Homework 3

1. Find the matrix representation for the following linear transformations.

(a) $T(x_1, x_2, x_3) = (3x_1 - x_2, x_2 + x_3, x_1 - x_2 - x_3)$

(b) T **maps** $\begin{bmatrix}1\\0\\1\end{bmatrix}$, **and** $\begin{bmatrix}0\\1\\0\end{bmatrix}$ **and** $\begin{bmatrix}0\\0\\1\end{bmatrix}$ **respectively to** $\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}$

(c) $T(x_1, x_2) = x_1 \begin{bmatrix}1\\1\\2\end{bmatrix} + x_2 \begin{bmatrix}-1\\1\\5\end{bmatrix}$

2. Sketch the image of the square formed by vertices $(0,0), (0,1), (1,0)$ **and** $(1,1)$ **under the linear transformation** $T(\mathbf{x}) = \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}\mathbf{x}$.

3. Sketch the image of the triangle formed by vertices $(-1,1), (1,0)$ **and** $(1,1)$ **under the linear transformation** $T(\mathbf{x}) = \begin{bmatrix}4 & -1\\0 & 1\end{bmatrix}\mathbf{x}$.

4. Let $A = \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix}$.

(a) Find A^2 **and** A^3.

(b) Find $2A^3 - 4A + 5I_2$ **and** $A^2 + 2A + 11I_2$.

5. Let $\mathbf{v} = \begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}, \mathbf{v}^T = \begin{bmatrix}v_1 & v_2 & v_3\end{bmatrix}$, **and** $v_1 \neq 0$.

(a) Find $\mathbf{v}^T\mathbf{v}$ **and** $\mathbf{v}\mathbf{v}^T$.

(b) If $\mathbf{v} \neq \mathbf{0}$, **verify that the rank of** $\mathbf{v}^T\mathbf{v}$ **and** $\mathbf{v}\mathbf{v}^T$ **are** 1.