Homework [redacted]

(All solution must be supplied with steps and justifications. Numerical answer without justifications will not be graded)

1. Find the matrix representation for the following linear transformations.

(a) $T(x_1, x_2, x_3) = (3x_1 - x_2, x_2 + x_3, x_1 - x_2 - x_3)$.

(b) T maps $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ respectively to $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $T(x_1, x_2) = x_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$.

2. Sketch the image of the square formed by vertices $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$ under the linear transformation $T(\mathbf{x}) = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \mathbf{x}$.

3. Sketch the image of the triangle formed by vertices $(-1,1)$, $(1,0)$ and $(1,1)$ under the linear transformation $T(\mathbf{x}) = \begin{bmatrix} 4 & -1 \\ 0 & 1 \end{bmatrix} \mathbf{x}$.

4. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$.

(a) Find A^2 and A^3.

(b) Find $2A^3 - 4A + 5I_2$ and $A^2 + 2A - 11I_2$.

5. Let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, $\mathbf{v}^T = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ and $v_1 \neq 0$.

(a) Find $\mathbf{v}^T\mathbf{v}$ and $\mathbf{v}\mathbf{v}^T$.

(b) If $\mathbf{v} \neq \mathbf{0}$, verify that the rank of $\mathbf{v}^T\mathbf{v}$ and $\mathbf{v}\mathbf{v}^T$ are 1.