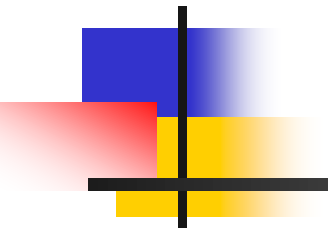


Model Essentials for Neural Net

Dr. Goutam Chakraborty

Outline

- Predict new cases.
- Select useful inputs.
- Optimize complexity.

Prediction formula

None

Stopped training

...

Model Essentials – Neural Networks

- Predict new cases.
- Select useful inputs.
- Optimize complexity.

Prediction formula
None
Stopped training

...

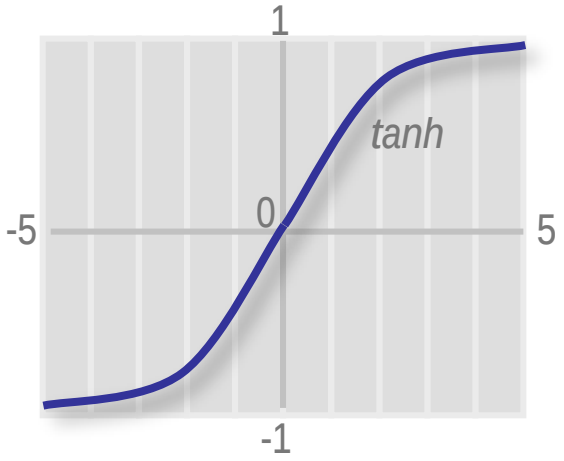
Neural Network Prediction Formula

hidden unit

prediction estimate

$$\hat{y} = \hat{w}_{00} + \hat{w}_{01} \cdot H_1 + \hat{w}_{02} \cdot H_2 + \hat{w}_{03} \cdot H_3$$

bias estimate *weight estimate*



$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

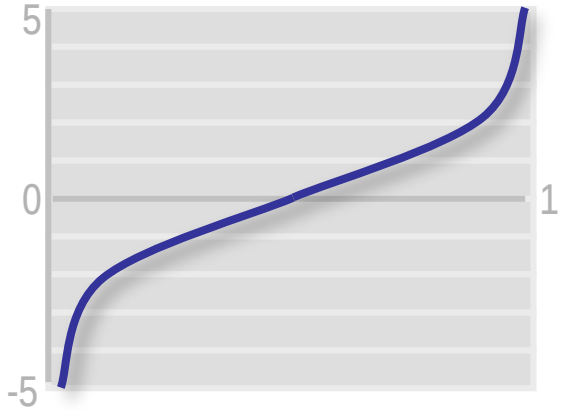
activation function

...

Neural Network Binary Prediction Formula

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

logit
link function



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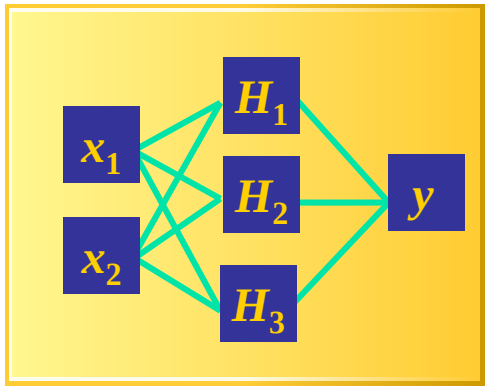
...

Neural Network Diagram

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$



input layer *hidden layer* *target layer*

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

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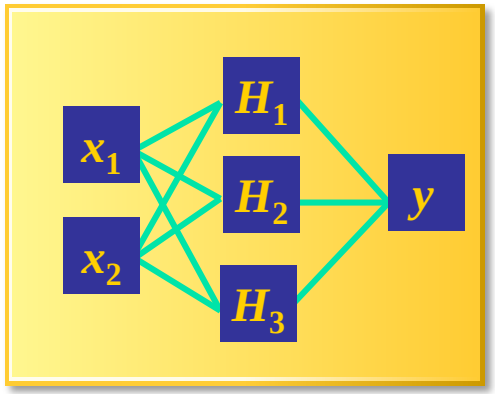

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...

Prediction Illustration – Neural Networks

logit equation

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

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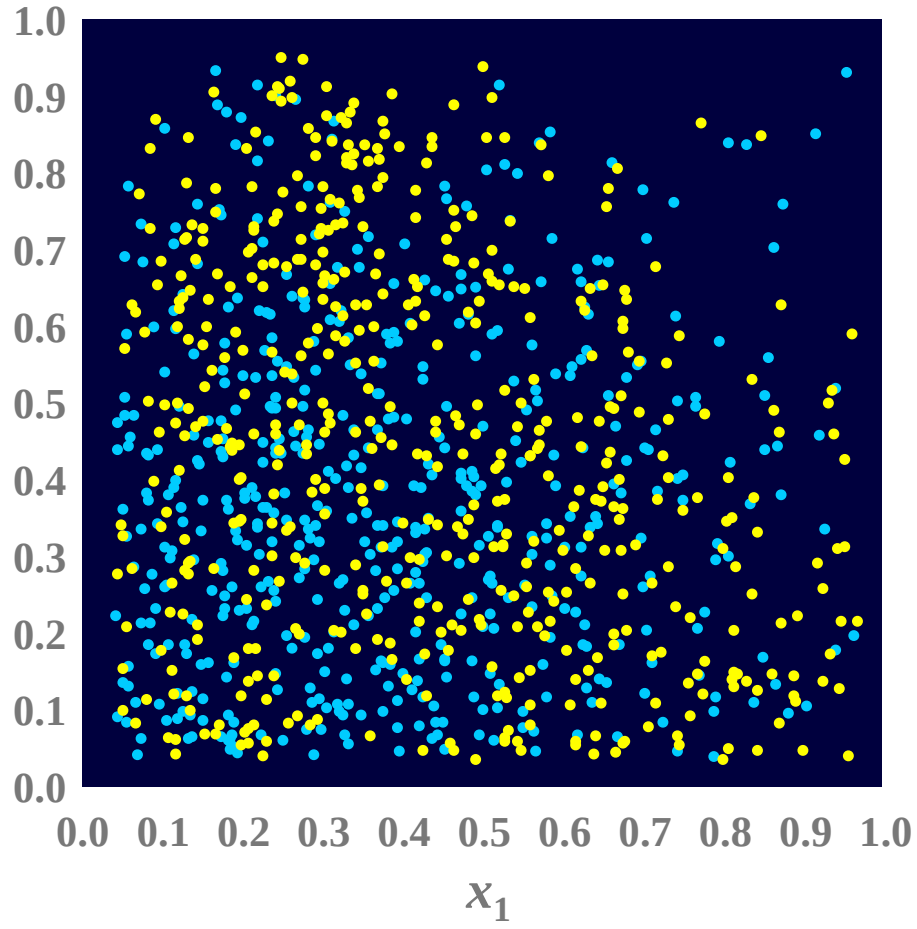

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Prediction Illustration – Neural Networks

logit equation

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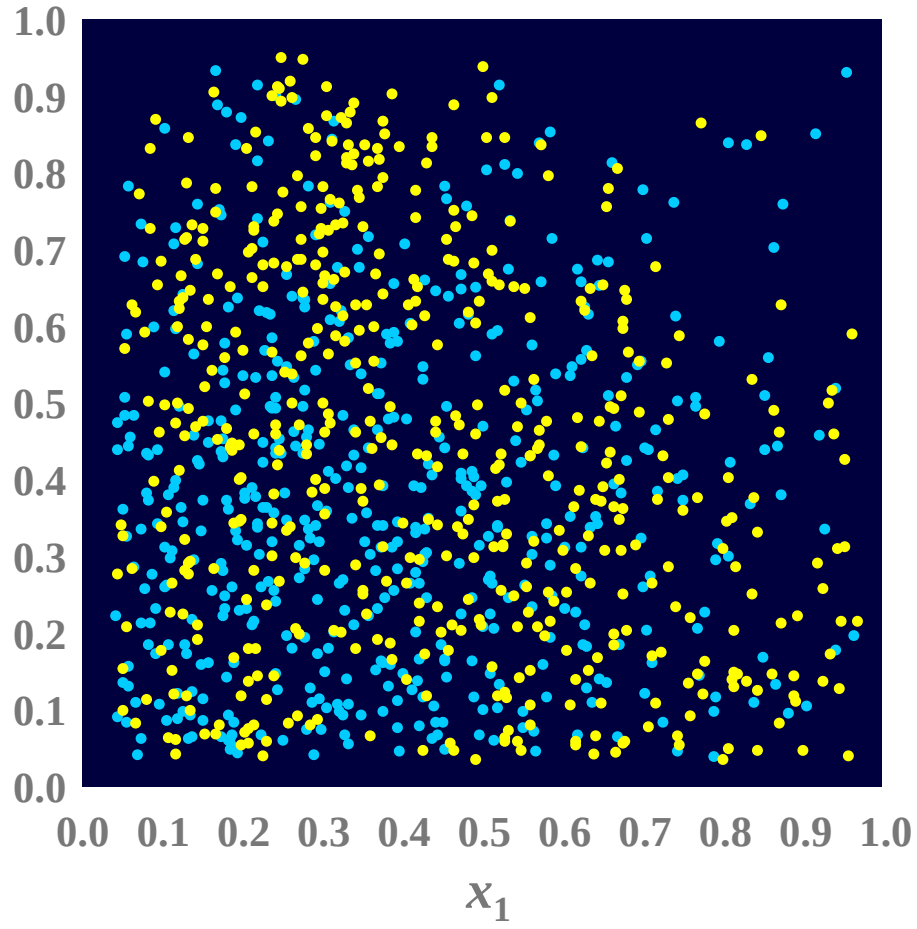
$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

Need weight estimates.



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Prediction Illustration – Neural Networks

logit equation

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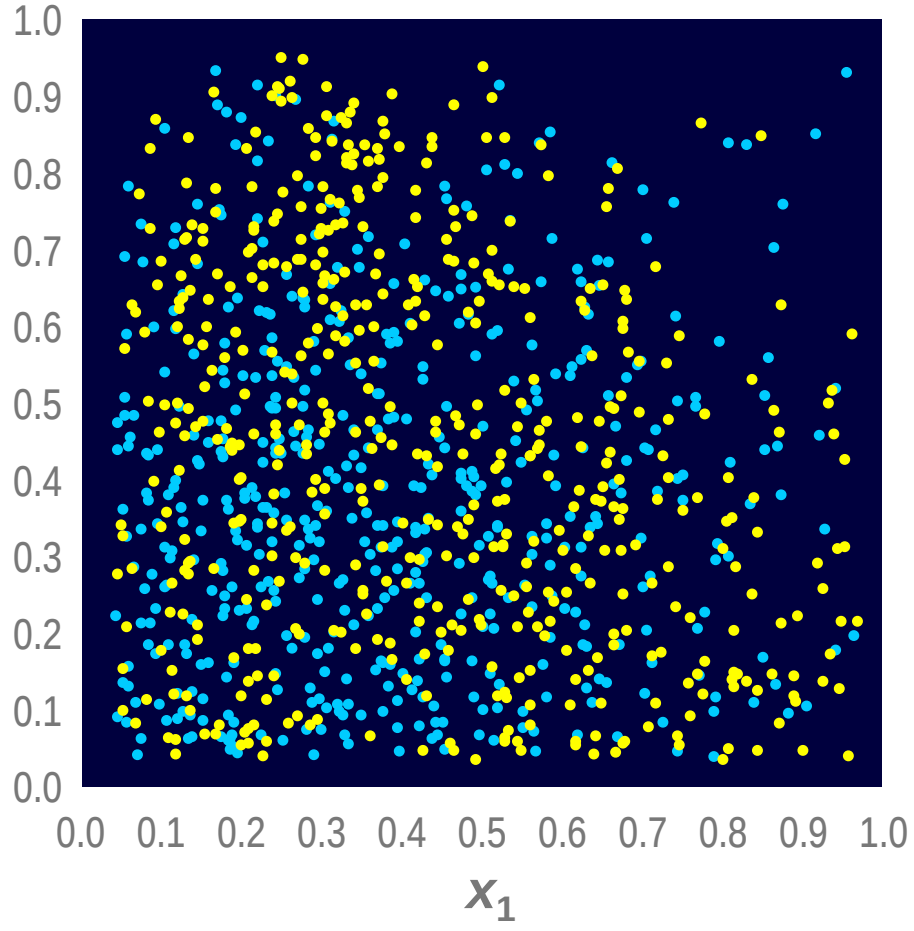
Weight estimates are found by *maximizing*:

$$\sum_{\substack{\text{primary} \\ \text{outcome} \\ \text{training cases}}} \log(\hat{p}_i) + \sum_{\substack{\text{secondary} \\ \text{outcome} \\ \text{training cases}}} \log(1 - \hat{p}_i)$$

Log-likelihood Function



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Prediction Illustration – Neural Networks

logit equation

$$\text{logit}(\hat{p}) = -0.5 + -2.6\ H_1 + -1.9\ H_2 + 0.63\ H_3$$

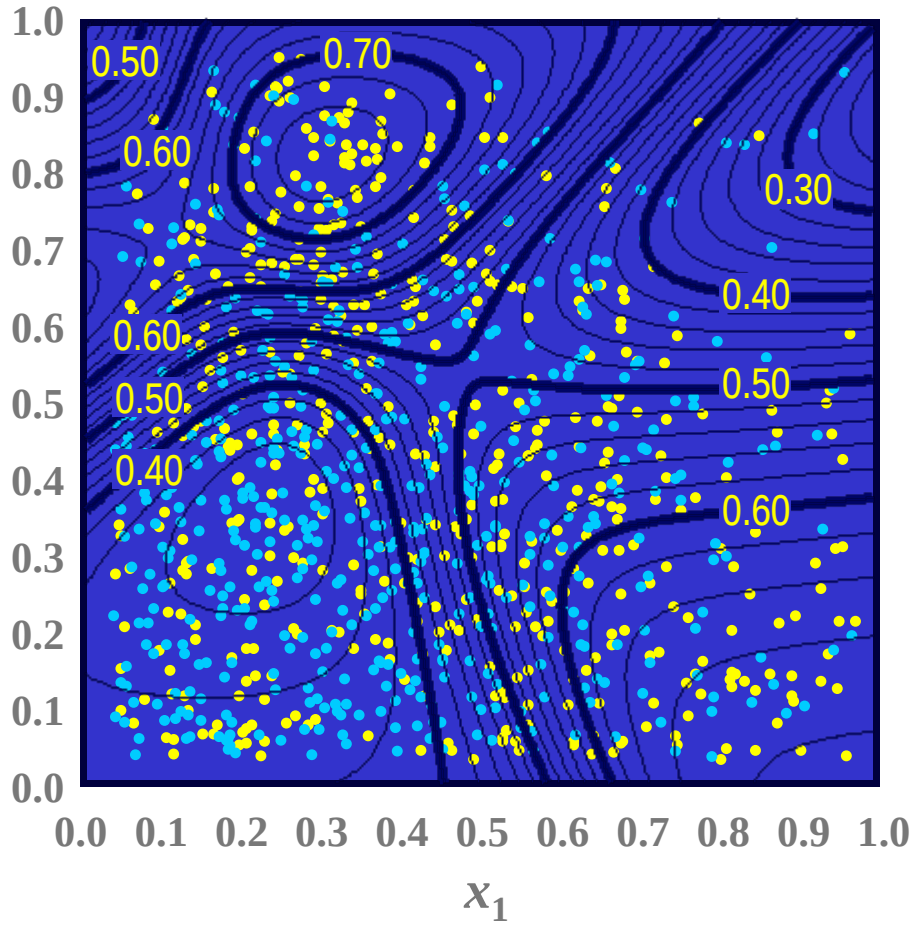
$$H_1 = \tanh(-1.8 + 0.25\ x_1 + -1.8\ x_2)$$

$$H_2 = \tanh(2.7 + 2.7\ x_1 + -5.3\ x_2)$$

$$H_3 = \tanh(-5.0 + 8.1\ x_1 + 4.3\ x_2)$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

Probability estimates are obtained by solving the logit equation for $\hat{p}$ for each (x_1, x_2).



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Neural Nets: Beyond the Prediction Formula

- Manage missing values.
- Handle extreme or unusual values.
- Use non-numeric inputs.
- Account for nonlinearities.
- Interpret the model.

...

Model Essentials – Neural Networks

- Predict new cases.
- Select useful inputs.
- Optimize complexity.

Prediction formula

None

Best model from sequence

Model Essentials – Neural Networks

- Predict new cases.
- Select useful inputs.
- Optimize complexity.

Prediction formula

Sequential selection

Stopped training

...

Fit Statistic versus Optimization Iteration

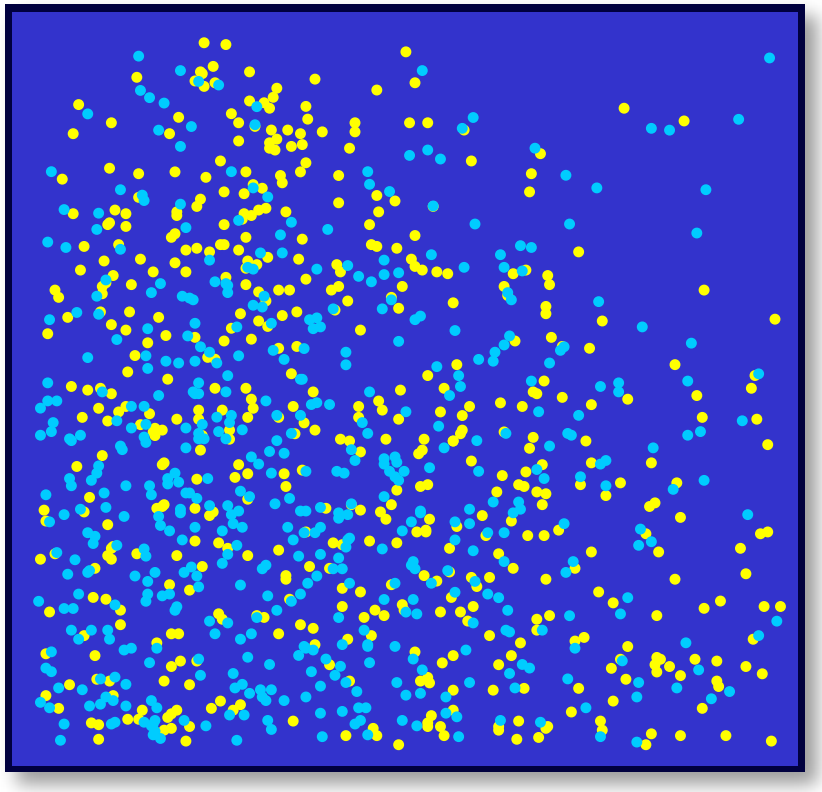
initial hidden unit weights

$$\text{logit}(\hat{p}) = 0 + 0 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3$$

$$H_1 = \tanh(-1.5 - .03x_1 - .07x_2)$$

$$H_2 = \tanh(.79 - .17x_1 - .16x_2)$$

$$H_3 = \tanh(.57 + .05x_1 + .35x_2)$$



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Fit Statistic versus Optimization Iteration

$$\text{logit}(\hat{p}) = 0 + 0 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3$$

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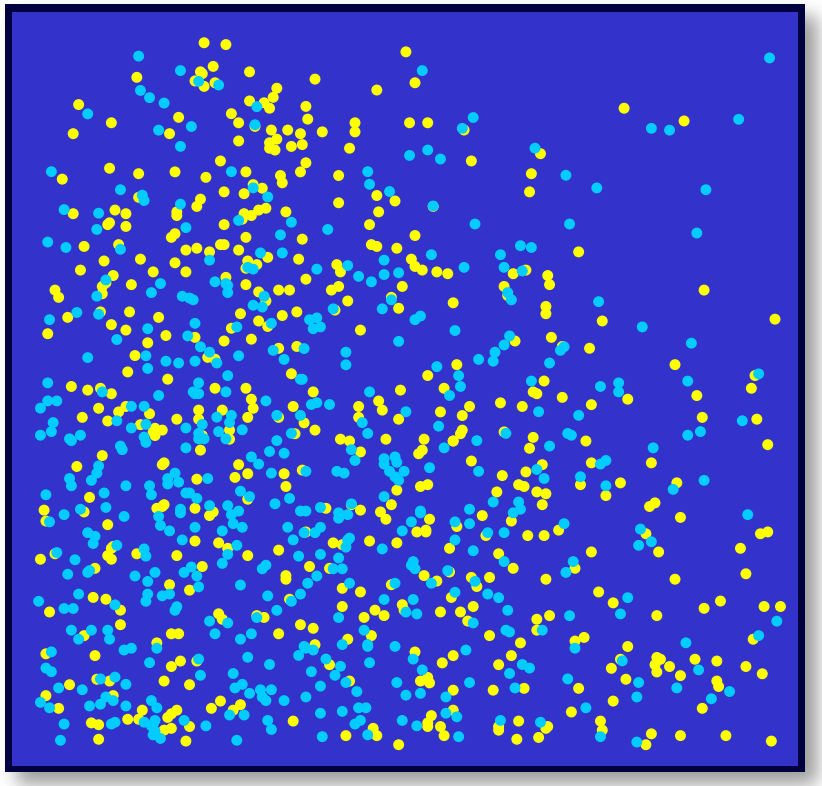
$$H_3 = \tanh(.57 + .05x_1 + .35x_2)$$

random initial
input weights and biases



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Fit Statistic versus Optimization Iteration

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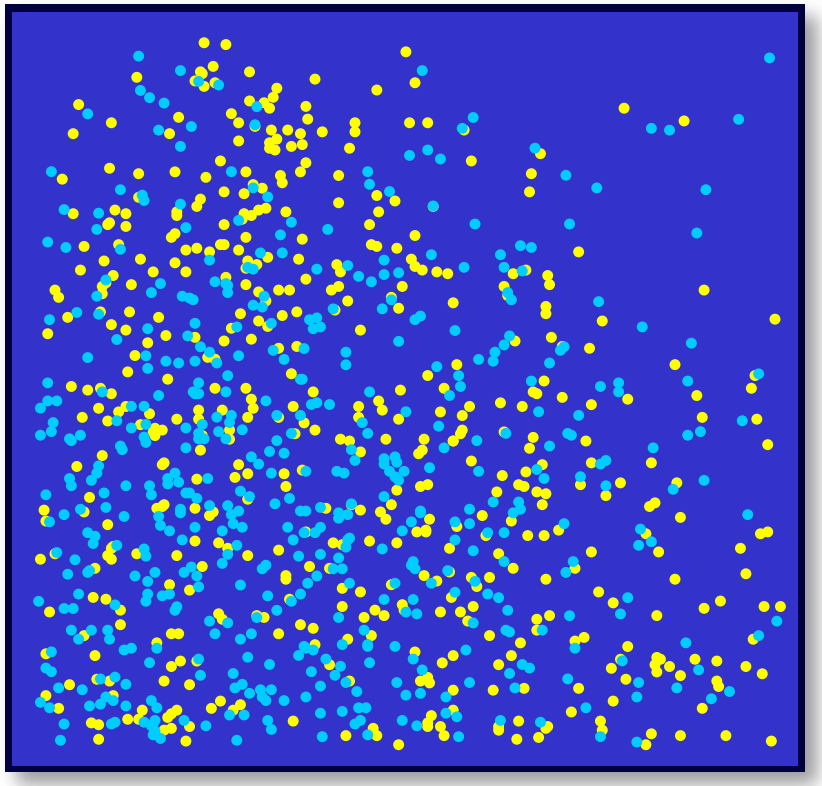

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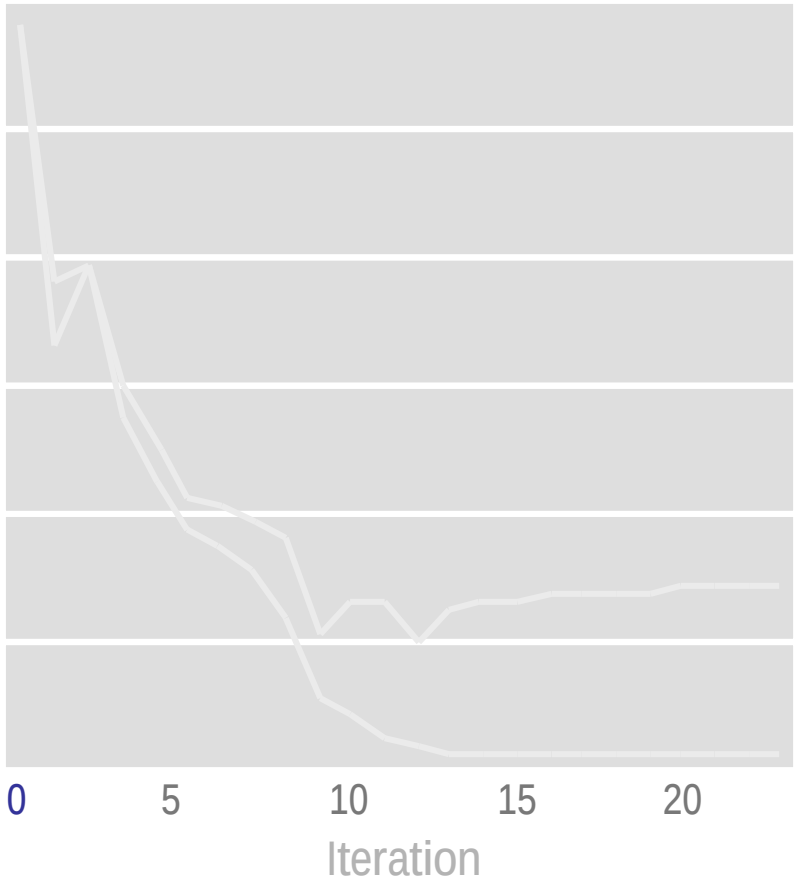
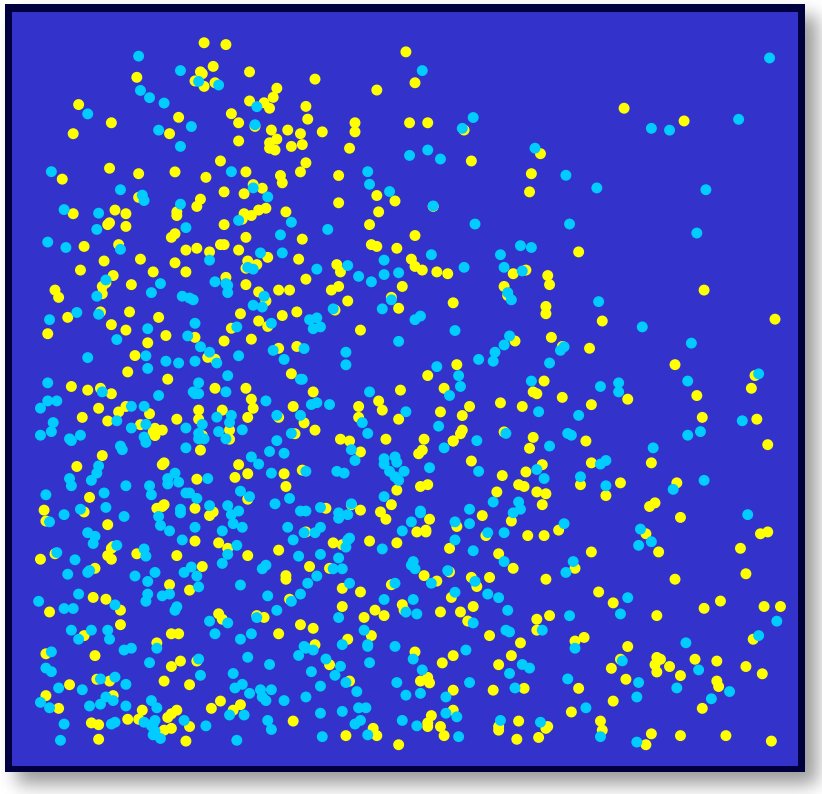
Fit Statistic versus Optimization Iteration





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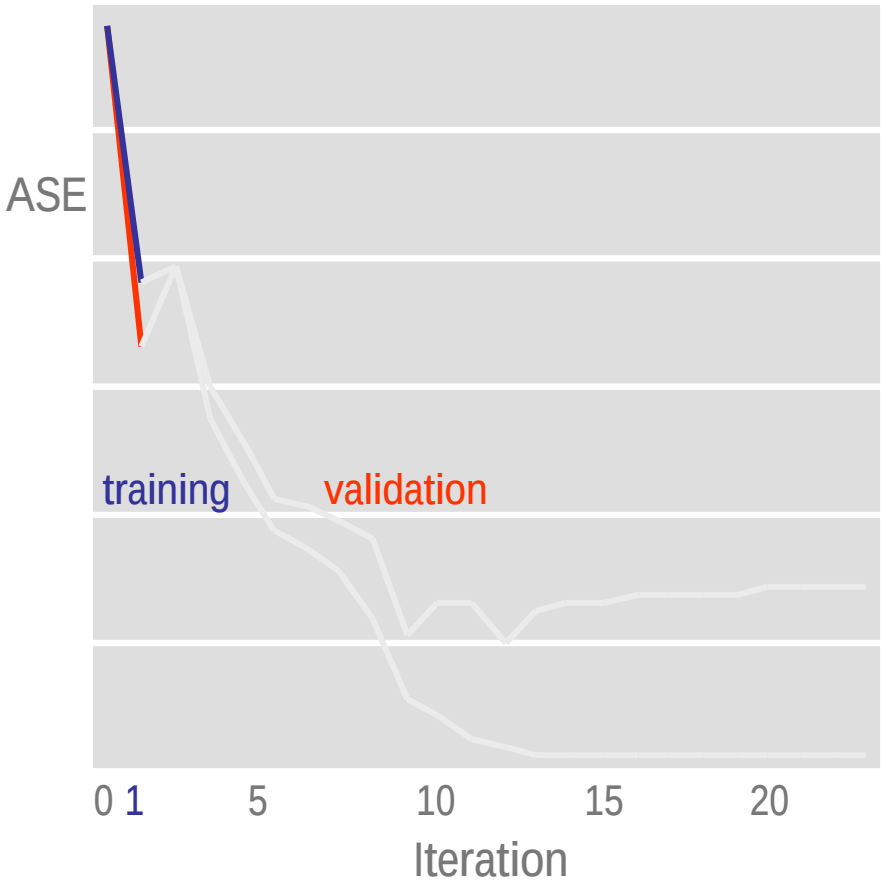
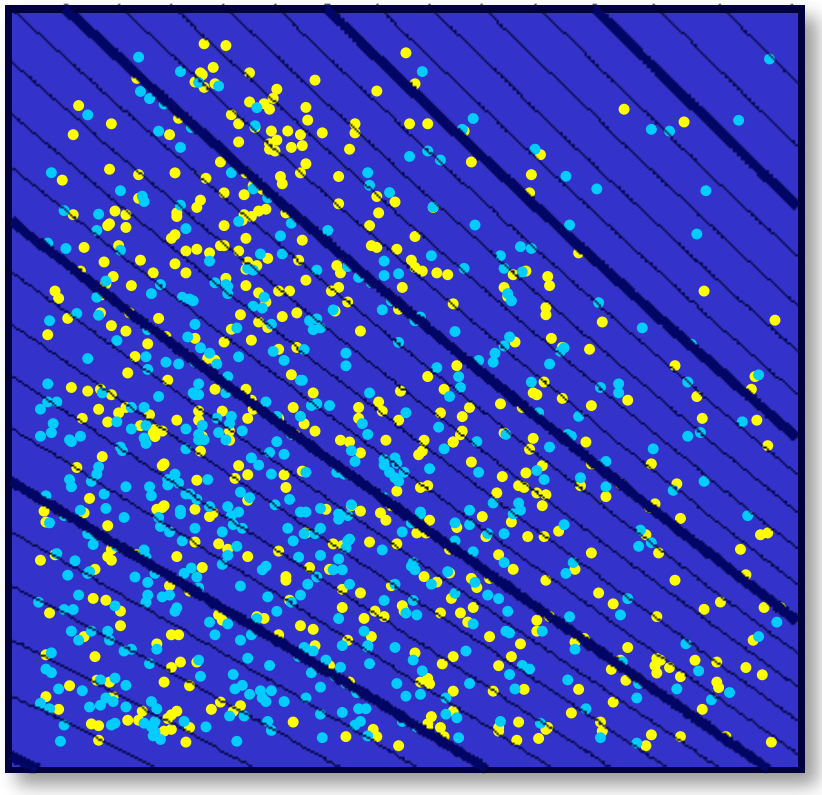
Fit Statistic versus Optimization Iteration





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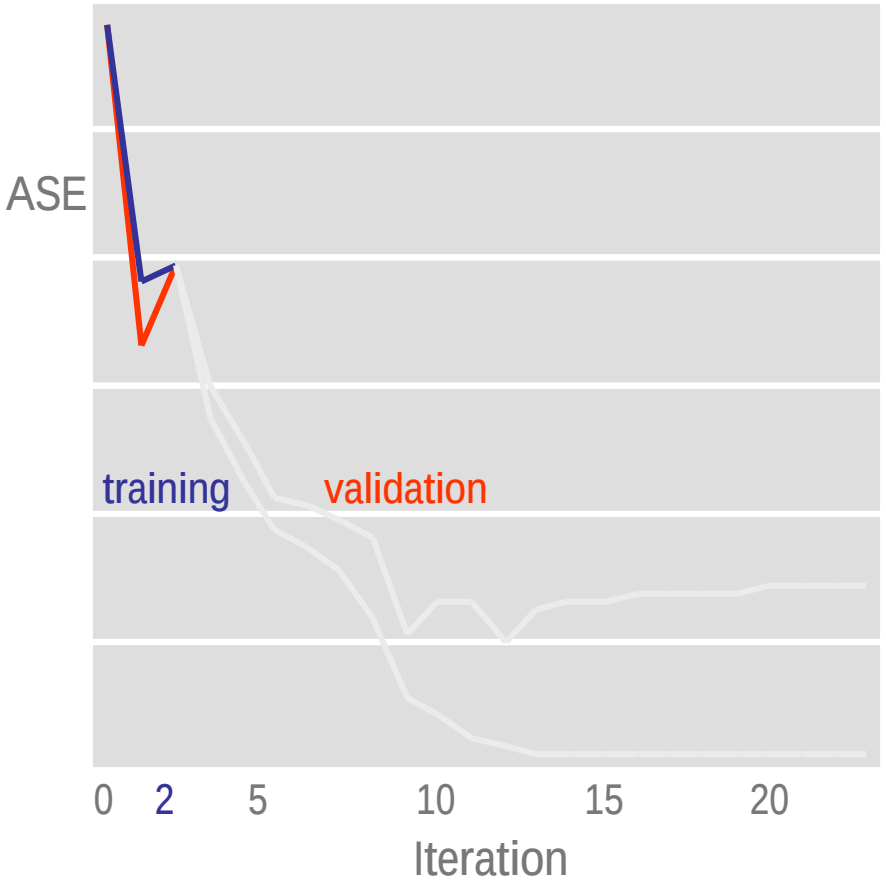

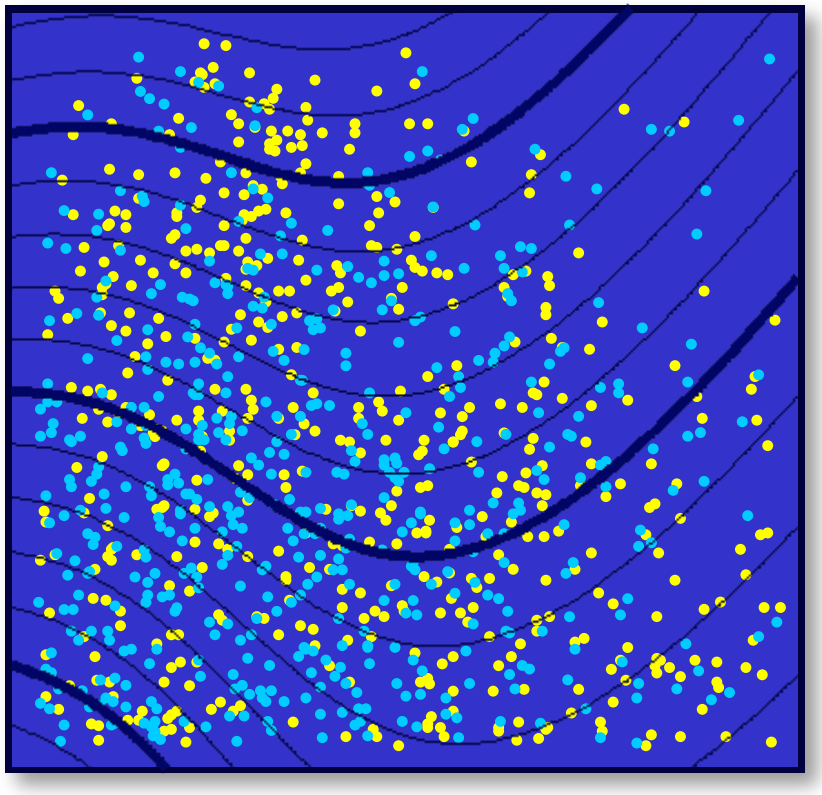

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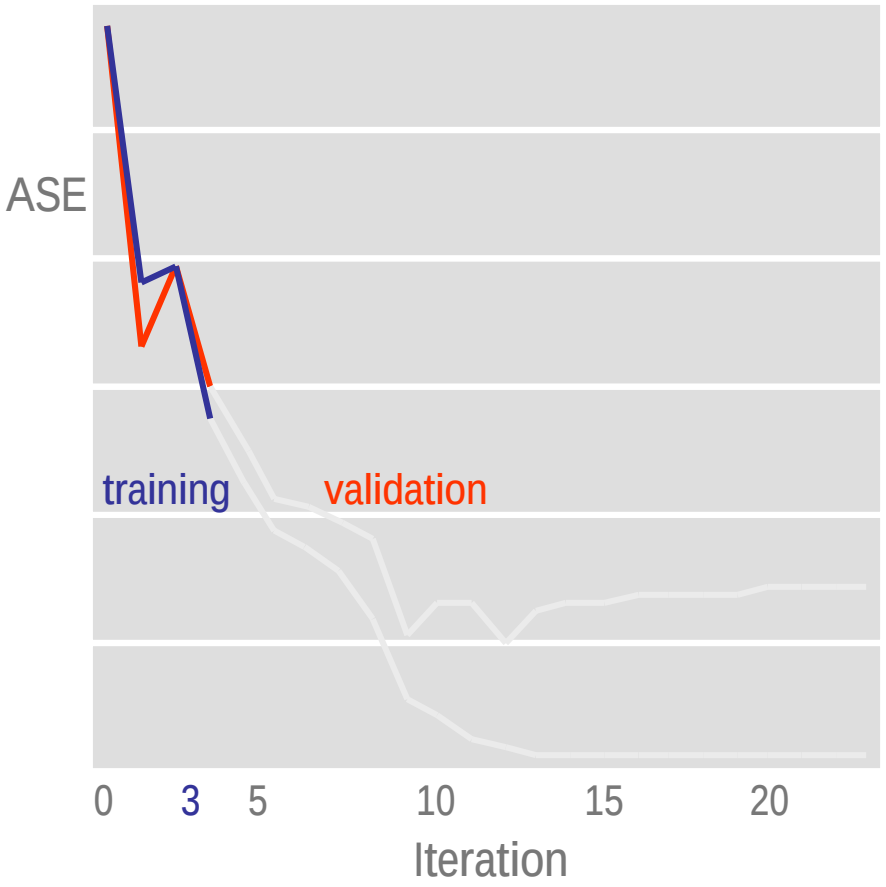

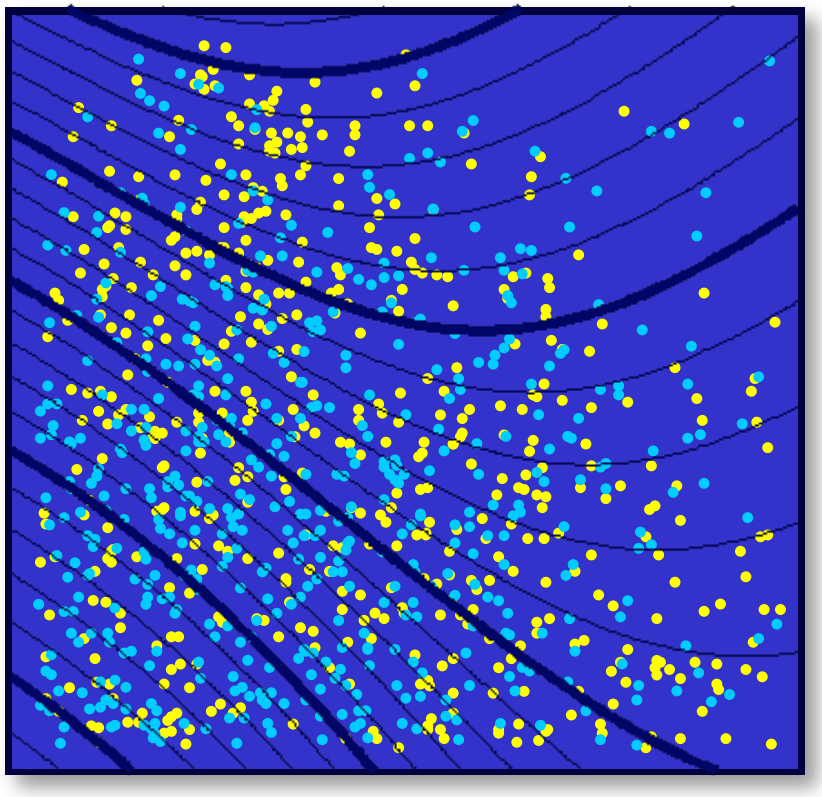

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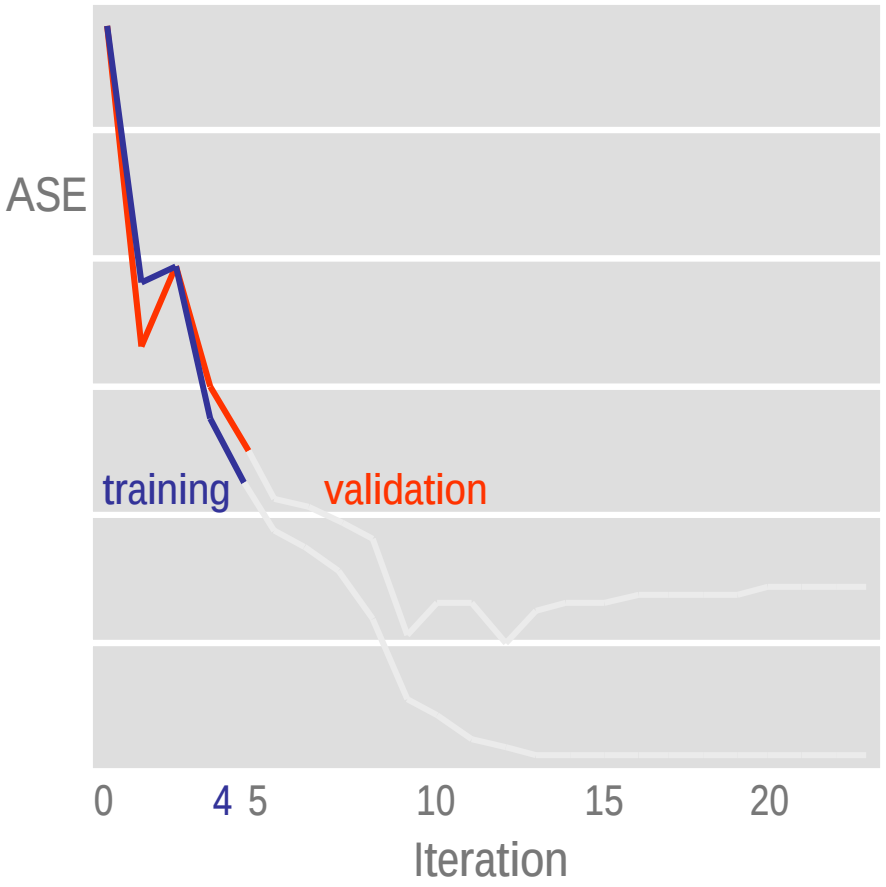

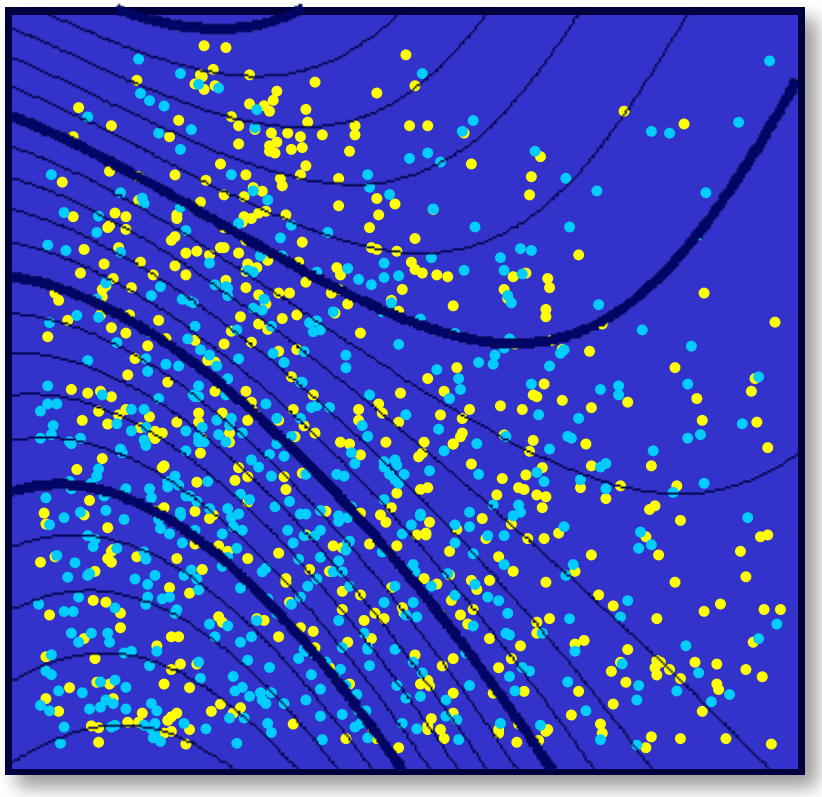

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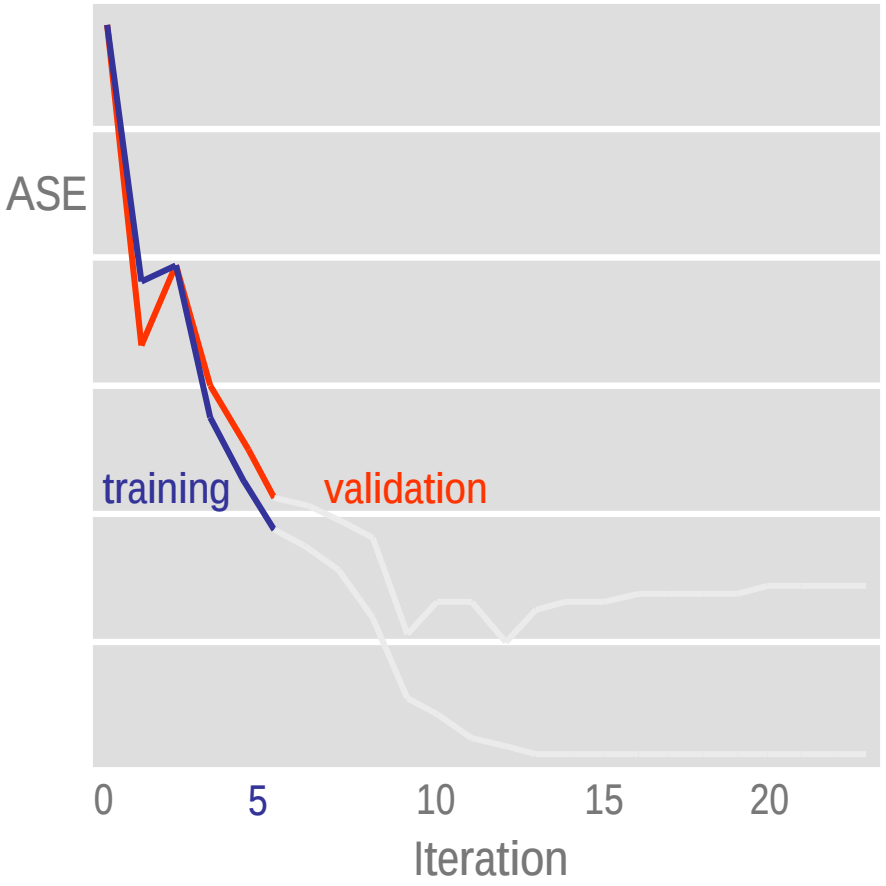

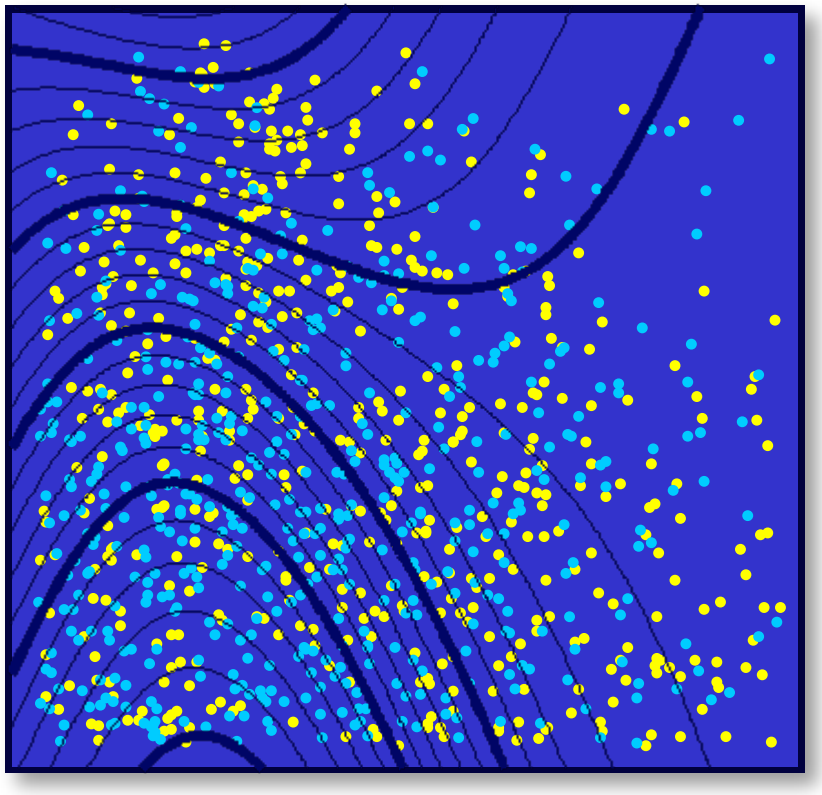

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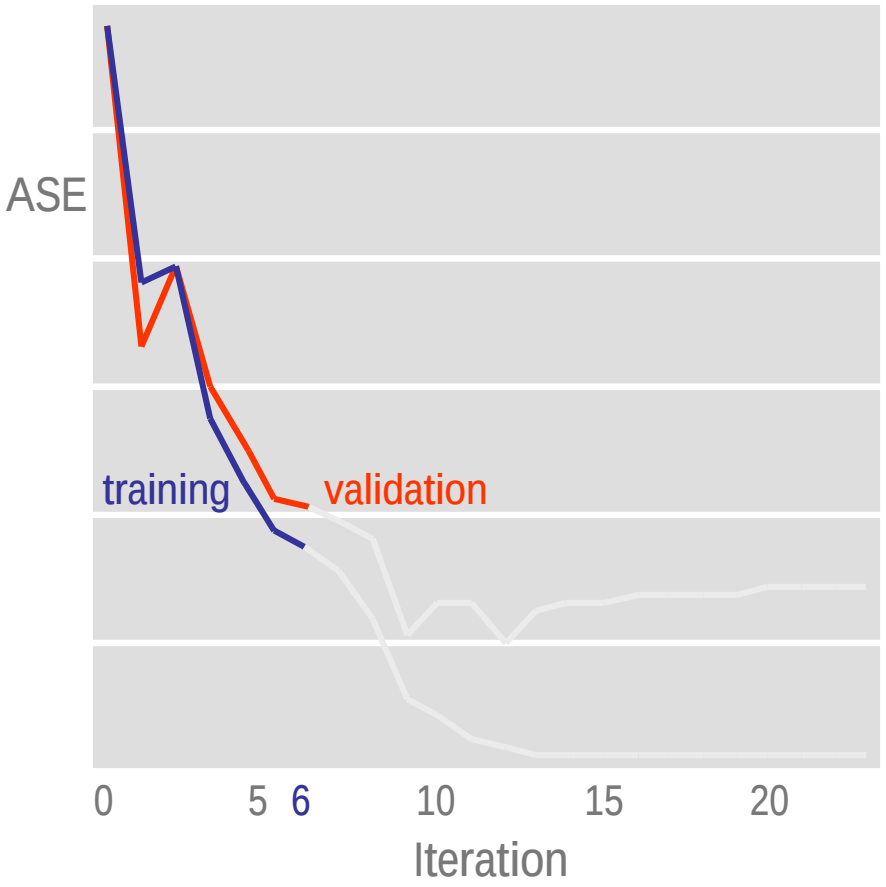

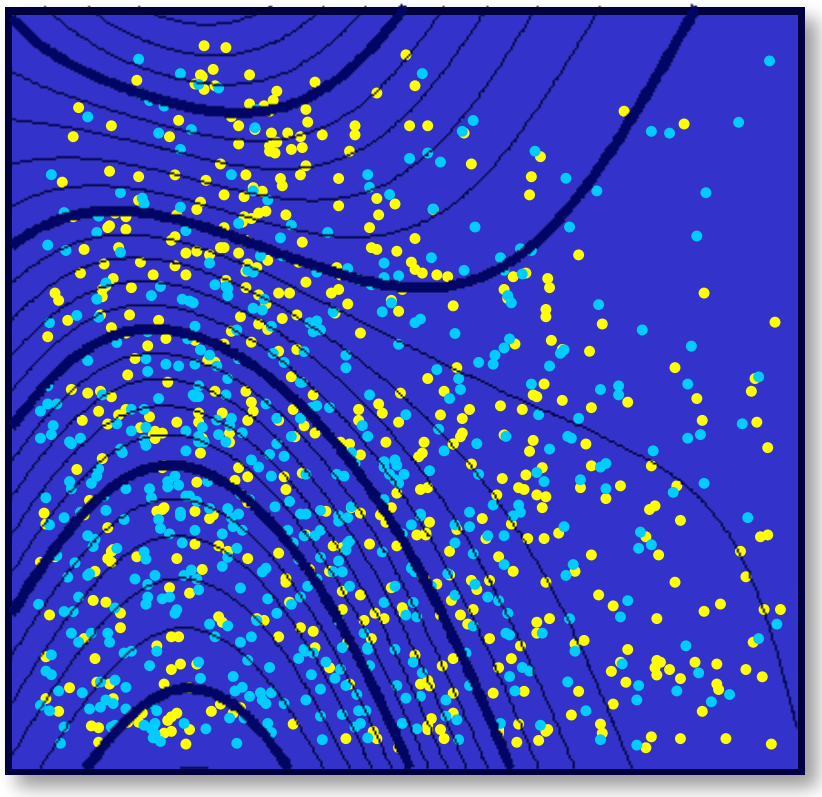

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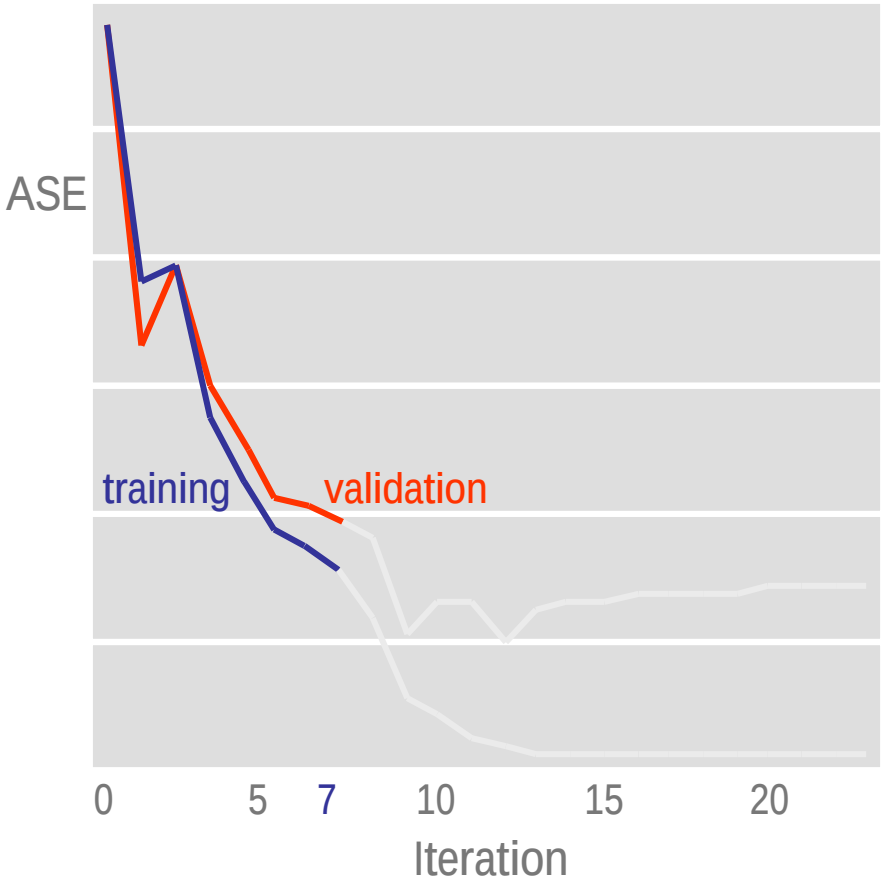

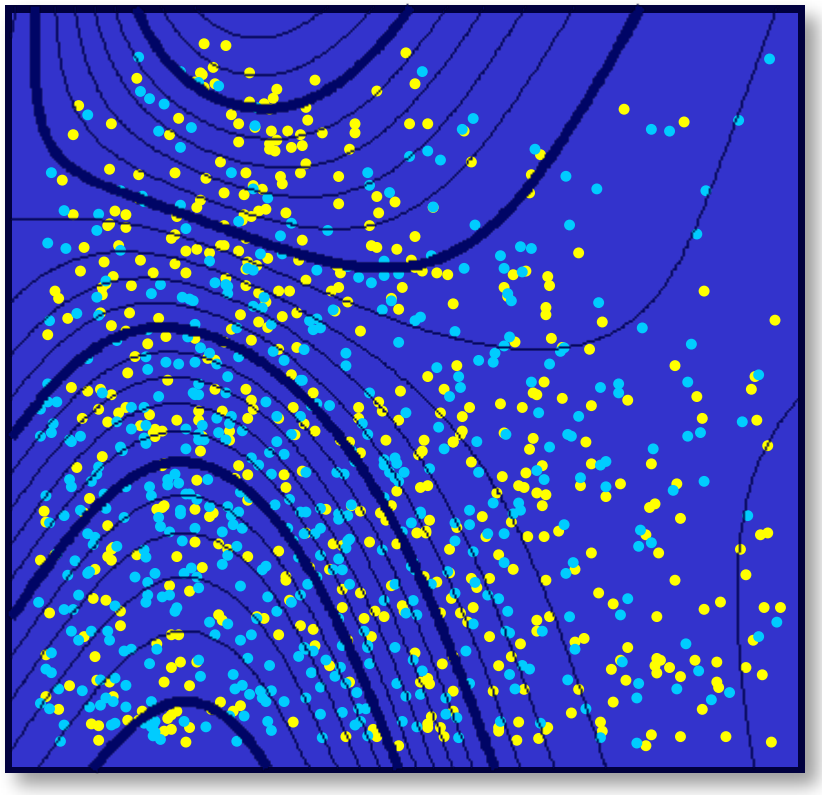

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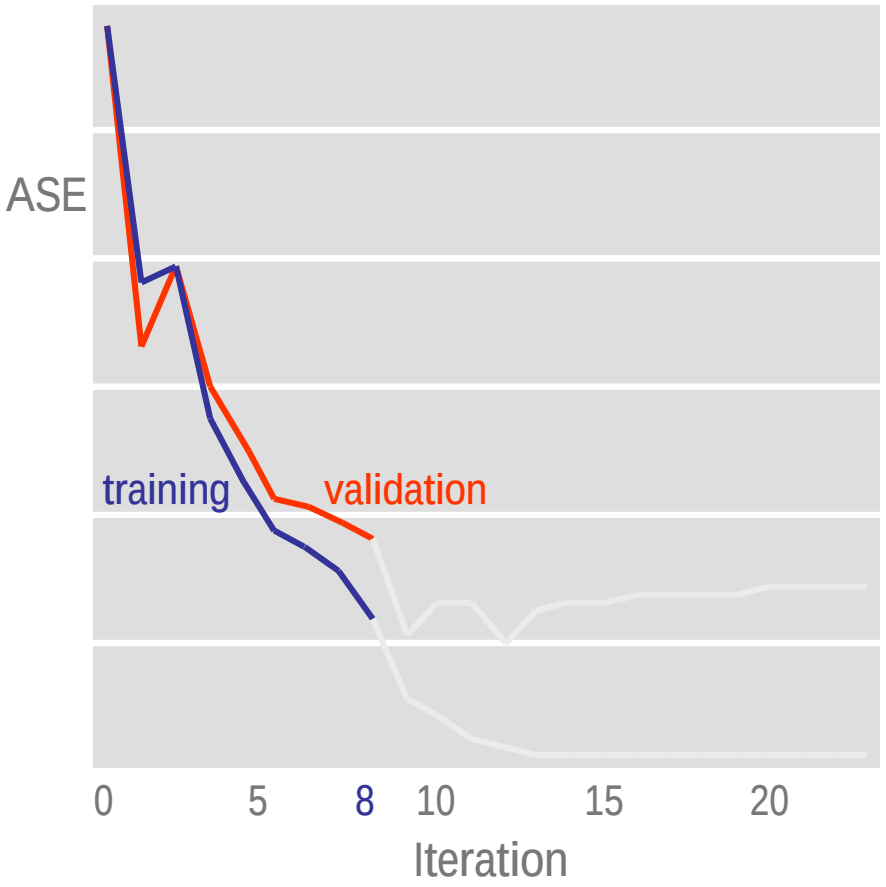

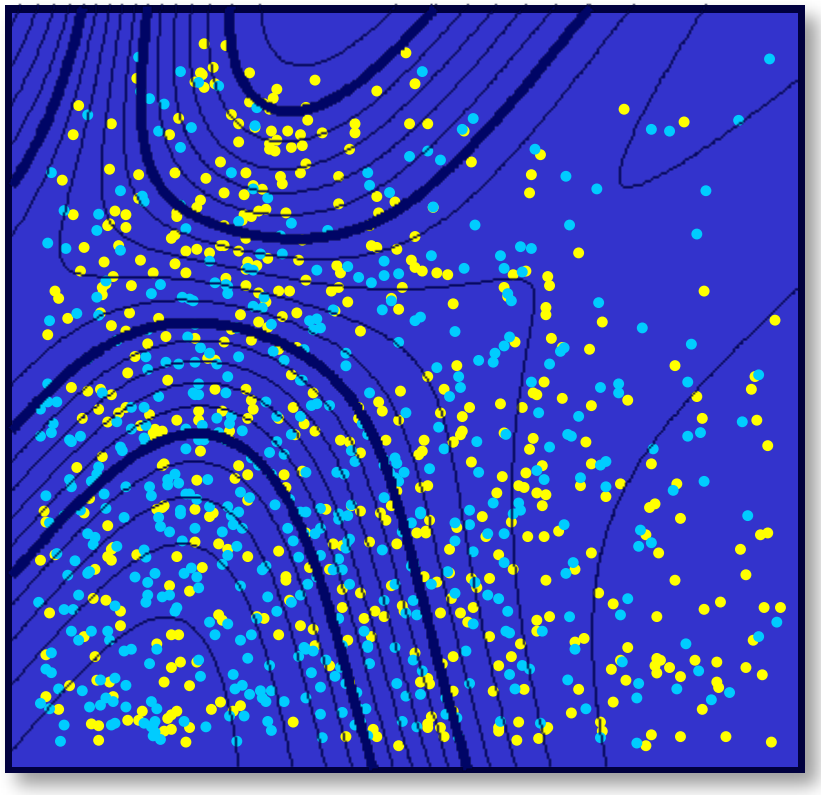

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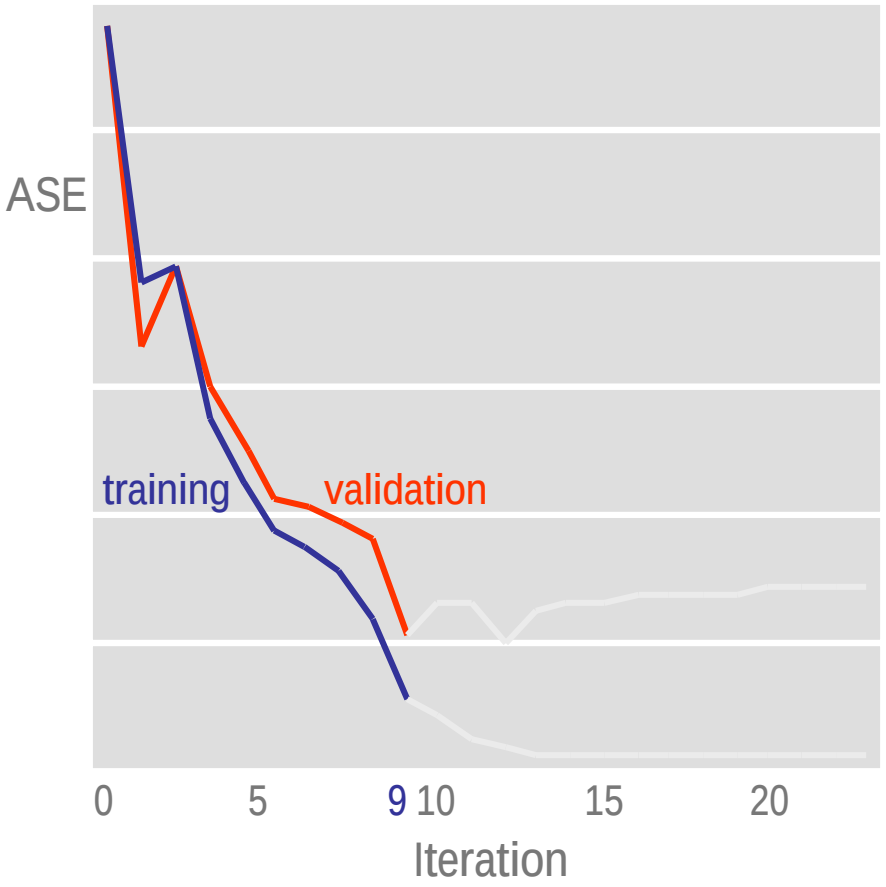

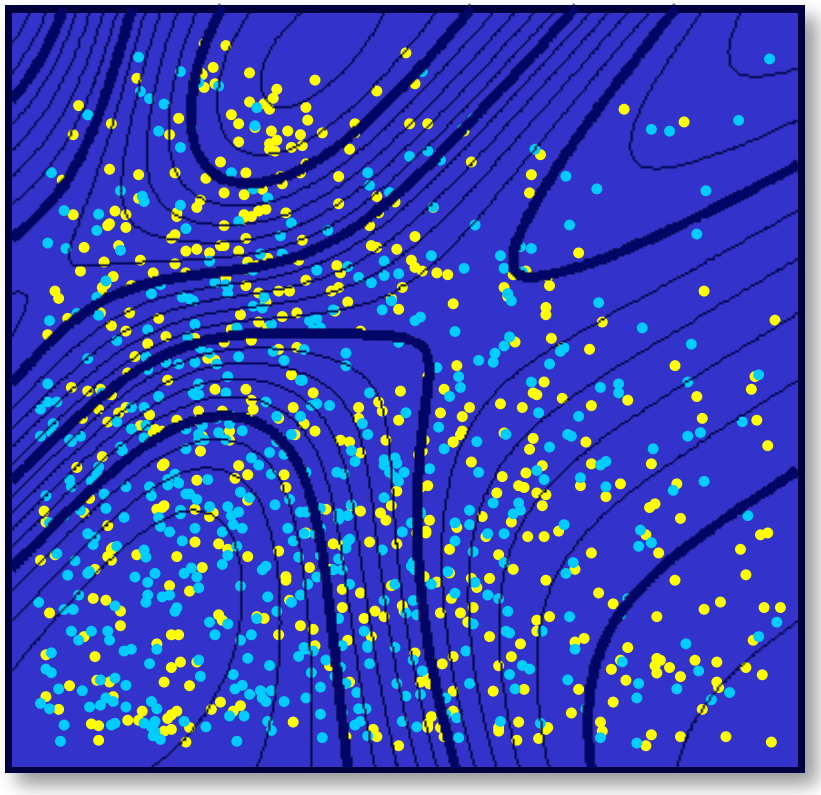

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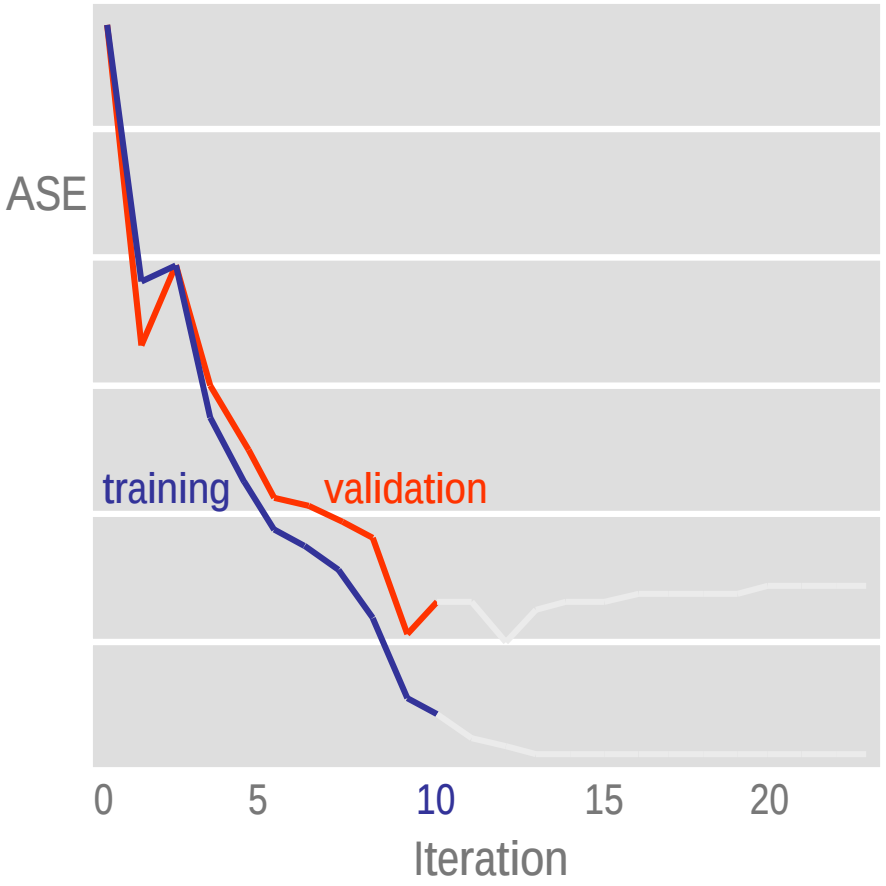

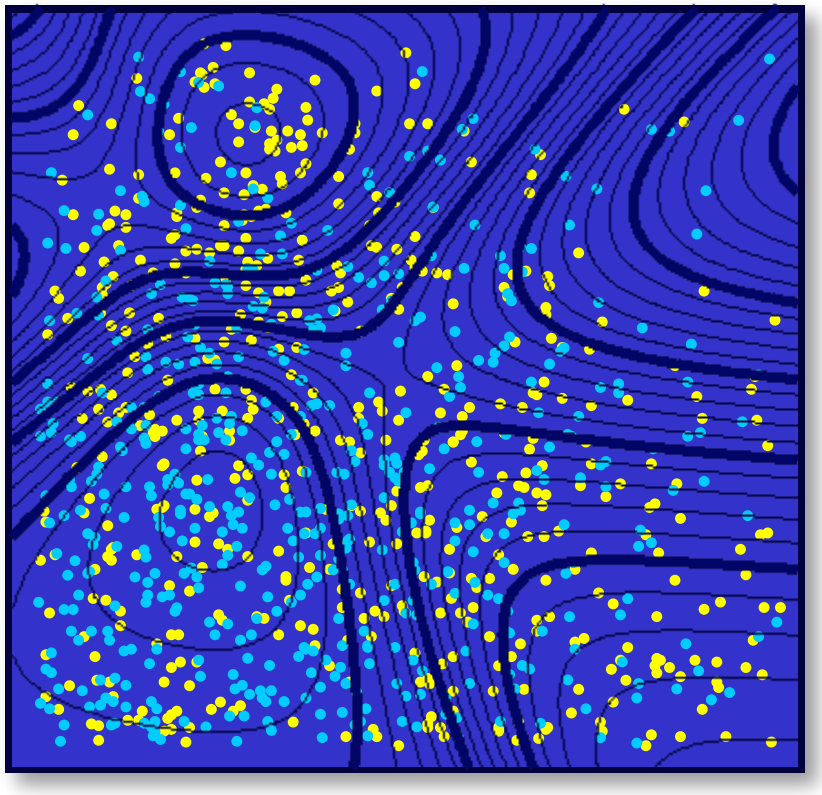

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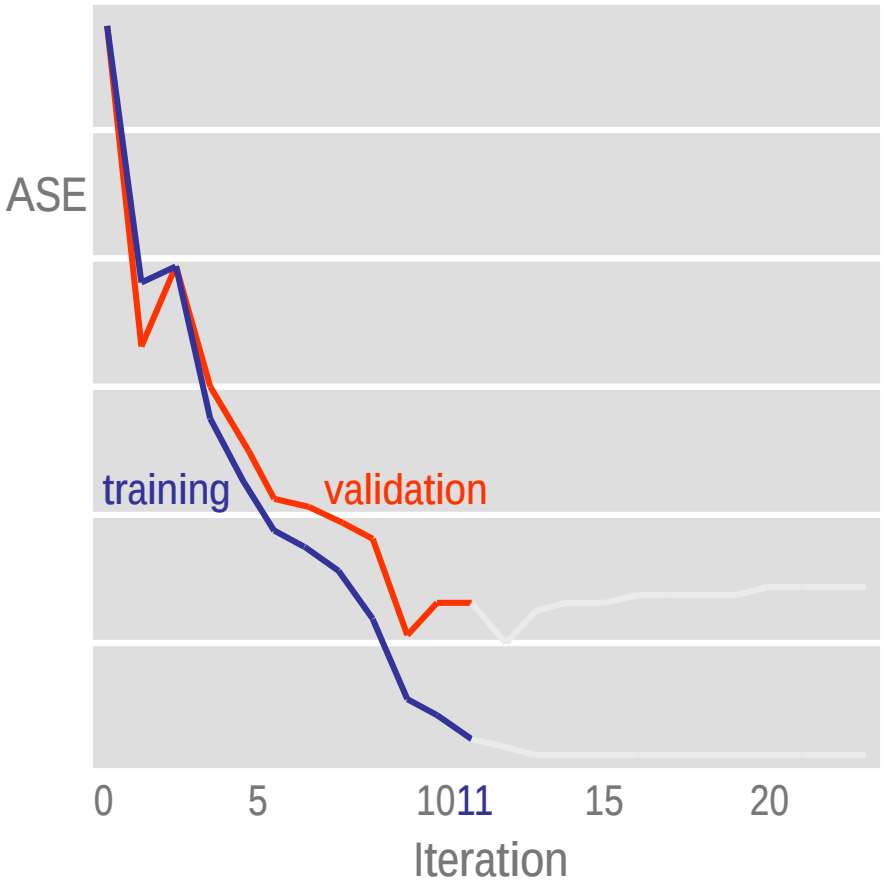

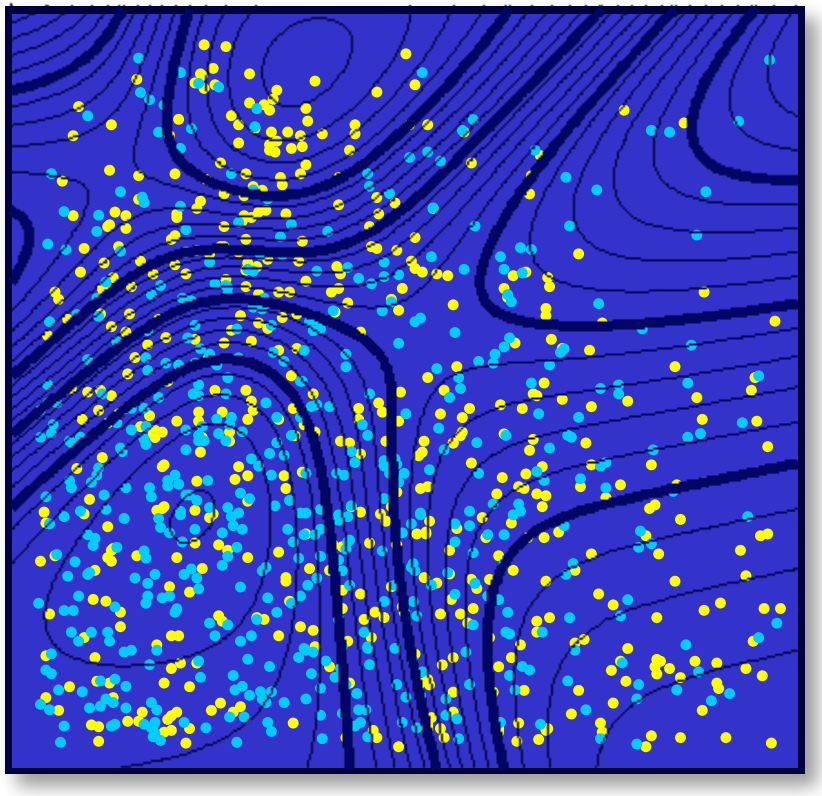

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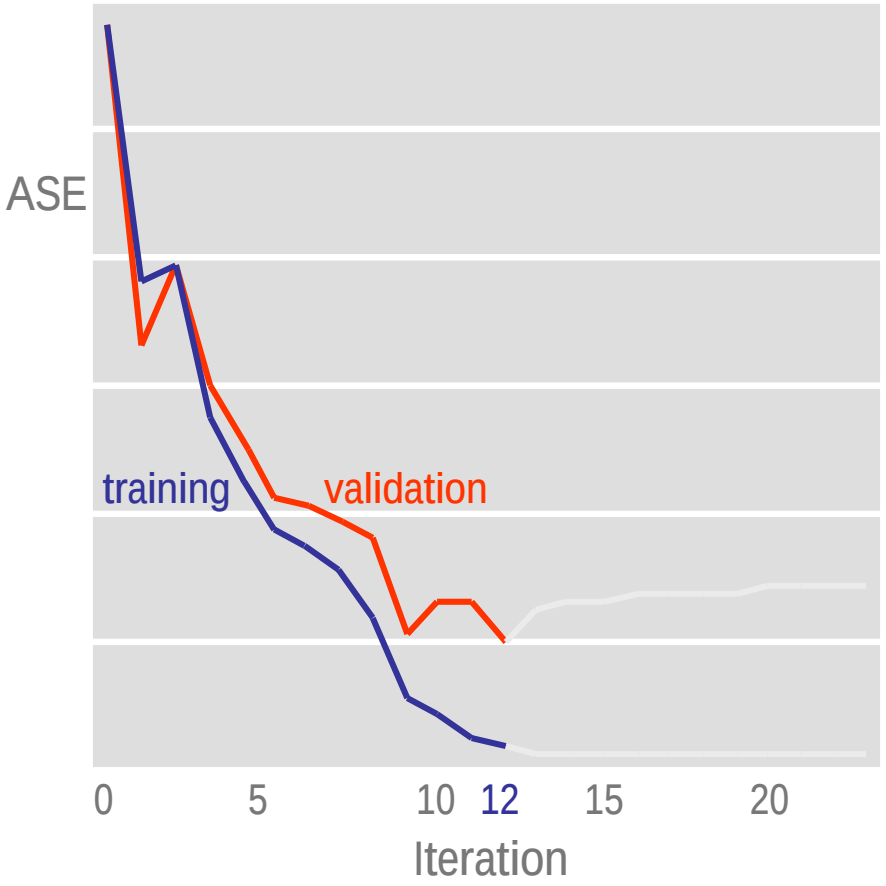
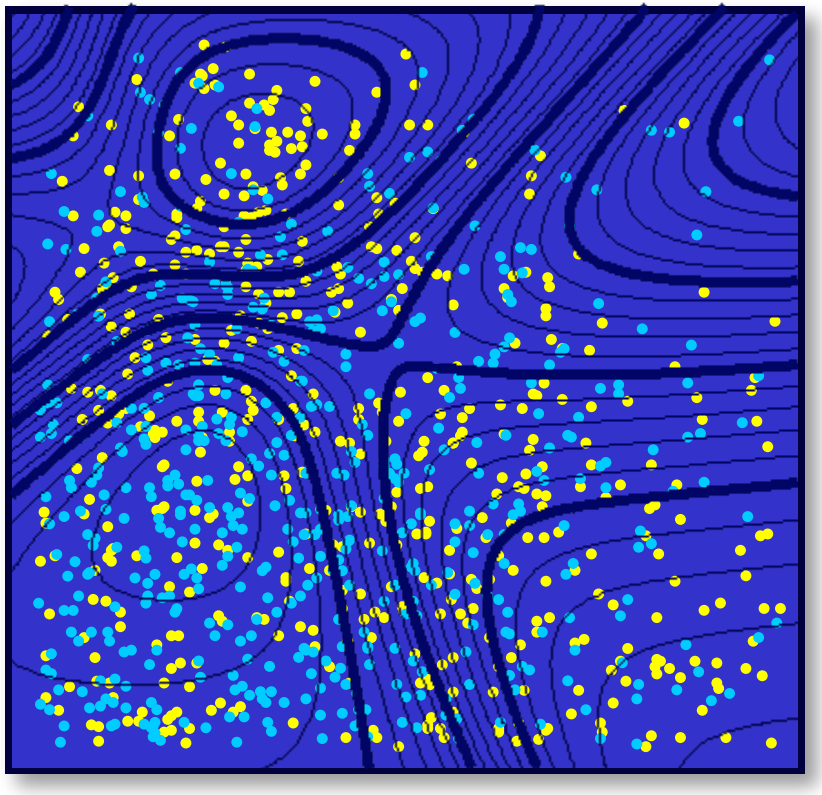
Fit Statistic versus Optimization Iteration





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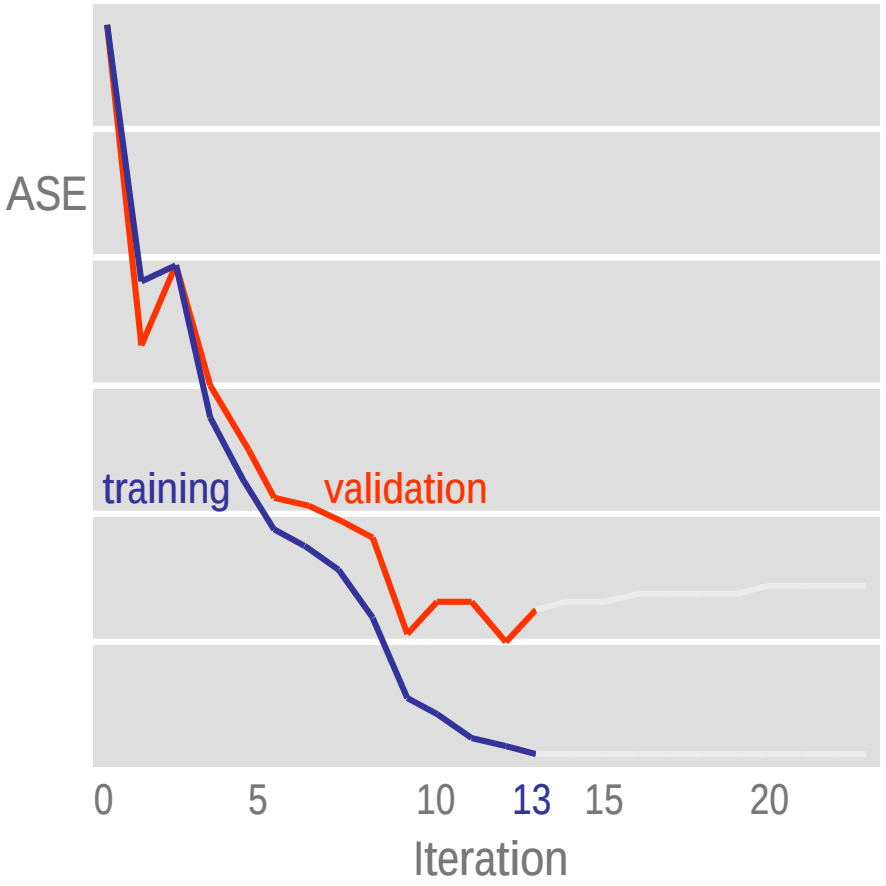
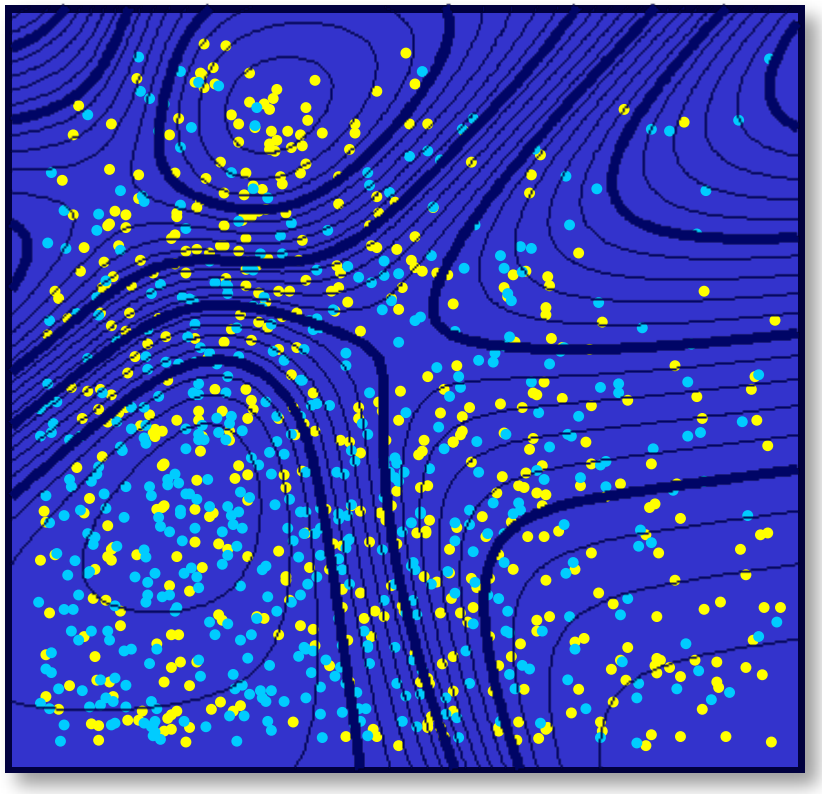
Fit Statistic versus Optimization Iteration





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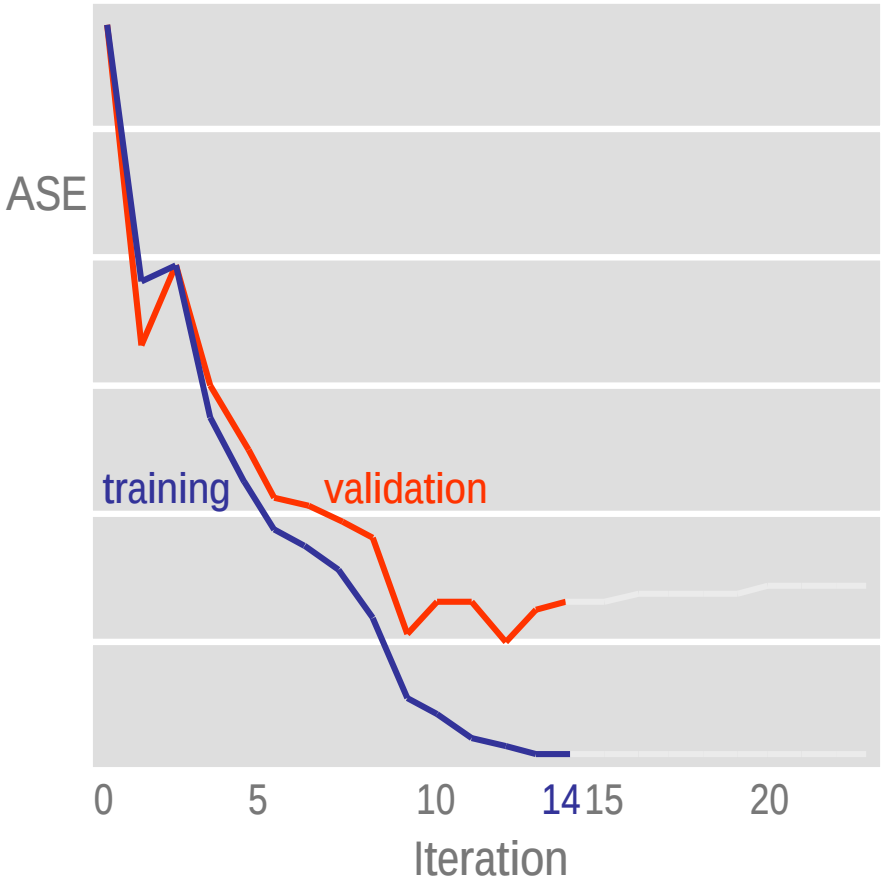
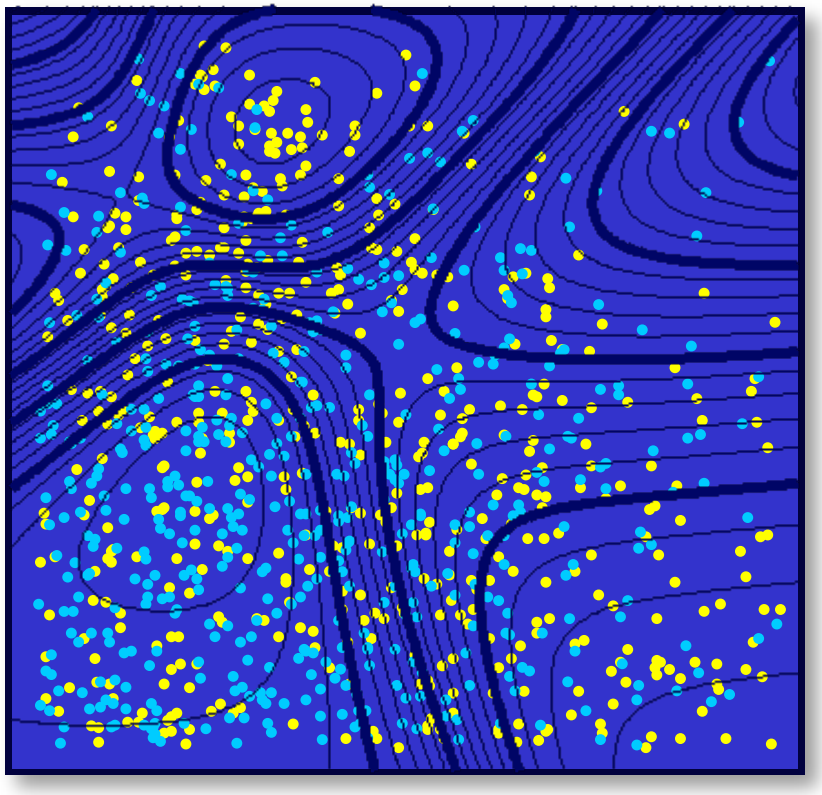
Fit Statistic versus Optimization Iteration





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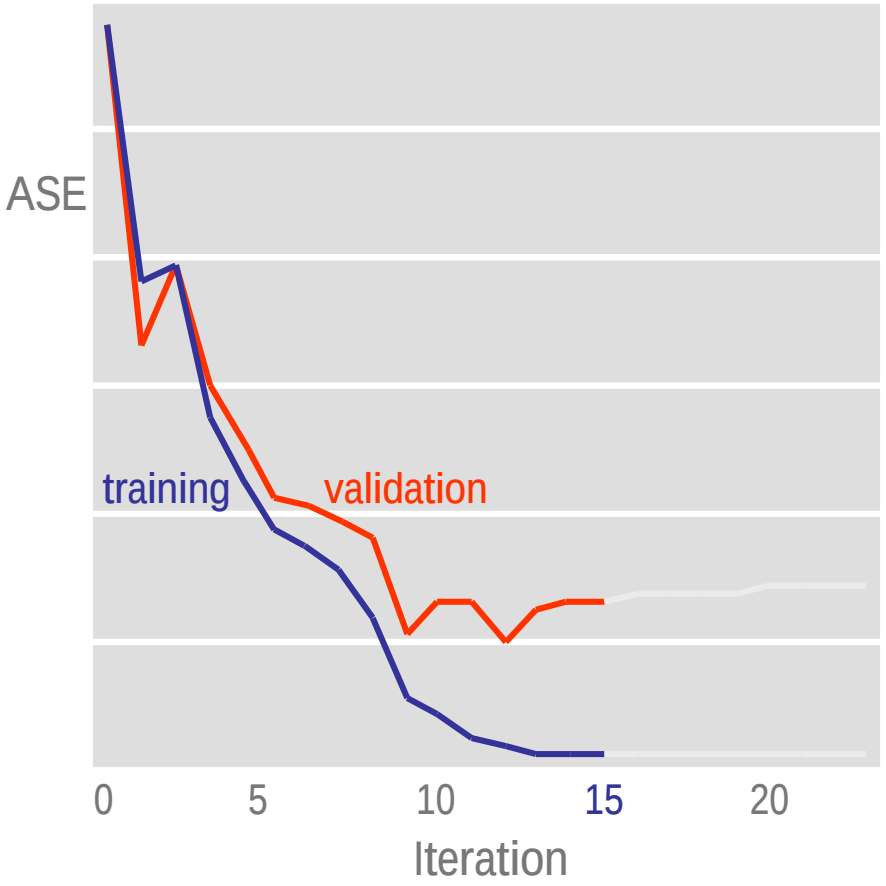
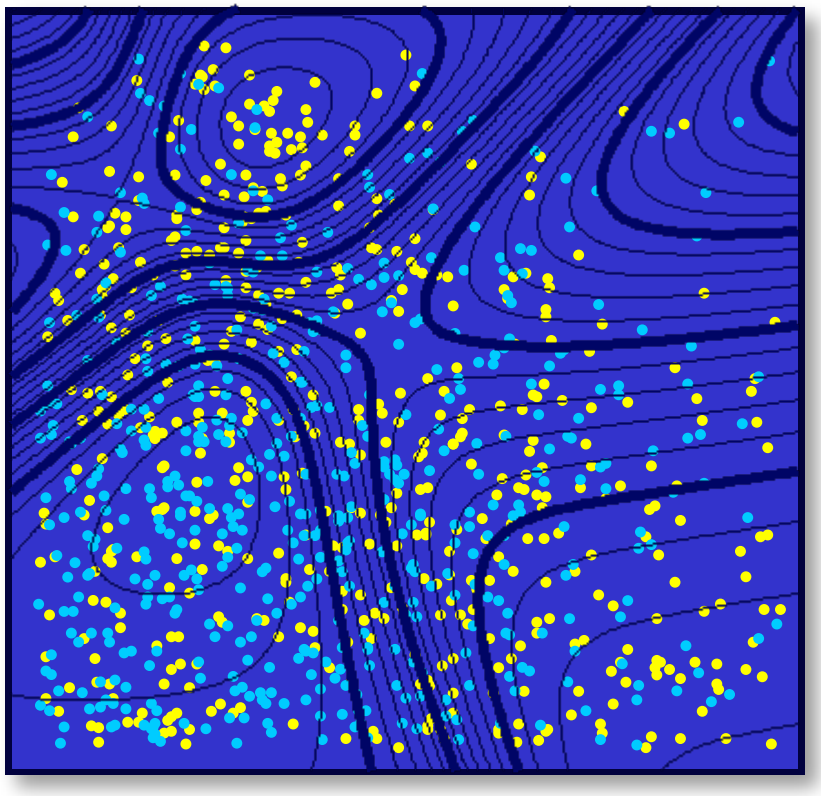
Fit Statistic versus Optimization Iteration





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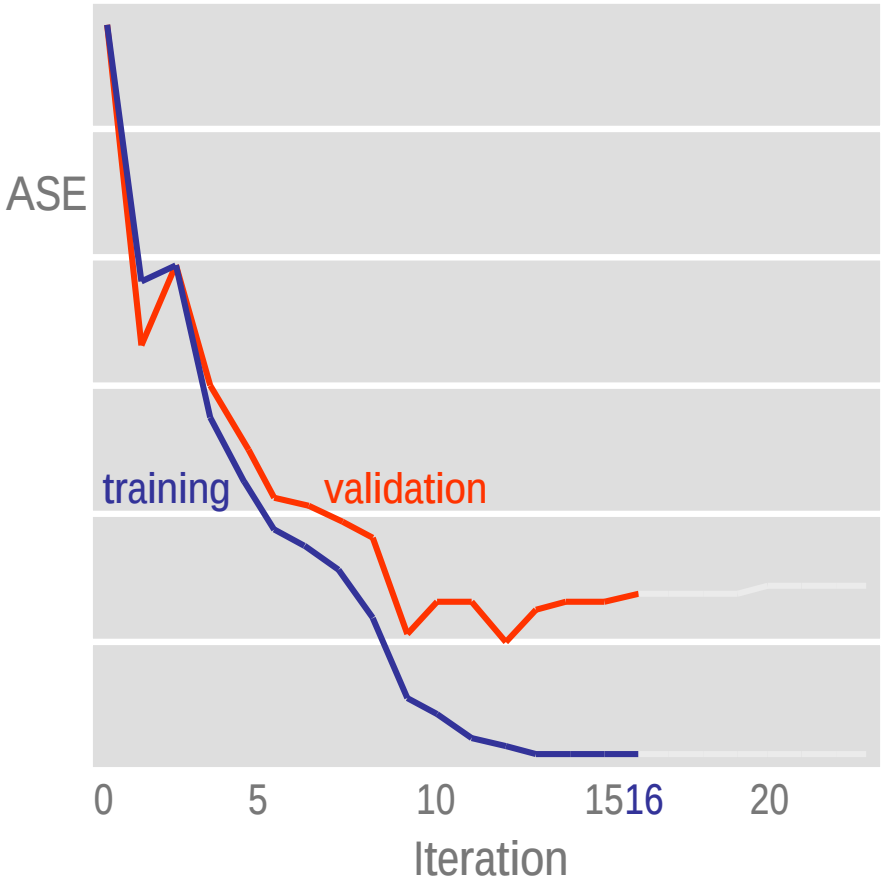

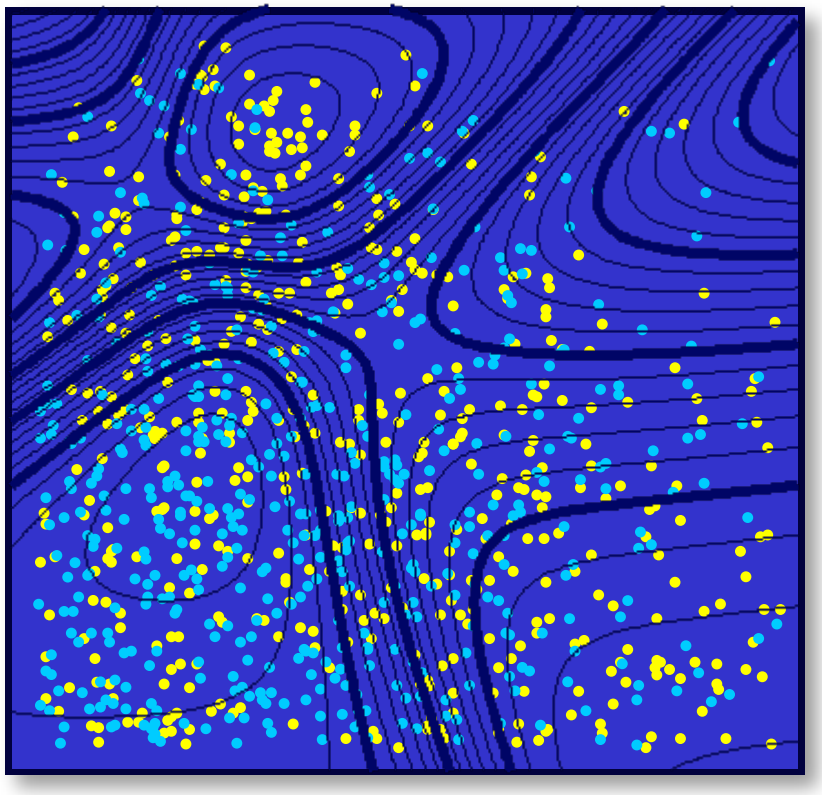

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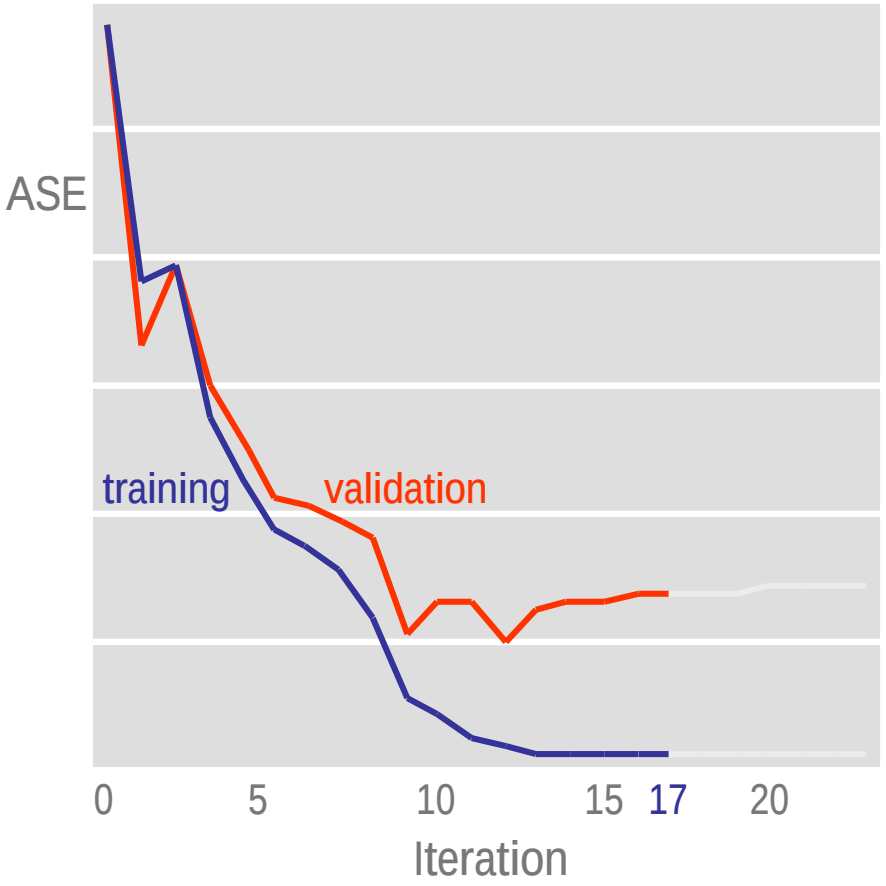

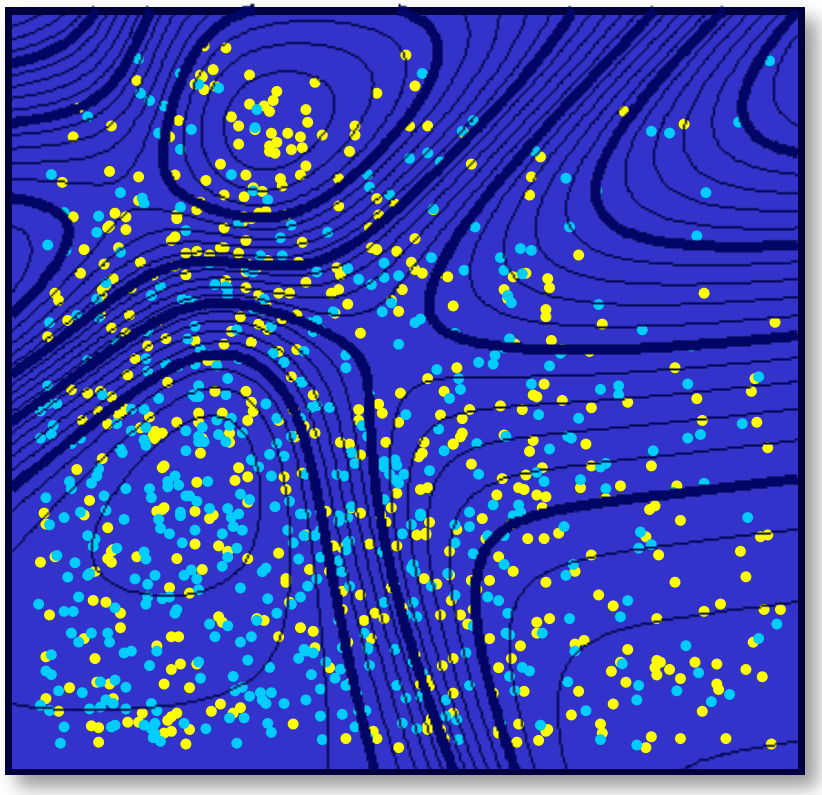

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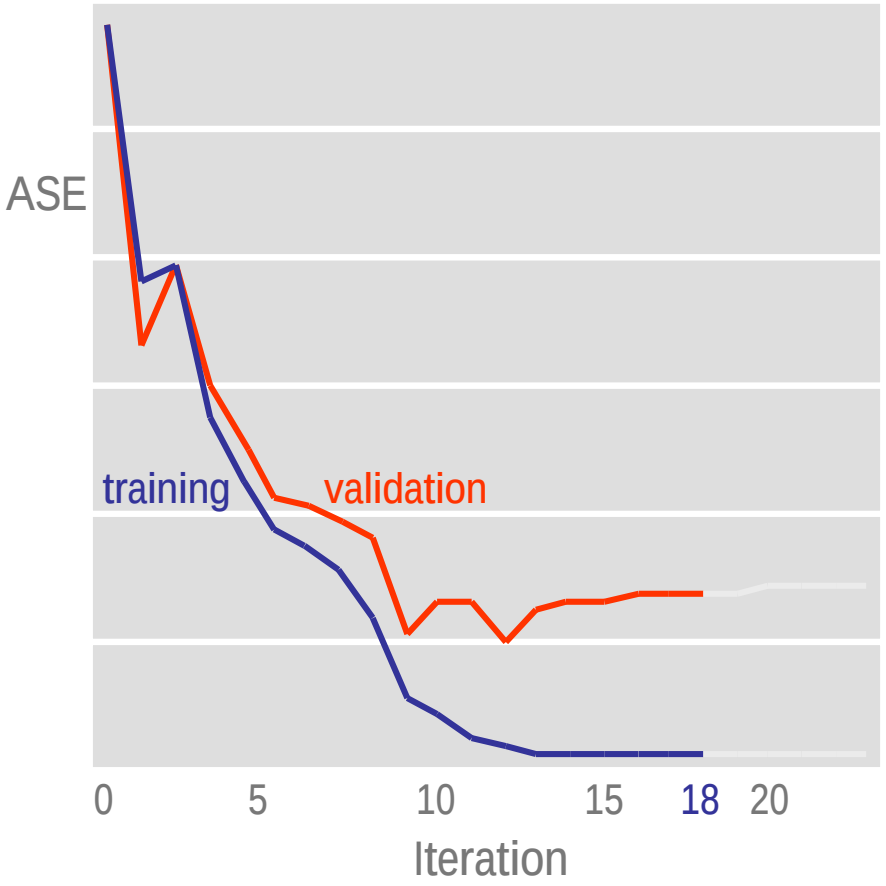

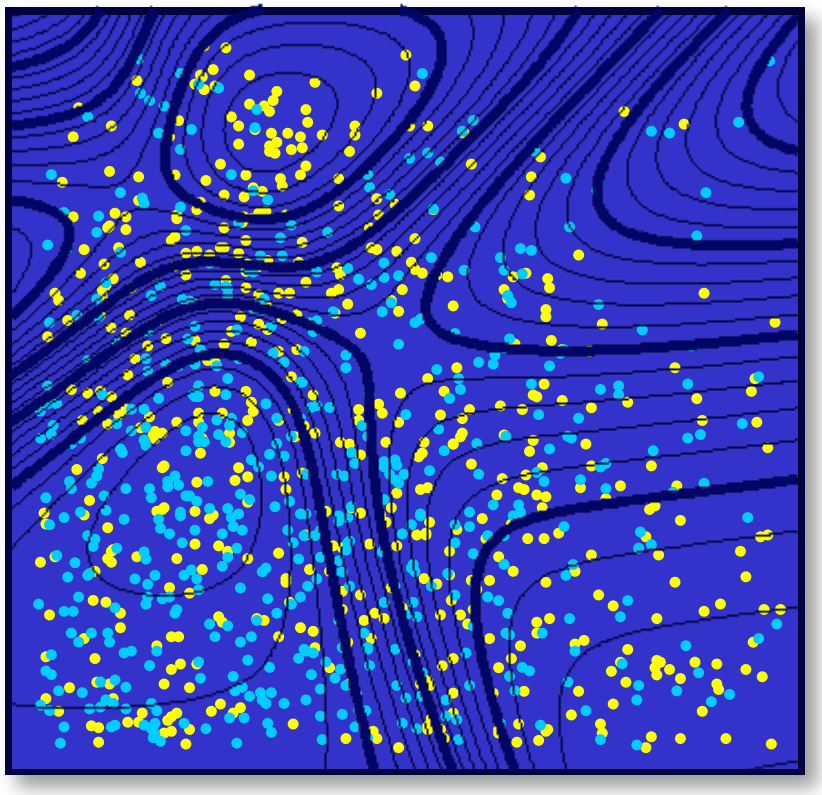

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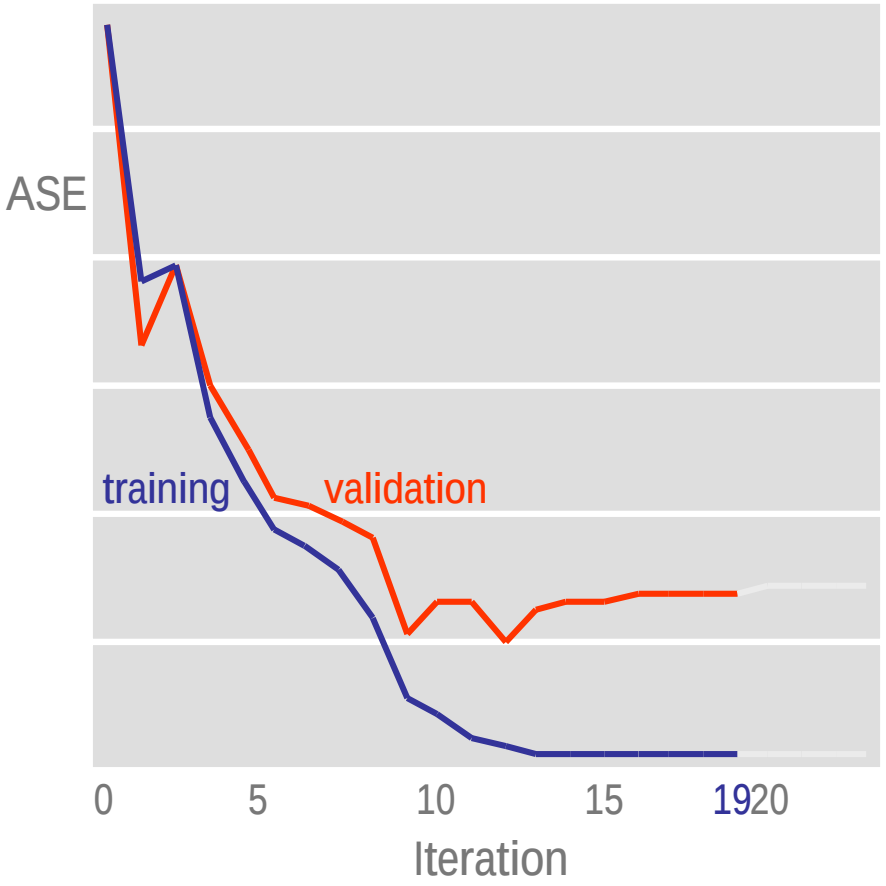

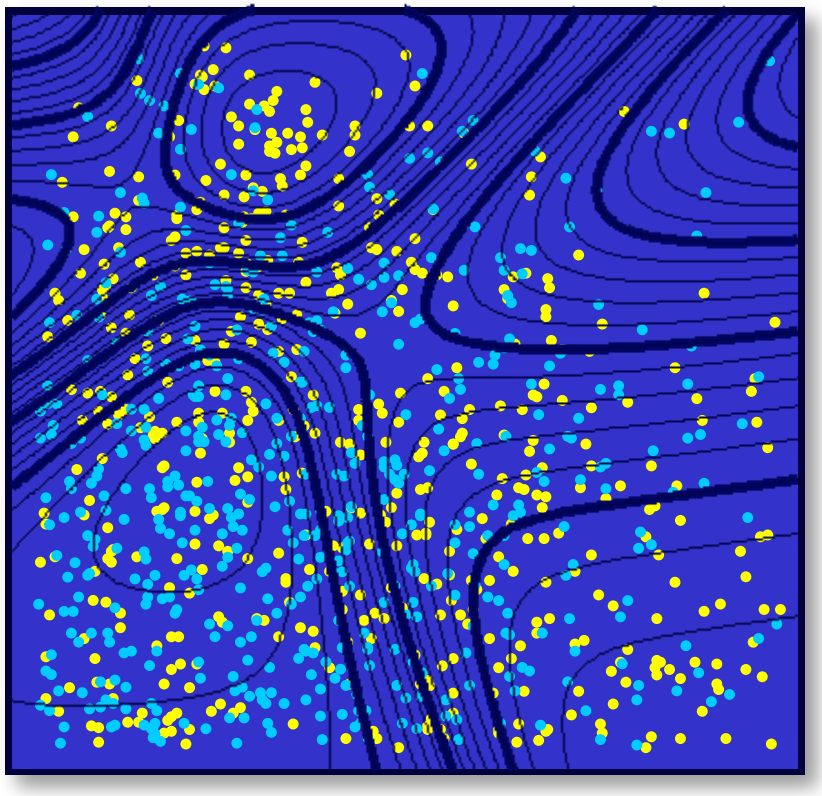

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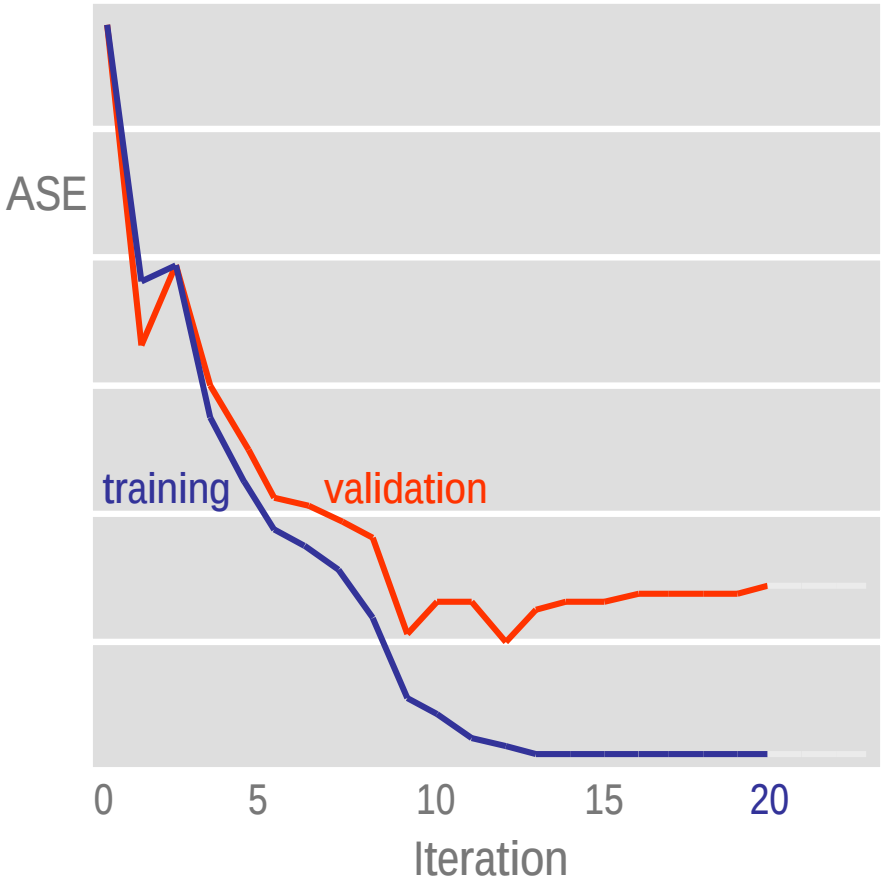

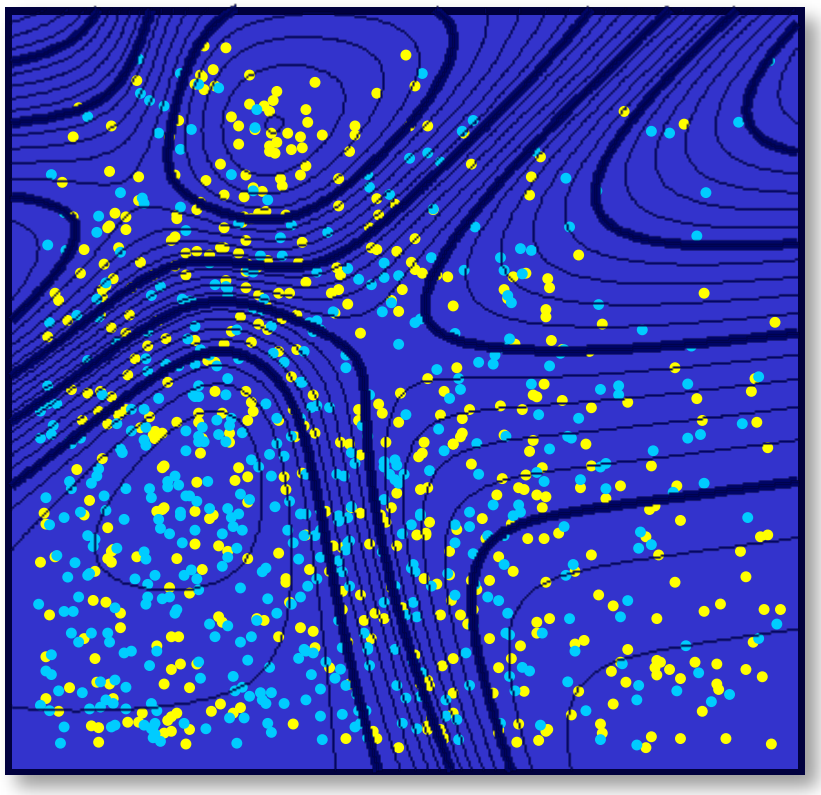

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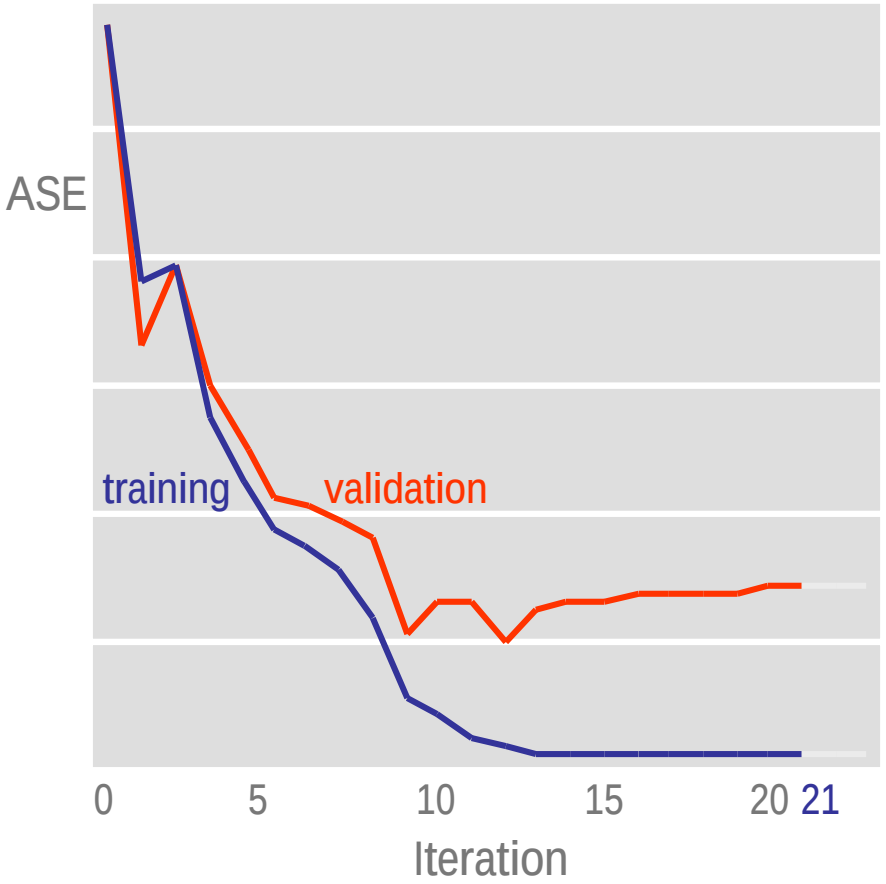

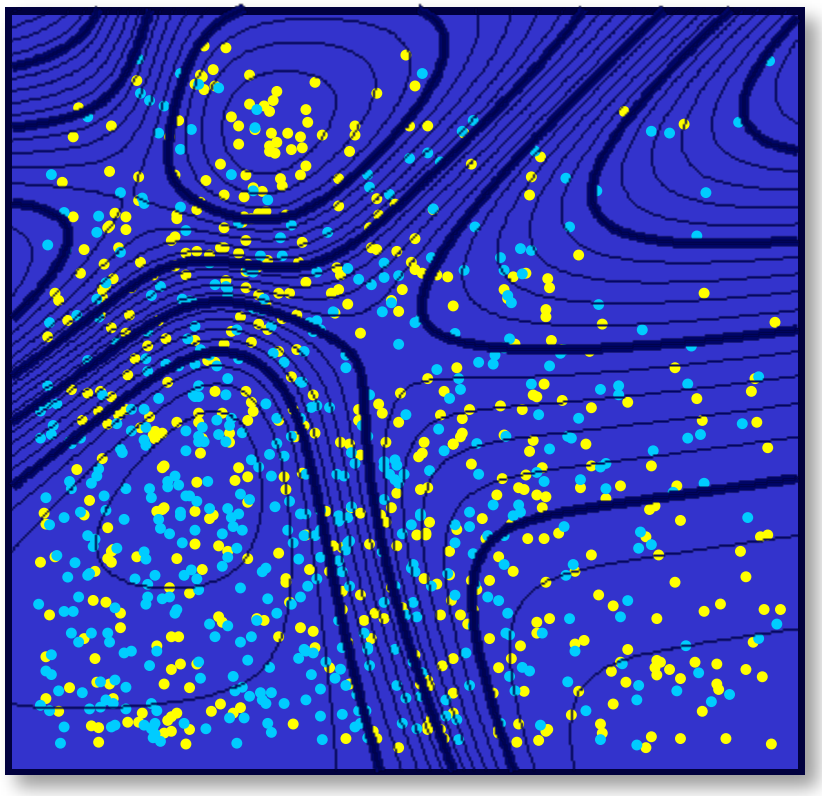

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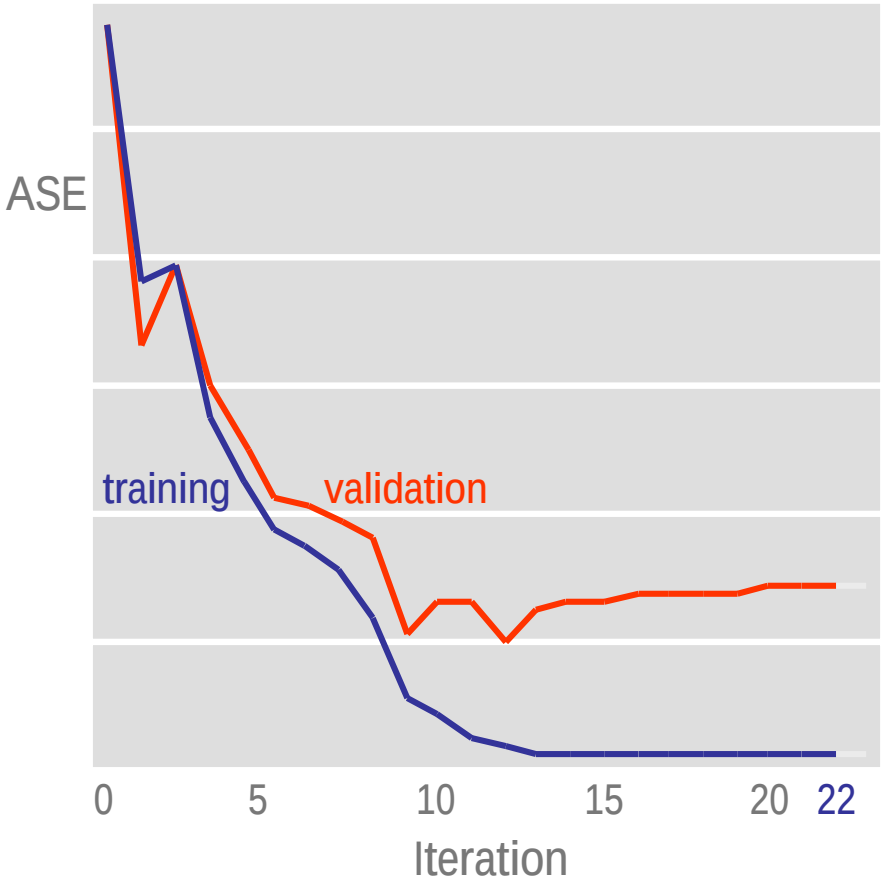

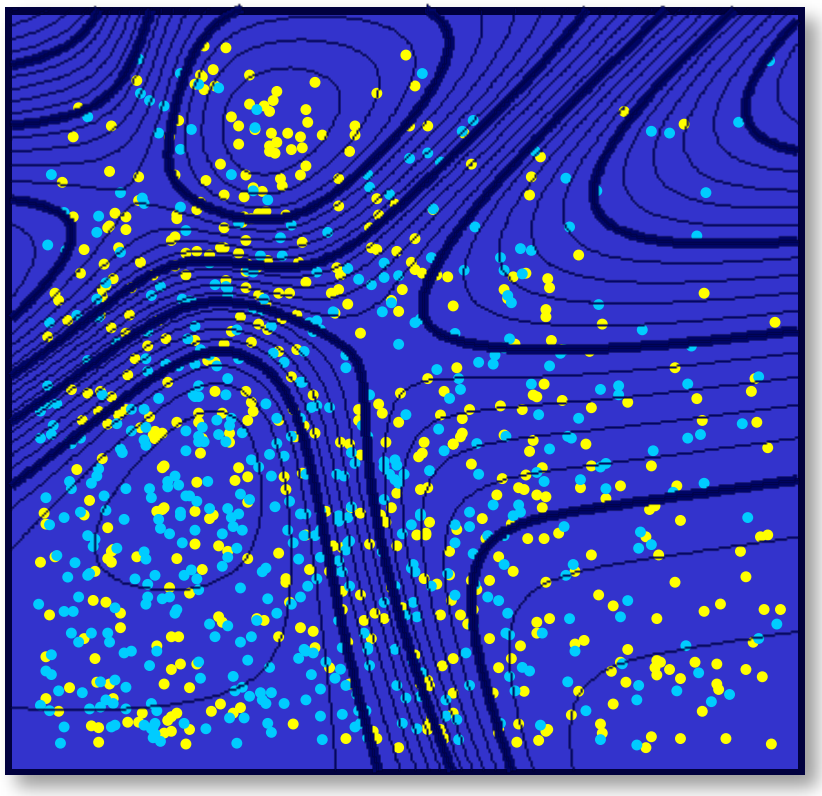

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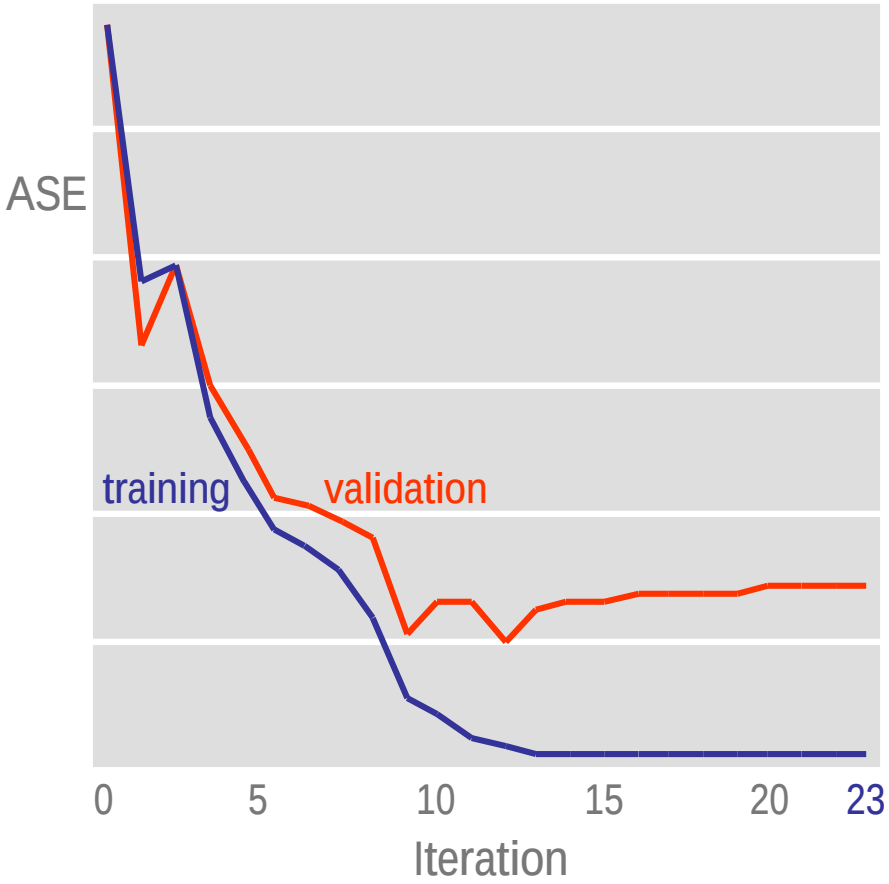

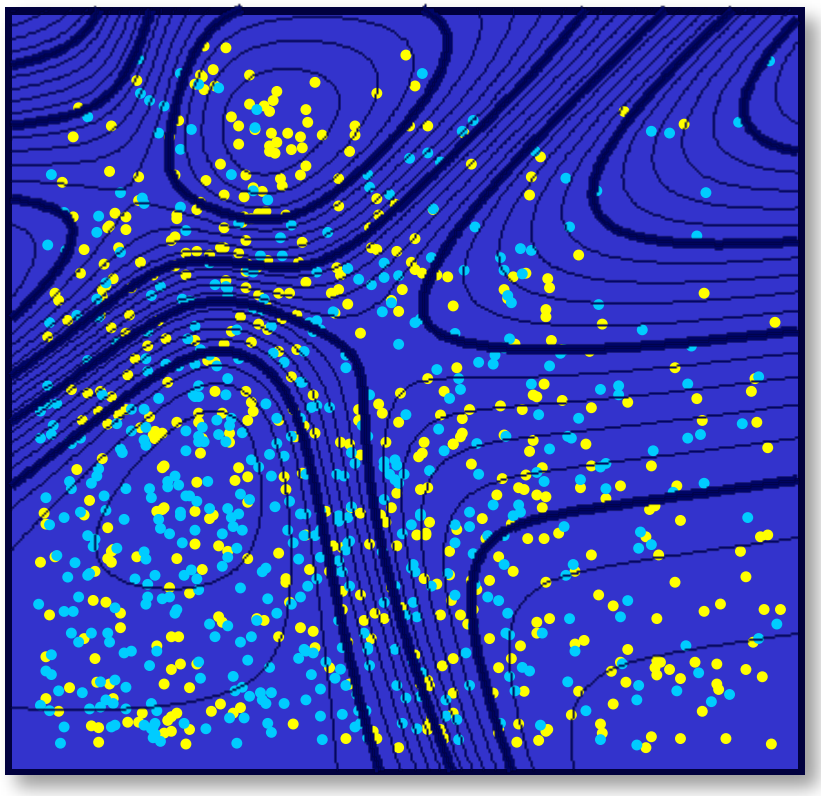

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