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Parallel Decision Fusion with Local Constraints

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***Abstract*—Motivated by an example by Tenney and Sandell (1981), we discuss the trade-off between performance of local detectors (LDs) and the combined LD/Data Fusion Center system in parallel decision fusion architectures. In these architectures the LDs make observations, translate these observations to local decisions, and send these local decisions forward to a Data Fusion Center (DFC). The DFC uses the local decisions to synthesize a global decision (in our context both local and global decisions are binary and pertain to binary hypothesis testing based on the LD observations; in other words, both LDs and DFC decide whether to accept or reject a hypothesis). The original example demonstrated how the minimization of a global performance index by the combined system may yield an alignment of the local detectors that avoids a high value of the performance index, but otherwise have no value at the LD level (the LDs are directed to make constant decisions that are almost independent of the observations, in order to avoid a local-decision combination that would incur a high penalty). If we require that the global performance index be minimized while the LDs are also allowed to minimize a local performance index (or have constraints on their error probabilities), a trade-off emerges between the local and global performances. In this paper we provide an example similar in nature to the Tenney-Sandell example, and proceed to analyze the impact of performance constraints on the LDs on the design and performance of the parallel decision fusion architecture. If we provide reasonable constraints on the performance of the LDs, a compromise can be established between the global performance index and the local LD performances.**

***Index Terms*—Decision Fusion, Data Fusion, Bayesian Risk, Neyman-Pearson Criterion**

Nomenclature

α	The upper bound constraint on the global probability of false alarm, $P_f \leq \alpha$
β	The common upper bound constraint on the local probabilities of false alarm at all identical local detectors, $P_{fi} = pf \leq \beta, i = 1, \ldots, n$
β_i	The upper bound constraint on the local probability of false alarm at the i^{th} local detector, $P_{fi} \leq \beta_i$
η_0	The threshold in the global decision rule
η_i	The threshold in the local decision rule at the i^{th} local detector
n	Number of local detectors in the parallel fusion architecture
P_d	Probability of detection of the DFC, $P(u_0 = 1 \mid H_1)$
$P_d(k, pd)$	Probability of detection of the DFC when the DFC uses a "k out of n" rule and $P_{di} = pd, i = 1, \ldots, n$
P_f	Probability of false alarm of the DFC, $P(u_0 = 1 \mid H_0)$
$P_f(k, pf)$	Probability of false alarm of the DFC when the DFC uses a "k out of n" rule and $P_{fi} = pf, i = 1, \ldots, n$
P_0	A priori probability of hypothesis H_0
P_1	A priori probability of hypothesis H_1
P_{di}	Probability of detection of the i^{th} local detector, $P(u_i = 1 \mid H_1)$
P_{fi}	Probability of false alarm of the i^{th} local detector, $P(u_i = 1 \mid H_0)$
u_0	Global decision by the data fusion center ($u_0 = 1$ or 0)
u_i	Local decision by the i^{th} detector ($u_i = 1$ or 0)
y_i	Local observations of the i^{th} detector

I. A Motivating Example

In [1], Tenney and Sandell have designed a binary detection system (shown in Fig. 1) consisting of two local detectors ($LD1$ and $LD2$) with respective local observations $y_1, y_2 \in \mathbb{R}$, and binary local decisions $u_1, u_2 \in \{0, 1\}$. $u_i = 1$ when LDi accepts hypothesis H_1 and $u_i = 0$ when LDi accepts hypothesis H_0. The (global) objective function that their system minimized was:

$$J = \sum_{u_1=0}^{1} \sum_{u_2=0}^{1} \sum_{H_j=0}^{1} J_{u_1 u_2 H_j} P(u_1, u_2, H_j), \tag{1}$$

where $j = 0, 1$ and $J_{u_1 u_2 H_j}$ is the cost for $LD1$ deciding u_1 and $LD2$ deciding u_2 when H_j is true. The cost for both LDs making correct decisions is 0, $J_{000} = J_{111} = 0$; The cost for only one LD making correct decision is 1, $J_{010} = J_{100} = J_{011} = J_{101} = 1$; The cost for both LDs making wrong decisions is k, $J_{110} = J_{001} = k, k > 1$.

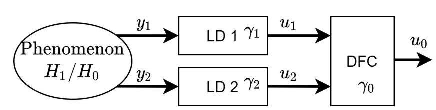

Fig. 1: Parallel decentralized detection architecture with two LDs.

Before we present the design of this system, let us preview the result. For a high value of k ($k =$ the cost of a double error) the local detector rules "arrange themselves" so that one of the LDs declares almost always u=0 and the other declares almost always u=1. This arrangement avoids the highest penalty (k) but renders the system otherwise useless when it comes to making a decision about the hypothesis.

Details of the Design. It is assumed the local observations $y_i, i \in \{1, 2\}$ are statistically independent, conditioned on the hypothesis, $P(y_1, y_2|H_j) = P(y_1|H_j)P(y_2|H_j), j \in \{0, 1\}$. When $P(y_i|H_0) \sim \mathcal{N}(0, \sigma^2)$, $P(y_i|H_1) \sim \mathcal{N}(m, \sigma^2), i \in \{1, 2\}$, the local decisions u_i can be expressed as threshold tests of the local observations $y_i, i \in \{1, 2\}$ [1, 2],

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$$u_i = \begin{cases} 0 & y_i < \tau_i \\ 1 & y_i \geq \tau_i, \end{cases} \quad (2)$$

where

$$\tau_i = \frac{\sigma^2}{m} \ln \eta_i + \frac{m}{2}, \quad (3)$$

$$\eta_1 = \frac{P(H_0)\{J_{110} - J_{010} + (1 - P_{f2})[J_{100} - J_{000} + J_{010} - J_{110}]\}}{P(H_1)\{J_{011} - J_{111} + (1 - P_{d2})[J_{001} - J_{101} + J_{111} - J_{011}]\}}, \quad (4)$$

$$\eta_2 = \frac{P(H_0)\{J_{110} - J_{010} + (1 - P_{f1})[J_{100} - J_{000} + J_{010} - J_{110}]\}}{P(H_1)\{J_{011} - J_{111} + (1 - P_{d1})[J_{001} - J_{101} + J_{111} - J_{011}]\}}. \quad (5)$$

$P_{fi} = P(u_i = 1|H_0)$ is the probability of false alarm of the i^{th} LD and $P_{di} = P(u_i = 1|H_1)$ is the probability of detection of the i^{th} LD. Both P_{fi} and P_{di} are determined by η_i. Therefore, η_1 in (4) and η_2 in (5) are coupled. When $m = 2.5, \sigma = 2$, we show the relations between (a) the optimal thresholds τ_i in (2) and the cost for both LDs making mistakes k; (b) J in (1) and k; (c) P_{fi} and k; and (d) P_{di} and k in Figs. 2a, 2b, 2c, and 2d, respectively (the first row of Fig. 2). In Fig. 2a, when $k \leq 5.7$, both optimal thresholds are $m/2 = 1.25$. When $k > 5.7$, one threshold (τ_1) is an increasing function of k while the other (τ_2) is a decreasing function of k. When $k >> 5.7$, $LD1$ almost always decide $u_1 = 0$ and $LD2$ almost always decide $u_2 = 1$ regardless of input observations. The reason for this counter intuitive behavior is that when the cost for both LDs making mistakes is very large, it makes sense to avoid it by preventing both LDs from coming to the same decision regardless of their observations. Such outcome is clearly counterproductive.

Adding local constraints. The reason for the counter-intuitive of the system we just analyzed is that we did not take into account any constraints on the LDs, that would allow them to develop their own local decisions about the observed data. Suppose we introduced upper bound constraints on the local false alarm probabilities $P_{fi}(P_{fi} \leq \beta)$, for some β. Continuing with our example, let the constraints $P_{fi} \leq \beta = 0.3$ be imposed while the cost in (1) is still minimized.

In Fig. 2 we demonstrate the impact of the constraints on the local detectors. The upper row (2a-2d) shows the values of key parameters versus k the cost of a double error. In Fig. 2b we see the global cost which never exceeds 1 (since k, the cost of a double error, is never exacted). The cost of this achievement is apparent in the other three graphs. In Fig. 2a we show the local thresholds to which the observation of the LD is compared. Apparently one threshold goes down as the double-error penalty k grows, imposing a "target present" decision on the respective LD. The other threshold grows with k, imposing a "target absent" decision on the other LD. The respective error performance graphs (Figs. 2c and 2d) show how one LD reaches $P_f = P_d = 1$ and the other $P_f = P_d = 0$, apparently providing no meaningful information on their original observations. Once the local constraints are introduced (in our case, $P_{fi} \leq 0.3$) the extreme behaviors of the thresholds and the performance probabilities are replaced by more meaningful local performance probabilities (see Figs. 2e, 2g, and 2h). However, the global cost is no longer limited to 1; occasionally, double errors occur and exact a higher penalty.

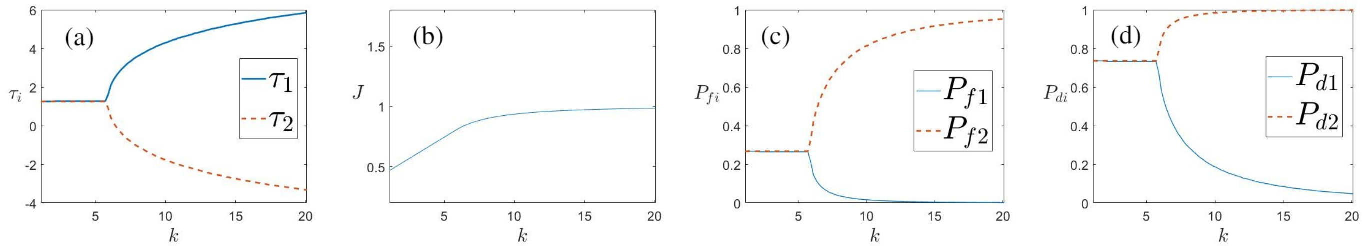
Organization of the paper. In Section II, we define the setting for the design of a parallel decentralized binary detection system. In Section III, we introduce the constraints on local probabilities of false alarm and consider the ramifications on design and performance. Section IV provides an example to illustrate how the constraints on local probabilities of false alarm affect the system performance.

II. Parallel Decentralized Binary Decision Fusion

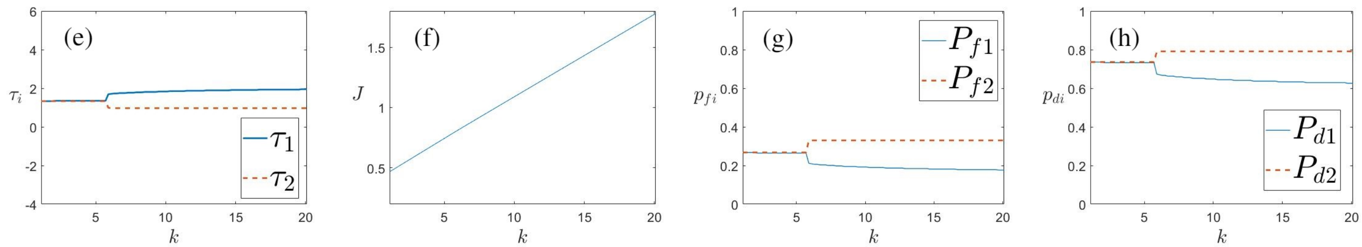
The parallel decentralized binary decision fusion architecture is shown in Fig. 3. The system uses a bank of n local detectors (LDs) to observe a phenomenon and accept a binary hypothesis (H_0 or H_1). P_0 ($P_1 = 1 - P_0$) is the a priori probability of $H_0(H_1)$. y_i is the local observations collected by LD i. In this paper, all the local observations are assumed to be statistically independent, conditioned on the hypothesis, i.e., $Prob.(y_1, \ldots, y_n|H_j) = \prod_i Prob.(y_i|H_j), i \in \{1, \ldots, n\}, j \in \{0, 1\}$. The i^{th} LD uses the local decision rule $u_i = \gamma_i(y_i)$ to compress its local observations y_i into a local decision u_i. Here $u_i = 0$ means that the i^{th} LD prefers hypothesis H_0, and $u_i = 1$ means that the i^{th} LD prefers hypothesis H_1. $U = \{u_1, \ldots, u_n\}$ is the set of all local decisions. A Data Fusion Center (DFC) uses the global decision rule $u_0 = \gamma_0(U)$ to integrate all local decisions and generate a global decision u_0, where $u_0 = 0$ indicates preference for hypothesis H_0, and $u_0 = 1$ indicates preference for hypothesis H_1. The probability of false alarm of the DFC is $P_f = Prob.(u_0 = 1|H_0)$. The probability of detection of the DFC is $P_d = Prob.(u_0 = 1|H_1)$. The tuple (P_f, P_d) is considered the *operating point* of the detection system (or the DFC). Similarly, the local operating point of the LD i is (P_{fi}, P_{di}), where $P_{fi} = Prob.(u_i = 1|H_0)$ and $P_{di} = Prob.(u_i = 1|H_1)$.

The design of the system includes determining n local decision rules $\gamma_i(.), i = 1, \ldots, n$ and one global decision rule $\gamma_0(.)$ ([3][4]). The combination of the global decision rule and all the local decision rules is called the *detection strategy*, $\gamma = \{\gamma_0, \ldots, \gamma_n\}$. We consider two performance criteria for designing the system in Fig. 3: (a) satisfying a Neyman-Pearson criterion [4–10] and (b) minimizing the Bayesian cost of the global decision u_0 [11–13].

When satisfying a Neyman-Pearson criterion, the system maximizes the probability of detection P_d while keeping the global probability of false alarm P_f no larger than a specified value α ($0 < \alpha < 1$). Hoballah and Varshney used a person-by-person optimization (PBPO) procedure to carry out the design of the system detection strategy ([5]). Acharya *et al.* calculated the optimal detection strategy of the architecture of Fig. 3 with identical LDs. Randomized detection strategy allows the system hopping between two operating points in order to achieve a higher probability of detection, which was studied in [4, 7–10].



(a) τ_i $vs.$ k. (b) J $vs.$ k. (c) P_{fi} $vs.$ k. (d) P_{di} $vs.$ k.



(e) τ_i $vs.$ k with local constraint $P_{fi} \leq 0.3$. (f) J $vs.$ k with local constraint $P_{fi} \leq 0.3$. (g) P_{fi} $vs.$ k with local constraint $P_{fi} \leq 0.3$. (h) P_{di} $vs.$ k with local constraint $P_{fi} \leq 0.3$.

Fig. 2: The effect of local constraints on the 2-LD example.

The authors of [11] developed the detection strategy aiming at minimizing a Bayesian cost of the global decision

$$\begin{aligned} J &= C_{00}P(u_0 = 0|H_0)P_0 + C_{01}P(u_0 = 0|H_1)P_1 \\ &+ C_{10}P(u_0 = 1|H_0)P_0 + C_{11}P(u_0 = 1|H_1)P_1 \\ &= P_0(C_{10} - C_{00})P_f - P_1(C_{01} - C_{11})P_d + P_1C_{01} + P_0C_{00}, \end{aligned} \tag{6}$$

where C_{ij} is the cost for the DFC to accept H_i ($u_0 = i$) when the true hypothesis is H_j and P_i is the a priori probability of H_i, $i \in \{0, 1\}, j \in \{0, 1\}$. Chair and Varshney presented the optimal global decision rule at the DFC while the performance of LDs is fixed ([12]).

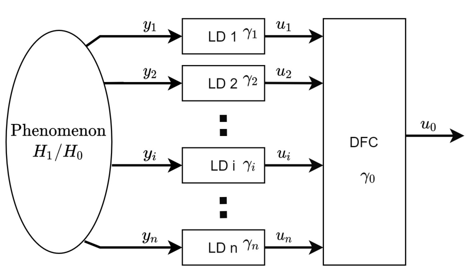


Fig. 3: Parallel decentralized decision fusion architecture.

When the local observations are statistically independent, conditioned on the hypothesis, both the local decision rules and the global decision rule are likelihood ratio tests of the form [4]:

$$u_i = \gamma_k(y_i) = \begin{cases} 0 & \Lambda(y_i) < \eta_i \\ 1 & \Lambda(y_i) \geq \eta_i, \end{cases} \tag{7}$$

$$u_0 = \gamma_0(U) = \begin{cases} 0 & \Lambda(U) < \eta_0 \\ 1 & \Lambda(U) \geq \eta_0, \end{cases} \tag{8}$$

where

$$\Lambda(x) = \frac{P(x|H_1)}{P(x|H_0)}. \tag{9}$$

η_0 is the global threshold in the global decision rule at the DFC and η_i is the local threshold in the local decision rule at the i^{th} LD. The global threshold η_0 and the local thresholds $\eta_i, i = 1, \ldots, n$ fully describe a detection strategy (global and local decision rules) and determine the trade-off between the probabilities of false alarm and the probabilities of detection at the DFC and the LDs.

The probability of false alarm and the probability of detection at the DFC are [14]:

$$P_f = Pr(u_0 = 1|H_0) = \sum_{\Lambda(U) \geq \eta_0} P(\Lambda(U)|H_0), \tag{10}$$

$$P_d = Pr(u_0 = 1|H_1) = \sum_{\Lambda(U) \geq \eta_0} P(\Lambda(U)|H_1). \tag{11}$$

The probability of false alarm and the probability of detection at the i^{th} LD are ([2] [15, pp. 567–568]):

$$P_{fi} = \Pr(u_i = 1|H_0) = \int_{\Lambda(y_i) \geq \eta_i} P(\Lambda(y_i)|H_0) d\Lambda(y_i), \tag{12}$$

$$P_{di} = \Pr(u_i = 1|H_0) = \int_{\Lambda(y_i) \geq \eta_i} P(\Lambda(y_i)|H_1) d\Lambda(y_i). \tag{13}$$

A. Identical Local Detectors

In the rest of the paper, we assume that the LDs in the system are identical, $(P_{fi}, P_{di}) = (p_f, p_d), i = 1, \ldots, n$. In this circumstance, the global decision rule is a *"k out of n"* rule ([4][14][15]), namely, if k or more LDs in the system decide '1', $u_0 = 1$; otherwise, $u_0 = 0$. There are totally $k + 2$

"k out of n" rules for $k = 0, \ldots, n+1$. When $k = 0$, the DFC always decides $u_0 = 1$ and the system operates at $(P_f, P_d) = (1, 1)$; when $k = n+1$, the DFC always decides $u_0 = 0$ and the system operates at $(P_f, P_d) = (0, 0)$. When $k = \{1, \ldots, n\}$, the probability of false alarm and the probability of detection of the system when using the "k out of n" rule are denoted as $P_f(k, p_f)$ and $P_d(k, p_d)$, respectively, where

$$P_f(k, p_f) = \sum_{k}^{n} \binom{n}{k} p_f^k (1 - p_f)^{(n-k)}, \text{and} \tag{14}$$

$$P_d(k, p_d) = \sum_{k}^{n} \binom{n}{k} p_d^k (1 - p_d)^{(n-k)}. \tag{15}$$

B. Upper bound constraints on local probabilities of false alarm

We are interested in minimizing a global performance index such as (6) under additional constraints on the LDs. Specifically, we impose upper bound constraints on the local probabilities of false alarm.

According to (12), the local probability of false alarm at the i^{th} LD P_{fi} is a function of the local threshold η_i in (7). We can convert the upper bound constraints on the local probabilities of false alarm to lower bound constraints on the local likelihood ratios.

Let the upper bound constraint on the local probability of false alarm at the i^{th} LD be $P_{fi} \leq \beta_i$. The lower bound constraint on the local likelihood ratio $\Lambda(y_i)$ is t_i; t_i satisfies

$$\beta_i = \int_{\Lambda(y_i) \geq t_i} P(\Lambda(y_i)|H_0) d\Lambda(y_i). \tag{16}$$

We shall assume $\beta_i = \beta, i = 1, \ldots, n$.

III. Parallel Decentralized Decision Fusion under Local Constraints

We now present the design of a parallel decentralized decision fusion system with n identical constrained LDs under (I) a global Neyman-Pearson criterion and (II) a global Bayesian criterion.

A. Neyman-Pearson criterion with local constraints

In this setting, the global probability of false alarm P_f of the system is kept no larger than a constant $\alpha, \alpha \in (0, 1)$ and the local probability of false alarm of the system $P_{fi} = p_f, i = 1, \ldots, n$ is kept no larger than a constant $\beta, \beta \in (0, 1)$. The objective is to maximize the P_d.

Recall that the global decision rule of a system with n identical LDs is a "k out of n" rule. Since $\alpha \in (0, 1)$, the range of k is $k \in \{1, \ldots, n\}$ $(k \neq 0, n+1)$. The probability of false alarm of the system can be calculated by (14). Ideally, it is desired to have

$$P_f(k, p_f) = \sum_{k}^{n} \binom{n}{k} p_f^k (1 - p_f)^{(n-k)} = \alpha, \tag{17}$$

where p_f is the local probability of false alarm.

In practice, since there are $n+2$ "k out of n" rules, for a given value of p_f, P_f has at most $n+2$ different values and $P_f = \alpha$ may not be achievable, which means (17) may not hold and P_f would have to be strictly smaller than α. To find the optimal detection strategy, we iterate over all the feasible "k out of n" rules, $k \in \{1, \ldots, n\}$ $(k \neq 0, n+1)$. For each "k out of n" rule, we first solve $P_f = \alpha$ for the local probabilities of false alarm $p_f = p_f^*$ by using (17). The local thresholds η_i is the greatest valid local threshold which provides $p_f \leq \beta$ and $p_f \leq p_f^*$ simultaneously ($p_f \leq min(\beta, p_f^*)$), where $p_f = \int_{\eta_i} P(\Lambda(y_i)|H_0) d\Lambda(y_i)$, calculated from (12). The first inequality constraint on p_f is the constraint on the local probability of false alarm. The second inequality constraint on p_f comes from the constraint on the global probability of false alarm $P_f = \sum_k^n \binom{n}{k} p_f^k (1 - p_f)^{(n-k)} \leq \alpha$. After calculating η_i, the local probabilities of false alarm p_f can be obtained from (12). The local probabilities of detection $P_{di} = p_d$ can be obtained by using (13). The global probability of detection P_d then can be calculated by using (15). The optimal global decision rule is the "k out of n" rule providing the highest probability of detection P_d. The corresponding local decision rules are determined by the corresponding local thresholds.

B. Bayesian criterion with local constraints

When the system is designed under a global Bayesian criterion, the cost (to be minimized) is given in (6). The Bayesian detection strategy is always deterministic [16]. A PBPO procedure can be used to carry out the deterministic detection strategy of the system [11][13]. For a system with n identical LDs, the global decision rule is a "k out of n" rule and the local thresholds $\eta_i, i = 1, \ldots, n$ are

$$\eta_i = \frac{P_0(C_{10} - C_{00}) \sum_{\mathbf{u^i}} A(\mathbf{u^i}) \prod_{j=1, j \neq i}^{n} P(u_j|H_0)}{P_1(C_{01} - C_{11}) \sum_{\mathbf{u^i}} A(\mathbf{u^i}) \prod_{j=1, j \neq i}^{n} P(u_j|H_1)}, \tag{18}$$

where

$$\mathbf{u^i} = \{u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n\},$$
$$A(\mathbf{u^i}) = P(u_0 = 1|\mathbf{u^{i1}}) - P(u_0 = 1|\mathbf{u^{i0}}),$$
$$\mathbf{u^{ij}} = \{u_1, \ldots, u_{i-1}, u_i = j, u_{i+1}, \ldots, u_n\}, j = 0, 1.$$

According to (16), an upper bound constraint on P_{fi} can be considered as a lower bound constraint on $\Lambda(y_i)$, $\Lambda(y_i) \geq t_i$. To determine the optimal Bayesian detection strategy, we can iterate over all the feasible "k out of n" rules, $k \in \{1, \ldots, n\}$ $(k \neq 0, n+1)$. For each "k out of n" rule, we can calculate the local thresholds η_i from (18). The local constraint $p_f \leq \beta$ requires $\eta_i = t_i$ if $\eta_i > t_i$. The calculated η_i can be used to find P_f and P_d from (14) and (15), respectively. The Bayesian cost for the "k out of n" rule can be calculated by using (6). We will use the detection strategy, which comprises the "k' out of n" rule and the local decision rules with local thresholds η_i', providing the minimized Bayesian cost in (6).

IV. Example

Let the local observations y_i be identical logistic random variables (as done in [9]). The conditional probability distributions of the local observations are:

$$
\begin{aligned}
P(y_i|H_0) &= \frac{1}{4} sech^2(\frac{y_i}{2}), \\
P(y_i|H_1) &= \frac{1}{4} sech^2(\frac{y_i - 2.5}{2})
\end{aligned}
\tag{19}
$$

which are shown in Fig. 4. Fig. 5 shows the receiver operating characteristic (ROC) curves [2] of a LD in the system without local constraints.

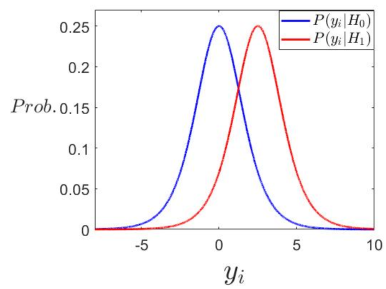


Fig. 4: Conditional probability distributions of the local observations.

We consider a system with five identical LDs. The constraints on the local probabilities of false alarm of the LD are $P_{fi} = p_f \le \beta = 0.04, i = 1, \ldots, n$. The ROC curve of a LD in the system without local constraints is shown in Fig. 6. The curve in Fig. 6 is a truncation of the curve in Fig. 5 for $P_{fi} \in [0, \beta = 0.04]$. When $p_f \le 0.04$, the highest p_d is achieved by the operating point $C = (0.04, 0.3348)$, shown as the red circle in Fig. 6. When the LD operates at point C, $\eta_i = 5.8468$.

A. Neyman-Pearson criterion

Under a Neyman-Pearson criterion, assuming the constraint on the global probability of false alarm is $P_f \le \alpha = 0.1$, the ROC curves of all five "k out of n" rules without constraints on local probabilities of false alarm are shown in Fig. 7. The operating point satisfying $P_f \le \alpha$ and maximizing the P_d is $(P_f, P_d) = (0.1, 0.9415)$, show as the black circle in Fig. 7, which is achieved when the DFC uses the "3 out of 5" rule and all five LDs operate at $(P_{fi}, P_{di}) = (p_f, p_d) = (0.2466, 0.7993)$.

When the local probabilities of false alarm $p_f \le \beta = 0.04$, the ROC curves of all five "k out of n" rules are shown in Fig. 8. Each curve in Fig. 8 is a truncation of the corresponding curve (of same color) in Fig. 7. In this example, the two endpoints of the ROC curve for a "k out of n" rule are $(0, 0)$ and $(P_f(k, \beta), P_d(k, \beta))$. The operating point satisfying $P_f \le \alpha, p_f \le \beta$ and maximizing the P_d is $(P_f, P_d) = (0.1, 0.6792)$, show as the black circle in Fig. 8, which is achieved when the DFC uses the "1 out of 5" rule and all five LDs operate at $(0.0209, 0.2034)$.

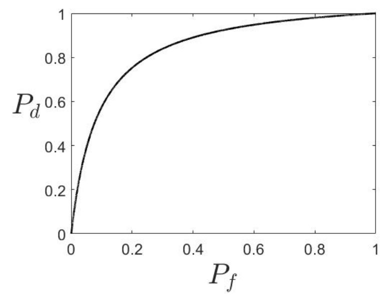


Fig. 5: ROC curve of the LD.

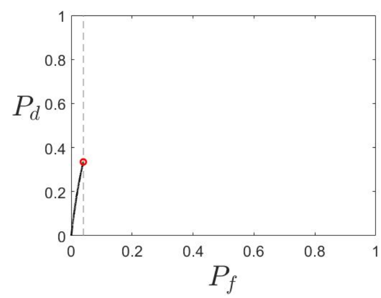


Fig. 6: ROC curve of the LD with $p_f \le 0.04$.

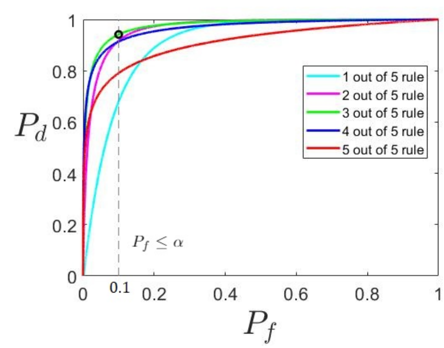


Fig. 7: ROC curves of five "k out of n" rules.

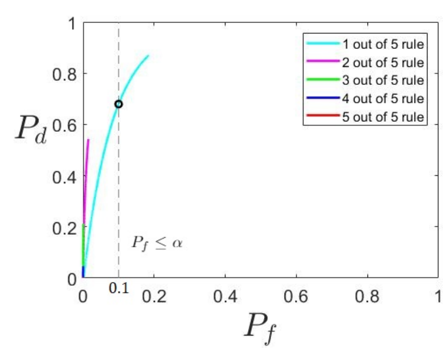


Fig. 8: ROC curves of five "k out of n" rules with $p_f \le 0.04$.

When $\alpha = 0.1$, the global probability of detection reduce from $P_d = 0.9415$ to $P_d = 0.6792$ because of the introduction of the constraints on the local probabilities of false alarm $P_{fi} = p_f \le \beta = 0.04$.

B. Bayesian criterion

In (6), let $C_{11} = C_{00} = 0$ and $C_{01} = C_{10} = 1$, the Bayesian cost in (6) becomes the probability of error:

$$P_e = (1 - P_d)(1 - P_0) + P_f P_0. \quad (20)$$

The solid curves in Fig. 9 shows the graphs of the probabilities

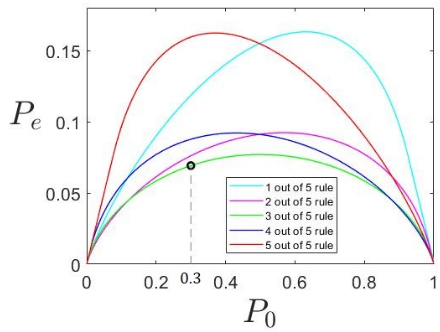


Fig. 9: P_e of five "k out of n" rules vs. P_0 without local constraints.

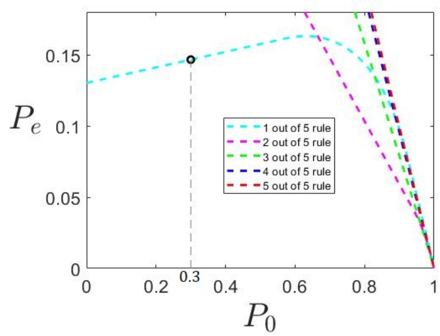


Fig. 10: P_e of five "k out of n" rules vs. P_0 with local constraints.

of error for five "k out of n" rules with different P_0.

Since the probability of error P_e is desired to be minimized, the graph of the probability of error P_e for the entire system is the lower boundary of the graphs of the probabilities of error for all five "k out of n" rules in Fig. 9.

When the local probabilities of false alarm $p_f \leq \beta = 0.04$, the graphs of the probabilities of error for five "k out of n" rules with different P_0 are shown by the dashed lines in Fig. 10. The probability of error P_e for the entire system with constraints on local probabilities of false alarm $P_{fi} \leq \beta$ is the lower boundary of the five curves.

When $P_0 = 0.3$, the probability of error P_e by the system without local constraints is $P_e = 0.0692$, which is achieved when the DFC uses the "3 out of 5" rule and all five LDs operate at $(0.2704, 0.8184)$, shown as the circle in Fig. 9; the probability of error P_e by the system with local constraints $P_{fi} \leq \beta = 0.04$ is $P_e = 0.1465$, shown as the circle in Fig. 10, which is achieved when the DFC uses the "1 out of 5" rule and all five LDs operate at $(0.04, 0.3348)$. Each dashed line in Fig. 10 consists of a part of the corresponding solid line in Fig. 9 and a tangent to that solid line. The reason is that when the Bayesian operating point is the right endpoint of the ROC curve of a "k out of n" rule shown in Fig. 8, $(P_f(k,\beta), P_d(k,\beta))$, P_e is an affine function of P_0 (from (20)).

V. Conclusion

Concerned about counterproductive behavior of decision fusion architectures with a global performance index only, we introduce constraints on the local detectors to temper this behavior. Specifically, upper bounds on the local probabilities of false alarm are imposed. These upper bounds on the local probabilities of false alarm translate into lower bounds on the local likelihood ratios computed by the local decision rules. We demonstrate the effect of the local constraints, and the resulting detection strategy which is a compromise between local and global requirements. Examples are provided to quantify the impact of the local constraints on the global performance.

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