Homework 7

1. Find the determinant of the matrices using Gaussian elimination.

$$\text{(a)} \begin{bmatrix} 2 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \text{(b)} \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}, \text{(c)} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & 2 & 0 & -2 \\ 2 & 0 & 2 & 1 \end{bmatrix} \text{(d)} \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}.$$

2. Given 5×5 matrices A, B, Q. Suppose that $\det A = 3$, $\det B = 2$ and Q is an invertible matrix. Find the determinant of $A^T B$, A^3, $2A$, ABA and $Q^{-1}AQ$.

3. Consider the following system of linear equations:

$$\begin{cases} px + y + z = 6, \\ 3x - y + 11z = 6, \\ 2x + y + 4z = q, \end{cases}$$

(a) Find the condition on p so that the system has unique solution (Hint: $\det(A) \neq 0$).

(b) Find the condition on p and q so that the system has infinitely many solutions (Hint: $\det(A) = 0$ and no inconsistent equations). Describe the solution set.

4. Show that if A is an $n \times n$ skew-symmetric matrix (i.e. $A^T = -A$) and n is an odd number, then $\det A = 0$.

5. Let

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}.$$

(i) Find the eigenvalues and eigenvectors of both A and B.

(ii) Diagonalize A and B.

(iii) Find A^{10} and B^3.