Homework 6

1. *(Although this question is just copying definitions, this is important to understand the whole concepts and I expect students should remember these definitions)*

Write down the definition of the following:

(a) W is a subspace of a vector space V

(b) span $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$.

(c) $\mathbf{v}_1, ..., \mathbf{v}_n$ are linearly independent.

(d) $\mathbf{v}_1, ..., \mathbf{v}_n$ are linearly dependent.

(e) $\mathbf{v}_1, ..., \mathbf{v}_n$ forms a basis of V.

(f) The dimension of a vector space V.

2. Determine if the following sets are subspaces of $\mathbb{R}^3$. Justify your answer.

(i) $W_1 = \{(x, y, z) : x = z + 2\}$.

(ii) $W_2 = \{(x, y, z) : x = 3y \text{ and } z = -y\}$.

(iii) $W_3 = \{(x, y, z) : z = x^2 + y^2\}$

3. For the following sets of vectors,

(i) $\mathbf{v}_1 = (0, 1, 1)$, $\mathbf{v}_2 = (1, -1, 0)$ and $\mathbf{v_3} = (3, -1, 2)$.

(ii) $\mathbf{v}_1 = (2, 1, 3)$, $\mathbf{v}_2 = (1, -2, 1)$, $\mathbf{v_3} = (2, -3, 0)$ and $\mathbf{v}_4 = (0, -1, 4)$.

(iii) $\mathbf{v}_1 = (1, 0, 2, 1)$, $\mathbf{v}_2 = (-2, 3, -1, 1)$ and $\mathbf{v_3} = (2, -2, 1, -1)$.

(a) Determine if the above set of vectors linearly dependent or linearly independent.

(b) For (i), determine if $\mathbf{w} = (1, 1, 1)$ lies in the span.

(c) For (ii), express $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.

4. Expand the kernel of the following matrices as span of vectors and then compute the dimension.

$$\text{(a)} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad \text{(b)} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{(c)} \begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & -2 & 2 \\ 1 & -1 & -2 & 0 & 3 \\ 2 & -2 & -1 & 3 & 4 \end{bmatrix}$$

5. Let $W = \{(x_1, x_2, x_3, x_4) : x_1 - x_2 + 2x_3 - x_4 = 0\}$. Find a basis for the subspace W.

5. Let $W = \left\{ (x_1, x_2, x_3, x_4) : \begin{cases} x_1 + x_2 - x_3 + x_4 = 0, \\ 2x_1 + 2x_2 - 2x_3 + x_4 = 0, \end{cases} \right\}$. Find a basis for the subspace W and what is its dimension?