Dependent Randomization in Parallel Binary Decision Fusion

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Weiqiang Dong and Moshe Kam

Department of Electrical and Computer Engineering

New Jersey Institute of Technology

Newark, New Jersey 07102

Email: wd35@njit.edu, kam@njit.edu

Supplement file, Part B

Numerical examples for loss of synchronization between the DFC and the group of LDs (Section IV-B)

In this file, the index of each Table/Figure/Equation has a prefix "B-"; the index of each Table/Figure/Equation that appears in the main paper has a prefix "P-".

SUPPLEMENT FILE, PART B

NUMERICAL EXAMPLES FOR LOSS OF SYNCHRONIZATION BETWEEN THE DFC AND THE GROUP OF LDS (SECTION IV-B)

Part B provides the numerical examples (2-LD and 3-LD systems) when the DFC loses synchronization with the LDs group. This part supplements Section IV-B of the paper.

A. 2-LD system

The design input and output of the 2-LD system (Section III-A of the paper) employing dependent randomization with $\alpha = 0.2009$, was shown in Table P-II of the paper. When the DFC loses synchronization with the LDs group, the 2-LD system performance before and after a corrective action is taken is shown in Table B-I (P-V) and Figure B-1 (P-8).

In Figure B-1a (P-8a), the black curve is the ROC curve of the 2-LD example in Section III-A when using dependent randomization. It comprises (from left to right) a segment that corresponds to the AND fusion rule at the DFC (left of point A); a straight line segment AB which is a common tangent of the ROC curves for the AND and OR fusion rules at the DFC; and a segment (to the right of point B) that corresponds to the OR fusion rule at the DFC. When $\alpha = 0.2009$, the operating point of the system is $C = (P_f^C, P_d^C) = (0.2009, 0.8261)$, shown by the black circle. Point C is generated by operating at $A = (P_f^A, P_d^A) = (0.1581, 0.7870)$ with probability $p = 0.5$ and at $B = (P_f^B, P_d^B) = (0.2437, 0.8652)$ with probability $1-p = 0.5$ (p was calculated using (P-11) in the paper). According to Section III-A of the paper, the system operates at A when both LDs operate at $(P_{f1}^A, P_{d1}^A) = (P_{f2}^A, P_{d2}^A) = (0.3976, 0.8871)$ and the DFC uses the AND fusion rule. The system operates at B when both LDs operate at $(P_{f1}^B, P_{d1}^B) = (P_{f2}^B, P_{d2}^B) = (0.1304, 0.6328)$ and the DFC uses the OR fusion rule. When the synchronization between the LDs group and the DFC is lost, the system may also operates (see Figure B-1 (P-8)) at $M1 = (P_f^{M1}, P_d^{M1}) = (0.6371.9873)$ and $M2 = (P_f^{M2}, P_d^{M2}) = (0.0170, 0.4004)$. The operating point of the non-synchronized system

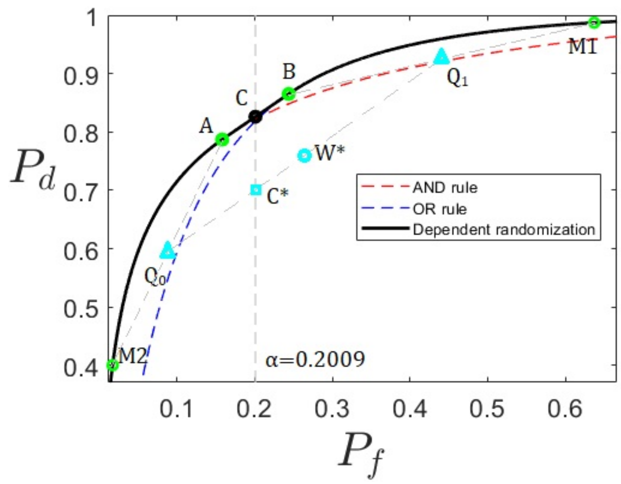
Output of the non-synchronized 2-LD system before the corrective action is taken ($\alpha = 0.2009$)
1. Four possible operating points when the DFC lost synchronization with the LDs groups, $A = (0.1581, 0.7870)$, $B = (0.2437, 0.8652)$, $M1 = (0.6371.9873)$, and $M2 = (0.0170, 0.4004)$.
2. The operating point of the non-synchronized system, $W^* = (0.2640, 0.7600)$, calculated by (P-17) and (P-18).
3. The fact that the probability of the false alarm of the non-synchronized system exceeds the constraint α, $P_f^{W^*} = 0.2640 > 0.2009 = \alpha$
Output of the non-synchronized 2-LD system after the corrective action is taken ($\alpha = 0.2009$)
1. The new probability for the DFC selecting γ_0^A, $q = 0.6787$, calculated by (P-25)
2. The fulfillment of the prerequisite of the correction action, $0 < q < 1$
3. The operating point of the non-synchronized system after the corrective action is taken, $C^* = (0.2009, 0.7005)$, calculated by (P-19) and (P-20).

Table B-I (P-V): The output of the 2-LD system employing dependent randomization when the DFC lost synchronization with the LDs group before and after a corrective action is taken.

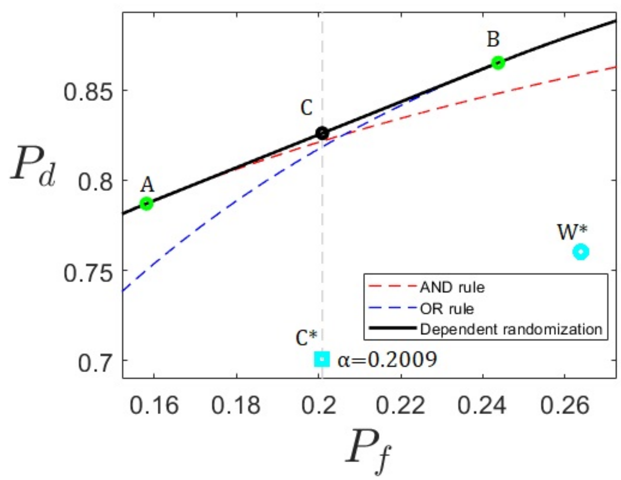
is $W^* = (P_f^{W^*}, P_d^{W^*}) = (0.2640, 0.7600)$, which can be calculated by equations (P-17) and (P-18) in the paper. The probability of false alarm of the non-synchronized system $P_f^{W^*} = 0.2640$ exceeds the $\alpha = 0.2009$ constraint. When the DFC realizes that synchronization was lost, the DFC can change the probability of randomization from p to q to satisfy the constraint on α. In this situation the system is moved to $C^* = (0.2009, 0.7005)$, calculated by (P-19) and (P-20) in the paper. The corresponding $q = 0.6787$ is calculated by (P-25) in the paper. The DFC needs to run a random selection (with probability $q = 0.6787$) between the two global fusion rules (AND and OR) and the LDs run a random selection (with probability $p = 0.5$) between two set of local decision rules, independently of the DFC. Due to the loss of synchronization the probability of detection under the constraint $P_f \leq \alpha = 0.2009$ has been reduced from 0.8261 to 0.7005.

B. 3-LD system

The design input and output of the 3-LD system (Section III-B of the paper) employing dependent randomization with $\alpha = 0.1708$ was shown in Table P-III. When the DFC lost synchronization with the LDs group in the 3-LD system (which employed dependent randomization), the performance



(a) ROC curve of the 2-LD system employing dependent randomization



(b) Zooming in on the ROC curve of the 2-LD system employing dependent randomization

Figure B-1 (P-8): A, B, $M1$ and $M2$, shown by green circles, are the possible operating points of the 2-LD system (Section III-A) when the synchronization between the LDs and the DFC is lost. The black circle, C, shows the operating point of the synchronized system. The magenta circle, W^*, shows the equivalent operating point of the system when it lost synchronization. C^* is the equivalent operating point after the corrective action is taken.

of the system (before and after a corrective action is taken) is shown in Table B-II (P-VI) and Figure B-2 (P-9).

In Figure B-2 (P-9), the black curve is the ROC curve of the 3-LD example in Section III-B when using dependent randomization (this is the curve developed in Figure P-7 of the paper and Figure A-5 of the supplemental file part A). Since $\alpha = 0.1708$, we are looking at the segment corresponding

Output of the non-synchronized 3-LD system before the corrective action is taken ($\alpha = 0.1708$)
1. Four possible operating points when the DFC lost synchronization with the LDs groups, $A = (0.1040, 0.7840)$, $B = (0.2710, 0.9360)$, $M1 = (0.4880, 0.9730)$, and $M2 = (0.0280, 0.6480)$.
2. The operating point of the non-synchronized system, $W^* = (0.2046, 0.8210)$, calculated by (P-17) and (P-18).
3. The fact that the probability of the false alarm of the non-synchronized system exceeds the constraint α, $P_f^{W^*} = 0.2046 > 0.1708 = \alpha$
Output of the non-synchronized 3-LD system after the corrective action is taken ($\alpha = 0.1708$)
1. The new probability for the DFC selecting γ_0^A, $q = 0.7033$, calculated by (P-25)
2. The fulfillment of the prerequisite of the correction action, $0 < q < 1$
3. The operating point of the non-synchronized system after the corrective action is taken, $C^* = (0.1708, 0.7974)$, calculated by (P-19) and (P-20).

Table B-II (P-VI): The output of the 3-LD system employing dependent randomization when the DFC lost synchronization with the LDs group before and after a corrective action is taken.

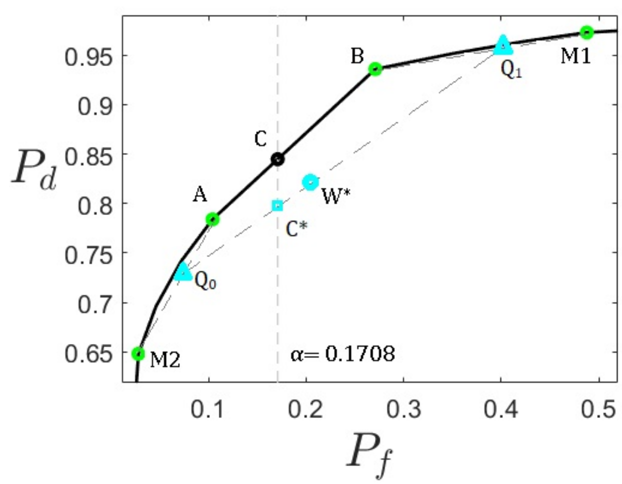


Figure B-2 (P-9): A, B, $M1$ and $M2$, shown by green circles, are the possible operating points of the 3-LD system (Section III-B) when the synchronization between the LDs and the DFC is lost. The black circle, C, shows the operating point of the synchronized system. The cyan circle, W^*, shows the equivalent operating point of the system when it lost synchronization. C^* is the equivalent operating point after the corrective action is taken.

to $0.1040 = P_f^A \leq P_f \leq P_f^B = 0.2710$. When the probability of false alarm $P_f = \alpha = 0.1708$, the operating point of the system is $C = (P_f^C, P_d^C) = (0.1708, 0.8448)$, shown as the black circle. C is generated by operating at $A = (P_f^A, P_d^A) = (0.1040, 0.7840)$ with probability $p = 0.6$ and at $B = (P_f^B, P_d^B) = (0.2710, 0.9360)$ with probability $1 - p = 0.4$, respectively (p was calculated using (P-11) in the paper. A is achieved when all 3 LDs operate at $(0.2, 0.7)$ and the DFC uses a "2 out of 3 rule". B is achieved when all 3 LDs operate at $(0.1, 0.6)$ and the DFC uses a "1 out of 3 rule". When the synchronization between the LDs group and the DFC is lost, the system may also operate at $M1 = (P_f^{M1}, P_d^{M1}) = (0.4880, 0.9730)$ (all 3 LDs operate at $(0.2, 0.7)$ and the DFC uses "1 out of 3 rule") and $M2 = (P_f^{M2}, P_d^{M2}) = (0.0280, 0.6480)$ (all 3 LDs operate at $(0.1, 0.6)$ and the DFC uses "2 out of 3 rule"). The equivalent operating point of the non-synchronized system is $W^* = (P_f^{W^*}, P_d^{W^*}) = (0.2046, 0.8210)$, which can be calculated by (P-17) and (P-18) in the paper. The probability of false alarm constraint $P_f \leq \alpha = 0.1708$ is no longer satisfied. When the DFC realizes that synchronization was lost, the DFC can change the probability of randomization from p to q to satisfy the constraint on α. In this situation $C* = (P_f^{C*}, P_d^{C*}) = (0.1708, 0.7974)$, calculated by (P-19) and (P-20) in the paper. The corresponding probability of randomization at the DFC $q = 0.7033$ can be calculated by (P-25) in the paper. The DFC needs to run a random selection (with probability $q = 0.7033$) between two global fusion rules and the LDs run a random selection (with probability $p = 0.6$) between two set of local decision rules independently. Due to the loss of synchronization, the probability of detection under the constraint $P_f \leq \alpha = 0.1708$ has been reduced from 0.8448 to 0.7974.

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