

Critical Thinking

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Course Outline

1. Linguistics: Ambiguity in Language, Language of Logic
2. Propositional logic: connectives, reasoning, causality.
3. Set Theory: Properties, Predicates, Relations
4. First Order Logic: Quantifiers
5. Modal Logic: temporal, modalities, knowledge, belief
6. Gödel's theorem: Truth and Proof.
7. Applications of Logic: Fallacies, Disambiguation, Paradoxes, Psychology
8. Philosophy of Logic: Rhetoric, Free will.

Discussion, References

- Join FB Group: facebook.com/jkshim.critical.thinking for questions, announcements, discussions.
- Class notes compiled from following sources

References: Google, Wikipedia.

Books used:

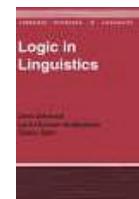
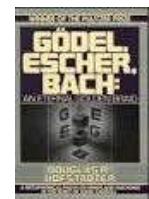
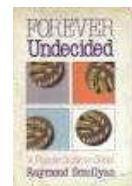
- Forever Undecided, by Smullyan.
- Gödel, Escher, Bach, by Hofstadter.
- Logic in Linguistics, by Allwood.
- How to Solve It, by Pólya.
- About Language, by Roberts and Turgeon, 1988.

Multimedia / movies shown are part of the coursework.

you don't need to read these books, just the class notes.



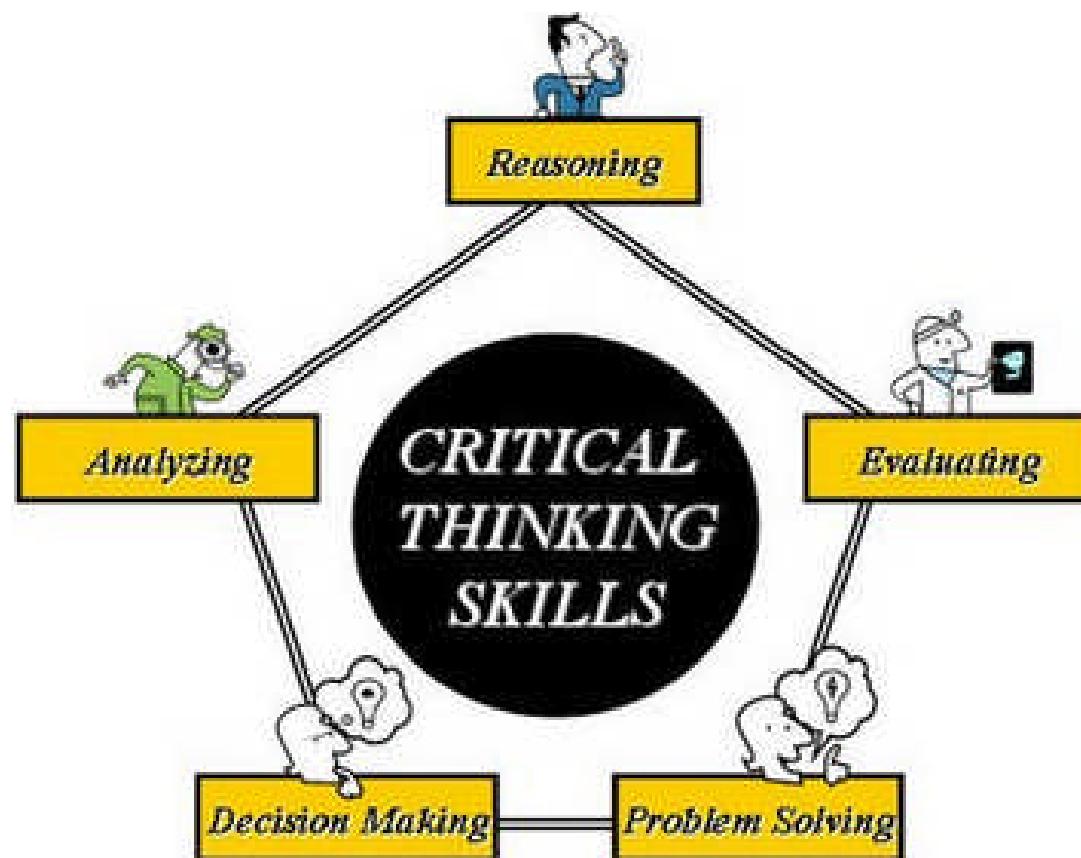
WIKIPEDIA
The Free Encyclopedia



Critical Thinking Introduction



Critical Thinking Skills



Language of logic

- We use language to communicate
- But language is ambiguous.

Examples:

- *Amit said 'bye' to Raja, as he got in a bus.*

Question: Who got in the bus?



Precise

- How can we make our words precise?
- What does it mean to say language is precise?
- Language is precise if listener/reader understands the meaning of the sentence as intended as by the speaker/writer.

Imprecise

- There are many problems with the language:
 - Words not understood by the listener
 - Wrong words used to confuse the listener
 - Wrong spelling
 - Wrong grammar.
 - Ambiguous grammar.
- Give examples of each type.

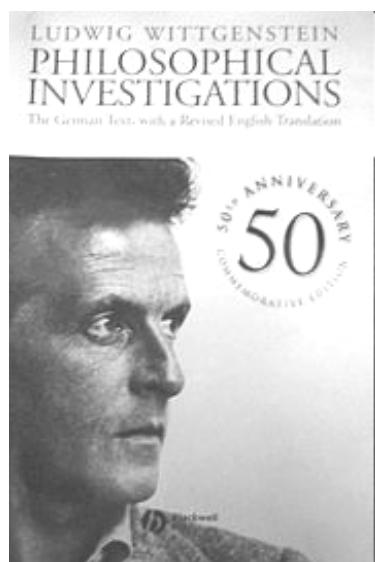
Same sentence can mean different things to different people



from facebook.com

Philosophy and Logic

"If a lion could talk, we could not understand him." - Ludwig Wittgenstein (Philosophical Investigations, 1953, page 223)

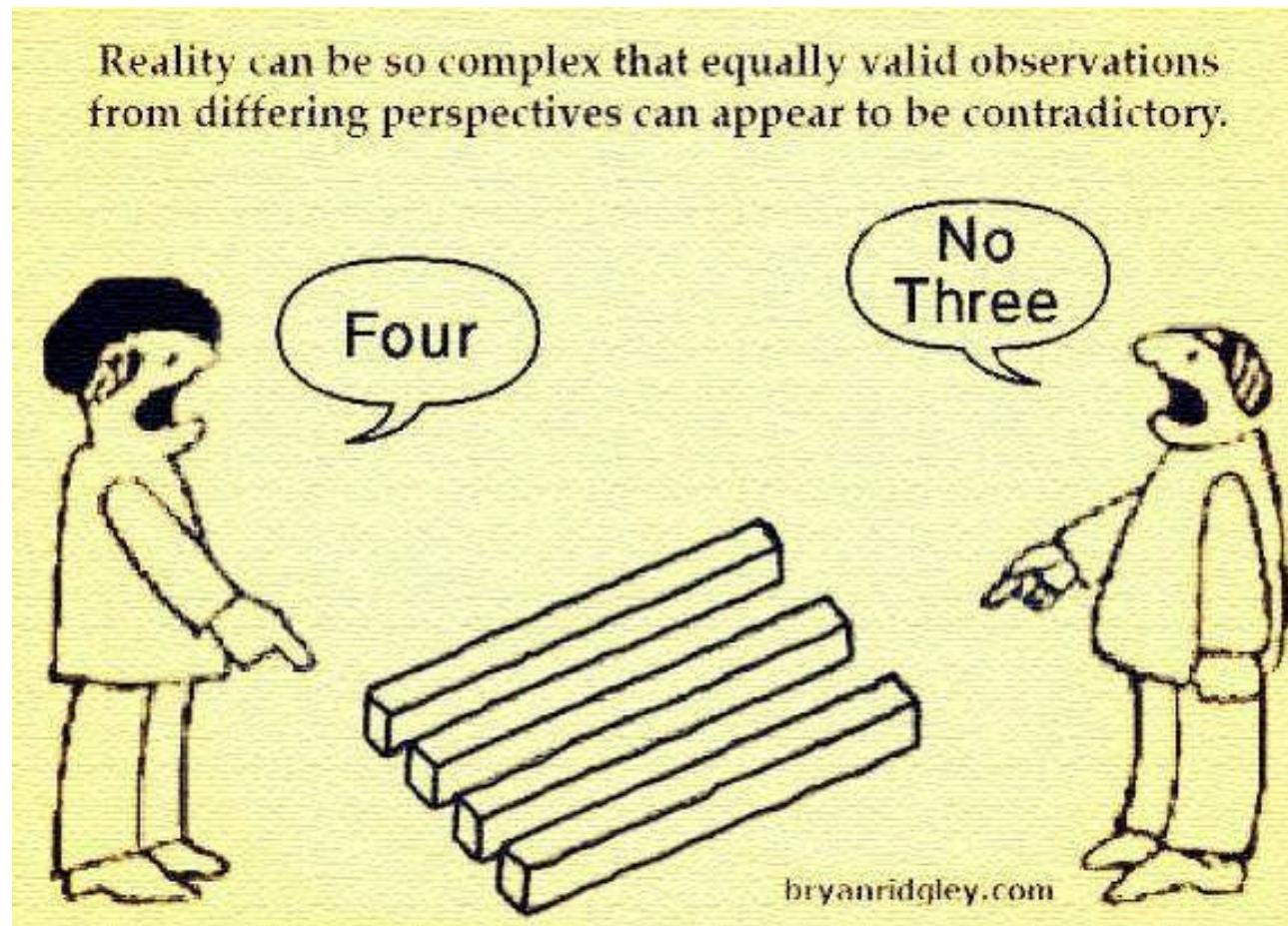


People hear what they want



- It is not enough to think you are right.
- You must also keep in mind what your listener is thinking.

Different points of views



Different news reports of same event



Against war



Original photo



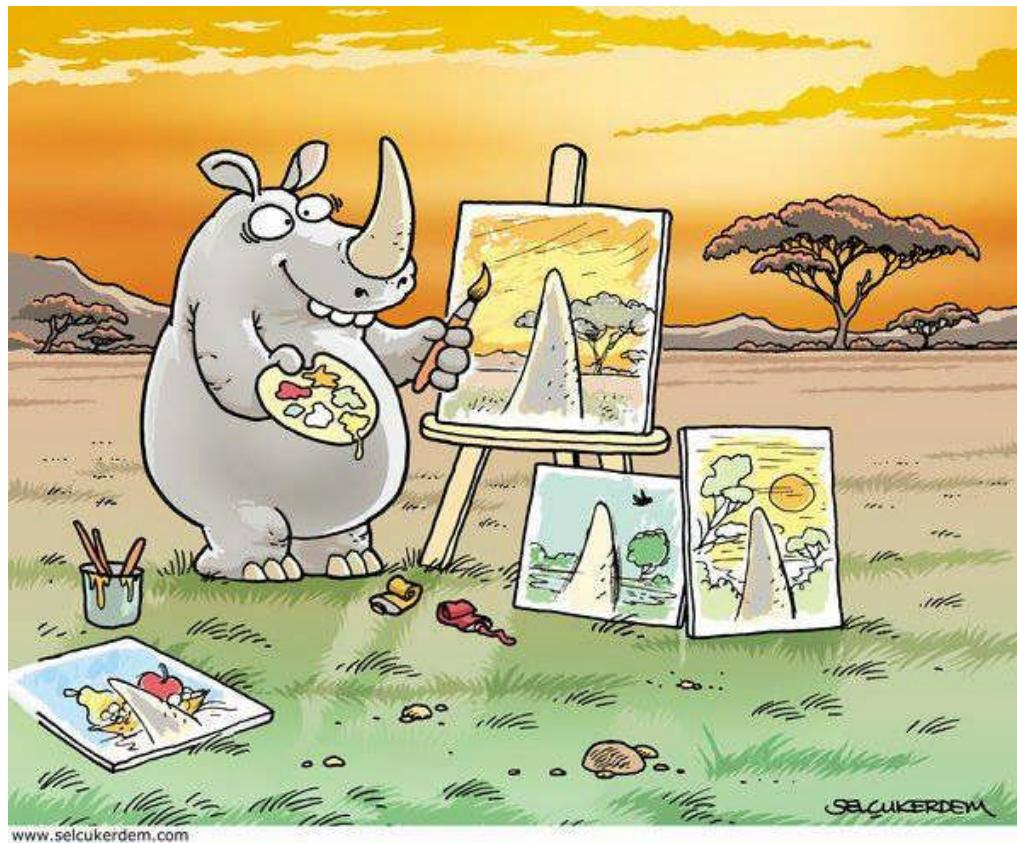
In Support of war

Calvin and Hobbes on News

Calvin and Hobbes by Bill Watterson



Biases



Bertrand Russell

Don't believe
everything you
hear, read, see
on

- TV, News
- Newspaper
- Magazines
- rumours
- books
- lectures

you must think at
all times.

**A stupid man's report of
what a clever man says
can never be accurate,
because he unconsciously
translates what he hears
into something he can
understand.**

Bertrand Russell

fb.com/PhilosophicalAtheism



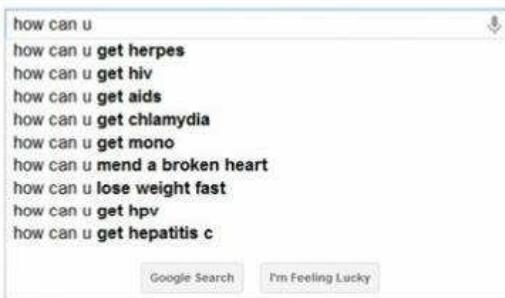
Language is culturally sensitive



- In African American lingo, double negation is same as single negation.
- But in Logic, double negation is positive.

Your grammar shows to your background

Google



Google



From Google suggest:

1. how can u ...
2. how can an individual

...

Punctuation matters



- An English teacher wrote these words on the whiteboard: "*woman without her man is nothing*", and then asked the students to punctuate the words correctly.



- The boys wrote: "Woman, *without her man*, is nothing."



- The girls wrote: "Woman! *Without her*, man is nothing."

Cognitive Dissonance

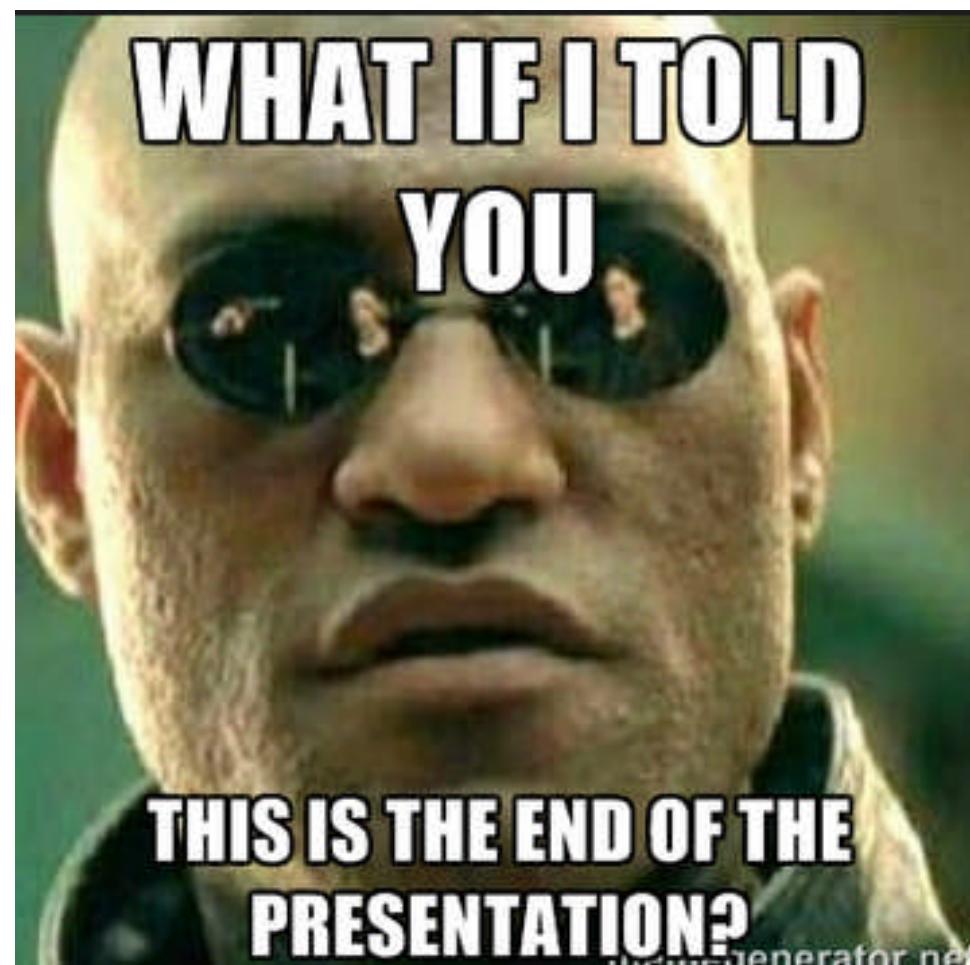


Cognitive dissonance is the discomfort experienced when simultaneously holding two or more conflicting cognitions: ideas, beliefs

Why is it hard?



Questions?



Propositional Logic



Propositions

- Propositions are sentences (statements) we make about the world.
- Examples:
 - It is raining.
 - The wall is yellow.
 - This is Ram.
 - I am Pai.
 - $2+2 = 3$
 - $2 > 3$

Truth of propositions

- Given a statement and a situation in the world, the statement is either "*True*" or "*False*".
- Notice the truth of a statement depends on the situation (location and time and speaker).
 - Example, "*This is Ram*", depends on whether Ram is there or not.
 - Example, "*I am Pai*", depends on who says it
 - But something like $(2+2=3)$, is *false* in every situation, no matter who says it.
- What about " $2+2=x$ " ?

Tautology and Invalid

- Something that is always **true** is called a *tautology*. E.g. $1=1$.
- Something that is always **false** is called *invalid* (or *contradiction*), E.g. $1=2$.

Logical Connectives

- Now that we have propositions, what can we do with them?
- Make more sentences by connecting with a logical operator like
- **OR, AND, NOT, IF, ONLY IF**

English: BUT, THEREFORE, IMPLIES, SO, BECAUSE, UNLESS,

OR

- OR connects two sentences, one of which must be true.
- Example: "*It is raining OR it is sunny.*"
- This example is made of 2 propositions, the sentence is true, if either of the two is true.
- Example: "x=1 or x=2 or x=3 or x=4", we can use as many OR as we want.

Logical Notation

- A, B, C = any propositions.
- 1 = T = True (top)
- 0 = F = False, \perp (bot, contradiction).
- & = AND, \wedge (conjunction).
- | = OR, \vee (disjunction)
- ! = NOT, \sim , \neg
- "A != B" means "A not equal B"
- "A == B" means "A equal to B".
- "A \equiv B" = "iff" = "if and only if"
- "A \rightarrow B" = A implies B, "if (A) then B"

From http://en.wikipedia.org/wiki/List_of_logic_symbols

Truth table of OR

A	B	A OR B
0	0	0
1	0	1
0	1	1
1	1	1

Properties of OR

For any propositions A, B, we have:

- $(A \text{ or } B) = (B \text{ or } A)$.. Commutative
- $(A \text{ or True}) = \text{True}$
- $(\text{True or } A) = \text{True}$
- $(\text{False or } A) = A$
- $(A \text{ or False}) = A$

AND, &

- AND connects two sentences, both of them must be true.
- Example: "*It is raining and the road is wet.*" This example is made of 2 propositions connected by AND.
- We can have as many AND in a sentence as we want: "5>1 & 5>2 & 5>3 & 5>4", is made of 4 sentences, all of them must be true for the whole sentence to be true.

Truth table of AND

A	B	A AND B
0	0	0
1	0	0
0	1	0
1	1	1

Properties of AND

For any propositions A, B, we have:

- $(A \text{ and } B) = (B \text{ and } A)$.. Commutative
- $(A \text{ and True}) = A$
- $(\text{True and } A) = A$
- $(\text{False and } A) = \text{False}$
- $(A \text{ and False}) = \text{False}$

Exercise

Give example of a non-commutative operator?

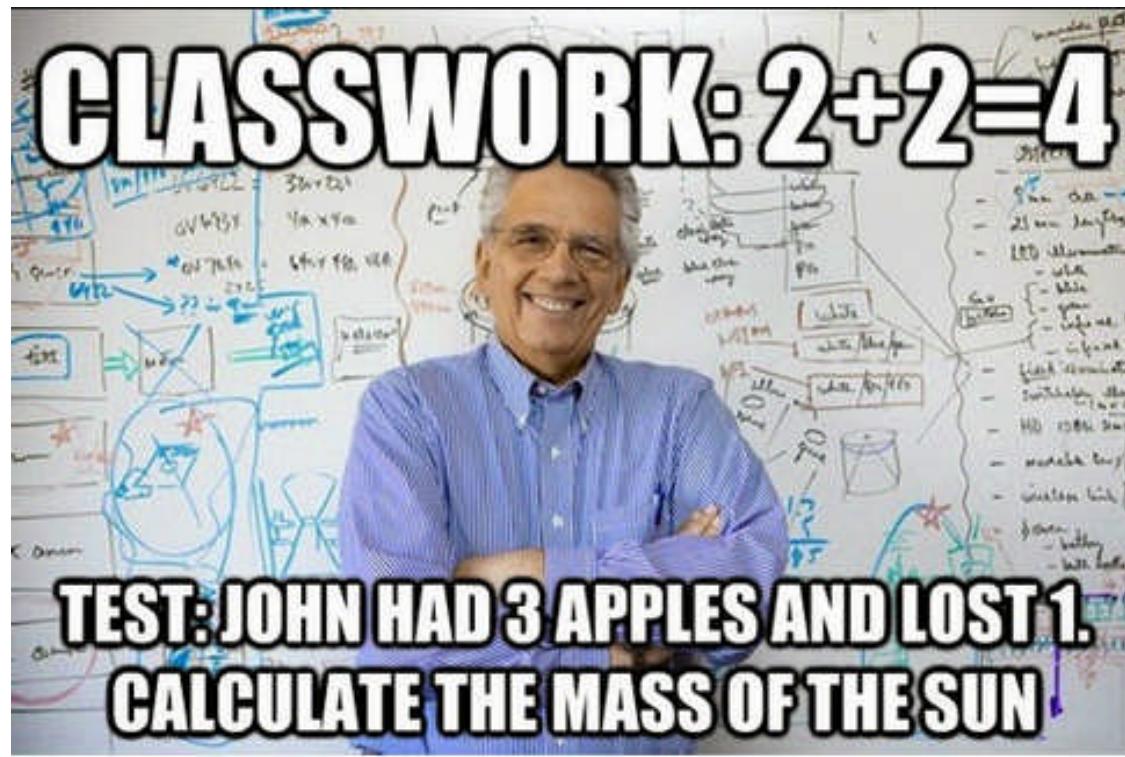
Exercise

Give example of a non-commutative operator?

Answer:

$1/2 \neq 2/1$.. division is not commutative.

Exams will be like



Non-commutative example

What about this?

- “Boil the water **and** drink it”
- “Drink the water **and** boil it”

Non-commutative example

What about this?

- “Boil the water **and** drink it”
- “Drink the water **and** boil it”

The actual operator is “**and then**”:

- “Boil the water **and then** drink it”
- “Drink the water **and then** boil it”

AND: But, Also.

Examples, compare:

- It is raining, **but** I am at home.
- It is raining **and** I am at home.
- It is raining, **so** I am at home.
- I got an umbrella, **also** I got a raincoat.

Comma "," can be AND/OR

Example 1. "I have a pen, paper **and** pencil".

This is same as: "*(I have a pen) AND (I have paper) AND (I have a pencil)*".

Example 2: "*Will you have tea, coffee **or** Juice?*".

This is same as "*Will you have tea or coffee or Juice?*"

Not, !

- The next connective is "not", it negates the meaning of the sentence.
- Example: "It is **not** raining" is opposite of "it is raining". Mathematically we can write: $\text{not}(\text{ "it is raining"})$.

Double Negation is Positive,
E.g. $\text{Not}(\text{ "Not raining"})$ is "raining"

Truth table of NOT

A	NOT A
0	1
1	0

Properties of NOT:

For any propositions A we have:

"not not A" is same as "A"

Exclusive or (**xor**, \oplus)

Anu: Will you have tea or coffee?

Biju: Yes please

Anu: choose Tea or coffee, not both.

Biju: Not both?

Anu: Tea **xor** Coffee

Example: $(x>0) \text{ xor } (x < 0) \text{ xor } (x=0)$.

The three possibilities are *mutually exclusive*.

Truth table of exclusive-or XOR

A	B	A XOR B
0	0	0
1	0	1
0	1	1
1	1	0

Properties of XOR

For any propositions A, B, we have:

- $(A \text{ xor } B) = (B \text{ xor } A)$.. Commutative
- $(A \text{ xor True}) = !A$
- $(A \text{ xor False}) = A$

XOR logically means "not equal".

$(A \text{ xor } B)$ logically means $(A \neq B)$

If, →, implies, therefore

“**If**” connects two sentences

Examples:

- **If** (it is raining), **then** (the ground is wet).
- (the ground is wet) **if** (it is raining).
- $x>5 \rightarrow x>4$ (read as $x>5$ **implies** $x>4$).

If, →, implies, therefore

Compare:

1. It is raining **therefore** the ground is wet.
2. ? The ground is wet **therefore** it is raining
3. ? The ground is wet because it is raining
4. ? It is raining because the ground is wet

Truth table of Implies → IF

A	B	$A \rightarrow B$, "if (A) then B"
0	0	1
1	0	0
0	1	1
1	1	1

Properties of \rightarrow IF

For any propositions A, B, we have:

- " $A \rightarrow B$ " \neq " $B \rightarrow A$ " .. NOT Commutative

False premise can imply anything:

- False \rightarrow Anything

Unless (if not)

Examples:

- 1 It is sunny **unless** it is raining.
2. It is sunny **if** it is **NOT** raining
3. Go home unless there is class
4. Go home if there is no class

Exercise: Make your own sentence

Contra positive

Rain \rightarrow Wet

is same as

$! \text{ Wet} \rightarrow ! \text{ Rain}$

Exercise: Make your own example

Only if

These two sentences are not equal

1. if (it is raining) then (the ground is wet).

Converse

2. (It is raining) only if (the ground is wet).

Only-if, <-, converse

- rain → wet

Converse is:

- rain <- wet

Converse is false, as the ground maybe wet,
even when it is not raining.

iff (if and only if)

- "A iff B" means
- $(A \rightarrow B) \ \& \ (B \rightarrow A)$
- "if(A) then B" AND "if(B) then A"

Example: $(X / X = 1) \text{ iff } (X \neq 0)$

Causality (cause -> effect)

It is raining, **SO** the road is wet.

(it is raining) -> (road is wet)

The road is wet **BECAUSE** it is raining.

(road is wet) <- (it is raining)

Correlation is not causation

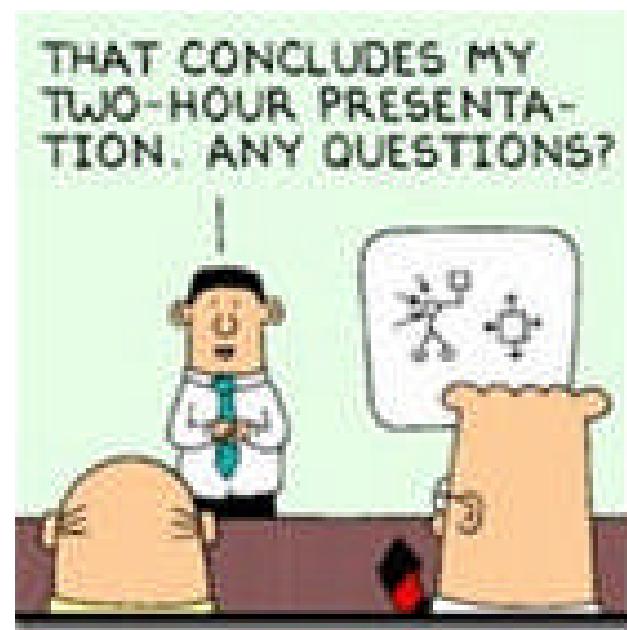


Amit: Everybody who went to the moon has eaten chicken.

Biju: Good Grief, chicken makes you go to the moon!

Fallacy: There is no cause and effect, just correlation.

Questions?



Deduction

(means: conclusion reached by mental inference, not subtraction)

Facts and Rules

- We have facts and rules about the world.

Examples:

fact: "*it is raining*"

rule: "*if it is raining then the ground is wet*".

Q. Can we use the **fact** and **rule** to conclude something new?

Modus Ponens Rule

MP rule is:

- Knowing both A and $A \rightarrow B$,
- we can conclude B is true.

Example:

1. It is raining [fact]
2. If it is raining, then we will get wet [rule]
3. We will get wet [conclusion]

Modus Tollens Rule

- Knowing B is false and $A \rightarrow B$,
- we can conclude A is false.

Example:

1. $\neg B$. The ground is dry (not wet).
2. $A \rightarrow B$. If it is raining then the ground is wet.
3. $\neg A$. It is not raining.

Disjunctive Syllogism (OR→)

From (A or B), not(B) conclude A.

1. Anu: "*It is raining or I am the queen.*"
2. But "*Anu is not the queen*",
3. Therefore, Anu is saying "*it is raining*".

Hypothetical Syllogism ($\rightarrow \rightarrow$)

- Given, $A \rightarrow B$ and $B \rightarrow C$
- Conclude: $A \rightarrow C$

Example:

1. If I get admission, I will pass MBA.
2. If I pass MBA, I will be happy.

Conclude:

3. If I get admission, I will be happy.

Simplification

From "A and B", conclude "A"

Remove irrelevant data from a sentence:

Example: "It is raining in China and India".

Conclude: "It is raining in India".

Addition

- From A, conclude A or B.

We can weaken a argument by adding extra information:

Example: "Anu got a prize"

Conclude: "Anu or Banu got a prize".

We can even generalize it

Conclude: "**Someone** got a prize".

This is conveying less information than the original sentence.

Reductio ad absurdum (Proof by Contradiction)

- If your assumption leads to an absurd (false) conclusion, then your assumption must be false.

Example:

Assume: It is raining.

Conclude: The road must be wet.

But: The road is not wet.

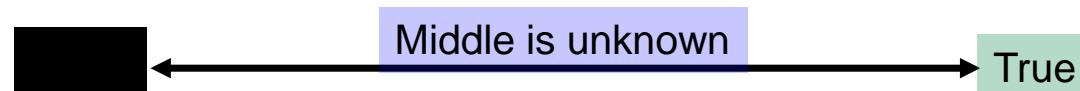
So, the assumption must be wrong.

that is: It is not raining.

Law of excluded middle

A statement is either true or false (no other possibility – there is no middle).

Third possibility is "A is unknown".



References: <http://www.stanford.edu/~bobonich/dictionary/dictionary.html>

Set Theory

Set

- A set is an unordered collection of objects.
- Set can be of any size.

Examples:

- Set of students in this college, is well defined.
- The set of stars, is a large set.
- Set of students in this college who live in China, is an empty set.

Special sets

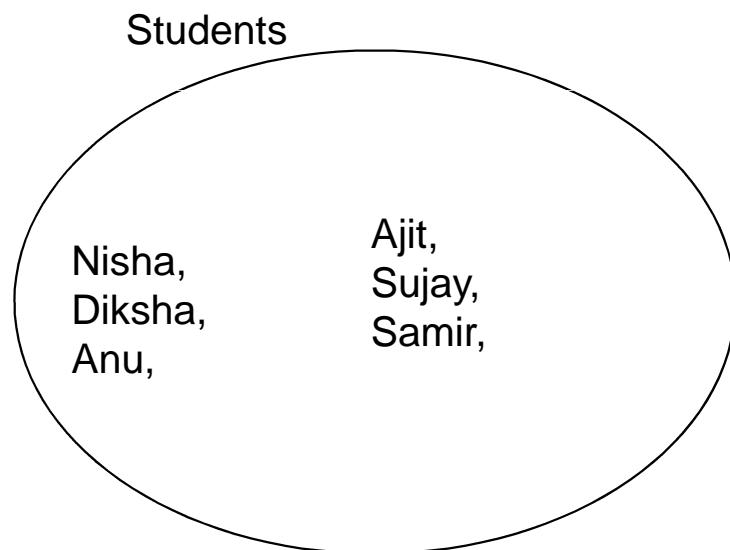
- Empty set = { } = \emptyset .
- Universal set Z , contains everything.
- Infinite set = {1, 2, 3, 4, ...}, we can list them one by one, but never finish.
- Uncountable set = {0, 0.1, 0.01, 3.14,...} the set of real numbers, we cannot even list them.

Subsets

- Set of boys in this college is a SUBSET of the set of students in this college.
 - Set of even numbers = {2, 4, 6, 8, ...} is a subset of natural numbers = {1, 2, 3, 4, 5, ...}.
 - Set of odd numbers = {1, 3, 5, 7, ...} is also a subset of natural numbers.
-
- Empty set φ is a subset of every set.
 - Every set is a subset of the universal set Z.

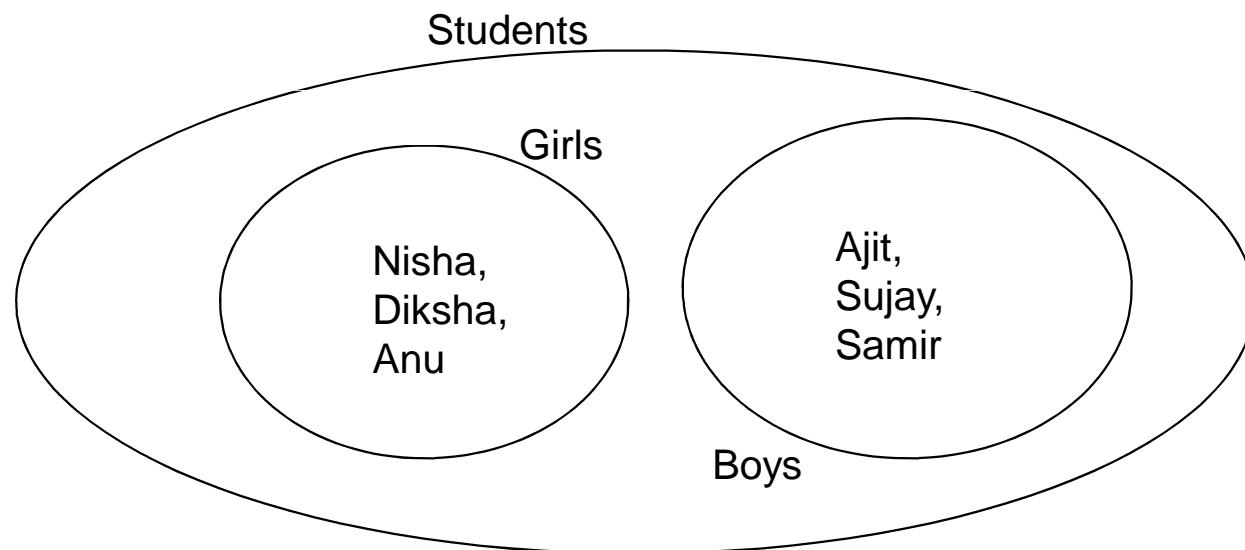
Drawing a set (Venn diagram)

- Set of students



Drawing sets (Venn diagram)

students = boys \cup girls



Notation

Let A, B, C be any sets

Z = Universal set contains everything.

{ } = \emptyset = Empty set containing nothing.

$x \in A$... x belongs to set A.

"A \cup B" = "A Union B", in A or B.

"A \cap B" = "A intersection B", in A and B.

"A - B" = Only in A, but not in B

A' .. complement of A is Z - A, not in A.

Union of sets

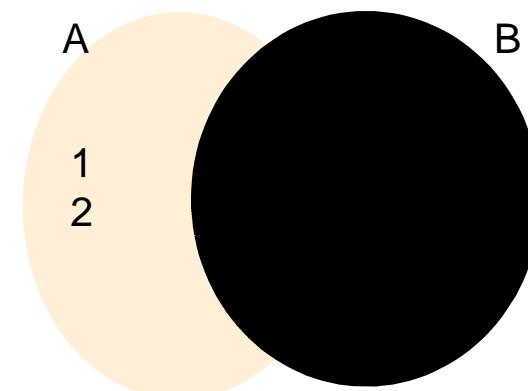
- Union: combine two sets
- $C = A \cup B$
- $C = A \cup B$

Example

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$



Intersection of sets

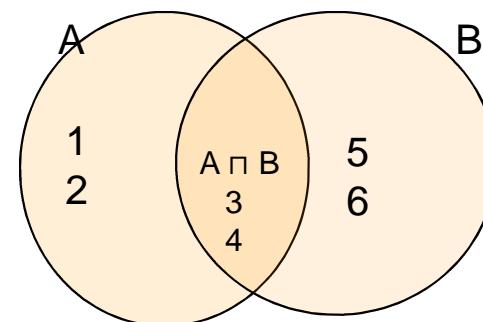
- Intersection: what is common to both:
- $C = A \text{ intersection } B$
- $C = A \cap B$

Example

$$A=\{1,2,3,4\}$$

$$B=\{3,4,5,6\}$$

$$A \cap B = \{3,4\}$$

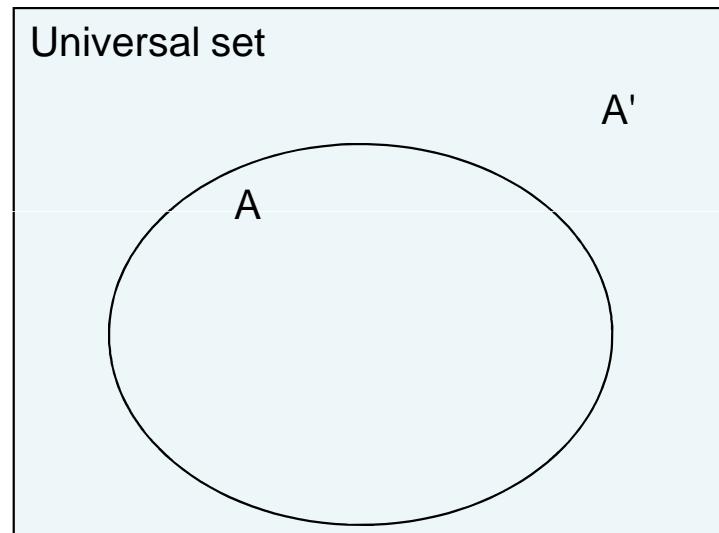


Complement (A')

- $A' = \text{Not in } A$

Examples:

- boys' = girls.
- girls' = boys.
- even' = odd
- odd' = even.

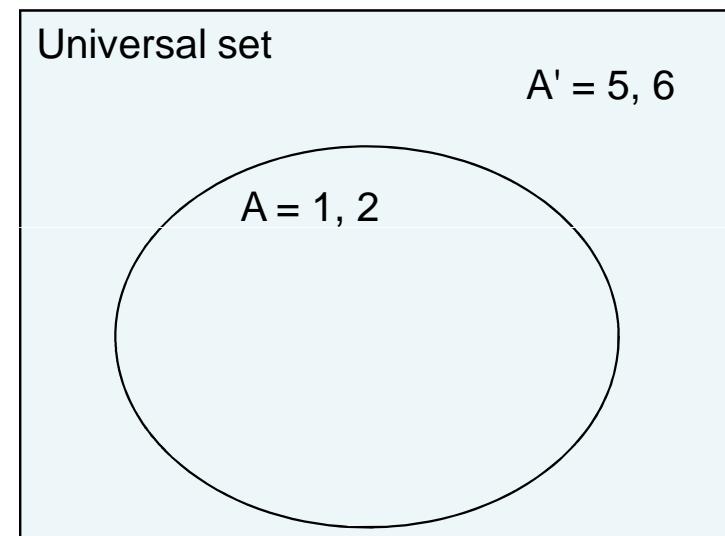


Complement (A')

Example:

$$A = \{1, 2\}$$

$$A' = \{5, 6\}$$



Set difference

$A-B$ = Subtraction =
Only in A but not in B.

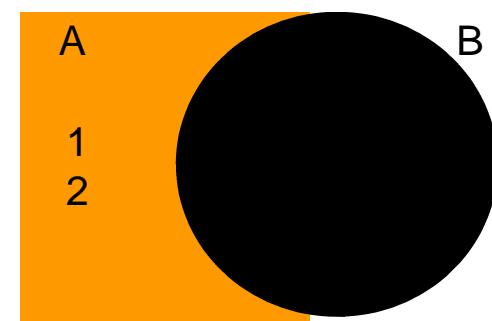
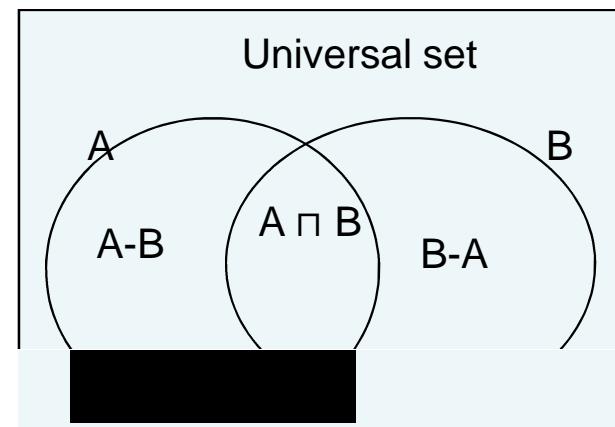
Example:

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

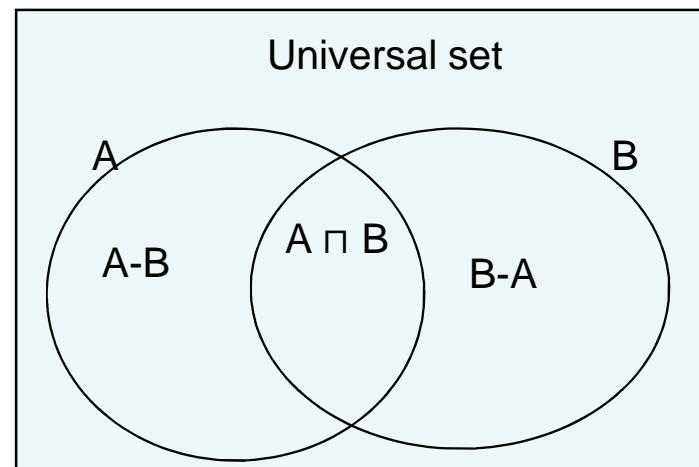
$$A-B = \{1, 2\}$$

$$B-A = \{5, 6\}$$



Properties

- $A = (A')'$.. double complement
- $A \cup B = (A-B) \cup (B-A) \cup (A \cap B)$
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$



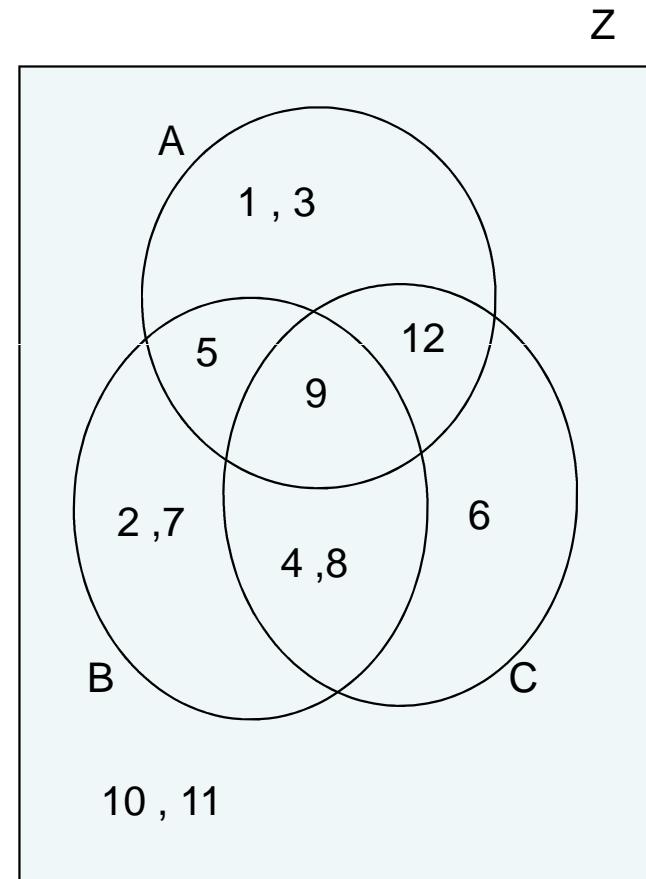
Exercise

Write:

1. A, B, C, Z,
2. $A \cup B$, $A \cup B \cup C$,
3. $A \cap B$, $A \cap B \cap C$,
4. $A - B$, $C - A$,
5. A' , C' ,
6. $(A \cup B \cup C)'$

Verify, by computing:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$



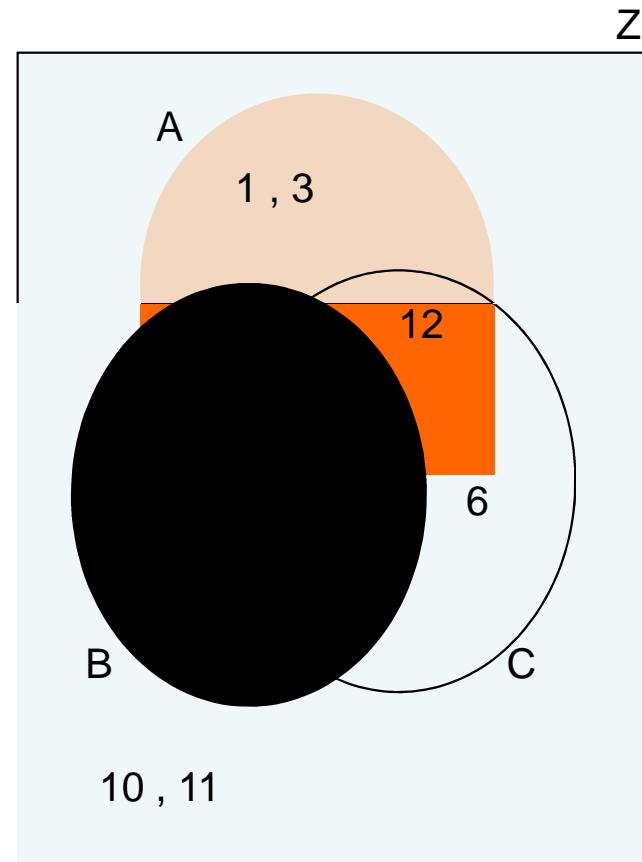
Hints: $(A \cup B)' = A' \cap B'$

$$(A \cup B)' = \{6, 10, 11\}$$

$$A' \cap B' =$$

$$\{2, 7, 4, 8, 6, 10, 11\} \cap \\ \{1, 3, 12, 6, 10, 11\}$$

$$= \{6, 10, 11\}$$



Hints: $(A \cap B)' = A' \cup B'$

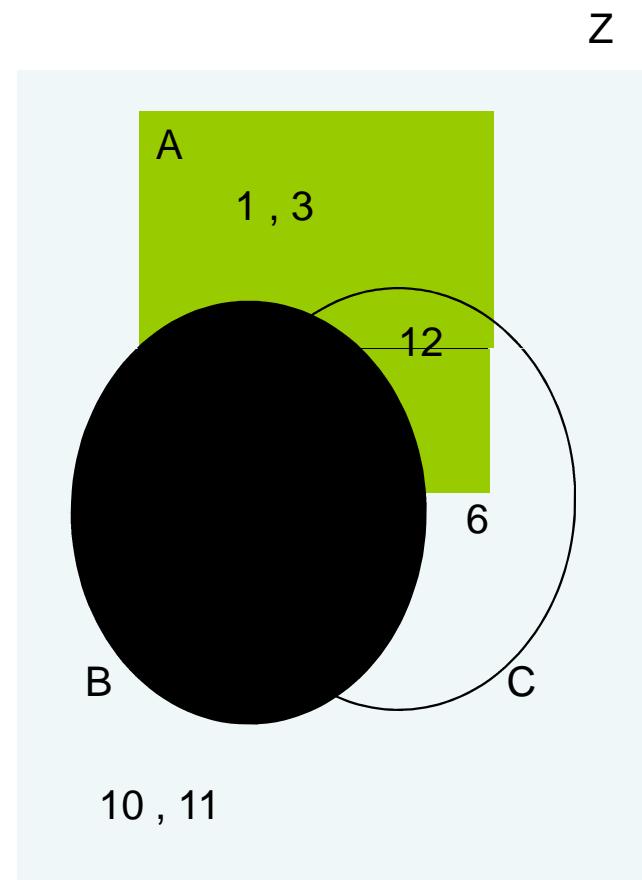
$$(A \cap B)' =$$

$$\{1, 3, 12, 6, 2, 7, 4, 8, 10, 11\}$$

$$A' \cup B' =$$

$$\{2, 7, 4, 8, 6, 10, 11\} \cup$$

$$\{1, 3, 12, 6, 10, 11\}$$



Properties

Objects, sets and properties

Example of objects and sets:

- Objects = Anu, Banu, Chetan, David, ...
- Sets = students, boys, girls, ...

Property notation:

- $p(x)$ means object x has property p .

Examples:

- $\text{girl}(\text{Anu})$, $\text{boy}(\text{David})$, $\text{student}(\text{David})$.
- **This** is a **blue** car: $\text{car}(x)$ and **blue**(x)

Sets defined by properties

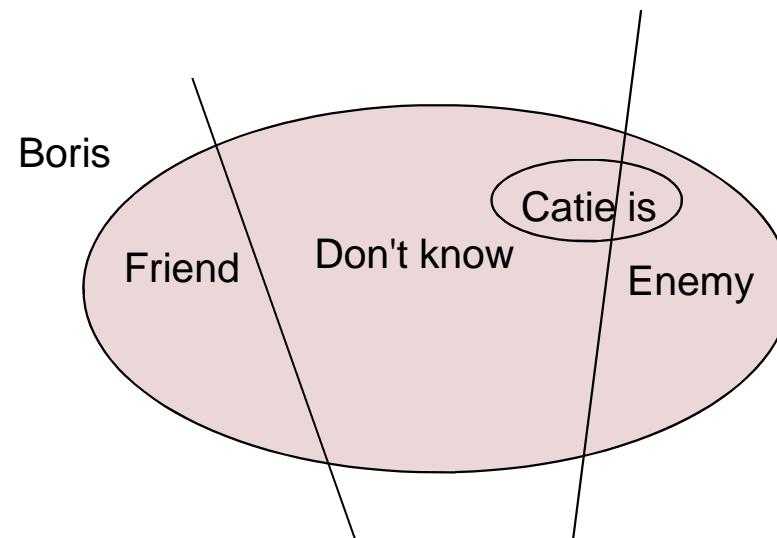
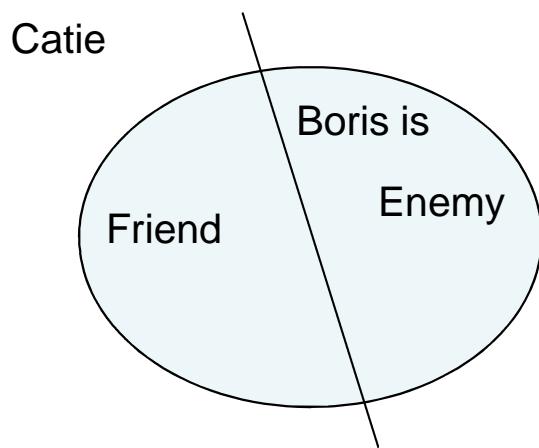
- Boys = { $x : \text{boy}(x)$ }
= set of objects x , such that x is a boy.
- Boy students = { $x : \text{boy}(x)$ and $\text{student}(x)$ }
- Girl students = { $x : \text{student}(x) \& ! \text{boy}(x)$ }.

We can now define sets by combining properties.

Negation of property

Does *Not Friend* means *enemy*?

- Boris: "Catie is NOT my friend."
- Catie: "Boris is my enemy."



Relations

Relations

Relations are properties that connect two or more objects.

Example:

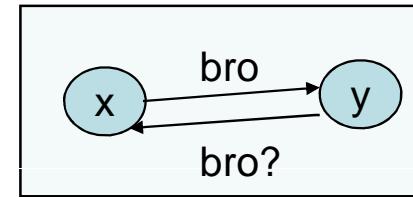
- Biju is brother of Chetan:
`brother(Biju, Chetan)`
`relative(Biju, Chetan)`
- Chetan is the son of David
`son(Chetan, David)`
`father(David, Chetan)`
- Eugene is a friend of Farid
`friend(Eugene, Farid)`
- Chetan like Chetna
`likes(Chetan, Chetna)`

Properties of relations: *symmetric*

Relation r is called symmetric:

$$r(x,y) = r(y,x) \text{ .. for all } x, y.$$

Question: is $\text{brother}(x,y)$ symmetric?
i.e. $\text{brother}(x,y) = \text{brother}(y,x)$?



Question:

1. Give examples of symmetric relations.
2. Are these symmetric?
friend, parent, relative, enemy, like, equal, \leq

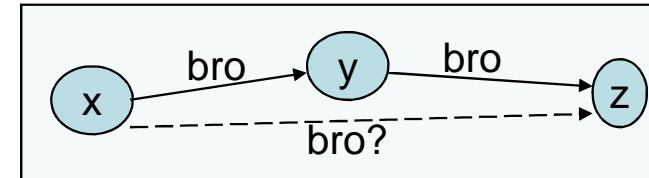
Properties of relations: *transitive*

Relation r is called transitive:

$$r(x,y) \& r(y,z) \rightarrow r(x,z) \dots \text{for all } x, y, z.$$

Question:

- is $\text{brother}(x,y)$ transitive?
i.e. $(\text{brother}(x,y) \& \text{brother}(y,z)) \rightarrow \text{brother}(x,z)$?
- Is *friend* transitive?
- Is *relative* transitive?
- Is *parent* transitive?
- Is taller than transitive?



Properties of relations: *reflexive*

Relation r is called reflexive, if $r(x,x)$ for all x .

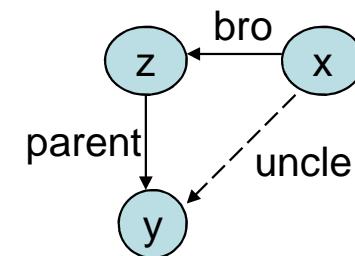
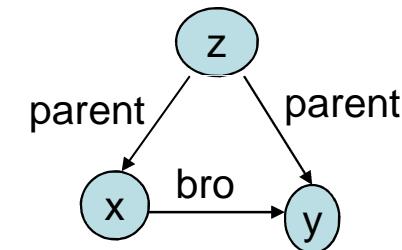
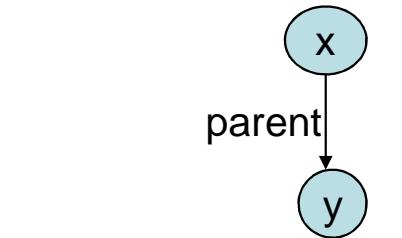
Example: \leq is reflexive, because $x \leq x$.

Question: Which other properties are reflexive?

Defining relations

- **parent(x,y)** = $\text{mother}(x,y)$ or $\text{father}(x,y)$.
- **brother(x,y)** = $\text{parent}(z,x) \ \& \ \text{parent}(z,y) \ \& \ \text{boy}(x)$
- **uncle(x,y)** = $\text{parent}(z,y) \ \& \ \text{brother}(x,z)$

z is someone unnamed.



Simplifying Gender

- $\text{sibling}(x,y)$ = $\text{brother}(x,y)$ or $\text{sister}(x,y)$
- $\text{parent}(x,y)$ = $\text{mother}(x,y)$ or $\text{father}(x,y)$
- $\text{child}(x,y)$ = $\text{son}(x,y)$ or $\text{daughter}(x,y)$
- $\text{spouse}(x,y)$ = $\text{husband}(x,y)$ or $\text{wife}(x,y)$

Exercise

Define these relations with a diagram:

- $\text{grandparent}(x,y)$
- $\text{aunt}(x,y)$
- $\text{niece}(x,y)$
- $\text{cousin}(x,y)$

Hints:

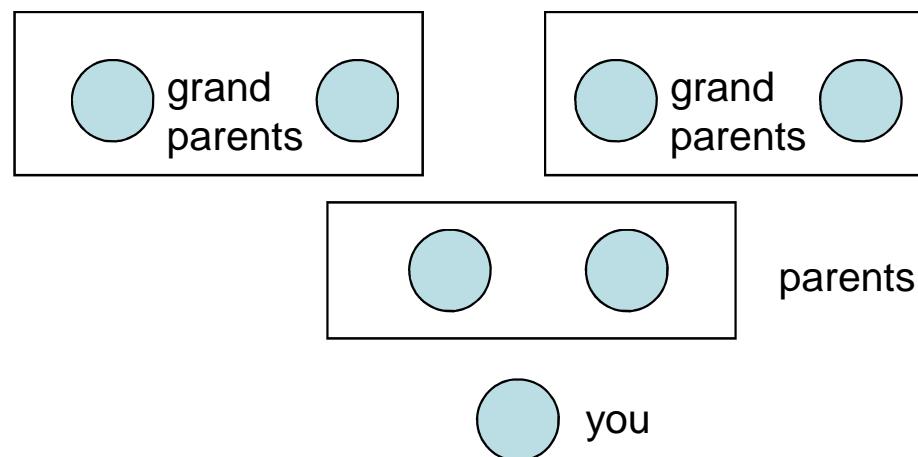
- Write the relationship in English first.
- Draw children **below** the parents.
- Use **new variables** like z , u , v for some un-named relatives.

Solutions

- $\text{grandparent}(x,y)$
= x is the parent of parent of y
= $\text{parent}(x,z) \ \& \ \text{parent}(z,y)$
- $\text{aunt}(x,y) = x$ is sister of y 's parent.
= $\text{sister}(x,z) \ \& \ \text{parent}(z,y)$
- $\text{niece}(x,y) = x$ is daughter of y 's sibling
= $\text{daughter}(x,z) \ \& \ \text{ sibling}(z,y)$
- $\text{cousin}(x,y) = \text{parents are siblings.}$
= $\text{parent}(z,x) \ \& \ \text{parent}(u,y) \ \& \ \text{ sibling}(z,u)$

Exercise: Ancestors

1. How would you define your ancestors?
2. Are your parents your ancestors?
3. Are your grandparents your ancestors?
4. Define ancestors in terms of parents?



Ancestor Solution

We can define ancestors **recursively** in terms of itself:

$$\begin{aligned}\text{ancestor}(x) &= \text{parents}(x) \\ &+ \text{ancestors}(\text{parents}(x))\end{aligned}$$

We can also define it as an infinite series:

$$\begin{aligned}\text{ancestor}(x) &= \text{parent}(x) \\ &+ \text{parents}(\text{parents}(x)) \\ &+ \text{parents}(\text{parents}(\text{parents}(x))) \\ &+ \text{parents}(\text{parents}(\text{parents}(\text{parents}(x)))) + \dots \infty\end{aligned}$$

Homework: Define these Relations in terms of simpler ones.

- nephew(x,y)
- uncle(x,y)
- grandson(x,y)
- mother-in-law(x,y)
- brother-in-law(x,y)
- relative(x,y) .. recursive definition?

First Order Logic 1: Quantifiers

Quantifiers in English:

All, Some, few, many, none, not all

Examples

- Some students are boys.
- Some students are girls.
- Not all students are boys.
- All boys are not students.
- No boy is a girl.
- Many boys are students.
- Few students have jobs.
- There is a boy.



How Many? Few?

Many and **Few** are not numerically quantifiable, logically they mean '**some**'

Example:

- "Many people were absent".
- "Few people were absent"
- How many is that?
- What percentage?

Quantifier notation

- Constants are names, e.g. John, Anu.
 - Variables x, y, z, ...
 - \forall All (upside down A)
 - \exists Some (upside down E, for "Exists").
-

- $\forall x \text{ boy}(x)$.. for all x : x is a boy, same as
 $\forall y \text{ boy}(y)$.. variable name don't matter.
-

- $\exists y \text{ boy}(y)$.. there exists y : y is a boy

The Four Aristotelian Forms

1. All P are Q
 $\forall x (P(x) \rightarrow Q(x))$
2. Some P are Q
 $\exists x (P(x) \& Q(x))$
3. No P are Q
 $\forall x (P(x) \rightarrow \neg Q(x))$
4. Some P are not Q
 $\exists x (P(x) \& \neg Q(x))$

Aristotle was a Greek Philosopher and Logician; he was a student of Plato,
and teacher of Alexander the Great in 300BC, see <http://en.wikipedia.org/wiki/Aristotle>

Exercise: Explain in words

1. $\forall x \text{ Mortal}(x)$
2. $\exists x \text{ Mortal}(x)$
3. $\forall x (\text{Human}(x) \rightarrow \text{Mortal}(x))$
4. $\exists x (\text{Human}(x) \ \& \ \neg \text{Mortal}(x))$

Note:

'Mortal' is opposite of Immortal (never die).

Exercise solution

1. $\forall x \text{ Mortal}(x)$ ‘Everyone is mortal’
2. $\exists x \text{ Mortal}(x)$ ‘Someone is mortal’
3. $\forall x (\text{Human}(x) \rightarrow \text{Mortal}(x))$
‘Every human is mortal’
4. $\exists x (\text{Human}(x) \ \& \ !\text{Mortal}(x))$
‘Some human is immortal’

Quantifier formula examples

1. boy(john) .. John is a boy
2. $\forall x \text{ boy}(x)$.. Everyone is a boy
3. $\exists x \text{ boy}(x)$.. There is a boy
4. $\neg \forall x \text{ boy}(x)$.. Not everyone is a boy
5. $\neg \exists x \text{ boy}(x)$.. There are no boys.
6. $\forall x \neg \text{boy}(x)$.. No one is a boy.
7. $\exists x \neg \text{boy}(x)$.. Someone is not a boy.
8. $\forall x (\text{boy}(x) \text{ or } \text{girl}(x))$.. everyone is a boy or a girl.
9. $\forall x \text{boy}(x) \text{ or } \forall x \text{girl}(x)$.. everyone is a boy or everyone is a girl.

Named constants

Constants are named objects

E.g. boy(John)

1. John is name of some boy
2. There is a boy named John
3. John is a boy

E.g. $\exists x (\text{boy}(x) \& x=\text{John})$

Means there is a boy and his name is John

E.g. (Anu != Banu) & (Anu = Anuradha) means Anu and Banu are different, and Anu is same as Anuradha.

Quantifiers on relations

We have

- Set of people,
- Relations between people
- Logical connectives ($\&$, or , \rightarrow , $!$)
- Quantifiers: \forall for-all, \exists there-exists.

Examples:

1. $\exists x \text{ likes}(\text{Jim}, x) \dots \text{ Jim likes someone.}$
2. $\exists x \text{ likes}(x, \text{ Jim}) \dots \text{ someone likes Jim.}$
3. $\forall x \text{ likes}(x, \text{ Jim}) \dots \text{ Everybody likes Jim.}$

Quantifier Exercises:

Write formulas using \forall and \exists

1. some students are boys.
2. some students are girls.
3. not all students are boys.
4. all boys are not students.
5. no boy is a girl.
6. some boys are students
7. some students have jobs
8. every student is a boy or a girl.

$\forall \exists$ Solution

- some students are boys: $\exists x (\text{student}(x) \ \& \ \text{boy}(x))$
- some students are girls. $\exists x (\text{student}(x) \ \& \ \text{girl}(x))$
- not all students are boys. $\neg \forall x (\text{student}(x) \rightarrow \text{boy}(x))$
 $\exists x (\text{student}(x) \ \& \ \neg \text{boy}(x))$
- all boys are not students. $\neg \forall x (\text{boy}(x) \rightarrow \text{student}(x))$
- no boy is a girl. $\neg \exists x (\text{boy}(x) \ \& \ \text{girl}(x))$
- some boys are students: $\exists x (\text{boy}(x) \ \& \ \text{student}(x))$
- some students have jobs $\exists x (\text{student}(x) \ \& \ \text{job}(x))$
- every student is a boy or a girl:
 $\forall x (\text{student}(x) \rightarrow (\text{boy}(x) \ \text{or} \ \text{girl}(x)))$

Examples of quantifiers use

1. John likes Kate: Likes(John, Kate)
2. John only likes Kate:
 $\forall x (\text{Likes}(\text{John}, x) \rightarrow x = \text{Kate})$
3. Only John Likes Kate:
 $\forall x (\text{Likes}(x, \text{Kate}) \rightarrow x = \text{John})$
4. Kate likes John's brother Larry
Likes(Kate, Larry) & brother(John,Larry)

Examples of double quantifiers

1. Everybody loves someone: $\forall x \exists y \text{ loves}(x,y)$
2. Someone loves everyone: $\exists x \forall y \text{ loves}(x,y)$
3. John loves no one: $\neg \exists y \text{ loves}(\text{John},y).$
4. Jim loves only himself:
 $\forall y (\text{loves}(\text{Jim},y) \rightarrow (y=\text{Jim}))$
5. God blesses those who pray to Him
 $\forall x (\text{pray}(x,\text{God}) \rightarrow \text{bless}(\text{God},x))$

Exercise: True or false? (Also write in English)

1. $\forall x : \text{likes}(x,x)$ _____ T/F
2. $\forall x \forall y : \text{likes}(x,y) \rightarrow \text{likes}(y,x)$ _____ T/F
3. $\forall x \forall y \forall z :$
 $\text{like}(x,y) \& \text{like}(y,z) \rightarrow \text{likes}(x,z)$ _____ T/F
4. $\text{likes}(\text{john},\text{kate}) \rightarrow \exists x \text{ likes}(\text{john},x)$ _____ T/F
5. $\text{likes}(\text{john},\text{kate}) \rightarrow \exists x \exists y \text{ likes}(x,y)$ _____ T/F
6. $\text{likes}(\text{john},\text{kate}) \rightarrow \exists x \forall y \text{ likes}(x,y)$ _____ T/F
7. $\text{likes}(\text{john},\text{kate}) \rightarrow \forall y \exists x \text{ likes}(x,y)$ _____ T/F

First Order Logic 2: Properties of Quantifiers and Duality

Properties of Quantifiers 1

$$\begin{aligned}\exists x (\text{boy}(x) \text{ or } \text{girl}(x)) \\ = (\exists x \text{ boy}(x)) \text{ or } (\exists x \text{ girl}(x))\end{aligned}$$

In English:

There is a boy or a girl.

= There is a boy OR

There is a girl.

Properties of Quantifiers 2

$$\begin{aligned}\forall x (\text{good}(x) \ \& \ \text{student}(x)) \\ = (\forall x \text{ good}(x)) \ \& \ (\forall x \text{ student}(x))\end{aligned}$$



“Everybody is a good student” is
same as

= Everybody is good AND
Everybody is a student.

Duality: Property of Quantifiers 3

1. Not everyone is a boy

\equiv There is someone who is not a boy.

- $\neg \forall x \text{ boy}(x) \equiv \exists x \neg \text{boy}(x)$

2. There does not exist a king

\equiv for all x , x is not a king.

- $\neg \exists x \text{ king}(x) \equiv \forall x \neg \text{king}(x)$

Compare with: $\exists x \neg \text{king}(x)$: there is someone who is not the king

Properties of Quantifiers 4

$\exists x (\text{good}(x) \ \& \ \text{boy}(x))$

$\rightarrow (\exists x \text{ good}(x)) \ \& \ (\exists x \text{ boy}(x))$

(There is a good boy) \rightarrow (There is a
good person **AND** There is a boy)

But converse is **not-true**:

$((\exists x \text{ good}(x) \ \& \ (\exists x \text{ boy}(x)))$

$\rightarrow \exists x (\text{good}(x) \ \& \ \text{boy}(x))$

False because the boy and good
maybe different people.

Properties of Quantifiers 5.

$(\forall x \text{ boy}(x)) \text{ or } (\forall x \text{ girl}(x))$

$\rightarrow \forall x (\text{boy}(x) \text{ or } \text{girl}(x))$

(Everyone is a boy OR Everyone is a girl)

$\rightarrow (\text{Everyone is a boy or a girl})$

But the converse is **not-true**:

$(\text{Everyone is a boy or a girl}) ! \rightarrow$

$((\text{Everyone is a boy}) \text{ or } (\text{Everyone is a girl}))$

Because the boy and girl can be different people.

Exercise

Can you write these as logic formula?

1. Everyone has a parent
2. Some people have brothers
3. Adam is the father of everyone.
4. There is a smallest number.
5. There is no largest number.
6. For every number there is a larger number.

Exercise: Solution

1. Everyone has a parent:

$$\forall x : \text{hasParent}(x) = \\ \forall x \exists p : \text{parent}(p, x)$$

2. Some people have brothers

$$\exists x : \text{HasBrother}(x) = \\ \exists x \exists b : \text{brother}(b, x)$$

3. Adam is the father of everyone.

$$\forall x : \text{father}(\text{Adam}, x)$$

4. There is a smallest number.

$$\exists s : \text{smallest}(s) = \\ \exists s \forall y : s \leq y.$$

5. There is no largest number.

$$! \exists b : \text{largest}(b) = \\ ! \exists b : \forall y : b \geq y$$

6. For every number there is a larger number.

$$\forall n \text{ there is a number } k \text{ larger than } n = \\ \forall n \exists k : k \geq n$$

First Order Logic 3: Uniqueness

Uniqueness

- There is a king:

$$\exists k \text{ king}(k)$$

- There is only one king:

$$\exists k (\text{ king}(k) \ \& \ \forall y (\text{ king}(y) \rightarrow y=k))$$


Counting

1. There are two cr (class representative):

$$\exists g \ \exists b (\text{cr}(g) \ \& \ \text{cr}(b) \ \& \ (g \neq b))$$



2. There are **only** two cr:

$$\begin{aligned} \exists g \ \exists b (\text{cr}(g) \ \& \ \text{cr}(b) \ \& \ (g \neq b) \ \& \\ \forall y (\text{cr}(y) \rightarrow (y=g \ \text{or} \ y=b)) \end{aligned}$$



Exercise

1. There is no king.
2. There are only 2 ministers.
3. Everybody likes two people (parents).
4. Two people like everyone.
5. Two people don't like each other.



Exercise: Solution

1. There is no king: $\neg \exists x : \text{king}(x)$

There are only 2 ministers.



2. $\exists m \exists n (\text{minister}(m) \ \& \ \text{minister}(n) \ \&$
 $(m \neq n) \ \&$
 $\forall z (\text{minister}(z) \rightarrow (z=m \text{ or } z=n))$



3. Everybody likes two people.

$\forall x \exists m \exists f (\text{likes}(x, m) \ \&$
 $\text{likes}(x, f) \ \&$
 $(m \neq f))$



Exercise: Solution

4. Two people like everyone.

$$\exists x \exists y \forall z (\text{likes}(x,z) \ \& \\ \text{likes}(y,z) \ \& \\ (x \neq y))$$



5. Two people don't like each other.

$$\exists x \exists y (\text{! likes}(x,y) \ \& \\ \text{! likes}(y,x) \ \& \\ (x \neq y))$$



Temporal Logic



Temporal Ambiguity

Anu says:

"We will meet Saturday or Sunday,
unless it rains."



Logically this means:

if (does not rain) then
((we will meet Saturday) xor
(we will meet on Sunday)).

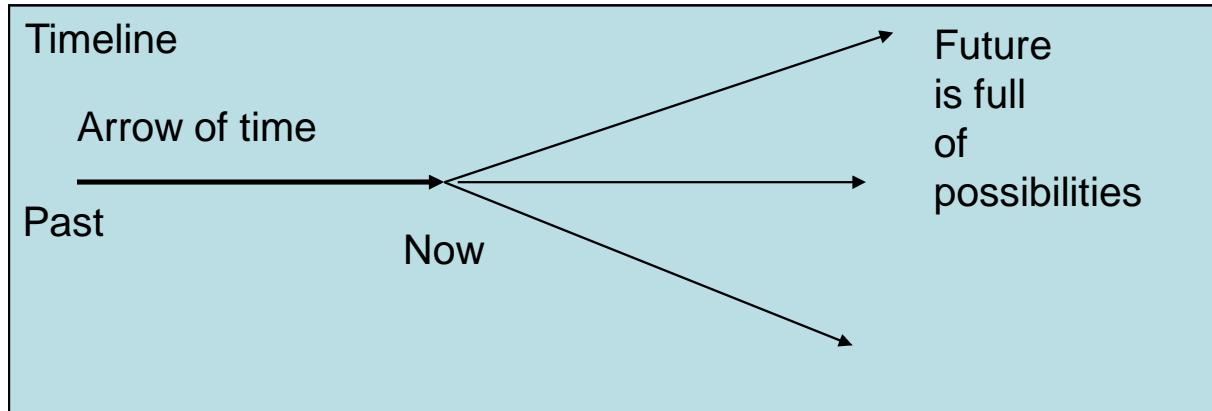


It is unclear **when** "it rains"



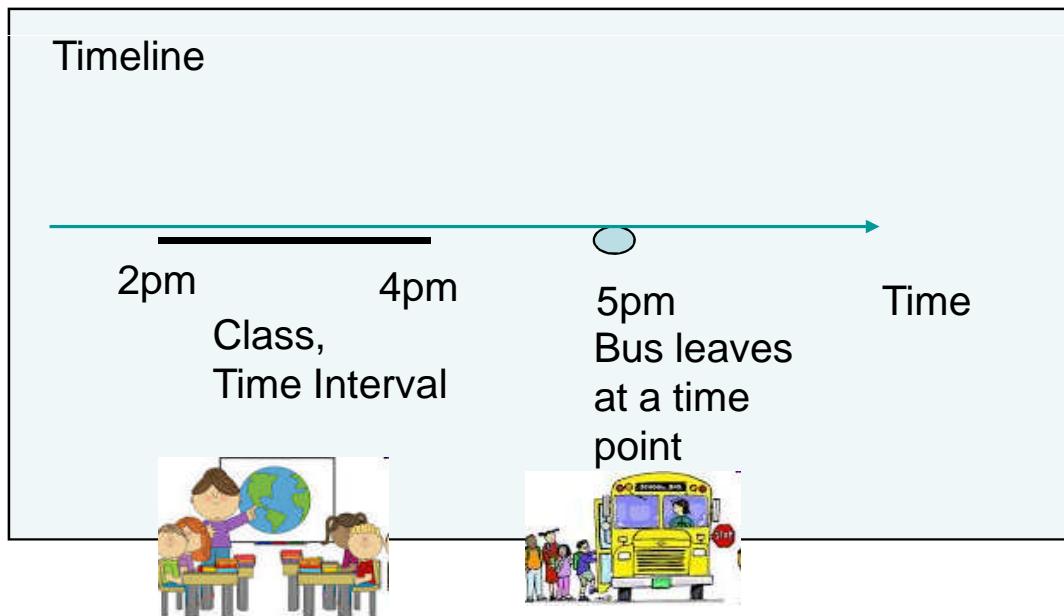
Time: past, present, future

- Speaking of time, we have past, present, future.
- There is one fixed past.
- There are multiple possibilities for the future.



Time point versus interval

- **Interval:** Class is from "2pm to 4pm"
- **Point:** Bus leaves at "5pm"

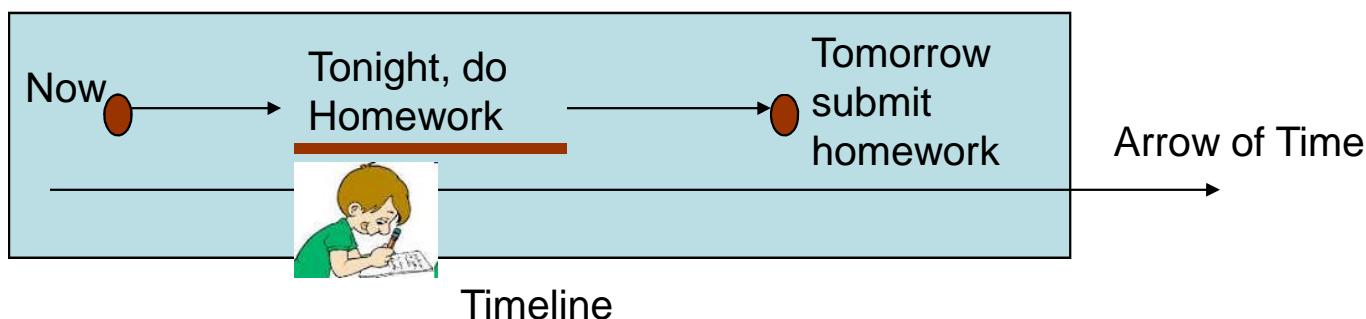


Example

- "I will submit the homework tomorrow, if I finish doing it tonight."

How many time points are there?

Now, Tonight interval, Tomorrow.



Exercise

Draw the timeline or write logical formula for following events:

1. Yesterday she said "someday, I will buy an umbrella, if it keeps raining."
2. Anu told Bina that she will win a lottery, if she has never committed a sin.
3. Chetan worried that if he had forgotten to close the tap, the house would be flooded.



Temporal modalities

1. [] Always, daily, weekly, monthly, yearly.
2. <> Sometimes, rarely, once
3. O, Next: Tomorrow, yesterday, next week.
4. U, Until:
5. While

Sequencing Events

Keywords: *then, before, after, later, earlier, next*

Question: Are 1,2,3 are same or different?

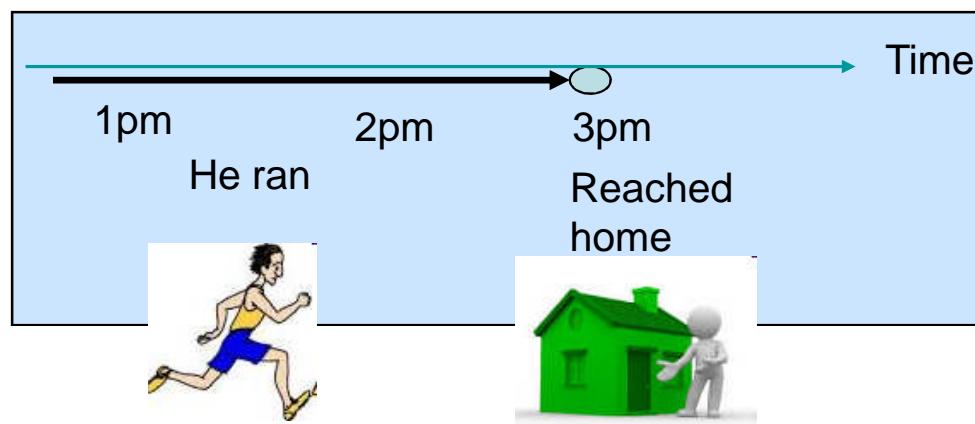
1. "Mary turned the car, and **then** she stopped".
2. "Mary turned the car, **before** she stopped."
3. "**Before** stopping the car, Mary turned.



Notation: (Mary turned the car) and
Next(Mary stopped)

Until

- "He ran *until* he reached home."
- "He kept running *till* he reached home."



Repeating events

- Keywords: **Always, daily**
"It rains **daily**" (once a day?)
"It is **always** raining" (non stop?)
"It **never** rains" (always not raining).



Notation:

- [] rains, (**always** is written as a box)
- [] !rains (**never** is box not)

Note: !rain = dry

Sometimes

1. "Sometimes it rains".
2. "It rained once".



Notation: \diamond rains (diamond is sometimes).

"Sometimes it rains, other times it is dry" is " \diamond rains & \diamond !rains"



Duality of Always and Sometimes

1. $\neg \Box \text{rain} \equiv \neg \Diamond \text{rain}$

"It is **not always** raining"
 \equiv "Sometimes dry"

2. $\Box \neg \text{rain} \equiv \neg \Diamond \text{rain}$

"Always dry"
 \equiv "not sometimes rains"
 \equiv "never rains"



Duality examples

1. $\neg \Box \neg \text{rain} \equiv \neg \Diamond \text{rain}$

"**not always** dry" \equiv "Sometimes
raining"

2. $\Box \text{rain} \equiv \neg \Diamond \neg \text{rain}$

"**Always** rain" \equiv "Not sometimes
dry"



Philosophical Questions

- Does the past exists?
- Does the future exists?
- Are there multiple futures or only one?
- Can we choose our future or is it fixed by the laws of physics?
- Is there free will, or is future decided by god?

Exercise: Complete and Explain these proverbs

1. Never say never
2. Time and tide ..
3. A stitch in time saves ..
4. Bird in hand ..
5. Don't cry over ..

More proverbs about time?

Knowledge and Belief

Examples

1. I **know** I have done well in my exam.
2. I **hope** you have done well in your exam.
3. I **believe** I will be the topper in the class.

4. "I **know** $2+2=4$."
5. "I **believe** $\pi=3.14$ "
6. "I **believe** cancer **is** curable."
7. "I **doubt** cancer **is** curable."
8. "I **hope** cancer **is** curable."

Notation

I believe it is rain(ing)

- $B(\text{rain})$
- $B(i, \text{rain}).$



I know it is raining

- $K(\text{raining})$
- $K(i, \text{raining})$

Anu: "I believe in Karma:

- $B(\text{Anu, karma}) .. \text{Anu believes in Karma.}$

Temporal beliefs



Anu: "I **believe** cancer *will be* curable."



Biju: "I **bet** a cure for cancer *will be* found."

Catie: "I **doubt** a cure for cancer *will be* found."

Temporal beliefs

Compare:



Anu: "I **think** cancer **is** curable."

Biju: "I **think** cancer **will be** curable."

Catie: "I **thought** cancer **is** curable."

Dave: "I **thought** cancer **was** curable."

Eva: "I **thought** cancer **will be** curable."

Temporal (past) beliefs

Beliefs about the past:

Anu: I **believe** Vikings **had** visited India.



Biju: I **guess** Vikings **had** conquered Russia.



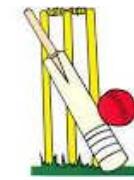
Catie: I **believe** Vikings **were** Germans.

Eva: I **hope** Vikings **lived** in Germany.



More beliefs

- I believe in cricket.
 $B(\text{cricket})$
 $B(i, \text{cricket})$
- I believe in nothing.
 $\forall.y \neg B(i, y)$
- Everyone believes in something:
 $\forall.x \exists.y B(x,y) \quad .. x \text{ is a person, } y \text{ is some belief.}$
- Those who believe in the aliens believe in everything.
 $\forall.x (B(x,\text{aliens}) \rightarrow \forall.y B(x,y))$



Knowing

Are these two different?

Anu: "I forgot where I kept the book."

Biju: "I don't know where the book is."

Explain

Solution

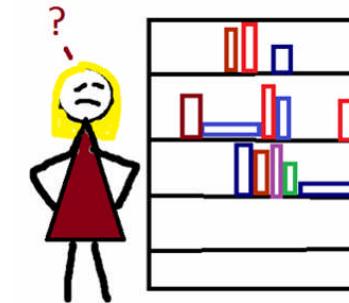
Anu: "I forgot where I kept the book."



Biju: "I don't know where the book is."



Anu: "I knew where the book was before, but now I forgot where it is."



Meta knowledge

1. I **know** you **know** the answer
 $K(i, K(\text{you}, \text{answer}))$.

2. I **know** you **believe** in Karma
 $K(i, B(\text{you}, \text{karma}))$ s.



3. I **think**, you **believe** that I am superman.
 $B(i, B(\text{you}, \text{I am superman}))$.



Meta beliefs

1. I believe, If I believe in God, then
God will help me.

$B(i, B(i, \text{God}) \rightarrow \text{"God will help me"})$.

2. I believe, those who believe in
magic, also believe in ghosts.

$B(i, \forall x (B(x, \text{magic}) \rightarrow B(x, \text{ghosts}))$.



Exercise: Write in your notebook



Anu: "I believe the believers will be saved."

Ask questions to Anu to **clarify** (write dialog):

Biju: ???

Catie: ???

Dave: ???

Eva: ???

Farid: ???

etc...

Exercise: Solution

Anu: "I believe the believers will be saved."

Questions to clarify the meaning:

Biju: Who are the believers?

Catie: What do they believe in?

Dave: What will they be saved from?

Eva: When will they be saved?

Farid: What about non-believers?

Ganesh: What about dis-believers?

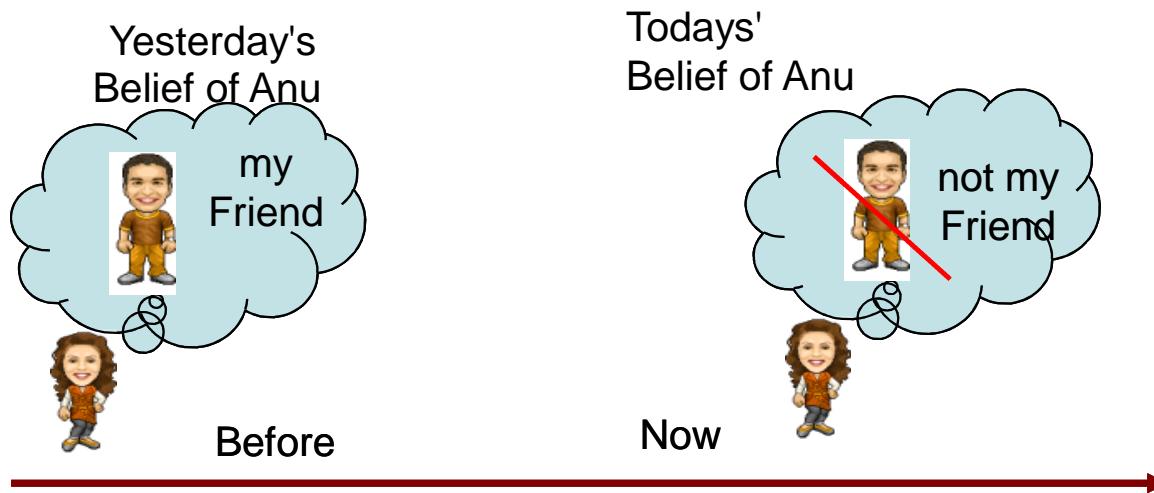
Exercise: Explain in your words

Anu: "I **used to believe**, Biju is my friend, but **not** anymore."

1. Explain in words first.
2. Then write it logically.

Explain: Solution

Anu: "I used to believe, Biju is my friend, but not anymore."



De-Dicto versus De-Re

Anu: "I believe **all** politicians are honest"

There are two interpretations of '**all**'

1. De-dicto:

$$B(anu, \forall x \text{ politician}(x) \rightarrow \text{honest}(x))$$

All the people Anu knows.

2. De-re:

$$\forall x \text{ politician}(x) \rightarrow B(anu, \text{honest}(x))$$

All the people in the world.

Counterfactuals and planning

Dreaming (future impossible):

- Anu: "If I was born in Germany, I would get a BMW car."



Past (regretting: past impossible):

- Biju: "If I had studied for my exam, I would have got 100%".



Present (thinking: present possible):

- Catie: "If I sleep now, I can study tomorrow."



Future (planning: future possible):

- Dave: "If we go early, we will get good seats."



Exercise

Make your own examples in your notebook

Dreaming (future impossible):

— ...

Past (regretting: past impossible):

— ...

Present (thinking: present possible):

— ...

Future (planning: future possible):

— ...

Types of Believers

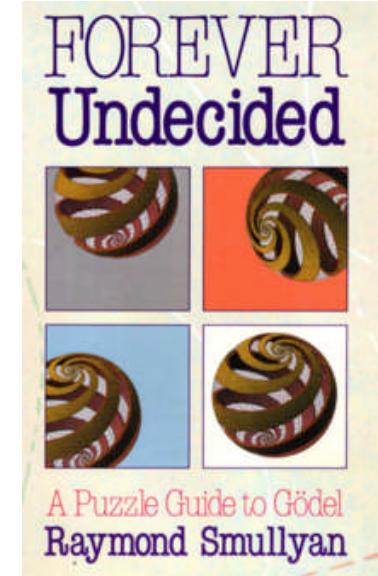
- $B(p) \& p$.. Correctly believe
- $Bp \rightarrow p$.. Accurate person
- $Bp \& \neg p$.. Inaccurate person
- $\neg B(\text{False})$.. Consistent Person
- $B(\text{False}), (Bp \& B(\neg p))$.. Unsound Person
- $Bp \rightarrow BBp$.. Normal Person
- $Bp \& B(\neg Bp)$.. Peculiar Person.
- $Bp \& B(B(\neg p))$.. Nutcase
- $B(Bp \rightarrow p) \rightarrow p$.. Modest believer.
- $B(Bp \rightarrow B(\text{False}))$.. Afraid to Believe p.
- $K(p) \& \text{tell}(\neg p)$.. Liar
- $K(p) \& \neg \text{tell}(p)$.. Secretive?
- $K(p) \& \text{tell}(p)$.. Honest

Logical Omniscience

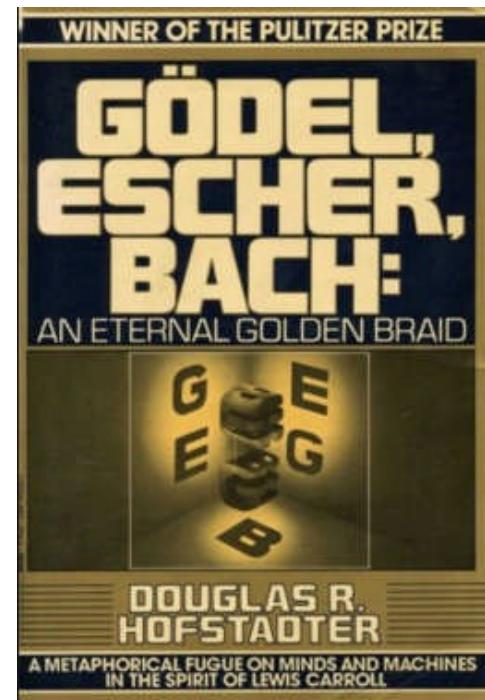
- A person who knows all the logical truths, and its consequences.

References:

Forever Undecided,
by Raymond Smullyan, 1986.
Google for Modal Logic.



Gödel's Theorem



Completeness: Can all truth be known?

- Hilbert was a great mathematician 100 years ago.
- Hilbert's program was to decide the truth of all mathematical sentences.
- But Gödel used self-reference to prove Hilbert's dream was impossible.

Incomplete: means there are truths that can not be proved.



David Hilbert



Kurt Gödel

Godel's 1st incompleteness theorem

G1. "Powerful logics are incomplete" ..
means in any powerful logic, there are truths that cannot be proved.

- More details:
 - Google: Wikipedia Godel's theorems
 - Book: Forever undecided by Smullyan.

Godel's 2nd incompleteness theorem

G2. "If a powerful logic claims to be correct, then it goes **wrong**."

- means, in any powerful logic
- We will never know whether it is correct or not
- There must always be doubts.
- As soon as we stop doubting, it will go wrong.

Beliefs

- We can never know if we are always correct.
- Anyone who claims to know everything is going to be wrong.
- Not all truth can be proved.
- If we stop doubting our knowledge, we can go wrong.
- A modest person cannot claim that "he/she is modest".

Truth and proof

What is truth and proof?

- Anu: "What is truth?"
- Chetan: "Truth is something that is universally true in all interpretations."
- Biju: "What is proof?"
- Chetan: "Proof is evidence for the truth."
- David: "Truth is independent of the speaker and listener, and the situation."
- Chetan: "[Anyone](#) can check the proof and verify if it is correct or not."

Reference: "Truth and Proof" by Alfred Tarski.

Truth

- Before you can say a sentence is true or false, you must be able to define every word in the sentence.
- If you don't understand a sentence, then you cannot proceed further.

Induction

Chetan: "Let's prove something, Anu?"

Anu: "**All crows are black!**" (she sees a crow).

Biju: "How can you be sure?"

Anu: "I have seen a million black crows."

Biju: "Now you are **exaggerating.**"

Chetan: "Anu is **proving by induction**".



Probable proof

Anu: "I am **99% sure** all crows are black."

Biju: "The crow you saw was black, there maybe **other possibilities?**"

Chetan: "Anu gave us a **probable proof**, it is probably true, unless we find a white crow?"

Circular reasoning

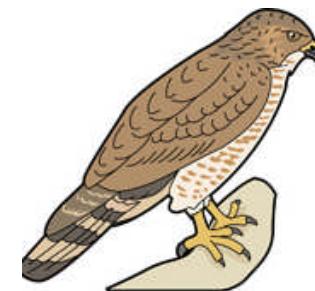
Chetan: "Would you call a Myna a brown crow?"

Anu: "But, all crows are black?"

Chetan: "You can't define crows as black bird, and then prove crows are black!"



Brown crow
or Myna



Brown crow
or Hawk?

Circular definition

Biju: "So what is a crow?"

([Definition of crow?](#))



Anu: "Crow is a black bird that eats garbage."

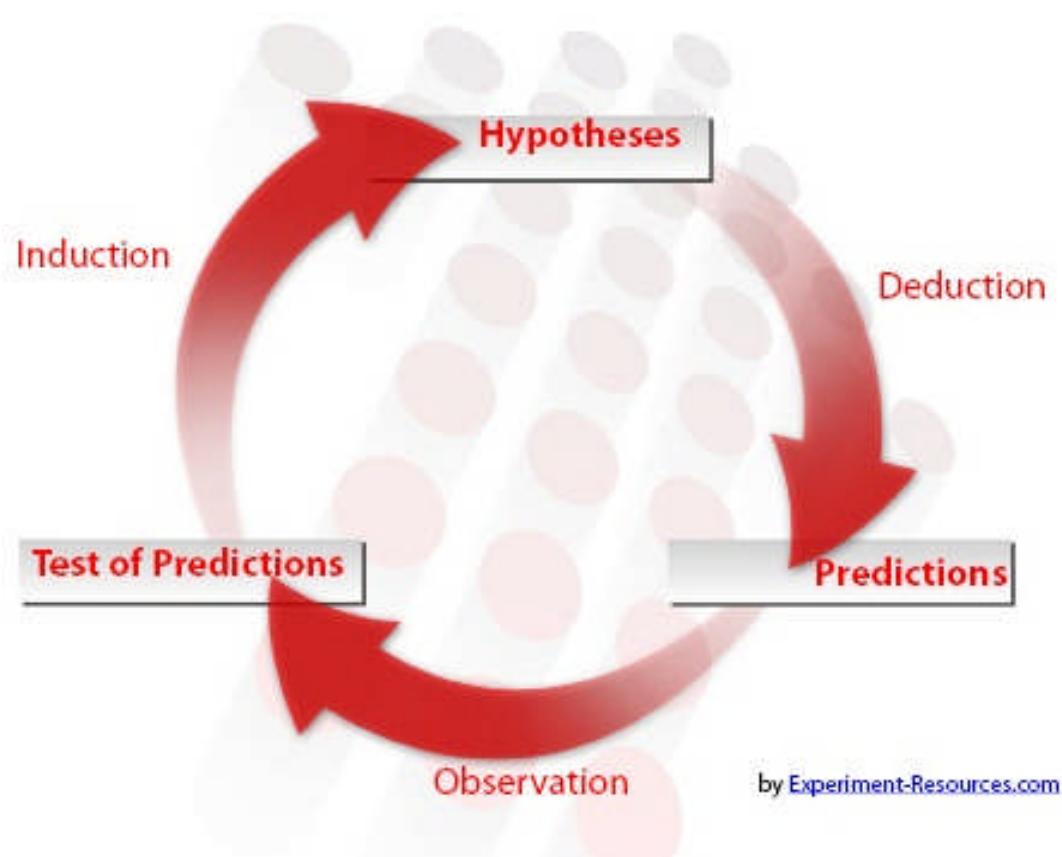
Chetan: "So by your definition, all crows are black!"

Dave: "So, what is the procedure to prove something?"

Define, Hypothesis, Prove.

1. "To prove something, you must first define it clearly in simple terms."
2. "Then you form a hypothesis."
3. "Then you prove the hypothesis to make a theory"
4. You use the theory to make predictions.
5. If new data invalidates the theory, you must revise the theory.

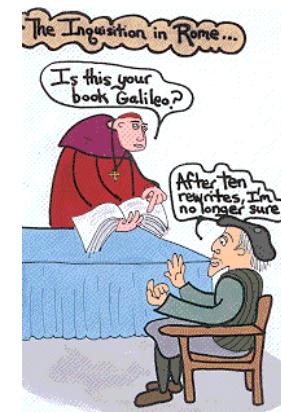
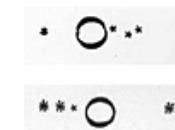
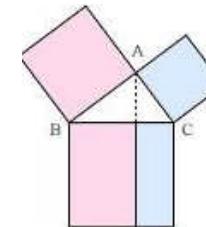
Circle of Knowledge



by Experiment-Resources.com

Example: Theory of Geometry

1. Euclidean geometry
2. Earth is center of the universe
3. Galileo and moons of Jupiter
4. Hyperbolic Geometry
5. Relativity.



<http://www.math.brown.edu/~banchoff/STG/ma8/papers/dstanke/Project/softball.html>

Counter example

Eva: "There are pied (black and white) crows in Africa".

Chetan: "This is a **counter-example** to the statement that **"All crows are black"**".

Anu: "So a single counterexample can invalidate a proof!"

Chetan: "So, it is not a fact."



Indian Crow



Pied African Crow



Rare albino crow

Qualify your assertions

- Anu: "So the theorem is false?"
- Chetan: "No, we can just add a condition, that **"all crows are black – *in this country.*"**
- Anu: "Are you sure?"
- Chetan: "*hmm, I believe, almost all crows are black in this country*".
- Anu: "So, now you don't have to prove it, because it is just your belief, others may or may-not agree with it!"
- Chetan: "Yes."

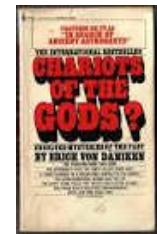
Fallacies

Introduction

- Fallacy is a wrong argument (logically wrong).
- There are many different types of fallacies and we must be able to recognize the logical problems, when presented in conversation, speeches, newspapers, tv, books.

Ad Hominem (Argument against the Man)

Example: "The book 'Chariots of God' is wrong about aliens, because the author was once jailed."



Anu "Be a vegetarian and kind to animals".

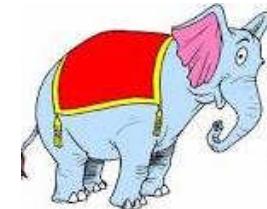


Biju "But you wear leather shoes, made from animal skin!"



Needling:

Chetan "I am kind to animals."



Dravid "You should be in the circus."

Chetan "Don't needle me, listen carefully."

Dravid "You are a joker."

(Name calling)



Inductive fallacy.

Anu: "All crows are black."

Biju: "Why?"

Anu: "Because I have seen a million crows, and all of them are black!"

Chetan: "This is an inductive fallacy, there are pied (black and white) crows in Africa."



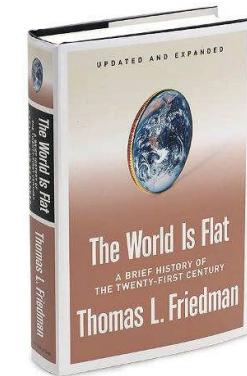
Appeal to authority

(Latin: *argumentum ad auctoritatem*)

- Eugene: "There are no trees on Saturn, because our teacher said so."



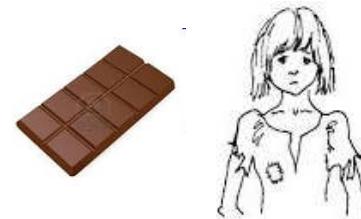
- Farid: "I know, the Earth is flat, because I saw it in the **NY Times** book review."





Straw Man (Fallacy Of Extension)

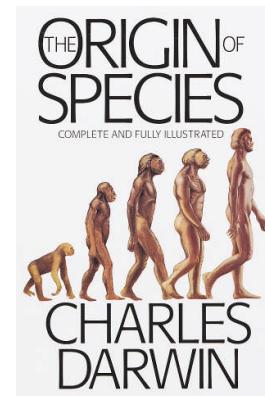
Gandhe: "Children should eat chocolate."



Hanifa: "It is bad for their health."

Gandhe: "You want children to be hungry?"

Darwin: "We descended from monkeys."



Ingrid: "Your father was a monkey?"



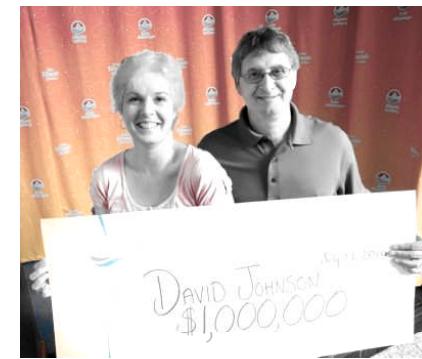
Appeal to Emotion

- Jamil: "Donate money for Hunger Project"
- Karen: "Tell me why they need money?"
- Jamil: "Do you want cute babies to suffer from it?"

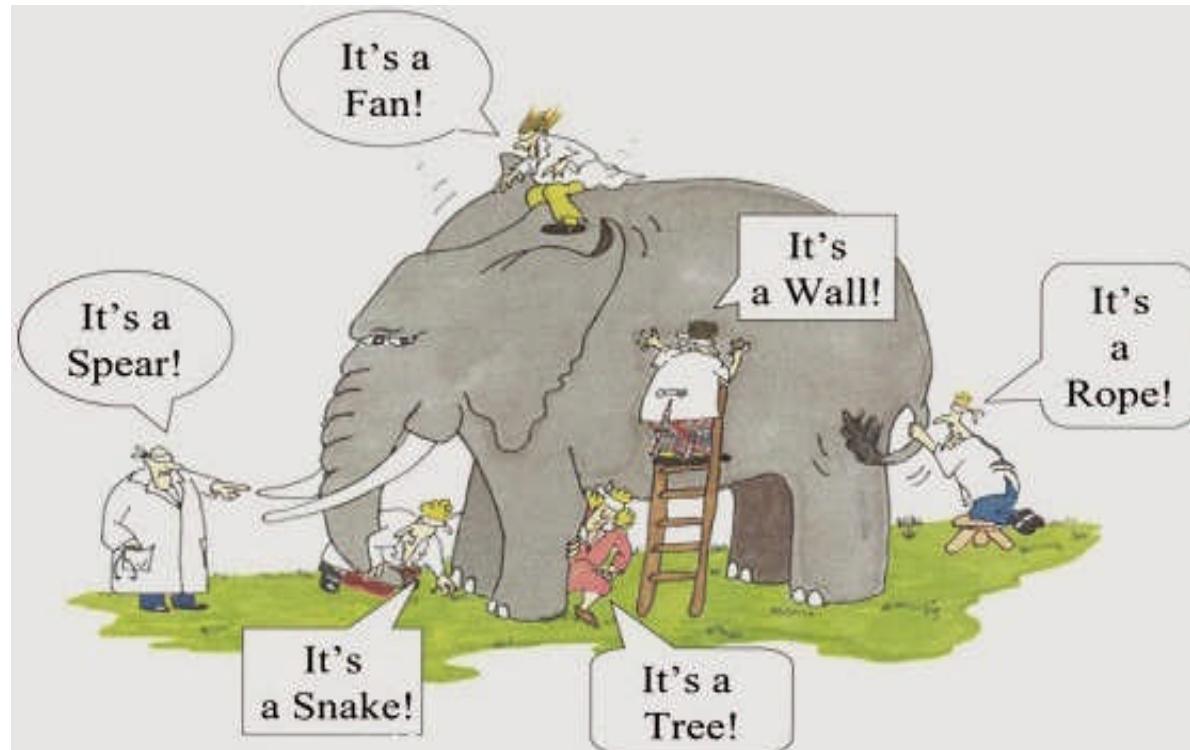


Argument By Selective Observation

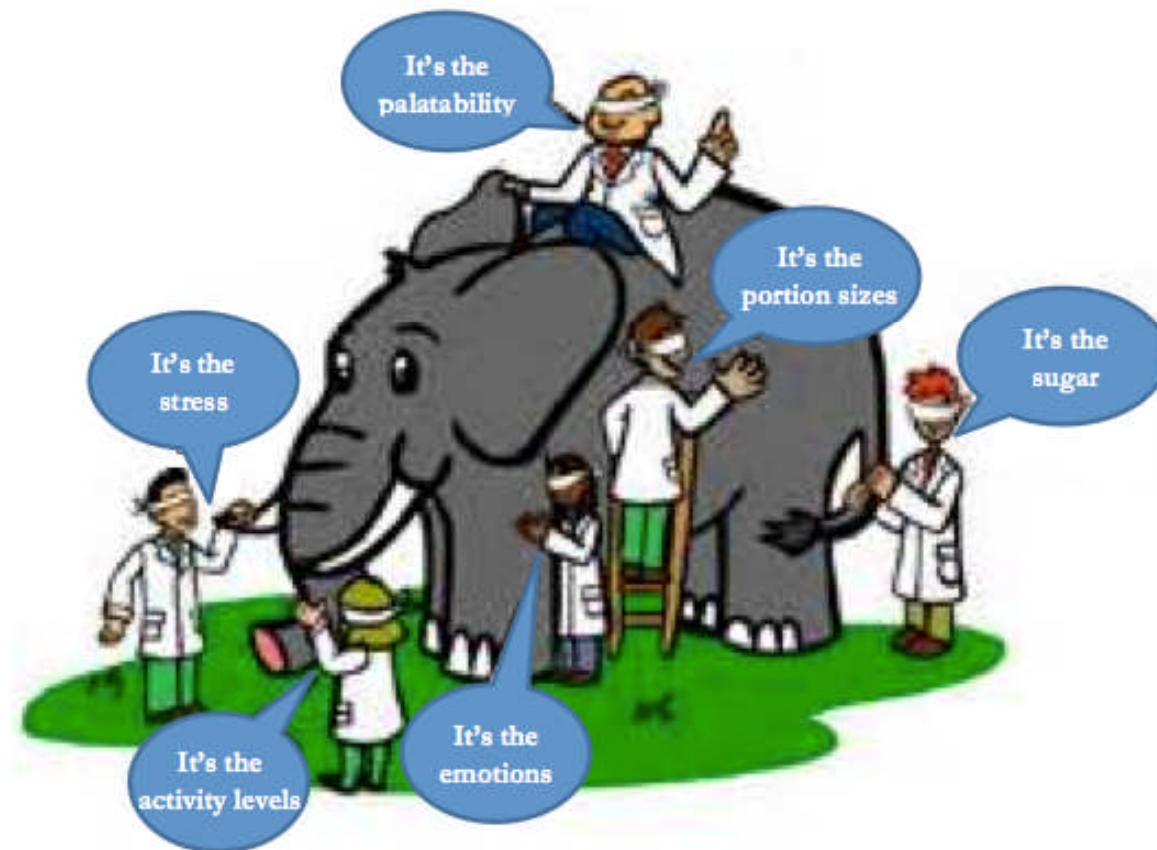
- Example: Lottery winners appear on TV, Gamblers who lose money, don't appear on TV.
- So people only see the one side of the issue in media.



Elephant and 5 blind men (selective observation)



Medical Diagnosis?



Correlation is not Causation

- Karen: "Children with bigger shoes size have higher IQ, so I am getting big shoes for my daughter."
- Lavina: "Older children have bigger shoes. Shoes are not the cause of higher IQ, they are just correlated."



Bandwagon Argument, Peer Pressure

- Manju: "I will vote for Green Peace, because my friends voted for it."
- Nadini, "If smoking was bad, why would millions of people still be smoking?"



Slippery Slope (Camel's nose)

- Omar: "If we allow smoking, soon people will smoke drugs."
- Parijat: "and soon they will be selling drugs."



But when the camel's nose appears under the tent, it is not long before the rest of the camel invites itself in.

Complex Question (Tying)

- Lawyer: "Answer Yes/No."
- Witness: "Yes"
- Lawyer: "Have you stopped drinking?"
- Witness: "Yes"
- Lawyer: "What was your favorite drink?"



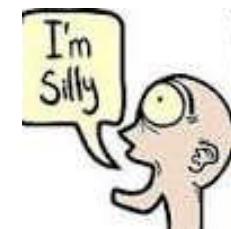
Non Sequitur: does not follow

- Ajit: "We all saw lights in the sky yesterday night, it must be spaceships visiting the Earth."
- More examples?



Meaningless Questions

- Qasim: "If Rajni-kant can't stop dowry, who can stop it?"
- Rajni: "If you are so smart, why do you have to study?"
- Sunil: "What kind of idiot are you?"
- Tina: "Why are you such a idiot?"
- Uma: "Why don't you get lost?"



Exercise: Give your own example.

...

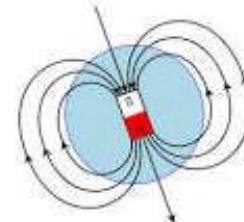
Bad Analogy

- Ophelia: "I am a vegetarian."
- Priya: "Then don't eat walnuts,
they are the eggs of walnut trees"
- Exercises: Your example?



Argument by Jargon

- Sujit: "The unified field theory proves that life may exists in other universe."



Exercise: Your own example?

$$\begin{aligned} & \int_0^{2\pi/5} \int_0^a \frac{ar}{\sqrt{a^2-r^2}} dr d\phi \\ &= a \int_0^{2\pi/5} \int_0^a \frac{r}{\sqrt{a^2-r^2}} dr d\phi \\ &= a \int_0^{2\pi/5} \left[-\sqrt{a^2-r^2} \right]_0^a d\phi \\ &= a \int_0^{2\pi/5} [(-\sqrt{a^2}) - (-\sqrt{a^2})] d\phi \\ &= a \int_0^{2\pi/5} [\sqrt{a^2}] d\phi = a \int_0^{2\pi/5} a d\phi \\ &= a^2 \int_0^{2\pi/5} d\phi = a^2 2\pi/5 \end{aligned}$$

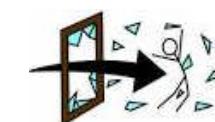
Medical Jargon



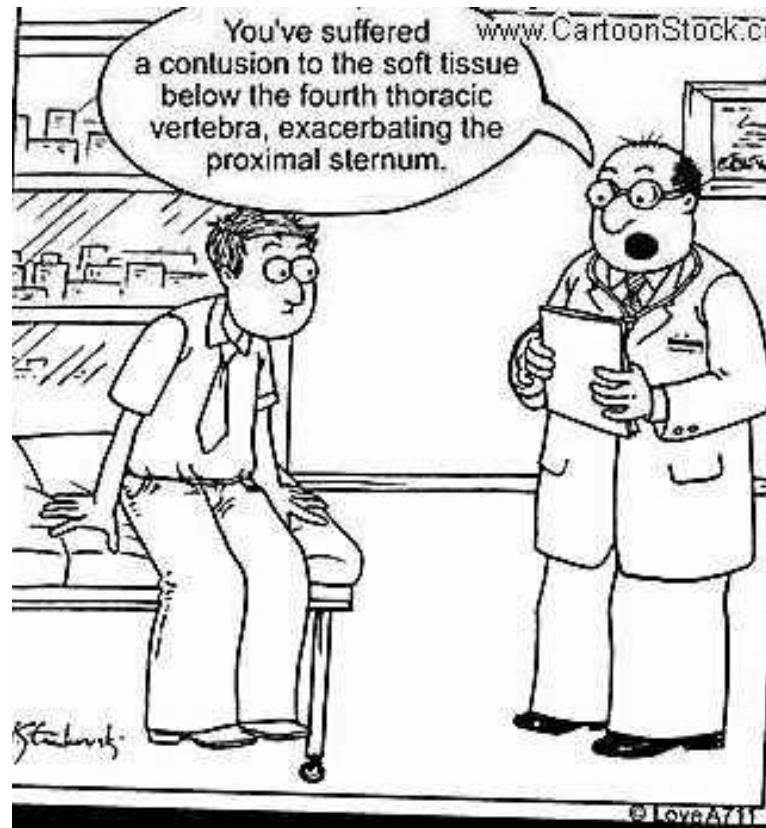
- Dr Tina: "The subject incurred lacerations, abrasions and a hematoma to the cranium after being defenestrated."



- Dr Mina: "he cut and hurt his head, when he fell out of the window?"



Medical Jargon Example



Translation: "You have a bruised rib."

Logic of Praying

Post-priori

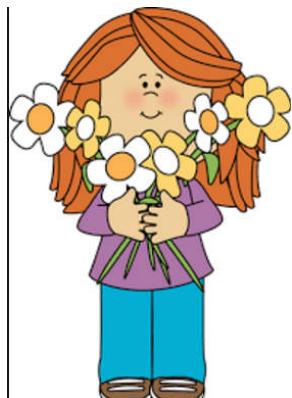
- Anu: [After exams] “Oh God, Please give me good marks. So my parents will be happy. I promise I will never ask anything again.”

Apriori

- Bina: [Before exams] “Oh God, Please help me study, and do well in the exams. So I am may be a better person in life with this learning.”

Logic of Love

- “When you like a flower, you just pluck it. But when you love a flower, you water it daily.”



St Paul on Love

“Love is always patient and kind;
Love is never jealous;
Love is not boastful or conceited,
it is never rude and never seeks its own advantage,
it does not take offence or store up grievances.

Love does not rejoice at wrongdoing,
but finds its joy in the truth.
It is always ready to make allowances,
to trust, to hope and to endure whatever comes.”

- St Paul, 1 Corinthians 13: 4-7.

Logic of Movies



Logic of Movies

If there is a fight in a movie
between the main guy and a load of
baddies, they will come at him one by one.
In an orderly fashion. So he can deal with them.



GRE English (sounds like insult)

- Alok: "Biju masticates at the dinner table."
- Biju: "Alok defenstrated his chappati."

GRE/GMAT English 1

NORMAL PERSON : People who live in glass houses should not throw stones.

GRE STUDENT : Individuals who make their abodes in vitreous edifices would be advised to refrain from catapulting perilous projectiles.

NORMAL PERSON : Twinkle, twinkle, little star

GRE STUDENT : Scintillate, scintillate, asteroid minim.

NORMAL PERSON : All that glitters is not gold.

GRE STUDENT : All articles that coruscate with resplendence are not truly auriferous.

NORMAL PERSON : Beggars are not choosers

GRE STUDENT : Sorting on the part of mendicants must be interdicted.

NORMAL PERSON : Beauty is only skin deep

GRE STUDENT : Pulchritude possesses solely cutaneous profundity.

NORMAL PERSON : Cleanliness is godliness

GRE STUDENT : Freedom from incrustations of grime is contiguous to rectitude.

NORMAL PERSON : There's no use crying over spilt milk

GRE STUDENT : It is fruitless to become lachrymose of precipitately departed lacteal fluid.

NORMAL PERSON : You can't try to teach an old dog new tricks

GRE STUDENT : It is fruitless to attempt to indoctrinate a superannuated canine with innovative maneuvers.

NORMAL PERSON : Look before you leap

GRE STUDENT : Surveillance should precede saltation.

GRE/GMAT English 2

NORMAL PERSON : Dead men tell no tales

GRE STUDENT : Male cadavers are incapable of rendering any testimony.

NORMAL PERSON : Beginner's luck

GRE STUDENT : Neophyte's serendipity.

NORMAL PERSON : A rolling stone gathers no moss

GRE STUDENT : A revolving lithic conglomerate accumulates no congeries of small, green, biophytic plant.

NORMAL PERSON : Birds of a feather flock together

GRE STUDENT : Members of an avian species of identical plumage tend to congregate.

NORMAL PERSON : He who laughs last, laughs best

GRE STUDENT : The person presenting the ultimate cachinnation possesses thereby the optimal cachinnation.

NORMAL PERSON : All work and no play makes Jack a dull boy.

GRE STUDENT : Exclusive dedication to necessitous chores without interludes of hedonistic diversion renders Jack a hebetudinous fellow.

NORMAL PERSON : Where there's smoke, there's fire!

GRE STUDENT : Where there are visible vapours having their provenance in ignited carbonaceous materials, there is conflagration.

Exercise: Are these correct?

1. Water is addictive, (people can't give up drinking water, even for a day).
2. Water is deadly, (everyone who drinks it will die).
3. John "We wasted money on the Life Insurance Policy", no one got sick last year.
4. Jim "We wasted money on the fire extinguisher, we never used it so far, let's not buy more."
5. Jena "This medicine is about to expire, let us eat it now, otherwise it will expire."

More Fallacies

Metaphorical fallacy

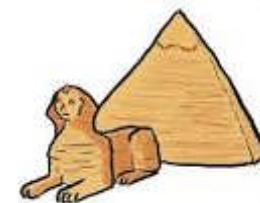
1. X is *metaphorically* Y.
2. Y is literally Z.
3. Therefore, X is literally Z.

Sam: "I am a computer" (metaphorically)

Tina: "Can I reboot you?" (literally)

Argument from incredulity

- Usha: "There is no life on the moon, because no one can live in outer space."
- Vinod: "I can't do badly in exams, therefore I must have done well in exams."
- Wajid: "Nobody can lift the pyramid stones, hence Pyramids must be built by Dinosaurs or Aliens."



Negative proof

Arguing that something must exist because there is no evidence it does not exist.

- Wanda: "I must be a genius, because no one has called me an idiot."
- Xerxes: "There are no aliens, because nobody has seen them."



Equivocation

Using different meanings of a word.

Xavier: "All rivers have a **bank**, therefore there must be money in the river."

Yatish: "Politicians are a **headache**, aspirin tablet will cure a **headache**, therefore aspirins can cure politicians."

Zahir: "**Average** family have 2.7 people, and Usha is **average**, so her family has 2.7 people."

Exercise: Your examples?

Association

Asma: "Hitler was a vegetarian, therefore vegetarians are dangerous."



Charlie Chaplin in the movie
"The Great Dictator"
about Hitler and Fascism.

Poisoning the well

Cathie: "Let's read about Microsoft, from a book written by Bill Gates, he never graduated from college."



Question: Identify the people in the photos -->



Manufacturing consent

Get Newspapers to print articles and take polls that present your viewpoint:



Headline: "Cricket match fixing is a bigger problem, than corruption, hunger and safety for women."





Red herring

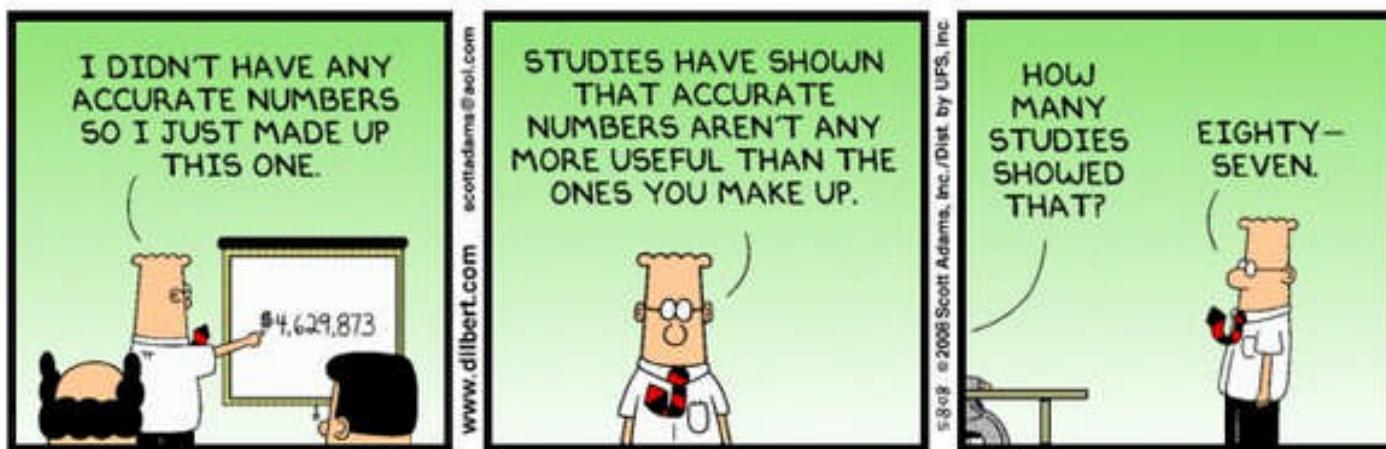


Divert attention from the main topic.

1. Ajit: "people are tired of up paying bribes to corrupt officers."
2. Banti: "Yes, corruption is a big problem in Cricket. I will arrest cricketers who take bribes."

Exercise: Your example?

Wrong use of statistics



Wrong use of statistics

1. Donna: "90% of the accidents happen in the kitchen, so I stopped cooking."
2. Eugene: "More people died in car accidents than in war, so I am protesting against car makers instead of war."
3. Falguni: "People who celebrate more birthdays live longer."
 - (which is cause and which is effect?)

Exercise: Your examples?



Gambler's fallacy

1. Ganesh: "We lost last six games, so we are likely to win the next six games."
2. Honavar: "I found a money on the road, when I was wearing a blue shirt, so I am wearing the same lucky blue shirt for my exams."



Exercise: Your example?

Begging the question

Saying the same thing: X → X

1. Ingrid: "Eating meat is bad, because it is bad."
2. Jayesh: "I will be the president, because I am the greatest."
3. Katie: "No truth is a lie."
4. Lenny: "Opium causes sleepiness because it is soporific."

Quoting

- Anu: "I don't think, we should drink tea at night."

Quoting out of context

- Bina: "Anu said, she doesn't think."
- Charu: "Anu said, we should drink at night."

Misquoting

- Dina: "Anu said, we should drink without thinking."



Misquoting

- Sachin: "I am afraid, we will lose the match unless we put up a good fight."

Misquotes:

1. Tarun: "Sachin admits he is afraid."
2. Vikas: "Sachin expects lose the match."
3. Walim: "Sachin wants to fight in match."
4. Xandu: "Sachin afraid to fight."
5. Yatish: "Sachin wants a good fight."

Exercise: Your examples?

False economy

- A **false economy** is an action that saves money initially, but in long run waste money. [short sightedness?]

Examples:

- Buying cheaper plastic cups to save money, instead of good ceramic cup. [why?]
- Bargain shopping to save Rs 100 [why?]
- Arguing with taxi-driver to save Rs 10 [why?]

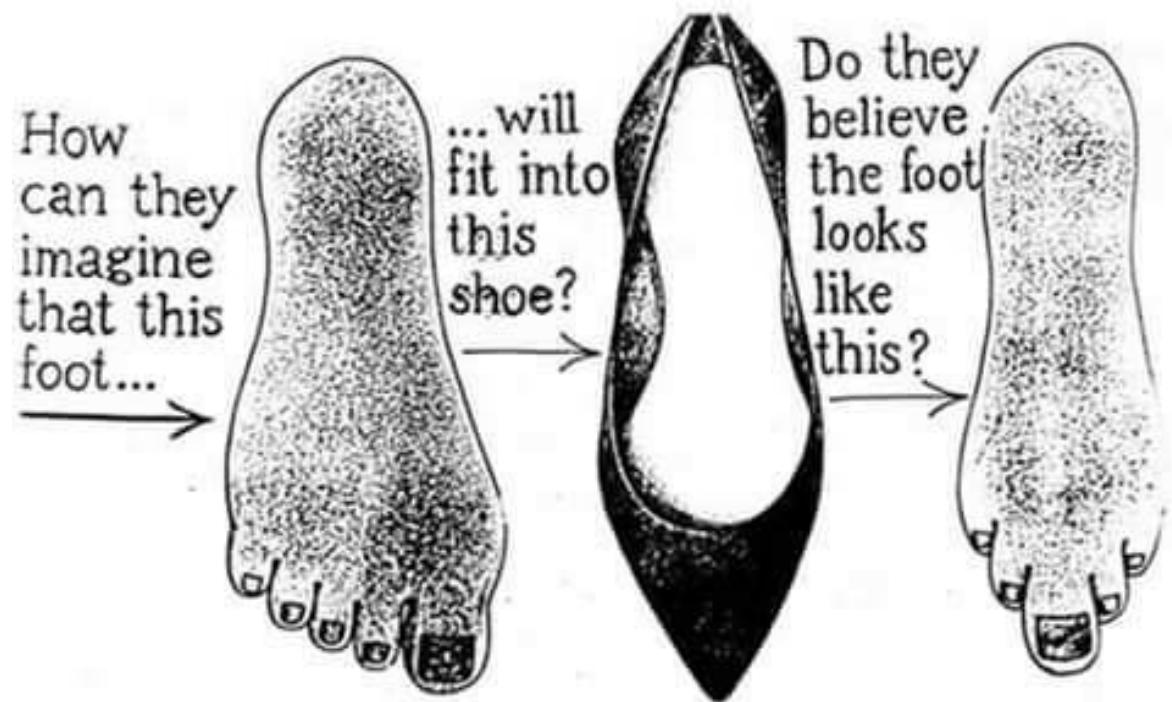


False Economy examples

1. Meena: Raju finish the medicines today, as they will expire tomorrow.
2. Anil: Our medical insurance has been a waste, we haven't got sick once last year.

Exercise: Write your own 2 examples in your notebook.

Logic of Fashion



]

Advertising



Advertising.



Reality.



- see <http://www.hongkiat.com/blog/52-worst-photoshop-mistakes-in-magazines/>

Marketing – Brand image



Dilbert.com · DilbertCartoonist@gmail.com

Exercise

Camel Ad

1. Do you agree?
2. Is it correct?
3. Is it legal?
4. Is it an effective ad?

A vintage Camel cigarette advertisement. At the top left, a yellow box contains the text: "He's one of the busiest men in town. While his door may say *Office Hours 2 to 4*, he's actually on call 24 hours a day. The doctor is a scientist, a diplomat, and a friendly sympathetic human being all in one, no matter how long and hard his schedule." Below this, a man with grey hair and a suit is smiling while holding a cigarette. The main headline reads: "According to a recent Nationwide survey: MORE DOCTORS SMOKE CAMELS THAN ANY OTHER CIGARETTE". Below the headline, there is a photograph of a woman with a cigarette and a pack of Camel cigarettes. Text next to the woman says: "Your 'T-Zone' Will Tell You... T for Taste... T for Throat... that's your proving ground for any cigarette. See if Camels don't suit your 'T-Zone' to a 'T'." A small note at the bottom right of the advertisement reads: "W. J. Morrissey Tobacco Company, Winston-Salem, N.C."

CAMELS Costlier Tobaccos

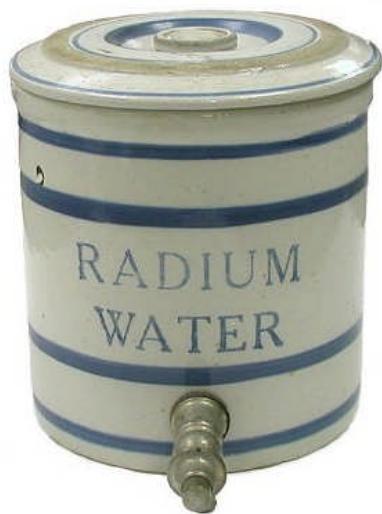
Exercise

Is this Ad misleading? How?

from <http://goretro.blogspot.in/2009/11/claims-that-went-up-in-smoke-look-at.html>



Quackery (bad science)



- Radioactive radium as a cure, more here:
<http://www.orau.org/ptp/collection/quackcures/quackcures.htm>

Creating Political Opinion

Unverifiable Anecdotes to create public sympathy

Propaganda by creating personal stories about unknown personal sufferings.

Also see

- Machiavelli's book "The Prince".
- Voltaire's book "Candide".
- Goebbels, Hitler's Minister for Propaganda:
 - "It is the absolute right of the state to supervise the formation of public opinion."
 - "We shall go down in history as the greatest statesmen of all time, or as the greatest criminals"



Socratic Questioning

Socratic questioning

- **Socratic questioning** is disciplined questioning that can be used to pursue thought in many directions and for many purposes
 - to explore complex ideas,
 - to get to the truth of things,
 - to open up issues and problems,
 - to uncover assumptions, to analyze concepts,
 - to distinguish what we know from what we don't know,
 - to follow out logical implications of thought, or
 - to control the discussion.
- from http://en.wikipedia.org/wiki/Socratic_questioning

Examples: Anu and the stock market



- Anu: "I don't understand the stock market, everyone makes money on it, but I lost all my savings on IPL stocks."



- Biju: "*Everyone?* How do you know that?"

- Anu: "I read in the newspaper, that -- *everyone makes money on stocks.*"



- Chetan: "Do you believe everything you hear or read?"

Anu and cliches

- Biju: "What is your strategy for trading stocks?"
- Anu: "*Buy low and sell high*, keep the difference as profits."
- Chetan: "Do you believe simple *clichés* can substitute for hard work in understanding Financial management."

Anu and statistics

- Anu: "On the *average*, stocks go up, so no matter what I buy, I should make profit?"
- Biju: "And your stocks are *average*?"
- Chetan: "Third mistake, not understanding statistics, just because the *average* goes up doesn't mean your profit also goes up."
- David: "What is the meaning of the word *Average*?" (using words in different sense).



Appeal to emotions

- Anu: "I always pray before buying my stocks. My mother said, luck favours the pious."
- Biju: "Do you think prayers affect the stock market?"

Wrong context

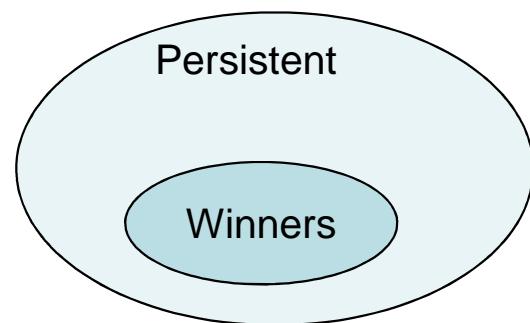
- Biju: "You better stop trading and cut your losses."
- Anu: "I won't give up, I read that '*Winners don't quit, and quitters don't win*'."
- Chetan: "Are you applying that lesson in the right **context**?"

Platitudes and Proverbs

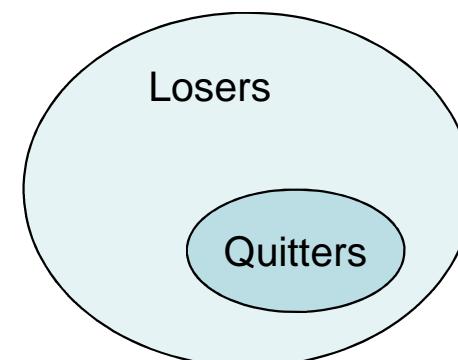
Winners don't quit, and quitters don't win



- Anu, "So, If I persist, will I win?"
- Chetna: "Is it a law of nature?"
- See <http://en.wikipedia.org/wiki/Saying>
 - Persistent = don't quit
 - Losers = don't win



Winners are persistent



quitters don't win = quitters are losers

Dogmatism (hard headed)



- Anu, "*Winners don't do different things, they do things differently.*" So I am going to change my strategy, and visit a new astrologer to advise me on stocks."
- Chetna: "Can you apply lessons blindly?"



Matter of viewpoint

- Chetan: "Luck! So how is this different from gambling?"
- Anu: "Gambling is illegal, but trading is legal, we pay taxes on profit? I want my taxes back."
- David: "What is the definition of Gambling?"
- Anu: "It is betting your money and depending on chance to make quick profit."
- David "Were you doing that?"
- Anu: "Yes, but God knows, I am honest."

Stretching the argument

Biju: "If you gamble, you have broken the law. And you can be arrested."

David: "If I bet for Rs 5, can I be arrested for gambling?"

Biju: "It will cost more file a case, it is also a matter of degree (amount)."

Exercise (30 minutes)

- Write your own dialogues in the style of **Socrates questioning** to clarify the logic of argument.
- Any topic with multiple viewpoints, imagine your friend's arguing.
- Hand written, more than 1 page.
- Example topics:
 - About the ethics of cricket match fixing?
 - Buying Gold versus Stocks?
 - Doing job versus studying for higher degree
 - [Your own topic].

Problem Solving

Out of the box thinking

Out of the box thinking

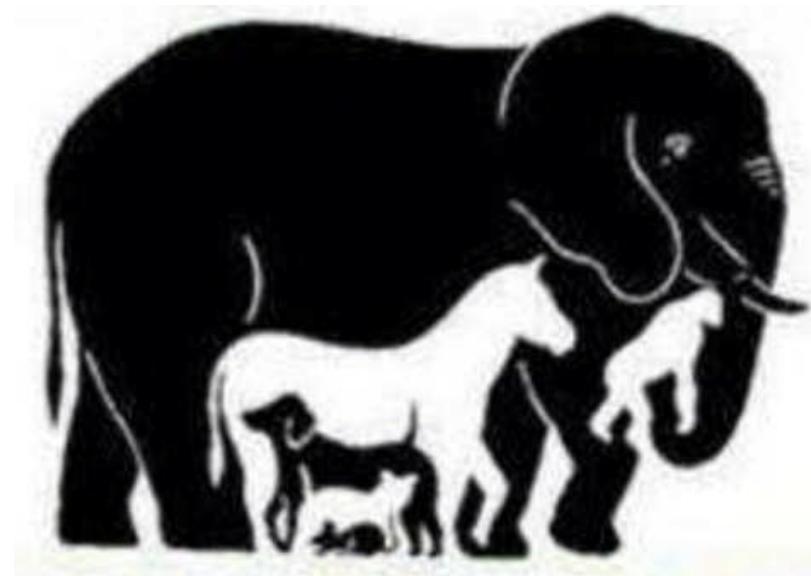
You are limited only by your imagination



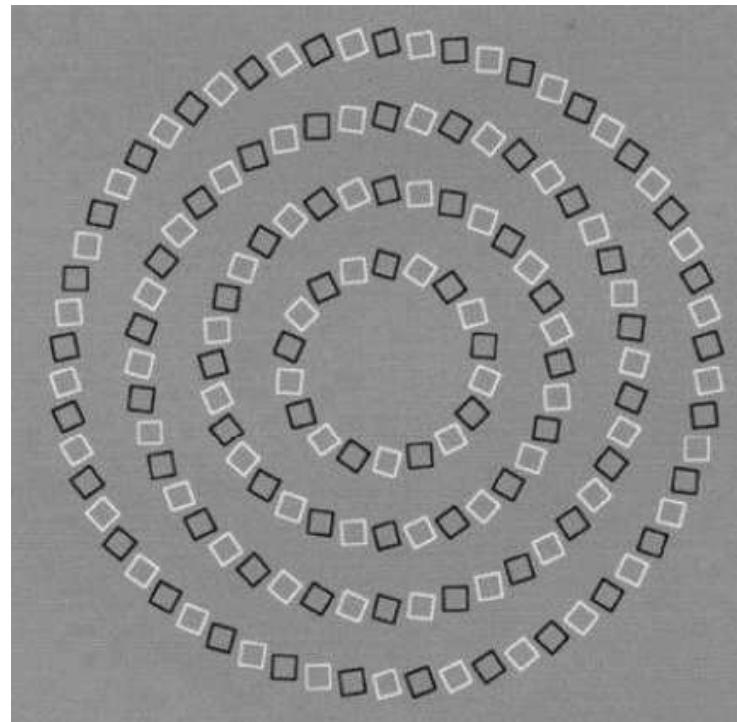
Strategies

- Do not rush to solve a problem
- Step back and understand the problem
- Ask for clarifications about the situation
- Ask for definition of words in the problem
- Check your assumptions
- Write down the problem
- Draw a diagram

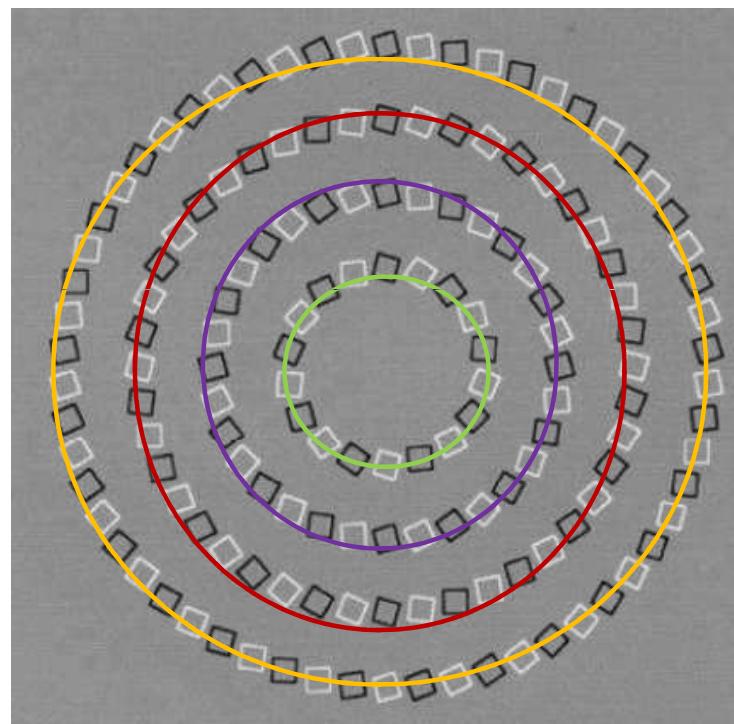
What do you see here?



What do you see here?
How many circles or spirals?

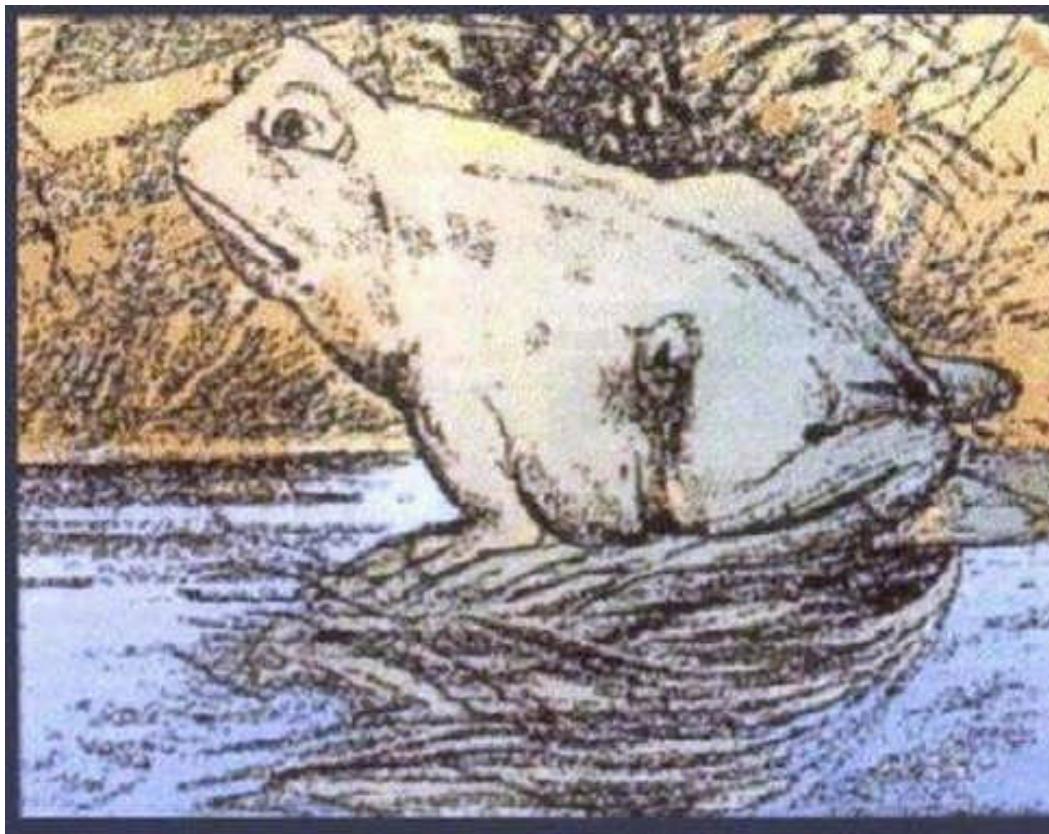


Solution



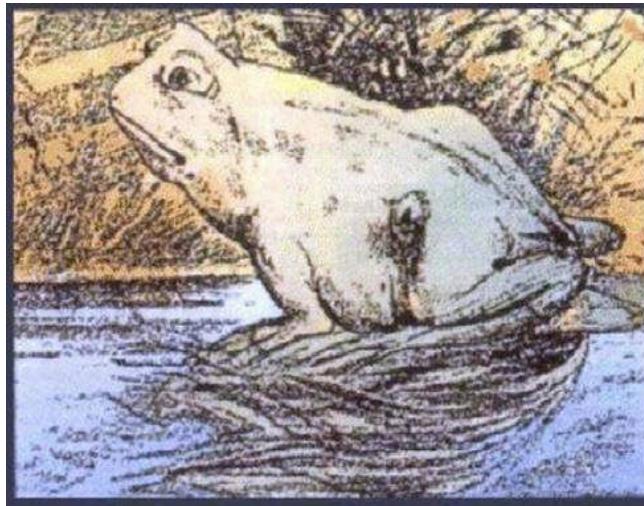
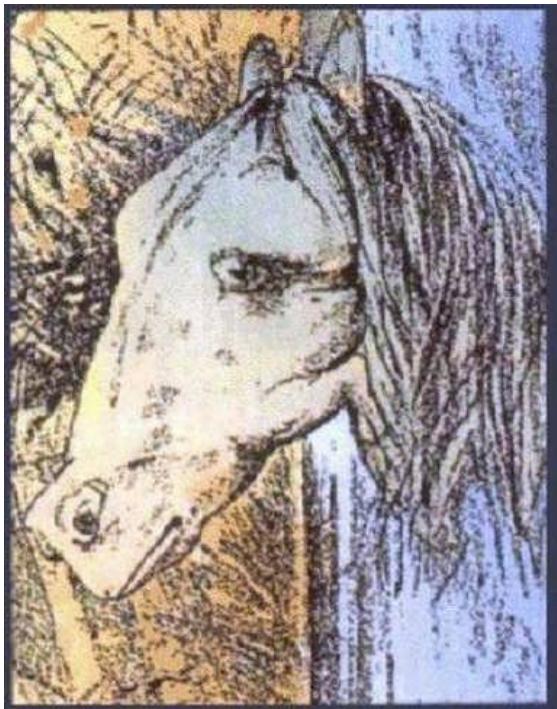
What do you see here?

- Thinking out of the box



What do you see here?

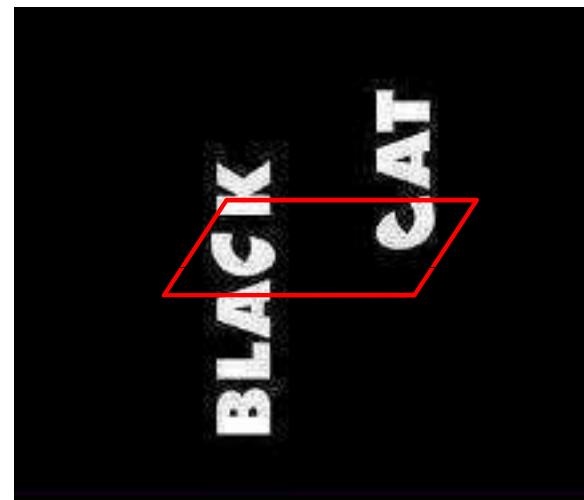
- Solution: Horse head or Frog?



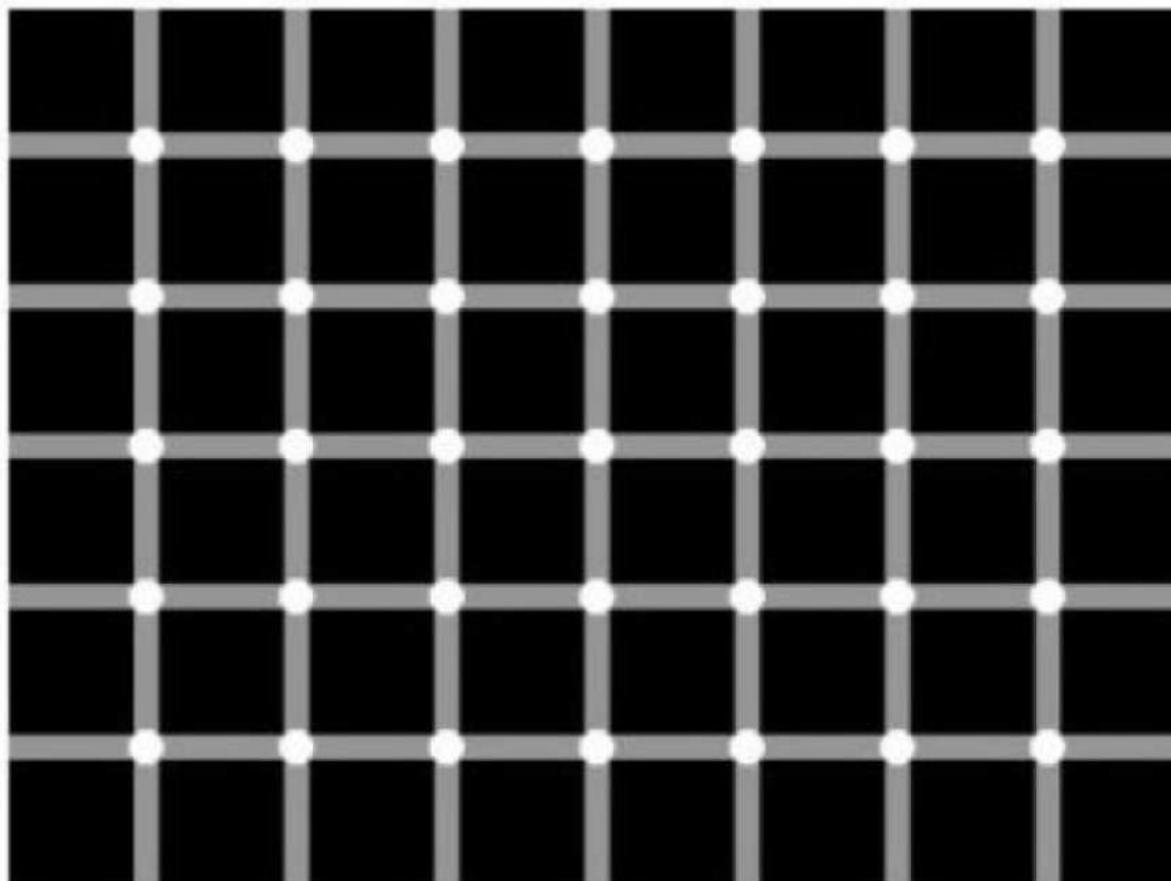
Can you see something here?



Solution: Cat eyes



Illusions (black or white dots?)



Strategies

- Brainstorm and list some solutions
- Bounce your ideas on a good friend
- Most problem/solutions are not 0/1 but have trade-offs.
- Create a profit/loss cost model of the problem and solution.
- Calculate labor, time, resource requirements of each solution
- Calculate the risks and estimate loss in case of failures.

Finding problems

- Most successful people are those who can see a problem and solve it before others can even see a problem.
- "If it ain't broke, don't fix it" – Traditional.
 - Enjoy the profits till 'time' beats you.
- "Change is the only constant" – Google
 - Improve things even before they break.
 - Bring change, try new solutions for solved problems, you never know!

Finding problems

- The most successful people are those who can spot a problem and solve it for their profit.
- Spotting problems requires an open mind.
Think: can X be done better / faster / cheaper / cleaner / ethically / ... ?
- Keeping up to date, reading news, talking to people in your field for new ideas.

Thinking out of the box



It is a metaphor that means to think differently, unconventionally, or from a new perspective. It refers to novel or lateral or creative thinking.

1. How can you make this equation correct?
 - $8 ? 8 = 91$
2. Sujit is talking to his lawyer in jail. They are very upset because the judge has refused to grant bail. Soon Sujit is allowed to go home. Why?

from <http://arkarthick.com/2010/09/06/tricky-and-witty-brain-teasers-to-blow-your-mind-away/>

Solution



1. How can you make this equation correct?
 - $8 ? 8 = 91$.. View it upside down.

$$16 = 8 + 8$$

2. Sujit is talking to his lawyer in jail. They are very upset because the judge has refused to grant bail. Soon Sujit is allowed to go home. Why?

Nine dots problem

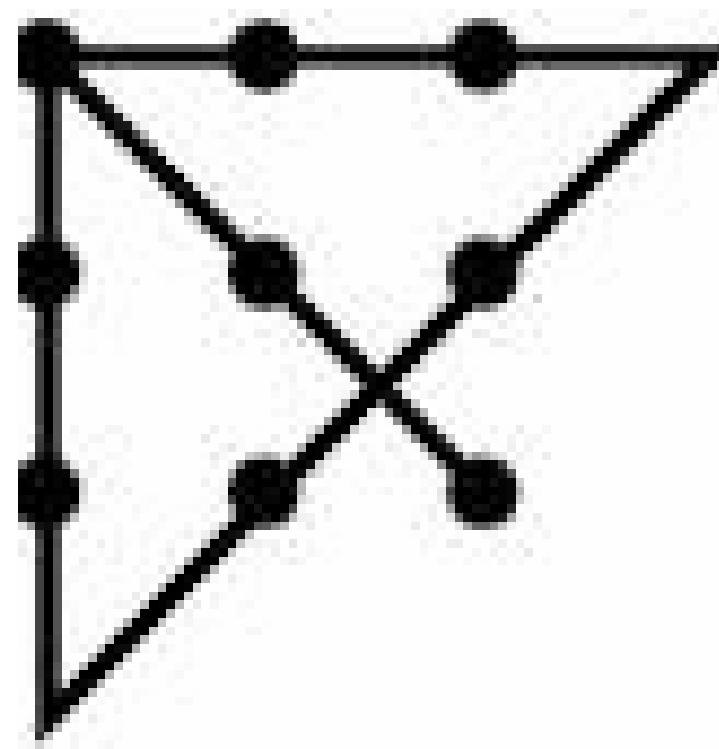
Draw four straight lines
which go through the
middle of all of the dots
without taking the pencil
off the paper.

If you were using a pencil,
you must start from any
position and draw the lines
one after the other without
taking your pencil off the
page.

Each line starts where the
last line finishes.



Nine dots solution



Lessons

- Look beyond the current definition of the problem.
- Analyze the definition to find out what is allowed and what is not.
- Are there any real rules to the problem anyway?
(especially valid in human related problems - there are only perceptions, not physical rules)
- Look for other definitions of problems.
- Do not accept other people's definitions of problems.
They may be either wrong or biased.
- If a problem definition is wrong, no number of solutions will solve the real problem.

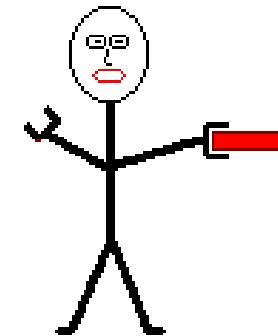
from <http://www.brainstorming.co.uk/puzzles/dropblock.html>

Block of wood - Context

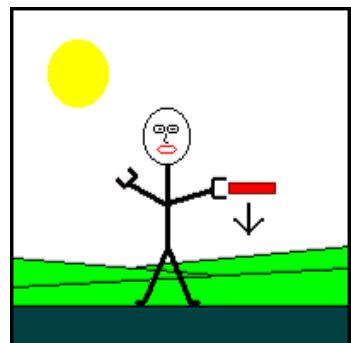
- What will happen to a block of wood, if a person holding it drops it?

Think:

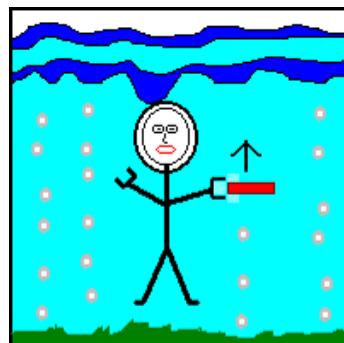
- What are your assumptions?
- ???



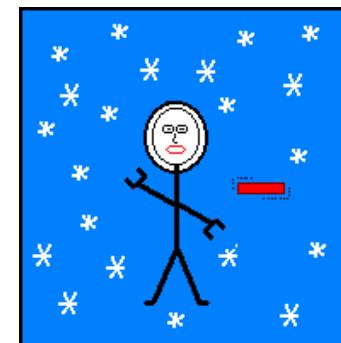
Block of wood - solution



on ground,
wood falls
down



under water,
wood floats up



in outer space,
wood floats

How to empty the wash basin test

- During a visit to a mental asylum, Ajay asked: “How do you decide if a patient needs treatment?”
- Doctor: “We give them the wash basin test. We fill a wash-basin with water, and give them a choice of **pipe**, **spoon** or **mug** to empty it.”
- Ajay: Oh, is it a “**out of the box thinking test**” for patients?



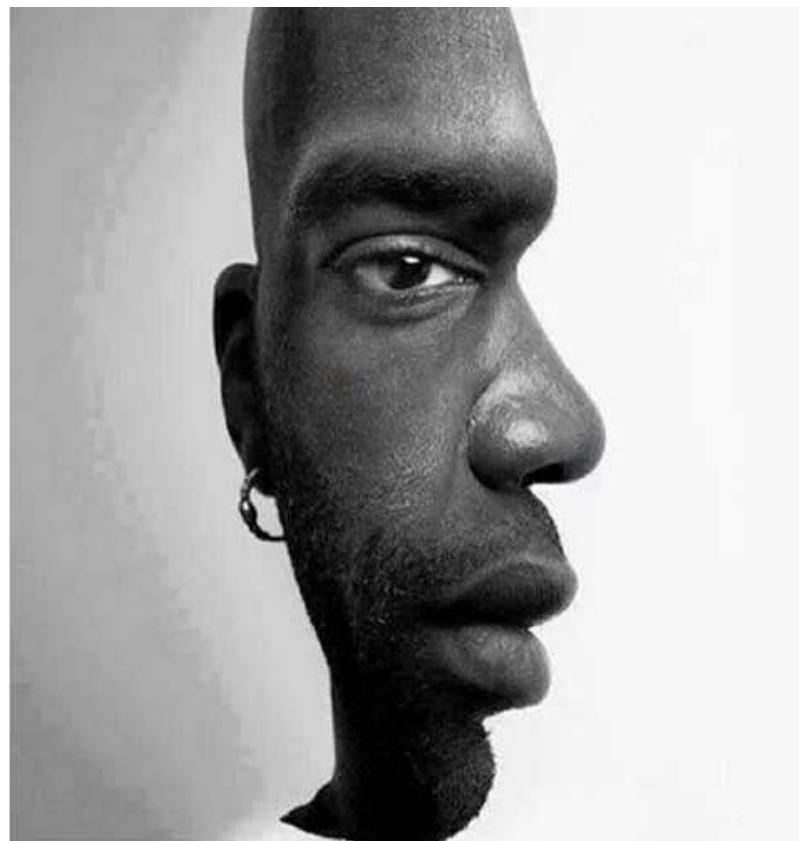
Wash basin test solution.

- Ajay: “I see, a normal person would choose a mug to empty the basin?”
- Director: “No a normal person will unplug the tub to empty it. Would you like a bed near the window?”

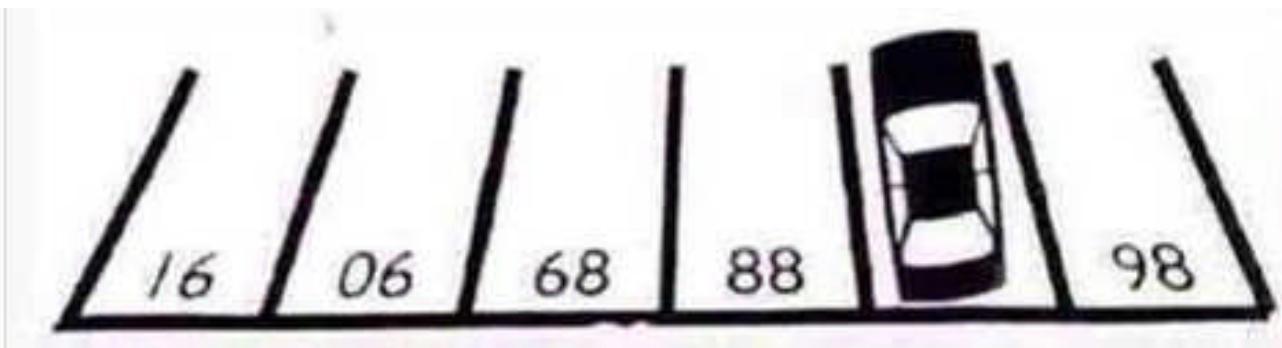


Which way is the man looking?

- Looking at us
- OR
- Looking to right



What the number of the car park?



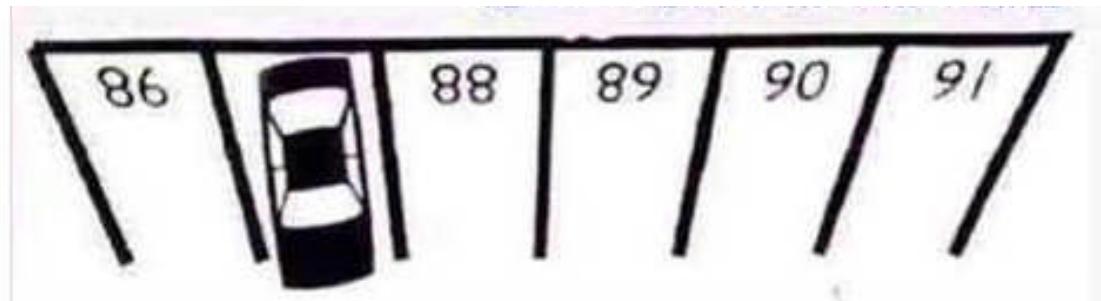
香港小学入学考试题：21題

This Math Question From A Hong Kong Elementary School Test Has Adults Stumped

You don't need a PhD to solve it.

What the number of the car park?

Solution: View it from other side
86 to 91.



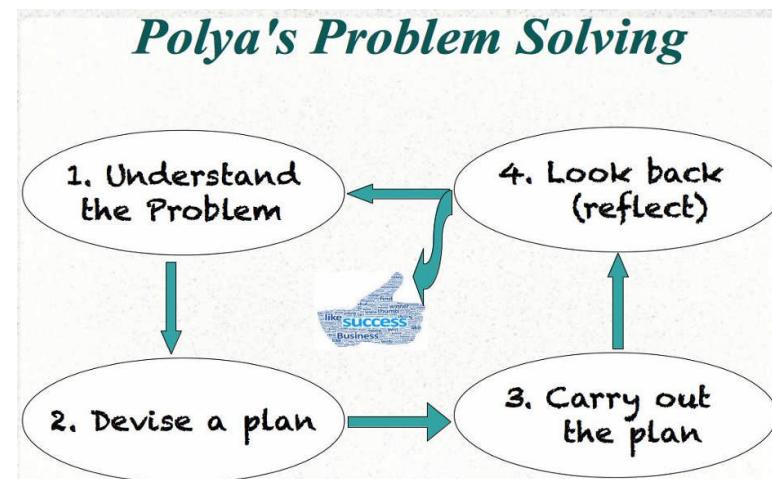
Is the train coming or going?



Problem solving with K T Analysis (Kepner Tregoe)

K T Methodology

1. Define
2. Generate Solutions
3. Make an Action plan
4. Execute the plan
5. Evaluate



K T Methodology

- Define the problem
 - Ask and clear doubts
 - Talk to experts
 - Visit, and get firsthand report.
 - Collect data (measure the problem),
 - Document before/after photos.
 - Write down the problem precisely
 - Add charts and data.
 - Decide, if this is the problem or something else?

Data collection

- Hypothesize -- test the hypothesis
- Brainstorm -- come up with several solutions.
- Calculate the costs of
 - each solution (multiple solutions).
 - no solution (ignore the issue).
 - delay in solving the issue (urgency).
 - Public opinion.

Blocks to the solution

- Mental blocks
 - Not accepting the problem.
 - Thinking inside the box.
 - Passing the buck (reponsibility of other dept).
 - Avoid the blame-game (fault of last person).
 - Fear of failure (will I be fired if this fails).
 - Fear of unknown (never done this before).
 - Communication issues (can't explain, cannot understand, others not showing interests).

Inaction to Action

- Meditate, sleep on the problem.
- Research the problem on internet/library/talk.
- Find out the experts, call them.

Action plan

- Come up with an action plan.
- Break up the solution into smaller tasks.
- Draw a dependency graphs.
- Find out which tasks are "*critical*".
- Make a time table, time estimates.
- Make a budget for each task.
- Enlist help for faster execution.
- Keep a backup plan.

Execution

- Make a proposal
- Make presentations and get feedback
- Get management support.
- Get manpower with skill sets.
- Create the team
- Create communication channels
- Visit site and talk to effected people

Measuring



Reliable
Not Valid

Low Validity
Low Reliability

Not Reliable
Not Valid

Both Reliable
and Valid

Implementation

- Keep a daily TO DO list.
- Tick off items as done
- Check schedule
- Communicate schedule.
- Keep notes of work done.

Evaluate

- Once work is done, evaluate it.
- Write completion report.
 - Note caveats (e.g. maintenance plan).
 - Give presentation (many will be unaware).
- Share credit.
- Update accounts.
- Celebrate.

Exercise

Apply KT method to

- Improve water situation in the college.
- Improve cleanliness of the college.
- Your college life more effective.

Questions:

- Define the problem.
- What data will you collect?
- Who are the experts?
- What the bottle necks?
- What are the solutions?
- Draw a timeline chart of action.

Problem Solving by Writing



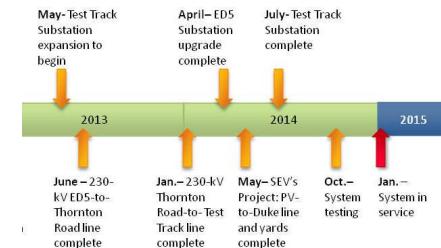
Exercise 30 minutes

Write in your notebook:

- Problem you want to resolve:
 - + ten plus points, possible solutions
 - ten minus points, possible problems.
- Work involved for each point.
- Timeline for each point, dependency of points.
- Cost (cash/work) for each point.
- Risk of failure in each point, alternate plans.
- Benefits of each point.



TIMELINE



Homework

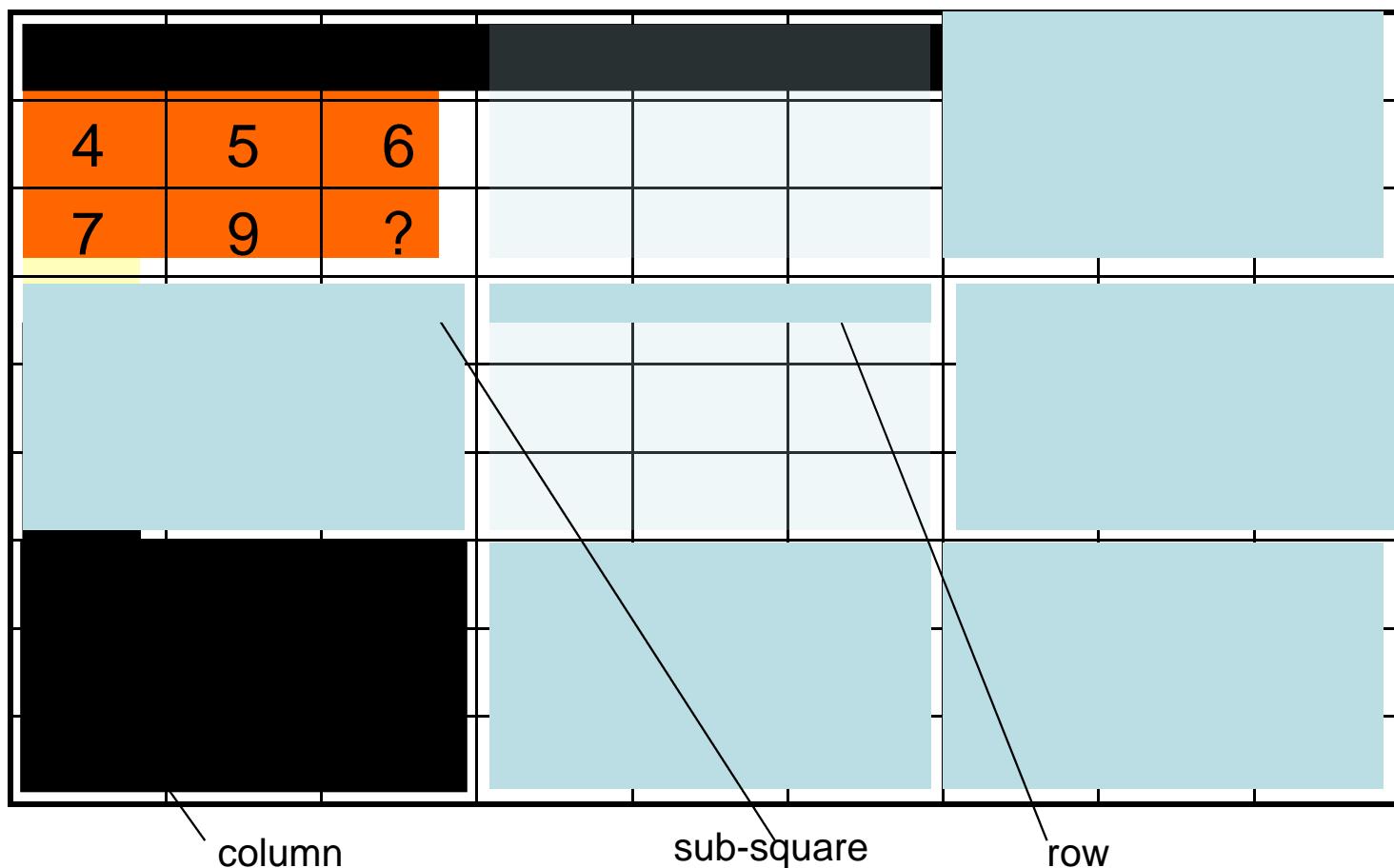
Read these pages:

References:

- <http://explorable.com/research-basics>
- <http://explorable.com/print/research-methodology>

Sudoku

Game of filling missing numbers



Rules

- The grid is made of 81 squares,
9 rows into 9 columns.
- Given a partially filled grid.
- Fill all the 81 squares with numbers $\{1, \dots, 9\}$
- Every Row must have $\{1, \dots, 9\}$
- Every Column must have $\{1, \dots, 9\}$
- Every Sub-Square must have $\{1, \dots, 9\}$

Strategy 1 to solve

- Repeat:

Examine each row, column, sub-square with 1 missing number.

Calculate the last missing number and fill it.

Exercise 1.

•	4	8	3		9	2	1		6	5	7
•	9	—	7		3	—	5		8	—	1
•	2	5	1		8	7	6		4	9	3
•	-----+-----+-----										
•	5	4	8		1	3	2		9	7	6
•	7	—	9		5	—	4		1	—	8
•	1	3	6		7	9	8		2	4	5
•	-----+-----+-----										
•	3	7	2		6	8	9		5	1	4
•	8	—	4		2	—	3		7	—	9
•	6	9	5		4	1	7		3	8	2

Solution 1

•	4	8	3		9	2	1		6	5	7
•	9	6	7		3	4	5		8	2	1
•	2	5	1		8	7	6		4	9	3
• -----+-----+-----											
•	5	4	8		1	3	2		9	7	6
•	7	2	9		5	6	4		1	3	8
•	1	3	6		7	9	8		2	4	5
• -----+-----+-----											
•	3	7	2		6	8	9		5	1	4
•	8	1	4		2	5	3		7	6	9
•	6	9	5		4	1	7		3	8	2

Exercise 2.

•	2	_	5		9	8	1		3	7	6
•	1	_	9		2	7	3		5	8	4
•	8	_	7		5	6	4		2	1	9
• -----+-----+-----											
•	_	_	_		_	_	_		_	_	_
•	5	1	3		4	9	8		6	2	7
•	4	8	2		7	3	6		9	5	1
• -----+-----+-----											
•	3	9	1		6	5	7		8	4	2
•	7	2	8		3	4	9		1	6	5
•	6	5	4		8	1	2		7	9	3

Solution 2

•	2	4	5		9	8	1		3	7	6
•	1	6	9		2	7	3		5	8	4
•	8	3	7		5	6	4		2	1	9
• -----+-----+-----											
•	9	7	6		1	2	5		4	3	8
•	5	1	3		4	9	8		6	2	7
•	4	8	2		7	3	6		9	5	1
• -----+-----+-----											
•	3	9	1		6	5	7		8	4	2
•	7	2	8		3	4	9		1	6	5
•	6	5	4		8	1	2		7	9	3

Exercise 3.

•	—	—	—		8	3	1		9	5	7
•	—	—	—		4	2	6		1	8	3
•	—	—	—		7	9	5		4	2	6
• -----+-----+-----											
•	1	7	3		9	8	4		2	6	5
•	6	5	9		3	1	2		7	4	8
•	2	4	8		5	6	7		3	1	9
• -----+-----+-----											
•	9	2	6		1	7	8		5	3	4
•	8	3	4		2	5	9		6	7	1
•	5	1	7		6	4	3		8	9	2

Solution 3.

•	4	6	2		8	3	1		9	5	7
•	7	9	5		4	2	6		1	8	3
•	3	8	1		7	9	5		4	2	6
• -----+-----+-----											
•	1	7	3		9	8	4		2	6	5
•	6	5	9		3	1	2		7	4	8
•	2	4	8		5	6	7		3	1	9
• -----+-----+-----											
•	9	2	6		1	7	8		5	3	4
•	8	3	4		2	5	9		6	7	1
•	5	1	7		6	4	3		8	9	2

Strategy 2

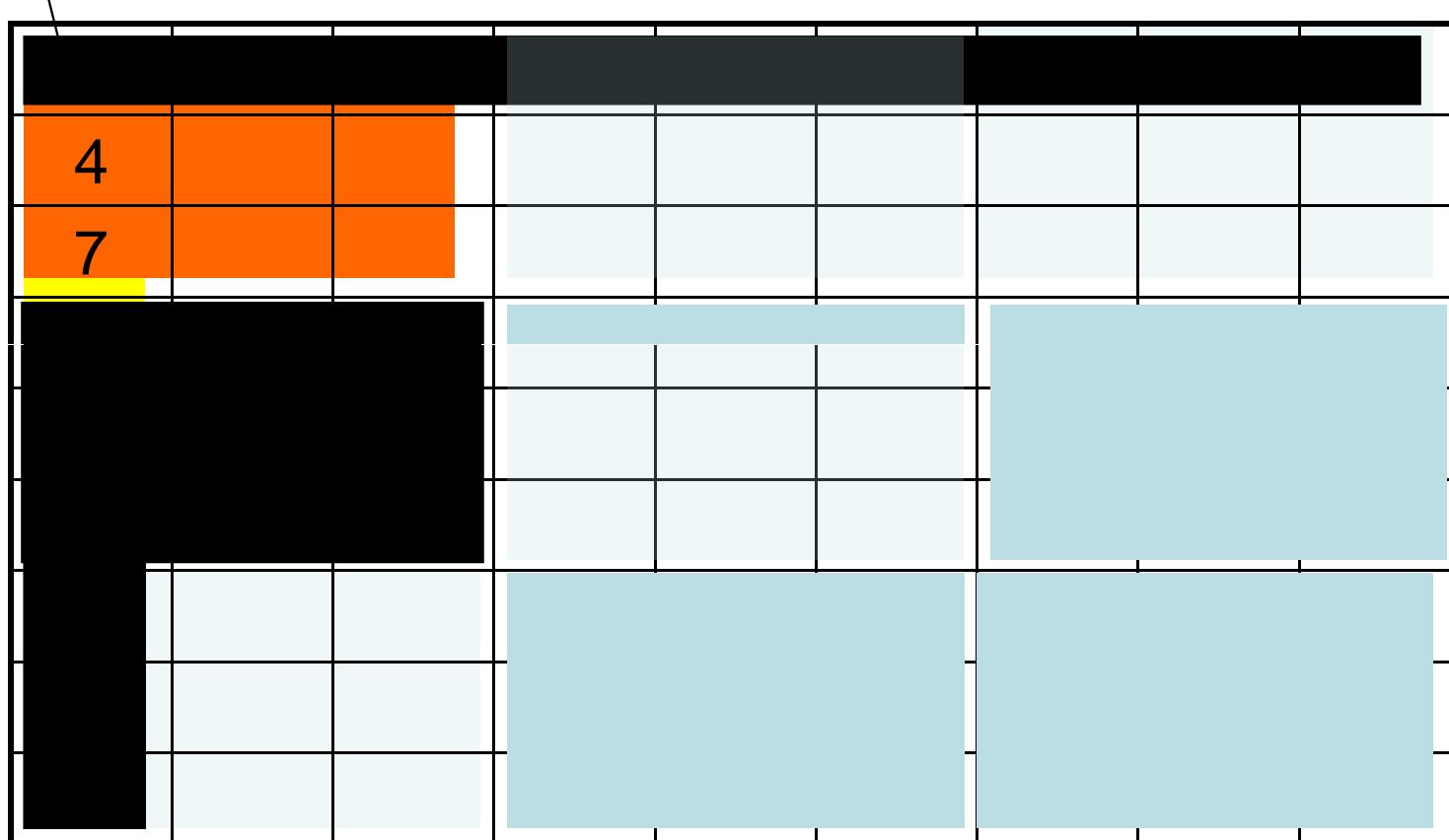
- Repeat:
 - Find an empty square.
 - Create a list={1,...,9} of choices to fill in it.
 1. Examine the row of the empty square, remove duplicate numbers from the list.
 2. Examine the column of the empty square, remove duplicates.
 3. Examine the sub-square, and remove duplicates.
 - If the list has single number left, fill it in the empty square.

{1,2,3,4,5,6,7,8,9}

{1,9}

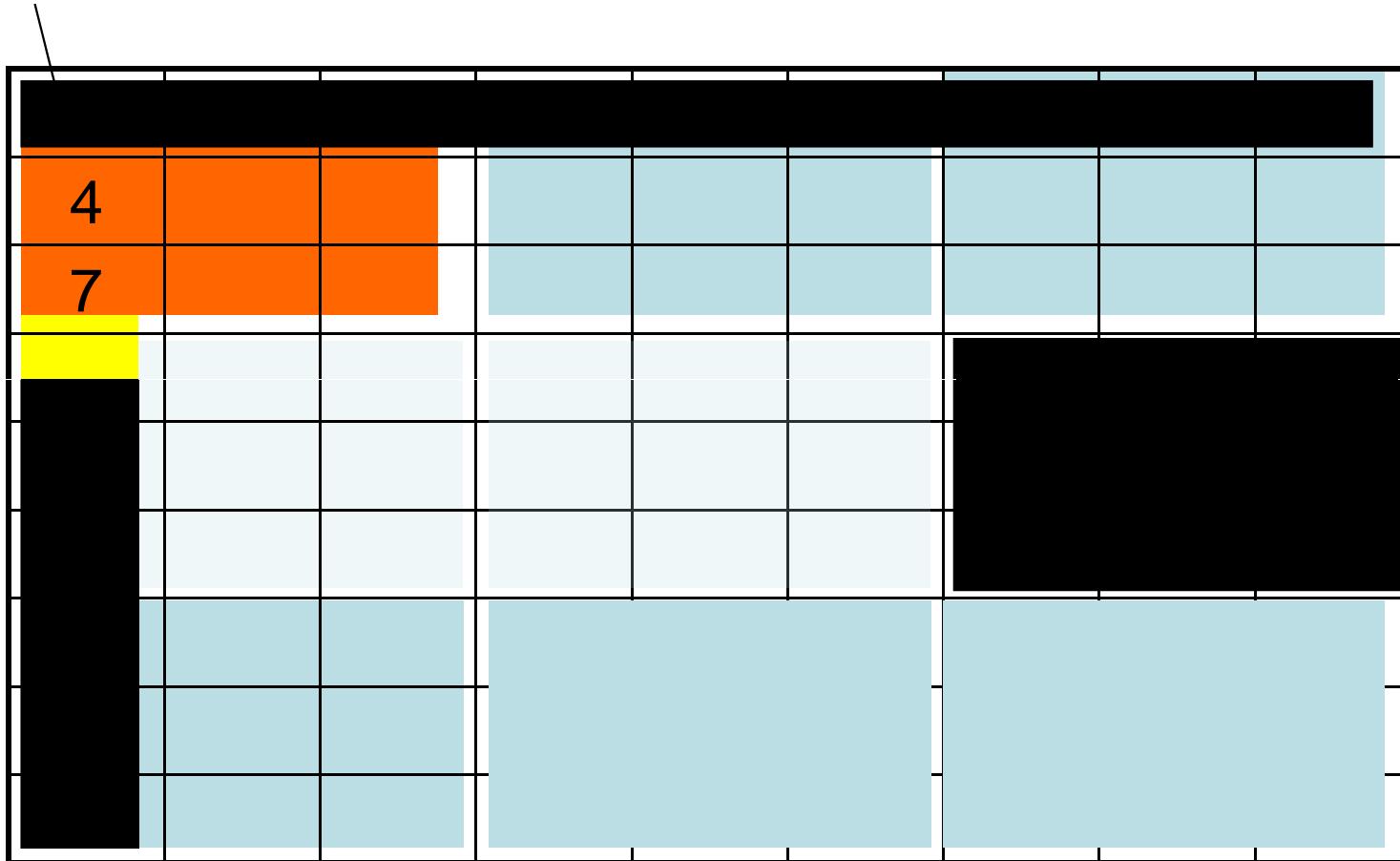
{1,3}

Strategy 2



$$\{1,2,3,4,5,6,7,8,9\} - \{4,7,2,9\} - \{3,5,6,8\} = \{ ? \}$$

Strategy 2



$$\{1,2,3,4,5,6,7,8,9\} - \{7,9\} - \{5,6,8\} - \{3,4,2,7\} = \{ ? \}$$

Strategy 2

A 10x10 grid diagram with colored cells. The first column has black cells at the top and bottom. The second column has orange cells at the top and bottom, with a yellow cell at row 4. The third column has orange cells at the top and bottom. The fourth column has light blue cells at the top and bottom. The fifth column has light blue cells at the top and bottom. The sixth column has light blue cells at the top and bottom. The seventh column has light blue cells at the top and bottom. The eighth column has light blue cells at the top and bottom. The ninth column has light blue cells at the top and bottom. The tenth column has light blue cells at the top and bottom.

Strategy 3

{?, ?}

Strategy 4

- If you still have blank squares.
- Write list of choices for these difficult blank squares.
- Try solving other easier blank squares.
- If any other square gets filled, go back to the difficult blank squares and update the list of choices.

Exercise 4, Homework

$$\bullet \begin{array}{r|rr} - & 5 & \\ \hline - & 8 & \end{array} \quad | \quad \begin{array}{r|l} 7 & 3 \end{array}$$

$$\bullet \begin{array}{r|rr} 7 & - & \\ \hline 2 & 4 & \end{array} \quad | \quad \begin{array}{l} - \\ - \\ 5 \end{array}$$

$$\bullet \begin{array}{r|rrr} 3 & 2 & - & \\ \hline 7 & 5 & 9 & \end{array} \quad | \quad \begin{array}{r|ll} 8 & 4 & \end{array}$$

• -----+-----+-----

$$\bullet \begin{array}{r|rrr} - & 6 & - & \\ \hline 1 & 7 & 5 & \end{array} \quad | \quad \begin{array}{r|l} 4 & - \end{array}$$

$$\bullet \begin{array}{r|rr} 1 & - & 8 \\ \hline 9 & 4 & 2 \end{array} \quad | \quad \begin{array}{r|ll} 5 & - & 6 \end{array}$$

$$\bullet \begin{array}{r|rrr} - & 7 & - & \\ \hline 8 & 6 & 3 & \end{array} \quad | \quad \begin{array}{r|ll} 1 & - & \end{array}$$

• -----+-----+-----

$$\bullet \begin{array}{r|rr} 4 & 5 & - \\ \hline - & 2 & - \end{array} \quad | \quad \begin{array}{r|ll} 9 & 1 & \end{array}$$

$$\bullet \begin{array}{r|rr} 6 & - & - \\ \hline 5 & - & 8 \end{array} \quad | \quad \begin{array}{r|ll} 3 & 7 & \end{array}$$

$$\bullet \begin{array}{r|rr} 8 & - & 3 \\ \hline - & 1 & - \end{array} \quad | \quad \begin{array}{r|ll} 6 & - & - \end{array}$$

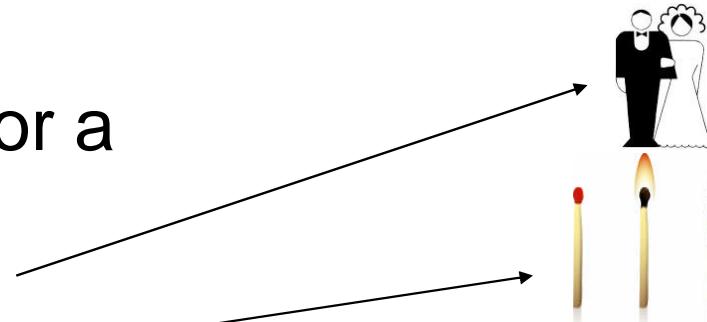
Solution 4.

•	9	4	5		6	8	1		7	2	3
•	7	8	1		2	3	4		9	6	5
•	3	2	6		7	5	9		1	8	4
• -----+-----+-----											
•	2	6	9		1	7	5		3	4	8
•	1	3	8		9	4	2		5	7	6
•	5	7	4		8	6	3		2	1	9
• -----+-----+-----											
•	4	5	7		3	2	6		8	9	1
•	6	1	2		5	9	8		4	3	7
•	8	9	3		4	1	7		6	5	2

Ambiguity

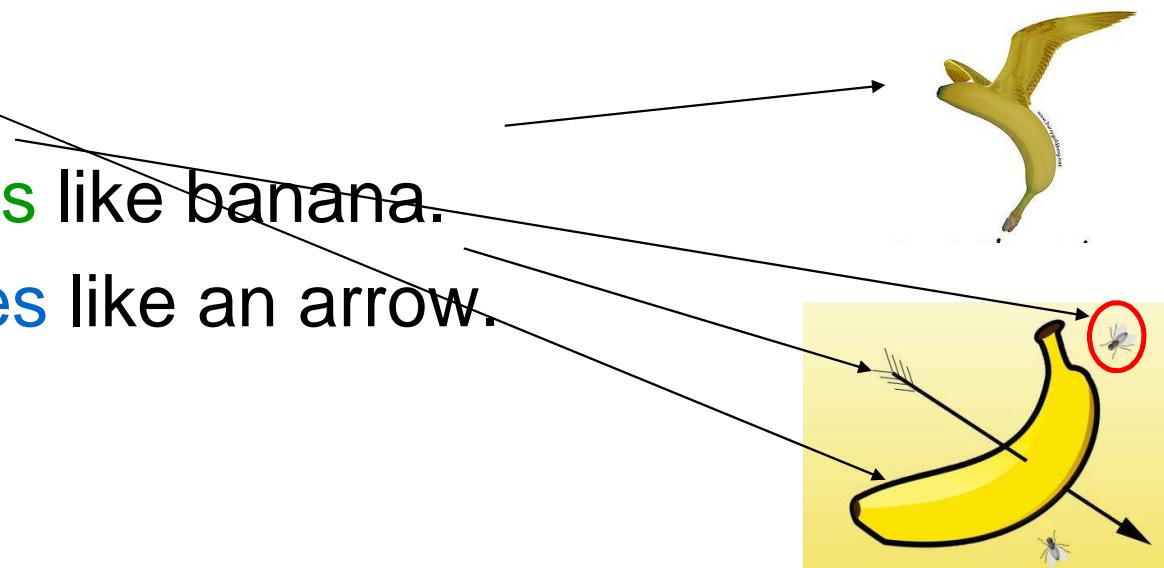
Words can mean different things

- Cathy: "I am looking for a *match*?"
 - David: "Did you try matrimonial ad?"
 - Cathy: "I need a matchbox to light the fire."



Compare:

- Fruit-**flies** like banana.
- Time **flies** like an arrow.



Alliteration (repetition)

- "Rose rose to pluck a rose rose from her rose roses." --Robert Baran.

Exercise: Explain meaning of each rose in above sentence.

See http://en.wikipedia.org/wiki/List_of_linguistic_example_sentences



Alliteration (repetition)

- "Rose rose to pluck a rose rose from her rose roses." --Robert Baran.
 - Rose [name of a girl] rose [stood up] to pluck a **rose** [pink-color] rose [flower] from her **rose** [pink] roses [garden].

See http://en.wikipedia.org/wiki/List_of_linguistic_example_sentences



Syntactic ambiguity

- Farid: "I once saw a dog riding my bicycle."
 - Explain



- Notice: "Wanted: a tutor for a boy, about fifteen years old."
 - Explain

Syntactic ambiguity

- Farid: "I once saw a dog riding my bicycle."
 - (who was riding the cycle?).



- Notice: "Wanted: a tutor for a boy, about fifteen years old."
 - (whose age is 15?).

Explain

1. *The word of the Lord came to Zechariah, son of Berekiah, son of Iddo, [pause] the prophet.*
2. "I haven't slept for ten days, [pause] because that would be too long." -- [Mitch Hedberg](#)
3. "She was good as cooks go, and as cooks go she went.
– Saki.
4. "I sleep eight hours a day and at least ten at night." –
Hicks.

See http://en.wikipedia.org/wiki/Crash_blossom and
<http://en.wikipedia.org/wiki/Paraprosdokian>

Exercise

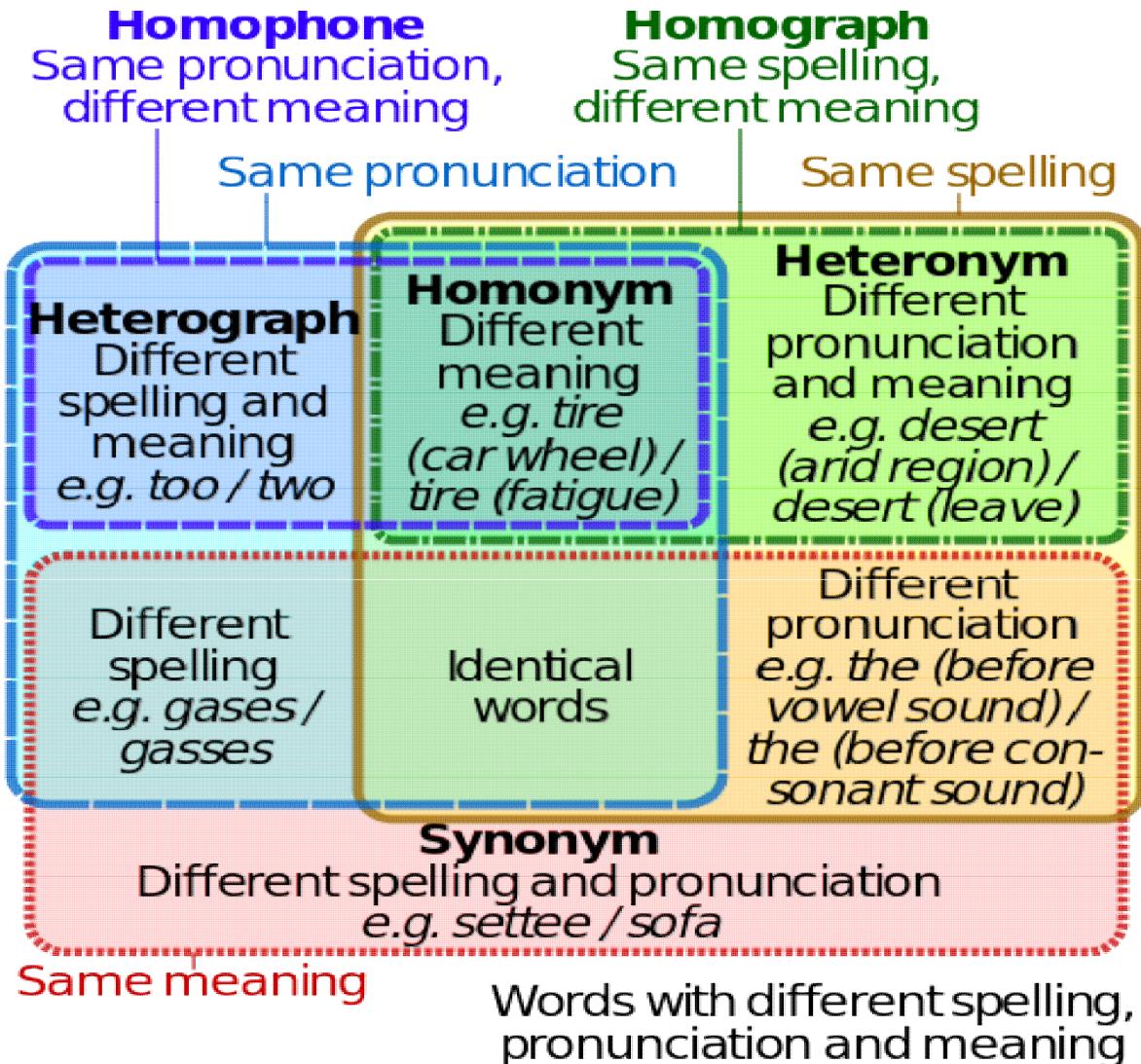
Explain the different meanings of:

- Visiting relatives can be boring.
- The priest married my sister.
- The old men and women stayed here.
- Anu made Bina wash her feet.

Phonologically ambiguous (when read aloud)

Explain ambiguity in each of these:

- He is famous for his feat
- The cobler will save our sole.
- I see sea.
- C is deep.
- Sue sew my socks.
- psychotherapist is not "psycho/the/rapist".



from <https://en.wikipedia.org/wiki/Homograph>

Exercise: Give more words

	Sound	Spell	meaning	Word1	Word2
Homophone	same	diff	diff	too	two
Homograph	diff	same	diff	wind	wind
Synonym	diff	diff	same	couch	Sofa
Multiple meanings	same	same	diff	Lead	Lead

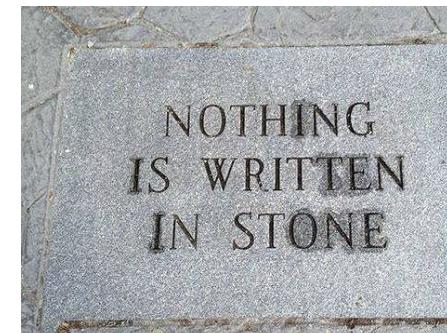
Figure of Speech

Oxymoron, Irony, Sarcasm

- **Oxymoron** is a figure of speech that combines contradictory terms.
 - Example: *dry ice, sweet sorrow.*
- **Irony**: Say opposite of what is intended.
"He was quiet like a sleeping snake."
- **Dramatic irony**: Audience knows before the doomed hero. (Examples from Hindi movies?).
- **Socratic irony**: Pretend ignorance in an argument.
- **Sarcasm**: Say nice thing, to insult someone.

Examples:

Irony examples



Rhetoric

Rhetoric is art of speaking well

Examples

- "If you owe your banker a thousand pounds, you are at his mercy. If you owe your banker a million pounds, he is at your mercy."

-- John Maynard Keynes



Keynes

Rhetoric Example with Puns



“It's neither in our principle to pay interest, Nor it's in our interest to pay the principal...”

- **Vijay Mallya**

- Author of The Theory of Borrowing

Rhetorical Question

.. is a question asked merely for effect
with no answer expected.

Examples:

Anu: "Who here does not want to
study?"

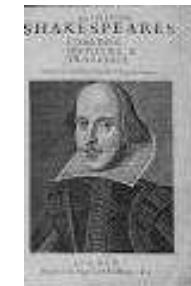
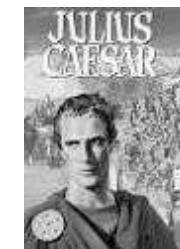
Biju: "Am I a fool to waste my time?"

see <http://public.wsu.edu/~brians/errors/rhetorical.html>

Reverse Psychology

Conclusion is opposite of beginning.

- Antony: "*I come here to bury Caesar, not to praise him ...*" Then Antony praises Caesar indirectly. In the play "Julius Caesar" by Shakespeare.



Shakespeare

Reverse Praise

Anu: "I don't like Sanjay, he is always sleepy."

Bina: "Yes, Sanjay is a very bad boy, he stays up late to study after cooking for his family."

Interviewer: "What are your bad habits?"

Jack: "I work so hard, that I forget to go home on time."

Katy: "And I am a bad girl. I secretly donate all my money to the poor, and I am too modest to praise myself."



Examples

- Anu: "Our minister is the a great man, during the floods, he prayed for the victims, day and night at home, till the flood ended. God heard his prayers."
- Biju: "During the famine when the children were hungry, he vowed to not eat two cups of ice-cream daily."
- Catie: "He made his servants donate their salaries to his relief fund."

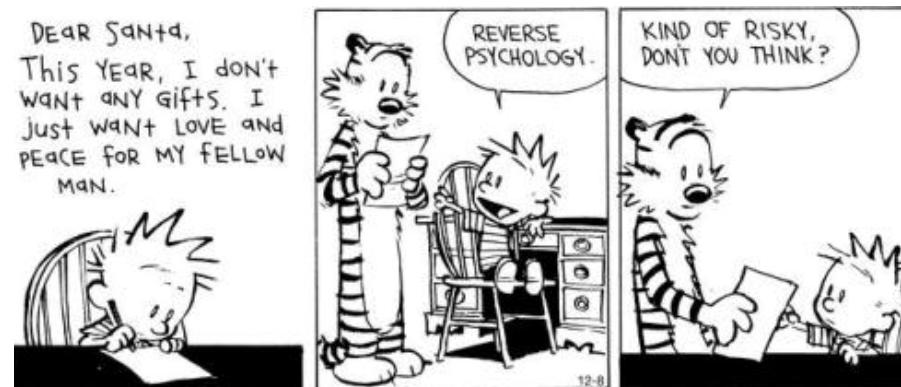


Reverse Psychology

Exercise: Make your own examples.



"And mark it 'strictly confidential', I want everybody to read it."



Calvin and Hobbes comic (Hobbes is his tiger)

Paradoxes

Paradox

- A *paradox* is an argument that produces an inconsistency, typically within logic or common sense.

Examples:

- Anu: "I am lying!"
- "This sentence is false."
- "The barber shaves only those who don't shave themselves" (Does he shave himself?).

"Surprise quiz this week"

- Teacher: "There will be a surprise quiz this week"
- Anu: "It can't be on Saturday, so it must be earlier."
- Biju: "So if it must be on Friday."
- Catie: "It won't be a surprise, so it must be before that."

Free will

- Anu "Has God created us? and is all knowing?"
- Biju: "Yes"
- Anu: "Why am I responsible for my sins?"
- Biju: "He gave you *free will* to test you."
- Anu: "But he created me, and already knows what I will do, so I have no free will
– I have to do as He knows I will do."

Ship of Theseus

- Anu: "If you replace a part on a ship, it is still the same ship?"
- Biju: "Yes."
- Anu: "So if we replace everypart of a ship, is it still the same ship?"
- Chetan: "If we take all the replaced parts and assemble it, we get first ship. Now we have two same ships!"

Inventor's paradox

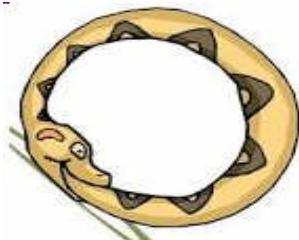
- It is easier to solve a general problem than a specific problem.

Time Travel Paradox?

Question:

1. Can we go into the future?
2. Can we go into the past?

Paradox: "What some goes back 100 years in time and changes his past, so that he is not born?"



Escher's 3D-cube illusion

- Search google images for more of Escher's artwork.



Quantum mechanics

Schrödinger's uncertainty principle: We cannot measure something without disturbing it.

Schrödinger's cat paradox: Things are not determined until we observe them.

Example: Whether you won the lottery or not, can change till you look it up, even though the result is already printed in the newspaper.

"If a tree fell in a forest, and no one heard it, does it make a sound?" does the world exists if there is no one to observe it?

Zen (Japanese)

Two monks were arguing about the temple flag waving in the wind.

A: "The flag moves."

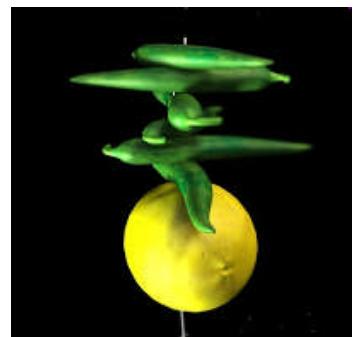
B: "The wind moves."

A&B argued back and forth but could not agree, when C arrived.

C: "Oh ignorant monks. It is not the wind that moves; it is not the flag that moves; it is your mind that moves."

The two monks were struck with awe.

Logic of superstitions



Why?

- One must not walk under a ladder



Why?

- Friday the 13th



Why?

- Mirror break
brings 7 years of
bad luck?



Why?

- Black cats are trouble?



Why?

A: Sneezes

B: "God Bless you"



Examples

- Do not go out during a solar eclipse
- Don't cut your nails at night.
- Don't open umbrella inside the house.
- Don't sit under tree at night
- Knock on wood for safety.
- Boiling milk
- Salt
- Fire?



Which superstitions are these?



SUPERSTITIONS



Exercise: find all the superstitions



Personal superstitions

- Lottery ticket with ending number 777 is lucky.
- I failed in exam because of my red shirt.



Exercise:

- List some of your superstitions.
- Can you explain why they exists?

Superstitions explained

- Historical
 - Ancient customs and diseases (black plague)
 - Embarrassment.
 - Safety lessons
-

- Unable to live with uncertainty.
- Unable to accept responsibility.
- Correlation is not cause.
- Bad logic, missing information, rumours.

Free Will?



?

Calvin
and
Hobbes



What is "free will"?

- **Free will** is the ability of agents to choose from available alternative futures.
- Humans have a strong sense of freedom, which leads us to believe that we have free **will**.
- Moral dilemma as well: How are we to assign responsibility for our actions if they are caused entirely by past events?
- Our feeling that we are free to choose an action is simply an illusion.

from http://en.wikipedia.org/wiki/Free_will

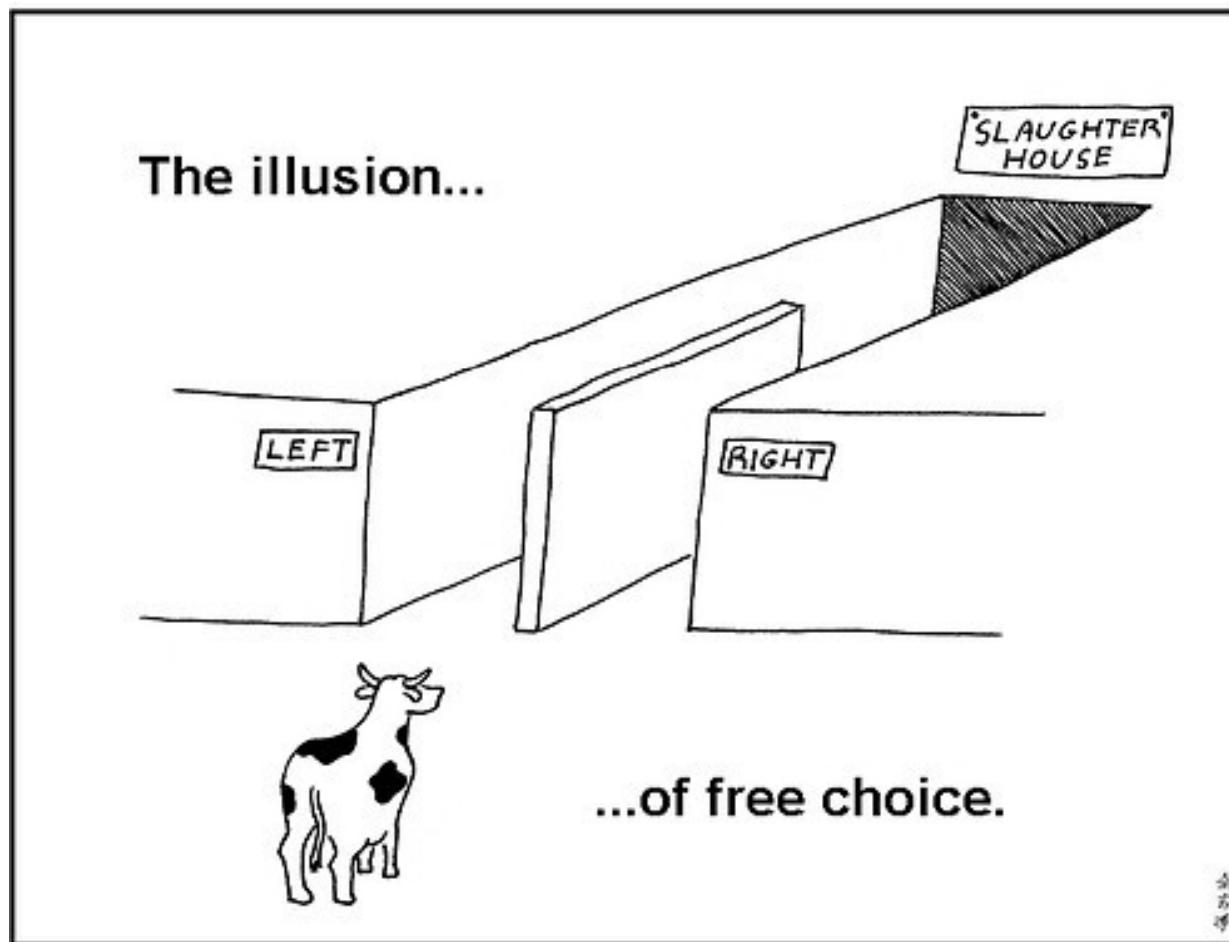
Why "bad things happen"

- We have free will to choose our future, but a lot of events are random (called "sh*t happens"), so if you fail the first few times, learn from it and try again and again.
- People rarely tell you about their initial failures in life; and in research we only publish the success stories.

Science of free will

- But in scientific view, the physical world can be explained to operate perfectly by deterministic physical laws.
- But quantum mechanics says the world is non-deterministic.
- The laws of physics (deterministic or not) have yet to resolve the hard problem of consciousness.

Illusion of free will



What is fate?

- Destiny or fate is a predetermined course of events.
 - It may be conceived as a predetermined future, whether in general or of an individual.
 - It is a concept based on the belief that there is a fixed natural order to the cosmos.
 - We have no control over fate
 - We can chose our destiny.

Who decides?

- Predeterminism is the philosophy that all events of history, past, present and future, have been decided or are known (by God, fate, or some other force), including human actions.
- Is there God? – Omniscient being – all knowing of past and future, all deciding for everyone?

Is there anything beyond physics?

- Cartesian dualism holds that the mind is a nonphysical substance, the seat of consciousness and intelligence, and raises the question of how mind and body interact.
- **Physicalism** is a philosophical theory holding that everything which exists is no more extensive than its physical properties; that is, that there are no non-physical substances (for example physically independent minds)

Philosophers

- “We believe ourselves free, simply because we are conscious of our actions, and unconscious of the causes of those actions.” - Spinoza in *Ethics*
- "You can do what you will, but in any choice - you can *will* only one definite thing and absolutely nothing other than that one thing. – Schopenhauer.
- "It is the coward and the fool who says this is his fate. But it is the strong man who stands up and says I will make my own fate." - Swami Vivekananda

Why is there evil in the world?

The Argument from Moral Evil

If God exists **then** God is omnipotent, omniscient, and benevolent.

If God were omnipotent, omniscient, and benevolent **then** the world would not contain moral evil.

Fact: The world contains moral evil.

Therefore: God does NOT exist.