Asynchronous Distributed Detection

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Binary parallel distributed-detection architectures employ a bank of local detectors to observe a common volume of surveillance, and form binary local decisions about the existence or nonexistence of a target in that volume. The local decisions are transmitted to a central detector, the data fusion center (DFC), which integrates them to a global *target* or *no target* decision. Most studies of distributed-detection systems assume that the local detectors are synchronized. In practice local decisions are made asynchronously, and the DFC has to update its global decision continually. In this study the number of local decisions observed by the central detector within any observation period is Poisson distributed. An optimal fusion rule is developed, and the sufficient statistic is shown to be a weighted sum of the local decisions collected by the DFC within the observation interval. The weights are functions of the individual local detector performance probabilities (i.e., probabilities of false alarm and detection). In this respect the decision rule is similar to the one developed by Chair and Varshney for the synchronized system. Unlike the Chair–Varshney rule, however, the DFC's decision threshold in the asynchronous system is time varying. Exact expressions and asymptotic approximations are developed for the detection performance with the optimal rule. These expressions allow performance prediction and assessment of tradeoffs in realistic decision fusion architectures which operate over modern communication networks.

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I. NOMENCLATURE

$\Phi(\cdot)$	Normal distribution: $\Phi(x) = 1/\sqrt{2\pi}\int_{-\infty}^{x}\exp(-y^2/2)\,dy$
φ_f	Variable used to describe the sign argument of Gaussian function in $\hat{P}_e[t]$
φ_d	Variable used to describe the sign argument of Gaussian function in $\hat{P}_e[t]$
η	DFC decision threshold
λ_j	Sum of local Poisson parameters under hypothesis $H_j : \lambda_j = \sum_{i=1}^{N}\tau_{i_j}$
τ_{i_j}	Poisson parameter of detector i under hypothesis H_j
(a,b)	Open interval between a and b on the real line
DFC	Data fusion center
$\mathbf{E}[\cdot]$	Statistical expectation operator
H_j	jth hypothesis
$\mathbf{L}(\cdot : \cdot)$	Discrimination function used to measure the information (cf. [1])
$k_i^{(t)}$	Total number of decisions transmitted to the DFC by the ith local detector during the interval $[0,t)$
N	Number of local detectors
$O(\cdot)$	Asymptotic analysis symbol: $f(x) = O(\phi(x))$ $(x \to \infty)$ means that there exist real numbers a and A such that $\lvert f(x)\rvert \le A\lvert\phi(x)\rvert$ whenever $a < x < \infty$
$o(\cdot)$	Asymptotic analysis symbol: $f(x) = o(g(x))$ $(x \to \infty)$ means that $f(x)/g(x)$ tends to 0 when $x \to \infty$
$\mathbf{P}[\cdot]$	Probability measure
$P_{dg}^{(t)}$	Global probability of detection at time instant t with known $\{k_i^{(t)}\}_{i=1}^{N}$
$P_{dg}[t]$	Global probability of detection at time instant t
P_{di}	Detection probability of the ith local detector
$P_e[t]$	Global probability of error at time instant t
$\hat{P}_e[t]$	Approximation to the global probability of error at time t
$P_{fg}^{(t)}$	Global probability of false alarm at time instant t with known $\{k_i^{(t)}\}_{i=1}^{N}$
$P_{fg}[t]$	Global probability of false alarm at time instant t
P_{fi}	False alarm probability of the ith local detector
t	Time instant
$\mathbf{U}_{-1}[\cdot]$	Unit step function: $\mathbf{U}_{-1}[x] = \begin{cases} 1, & \text{if } x \ge 0 \\ 0, & \text{if } x < 0 \end{cases}$
$u_g^{(t)}$	Decision of the DFC at time instant t
u_i	Decision of the ith local detector
$\underline{y}_i$	Set of observations collected by the ith local detector

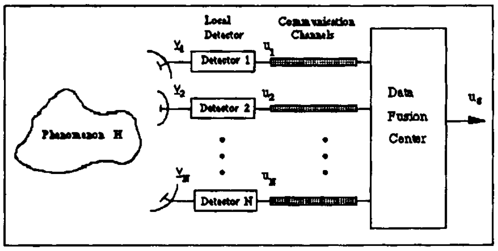

Fig. 1. Decentralized detection structure.

II. INTRODUCTION

Binary parallel distributed-detection architectures (Fig. 1) employ a bank of local detectors which observe a common volume of surveillance, and form binary local decisions about the existence or nonexistence of a target in the surveyed volume. The local decisions are transmitted to a central detector, the data fusion center (DFC), which integrates these decisions to a global *target or no target* decision. Many local-detector and objective-function combinations have been studied over the years: Bayesian (e.g., Tenney and Sandell [11], Chair and Varshney [2], Reibman and Nolte [9], Hoballah and Varshney [6], Kam, et al. [7]), Neyman–Pearson (e.g., Thomopoulus, et al. [12]), minimax (e.g., Geraniotis and Chau [3]), Bayesian-sequential (e.g., Hashemi and Rhodes [4], Teneketzis [10]), max-divergence (e.g., Lee and Chao [8]) and information theoretic (e.g., Hoballah and Varshney [5]). Almost all studies assume that the local detectors are synchronized, namely the local decisions are transmitted to the DFC in common predetermined time instants. This assumption is made even in some studies of decentralized sequential detection systems (e.g. [4]).

In the configuration studied here, each local detector employs a fixed decision rule and makes its decisions asynchronously and independently of all other detectors. Every local detector that reaches a decision transmits it to the central detector instantaneously and proceeds to collect new observations for a subsequent decision. During any interval of observation, the number of decisions that each dectector makes is Poisson distributed. Consequently, the total number of local decisions received by the DFC is also Poisson distributed. We develop the sufficient statistic of the DFC for an optimal threshold test using the Bayes risk criterion and show that it is a weighted sum of the local decisions collected by the DFC over the observation period. The weights are functions of the local detector performance probabilities, namely the probability of false alarm and the probability of detection. The sufficient statistic is compared with a (time-varying) threshold which depends on the total number of local decisions received, the local-decision performance probabilities, the decision-rate (Poisson) parameters, the hypothesis a priori probabilities, the Bayesian costs, and time. In addition we find exact expressions for the global performance probabilities, and numerically efficient tight approximations for them.

In Section III we define the asynchronous distributed detection system and develop the DFC's optimal fusion rule for a global Bayes risk criterion. In Section IV exact expressions for the global probabilities of false alarm and detection as function of time are obtained. Asymptotic approximations to these probabilities are developed in terms of the local-detector performance and the parameters of the Poisson distribution which describe the local decision rates. The approximations become more accurate as the observation period becomes longer. Specifically we show that when the global objective function is the probability of error, this probability tends asymptotically to zero with time.

III. DISTRIBUTED DETECTION WITH ASYNCHRONOUS LOCAL DETECTORS

The asynchronous distributed-detection system (Fig. 1) comprises N *local detectors* and a DFC. Each local detector makes a decision about a binary hypothesis (H_0, H_1): the ith local detector collects the observations $\underline{v}_i$, and declares $u_i = 0$ if it accepts H_0 on the basis of its observations $\underline{v}_i$, or $u_i = 1$ if it accepts H_1. During an observation interval, each local detector makes several (possibly zero) binary decisions, and sends each one of them to the DFC immediately after it was made.[1] The DFC combines these local decisions into global decisions about the hypotheses, by continually updating its global decision $u_g^{(t)}$. $u_g^{(t)} = 0$ means that the DFC decides in favor of H_0 at time instant t; $u_g^{(t)} = 1$ means that the DFC decides in favor of H_1 at time t. The number of decisions that the ith local detector makes within any observation period is assumed to be Poisson distributed with parameter τ_{i_0} (> 0) under hypothesis H_0, and τ_{i_1} (> 0) under hypothesis H_1. Moreover, each decision of a local detector is assumed to be conditionally statistically independent of all other decisions (made by the same detector or by other detectors) under hypothesis H_j ($j = 0,1$). The conditional probability that the ith local detector makes $k_i^{(t)}$ (≥ 0) decisions during the interval $[0,t]$ is given by

$$\mathbf{P}[k_i^{(t)} \text{ decisions in } [0,t] \mid H_j] = \mathrm{e}^{-\tau_{i_j} t} \frac{(\tau_{i_j} t)^{k_i^{(t)}}}{k_i^{(t)}!},$$

$$\text{for} \quad i = 1,\ldots,N \quad \text{and} \quad j = 0,1. \qquad (1)$$

The variable $\mathbf{P}[\cdot]$ is a probability measure. The set $\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} = \{u_i^{(1)}, u_i^{(2)}, \ldots, u_i^{(k_i^{(t)})}\}$ contains $k_i^{(t)}$ local

[1] We assume that the true hypothesis did not change during the observation time interval.

decisions generated by detector i during the interval $[0,t]$.

Let P_{fi} and P_{di} $(0 < P_{fi},\ P_{di} < 1)$ be the (stationary) probability of false alarm and probability of detection, respectively, of the ith local detector (i.e., $P_{fi} = \mathbf{P}[u_i^{(k)} = 1 \mid H_0]$, $P_{di} = \mathbf{P}[u_i^{(k)} = 1 \mid H_1]$ for all $k = 1,2,\ldots,k_i^{(t)}$). The global performance criterion is the Bayes risk at time instant t,

$$\beta(t) = C_{00}\mathbf{P}[u_g^{(t)} = 0, H_0] + C_{11}\mathbf{P}[u_g^{(t)} = 1, H_1] + C_{10}\mathbf{P}[u_g^{(t)} = 1, H_0] + C_{01}\mathbf{P}[u_g^{(t)} = 0, H_1] \tag{2a}$$

where the costs C_{ij} $(i,j = 0,1)$ are specified. It specializes to the *global probability of error* when $C_{10} = C_{01} = 1$ and $C_{00} = C_{11} = 0$, namely,

$$\begin{aligned} P_e[t] &= P_0\mathbf{P}[u_g^{(t)} = 1 \mid H_0] + P_1(1 - \mathbf{P}[\mathbf{u}_g^{(t)} = 1 \mid H_1]) \\ &= P_0 P_{fg}[t] + P_1(1 - P_{dg}[t]) \end{aligned} \tag{2b}$$

where P_j is the a priori probability of hypothesis H_j $(j = 0,1)$. The DFC integration rule is derived in the following theorem.

THEOREM 1 *An asynchronous distributed detection system with the objective of minimizing the global Bayes risk (see (2a)) comprises N local detectors and a DFC. The number of decisions that the ith local detector makes within its observation period is Poisson distributed with parameter* τ_{i_j} (> 0) *under hypothesis* H_j $(j = 0,1)$. *If all local decisions made up to time t are known to the DFC, and are conditionally statistically independent, then the optimal DFC decision rule is*

$$u_g^{(t)} = \mathbf{U}_{-1}\left[\left(\sum_{i=1}^{N}\sum_{j=1}^{k_i^{(t)}} a_i u_i^{(j)}\right) - a_g^{(t)}\right] \tag{3}$$

where

$$a_i = \ln\frac{P_{di}(1 - P_{fi})}{P_{fi}(1 - P_{di})}, \tag{4a}$$

$$a_g^{(t)} = \ln\eta + \sum_{i=1}^{N} k_i^{(t)} \ln\frac{\tau_{i_0}(1 - P_{fi})}{\tau_{i_1}(1 - P_{di})} + \sum_{i=1}^{N}(\tau_{i_1} - \tau_{i_0})t \tag{4b}$$

with $\eta = (P_0(C_{10} - C_{00})/P_1(C_{01} - C_{11}))$, $\mathbf{U}_{-1}[\cdot]$ *is the unit step function:*

$$\mathbf{U}_{-1}[x] = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases},$$

and $k_i^{(t)}$ (≥ 0) *is the number of decisions that the ith local detector made during the interval* $[0,t]$.

PROOF. See Appendix A.

The fusion rule (3) is a generalization of the rule obtained by Chair and Varshney [2] for the synchronous case and for minimum probability of error. Indeed, Chair and Varshney's rule can be obtained from (3) with the appropriate costs, $k_i^{(t)} = 1$ and $\tau_{i_0} = \tau_{i_1}$ for $i = 1,\ldots,N$. We also note that the threshold, and hence the decision rule (3), is time varying, i.e., the DFC's global decision may change even if no new local decisions have arrived.

The global probability of false alarm at time t is defined as

$$\begin{aligned} P_{fg}[t] &= \mathbf{P}[u_g^{(t)} = 1 \mid H_0] \\ &= \mathbf{E}[\mathbf{P}[u_g^{(t)} = 1 \mid H_0,\ \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}},\ i = 1,\ldots,N]] \end{aligned} \tag{5a}$$

and the global probability of detection at time t is defined as

$$\begin{aligned} P_{dg}[t] &= \mathbf{P}[u_g^{(t)} = 1 \mid H_1] \\ &= \mathbf{E}[\mathbf{P}[u_g^{(t)} = 1 \mid H_1,\ \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}},\ i = 1,\ldots,N]] \end{aligned} \tag{5b}$$

where $\mathbf{E}[\cdot]$ is the expectation function over all possible $\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}$ for all $i = 1,\ldots,N$. The global probabilities of false alarm and detection at time instant t thus can be expressed explicitly as (see Appendix B for details)

$$P_{fg}[t] = \sum_{k_1^{(t)}=0}^{\infty}\sum_{k_2^{(t)}=0}^{\infty}\cdots\sum_{k_N^{(t)}=0}^{\infty} \mathbf{e}^{-\lambda_0 t}\prod_{i=1}^{N}\frac{(\tau_{i_0}t)^{k_i^{(t)}}}{k_i^{(t)}!}P_{fg}^{(t)} \tag{6a}$$

$$P_{dg}[t] = \sum_{k_1^{(t)}=0}^{\infty}\sum_{k_2^{(t)}=0}^{\infty}\cdots\sum_{k_N^{(t)}=0}^{\infty} \mathbf{e}^{-\lambda_1 t}\prod_{i=1}^{N}\frac{(\tau_{i_1}t)^{k_i^{(t)}}}{k_i^{(t)}!}P_{dg}^{(t)} \tag{6b}$$

where $\lambda_j = \sum_{i=1}^{N}\tau_{i_j}$ for $j = 0,1$

$$P_{fg}^{(t)} = \sum_{\mathbf{H}}\left\{\prod_{i=1}^{N}\prod_{j=1}^{k_i^{(t)}}|1 - h_i^{(j)} - P_{fi}|\mathbf{U}_{-1}\left[\sum_{i=1}^{N}\sum_{j=1}^{k_i^{(t)}} a_i h_i^{(j)} - a_g^{(t)}\right]\right\} \tag{6c}$$

$$P_{dg}^{(t)} = \sum_{\mathbf{H}}\left\{\prod_{i=1}^{N}\prod_{j=1}^{k_i^{(t)}}|1 - h_i^{(j)} - P_{di}|\mathbf{U}_{-1}\left[\sum_{i=1}^{N}\sum_{j=1}^{k_i^{(t)}} a_i h_i^{(j)} - a_g^{(t)}\right]\right\} \tag{6d}$$

and the summations $\sum_{\mathbf{H}}$ are performed over all possible $2^{\sum_{i=1}^{N} k_i^{(t)}}$ binary tuples in $\mathbf{H} = \{\{h_i^{(j)}\}_{j=1}^{k_i(t)}\}_{i=1}^{N}$ with $h_i^{(j)} \in \{0,1\}$ (**H** corresponds to all possible combinations of local decisions that can be transmitted to the DFC during the time interval $[0,t]$). Equations (6a)–(6d) require only that the DFC know the performance probabilities (P_{fi}, P_{di}) and the Poisson parameters (τ_{i_j}) of each local detector.

In general, equations (6) are difficult to evaluate. They are easier to compute and approximate when all local detectors are identical, namely, all detectors have the same probabilities of false alarm and detection: $P_{fi} = P_f$ and $P_{di} = P_d$, $i = 1,2,\ldots,N$; and all detectors have the same Poisson parameters: $\tau_{i_1} = \tau_1$ and $\tau_{i_0} = \tau_0$, $i = 1,2,\ldots,N$. Next, we concentrate on this case.

IV. GLOBAL PERFORMANCE WITH IDENTICAL LOCAL DETECTORS

For identical local detectors, alternative expressions for the global probability of false alarm ($P_{fg}[t]$ in (6a)) and the global probability of detection ($P_{dg}[t]$ in (6b)) can be obtained by using the well-known multiple Poisson distribution formula. The probabilities are

$$P_{fg}[t] = \sum_{m=0}^{\infty} e^{-\lambda_0 t} \frac{(\lambda_0 t)^m}{m!} \sum_{k=0}^{m} \binom{m}{k} P_f^k (1-P_f)^{m-k} \cdot \mathbf{U}_{-1}\left[k \ln \frac{P_d(1-P_f)}{P_f(1-P_d)} - \left(m \ln \frac{\lambda_0(1-P_f)}{\lambda_1(1-P_d)} + (\lambda_1-\lambda_0)t + \ln\eta\right)\right] \tag{7a}$$

$$P_{dg}[t] = \sum_{m=0}^{\infty} e^{-\lambda_1 t} \frac{(\lambda_1 t)^m}{m!} \sum_{k=0}^{m} \binom{m}{k} P_d^k (1-P_d)^{m-k} \cdot \mathbf{U}_{-1}\left[k \ln \frac{P_d(1-P_f)}{P_f(1-P_d)} - \left(m \ln \frac{\lambda_0(1-P_f)}{\lambda_1(1-P_d)} + (\lambda_1-\lambda_0)t + \ln\eta\right)\right] \tag{7b}$$

where $m = \sum_{i=1}^{N} k_i^{(t)}$. Moreover, by interchanging indices m and k, and setting $w = m - k$, equations (7) can be rewritten as

$$P_{fg}[t] = \sum_{k=0}^{\infty} e^{-\lambda_0 t P_f} \frac{(\lambda_0 t P_f)^k}{k!} \sum_{w=0}^{\infty} e^{-\lambda_0 t(1-P_f)} \frac{(\lambda_0 t(1-P_f))^w}{w!} \cdot \mathbf{U}_{-1}\left[\frac{k}{t} \ln \frac{\lambda_1 P_d}{\lambda_0 P_f} + \frac{w}{t} \ln \frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)} - \frac{1}{t}\ln\eta - (\lambda_1 - \lambda_0)\right] \tag{8a}$$

$$P_{dg}[t] = \sum_{k=0}^{\infty} e^{-\lambda_1 t P_d} \frac{(\lambda_1 t P_d)^k}{k!} \sum_{w=0}^{\infty} e^{-\lambda_1 t(1-P_d)} \frac{(\lambda_1 t(1-P_d))^w}{w!} \cdot \mathbf{U}_{-1}\left[\frac{k}{t} \ln \frac{\lambda_1 P_d}{\lambda_0 P_f} + \frac{w}{t} \ln \frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)} - \frac{1}{t}\ln\eta - (\lambda_1 - \lambda_0)\right]. \tag{8b}$$

To simplify (8), we use the following theorem.

THEOREM 2 *Let two independent random variables X and Y have Poisson distributions with parameters λ_x and λ_y, respectively. Let $Z = (aX + bY + c)/t$, where a, b and $c \in (-\infty,\infty)$ are fixed and $t \in (0,\infty)$. Then for each fixed $z \in (-\infty,\infty)$*

$$\mathbf{P}\left[\frac{Z-\mu_z}{\sigma_z} \le z\right] \text{ converges to } \Phi(z) \text{ as } t \text{ approaches } \infty$$

where $\mu_z = (a\lambda_x + b\lambda_y + c)/t$, $\sigma_z^2 = (a^2\lambda_x + b^2\lambda_y)/t^2$ and $\Phi(x) = 1/\sqrt{2\pi}\int_{-\infty}^{x} \exp(-y^2/2)\,dy$.

PROOF. See Appendix C.

Using Theorem 2, the global probability of false alarm in (8a) can be expressed as

$$P_{fg}[t] = \mathbf{P}\left[\frac{aX + bY + c}{t} \ge z\right]. \tag{9a}$$

The random variables X and Y are independent and Poisson distributed, with parameters $\lambda_0 t P_f$ and $\lambda_0 t(1-P_f)$, respectively; $a = \ln(\lambda_1 P_d/\lambda_0 P_f)$, $b = \ln(\lambda_1(1-P_d)/\lambda_0(1-P_f))$, $c = -\ln\eta$ and $z = \lambda_1 - \lambda_0$. Similarly, the global probability of detection in (8b) can be expressed as

$$P_{dg}[t] = \mathbf{P}\left[\frac{aX + bY + c}{t} \ge z\right] \tag{9b}$$

with the random variables X and Y being independent and Poisson distributed with parameters $\lambda_1 t P_d$ and $\lambda_1 t(1-P_d)$, respectively. From Theorem 2, the global probabilities of false alarm (6a) and detection (6b) tend asymptotically to

$$\beta_{fg}[t] = \Phi\left(\frac{\mu_f - z}{\sigma_f}\right) \quad \text{and} \quad \beta_{dg}[t] = \Phi\left(\frac{\mu_d - z}{\sigma_d}\right) \tag{10}$$

where

$$\mu_f = \lambda_0 P_f \ln \frac{\lambda_1 P_d}{\lambda_0 P_f} + \lambda_0(1-P_f) \ln \frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)} - \frac{1}{t}\ln\eta \tag{11a}$$

$$\sigma_f^2 = \frac{\lambda_0 P_f}{t}\left(\ln\frac{\lambda_1 P_d}{\lambda_0 P_f}\right)^2 + \frac{\lambda_0(1-P_f)}{t}\left(\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)}\right)^2 \tag{11b}$$

$$\mu_d = \lambda_1 P_d \ln\frac{\lambda_1 P_d}{\lambda_0 P_f} + \lambda_1(1-P_d)\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)} - \frac{1}{t}\ln\eta \tag{11c}$$

$$\sigma_d^2 = \frac{\lambda_1 P_d}{t}\left(\ln\frac{\lambda_1 P_d}{\lambda_0 P_f}\right)^2 + \frac{\lambda_1(1-P_d)}{t}\left(\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)}\right)^2. \tag{11d}$$

For large t, the global probability of error $P_e[t]$ in (2b) can now be approximated by

$$\hat{P}_e[t] = P_0\Phi\left(\frac{\mu_f - z}{\sigma_f}\right) + (1-P_0)\Phi\left(\frac{z-\mu_d}{\sigma_d}\right). \tag{12}$$

In (12) the (approximated) global probability of error of the asynchronous system $\hat{P}_e[t]$ is completely characterized by the means (μ_f and μ_d) and variances (σ_f^2 and σ_d^2) of the two Gaussian functions. The moments μ_f, μ_d, σ_f^2, and σ_d^2 are in turn functions of the parameters P_f, P_d, λ_0, λ_1, η, and t (cf. (11a)–(11d)). It is of course of interest to analyze the asymptotic behavior of $\hat{P}_e[t]$ as $t \to \infty$. Next, we show that $\hat{P}_e[t] \to 0$ as $t \to \infty$.

From examination of the arguments of the Gaussian functions in $\hat{P}_e[t]$ ((12)) it is easy to verify that[2]

$$\frac{\mu_f - z}{\sigma_f} = \varphi_f\sqrt{t}(1+o(1)) \quad (t\to\infty)^2 \tag{13a}$$

$$\frac{z-\mu_d}{\sigma_d} = \varphi_d\sqrt{t}(1+o(1)) \quad (t\to\infty) \tag{13b}$$

where

$$\varphi_f = \frac{\lambda_0 P_f \ln\frac{\lambda_1 P_d}{\lambda_0 P_f} + \lambda_0(1-P_f)\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)} - (\lambda_1-\lambda_0)}{\left[\lambda_0 P_f\left(\ln\frac{\lambda_1 P_d}{\lambda_0 P_f}\right)^2 + \lambda_0(1-P_f)\left(\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)}\right)^2\right]^{1/2}} \tag{14a}$$

$$\varphi_d = \frac{(\lambda_1-\lambda_0) - \left[\lambda_1 P_d \ln\frac{\lambda_1 P_d}{\lambda_0 P_f} + \lambda_1(1-P_d)\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)}\right]}{\left[\lambda_1 P_d\left(\ln\frac{\lambda_1 P_d}{\lambda_0 P_f}\right)^2 + \lambda_1(1-P_d)\left(\ln\frac{\lambda_1(1-P_d)}{\lambda_0(1-P_f)}\right)^2\right]^{1/2}}. \tag{14b}$$

[2] $f(x) = o(g(x))(x\to\infty)$ means that $f(x)/g(x)$ tends to zero as $x \to \infty$.

The asymptotic behavior of $\hat{P}_e[t]$ is then determined by the signs of φ_f and φ_d, namely,[3]

if $\varphi_f < 0$ and $\varphi_d < 0$, then $\hat{P}_e[t]$ approaches 0 as $t\to\infty$ (15a)

if $\varphi_f > 0$ and $\varphi_d < 0$, then $\hat{P}_e[t]$ approaches P_0 as $t\to\infty$ (15b)

if $\varphi_f < 0$ and $\varphi_d > 0$, then $\hat{P}_e[t]$ approaches $1-P_0$ as $t\to\infty$ (15c)

if $\varphi_f > 0$ and $\varphi_d > 0$, then $\hat{P}_e[t]$ approaches 1 as $t\to\infty$. (15d)

Using properties of the discrimination function (cf. [1]), we show in Appendix D that (15a) is the only one of practical interest. Consequently, we obtain the following theorem for the asymptotic performance of the asynchronous system as $t\to\infty$.

THEOREM 3 *The global probability of error for the asynchronous distributed detection system converges asymptotically to zero as $t\to\infty$ with probability one.*

PROOF. See Appendix D.

Using $\Phi(x) = 1-(1/\sqrt{2\pi}x)\exp(-x^2/2) + O(x^{-3}\exp(-x^2/2))$ $(x\to\infty)$,[4] the global probability of error $\hat{P}_e[t]$ in (12) can be approximated further by

$$\hat{P}_e[t] = \left[\frac{P_0\sigma_f}{\sqrt{2\pi}(z-\mu_f)}\exp\left[-\frac{(z-\mu_f)^2}{2\sigma_f^2}\right] + \frac{(1-P_0)\sigma_d}{\sqrt{2\pi}(\mu_d-z)}\exp\left[-\frac{(z-\mu_d)^2}{2\sigma_d^2}\right]\right] + O(t^{-3/2}\exp(-ct/2))(t\to\infty) \tag{16}$$

where $c = \min\{\varphi_f^2, \varphi_d^2\}$.

Fig. 2 shows the global probability of error (12) versus the observation time t for a decentralized asynchronous detection system with three identical local detectors (LDs) and with a DFC that minimizes the global probability of error. The parameters of the LDs are shown in Table I. In Fig. 2, values of $\hat{P}_e[t]$ for large t ($t \geq 50$) were calculated using approximation (16).

Comparing case I to case II we see that the rate of decrease in the global probability of error for case II is much faster than that for case I, because system II has better LDs.

Comparing case II to case III we see that the rate of decrease in the global probability of error for case III is faster than that for case II. This is because

[3] The cases $\varphi_f = 0$ and/or $\varphi_d = 0$ are analyzed in Appendix D. It is shown there that these are nongeneric cases of no practical interest.

[4] $f(x) = O(\phi(x))$ $(x\to\infty)$ means that there exist real numbers a and A such that $|f(x)| \leq A|\phi(x)|$ whenever $a < x < \infty$.

TABLE I
Parameters of LDMs in Three Asynchronous Decision Fusion Architectures

Case No.	P_f	P_d	P_0	λ_0	λ_1	φ_f	φ_d
I	0.4	0.7	0.4	0.8	2.0	-0.730...	-0.569...
II	0.1	0.7	0.4	0.8	2.0	-1.337...	-0.795...
III	0.1	0.7	0.4	3.0	2.0	-0.995...	-1.042...

Note: Roman characters here correspond to italic figures in text.

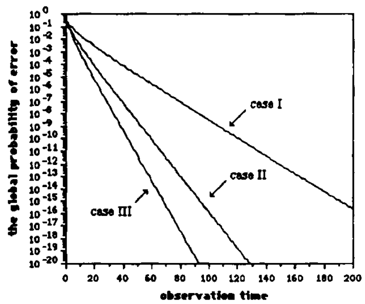

Fig. 2. Global probability of error versus observation time for asynchronous systems described in Table I.

system III gets a higher volume of reliable information than system II when the rate of local decision making under hypothesis H_0 is increased (from 0.8 to 3.0): $P_f = 0.1$ for H_0 but $P_m = 1 - P_d = 0.3$ for H_1.

Expression (12), along with the asymptotic expression (16), allow the generation of performance graphs (such as the ones in Fig. 2) for asynchronous systems with similar detectors. When the local detectors are not similar, it is at least possible to get upper bounds on expected performance assuming that all local detectors posses the worst detector performance probabilities. Asymptotic global performance of asynchronous systems can be predicted, and asynchronous distributed detection and fusion models can be compared.

V. CONCLUSION

A model of asynchronous decision making in distributed-detection systems is studied, when the number of local decisions during any observation interval is Poisson distributed. The optimal Bayesian fusion decision rule is developed, and the sufficient statistic is shown to be a weighted sum of local decisions collected by the DFC during the observation period. The weights are functions of the local-detector performance probabilities. The DFC decision rule for the asynchronous system is a generalization of the rule obtained by Chair and Varshney for the synchronized system. The main quantitative difference is the decision threshold in the synchronous scheme is time invariant, whereas in the asynchronous scheme the threshold is time varying. Exact expressions and asymptotic approximations for the DFC performance probabilities are calculated. The approximations allow the assessment and prediction of DFC performance for large observation intervals, and can be used to assess parameter tradeoffs in the design of asynchronous distributed detection architectures.

APPENDIX A. PROOF OF THEORM 1

Consider an asynchronous distributed detection system with N local detectors. The set $\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} = \{u_i^{(1)}, u_i^{(2)}, \ldots, u_i^{(k_i^{(t)})}\}$ contains the $k_i^{(t)}$ decisions made by the ith local detector and received by the DFC during the observation interval $[0,t]$. The number of decisions, $k_i^{(t)}$, is Poisson distributed (see (1)) with parameter τ_{i_j} under hypothesis H_j $(j = 0,1)$. Let η be the global threshold corresponding to the Bayes risk (2a), namely $\eta = (P_0(C_{10} - C_{00})/P_1(C_{01} - C_{11}))$. Then the optimal fusion rule at the DFC is given by a likelihood ratio test:

$$\frac{P(\{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}\}_{i=1}^{N} \mid H_1)}{P(\{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}\}_{i=1}^{N} \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{17}$$

Using Bayes rule on the left-hand side, we obtain

$$\prod_{i=1}^{N} \frac{P(k_i^{(t)} \mid H_1) P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)}, H_1)}{P(k_i^{(t)} \mid H_0) P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)}, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{18}$$

Using (1), (18) is reduced to

$$\sum_{i=1}^{N} \ln \left[\frac{P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)}, H_1)}{P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)}, H_0)} \right] \underset{H_0}{\overset{H_1}{\gtrless}} \ln \eta + \sum_{i=1}^{N} k_i^{(t)} \ln \frac{\tau_{i_0}}{\tau_{i_1}} + \sum_{i=1}^{N} (\tau_{i_1} - \tau_{i_0}) t. \tag{19}$$

Assuming the conditional statistical independence of the local-detector decisions $u_i^{(j)} \in \{0,1\}$, $j = 1, 2, \ldots, k_i^{(t)}$, and expanding each term in the left-hand side of (19) over $u_i^{(j)}$, the log-likelihood of the ith local detector becomes

$$\ln\left[\frac{P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)},H_1)}{P(\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid k_i^{(t)},H_0)}\right] = \sum_{j=1}^{k_i^{(t)}} \ln\left[\frac{P_{di}(1-P_{fi})}{P_{fi}(1-P_{di})}\right] u_i^{(j)} + k_i^{(t)} \ln\left(\frac{1-P_{di}}{1-P_{fi}}\right). \tag{20}$$

The optimal fusion rule (see (3)) is then obtained by combining (19) and (20).

APPENDIX B. DERIVATION OF EQUATIONS (6)

We derive (6a); (6b) follows similarly. By the definition of the global false alarm probability at time t

$$P_{fg}[t] = \mathbf{P}[u_g^{(t)} = 1 \mid H_0]. \tag{21}$$

Equivalently,

$$\begin{aligned} P_{fg}[t] &= \sum_{\{k_i^{(t)}\}_{i=1}^N} \sum_{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}} \mathbf{P}[u_g^{(t)} = 1, \{k_i^{(t)}\}_{i=1}^N, \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid H_0] \\ &= \sum_{\{k_i^{(t)}\}_{i=1}^N} \mathbf{P}[\{k_i^{(t)}\}_{i=1}^N \mid H_0] \\ &\quad \times \sum_{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}} \mathbf{P}[u_g^{(t)} = 1, \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid H_0; \{k_i^{(t)}\}_{i=1}^N]. \end{aligned} \tag{22}$$

Equation (22) is obtained using Bayes' rule. In (22), let

$$\begin{aligned} P_{fg}^{(t)} &= \sum_{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}} \mathbf{P}[u_g^{(t)} = 1, \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid H_0; \{k_i^{(t)}\}_{i=1}^N] \\ &= \sum_{\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}} \mathbf{P}[\{u_i^{(j)}\}_{j=1}^{k_i^{(t)}} \mid H_0; \{k_i^{(t)}\}_{i=1}^N] \\ &\quad \times \mathbf{P}[u_g^{(t)} = 1 \mid H_0; \{k_i^{(t)}\}_{i=1}^N, \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}]. \end{aligned} \tag{23}$$

Moreover, since

$$\mathbf{P}[u_g^{(t)} = 1 \mid H_0; \{k_i^{(t)}\}_{i=1}^N, \{u_i^{(j)}\}_{j=1}^{k_i^{(t)}}] = \mathbf{U}_{-1}\left[\sum_{i=1}^N \sum_{j=1}^{k_i^{(t)}} a_i u_i^{(j)} - a_g^{(t)}\right] \tag{24}$$

equation (23) can be rewritten as

$$P_{fg}^{(t)} = \sum_{\mathbf{H}} \left\{ \prod_{i=1}^N \prod_{j=1}^{k_i^{(t)}} |1 - h_i^{(j)} - P_{fi}| \times \mathbf{U}_{-1}\left[\sum_{i=1}^N \sum_{j=1}^{k_i^{(t)}} a_i h_i^{(j)} - a_g^{(t)}\right] \right\} \tag{25}$$

where $\mathbf{H} = \{\{h_i^{(j)}\}_{j=1}^{k_i(t)}\}_{i=1}^N$ with $h_i^{(j)} \in \{0,1\}$. By the Poisson distribution assumption for $\{k_i^{(t)}\}_{i=1}^N$ in (22) and (25), the global probability of false alarm in (21) can now be expressed as

$$P_{fg}[t] = \sum_{k_1^{(t)}=0}^{\infty} \sum_{k_2^{(t)}=0}^{\infty} \cdots \sum_{k_N^{(t)}=0}^{\infty} \mathbf{e}^{-\lambda_0 t} \prod_{i=1}^N \frac{(\tau_{i0} t)^{k_i^{(t)}}}{k_i^{(t)}!} P_{fg}^{(t)}. \tag{26}$$

APPENDIX C. PROOF OF THEOREM 2

Assume that two independent random variables X and Y have Poisson distribution with parameters λ_x and λ_y, respectively. Let $Z = (aX + bY + c)/t$, where a, b, and $c \in (-\infty,\infty)$ are fixed and $t \in (0,\infty)$. Let $m_Z(s)$ be the moment-generating function of the random variable Z:

$$m_Z(s) = \mathbf{E}[\exp(sZ)]. \tag{27}$$

Then,

$$\begin{aligned} m_Z(s) &= \exp\left(s\frac{c}{t}\right) \mathbf{E}\left[\exp\left(s\frac{aX}{t}\right)\right] \mathbf{E}\left[\exp\left(s\frac{bY}{t}\right)\right] \\ &= \exp\left(s\frac{c}{t}\right) \cdot \exp\left\{\lambda_x\left[\exp\left(s\frac{a}{t}\right) - 1\right]\right\} \\ &\quad \cdot \exp\left\{\lambda_y\left[\exp\left(s\frac{b}{t}\right) - 1\right]\right\}. \end{aligned} \tag{28}$$

The first equality uses the independence of X and Y, and the second is a consequence of X and Y being Poisson random variables with $m_{(aX/t)}(s) = \exp\{\lambda_x[\exp(s(a/t)) - 1]\}$ and $m_{(bY/t)}(s) = \exp\{\lambda_y[\exp(s(b/t)) - 1]\}$. Using a Taylor expansion of $\exp(z) = 1 + z + z^2/2! + O(z^3)(z \to 0)$, (28) can be rewritten as

$$m_Z(s) = \exp\left[\left(\frac{a\lambda_x + b\lambda_y + c}{t}\right) s + \left(\frac{a^2\lambda_x + b^2\lambda_y}{t^2}\right)\frac{s^2}{2} + O(t^{-3})(t \to \infty)\right]. \tag{29}$$

Thus, from (29), the moment-generating function of the random variable Z converges to a normal generating function $\exp(\mu_z s + \sigma_z^2(s^2/2))$ as $t \to \infty$ with the mean $\mu_z = (a\lambda_x + b\lambda_y + c)/t$ and the variance $\sigma_z^2 = (a^2\lambda_x + b^2\lambda_y)/t^2$. Hence, for each fixed $z \in$

$(-\infty,\infty)$, $\mathbf{P}[(Z-\mu_z)/\sigma_z \leq z]$ converges to $\Phi(z)$ as t approaches ∞.

APPENDIX D. PROOF OF THEOREM 3

With some algebra, expression (14a) becomes

$$\varphi_f = c_f \left[g(P_f, P_d) - \left(\frac{\lambda_1}{\lambda_0} + \ln \frac{\lambda_0}{\lambda_1} - 1 \right) \right] \tag{30}$$

where

$$g(P_f, P_d) = P_f \ln \frac{P_d}{P_f} + (1-P_f) \ln \frac{1-P_d}{1-P_f}$$

$$c_f = \frac{\sqrt{\lambda_0}}{\left[P_f \left(\ln \frac{\lambda_1 P_d}{\lambda_0 P_f} \right)^2 + (1-P_f) \left(\ln \frac{\lambda_1 (1-P_d)}{\lambda_0 (1-P_f)} \right)^2 \right]^{1/2}}$$

> 0 for all values of P_f, P_d, λ_0, and λ_1.

In (30) we recognize the term $g(P_f, P_d)$ is the negative of the expected value of the log likelihood ratio $P(u_i \mid H_1)/P(u_i \mid H_0)$ with respect to hypothesis H_0. Therefore, $g(P_f, P_d)$ is the negative of the *discrimination function* (cf. [1, 107–113]), and as such satisfies

$$g(P_f, P_d) \leq 0. \tag{31}$$

Furthermore, using the inequality

$$1 - \frac{1}{x} \leq \ln x \leq x - 1 \tag{32}$$

we obtain

$$0 \leq \frac{\lambda_1}{\lambda_0} + \ln \frac{\lambda_0}{\lambda_1} - 1. \tag{33}$$

From (31) and (33), it follows that for all values of P_f, P_d, λ_0, and λ_1,

$$P_f \ln \frac{P_d}{P_f} + (1-P_f) \ln \frac{1-P_d}{1-P_f} \leq \frac{\lambda_1}{\lambda_0} + \ln \frac{\lambda_0}{\lambda_1} - 1. \tag{34}$$

Equality in equation (34) occurs if and only if $P_f = P_d$ and $\lambda_0 = \lambda_1$, a case of no practical interest. Since equality in (34) is nongeneric, $\varphi_f < 0$ with probability 1. Similarly, we can show that $\varphi_d < 0$ with probability 1.

Consequently, (15a) is the only one of interest. Theorem 3 follows.

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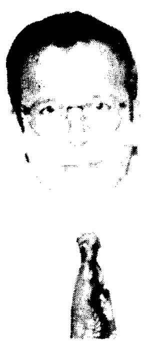

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