

### Homework 4

1. Compute the inverse of the following matrices.

(a)  $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ , (b)  $\begin{bmatrix} 1 & 2 & 30 \\ 0 & 5 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ , (c)  $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 6 \\ -1 & 2 & 2 \end{bmatrix}$ , (d)  $\begin{bmatrix} 1 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ,  
(e)  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

2. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transformation that maps the standard vector  $\mathbf{e}_1$ ,  $\mathbf{e}_2$  and  $\mathbf{e}_3$  to  $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ . Is  $T$  invertible? Explain.

3. (a) Find invertible matrix  $A, B$  such that  $A + B$  is not invertible.

(b) Find non-invertible matrix  $A, B$  such that  $A + B$  is invertible.

4. For which values of the constant  $a, b$  is the following matrix not invertible?

$$\begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

(Hint: find its row echelon form)

5. Determine if the following statement true or false.

(a) If a matrix  $A$  has a completely zero row, then it is not invertible.

(b) Upper triangular matrices are always invertible.

(c) If  $A$  is invertible, then  $Ax = 0$  may have non-trivial solution.

(d) If  $AB$  is invertible, then  $A$  is invertible.

6. For the matrices  $A, B$  are invertible, Is the following true? If it is true, verify it. If it is false, give an example to explain why it is false.

(i)  $(A^2)^{-1} = (A^{-1})^2$

(ii)  $(A + B)^{-1} = A^{-1} + B^{-1}$ .

- 7 (i) Let  $A$  be an  $n \times n$  matrix. Expand  $(I - A)(I + A + A^2)$ .
- (ii) Suppose that  $A^3 = O$ , the zero matrix. Use (i), find  $(I - A)^{-1}$  in terms of  $A$ .
- (iv) (Bonus 1 point) If  $A^k = O$ , find  $(I - A)^{-1}$ .