Hardware Complexity of Binary Distributed Detection Systems with Isolated Local Bayesian Detectors

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***Abstract*—Two multisensor multiobservation detection schemes are analyzed and compared, and their hardware complexity (= number of sensors) is discussed. The studied schemes are: a Bayesian optimal parallel-sensor centralized architecture and a suboptimal binary distributed-detection system. Both systems are to have the same performance, as measured in terms of a Bayesian risk. In the optimal system sensors transmit their raw measurements to a decision maker that minimizes a global Bayesian risk. In the suboptimal architecture each sensor acts as a local detector: it minimizes its own Bayesian risk *locally*, and submits a binary decision to a data fusion center that minimizes the same global risk (for the given fixed architectures of the local detectors). Two specific cases are studied: 1) discrimination between two Gaussian populations that differ in their means; and 2) discrimination between two Poisson populations that differ in their parameters. The tradeoff between performance and hardware complexity is demonstrated, and the cost (in terms of hardware units) of the design simplicity that characterizes the suboptimal system is calculated. Results are useful for comparing different distributed-sensor detection schemes. It is shown that in the Gaussian case, a high signal-to-noise ratio (SNR) decentralized system with $2N$ sensor/detectors performs at least as well as the centralized system with N sensors and a single detector.**

NOMENCLATURE

$\beta^{(k)}$	Bayesian risk of the kth decision maker in a binary decision system (as a function of the cost coefficients C_{ij}) i.e., $\beta^{(k)} = \beta^{(k)}\left(C_{00}^{(k)}, C_{01}^{(k)}, C_{10}^{(k)}, C_{11}^{(k)}\right)$.
β_o	Desirable global Bayesian risk in a binary detection system.
$\lambda(z)$	Likelihood ratio of the observation vector z.
σ	Standard deviation.
τ	Threshold for comparison of likelihood ratio.
A1	Optimal min-β centralized architecture (e.g., [10, sect. 5.3]).
A2	Suboptimal (local min-β) decentralized system.
A3	Optimal min-β decentralized architecture [8].
floor $[x]$	Largest integer smaller than or equal to x: $x-1 <$ floor $[x] \le x$.
int $\{x\}$	Smallest integer larger than or equal to x: $x \le$ int $\{x\} < x+1$.
K	Number of observations collected by a sensor in A1 and A2.
m	The mean of the observation (z and z_i) under H_1 in the examples.
n_o	Number of sensors used in the optimal min-Bayesian risk centralized system A1.
n_{so}	Number of sensors used in the decentralized detection system A2.
p_e	Probability of error $p_e = P_r(H_0$ is true, $D_1) + P_r$ $(H_1$ is true, $D_0)$.
p_{e_o}	Prespecified desirable probability of error for detection systems A1.
$p_{e_{so}}$	Prespecified desirable probability of error for detection systems A2.
$P_{F_{\rm DFC}}$	Probability of false alarm by the data fusion center: $P_r(H_0$ is true, $u_o = 1)$.
P_{F_i}	Probability of false alarm by the ith detector: $P_r(H_0$ is true, $u_i = 1)$.
P_{H_i}	*A priori* probability of hypothesis H_i for $i = 0, 1$.
$P_{M_{\rm DFC}}$	Probability of missed detection by the data fusion center: $P_r(H_1$ is true, $u_o = -1)$.
P_{M_i}	Probability of missed detection by the ith detector: $P_r(H_1$ is true, $u_i = -1)$.
$S(z)$	Sufficient statistic of the observation vector z.
sgn	Algebraic sign function.
SNR	Signal to noise ratio: m/σ.
t	Number of observations at the local detector in the Poisson distribution case.
U_{-1}	Unit step function.
z	Set of observations.

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I. INTRODUCTION

THE PROBLEM of binary parallel distributed detection with a Bayesian risk function has been the objective of several studies which aimed at devising an optimal design for the components of the distributed architecture (see [2]–[6], [8], [9], [11]–[13], [15]). The system comprises N *local detectors*, each making a decision about a binary hypothesis (H_0, H_1), and a *data fusion center* (DFC) that uses these local decisions $\{u_1, u_2, \cdots, u_N\}$ for a global decision about the hypothesis. K observations are collected by the ith detector $(z_i = \{z_{i_1}, z_{i_2}, \cdots, z_{i_K}\})$ before it makes its decision, u_i. The decision is $u_i = 1$ if the detector decides in favor of H_1 (decision D_1), and $u_i = -1$ if it decides in favor of H_0

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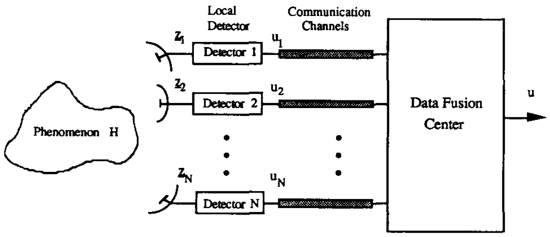

Fig. 1. Decentralized detection structure.

(decision D_0). The DFC collects the N decisions of the local detectors through ideal communication channels, and uses them in order to decide in favor of $H_0(u = -1)$ or in favor of $H_1(u = 1)$. Fig. 1 shows the architecture and the associated symbols.

The *Bayesian risk* $\beta^{(k)}$ $(C_{00}, C_{01}, C_{10}, C_{11})$ is defined for the kth decision maker in the system as

$$\begin{aligned}\beta^{(k)}(C_{00}, C_{01}, C_{10}, C_{11}) = {} & C_{00}^{(k)} P_r(H_0, D_0) \\ & + C_{10}^{(k)} P_r(H_0, D_1) \\ & + C_{01}^{(k)} P_r(H_1, D_0) \\ & + C_{11}^{(k)} P_r(H_1, D_1) \qquad (1)\end{aligned}$$

where $C_{00}^{(k)}$, $C_{01}^{(k)}$, $C_{10}^{(k)}$, and $C_{11}^{(k)}$ are the prespecified cost coefficients of decision maker k for each combination of hypothesis and detector decision: $C_{ij}^{(k)}$ is the cost incurred when the kth decision maker decides D_i when H_j is true. For the cost combination $C_{00}^{(k)} = C_{11}^{(k)} = 0$ and $C_{10}^{(k)} = C_{01}^{(k)} = 1$, the Bayesian cost becomes the *probability of error*. We consider a *suboptimal* system where each detector $(k = 1, 2, \cdots, n_{so})$ minimizes *locally* a Bayesian risk $\beta^{(k)}$ and the DFC $(k = 0)$ is optimal with respect to $\beta^{(0)}$, *given the local-detector design.* In the subsequent derivation we assume $\beta^{(k)} = \beta^{(0)}$, $k = 1, 2, \cdots, n_{so}$ (all local detectors minimize the same Bayesian risk) and the superscript k is, therefore, omitted. As shown later, the local risks could be different without affecting the derivation. The overall system is suboptimal with respect to β, but this deficiency is offset partially by the fact that it is much easier to design this system than the optimal decentralized system. This advantage is due to the fact that in the suboptimal system each detector is designed independently of the other detectors, while the design of the detectors in the optimal system is coupled (the threshold of each local detector is derived from the decision rules of the other detectors [11]). Hence, unlike the design of the optimal system, the removal or sudden failure of some local detectors does not affect the design of all other detectors in the system (but of course the performance would be affected).

Hardware complexity can be measured in terms of the number of local detectors which are necessary in order to achieve a prespecified global Bayesian risk $\beta = \beta_o$. We shall compare the hardware complexity of two architectures:

$A1$: A centralized system, where a *single* central detector makes a global decision u_o ($u_o = 1$ for $D_1, u_o = -1$ for D_0) on the basis of L statistically independent observations $z_1, \cdots, z_L$—using a minimum Bayesian risk criterion. These L observations can be generated by $n_o = L/K$ parallel sensors each taking K measurements, where K is a divisor of L and $1 \le K \le L$. The hardware complexity in this case is the number of parallel sensors, n_o, which is strongly dependent on β_o.

$A2$: A decentralized system where n_{so} parallel sensors are followed by detectors which make local decisions u_i, $i = 1, 2, \cdots, n_{so}$ ($u_i = 1$ for $D_1, u_i = -1$ for D_0). The ith detector uses K observations $z_{i_1}, z_{i_2}, \cdots, z_{i_K}$ and forms its decision to minimize the Bayesian risk locally. The data fusion center makes its decision u_{so} ($u_{so} = 1$ for $D_1, u_{so} = -1$ for D_0) on the basis of the decisions of the n_{so} local detectors $u_1, u_2, \cdots, u_{n_{so}}$. It minimizes the Bayesian risk, *given the fixed local min-β detectors.* The hardware complexity in this case is the number of parallel sensors that feed the local detectors (= number of local detectors), n_{so}, which is strongly dependent on β_o.

In this context, the *optimal* decentralized system is

$A3$: A decentralized system like A2, except that the local detectors, as well as the DFC, are designed such that the *overall* system is optimal with respect to β_o (i.e., according to the procedure in [8]).

We are interested in comparing the number of sensors/detectors that are required for achieving a specified global value of the Bayesian risk using A1 and A2. It is clearly of interest to compare them to the number of detectors required by A3 for the same task, but the calculation of the number of detectors required for architecture A3 is not practical for systems with more than five detectors (see [13]). For a prespecified risk β_o, we designate the number of required sensors for A1 as $n_o(\beta_o)$ and for A2 as $n_{so}(\beta_o)$. The difference between these numbers is a measure of the cost differential that one has to pay for replacing a centralized architecture (A1) by the suboptimal decentralized alternative (A2).

Application of the proposed architectures are expected primarily in the area of diversity in digital communications. The main reason for employing diversity in this context is to improve communication performance under jamming (e.g., [1], [7]). Additional areas of applicability are countercrime measures (intrusion alarm systems and automated entry control) and military surveillance (especially in fusing radar signals from several early warning stations).

II. Decision Rules and Hardware Complexity

We describe the decision rules for components of architectures A1 (centralized architecture) and A2 (decentralized suboptimal architecture). Details about the decision rules of the globally optimal system A3 can be found in [8].

A. Architecture A1

The single decision maker decides on the value of its decision u_o on the basis of L statistically independent observations

$z = \{z_1, \cdots, z_L\}$. The decision rule is the standard multi-observation likelihood ratio test (see [14]):

$$u_o = \begin{cases} 1, & \text{if } \lambda(z) \geq \tau \\ -1, & \text{if } \lambda(z) < \tau. \end{cases} \tag{2a}$$

The likelihood ratio $\lambda(z)$ and the threshold τ are

$$\lambda(z) = \frac{p_{z\,|\,H}(z\,|\,H_1)}{p_{z\,|\,H}(z\,|\,H_0)}; \qquad \tau = \frac{P_{H_0}(C_{10} - C_{00})}{P_{H_1}(C_{01} - C_{11})} \tag{2b}$$

Let $S(z)$ be the *sufficient statistic* of the observation vector z. The likelihood ratio $\lambda(z)$ in (2b) can be replaced by $p_{S\,|\,H}(S\,|\,H_1)/p_{S\,|\,H}(S\,|\,H_0)$, where $p_{S\,|\,H}(S\,|\,H_i)$ is the probability density of the sufficient statistic under hypothesis H_i $(i = 0, 1)$, which depends on the number of sensors $n_o(\beta_o)$ [10, p. 127]. The number of sensors required in order to achieve Bayesian risk of β_o is the $n_o(\beta_o)$ which solves

$$\begin{aligned}\beta_o = {} & C_{00}P_{H_0} \int_{-\infty}^{\tau} p_{S\,|\,H}(S\,|\,H_0)\,dS \\ & + C_{01}P_{H_1} \int_{-\infty}^{\tau} p_{S\,|\,H}(S\,|\,H_1)\,dS \\ & + C_{10}P_{H_0} \int_{\tau}^{\infty} p_{S\,|\,H}(S\,|\,H_0)\,dS \\ & + C_{11}P_{H_1} \int_{\tau}^{\infty} p_{S\,|\,H}(S\,|\,H_1)\,dS. \end{aligned} \tag{3}$$

B. Architecture A2

The ith local detector makes its local min-Bayesian risk decision u_i on the basis of the K observations $z_i = [z_{i_1}, z_{i_2}, \cdots, z_{i_K}]$. The DFC makes its decision u_{so}, on the basis of the observation vector $[u_1, u_2, \cdots, u_{n_{so}}]$ (see [2]):

$$\begin{aligned} u_{so} = \operatorname{sgn}\Bigg\{ \sum_{i=1}^{n_{so}} \Bigg[& \log \frac{1 - P_{M_i}}{P_{F_i}}\, U_{-1}(u_i) \\ & + \log \frac{1 - P_{F_i}}{P_{M_i}}\, U_{-1}(-u_i) \Bigg] u_i - \log \tau \Bigg\} \end{aligned} \tag{4}$$

where

- n_{so} number of local detectors,
- P_{F_i} probability of accepting H_1 at detector i when H_0 is true (false alarm),
- P_{M_i} probability of accepting H_0 at detector i when H_1 is true (missed detection),
- τ decision threshold (see (2b)),
- sgn algebraic sign function:

$$\operatorname{sgn}(x) = \begin{cases} -1, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

- U_{-1} the unit step function:

$$U_{-1}(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0. \end{cases}$$

The decision (4) is optimal with respect to the min-Bayesian risk criterion, given fixed local detectors with specified P_{M_i} and P_{F_i}.

The number of detectors required to achieve an overall risk of β_o is the solution to

$$\begin{aligned}\beta_o = {} & C_{00}P_{H_0} + C_{11}P_{H_1} + (C_{10} - C_{00})P_{H_0}P_{F_{\mathrm{DFC}}} \\ & + (C_{01} - C_{11})P_{H_1}P_{M_{\mathrm{DFC}}} \end{aligned} \tag{5}$$

where the dependence on n_{so} is through $P_{F_{\mathrm{DFC}}}$ and $P_{M_{\mathrm{DFC}}}$:

$$\begin{aligned} P_{F_{\mathrm{DFC}}} = {} & \sum_{h_1=0}^{1} \sum_{h_2=0}^{1} \cdots \sum_{h_{n_{so}}=0}^{1} \left| \prod_{i=1}^{n_{so}} (h_i - P_{F_i}) \right| U_{-1} \\ & \cdot \left[\prod_{i=1}^{n_{so}} \left(\frac{1 - P_{M_i}}{P_{F_i}} \right)^{1-h_i} \left(\frac{P_{M_i}}{1 - P_{F_i}} \right)^{h_i} - \tau \right] \end{aligned} \tag{6a}$$

$$\begin{aligned} P_{M_{\mathrm{DFC}}} = {} & \sum_{h_1=0}^{1} \sum_{h_2=0}^{1} \cdots \sum_{h_{n_{so}}=0}^{1} \left| \prod_{i=1}^{n_{so}} (h_i - P_{M_i}) \right| U_{-1} \\ & \cdot \left[\tau - \prod_{i=1}^{n_{so}} \left(\frac{1 - P_{M_i}}{P_{F_i}} \right)^{h_i} \left(\frac{P_{M_i}}{1 - P_{F_i}} \right)^{1-h_i} \right]. \end{aligned} \tag{6b}$$

In (6), we sum over all possible combinations of $[h_1, h_2, \cdots, h_{n_{so}}]$, $h_i \in \{0, 1\}$, which correspond to all possible combinations of local decisions. These expressions hold for any local detectors (regardless of specific input statistical densities and decisions criteria) as long as the observations are statistically independent.

Next we consider two specific cases:

Case 1—Decentralized architecture A2 with identical detectors:[1] We assume that $P_{F_i} = P_F$ and $P_{M_i} = P_M$, $i = 1, 2, \cdots, n_{so}$ and $(1 - P_F)(1 - P_M) > P_F P_M$. The global risk is calculated with

$$P_{F_{\mathrm{DFC}}} = \sum_{i=J_F}^{n_{so}} \binom{n_{so}}{i} P_F^i (1 - P_F)^{n_{so}-i} \tag{7a}$$

and

$$P_{M_{\mathrm{DFC}}} = \sum_{i=J_M}^{n_{so}} \binom{n_{so}}{i} P_M^i (1 - P_M)^{n_{so}-i} \tag{7b}$$

where

$$J_F = \operatorname{int}\left\{ \frac{\ln \tau + n_{so}[\ln(1 - P_F) - \ln P_M]}{[\ln(1 - P_M) - \ln P_F] + [\ln(1 - P_F) - \ln P_M]} \right\} \tag{7c}$$

$$J_M = \operatorname{int}\left\{ \frac{n_{so}[\ln(1 - P_M) - \ln P_F] - \ln \tau}{[\ln(1 - P_M) - \ln P_F] + [\ln(1 - P_F) - \ln P_M]} \right\} \tag{7d}$$

[1] A similar architecture, using a Neyman–Pearson test, was studied in [12]. Equations (13) and (14) in [12] are closely related to (7a) and (7b) in this paper.

TABLE I
THE PROBABILITY OF ERROR VERSUS NUMBER OF SENSORS REQUIRED FOR ARCHITECTURE A1, EXAMPLE 1

p_e:	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}
$n_o\left(\sqrt{K}\,\frac{m}{\sigma}=1\right)$:	39	56	73	91	109	126	144	162	180	198
$n_o\left(\sqrt{K}\,\frac{m}{\sigma}=3\right)$:	5	7	9	11	13	14	16	18	20	22
$n_o\left(\sqrt{K}\,\frac{m}{\sigma}=6\right)$:	2	2	3	3	4	4	4	5	5	6
$n_o\left(\sqrt{K}\,\frac{m}{\sigma}=9\right)$:				2	2	2	2	2	3	3

and $\text{int}\{x\}$ is the smallest integer larger than or equal to x: $x \le \text{int}\{x\} < x+1$. Moreover, if $J_F > n_{so}P_F - 1/2$ and $J_M > n_{so}P_M - 1/2$, $P_{F_{\text{DFC}}}$ and $P_{M_{\text{DFC}}}$ can be bounded from above (see the Appendix) by

$$P_{F_{\text{DFC}}} \le \binom{n_{so}}{J_F} P_F^{J_F}(1-P_F)^{n_{so}-J_F} \cdot \left[\frac{(J_F+1)(1-P_F)}{(J_F+1)-(n_{so}+1)P_F}\right] \tag{8a}$$

$$P_{M_{\text{DFC}}} \le \binom{n_{so}}{J_M} P_M^{J_M}(1-P_M)^{n_{so}-J_M} \cdot \left[\frac{(J_M+1)(1-P_M)}{(J_M+1)-(n_{so}+1)P_M}\right]. \tag{8b}$$

Case 2—Identical sensors, equally likely hypotheses, probability of error criterion: We now assume $C_{00} = C_{11} = 0$, $C_{01} = C_{10} = 1$ and $P_{H_0} = P_{H_1} = 1/2$. These assumptions imply $P_{F_i} = P_{M_i} = P_F = P_M = p$ and $p < 1/2$ for all i. Also, $\tau = 1$ and $J_F = J_M = J$. We minimize the probability of error $1/2P_{F_{\text{DFC}}} + 1/2P_{M_{\text{DFC}}}$ by invoking Stirling's formula:

$$n! \sim \left(\sqrt{2\pi}\; n^{n+\frac{1}{2}} e^{\left(-n+\frac{1}{12n}\right)}\right). \tag{9}$$

We get (from (8)) an approximation of the upper bound for the probability of error (with $q = 1-p$)

$$\overline{p_{e_{so}}} \approx \frac{2^{n_{so}+1}(pq)^{(n_{so}/2)}}{\sqrt{2n_{so}\pi}\left(1-\frac{n_{so}p}{(n_{so}+2)q}\right)} e^{-\frac{1}{4n_{so}}}. \tag{10a}$$

A (sufficiently large) n_{so} can be obtained as a (successive-approximation) solution of

$$x = \frac{\ln\left[\frac{\pi}{2}\overline{p_{e_{so}}}^{\,2}\left(1-\frac{xp}{(x+2)q}\right)^2\right] + \ln x + \frac{1}{2x}}{\ln(4pq)} \tag{10b}$$

by taking $n_{so} = \text{int}\{x\}$.

III. EXAMPLES

A. Example 1: Discrimination between Two Gaussian Populations

We assume that the observations are drawn from one of two Gaussian populations with the same variance (σ^2) but different means (zero for H_0, m for H_1). The observations are statistically independent.

In *architecture A1*, we employ n_o parallel sensors, each collecting K observations and transmitting them to the central detector. The performance of this architecture is the same as that of a single detector, using Kn_o observations for its decision (see [14, pp. 27–28; 36–38], [10, pp. 129–131]). If the hypotheses are equally likely, the probability of error is

$$p_{e_o} = 1 - \Phi\left(\frac{\sqrt{Kn_o}\,m}{2\sigma}\right) \tag{11a}$$

where

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)\,dy \quad \text{and} \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \tag{11b}$$

For $x = \left(\sqrt{Kn_o}\,m\right)/2\sigma > 3$ we have

$$\Phi(x) \sim \left[1 - \phi(x)\left(\frac{1}{x} - \frac{1}{x^3}\right)\right] \tag{12}$$

and n_o can be obtained as a (successive-approximation) solution of

$$y = \frac{4\sigma^2}{Km^2}\ln\left(\frac{2\sigma^2}{K\pi m^2 p_{e_o}^2}\right) - \frac{4\sigma^2}{Km^2}\ln y + \frac{8\sigma^2}{Km^2}\ln\left(1-\frac{4\sigma^2}{Km^2 y}\right) \tag{13}$$

by taking $n_o = \text{int}\{y\}$.

Table I shows the required number of sensors, n_o, for p_{e_o} values ranging from 10^{-3} to 10^{-12}. We show the results for several values of the signal to noise ratio $(\sqrt{K}(m/\sigma))$, at each sensor.

In architecture A2, continuing with Example 1, we employ n_{so} local detectors, each collecting K observations and transmitting the local decisions u_i, $i = 1, 2, \cdots, n_{so}$, to the DFC. The error probabilities of the local detectors are

$$P_{F_i} = 1 - \Phi\left(\frac{\sigma \ln \tau}{m\sqrt{K}} + \frac{m\sqrt{K}}{2\sigma}\right) \tag{14a}$$

$$P_{M_i} = \Phi\left(\frac{\sigma \ln \tau}{m\sqrt{K}} - \frac{m\sqrt{K}}{2\sigma}\right). \tag{14b}$$

Table II shows the sufficient number of sensors, n_{so} (calculated with (10)) for a min-p_e system with equally likely hypotheses ($\tau = 1$) and required p_e values ranging from 10^{-3}

TABLE II
THE PROBABILITY OF ERROR VERSUS NUMBER OF SENSORS REQUIRED FOR ARICHITECTURE A2, EXAMPLE 1

p_e:	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}
$n_{so}\left(\sqrt{K}\,\frac{m}{\sigma}=1\right)$:	66	93	120	148	176	204	232	260	289	317
$n_{so}\left(\sqrt{K}\,\frac{m}{\sigma}=3\right)$:	9	12	15	18	21	25	28	31	34	37
$n_{so}\left(\sqrt{K}\,\frac{m}{\sigma}=6\right)$:	3	3	5	5	6	7	7	9	9	11
$n_{so}\left(\sqrt{K}\,\frac{m}{\sigma}=9\right)$:				3	3	3	3	3	5	5

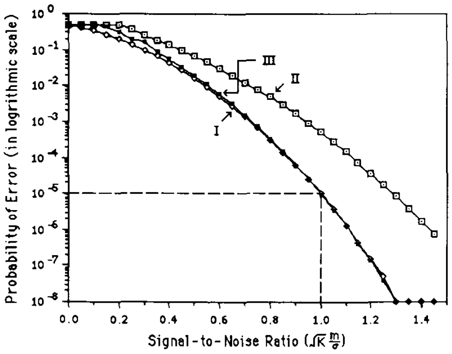

Fig. 2. Probability of error versus SNR for centralized ($n_o = 73$) and decentralized ($n_{so} = 73$ and 120) systems. I: Centralized system with 73 sensors; II: Decentralized system with 73 sensors; III: Decentralized system with 120 sensors.

to 10^{-12}. The signal-to-noise (SNR) ratios $\left(\sqrt{K}\,(m/\sigma)\right)$ are the same as in Table I.

Fig. 2 shows the global probability of error (p_e) versus the signal to noise ratio $\left(\sqrt{K}\,(m/\sigma)\right)$ for a centralized system with 73 sensors and decentralized systems with 73 and 120 sensors. Clearly the 73-sensor decentralized system has worse performance, and the 120-sensor decentralized system is comparable to the 73-sensor centralized system in the region of interest (the systems have identical performance for $\mathrm{SNR} = 1$, with $p_e = 10^{-5}$).

1) Hardware Requirements for High SNR: The ratio between the number of detectors required by architecture A1, and the number of detectors required by architecture A2, can be calculated from the table. For the Gaussian case this ratio can be shown to be between $1/2$ and 1. Since $\lim\, y/x \to 1/2$ as $\sqrt{K}\,(m/\sigma) \to \infty$ (y of (13), x of (10b)), we conclude that for a large SNR a decentralized system with $2N$ sensor/detectors will have performance that is at least as good as that of a centralized system with N sensors.

2) Notes on Applicability

a) Different local risks: In the discussion of A2 we have assumed that the *same* Bayesian risk is used by all local detectors and the DFC to synthesize a decision rule. The results are applicable for any (possibly different) Bayesian risks, as long as the local thresholds are designed according to the local risks, and the resulting P_{F_i} and P_{M_i} of the ith detector are used in designing the DFC.

b) Nonidentical sensors: We have assumed that the local sensors are identical. If they are not identical, we choose the worst-case sensor and assume that all sensors have the same poor performance. Thus we obtain an upper bound on the number of hardware units required. The assumption $P_{H_0} = P_{H_1} = 1/2$ is also a worst-case assumption about the *a priori* probabilities, which would yield an overestimation of the required hardware volume.

B. Example 2—Discrimination Between Two Poisson Populations

We consider discrimination between two Poisson populations which differ in their parameters (m_0 under H_0, m_1 under H_1).

In the centralized case (architecture A1) we have n_o sensors. Each of these sensors has K *independent* inputs, each receiving a Poisson-distributed sequence. All incoming sequences during a given experiment are from the same population (H_0 or H_1). The detector counts the number of events in each sequence, and sums over all Kn_o sequences to obtain the total number of events in the input sequences t. The detector then compares the total number of events to a threshold, and makes a decision.

In the decentralized case (architecture A2) we have n_{so} local detectors. Each of these has K independent inputs, each receiving a Poisson distributed sequence. All incoming sequences during a given experiment are from the same population (H_0 or H_1). Each detector counts the total number of events in its input sequences, and compares this number to a threshold in order to make a local decision about its input. It then sends this decision to the DFC that synthesizes a global decision.

For architecture A1 the decision rule of the central detector is [14, p. 30]

$$u_o = \operatorname{sgn}\left[t - \frac{\ln \tau + Kn_o(m_1 - m_0)}{\ln m_1 - \ln m_0}\right]. \qquad (15)$$

In this case the global $P_{F_{\mathrm{DFC}}}$ and $P_{M_{\mathrm{DFC}}}$ are

$$P_{F_{\mathrm{DFC}}} = 1 - \sum_{t=0}^{\gamma-1} \frac{(Kn_o m_0)^t}{t!} e^{-Kn_o m_0} \qquad (16a)$$

$$P_{M_{\mathrm{DFC}}} = \sum_{t=0}^{\gamma-1} \frac{(Kn_o m_1)^t}{t!} e^{-Kn_o m_1} \qquad (16b)$$

where

$$\gamma = \operatorname{floor}\left[\frac{\ln \tau + Kn_o(m_1 - m_0)}{\ln m_1 - \ln m_0}\right]$$

and floor $[x]$ is the largest integer smaller than or equal to x: $x - 1 < \text{floor}\,[x] \le x$.

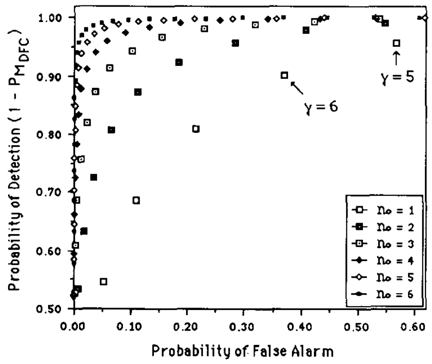

Fig. 3. The receiver operating characteristics of architecture A1 for values of $n_o = 1, 2, 3, 4, 5, 6$ with $K = 2$, $m_0 = 2$, $m_1 = 4$.

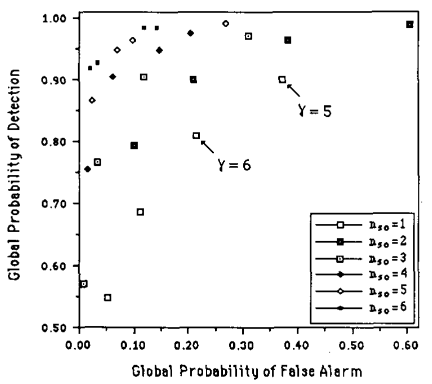

Fig. 4. The global receiver operating characteristics of architecture A2 for values of $n_{so} = 1, 2, 3, 4, 5, 6$ with $K = 2$, $m_0 = 2$ and $m_1 = 4$.

Fig. 3 shows the receiver operating characteristics for $P_{F_{\rm DFC}} < 1/2$ and $P_{M_{\rm DFC}} < 1/2$. The global probability of error is

$$p_{e_o} = P_{H_0}\left(1 - \sum_{t=0}^{\gamma-1} \frac{(Kn_o m_0)^t}{t!} e^{-Kn_o m_0}\right) + P_{H_1} \sum_{t=0}^{\gamma-1} \frac{(Kn_o m_1)^t}{t!} e^{-Kn_o m_1}. \tag{17}$$

For architecture A2 with identical local detectors ($P_{F_i} = P_F$ and $P_{M_i} = P_M$) we obtain

$$P_F = 1 - \sum_{t=0}^{\gamma-1} \frac{(Km_0)^t}{t!} e^{-Km_0} \tag{18a}$$

$$P_M = \sum_{t=0}^{\gamma-1} \frac{(Km_1)^t}{t!} e^{-Km_1} \tag{18b}$$

where

$$\gamma = \text{floor}\left[\frac{\ln \tau + K(m_1 - m_0)}{\ln m_1 - \ln m_0}\right]. \tag{18c}$$

Fig. 4 shows the receiver operating characteristics for architecture A2 with $K = 2$, $m_0 = 2$ and $m_1 = 4$. $P_{F_{\rm DFC}}$ and $P_{M_{\rm DFC}}$ can be obtained by (7). These characteristics are significantly worse than the A1 characteristics of Fig. 3.

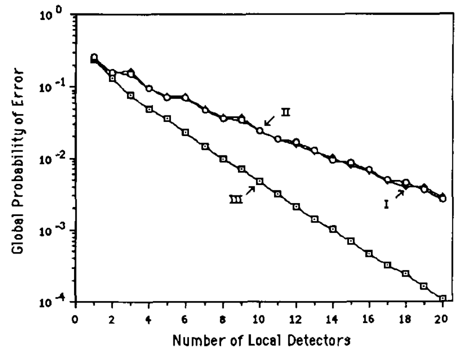

Fig. 5. The global probability of error versus the number of local detectors with $K = 2$, $m_0 = 2$, $m_1 = 4$ and $\tau = 1$. I: Simulated global probability of error for the decentralized system; II: Approximated upper bound of the global probability of error (see (19) and (20)) for the decentralized system; III: Global probability of error (see (17)) for the centralized system.

If $J_F > n_{so}P_F - 1/2$ and $J_M > n_{so}P_M - 1/2$ (see (7)), then $P_{F_{\rm DFC}}$ and $P_{M_{\rm DFC}}$ can be bounded from above as in (8); by invoking Stirling's formula, we obtain an approximation of the upper bounds for $P_{F_{\rm DFC}}$ and $P_{M_{\rm DFC}}$:

$$\overline{P_{F_{\rm DFC}}} \approx \frac{(J_F+1)n_{so}^{n_{so}+\frac{1}{2}}\, P_F^{J_F}(1-P_F)^{n_{so}-J_F+1}}{\sqrt{2\pi}\, J_F^{J_F+\frac{1}{2}}(n_{so}-J_F)^{n_{so}-J_F+\frac{1}{2}}[J_F+1-(n_{so}+1)P_F]} \cdot \exp\left(\frac{1}{12n_{so}} - \frac{1}{12J_F} - \frac{1}{12(n_{so}-J_F)}\right) \tag{19a}$$

$$\overline{P_{M_{\rm DFC}}} \approx \frac{(J_M+1)n_{so}^{n_{so}+\frac{1}{2}}\, P_M^{J_M}(1-P_M)^{n_{so}-J_M+1}}{\sqrt{2\pi}\, J_M^{J_M+\frac{1}{2}}(n_{so}-J_M)^{n_{so}-J_M+\frac{1}{2}}[J_M+1-(n_{so}+1)P_M]} \cdot \exp\left(\frac{1}{12n_{so}} - \frac{1}{12J_M} - \frac{1}{12(n_{so}-J_M)}\right) \tag{19b}$$

An upper bound on the probability of error $p_{e_{so}}$ is

$$\overline{p_{e_{so}}} = P_{H_0}\overline{P_{F_{\rm DFC}}} + P_{H_1}\overline{P_{M_{\rm DFC}}}\,. \tag{20}$$

In Fig. 5 we compare the global probability of error of the centralized and decentralized architectures versus the number of sensors. The graphs allow the assessment of hardware volume necessary for prespecified performance using the two architectures. We note that the simulated probability of error is close to the approximated upper bound.

IV. Conclusion

We have devised a simple suboptimal decentralized binary detection system where each local detector independently, and the data fusion center, are minimizing Bayesian risks (not necessary identical). We compared the hardware volume of a system with identical local risks to that of an optimal centralized system which minimizes the same global risk for several mathematically tractable cases. The results can be used by designers of distributed detection systems (even less restricted than the ones that we consider here) to obtain upper bounds on the number of (decentralized) detectors which are required in order to guarantee prespecified performance.

Appendix A
Upper Bound on DFC Performance

Let

$$\pi = \sum_{i=J}^{n} \pi_i, \quad \text{where } \pi_i = \binom{n}{i} p^i (1-p)^{n-i}.$$

π can be written as:

$$\pi = \pi_J \left[1 + \sum_{m=1}^{n-J} \left(\frac{p}{1-p} \right)^m \prod_{k=0}^{m-1} \frac{n-J-k}{J+1+k} \right]$$
$$\leq \pi_J \left[1 + \sum_{m=1}^{n-J} \left(\frac{p}{1-p} \frac{n-J}{J+1} \right)^m \right]. \tag{21}$$

Provided $P < 1/2$, $J > np - 1/2$ and $\frac{p}{1-p} \frac{n-J}{J+1} < 1$,

$$\pi < \pi_J \sum_{m=0}^{\infty} \left(\frac{p}{1-p} \frac{n-J}{J+1} \right)^m = \pi_J \left[\frac{(J+1)(1-p)}{(J+1)-(n+1)p} \right]. \tag{22}$$

Equations (8) follows from (22).

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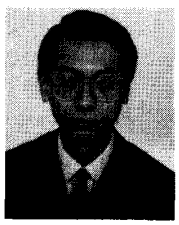

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