

# VINS online extrinsic calibration

Saturday, January 30, 2021 10:58 AM

extrinsics: from camera frame to IMU frame:  $bT_c$

We construct least squares  $Ax=0$ , and use SVD to find the smallest singular value.  
It's corresponding eigenvector is the solution.

To construct least squares:

between time  $k, k+1$ , we have IMU transformation  $q_{bkb_{k+1}}$ , camera transformation  $q_{c_k c_{k+1}}$

$$q_{bkb_{k+1}} \otimes q_{c_k} = q_{bkb_{k+1}} \otimes q_{c_{k+1}}^T = q_{bkb_{k+1}}^{c_k} \quad \text{from } c_k \text{ to } b_{k+1} \text{ transformation.}$$

$$\therefore [q_{bkb_{k+1}}]_L - [q_{c_k c_{k+1}}]_R q_{bc} = q_{bkb_{k+1}}^k q_{bc} = 0. \quad (6)$$

$[\cdot]_L, [\cdot]_R$  are left/right quaternion multiplication.

Stack the equations at multiple times, and use the robust kernel cost function:

$$\begin{bmatrix} w_1^0 \cdot Q_1^0 \\ w_2^1 \cdot Q_2^1 \\ \vdots \\ w_n^{n-1} \cdot Q_n^{n-1} \end{bmatrix} q_{bc} = Q_N \cdot q_{bc} = 0.$$

$$\text{where } w_{k+1}^k = \begin{cases} 1 & r_{k+1}^k < \text{threshold} \\ \frac{r_{k+1}^k}{\text{threshold}} & \text{otherwise} \end{cases}$$

for rotation residual  $r$ :

$$r_{k+1}^k = \text{acos}\left(\text{tr}\left(\hat{R}_{bc}^{-1} R_{bkb_{k+1}}^{-1} \hat{R}_{bc} R_{c_k c_{k+1}}\right) - 1\right) / 2$$

In code:

```
if (initial_ex_rotation, CalibrationExRotation (corres, pre_integrations[frame_count] → delta_q, calib_ric))
```

corres: feature correspondence between two frames, normalized coordinates, a vector

delta\_q: relative rotation from pre integration.

```
bool InitialExRotation::CalibrationExRotation (vector<pair<Vector3d, Vector3d>> corres,
```

Quaterniond delta\_q\_imu,

Matrix3d &calib\_ric\_result)

{

// record the time we enter this function

// each time construct one row in Eq(6)

frame\_count++;

// use matched features to find essential matrix

// decompose it to get rotation.

Rc.push\_back(solveRelativeR (corres));

Rimu.push\_back(delta\_q\_imu.toRotationMatrix());

// convert  $R_{bkb_{k+1}}^k$  to  $R_{c_k c_{k+1}}^k$ :  $R_{c_k c_{k+1}}^k = R_c^b R_{bkb_{k+1}}^k R_c^c$

Rc\_g.push\_back(ric.inverse() \* delta\_q\_imu \* ric);

// construct A matrix

Eigen::MatrixXd A(frame\_count \* 4, 4);

A.setZero();

int sum\_ok = 0;

for (int i=1; i<frame\_count; i++)

{

Quaterniond r1(Rc[i]);

Quaterniond r2(Rc\_g[i]);

// angularDistance is the relative transformation theta

// used in angle-axis  $\theta_u$ .

// this is to compute weights for robust kernel.

double angular\_distance = 180 / M\_PI \* r1.angularDistance(r2);

double huber = angular\_distance > 5.0 ? 5.0 / angular\_distance : 1.0;

sum\_ok++;

Matrix4d L, R;

double w = Quaterniond (Rc[i]).w();

Vector3d q = Quaterniond (Rc[i]).vec();

L.block<3,3>(0, 0) = w \* Matrix3d::Identity() + Utility::skewSymmetric(q);

L.block<3,1>(0, 3) = q;

L.block<1,3>(3, 0) = -q.transpose();

L(3, 3) = w;

Quaterniond R\_ij(Rimu[i]);

w = R\_ij.w();

q = R\_ij.vec();

R.block<3,3>(0, 0) = w \* Matrix3d::Identity() - Utility::skewSymmetric(q);

R.block<3,1>(0, 3) = q;

R.block<1,3>(3, 0) = -q.transpose();

R(3, 3) = w;

A.block<4,4>(i-1) \* 4, 0) = huber \* (L - R);

}

// use svd to solve the least squares system.

JacobiSVD<MatrixXd> svd(A, ComputeFullU | ComputeFullV);

// quaternion is  $[x, y, z, w]^T$ , i.e.  $[q_x, q_y, q_z, q_w]^T$

Matrix<double, 4, 1> x = svd.matrixV().col(3);

Quaterniond estimated\_R(x);

Ric = estimated\_R.toRotationMatrix().inverse();

Vector3d ric\_cov;

ric\_cov = svd.singularValues().tail<3>();

// iterate WINDOW\_SIZE times, and the second smallest singular value is larger than 0.25

if (frame\_count >= WINDOW\_SIZE && ric\_cov(1) > 0.25)

{

calib\_ric\_result = ric;

return true;

}

else

return false;

}