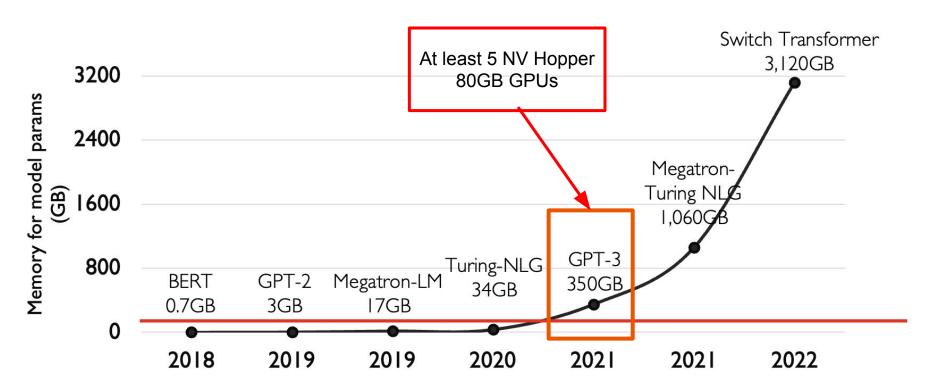
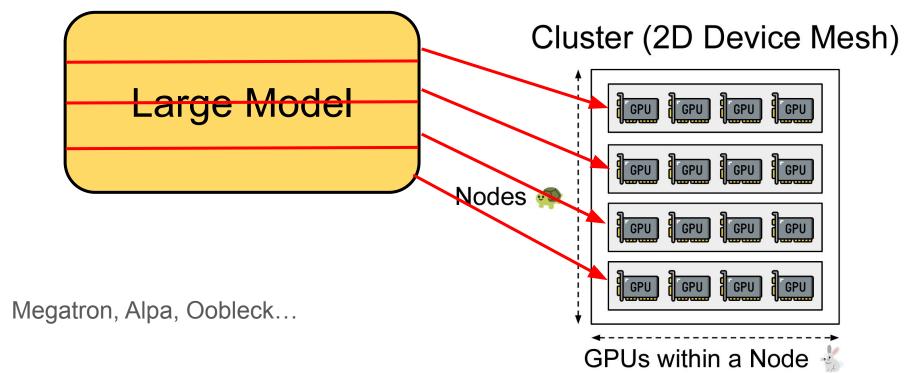
Making Inference Faster

Zhenyan(Luke) Zhu, Daniel Hou, Oh Jun Kweon

Background: Models are getting Larger!

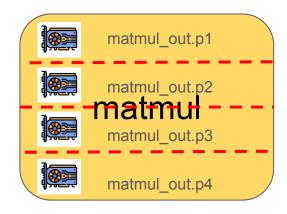


Background: Need to partition the model

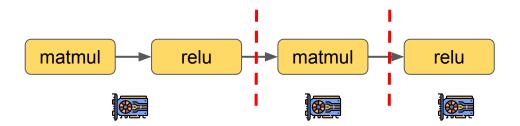


Background: recap on tensor/pipeline level parallelism

Tensor level parallelism: partition a layer to multiple devices



Background: recap on tensor/pipeline level parallelism

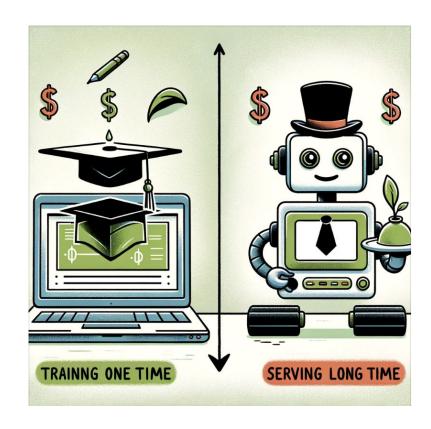


Pipeline level parallelism: partition a model to different stages, assign stages to devices.

Terminology

We use model level parallelism to refer both Tensor-level parallelism and Pipeline-level parallelism.

Background: Training VS Serving



7

AlpaServe: Statistical Multiplexing with Model Parallelism for Deep Learning Serving

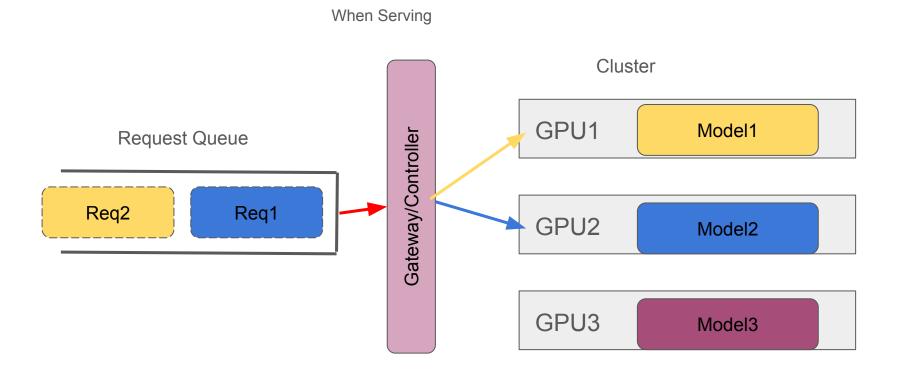
Zhuohan Li, UC Berkeley, Lianmin Zheng, UC Berkeley, Yinmin Zhong, Peking University, Vincent Liu, University of Pennsylvania, Ying Sheng, Stanford University, Xin Jin, Peking University, Yanping Huang, Google, Zhifeng Chen, Google, Hao Zhang

Presenter: Zhenyan(Luke) Zhu

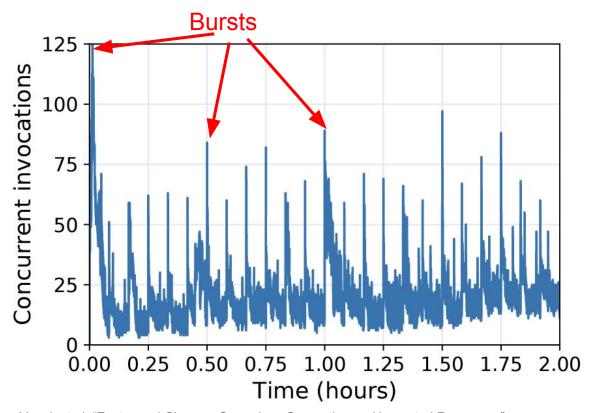
Contents:

- Background & Problem
- 2. How AlpaServe solves the problem
- 3. Evaluation
- 4. Discussion

Background: simple placement

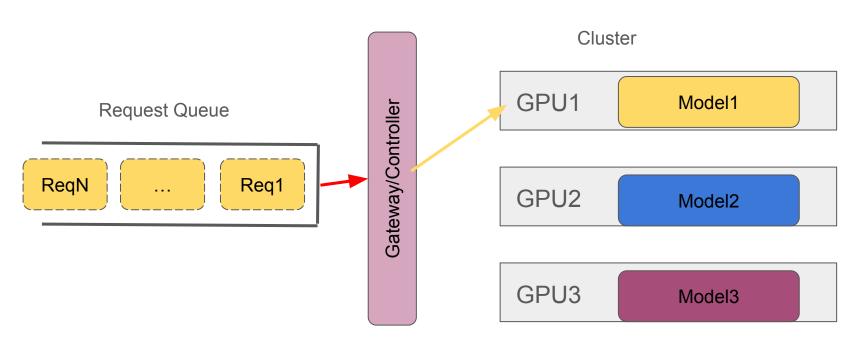


Background: Serving Request Patterns

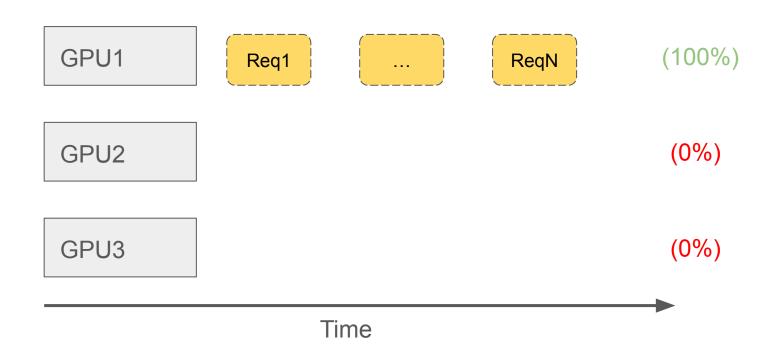


Background: Real-world Serving Pattern

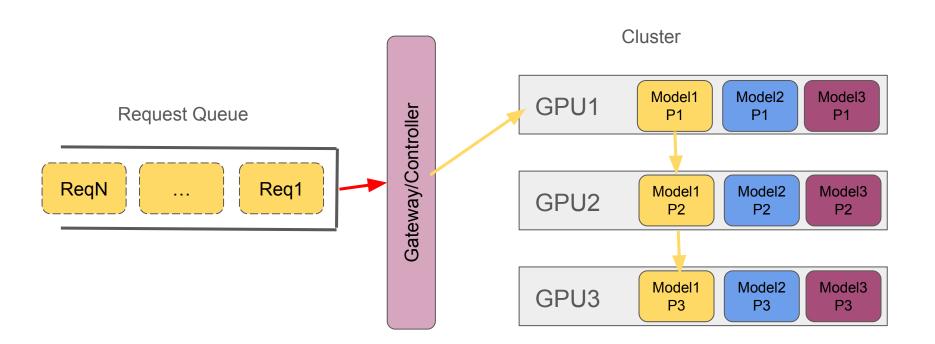
When Serving at peak time



Background: The Problem

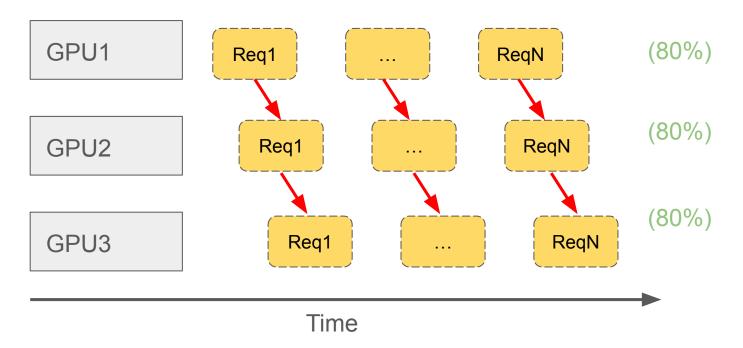


Motivation: Multiplex the GPU fully utilize resource



Motivation: Multiplex the GPU fully utilize resource

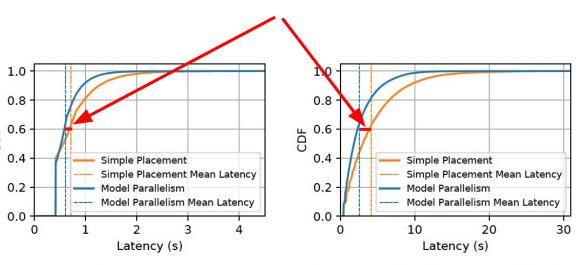
≈ 3x throughput!



2 Model Example Comparison

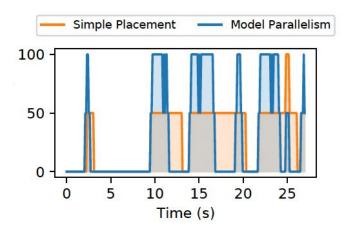
- 2 Transformer Model(13.4GB)
- **2 16GB GPU**

Model Parallelism outperforms greatly if request are brusty

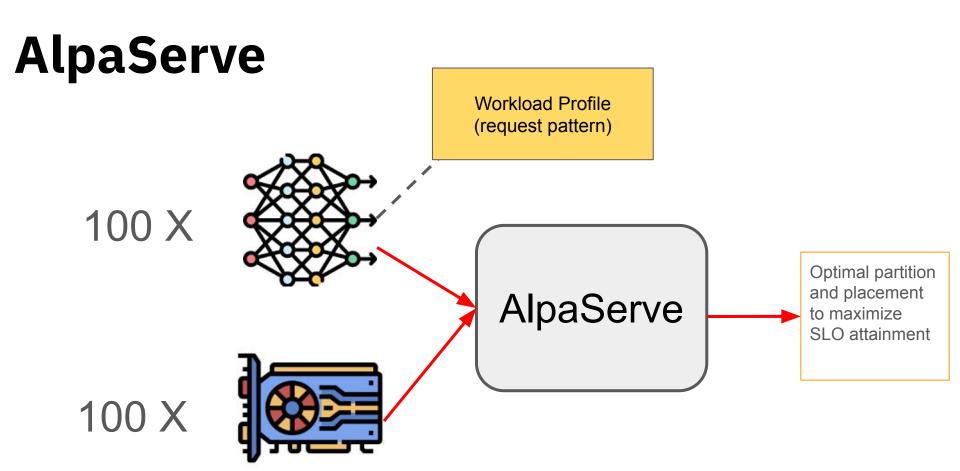


(a) Poisson arrival.





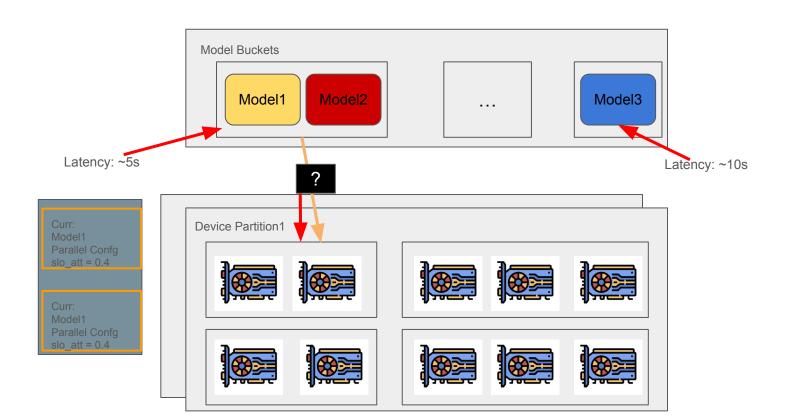
(d) Cluster utilization.

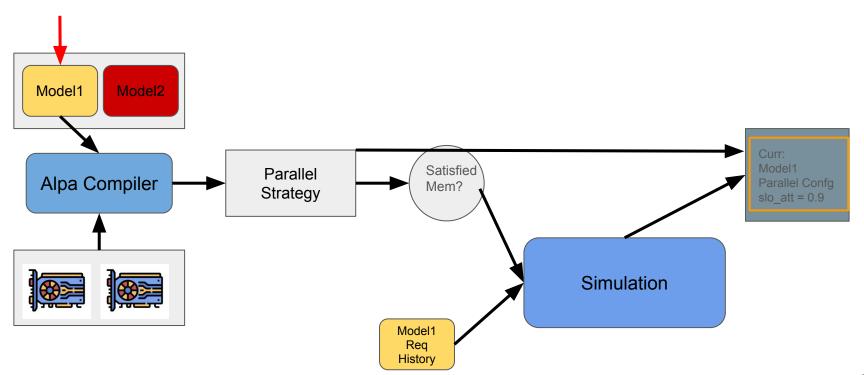


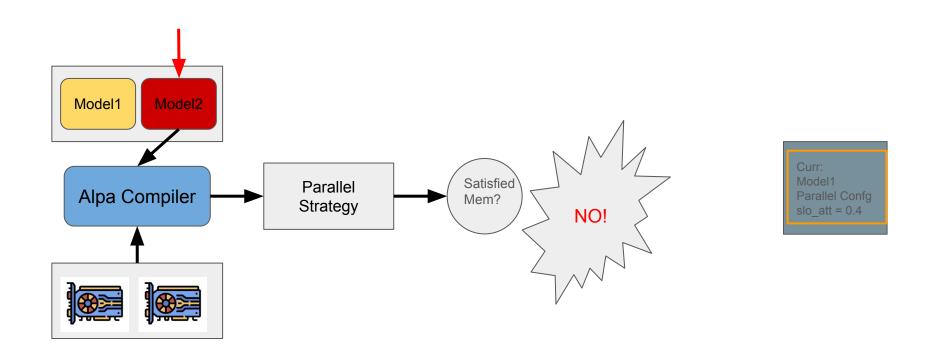
What is SLO attainment?

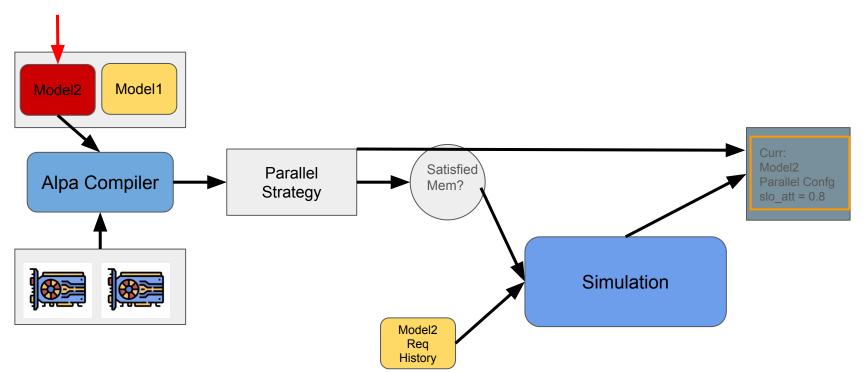
SLO: Service Level Objective (normally on latency)

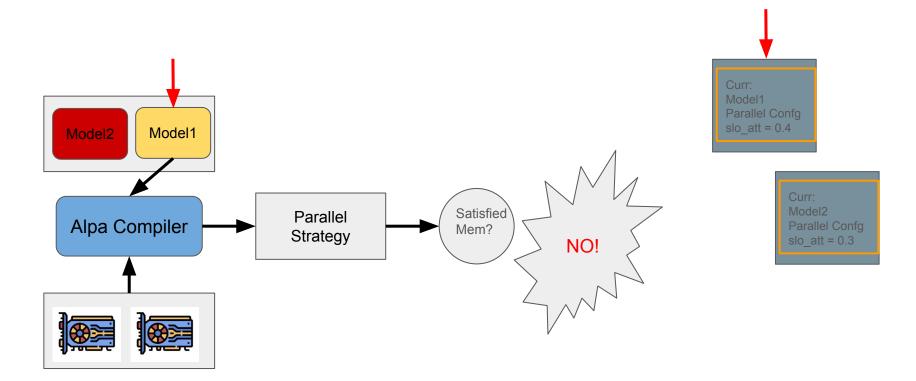
Different requests may have different SLO, SLO attainment is the percentage of requests you serve within SLO.

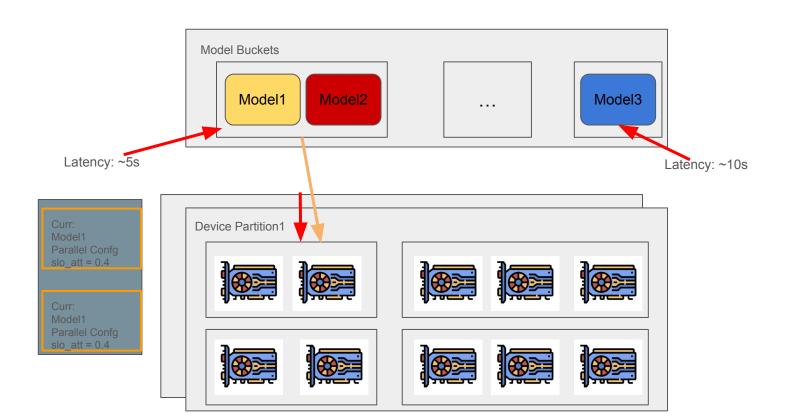


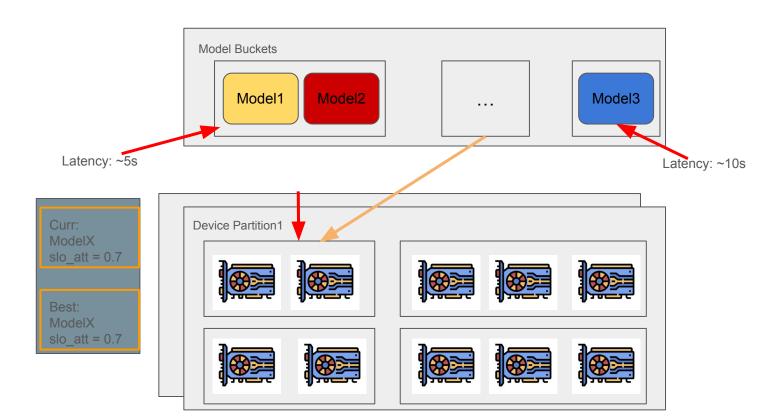


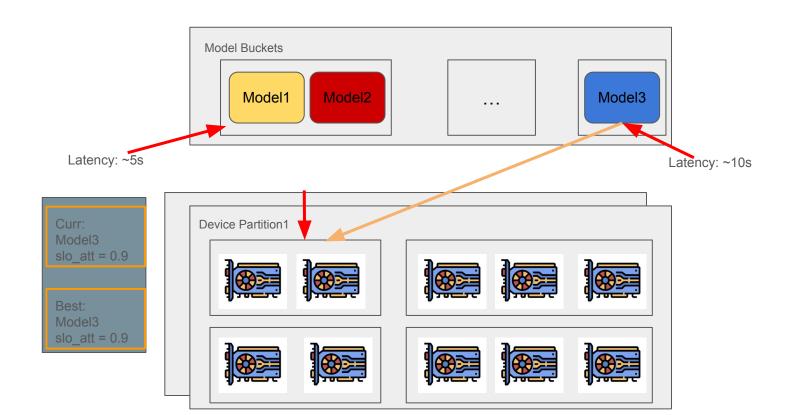


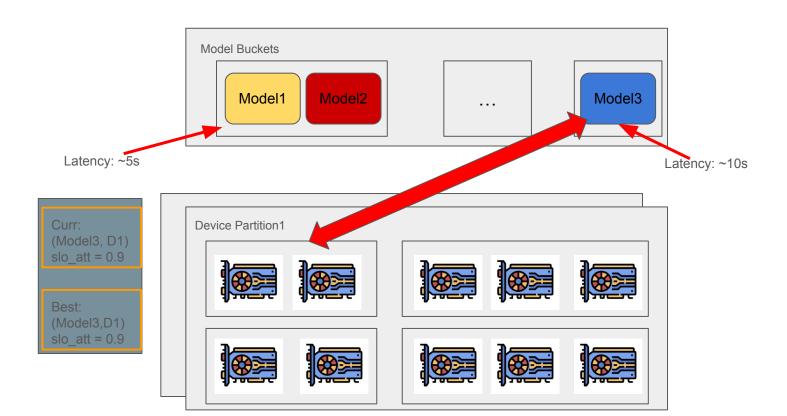


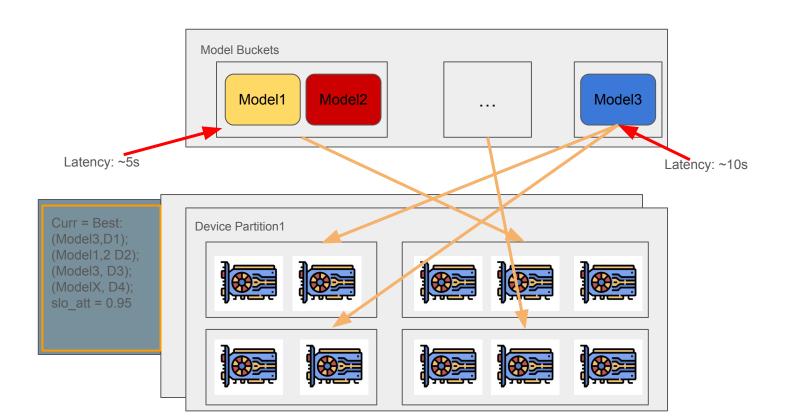


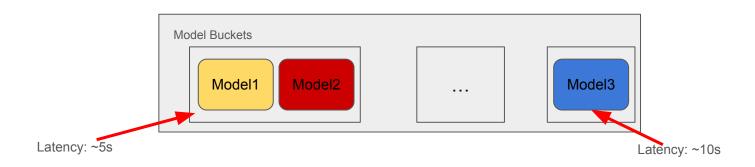




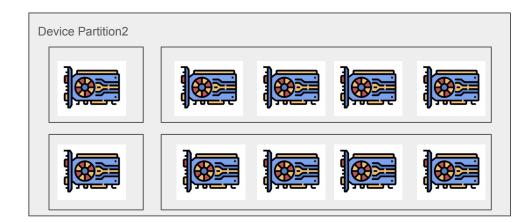








Iterate on all possible Device Buckets...



Evaluation Setup

Cluster: 8 nodes and 64 GPUs. Each node is an AWS EC2 p3.16xlarge instance with 8 NVIDIA Tesla V100 (16GB) GPUs

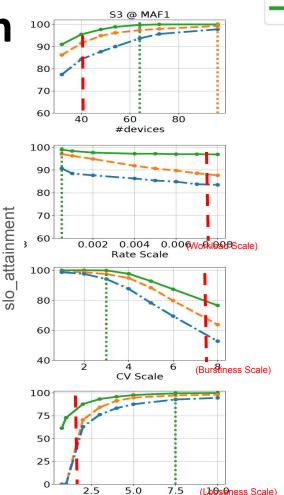
Model: Many BERT and Transformer MoE models of 6 different sizes/latency

Req History: 2 Microsoft Azure function traces(one steady and one brusty)

Evaluation Setup

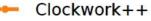
Name	Size	Latency (ms)	S 1	S2	S 3	S 4
BERT-1.3B	2.4 GB	151	32	0	10	0
BERT-2.7B	5.4 GB	238	0	0	10	0
BERT-6.7B	13.4 GB	395	0	32	10	0
BERT-104B	208 GB	4600	0	0	0	4
MoE-1.3B	2.6 GB	150	0	0	10	0
MoE-2.4B	4.8 GB	171	0	0	10	0
MoE-5.3B	10.6 GB	234	0	0	10	0

Evaluation



SLO Scale

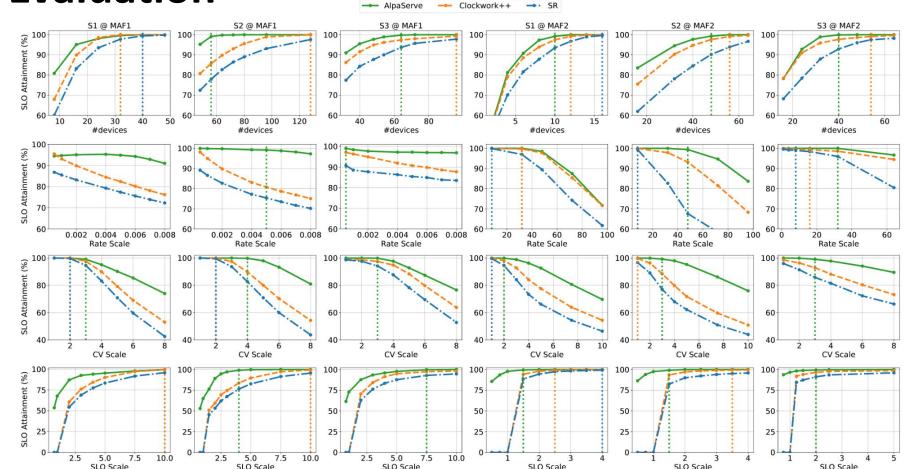






- AlpaServe uses fewer devices to achieve a relatively high SLO Attainment
- As the number of requests increase, AlpaServe's SLO Attainment rate won't decrease a lot
- As the requests become more bursty, AlpaServe can also achieve high attainment rate
- When SLO is small, AlpaServe can achieve relatively high SLO Attainment

Evaluation



Evaluation: Very Large Model (Set 4)

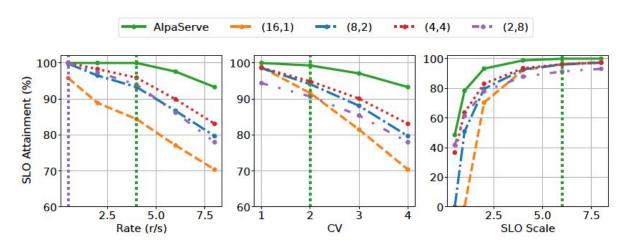
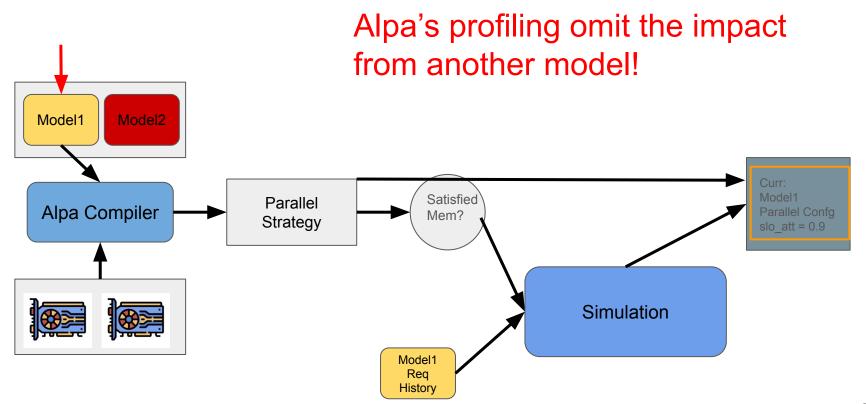


Figure 13: SLO attainment as we vary the rate, CV, and SLO scale. (8,2) means 8-way inter-op parallelism and in each pipeline stage using 2-way intra-op parallelism.

Discussion

1. When using Alpa Compiler to generate best parallel strategy, does it consider the interaction/impact between other models?



Discussion

- 1. When using Alpa Compiler to generate best parallel strategy, it will do profiling. Does it consider the interaction between other models?
- 2. Planning time is not included in their evaluation. Given such a high complexity, it cannot be directly ignored.

Discussion

- 1. When using Alpa Compiler to generate best parallel strategy, it will do profiling. Does it consider the interaction between other models?
- 2. Planning time is not included in their evaluation. Given such a high complexity, it cannot be directly ignored.
- 3. When model-level parallelism is most beneficial?

Discussion

- A1: Model parallelism benefits model serving through statistical multiplexing at the following scenario:
 - Limited per-device memory
 - Below system peak throughput overall request rate
 - High request burstiness
 - Tight SLO

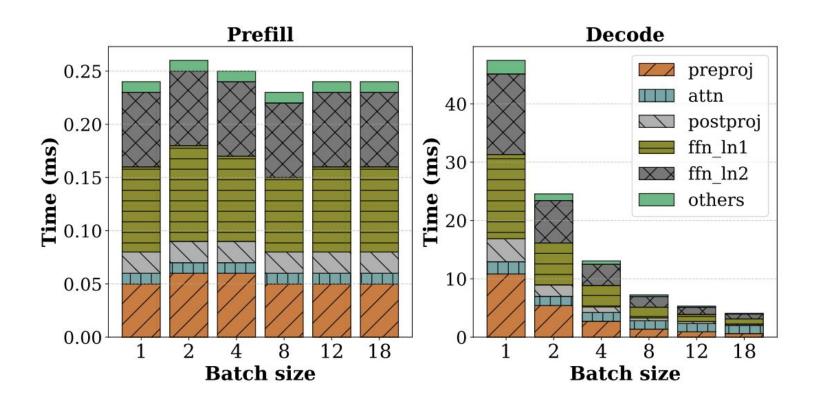
SARATHI: Efficient LLM Inference by Piggybacking Decodes with Chunked Prefills

Amey Agrwal, Ashish Panwar, Jayashree Mohan, Nipun Kwatra, Bhargav S. Gulavani, Ramachandran Ramjee (Microsoft Research India, Georgia Institute of Technology)

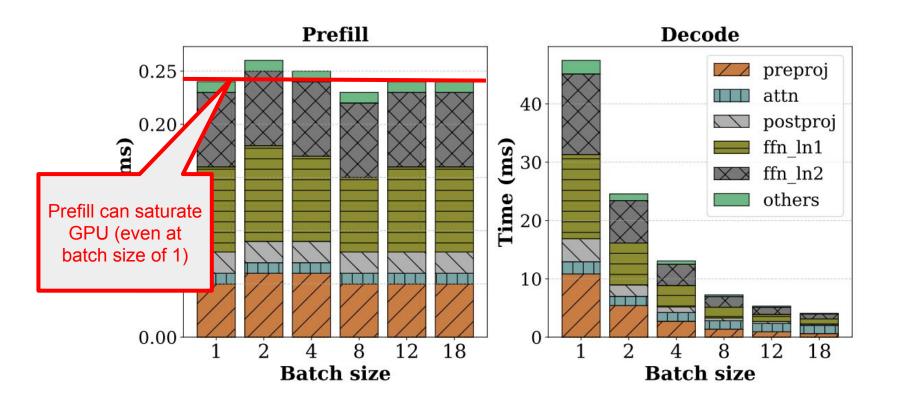
Recall: Prefill and Decode

- Transformer Architecture: 2 phases
 - Prefill
 - Decode
- Prefill
 - Digest Prompt → Process all tokens in input sequence in parallel
 - High GPU utilization for small batch sizes
- Decode
 - Predict Next Token → Processes only a single token in each autoregressive pass
 - Low GPU utilization at small batch sizes
- A single request goes through 1 prefill pass and multiple decode pass
 - Prefill pass + decode passes = inefficient

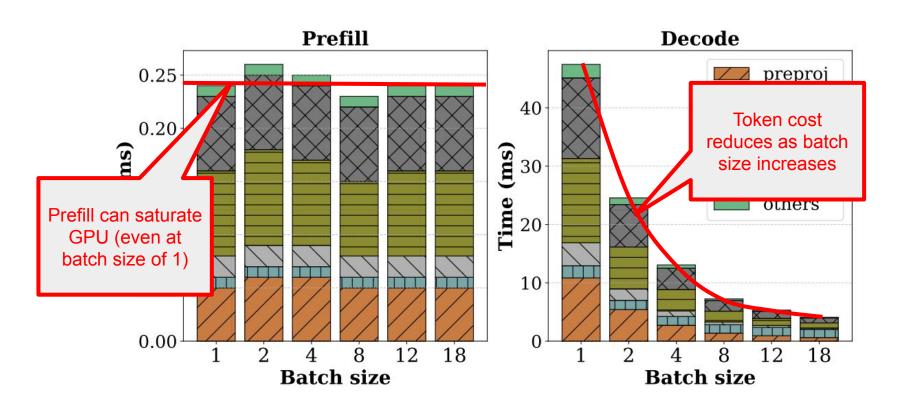
Prefill and Decode: Per token time



Prefill and Decode: Per token time



Prefill and Decode: Per token time

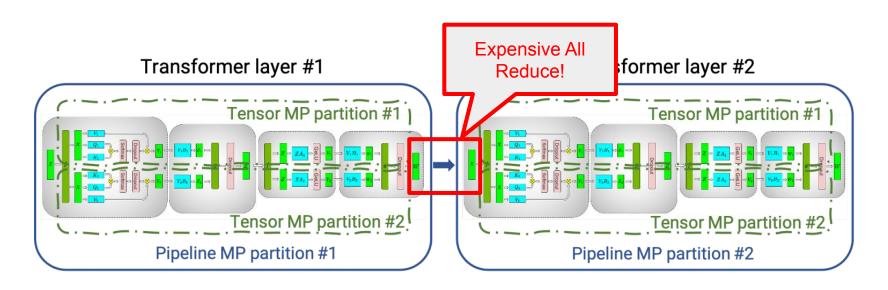


Parallelism: Improving LLM Inefficiencies

- Tensor Parallelism
- Pipeline Parallelism

Tensor Parallelism

Shard each individual layer across multiple GPU



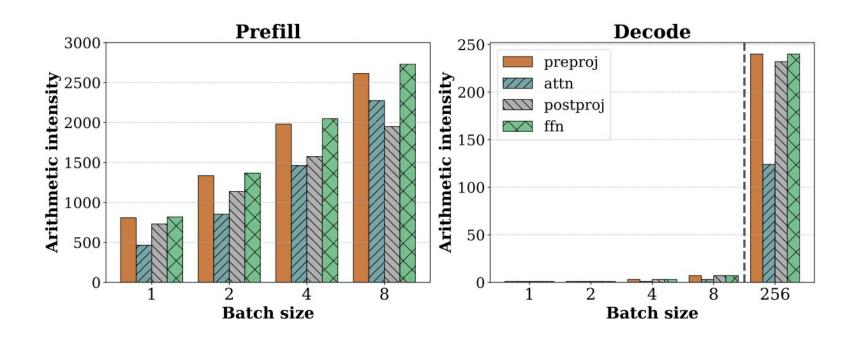
Pipeline Parallelism Bubbles lead to idle time! Bubbles occur from different compute times Shard model layer wise → each GPU responsible $A_{d1}B$ $C_{d1}D_{d1}$ $A_{d2}B_{d2}$ GPU1 B_p Cp Dp Bubble Bubble GPU2 B_p Cp Dp time (a) Baseline iteration-level scheduling Prefill Bubble Decode

Parallelism Insights

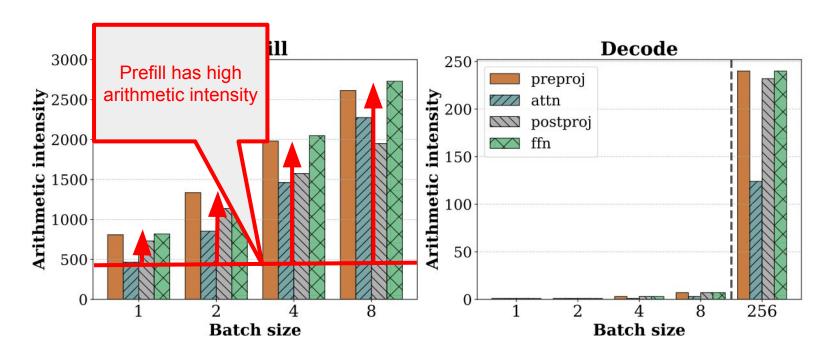
- Tensor Parallelism
 - Communication needed for all-reduce operation
 - Only viable with high-bandwidth connectivity
- Pipeline Parallelism
 - No all-reduce operation needed
 - Viable even if high-bandwidth connectivity not available
 - Pipeline Bubbles from **varying compute times** → Inefficiencies

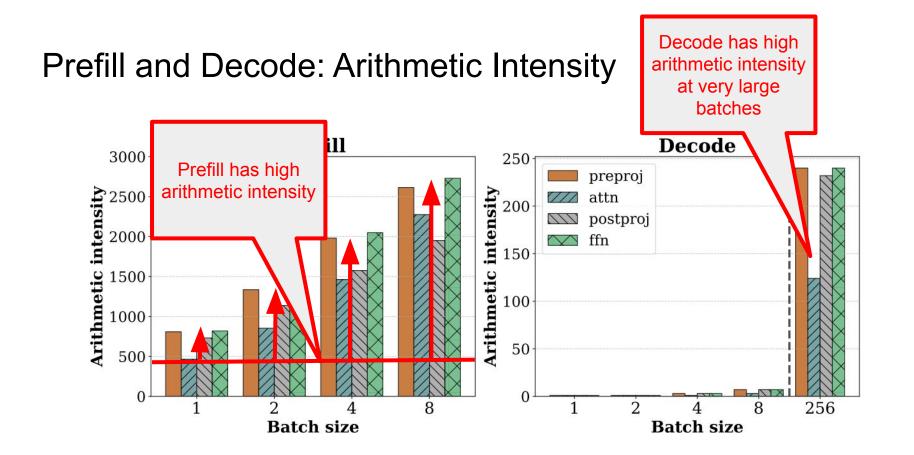
1. Pipeline Parallelism leads to big bubbles

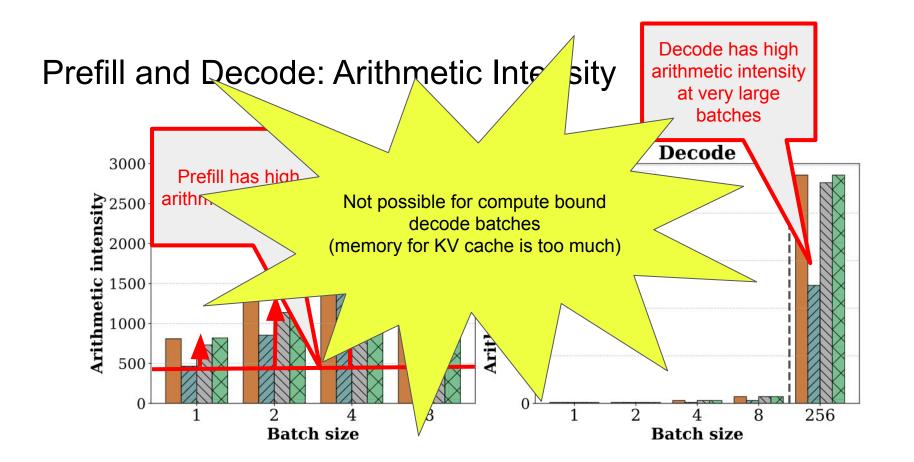
Prefill and Decode: Arithmetic Intensity



Prefill and Decode: Arithmetic Intensity





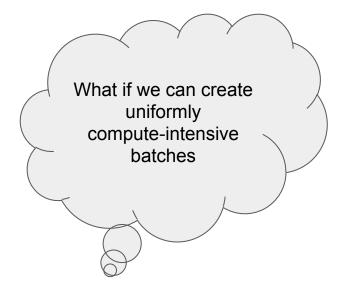


Notes

- Increasing batch size of prefill does not affect cost (per-token)
- Increasing batch size of decode reduces cost (per-token)
- Decode can be compute-bound at very high batch sizes
 - Very high batch sizes not practical today
 - Memory needed for KV-cache for each individual request → a lot

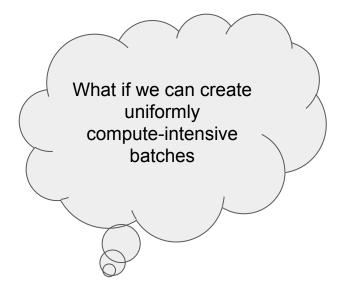
- 1. Pipeline Parallelism leads to big bubbles
- 2. Decoding is memory-bound

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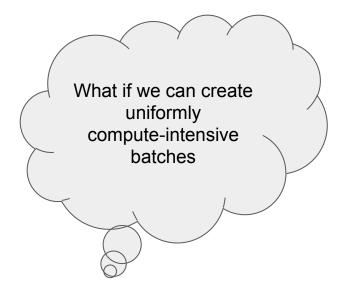


- 1. Pipeline Parallelism will have smaller bubbles
- 2. Decoding is memory-bound





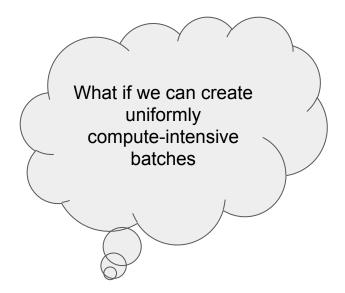
- 1. Pipeline Parallelism will have smaller bubbles
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LLM Inefficiencies (Gone)

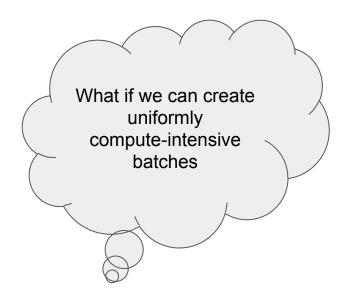
- 1. Pipeline Parallelism will have smaller bubbles
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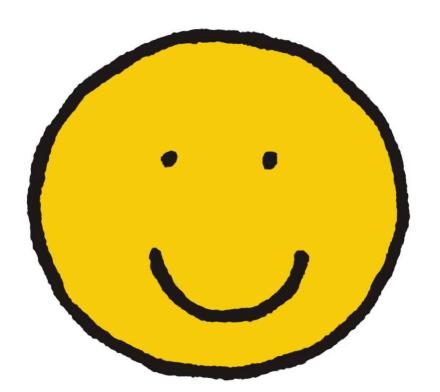




LLM Inefficiencies (Gone)

- 1. Pipeline Parallelism will have smaller bubbles
- 2. Decoding is compute-bound





Proposal for 2 new concepts:

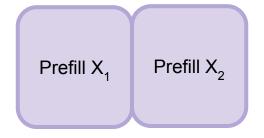
- **Chunked-Prefills:** Split a prefill into multiple chunks
- Decode-Maximal Batching: Construct a batch by using a single prefill chunk and "piggybacking" that chunk with multiple decode tokens

Chunked-Prefills + Decode-Maximal Batching → Hybrid batches of prefill and decode

Given Prefill X:

Prefill X

We can split X



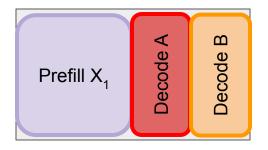
With a predetermined batch size,

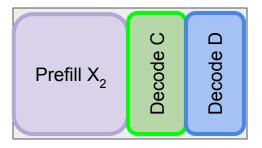




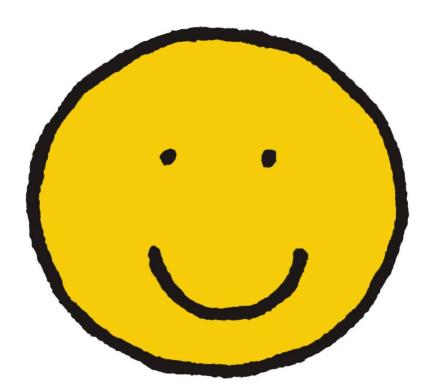
We can fill the remaining space in batch with "piggybacked" decodes

Uniform Hybrid Batches:



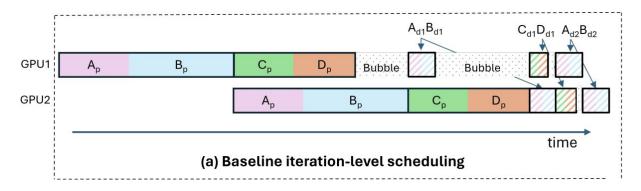


- 1. Pipeline Parallelism leads to large bubbles
 - a. Uniform Hybrid batch size
- 2. Decoding is memory-bound

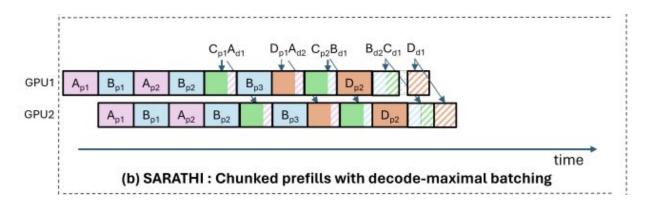


SARATHI: Results

Baseline Iteration (like ORCA):



SARATHI:



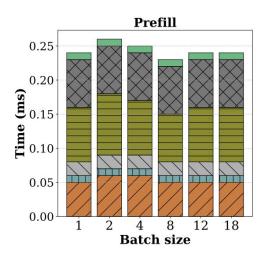
SARATHI: Chunked Prefills

Hinged on 2 key insights:

- Decreasing # of prefill tokens → gives around the same throughput
- Larger hidden dimension → chunk size of prefill needed to saturate
 GPU lowers
 - More computations per data element (in input)

Conclusion: Compute-saturating batch is possible

- Most practical scenarios: prefill is large
- Can easily split prefill into multiple chunks



SARATHI: Decode-Maximal Batching

Combine linear operation computations for prefill chunk and decodes into single operation

• Result: Decodes are more efficient because of GPU-saturating prefill computation

What is maximum possible number of decodes?

$$B = \left\lfloor \left(\frac{M_G - M_S}{L * m_{kv}} \right) \right\rfloor$$

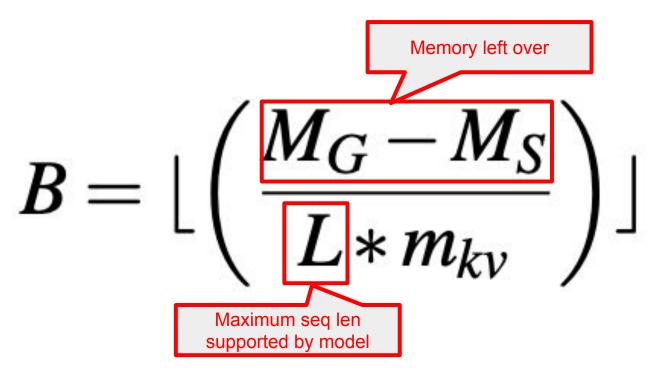
What is maximum possible number of decodes?

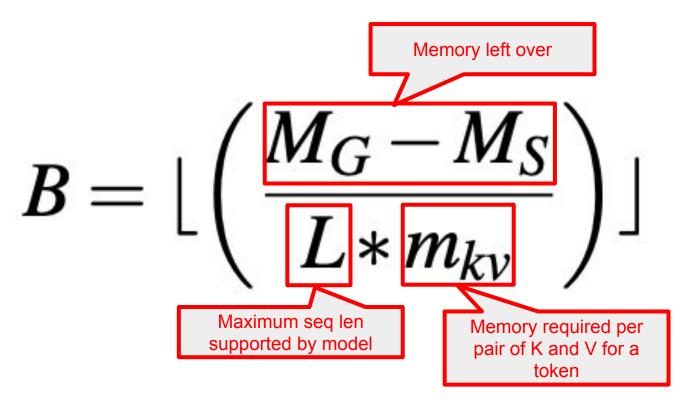
$$B = \lfloor \left(rac{M_G - M_S}{L*m_{kv}}
ight)
floor$$

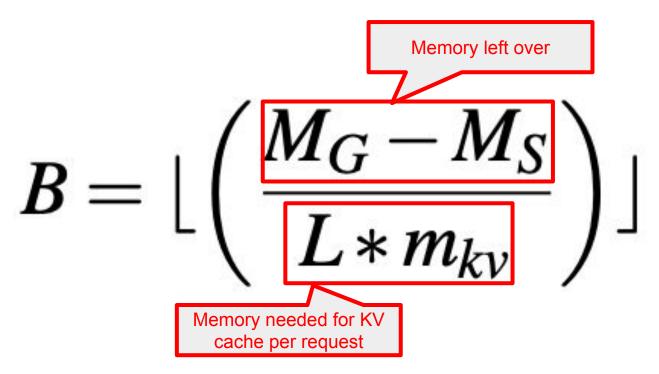
What is maximum possible number of decodes?

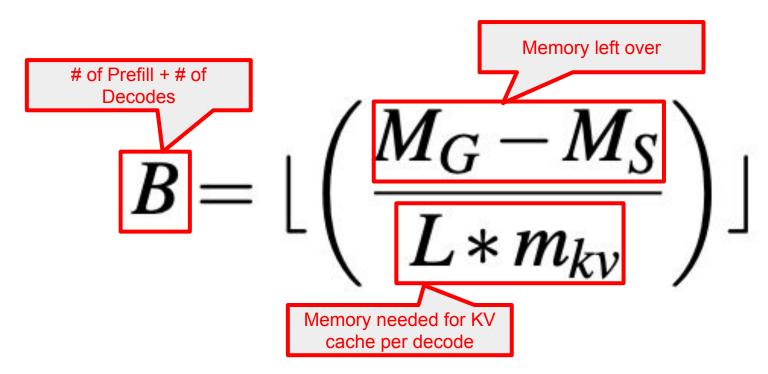
$$B=\lfloor egin{pmatrix} ext{Available GPU} & ext{Model Parameter Memory Requirement} \ ext{$MG-MS$} \ ext{$L*m_{kv}$} \end{pmatrix}
floor$$

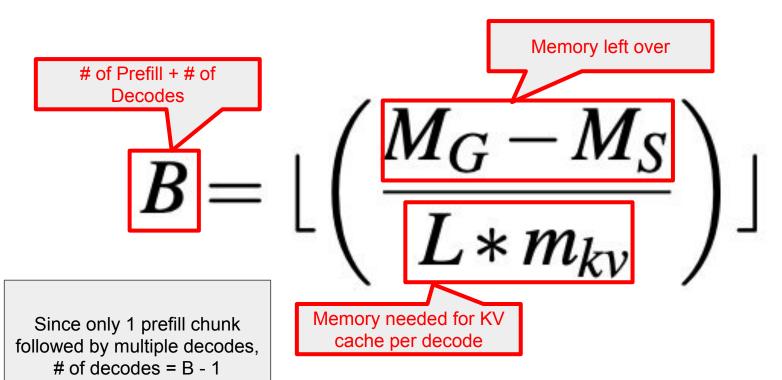
$$B = \lfloor \left(rac{M_G - M_S}{L*m_{kv}}
ight)
floor$$











78

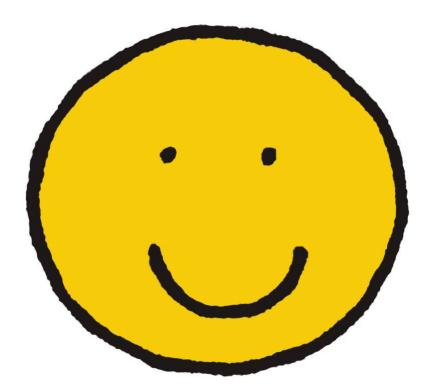
SARATHI: Decode-Maximal Batching

Prefill and Decode phases follow same computation path

- Linear operations use same weight tensors in both prefill and decode phases
 - Only need to load model weights once onto GPU!
 - Before: had to separately load weights for decoding
 - After: Decoding becomes compute-bound instead of memory bound

LLM Inefficiencies (Gone)

- 1. Pipeline Parallelism leads to large bubbles
 - a. Uniform Hybrid batch size
- 2. Decoding is memory bound
 - a. Loading weights only once
 - b. Efficient combination of linear ops
 - c. Larger batch of decodes



SARATHI: Tradeoff to prefill chunk size

As prefill chunk size decrease:

- More decode tokens can be "piggybacked" (Good)
- Arithmetic intensity of chunked-prefills computation decrease (Bad)

How to determine optimal prefill chunk size?

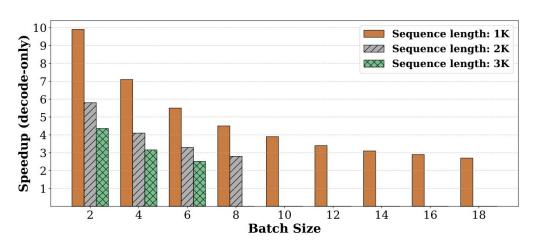
- What is expected of workload?
 - More prefills or decodes?
- Varies depending on model and hardware

Answer: **Profiling**

Evaluation

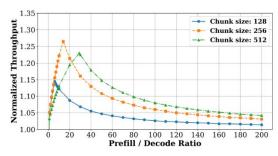
Batching	Operat	ion(s)	Total Per-token		ken Time
Scheme	Linear	Attn	Time	Prefill	Decode
Prefill-only	224.8	10	234.8	0.229	-
Decode-only	44.28	5.68	49.96	_	12.49
Decode-maximal	223.2	15.2	238.4	0.229	1.2

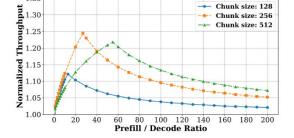
Evaluation

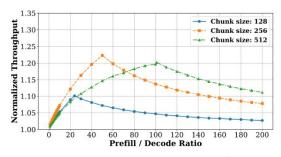


Model (GPU)	Sequence Length	Batch Size	P:D Ratio	Decode Speedup	Throughput Gain
LLaMA-13B	1 K	6	50:1	5.45×	1.33×
(A6000)	2K	6	50:1	3.26×	1.26×
	3K	6	50:1	2.51×	1.22×
LLaMA-33B	1 K	10	28:1	3.83×	1.25×
(A100)	2K	5	63:1	4.25×	1.22×
	3K	3	127:1	3.51×	1.14×

Evaluation over different workloads

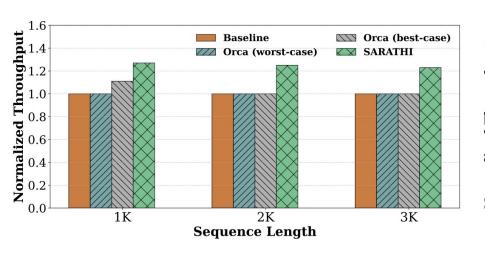


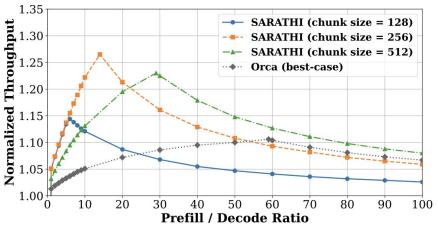




- (a) Sequence length = 1K, batch size = 18.
- (b) Sequence length = 2K, batch size = 10.
- (c) Sequence length = 3K, batch size = 6.

Comparison to Iteration Level Scheduling (ORCA)





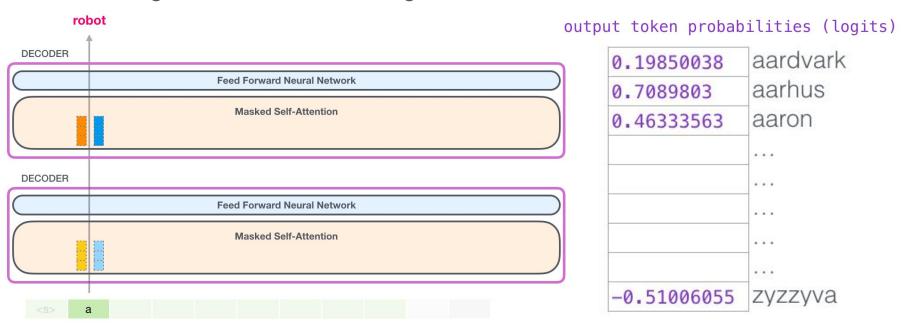
Accelerating Large Language Model Decoding with Speculative Sampling

Charlie Chen, Sebastian Borgeaud, Geoffrey Irving, Jean-Baptiste Lespiau, Laurent Sifre, Jon Jumper (DeepMind)

Feb 2023

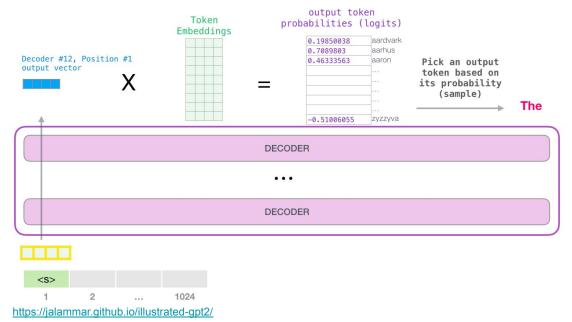
LLM Inference

- Generating the "next words" through transformer decoders



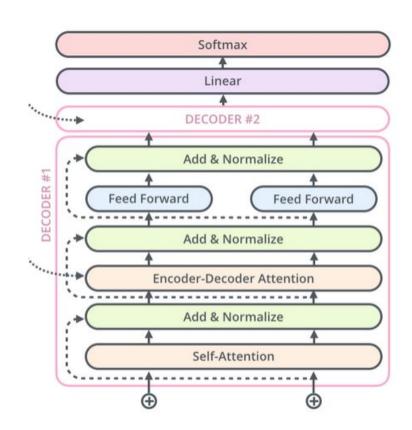
LLM Inference

- Transformers are cool: multiple requests can be run in parallel through elegant matrix multiplications
 - New request = "add a row to the matrix"



Problem: LLM Inference is slow

Generating response of k tokens requires
 k sequential decoder runs.



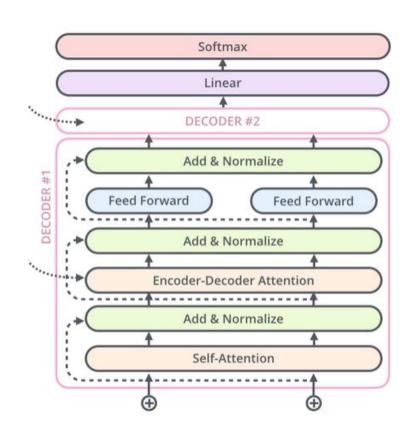
https://jalammar.github.io/illustrated-transformer/

Problem: LLM Inference is slow

Generating response of k tokens requires
 k sequential decoder runs.

Can we do this in parallel?

- Well, we would need to know the future...



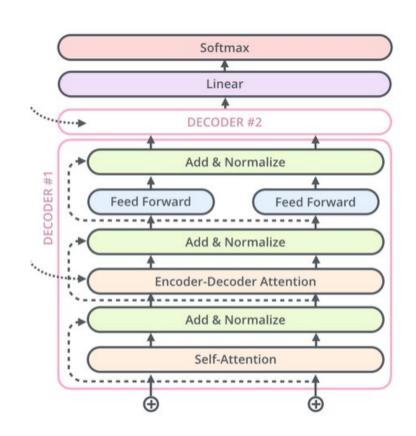
Problem: LLM Inference is slow

Generating response of k tokens requires
 k sequential decoder runs.

Can we do this in parallel?

- Well, we would need to know the future...

Can we speculate the future?



Observations

- Some inference steps are "easier" than others
- LLM inference is not bottlenecked by arithmetic operations
 - Bottleneck is memory bandwidth and communication

Observations

- Some inference steps are "easier" than others
- LLM inference is not bottlenecked by arithmetic operations
 - Bottleneck is memory bandwidth and communication

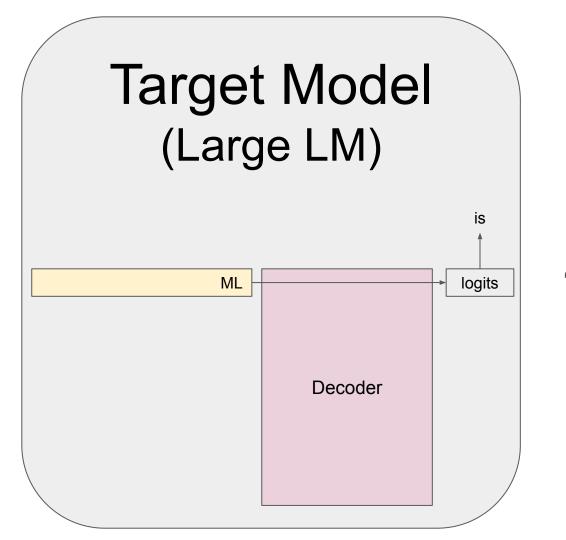
Idea: Use a smaller model to speculate the future

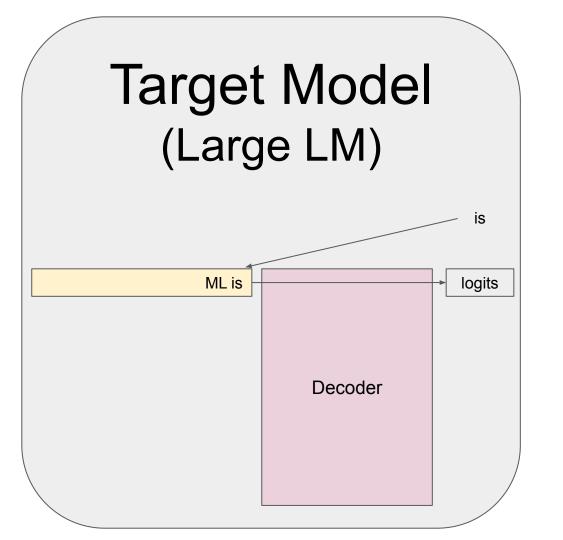
Target Model (Large LM)

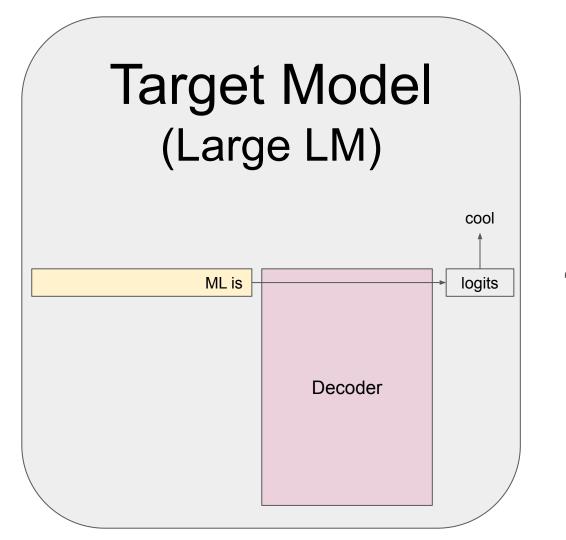
Draft
Model
(Smaller LM)

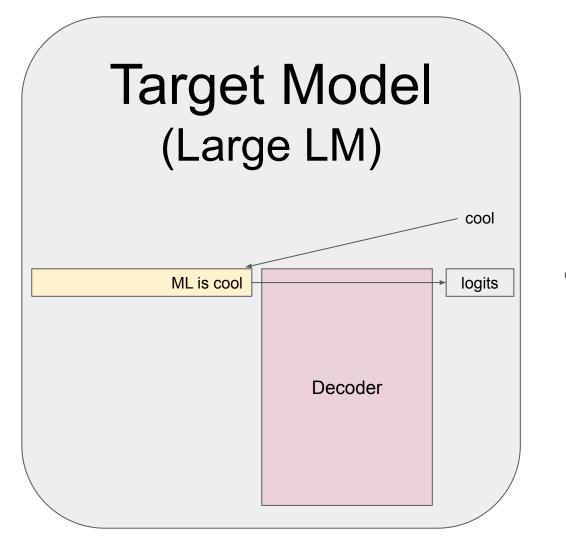
Target Model (Large LM) ML logits Decoder

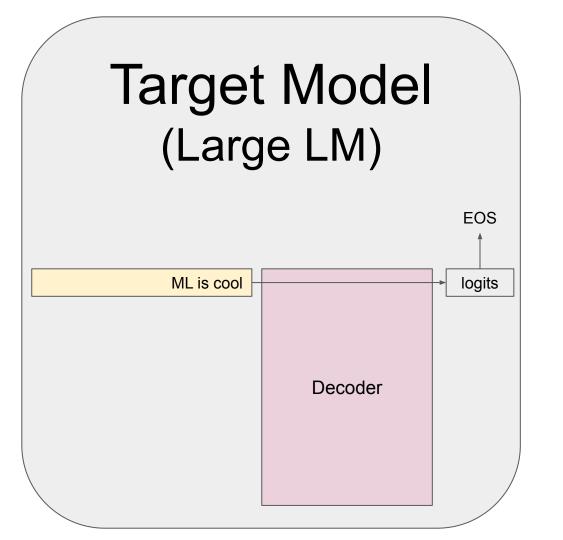
Draft
Model
(Smaller LM)

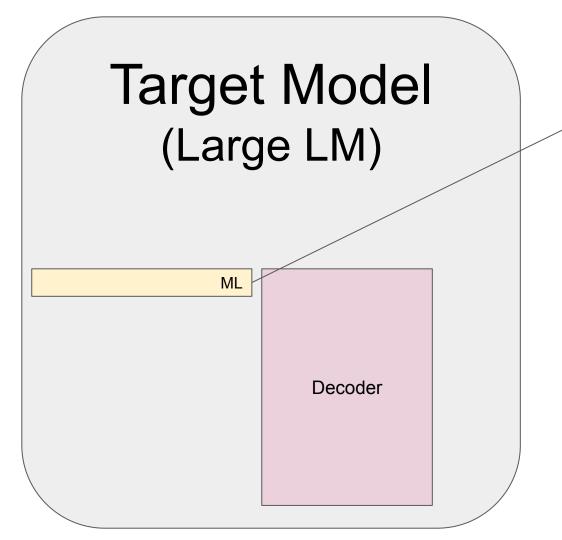




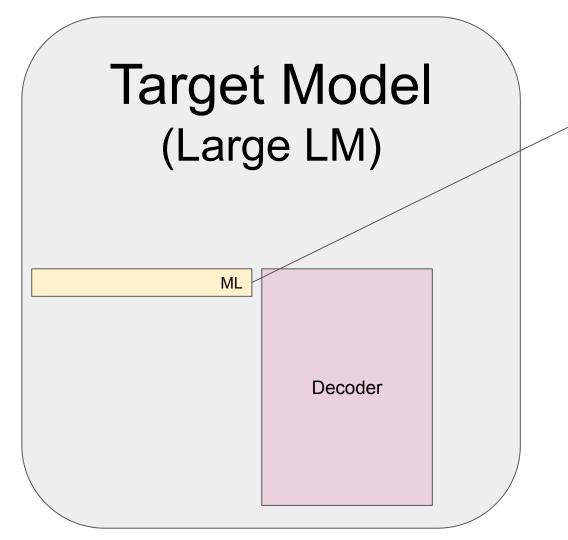








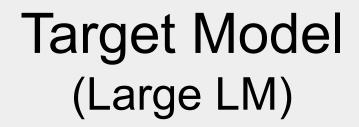
Speculative sampling



(k = 5)

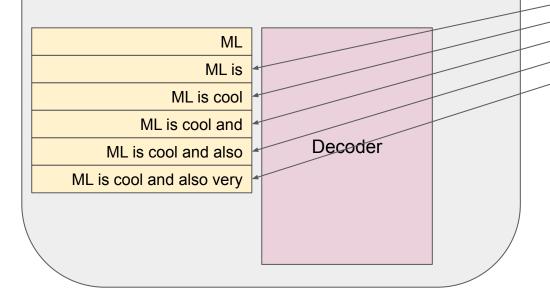
						4	4
	log	its	log	jits	logits	logits	logits
/	į	3	CC	100	and	also	very

*simplifying assumption: 1 word = 1 token



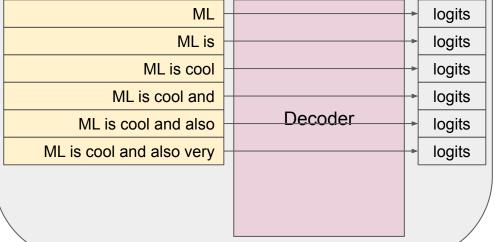
(k = 5)

logits	logits	logits	logits	logits
is	cool	and	also	very



Target Model (Large LM)

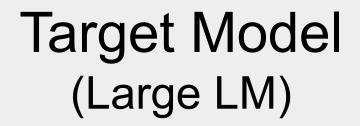
(in parallel)

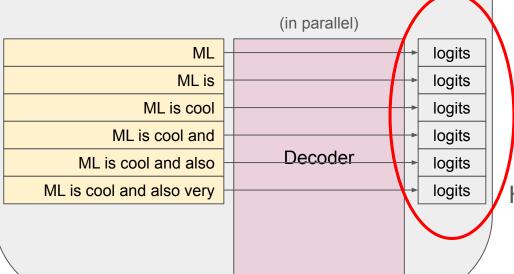


Draft Model (Smaller LM)

(k = 5)

logits	logits	logits	logits	logits
is	cool	and	also	very





(k = 5)

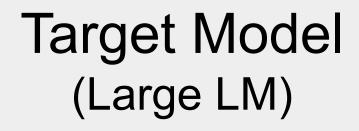
logitslogitslogitslogitslogitsiscoolandalsovery

Faster to compute

Higher quality

Goal:

Keep as many of the draft tokens as possible without compromising quality



(k = 5)

logits	logits	logits	logits	logits
is	cool	and	also	very

(in parallel)

ML

ML is

ML is

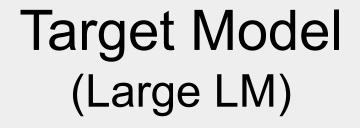
ML is cool

ML is cool and

ML is cool and also

sample "is"

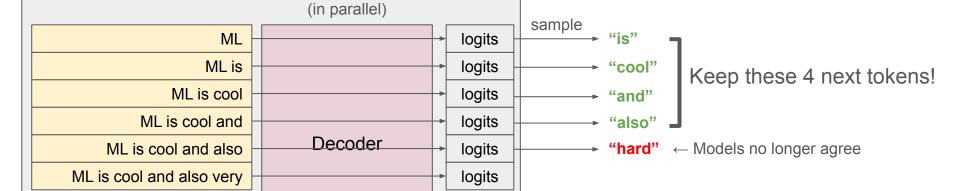
Intuitive idea: check if models agree





(k = 5)

logits	logits	logits	logits	logits
is	cool	and	also	very

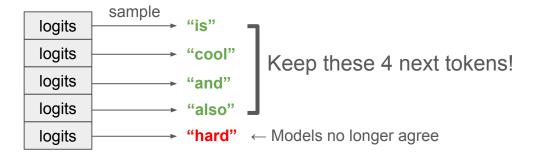


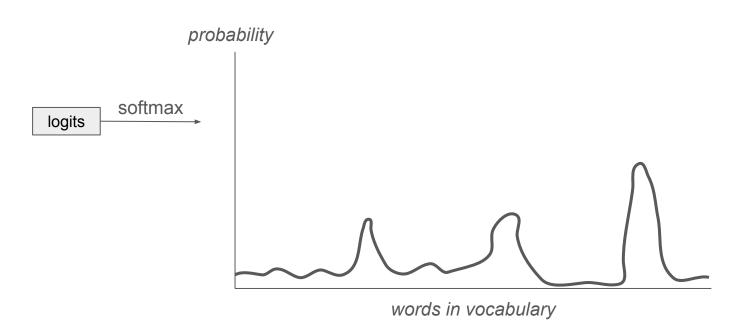
Intuitive idea: check if models agree

How do we do this efficiently?

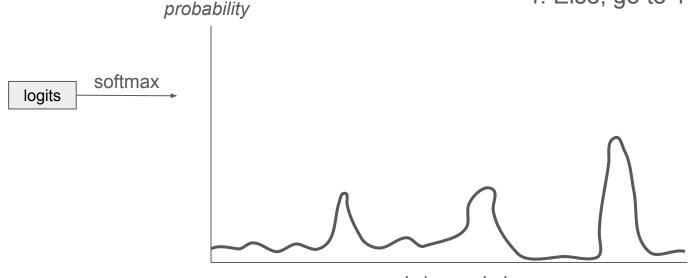
>> Modified Rejection Sampling

logits	logits	logits	logits	logits
is	cool	and	also	very



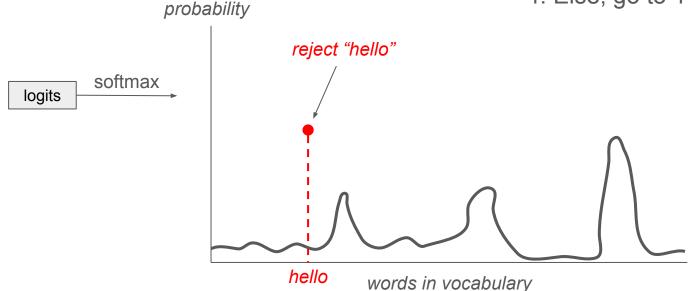


- 1. Pick a word w
- 2. Pick $r \sim U(0, 1)$
- 3. If $r \leq P(w)$, choose w
- 4. Else, go to 1

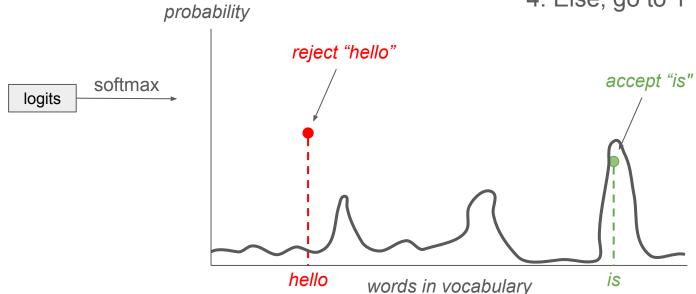


words in vocabulary

- 1. Pick a word w
- 2. Pick $r \sim U(0, 1)$
- 3. If $r \leq P(w)$, choose w
- 4. Else, go to 1



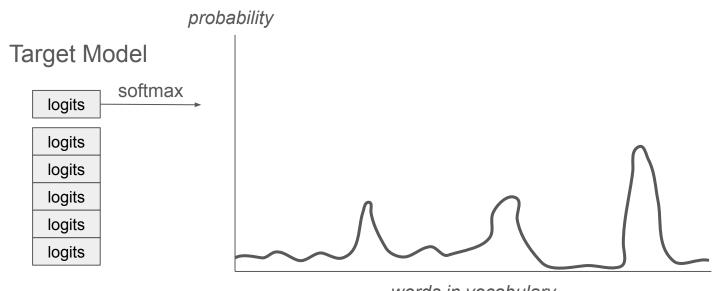
- 1. Pick a word w
- 2. Pick $r \sim U(0, 1)$
- 3. If $r \le P(w)$, choose w
- 4. Else, go to 1



>> Modified Rejection Sampling

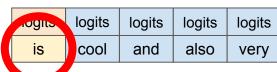
Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very

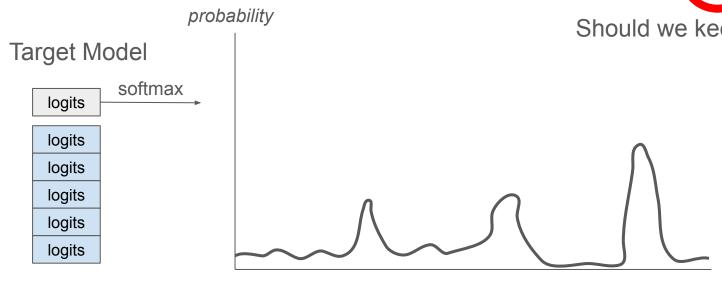


>> Modified Rejection Sampling

Draft Model



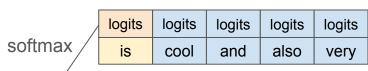
Should we keep this draft token?

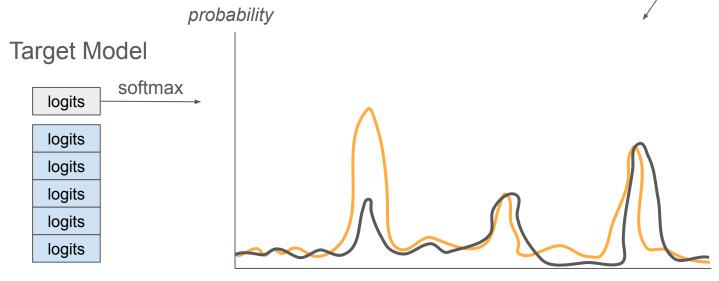


words in vocabulary

>> Modified Rejection Sampling

Draft Model



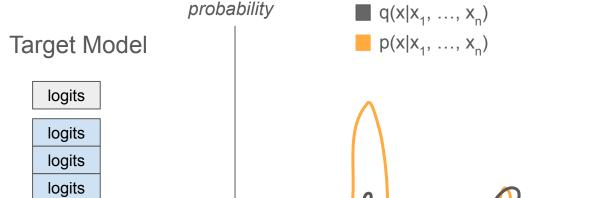


words in vocabulary

logits

logits

>> Modified Rejection Sampling



Draft Model

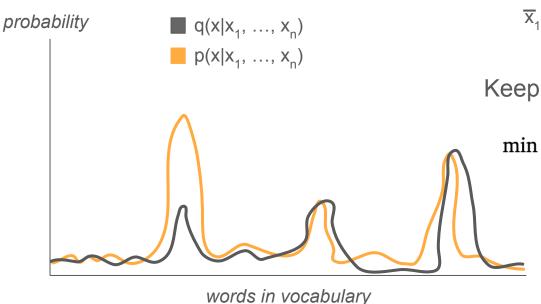
logits	logits	logits	logits	logits
is	cool	and	also	very
\overline{X}_1	\overline{X}_2	\overline{X}_3	\overline{X}_4	\overline{X}_5

words in vocabulary

>> Modified Rejection Sampling



logits
logits
logits
logits
logits
logits



Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very
\overline{X}_1	\overline{X}_2	\overline{X}_3	\overline{X}_{4}	\overline{X}_5

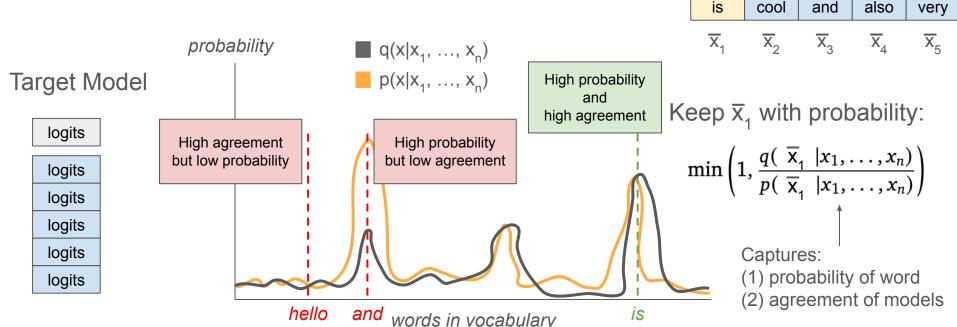
Keep \overline{x}_1 with probability:

 $\min\left(1,\frac{q(\overline{\mathbf{x}}_1 \mid x_1,\ldots,x_n)}{p(\overline{\mathbf{x}}_1 \mid x_1,\ldots,x_n)}\right)$

Captures:

- (1) probability of word
- (2) agreement of models

>> Modified Rejection Sampling



Draft Model

logits

logits

logits

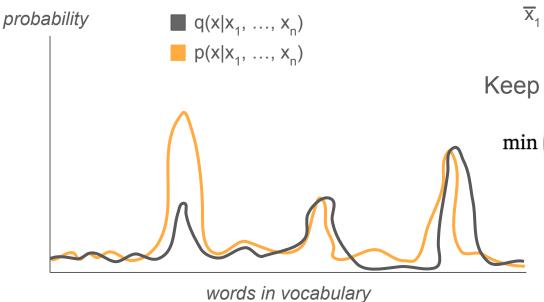
logits

logits

>> Modified Rejection Sampling







Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very
\overline{X}_1	\overline{X}_2	\overline{X}_3	\overline{X}_{4}	\overline{X}_5

Keep \overline{x}_1 with probability:

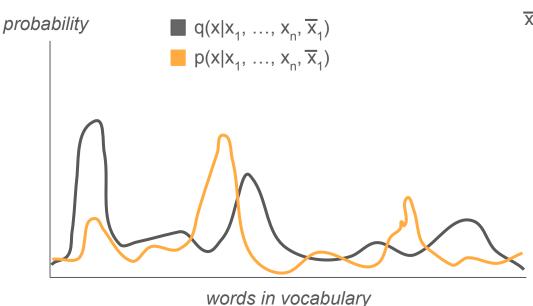
 $\min\left(1,\frac{q(\overline{X}_1 | x_1,\ldots,x_n)}{p(\overline{X}_1 | x_1,\ldots,x_n)}\right)$

If accept, continue!

>> Modified Rejection Sampling

Target Model





Draft Model

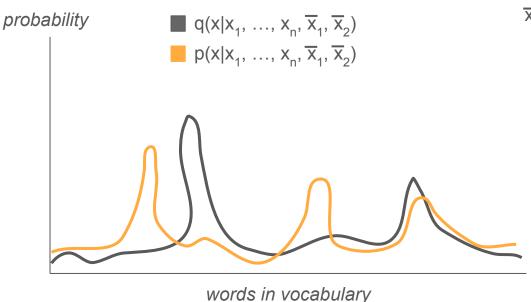
logits	logits	logits	logits	logits
is	cool	and	also	very
\overline{X}_{i}	X	X	\overline{X} .	X

If accept, continue!

>> Modified Rejection Sampling

Target Model

logits
logits
logits
logits
logits
logits



Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very
X	X	X	X	X

If accept, continue!

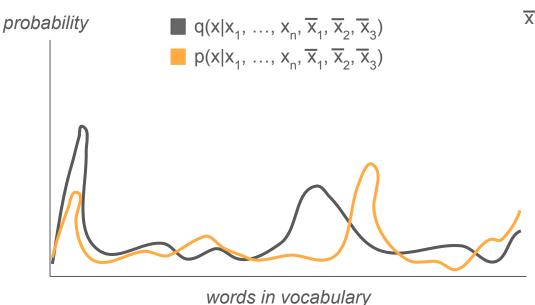
>> Modified Rejection Sampling

Target Model

logits
logits
logits

logits

logits



Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very
\overline{X}_{4}	\overline{X}_{0}	\overline{X}_{2}	\overline{X}_{4}	\overline{X}_{r}

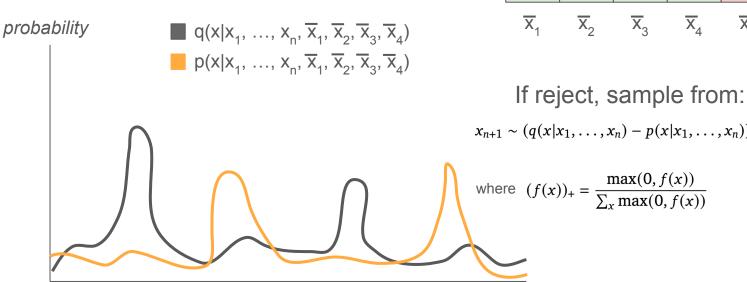
continue!

If accept,

>> Modified Rejection Sampling

Target Model

logits logits logits logits logits logits



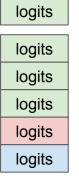
logits	logits	logits	logits	logits
is	cool	and	also	very
			-	-

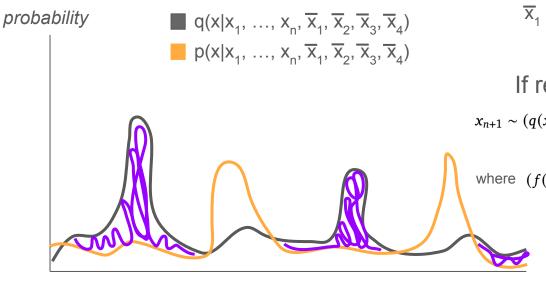
$$x_{n+1} \sim (q(x|x_1,\ldots,x_n) - p(x|x_1,\ldots,x_n))_+$$

words in vocabulary

>> Modified Rejection Sampling

Target Model





words in vocabulary

Draft Model

logits	logits	logits	logits	logits
is	cool	and	also	very
$\overline{\nabla}$	~	~	\overline{v}	$\overline{\mathbf{v}}$

If reject, sample from:

$$x_{n+1} \sim (q(x|x_1,\ldots,x_n) - p(x|x_1,\ldots,x_n))_+$$

where $(f(x))_{+} = \frac{\max(0, f(x))}{\sum_{x} \max(0, f(x))}$

Intuitively, this is like rejection sampling on purple regions

TL;DR - Modified Rejection Sampling

- Worst case: guarantees 1 token is generated
- Best case: can generate *k* tokens at once

- Mathematically proven that tokens are sampled according to target model distribution (Theorem 1 in Appendix)
 - a.k.a No reduction in model quality!

TL;DR - Speculative Sampling

- Key insights:
 - Decode k tokens in 1 step (instead of just 1 token)
 - Use a **smaller model** to speculate the larger model
- Key assumption:
 - Computing logits of *k* tokens in parallel **has similar latency** to sampling 1 token
- No modification to original (target) model
- Effective for small batch sizes
 - Batch size of n requires n*k speculative logits computations
 - \rightarrow OOM for large *n*

Evaluation - Set Up

- Target model: Chinchilla (70b params)
- Draft model: smaller Chinchilla (4b params)
 - Generally, smaller version of target model works well
 - But must be careful in distributed setups

- Eg. Chinchilla-70b used 16 TPU v4s
 - Naively sharding Chichilla-4b across 16 TPUs **increases** latency
 - Chinchilla-4b has lowest latency on 4 TPUs
 - So a "wider" version of Chinchilla was used for draft, allowing it to be sharded across 16 TPUs effectively

Evaluation - Performance & Speed

Table 1 | Chinchilla performance and speed on XSum and HumanEval with naive and speculative sampling at batch size 1 and K = 4. XSum was executed with nucleus parameter p = 0.8, and HumanEval with p = 0.95 and temperature 0.8.

Sampling Method	Benchmark	Result	Mean Token Time	Speed Up
ArS (Nucleus)	XSum (ROUGE-2)	0.112	14.1ms/Token	1×
SpS (Nucleus)		0.114	7.52ms/Token	1.92×
ArS (Greedy)	XSum (ROUGE-2)	0.157	14.1ms/Token	1×
SpS (Greedy)		0.156	7.00ms/Token	2.01×
ArS (Nucleus)	HumanEval (100 Shot)	45.1%	14.1ms/Token	1×
SpS (Nucleus)		47.0%	5.73ms/Token	2.46×

ArS: Autoregressive Sampling ("Normal" Sampling)

SpS: Speculative Sampling

Sampling methods: https://towardsdatascience.com/decoding-strategies-that-you-need-to-know-for-response-generation-ba95ee0faadc

Evaluation - Trade offs with larger *k*

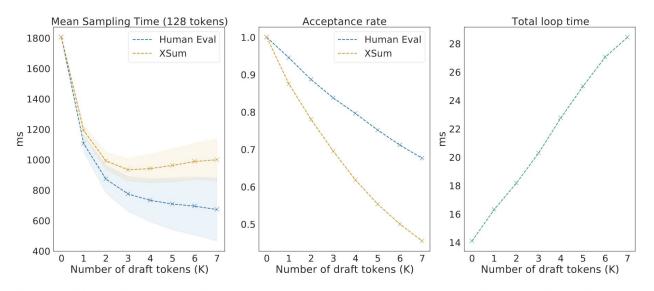


Figure 1 | **Left:** The average time to generate 128 tokens, with standard deviation. Note that as K increases, the overall speedup plateaus or even regresses, with XSum being optimal at K = 3. The variance consistently increases with K. **Middle:** The average number of tokens accepted divided by K + 1 — this serves as a measure of the overall efficiency of the modified rejection scheme, which decreases with the lookahead. **Right:** Average time per loop increases approximately linearly with K due to the increased number of model calls. Note that the gradient is slightly higher than the sampling speed of the draft model, due to additional overheads in nucleus decoding.

Conclusion

Speculative sampling generates multiple tokens in 1 step

- with no reduction in quality
- by speculating on a smaller model
- using a novel **modified rejection sampling** technique
- achieving **2-2.5x decoding speedup** in a distributed setup

Questions?

(hidden) Proof that target distribution is preserved

 $= \min(p(x), q(x))$

For the second conditional term, we apply the resampling rule:

$$\mathbb{P}(X = x | \tilde{x} \ rejected) = (q(x) - p(x))_{+}$$

Where (.) denotes:

$$(f(x))_{+} = \frac{\max(0, f(x))}{\sum_{x} \max(0, f(x))}$$

Finally, we calculate the probability of rejection:

Proofs

Theorem 1 (Modified Rejection Sampling recovers the target distribution). *Given discrete distributions* q, p and a single draft sample $\tilde{x} \sim p$, let X be the final resulting sample. For X = x to be true, we must either sample $\tilde{x} = x$ and then accept it, or resample it after \tilde{x} (of any value) is rejected. Hence:

$$\mathbb{P}(X=x)$$

 $= \mathbb{P}(\tilde{x} = x)\mathbb{P}(\tilde{x} \text{ accepted}|\tilde{x} = x) + \mathbb{P}(\tilde{x} \text{ rejected})\mathbb{P}(X = x|\tilde{x} \text{ rejected})$

For the first term, we apply the acceptance rule:

$$\mathbb{P}(\tilde{x} = x)\mathbb{P}(\tilde{x} \ accepted | \tilde{x} = x)$$

$$= p(x) \min\left(1, \frac{q(x)}{p(x)}\right)$$

 $\mathbb{P}(\tilde{x} \text{ rejected}) = 1 - \mathbb{P}(\tilde{x} \text{ accepted})$

$$=1-\sum_{x'}\mathbb{P}(X=x',\tilde{x} \text{ accepted})$$

$$=1-\sum_{x'}\min(p(x'),q(x'))$$

$$=\sum_{x'}\max(0,q(x')-p(x'))$$

$$= \sum_{x'} q(x') - \min(p(x'), q(x'))$$

$$=\sum_{x'}\max(0,q(x')-p(x'))$$

This is equal to the denominator of $(q(x) - p(x))_+$, so:

$$\mathbb{P}(\tilde{x} \ rejected)\mathbb{P}(X = x | \tilde{x} \ rejected) = \max(0, q(x) - p(x))$$

Hence:

$$\mathbb{P}(X = x)$$

$$= \min(p(x), q(x)) + \max(0, q(x) - p(x))$$

$$=q(x)$$

and we have recovered the desired target.

(hidden) Real world implementations

TGI gh issue:

- https://github.com/huggingface/text-generation-inference/issues/729
- https://github.com/huggingface/text-generation-inference/issues/1169
- Discusses using Medusa (multi-head) instead of separate draft model

vLLM gh issue:

- https://github.com/vllm-project/vllm/pull/2188
- Discusses implementing with draft model