people who went on the field trip (34).

What do these equations look like?

The correct equations are the following:

COST: $6A + 4K = 160$ (adult meals cost \$6, kids \$4, total spent \$160)

PEOPLE: $1A + 1K = 34$ (adults plus kids equal 34)

So, let's summarize what we have done so far in the first story problem taken from the shelves of our special library. We've identified that we want to find the value of two unknown variables, the number of adults (A) and the number of kids (K), and that they are

related by two equations that define the amount of money spent (COST) and the number of PEOPLE who went on the trip.

Thus, we've created our first mathematical (algebraic) model of the semester – two decisions, two unknowns. Our mantra says we can therefore solve for the values of A and K. The next section walks us through this solution process.

Reading Material: 1.4 – The PTA example solved

Even by now, you can tell that this book is more colloquial and conversational than typical book-speak (if the editors let me continue in this fashion). So we're going to resist the temptation to get all formal in solving for the values of A and K in our system of linear equations.

There is a lot of terminology that one could use as it relates to linear algebra (that's what we're dealing with right now). We will try to use as little as possible, but certainly not invent our own (terminology, not algebra) if at all possible.

We know that in a system of linear equations, each variable (in this case A and K) represents the same value in each equation. That is, A in the cost equation represents the same value as A in the people equation. Our strategy will be to try to isolate one variable in order to get it to give up its value, and then, once it is discovered, "back-substitute" it into any equation to solve for the other (I've seen this isolation technique work in many domains, especially in interrogations on TV crime dramas like CSI and Bones).

To do this, we need to recall some basic algebraic manipulations/operations. Consider the PEOPLE equation/constraint ($A + K = 34$).

Which of the following equations (a, b, c, or d) is equivalent to the original equation?

- a. $A = 34 - K$
- b. $K = 34 - A$
- c. $2A + 2K = 68$
- d. $3K = 102 - 3A$

Okay, so it was a trick question – all of them are equivalent. The point is that we can rearrange terms to our heart's content in trying to get one equation to a nice form that will allow us to take two equations and two unknowns and simplify them to one equation and one unknown (isolation). Once we isolate, we can solve for the variable and solve the whole thing.

So, let us take the first rewritten equation ($A = 34 - K$). This conveniently relates variable A with variable K. We can take the statement $(34 - K)$ and replace that for every occurrence of A in the first equation. This removes A from that equation, and we are left with a single equation and one variable. It looks like:

Original: $6A + 4K = 160$

Substitute $34 - K = A$ into this equation for: $6 * (34 - K) + 4K = 160$.

Simplify $6 * (34 - K) + 4K = 160$ to: $204 - 6K + 4K = 160$.

Get Ks on left hand side of equation and numbers on the right

First step: equation simplifies to $204 - 2K = 160$.

Second step: simplifies to $-2K = 160 - 204$, which simplifies to: $-2K = -44$.

We have therefore isolated K and because $-2K = -44$, $K = 22$.

Once we know $K = 22$, we can go back to any equation from our model, substitute the known value of K, and solve for A.

Let's use $A = 34 - K$. This simplifies to $A = 34 - 22 = 12$.

So, we have found that there were 12 adults and 22 kids who traveled to McAlister's on the field trip. A total of 34 people who spent $12 * 6 + 22 * 4 = 72 + 88 = \160 on food.

Looks like we have solved our first story problem of the semester!

Reading Material: 1.5 – A Step back from the PTA problem

Let's review a little bit:

1. There are a lot of different ways in which one can solve a system of linear equations. Matrix algebra (linear algebra) uses something called Gaussian elimination, but all of the approaches use the same basic premise of equation manipulation to isolate single variables. You may recall a different "style" of approach that you previously used. There is nothing wrong with that as long as you obey the universal rules of algebra.
2. The level of algebraic sophistication needed to solve this two-variable problem is perhaps the most difficult mathematical concept we need to use during the semester. That doesn't mean we won't be creating models with hundreds of variables and doing sophisticated solutions . . . it means the core algebra foundational skills needed to succeed in this class stem from what we have just reviewed. This is said to give you warm fuzzies.
3. Occasionally you have seen the word *linear* bantered about in the discussion. For the most part, we will only be constructing linear models during the majority of the course – what we lose in accuracy in modeling reality in a linear fashion (little) we gain in solution certainty (a lot!). More about this later as we move into the next few modules.

At the end of the module are three more practice problems that are meant as additional warm-up. I suggest you take a look at them and be confident in your ability to derive a system of linear equations of two or three variables and solve them. Please make sure you look at the problem related to cookies for the BCS Championship game – that problem, in a different incarnation, will appear again later as a practice problem.

We have reached the end of the module. I normally do these four problems as an in-class exercise on the first day of class, with only the Far Side as my syllabus. Normally students find it a fun warm-up (after they pick their jaws up off the desks in front of them when they realize that I really am going to start with Gary Larson's cartoon as their syllabus!). It speaks to two main points: We are going to do word problems in this class. More importantly, we are only going to need to recall solid algebra skills to succeed in this class. But with these skills that Gary Larson has helped us look at, we can do great things with a technique called linear programming that we will ultimately implement in EXCEL via the add-in "the SOLVER." But part of our foundation for doing great things will need competent algebra skills. A successful completion of Module 1 will start us on that path.

Much more powerful and useful models await us in subsequent modules. The next module will set the stage for our future exploration of linear programming and other management science techniques. Welcome aboard!