Dependent Randomization in Parallel Binary Decision Fusion

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Weiqiang Dong and Moshe Kam

Department of Electrical and Computer Engineering

New Jersey Institute of Technology

Newark, New Jersey 07102

Email: wd35@njit.edu, kam@njit.edu

Supplement file, Part A

Design of a 3-LD system example (Section III-B)

In this file, the index of each Table/Figure/Equation has a prefix “A-”; the index of each Table/Figure/Equation that appears in the main paper has a prefix “P-”.

SUPPLEMENT FILE, PART A

DESIGN OF A 3-LD SYSTEM EXAMPLE (SECTION III-B)

We provide design detail of a 3-LD system with discrete local observations. The system employs three approaches (1) deterministic strategy, and (2) randomization at the DFC, and (3) deterministic strategy. This part supplements Section III-B of the paper.

We consider the 3-LD implementation of the structure shown in Figure 1 of the main paper (with $n = 3$). The local observations of the three LDs in the system have identical discrete probability distributions, as shown in Figure A-1 (P-6), where the conditional probabilities $P(y_k|H_i)$ are given for $k = 1, 2, 3$ and $i = 0, 1$. We assume that the local observations are statistically independent,

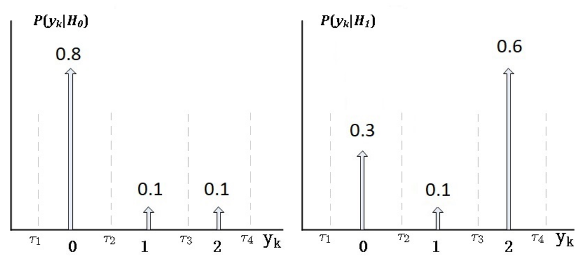


Figure A-1 (P-6): The conditional probability distributions of the local observations.

conditioned on the hypothesis. From Figure A-1 (P-6), each LD has 4 distinct local decision rules, corresponding to 4 distinct local observation thresholds, τ_1 (anywhere in the range $\tau_1 < 0$), τ_2 $(0 < \tau_2 < 1)$, τ_3 $(1 < \tau_3 < 2), \tau_4$ $(\tau_4 > 2)$. At the k^{th} LD, $k = 1, 2, 3$, if τ_1 is used, $P_{fk} = 1, P_{dk} = 1$; if τ_2 is used, $P_{fk} = 0.2, P_{dk} = 0.7$; if τ_3 is used, $P_{fk} = 0.1, P_{dk} = 0.6$; if τ_4 is used, $P_{fk} = 0, P_{dk} = 0$. Each LD has 4 possible local operating points (P_{fk}, P_{dk}), which are $(0, 0)$, $(0.1, 0.6)$, $(0.2, 0.7)$, and $(1, 1)$.

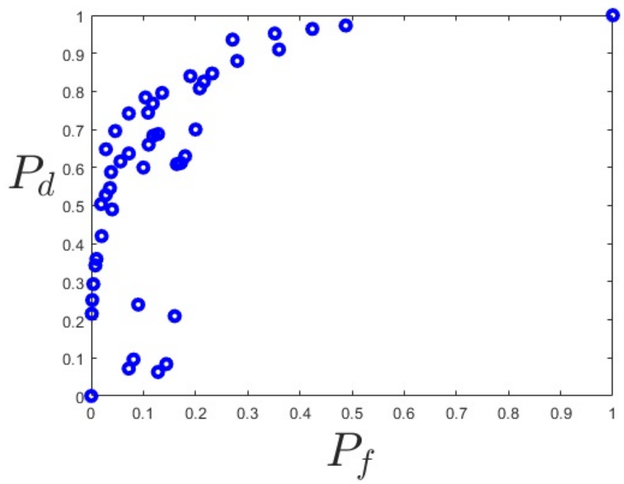
Figure A-6 (P-7) shows the ROC curves of the 3-LD system with three different designs for $0.75 < P_f < 0.3$. Under a Neyman-Pearson criterion with the probability of false alarm constraint $\alpha = 0.1708$, we show the operating points of the 3-LD system employing (1) deterministic strategy leading to $G = (0.1360, 0.7960)$ (blue circle), (2) randomization at the DFC leading to $E = (0.1708, 0.8208)$ (red circle), and (3) dependent randomization leading to $C = (0.1708, 0.8448)$ (black circle).

1) Deterministic strategy (Section II-A): A 3-LD system has 20 monotonic global decision rules [2], which are shown in Table A-I. Since each LD has four (4) possible operating points, there are $4^3 = 64$ combinations of local operating points. Overall, the system has $4^3 \cdot 20 = 1280$ operating points. Since some operating points coincide with others the total number is less than 1280.

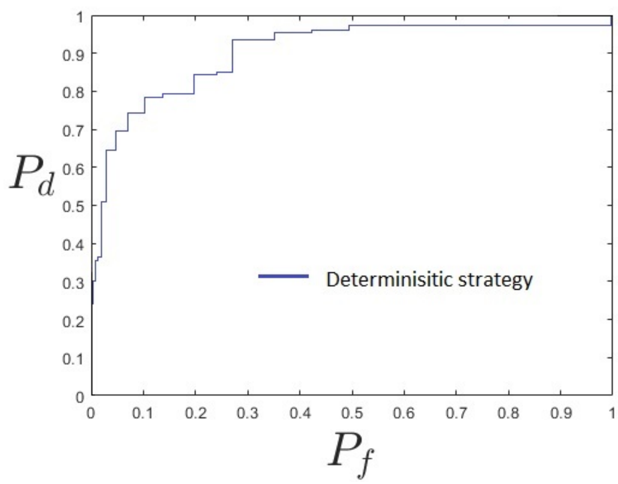
Figure A-2a shows all the distinct operating points of the 3-LD system with deterministic strategy. Since the distribution of the local observations in our example is discrete, the operating points of the system with deterministic strategy are isolated. As a result, in most circumstances the given probability of false alarm constraint α may not be achievable and the system will have to operate at a lower (realizable) rate of probability of false alarm in order not to violate the constraint $P_f \leq \alpha$. The ROC curve of the system has a the staircase form (the blue curve in Figure A-2b). Clearly, this ROC curve is not concave.

2) Strategy with randomization at the DFC only (Section II-B1): Randomization at the DFC allows the system to operate on the line segments connecting the operating points which are generated by same combination of local operating points. For this 3-LD system, there are overall $4^3 = 64$ combinations of local operating points.

When the local operating points are fixed, since there is finite number of monotonic global decision rules, the operating points of the system would be discrete. The authors of [10] point out that under this circumstance (fixed local operating points) the ROC curve of the system of Figure P-1 is a concave piecewise linear curve (i.e., the upper boundary of the convex hull of the discrete operating points is piecewise linear concave). Figure A-3 shows the ROC curve of the 3-LD system when all three LDs operates at $(0.1, 0.6)$ (which is one of the 64 combinations of local operating



(a) All the operating points of the system with deterministic strategy (blue circles).



(b) The team ROC curve of the system with the deterministic strategy.

Figure A-2: The 3-LD system employing deterministic strategy

points). The blue circles are the operating points of the system employing deterministic fusion rules. The red curve is the concave piecewise linear ROC curve of the 3-LD system when the DFC employs randomization.

Figure A-4a shows all the ROC curves of the system when the DFC applies randomization at each one of the 64 combinations of the local operating points. Each ROC curve in Figure A-4a, corresponding to one of the 64 combinations of local operating points, is concave piecewise linear (some ROC curves may coincide with others). In Figure A-4a, all the operating points that were used to generate the 64 ROC curves are shown as blue circles. Figure A-4b shows the team ROC curve of the system with DFC randomization (red piecewise linear curve). It is the upper boundary of all the ROC curves in Figure A-4a. Figure A-4b also shows the team ROC curve of the system with deterministic strategy (blue). We note that neither ROC is concave.

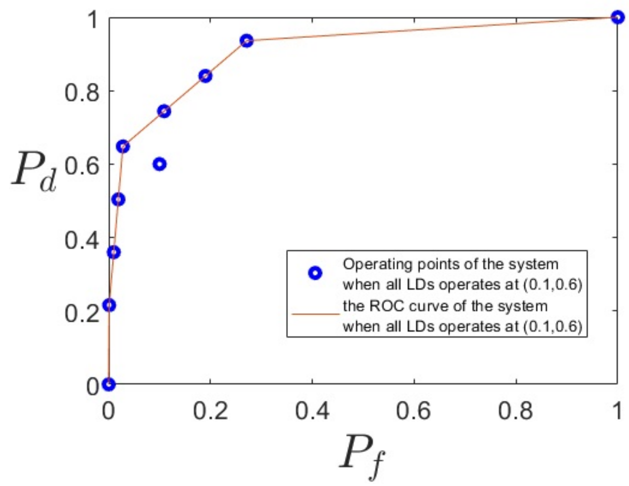
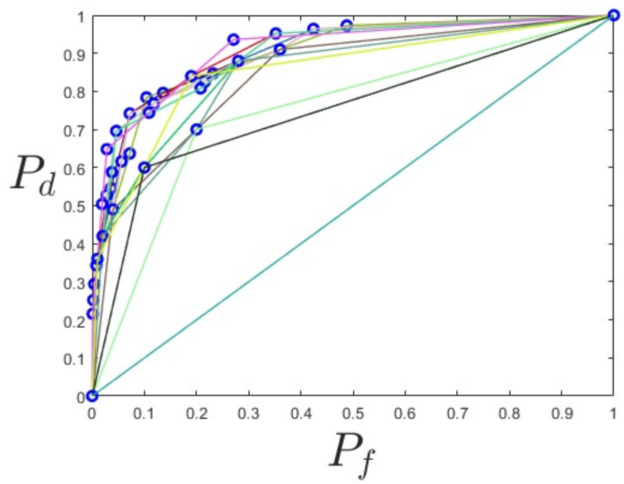


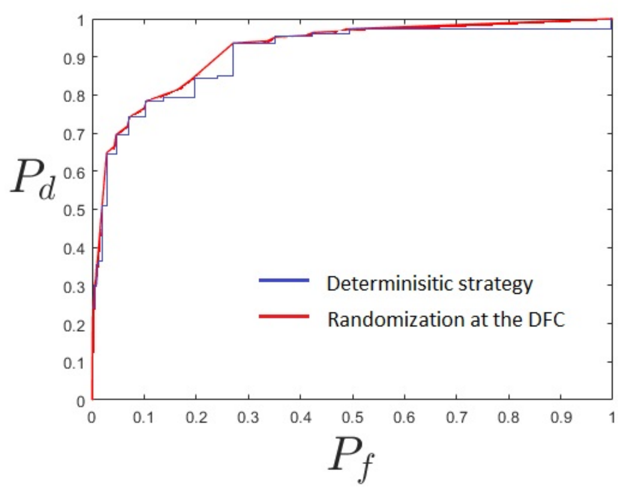
Figure A-3: The operating points (blue circles) and the ROC curve (red curve) of the 3-LD system when all three LDs operates at $(0.1, 0.6)$. When the local operating points are fixed, the ROC curve of the system employing randomization at the DFC is a concave piecewise linear curve.

3) Dependent randomization (Section II-B2): Dependent randomization allows the system to operate on the line segment connecting any two operating points generated by deterministic strategy. Therefore the ROC curve of the system with dependent randomization is the upper boundary of the convex hull of all the operating points in Figure A-2a.

Figure A-5 shows the team ROC curves of the systems with three different detection strategies: (1) deterministic strategy (blue); (2) randomization at the DFC (red); (3) dependent randomization (black). Only the last one is concave. The ROC curve of the strategy with dependent randomization "covers" the ROC curve of the strategy with randomization at the DFC; the ROC curve of the strategy with randomization at the DFC "covers" the ROC curve of the deterministic strategy. As



(a) All the ROC curves of the system when applying the strategies with randomization at the DFC only (each ROC curve connects the operating points if they correspond to the same local operating points).



(b) The team ROC curves of the deterministic strategy (blue) and the strategy with randomization at the DFC only (red)

Figure A-4: The 3-LD system employing randomization at the DFC

expected, dependent randomization performs at least as well as the strategy with randomization at the DFC; the strategy with randomization at the DFC performs at least as well as the deterministic strategy.

Figure A-6 (P-7) shows the operating points of the 3-LD system employing (1) deterministic strategy ($G = (0.1360, 0.7960)$, blue circle); (2) randomization at the DFC ($E = (0.1708, 0.8208)$,

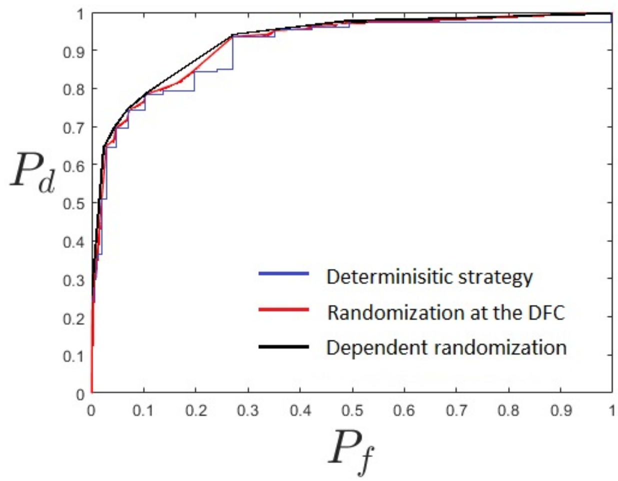


Figure A-5: The team ROC curves of the systems with (1) dependent randomization (black); (2) randomization at the DFC only (red); (3) deterministic strategy (blue).

red circle); (3) deterministic strategy ($C = (0.1708, 0.8448)$, black circle) under a Neyman-Pearson criterion with the probability of false alarm constraint $\alpha = 0.1708$. Table A-I shows the probability of false alarm and the probability of detection of the 3-LD system for the 20 applicable monotonic fusion rules (γ_0^1 to γ_0^{20}).

The operating point of the system employing deterministic strategy, G, can be achieved by using three different deterministic strategies: (1) the LDs operate at $\{(0.1, 0.6), (0.2, 0.7), (0.2, 0.7)\}$ while the DFC uses the fusion rule γ_0^{13}; (2) the LDs operate at $\{(0.2, 0.7), (0.1, 0.6), (0.2, 0.7)\}$ while the DFC uses the fusion rule γ_0^{14}; (3) the LDs operate at $\{(0.2, 0.7), (0.2, 0.7), (0.1, 0.6)\}$ while the DFC uses the fusion rule γ_0^{15}.

The operating point of the system employing randomization at the DFC, E, can be achieved when the system operates at point $(0.1180, 0.768)$ with probability 0.2667 and operating at point $(0.1900, 0.8400)$ with probability 0.7333. These two points used for achieving E are generated when the LDs operate at $(0.2, 0.7), (0.2, 0.7), (0.1, 0.6)$. The DFC can use the fusion rule γ_0^{14} to achieve point $(0.1180, 0.7680)$ and use the fusion rule γ_0^{16} to achieve point $(0.1900, 0.8400)$.

The operating point of the system employing dependent randomization $C = (0.1708, 0.8448)$

is achieved when the system operates at $A = (0.104, 0.784)$ with probability $p = 0.6$ and at $B = (0.271, 0.936)$ with probability $1 - p = 0.4$, respectively. A is achieved when all 3 LDs operate at $(0.2, 0.7)$, i.e. $(P_{fk}, P_{dk}) = (0.2, 0.7), k = 1, 2, 3$, and the DFC uses a “2 out of 3 rule” (γ_0^{12}) (namely, if any two LDs or more decide 1, then $u_0 = 1$; otherwise $u_0 = 0$). B is achieved when all 3 LDs operate at $(0.1, 0.6)$, i.e. $(P_{fk}, P_{dk}) = (0.1, 0.6), k = 1, 2, 3$, and the DFC uses a “1 out of 3 rule” (γ_0^{19}) (namely, if any one LD or more decides 1, then $u_0 = 1$; otherwise $u_0 = 0$.

$\alpha = 0.1708$	γ_0^1	γ_0^2	γ_0^3	γ_0^4	γ_0^5	γ_0^6	γ_0^7	γ_0^8	γ_0^9	γ_0^{10}	γ_0^{11}	γ_0^{12}	γ_0^{13}	γ_0^{14}	γ_0^{15}	γ_0^{16}	γ_0^{17}	γ_0^{18}	γ_0^{19}	γ_0^{20}
(u_1, u_2, u_3)	u_0																			
(0,0,0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
(0,0,1)	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	1	1	1
(0,1,0)	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0	1	1
(1,0,0)	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1	1
(0,1,1)	0	0	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1
(1,0,1)	0	0	0	1	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1
(1,1,0)	0	0	1	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1
(1,1,1)	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
P_f	0	0.1										0.104	**0.136**			0.1				1
P_d	0	0.6										0.784	**0.796**			0.6				1

Table A-I: All twenty monotonic fusion rules of the 3-LD system in Section and the corresponding P_f, P_d when $\alpha = 0.1708$.

Under the Neyman-Pearson criterion with $\alpha = 0.1708$, the input and output of three different designs of the 3-LD system (corresponding to Table P-I) are shown in Table A-II (P-III).

Input for the design	
	1. The number of local detectors, $n = 3$ 2. The probability of false alarm constraint, $\alpha = 0.1708$ 3. Conditional probability distributions of the local observations, $P(y_k \mid H_0)$ and $P(y_k \mid H_1)$, $k = 1, 2, 3$, shown in Figure A-1 (P-6)
Output of a design	
Deterministic strategy	1. System operating point $\boldsymbol{G = (P_f^G, P_d^G) = (0.1360, 0.7960)}$ 2. One way to achieve G is that the LDs operate at $\{(0.1, 0.6), (0.2, 0.7), (0.2, 0.7)\}$ while the DFC uses the fusion rule γ_0^{13} in Table A-I
Randomization at the DFC	1. Two operating points $A = (P_f^A, P_d^A) = (0.1180, 0.7680)$ and $B = (P_f^B, P_d^B) = (0.1900, 0.8400)$ 2. When the LDs operate at $(0.2, 0.7), (0.2, 0.7), (0.1, 0.6)$, the DFC can use the fusion rule γ_0^{14} (Table A-I) to achieve point $A = (0.1180, 0.7680)$ and use the fusion rule γ_0^{16} (Table A-I) to achieve point $B = (0.1900, 0.8400)$. 3. The probability of selecting A is $p = 0.2667$ 4. The resulting operating point is $\boldsymbol{E = (P_f^E, P_d^E) = (0.1708, 0.8208)}$
Dependent randomization	1. Two operating points $A = (P_f^A, P_d^A) = (0.104, 0.784)$ and $B = (P_f^B, P_d^B) = (0.271, 0.936)$ 2. A is achieved when $(P_{fk}, P_{dk}) = (0.2, 0.7), k = 1, 2, 3,$ and the DFC uses a "2 out of 3 rule" (γ_0^{12}) B is achieved when $(P_{fk}, P_{dk}) = (0.1, 0.6), k = 1, 2, 3,$ and the DFC uses a "1 out of 3 rule" (γ_0^{19}) 3. The probability of selecting A is $p = 0.6$ 4. The resulting operating point is $\boldsymbol{C = (P_f^C, P_d^C) = (0.1708, 0.8448)}$

Table A-II (P-III): Input and output of three different designs of a 3-LD system.

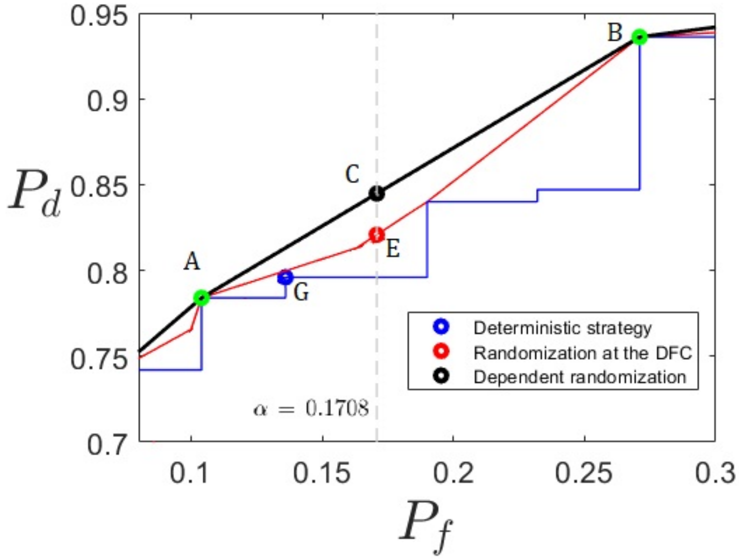


Figure A-6 (P-7): The operating points of the 3-LD system employing different detection strategies under the Neyman-Pearson criterion with $\alpha = 0.1708$: (1) deterministic strategy (blue circle); (2)randomization at the DFC (red circle); (3) dependent randomization (black circle). The ROC curves of the 3-LD system employing different detection strategies: (1) deterministic strategy (blue); (2) randomization at the DFC (red); (3) dependent randomization (black).

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