Math 425 Applied & Comput. Lin. Algebra   Fall 2024 Handout

Guide to Final Exam

The in-class final exam is on Thursday, December 19, 8:00-10:00. We will use the whole two hours. The exam will have two parts.

Part I: The first part of the exam is the `MATLAB` component. It will last 45 minutes. For this part, you need to have a device with which you can use `MATLAB`. The ideal set-up is a laptop on which `MATLAB` is installed. If you do not own a laptop, you should contact Academic Technologies located in the ground floor of the library to borrow a working laptop. Make sure to ask/request that `MATLAB` is installed on the computer. On the day of the exam, arrive to class 10 minutes early (I will open the room at 7:45 am) so that you can fire up `MATLAB` and make sure it is working. Besides your device nothing else is allowed during this portion as help.

This portion of the final exam is not much about your knowledge of `MATLAB`. In particular, I am not going to ask you to write code. But you need to be familiar with all the basic commands, in particular, commands needed linear algebra computations. Besides things you need to manipulate matrices and vectors, examples include commands such as `qr`, `eig`, `svd`. You will be asked to use these basic commands to answer linear algebra questions. Here is a typical scenario to give you an idea: I might give you a symmetric matrix A and ask you to compute the rank of A using the command `eig`.
Your answers need to be recorded in a `MATLAB` file where I can see the computations you have done. Parts of your answers that require explanations will be in the `MATLAB` file as well written as comments. Then this file will be uploaded onto Canvas. The instructions will be given during the exam.

Part II: The second part will start at 8:45 am, right after the first part. It will be a closed-book and closed-notes exam. You are *not* allowed to bring a "cheat-sheet". There should be enough space for your answers on the papers that the exam will be printed on. Use a pencil instead of an ink pen and give your best for a clean hand-writing.

This portion of the exam is a comprehensive test where about 30 % of questions will be related to the topics of the midterm and 70 % will be related to the topics we have covered since the midterm. For the midterm topics, consult the "Guide to Midterm" document on Canvas. Below is a list of additional topics you should pay attention to when studying for the final. Study also the notes I have posted on Canvas as well as your own notes since I point out/emphasize items that I think the book can do a better job expanding on. The questions in the homeworks will be also a good guide. *However, you should not expect some sort of replica of these exercises and questions in the test.* There are ideas that I want you to develop while doing the homework that is part of the learning in the course, so you are responsible to know the material in the homework as well.
As in the midterm, I will emphasize knowledge of concepts such as: tell me about LU-factorization, i.e., describe what it is, when is it a good idea to use it, are there any situations when you should not attempt to compute the LU-factorization? Make sure you know the definitions of things we have been studing

(what is a regular matrix? what is a nonsingular matrix?). While I am not going to ask you to code using `MATLAB`, there might be questions asking you to write the pseudocode of an algorithm, such as a high level pseudocode of LU-factorization of an in put matrix A. There will be small bits of questions where you will need to show your knowledge by justifying/proving simple statements, such as: show that if Q_1 and Q_2 are orthogonal matrices then $Q_1 Q_2$ is also an orthogonal matrix.

Orthogonal Projections and Orthogonal Subspaces: Section 4.4: Definitions 4.30 and 4.31, and Theorem 4.32. Example 4.33 is worth studying. Suggested exercises: 4.4.1-4.4.5.
Closest Point: Section 5.3: Read the introduction on page 245 and then skip to page 248 and read starting from "Remark". Theorem 5.7, and the two examples after that. Suggested exercises: 5.3.1, 5.3.3, 5.3.5, 5.3.13-5.3.14.
Least Squares: Section 5.4: Definiton 5.10, Theorem 5.11. Suggested exercises: 5.4.3-5.4.5.
Data Fitting and Interpolation: Section 5.5: The whole section through page 259. Suggested exercises: 5.5.1-5.5.12.
DFT and FFT: Section 5.6: The whole section through page 291. Then skip to the FFT section and read till the middle of page 298. Besides reading these use the notes you have taken in class. I have provided examples of FFT. Study also the examples in the book. Pay attention to the related homework. Suggested exercises: 5.6.3-5.6.4, 5.6.8, 5.6.17.
Eigenvalues and Eigenvectors: Section 8.2: Definition 8.2, Theorem 8.3, Corollary 8.4, Proposition 8.7, Proposition 8.10, and all the related examples. Theorem 8.11, Proposition 8.12. Suggested exercises: 8.2.1, 8.2.2, 8.2.5, 8.2.6, 8.2.17, 8.2.19-8.2.24, 8.2.32, 8.2.35, 8.2.38, 8.2.42.
Eigenvector bases: Section 8.3: Lemma 8.20, Theorem 8.21, Definition 8.22, Theorem 8.23, Definition 8.24, Theorem 8.25. Suggested exercises: 8.3.1-8.3.3, 8.3.6-8.3.8, 8.3.13-8.3.15, 8.3.19.
Incomplete Matrices: Section 8.6: Just the section on Schur Decomposition. Suggested exercise: 8.6.1.
Eigenvalues of Symmetric Matrices: Section 8.5: Theorem 8.2. I have given a proof of this theorem based on the Schur decomposition, so ignore the proof in this section but look at your notes for my proof. Examples 8.33-8.34. Proposition 8.37, Theorem 8.38 (very important). Suggested exercises: 8.5.1, 8.5.5 (a), 8.5.13-8.5.14, 8.5.18.
Computing Eigenvalues: I have given an algorithm based on QR factorizations. I have not proved much here, but you should know this algorithm. We also had a class activity about this. Make sure you understand what is in that class activity. If you want to read more about this I highly recommend that you look at Section 9.5 pages 526-530.
Singular Values: Section 8.7: Definition 8.60, Theorem 8.63, Corollary 8.64, Proposition 8.66, Eckhart-Minsky-Young Theorem (see notes), Theorem 8.71, Definition 8.73, Theorem 8.74. Suggested exercises: 8.7.1-8.7.5, 8.7.12.