Homework 2

(All solution must be supplied with steps and justifications. Numerical answer without justifications will not be graded)

1. Find the reduced row echelon form of the following matrices and compute the rank.

(a) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$,
(b) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 2 & -3 & -1 \\ 4 & -1 & 3 & 2 \\ 0 & 3 & -5 & -2 \end{bmatrix}$,

(c) $\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}$,
(d) $\begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$.

2. For which values of a, b, c, d, e is the following matrix in reduced row echelon form?

$$\begin{bmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & 0 & e & 0 & 1 & 1 \end{bmatrix}$$

3. If the rank of a 4×4 matrix A is 4, what is its $\operatorname{rref}(A)$?

4. Find all the possible solutions of the following systems.

(i) $\begin{cases} x - 2y + 2z - w = 3, \\ 3x + y + 6z + 11w = 16, \\ 2x - y + 4z + w = 9. \end{cases}$
(ii) $\begin{cases} x + y - 2z = -3, \\ 2x - y + 3z = 7, \\ x - 2y + 5z = 1. \end{cases}$

(iii) $\begin{cases} x + 2y = 1, \\ 2x + 5y = 2, \\ 3x + 6y = 3. \end{cases}$
(iv) $\begin{cases} x_1 + x_2 + x_3 + 9x_4 = 8, \\ x_2 + 2x_3 + 8x_4 = 7, \\ -3x_1 + x_3 - 7x_4 = 9. \end{cases}$

5. Determine k for which the following system has infinitely many solutions.

$$\begin{cases} x + y = 0, \\ 2y + 2kz = 1, \\ y + kz = 2k. \end{cases}$$

6. (True or False) Determine if the following statements are true or false. If it is true, explain and prove it. If it is false, give a counterexample.

Let A be an 3×5 matrix, then
(i) $Ax = b$ always has a solution.

(ii) $Ax = 0$ always has a solution.

(iii) If a system $Ax = b$ has no solution, then rank$(A) < 3$.

(iv) There are always infinitely many solutions to the system $Ax = 0$.

(v) It is possible that the system $Ax = b$ has a unique solution.