بنام یگانه خالق

مصطفی نفقلی نجوی ۹۸۲۲۸۰۲

Subject:

Year. Month. Day.

فصل ۴ - آمار و احتمال مهندسی

$f(x)=\frac{|x-2|}{7}$ $x=-1,0,1,3 \Rightarrow f(-1)=\frac{3}{7}, f(0)=\frac{2}{7}, f(1)=\frac{1}{7}, f(3)=\frac{1}{7}$ ۴,۴

$(-1)(\frac{3}{7})+(0)(\frac{2}{7})+1(\frac{1}{7})+3(\frac{1}{7})=-\frac{3}{7}+\frac{3}{7}+\frac{1}{7}+\frac{1}{7}=\frac{1}{7}$

۴-۶

$$f(y)=\begin{cases}\frac{1}{8}(y+1) & 2<y<4\\ 0 & \text{سایر جاها}\end{cases}$$

$\Rightarrow \int_2^4 \frac{1}{8}(y+1)(y)dy=\frac{1}{8}\int_2^4 y^2+y\,dy=\frac{1}{8}\times\frac{y^3}{3}+\frac{y^2}{2}\Big|_2^4=\frac{1}{8}(\frac{64}{3}+\frac{16}{2})-\frac{1}{8}(\frac{8}{3}+\frac{4}{2})=\frac{1}{8}(\frac{64+24}{3})-\frac{1}{8}(\frac{8+6}{3})=\frac{88}{24}-\frac{14}{24}=\frac{74}{24}=\frac{37}{12}$

۴-۹

$E(X)=0(\frac{1}{125})+1(\frac{12}{125})+2(\frac{48}{125})+3(\frac{64}{125})=\frac{300}{125}=\frac{12}{5}=\frac{24}{10}=2.4$

$E(X^2)=0(\frac{1}{125})+1(\frac{12}{125})+4(\frac{48}{125})+9(\frac{64}{125})=\frac{780}{125}=6.24$

۴-۵۷

$$(x,y)=\begin{cases}(-3,-5) & E(X)=0\\ (-1,-1) & E(Y)=0\\ (1,1) & E(XY)=(3\times5)(\frac{1}{4})+(-1\times-1)\frac{1}{4}+(1\times1)(\frac{1}{4})+(3\times5)\frac{1}{4}=8\\ (3,5) & \text{cov}(X,Y)=8-0\times0=8\end{cases}$$

۴-۶۲ الف)

$V=X^2$, $U=X$

$$F(x)=\begin{cases}1+x & -1<x\le0\\ 1-x & 0<x<1\\ 0 & \text{سایر جاها}\end{cases}$$

$E(U)=\int_{-1}^{0}x(1+x)dx+\int_0^1(x-x^2)dx=-\frac{1}{2}+\frac{1}{3}+\frac{1}{2}-\frac{1}{3}=0$

$E(V)=\int_{-1}^{0}(x^2+x^3)dx+\int_0^1(x^2-x^3)dx=\frac{1}{3}-\frac{1}{4}+\frac{1}{3}-\frac{1}{4}=\frac{1}{6}$

$E(VU)=\int_{-1}^{0}(x^3+x^4)dx+\int_0^1(x^3-x^4)dx=-\frac{1}{4}+\frac{1}{5}+\frac{1}{4}-\frac{1}{5}=0$

$\Rightarrow \text{cov}(U,V)=0$ ۴-۶۸ ب) واابسته هستند. $\begin{cases}U=x\\ V=x^2\end{cases}\Rightarrow V=U^2$

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$$\begin{cases} X_1 = 4 & 3 \\ X_2 = 9 & 7 \\ X_3 = 3 & 5 \end{cases}$$

ارزش‌های $-3X_2 + 2X_1 + 4X_3 = Y$ الف ـ 66 ـ 4

$(-3)(9) + (2(4)) + 3(4) = -7$

$(9)(7) + 4(3) + 16(5) = 155$

$Z = X_1 + 2X_2 - X_3 \Rightarrow 4 + 2(9) + (-3) = 19$ ب

$3 + 4(7) + (1(5)) = 36$

$$F(x,y) = \begin{cases} \frac{1}{3}(x+y) & 0<x<1,\ 0<y<2 \\ 0 & \text{سایر جاها} \end{cases}$$ ـ 66 ـ 4

$$E(x) = \frac{1}{3}\int_0^1\int_0^2 x(x+y)\,dy\,dx = \frac{1}{3}\int_0^1 (2x^2+2x)\,dx = \frac{1}{3}\left(\frac{2}{3}+1\right) = \frac{5}{9}$$

$$E(x^2) = \frac{1}{3}\int_0^1\int_0^2 (x)(x^2+x^2y)\,dy\,dx = \frac{1}{3}\int_0^1 (2x^3)+2x^2\,dx = \frac{1}{3}\left(\frac{1}{2}+\frac{2}{3}\right) = \frac{7}{18}$$

$$\sigma_x^2 = \frac{7}{18} - \frac{25}{81} = \frac{13}{162}$$

$$E(Y) = \frac{1}{3}\int_0^2\int_0^1 (xy+y^2)\,dx\,dy = \frac{1}{3}\int_0^2 \left(\frac{1}{2}y+y^2\right)dy = \frac{1}{3}\left(1+\frac{8}{3}\right) = \frac{11}{9}$$

$$E(Y^2) = \frac{1}{3}\int_0^2\int_0^1 (xy^2+y^2)\,dx\,dy = \frac{1}{3}\int_0^2 \left(\frac{1}{2}y^2+y^3\right)dy = \frac{16}{9}$$

$$\sigma_Y^2 = \frac{16}{9} - \frac{121}{81} = \frac{144-121}{81} = \frac{23}{81}$$

$$E(XY) = \frac{1}{3}\int_0^1\int_0^2 (x^2y+xy^2)\,dy\,dx = \frac{1}{3}\int_0^1 \left(2x^2+\frac{8}{3}x\right)dx = \frac{2}{3}$$

$$cov(X,Y) = \frac{2}{3} - \left(\frac{5}{9}\times\frac{11}{9}\right) = -\frac{1}{81}$$