مصطفی فضلی نهری     ۹۸۸۲۲۸.۳

حل تمرینات آمار و احتمال هندسی - فصل ۳ و ۴

فصل ۴

**53**

$$f(x) = (1-k) k^x \quad , \quad x = 0, 1, 2, 3, \dots$$

$x = 0 \to (1-k) 1 = 1-k \geq 0 \to k < 1$

$x = 1 \to (1-k) k = k - k^2 \geq 0 \to k^2 \leq k$ $\Big\}\cap \Rightarrow 0 < k < 1$

$\vdots$

**93**

$F(1) = 0.13$, $F(2) = 0.10$, $F(3) = 0.8$, $F(4) = 0.12$ $\Leftarrow$ $F(4) > 1$ $\Leftarrow$ نمی‌توان به کار برد. ✗

$F(1) = 0.5$, $F(2) = 0.4$, $F(3) = 0.7$, $F(4) = 1$ $\Leftarrow$ $F(2) < F(1)$ $\Leftarrow$ نمی‌توان به کار برد ✗

$F(1) = 0.25$, $F(2) = 0.61$, $F(3) = 0.83$, $F(4) = 1$ $\Leftarrow$ می‌توان به کار برد. ✓

**11.3**

$$f(x) = \frac{x}{15} \qquad x = 1, 2, 3, 4, 5$$

$$F(x) = \begin{cases} \frac{0}{15} = 0 & x < 1 \\ \frac{1}{15} & 1 \leq x < 2 \\ \frac{1}{15} + \frac{2}{15} = \frac{3}{15} = \frac{1}{5} & 2 \leq x < 3 \\ \frac{1}{15} + \frac{2}{15} + \frac{3}{15} = \frac{6}{15} = \frac{2}{5} & 3 \leq x < 4 \\ \frac{1}{15} + \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = \frac{10}{15} = \frac{2}{3} & 4 \leq x < 5 \\ \frac{1}{15} + \frac{2}{15} + \frac{3}{15} + \frac{4}{15} + \frac{5}{15} = \frac{15}{15} = 1 & x \geq 5 \end{cases}$$

**12.3**

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{3} & 1 \leq x < 4 \\ \frac{1}{2} & 4 \leq x < 6 \\ \frac{5}{6} & 6 \leq x < 10 \\ 1 & x \geq 10 \end{cases}$$

الف) $\frac{5}{6} - \frac{1}{3} = \frac{5-2}{6} = \frac{1}{2}$ $\Leftarrow P(2 < x \leq 6)$

ب) $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ $\Leftarrow P(x = 4)$

ج) توزیع احتمال $X$

$f(1) = \frac{1}{3}$ $\qquad f(4) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$f(6) = \frac{5}{6} - \frac{1}{2} = \frac{5-3}{6} = \frac{2}{6} = \frac{1}{3}$ $\qquad f(10) = \frac{1}{6}$

**18.3**

$$\binom{3}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^n \qquad P(H) = \frac{2}{3}, \quad \frac{1}{3} = \frac{1}{2+1} = P(T) \quad \Leftarrow P(H) = 2P(T)$$

الف)

| S | HHH | HTH | HHT | THH | TTH | THT | HTT | TTT |
|---|---|---|---|---|---|---|---|---|
| X | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 |
| P(x) | $\left(\frac{2}{3}\right)^3$ | $\frac{4}{27}$ | $\frac{4}{27}$ | $\frac{4}{27}$ | $\frac{2}{27}$ | $\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)\frac{2}{27}$ | $\frac{2}{27}$ | $\left(\frac{1}{3}\right)^3\frac{1}{27}$ |

$\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)$

$\frac{1}{27} = 0 \quad \frac{6}{27} = 1 \quad \frac{12}{27} = 2 \quad \frac{9}{27} = 3$

ب) $\frac{1}{27} + \frac{6}{27} + \frac{12}{27} = \frac{19}{27}$

$$F(y) = \begin{cases} 1 - \dfrac{1}{y^2} & y > 3 \\ 0 & \text{سائر جاها} \end{cases}$$

$$1 - \frac{9}{25} = \frac{16}{25} \quad \leftarrow P(Y \leqslant 5) \qquad 38 \quad 3$$

$$1 - \left(1 - \frac{9}{y^2}\right) = \cancel{1} - \left(\cancel{1} - \frac{9}{64}\right) = \frac{9}{64} \quad \leftarrow P(Y > 8)$$

$$\frac{-2+4}{8} = \frac{2}{8} = \frac{1}{4} \quad \leftarrow P(Z = -2) \qquad 45 \quad 3$$

$$F(Z) = \begin{cases} 0 & x < -2 \\ \dfrac{Z+4}{8} & -2 \leqslant x < 2 \\ 1 & Z \geqslant 2 \end{cases}$$

$$1 - \left(\frac{2+4}{8}\right) = 1 - \frac{3}{4} = \frac{1}{4} \quad \leftarrow P(Z = 2)$$

$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8} \quad \leftarrow P(-2 < Z < 1)$$

$$1 - \frac{1}{2} = \frac{1}{2} \quad \leftarrow P(0 \leqslant Z \leqslant 2)$$

$$F(x,y) = \begin{cases} Kx(x-y) & 0 < x < 1,\ -x \leqslant y \leqslant x \\ 0 & \text{سائر جاها} \end{cases} \qquad 59 \quad 3$$

$$\int_0^1 \int_{-x}^{x} K(x)(x-y)\,dy\,dx = K\int \left(x^2 y - \frac{xy^2}{2}\right)\Big|_{-x}^{x} dx = K\int_0^1 \left(x^3 - \frac{x^3}{2} + x^3 + \frac{x^3}{2}\right)dx$$

$$= K\int_0^1 2x^3\,dx \Rightarrow \frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$$

$$F(x,y) = \begin{cases} 24xy & 0 < x < 1,\ 0 < y \leqslant 1,\ x+y < 1 \\ 0 & \text{سائر جاها} \end{cases} \qquad 60 \quad 3$$

$$\int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-x} 24xy\,dy\,dx = 24\int_0^{\frac{1}{2}} \frac{xy^2}{2}\Big|_0^{\frac{1}{2}-x} dx = 12\int_0^{\frac{1}{2}} \left(\frac{x}{4} - x^2 + x^3\right)dx =$$

$$12\left\{\frac{x^2}{8} - \frac{x^3}{3} + \frac{x^6}{4}\right\}\Big|_0^{\frac{1}{2}} = 12\left(\frac{1}{32} + \frac{1}{24} + \frac{1}{64}\right) = \boxed{\frac{1}{16}}$$

$$F(x,y,Z) = \frac{xyz}{108} \qquad x = 1,2,3 \quad y = 1,2,3 \quad Z = 1,2 \qquad 87 \quad 3$$

$$\frac{x}{36}(1+2+3) \quad (? \qquad \frac{xz}{108}(1+2+3)(\ ) \qquad \frac{xy}{108}(1+2) \qquad \text{حاشیة}$$

$$\frac{yz}{36} \qquad \frac{yz}{18} \qquad \qquad (\Rightarrow \qquad \qquad \frac{\frac{z}{54}}{\frac{2}{36}} = \frac{z}{3} \quad (\Rightarrow$$

$$\frac{\frac{yz}{36}}{\frac{1}{2}} = \frac{yz}{18}$$

82