

Homework 12

(All solution must be supplied with steps and justifications. Numerical answer without justifications will not be graded)

1. Find the singular value decomposition of the following two matrices.

(a) $\begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 0 & -1 & 1 & -1 \end{bmatrix}$

2. A famous application of spectral theorem and SVD is the spectral graph theory. A graph (V, E) is a set of vertices V with edge set E . We say that for $x, y \in V$, $x \sim y$ if x and y are connected by an edge in E . The **Graph Laplacian** is defined to be a matrix A of size $|V| \times |V|$

$$A_{x,y} = \begin{cases} 1 & \text{if } x \sim y \\ -(\text{number of edges starting from } x) & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

For example, a triangle with three vertices

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

(a) Find the SVD for the Laplacian of the triangle graph .

(b) Write down the graph Laplacian matrix for the square graph and then find its SVD.

(You can use any online calculator to solve your questions)

