Homework 8

(All solution must be supplied with steps and justifications. Numerical answer without justifications will not be graded)

1. Find the algebraic and geometric multiplicity of the eigenvalues of the following matrices.

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 1 & 3 \\ 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Are they diagonalizable? Explain.

2. Find the conditions on a, b, c so that the following matrix is diagonalizable.

$$\begin{bmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 1 \end{bmatrix}.$$

3. Consider the set of all 3×3 upper triangular matrices

$$\mathcal{U} = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} : a, b, c, d, e, f \in \mathbb{R} \right\}.$$

(a) Show that $\mathcal{U}$ is a subspace of $\mathcal{M}_{3\times 3}$.

(b) What is the dimension of U?

(c) (i) Is it true that any 7 matrices taken from $\mathcal{U}$ must be linearly dependent? Explain

(ii) Is it true that any 6 matrices taken from $\mathcal{U}$ must be a basis for $\mathcal{U}$? Explain

4. Consider the set of all continuous functions on the interval $[a, b]$, denoted by $C([a, b])$. Show that the set of all functions with mean value zero, i.e.

$$M = \left\{ f : \frac{1}{b-a} \int_a^b f(x)dx = 0 \right\}$$

is a subspace of $C([a, b])$.

5. Determine if the following sets of vectors linearly independent in their own vector space.

(i) $x^2 - 3$, $2 - x$, $(x - 1)^2$ on $\mathcal{P}_2$.

(ii) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}$ on $\mathcal{M}_{2\times 2}$.

(iii) e^x, e^{3x} on $C([0,1])$.

6. Let

$$M = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} : a_{11} + a_{22} + a_{33} = 0 \right\}.$$

(i) Show that M is a subspace for $\mathcal{M}_{3\times 3}$.

(ii) Find a basis for M and what is the dimension of M?