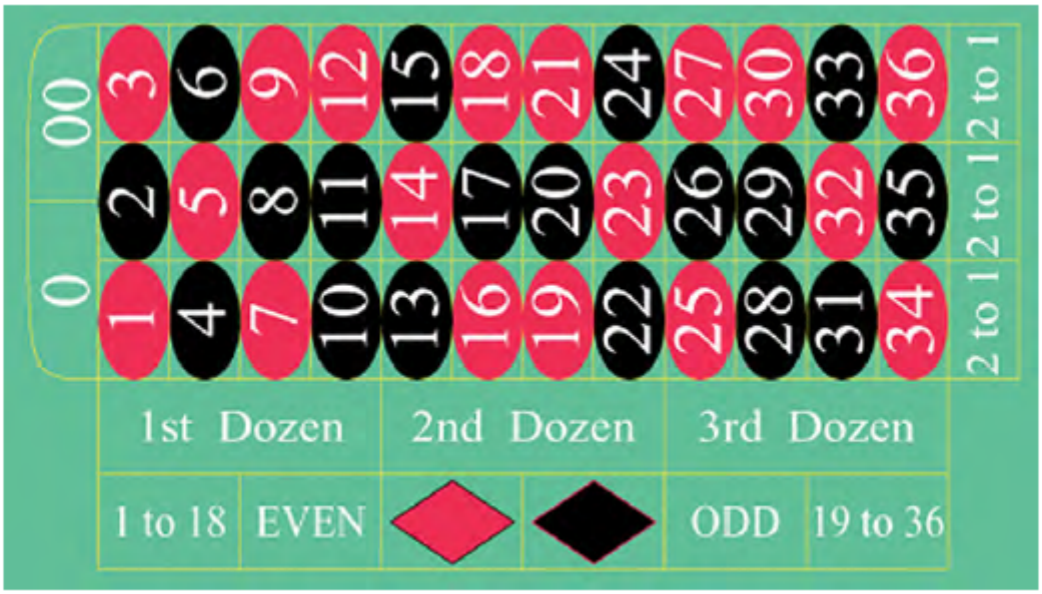
MSIS 5503 – Statistics for Data Science – Fall 2021- Assignment 1
Solution
(5 points)

1. The casino game, roulette, allows the gambler to bet on the probability of a ball, which spins in the roulette wheel, landing on a particular color, number, or range of numbers. The table used to place bets contains of 38 numbers, and each number is assigned to a color and a range.



Solution:

a. List the sample space of the 38 possible outcomes in roulette.
{0, 00, 1,2,…,36}

b. You bet on red. List the outcomes and find P(red).
Outcomes in Event (A = "Red") are {1,3,5,7,9,12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 26}
P("Red") = 18/38

c. You bet on -1st 12- (1st Dozen). List the outcomes and find P(-1st 12-).
Outcomes in Event (B = "-1st 12-") = {1,2,…,12}
P(-1st 12-) = 12/38

d. You bet on an even number. List the outcomes and find P(even number).
Outcomes in Event (C = "even number") = {2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36}
P("C = "even number") = 18/38

e. Is getting an odd number the complement of getting an even number? Why?
No; because the sample space still contains 0 and 00. The complement of Event(D = "odd number") is {Event (C = "even number") ***OR*** *Outcome (0)* ***OR*** *Outcome (00)}*

f. Find two mutually exclusive events.
Several answers possible. Example: "Odd" and "Even" numbers; "Red" and "Black"

g. Are the events Even and 1st Dozen independent?
No. Because P("Even") = 18/38; P("Even|1st Dozen"} = 6/12. Since P("Even") ≠ P("Even|1st Dozen"} the two events are not independent.

2. Consider the following scenario:
   Let P(C) = 0.4.
   Let P(D) = 0.5.
   Let P(C|D) = 0.6.

<u>Solution:</u>

a. Find P(C AND D).
   *P(C|D) = P(C AND D)/P(D) = 0.6 so P(C AND D) = P(C|D) * P(D) = 0.6*0.5 = 0.30*
b. Are C and D mutually exclusive? Why or why not?
   No because if they were mutually exclusive P(C AND D) = 0.
c. Are C and D independent events? Why or why not?
   No because P(C|D) = 0.6 ≠ P(C) = 0.4.
d. Find P(C OR D).
   P(C OR D) = P(C) + P(D) – P(C AND D) = 0.4+0.5 – 0.3 = 0.6
e. Find P(D|C).
   P(D|C) = P(C AND D)/P(C) = 0.3/04 = 0.75.

3. The Table below identifies a group of children by one of four hair colors, and by type of hair.

<u>Solution:</u>

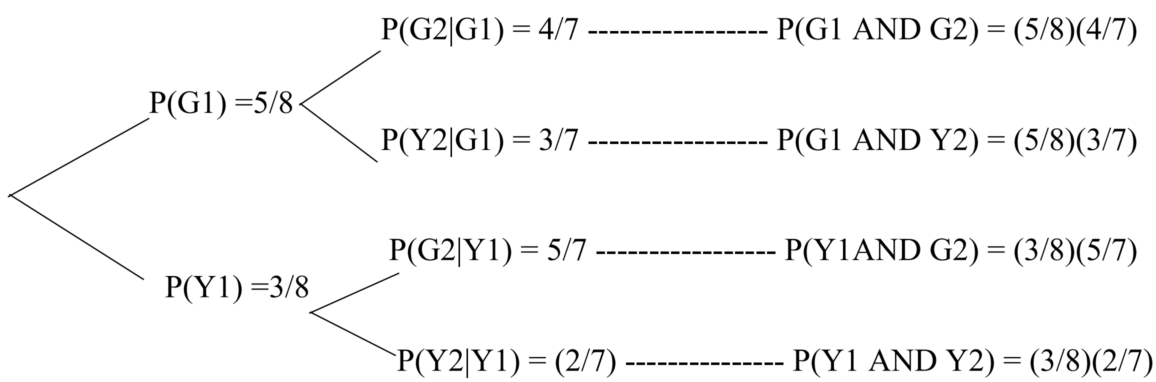
Hair Type	Brown	Blond	Black	Red	Totals
Wavy	20	5	15	3	43
Straight	80	15	65	12	172
Totals	100	20	80	15	215

a. Complete the table. (*See completed Table above*)
b. What is the probability that a randomly selected child will have wavy hair?
   P(Wavy) =43/215 = 0.20
c. What is the probability that a randomly selected child will have either brown or blond hair?
   P(Brown) = 100/215 P(Blond) = 20/215. So P(Brown or Blond) = 120/215 = 0.558 (Brown and Blond are mutually exclusive so you simply add the probabilities)
d. What is the probability that a randomly selected child will have wavy brown hair?
   P(Wavy Brown) = P(Wavy AND Brown) = 20/215 = 0.093
e. What is the probability that a randomly selected child will have red hair, given that he or she has straight hair?
   P(Red|Straight) = 12/172 = 0.07 (You only need to look across the "Straight" row)
f. If B is the event of a child having brown hair, find the probability of the complement of B.
   $P(B^c)$ = 1 – P(B) = 1 – (100/215) = 1 – 0.463 = 0.53
g. In words, what does the complement of B represent?
   The complement of B represents all children with hair of other colors (Blond, Black, Red)

4. Suppose that you have eight cards. Five are green and three are yellow. The cards are well shuffled. Suppose that you randomly draw **two cards**, one at a time, *without replacement*.

G1 = first card is green
G2 = second card is green

a. Draw a tree diagram of the situation.
b. Find P(G1 AND G2).
c. Find P(at least one green).
d. Find P(G2|G1).
e. Are G2 and G1 independent events? Explain why or why not.



Solution:

a. See diagram above

b. P(G1 and G2) = = (5/8)(4/7) = 20/56

c. P(at least 1 Green) = 1 – P(Y1 and Y2) = 1 – (3/8)(2/7) = 1 – 5/56 = 50/56

d. P(G2|G1) = P(G1 and G2)/P(G1) = (20/56)/(5/8) = 4/7

e. G2 and G1 are ***not independent*** *because*
P(G2|G1) =4/7= 32/56
P(G2) = second card is green = P(G1 AND G2) + P(Y1 AND G2) = 20/56 + 15/56 = 35/56
P(G2|G1) ≠ P(G2)

5. At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. Let **F** be the event that a course has a final exam. Let **R** be the event that a course requires a research paper.

a. Complete the Contingency Table for this problem.
b. Find the probability that a course has a final exam or a research project.
c. Find the probability that a course has NEITHER of these two requirements.

	Final Exam (F)	No Final Exam (F^c)	
Research Paper (R)	0.32	0.14	0.46
No Research Paper (R^c)	0.40	0.14	0.54
	0.72	0.28	1.00

Solution:

a. See Completed Table above

b. P(F OR R) = P(F) + P(R) – P(F AND R) = 0.72 + 0.46 – 0.32 = 0.86

c. *NEITHER requirement means that there is no requirement F OR R. So it is 1 – P(F OR R) = 0.14. Also, since 1 – P(F OR R) = P(F OR R)^c = P(F^c AND R^c) = 0.14 (From Table).*