Homework 1

1. Solve the following systems of linear equations by Gaussian elimination:

(a) $\begin{cases} 2y - 8z = 8, \\ x - 2y + z = 0, \\ -4x + 5y + 9z = -9. \end{cases}$ (b) $\begin{cases} x_1 - 2x_3 = -1, \\ x_2 - x_4 = 2, \\ -3x_2 + 2x_3 = 0, \\ -4x_1 + 7x_4 = -5. \end{cases}$

(Try to do it with your own hand, then check with an online program)

2. The sum of any two of three real numbers are 24,28,30. Find these three numbers.

3. Find the polynomial of degree 2 $f(t) = a + bt + ct^2$ whose graph passes through $(1, -1), (2, 3)$ and $(3, 13)$.

4. Use some online program, write down the echelon form of the following system and solve the system as well. $\begin{cases} x - 2y + 3z - 4w + 5v = -1, \\ 2x + 3y + 4z + 5w - 6v = 2, \\ 2x - 2y + 3z - 3w + 6v = 0, \\ x + y - z - w + 3v = 2. \\ 3x + 4y + 5z - 6w - 4v = 0 \end{cases}$

5. Find the following products. Explain why if it is undefined.

(a) $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix}$,

(c) $\begin{bmatrix} 0 & 1 \\ 3 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, (d) $\begin{bmatrix} 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 4 \end{bmatrix}$, (e) $\begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 4 \end{bmatrix}$.

6. Express the vector $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ as a linear combination of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

7. Can the vector $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$? Explain.