

Question 1

Let A be a matrix composed of the column vectors, such that $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -1 & -2 & 3 \\ 0 & -1 & 2 \end{pmatrix}$. We first note that the system contains no solution:

$$\text{rref} \left(A \mid \vec{\mathbf{b}} \right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Using $A \backslash \mathbf{b}$, we find that the least-squares solution is $\vec{\mathbf{x}}^* = A \backslash \mathbf{b} = \begin{pmatrix} 1/4 \\ 0 \\ 1/4 \end{pmatrix}$. Hence, the closest

point from $\vec{\mathbf{b}}$ is $A\vec{\mathbf{x}}^* = A * (A \backslash \mathbf{b}) = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$.

Question 2

Refer to section %% Question 2 in the math425hw5.m file for the relevant code.

Method one

This method involves using the QR decomposition. Where $A = QR = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & -2 & 3 \\ 0 & -1 & 2 \end{pmatrix}$, we

have that

$$\begin{aligned} A\vec{\mathbf{x}} &= \vec{\mathbf{b}} \\ QR\vec{\mathbf{x}} &= \vec{\mathbf{b}} \\ \cancel{Q}^T \cancel{Q} R\vec{\mathbf{x}} &= \cancel{Q}^T \vec{\mathbf{b}} \\ R\vec{\mathbf{x}} &= Q^T \vec{\mathbf{b}}. \end{aligned}$$

Since Q is orthogonal, $Q^T Q = I$. And since R is upper triangular, we can simply solve for $\vec{\mathbf{x}}$ by performing backward substitution. With our ol' reliable `myBackwardSubstitution` and the QR factorization of A , the least-squares solution is given by

$$\vec{\mathbf{x}} = \text{myBackwardSubstitution}(R, Q' * \mathbf{b}) = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}.$$

Method two

This method involves using the Cholesky decomposition. Since $A^\top A$ is symmetric, Using MATLAB, we can find the Cholesky factorization using `chol(A' A)`. Where R is an upper triangular matrix such that $R^\top R = A^\top A$, we have that

$$\begin{aligned} A^\top A \vec{x} &= A^\top \vec{b} \iff R^\top R \vec{x} = A^\top \vec{b} \\ &\iff R^\top \vec{y} = A^\top \vec{b} \end{aligned}$$

where $\vec{y} = R\vec{x}$.

As such, we can compute the least-squares solution in two steps. First, solve the lower triangular system for \vec{y} :

$$\vec{y} = \text{fixed.forwardSubstitute}(R', A' * b) = \begin{pmatrix} -5.4272... \\ 6.0554... \\ 7.6731... \end{pmatrix}$$

Then, solve the upper triangular system, giving us the least-squares solution

$$\vec{x} = \text{fixed.backwardSubstitute}(R, y) = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}.$$

Question 3

Refer to section `% Question 3` in the `math425hw5.m` file for the relevant code.