Math 425 Applied & Comput. Lin. Algebra   Fall 2024 Handout

Guide to Midterm

The in-class exam is on Thursday, October 17 during the lecture. It will be 75 minutes long. Please come to class on time so that we can start the exam right away. It is a closed-book and closed-notes exam. You are *not* allowed to bring a "cheat-sheet". There should be enough space for your answers on the papers that the exam will be printed on. Use a pencil instead of an ink pen and give your best for a clean hand-writing. Anything related to `MATLAB` will not be part of the midterm and you will not need your computer/device to do `MATLAB` computations.

The midterm will cover the material that we have learned since the beginning of the semester through the topic of finding an orthonormal basis and QR-factorization using Householder matrices. Below is a list of topics you should pay attention to when studying for the exam. Study also the notes I have posted on Canvas as well as your own notes since I point out/emphasize items that I think the book can do a better job expanding on. The questions in the homeworks will be also a good guide. *However, you should not expect some sort of replica of these exercises and questions in the test.* There are ideas that I want you to develop while doing the homework that is part of the learning in the course, so you are responsible to know the material in the homework as well.

I will emphasize knowledge of concepts such as: tell me about LU-factorization, i.e., describe what it is, when is it a good idea to use it, are there any situations when you should not attempt to compute the LU-factorization? Make sure you know the definitions of things we have been studing (what is a regular matrix? what is a nonsingular matrix?). While I am not going to ask you to code using `MATLAB`, there might be questions asking you to write the pseudocode of an algorithm, such as a high level pseudocode of LU-factorization of an in put matrix A. There will be small bits of questions where you will need to show your knowledge by justifying/proving simple statements, such as: show that if Q_1 and Q_2 are orthogonal matrices then Q_1Q_2 is also an orthogonal matrix.

Review of Matrix Arithmetic, Gaussian Elimination and LU factorization : Section 1.2, Section 1.3 Gaussian elimination and backward substitution, regular matrix; elementary matrices, their inverses, how to compute the L in LU decomposition. Section 1.4 just the part until subsection on permutations. Section 1.7 pivoting rules. Suggested exercises: 1.2.15, 1.2.19-20, 1.2.23, 1.2.31, 1.3.12, 1.3.19, 1.3.24, 1.3.26, 1.3.27, 1.4.4.
Matrix Inverses: Section 1.5: results Theorem 1.18 through Lemma 1.21. Gauss-Jordan elimination to compute the inverse of a matrix. How to solve a system using matrix inverses. Suggested exercises: 1.5.8, 1.5.11-1.5.13, 1.5.15-1.5.17.
Transposes and Symmetric Matrices: Section 1.6 until factorizations; matrix rules for transposition. Suggested exercises: 1.6.7, 1.6.9-10, 1.6.12, 1.6.16.
Span and Linear Independence: Section 2.3, very crucial! Except the examples involving trigonometric polynomials make sure you know the contents of this section. Suggested exercises: 2.3.25, 2.3.28.
Basis and dimension: Section 2.4, also very crucial! I expect you to know all the content of this section.

Suggested exercises: 2.4.21-23, 2.4.25-2.4.26.

Orthogonal and Orthonormal Bases: Section 4.1: Proposition 4.4, Theorem 4.5, Theorem 4.7, Theorem 4.9. Suggested exercises: 4.1.11, 4.1.15-16, 4.1.24

Gram-Schmidt Process: Section 4.2, the basic algorithm on pages 193-196; modifications of GS on pages 197-199. Suggested exercises: 4.2.12, 4.2.13

Orthogonal Matrices and QR-factorization: Section 4.3: Proposition 4.19, Proposition 4.23. Definition of QR factorization, Theorem 4.24, examples. Construction of Householder matrices, Lemma 4.28, using Householder matrices to compute the QR-factorization. Suggested exercises: 4.3.10, 4.3.11-4.3.16, 4.3.30.